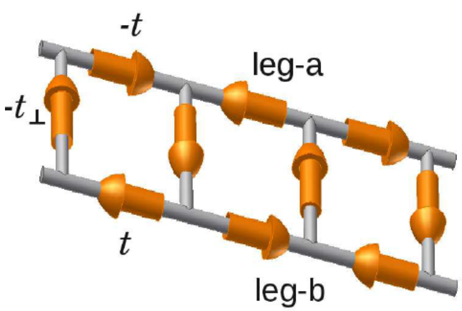
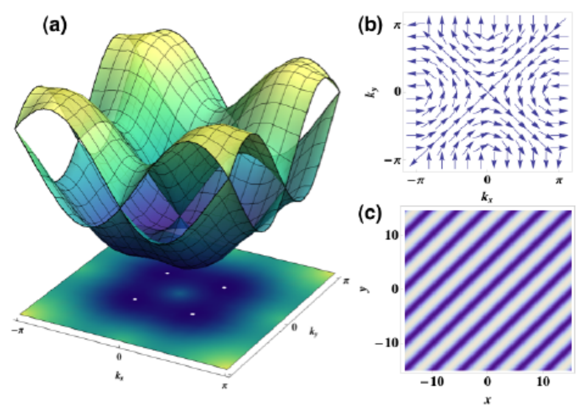
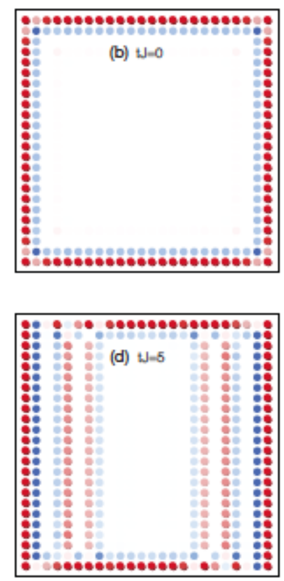
# Synthetic gauge fields for ultracold atoms
# Strongly Correlated States and Atom Current Probes

**Arun Paramekanti**
**(University of Toronto)**



ICTP Workshop (12-17 May, 2013)

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Synthetic gauge fields for ultracold atoms

# Strongly Correlated States and Atom Current Probes

**Collaborators**

**Matthew Killi**, **Ciaran Hickey**, Ian Kivlichan, Stefan Trotzky (Toronto)

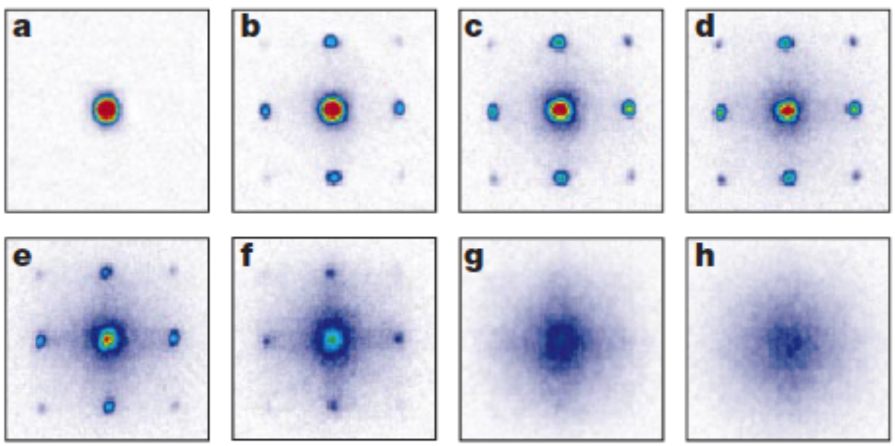
**Arya Dhar**, Tapan Mishra, M. Maji, R. V. Pai, S. Mukerjee (Bangalore)

**William S. Cole**, S. Zhang, N. Trivedi (Ohio State University)

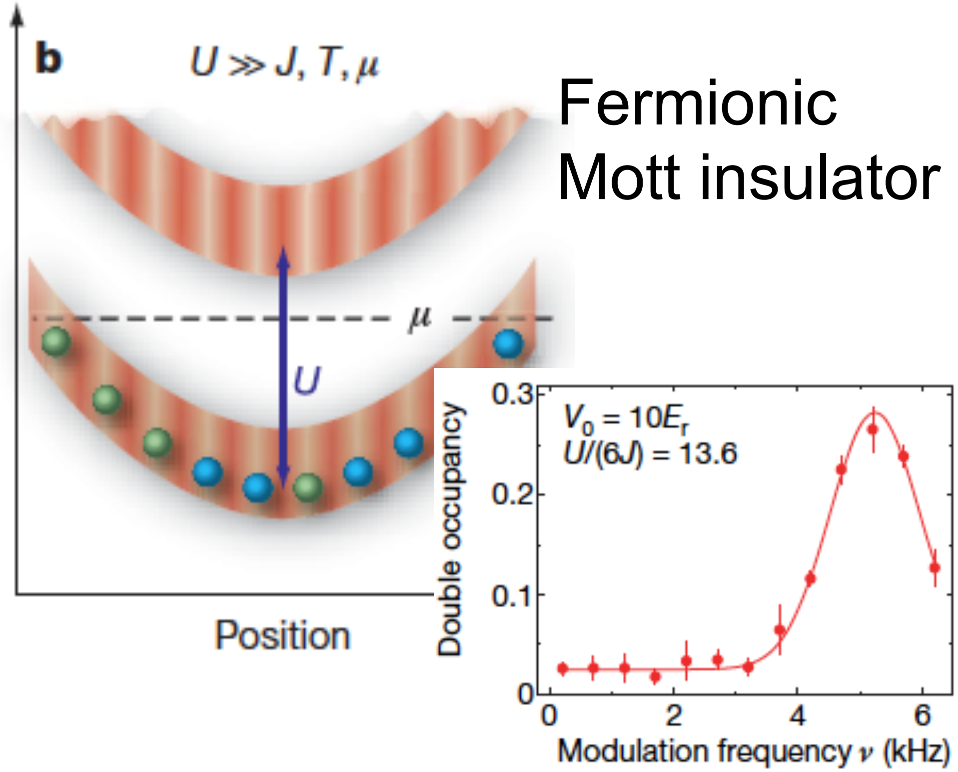
# Optical Lattices
## Strongly Correlated Phases and Quantum Phase Transitions

**Quantum phase transition from a superfluid to a Mott insulator in a gas of ultracold atoms**

Markus Greiner*, Olaf Mandel*, Tilman Esslinger†, Theodor W. Hänsch* & Immanuel Bloch*




Fermionic Mott insulator



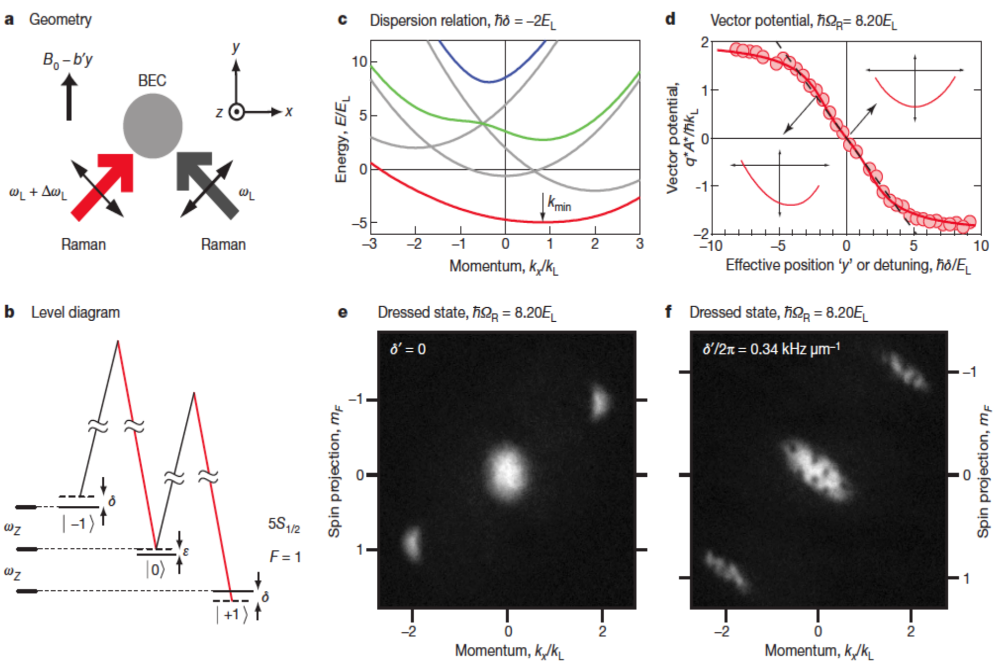

R. Jordens, N. Strohmaier, K. Gunter, H. Moritz, T. Esslinger (Nature, 2008)

**Expect more interesting states of matter arise in the presence of magnetic fields or spin-orbit coupling**

- **Interacting Quantum Hall and Chern insulators**
- **Interacting Topological insulators or semimetals**

# Synthetic magnetic fields: Experimental Progress

Raman induced "gauge field" for atoms in the continuum

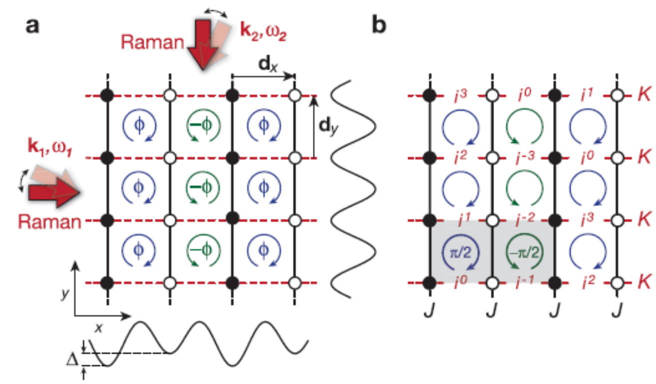


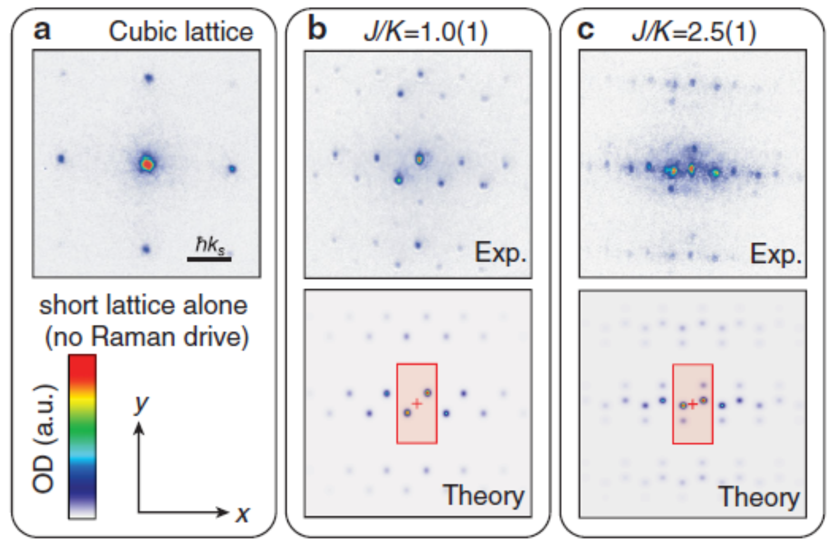

Vol 462|3 December 2009|doi:10.1038/nature08609

**Synthetic magnetic fields for ultracold neutral atoms**

Y.-J. Lin[1], R. L. Compton[1], K. Jiménez-García[1,2], J. V. Porto[1] & I. B. Spielman[1]

Raman assisted tunneling in a lattice




Large staggered fluxes in an optical lattice
M. Aidelsburger, et al (PRL 2011)

- N. Cooper proposal of optical flux lattices
- ENS work + review: J. Dalibard, F. Gerbier, G. Juzeliunas,P. Ohberg, RMP (2011)

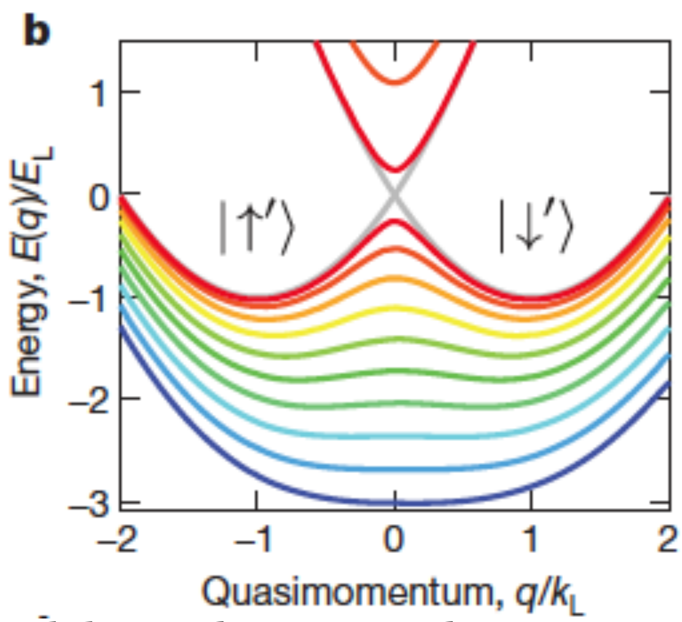
# Synthetic spin-orbit coupling: Experimental Progress

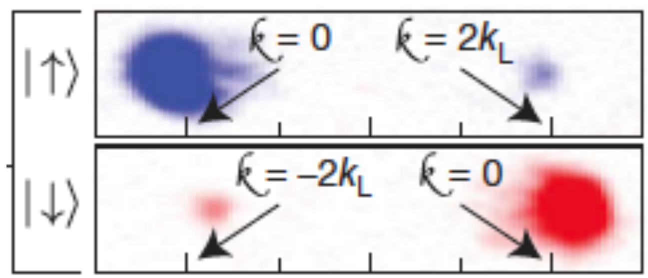
## LETTER

doi:10.1038/nature09887

### Spin–orbit-coupled Bose–Einstein condensates

Y.-J. Lin[1], K. Jiménez-García[1,2] & I. B. Spielman[1]




$$\hat{H} = \frac{\hbar^2 \hat{k}^2}{2m} \breve{1} + \frac{\Omega}{2}\breve{\sigma}_z + \frac{\delta}{2}\breve{\sigma}_y + 2\alpha \hat{k}_x \breve{\sigma}_y$$

Rashba: $k_x \sigma_y - k_y \sigma_x$

Dresselhaus: $k_x \sigma_y + k_y \sigma_x$

Equal Rashba/Dresselhaus

More general SO coupling possible?

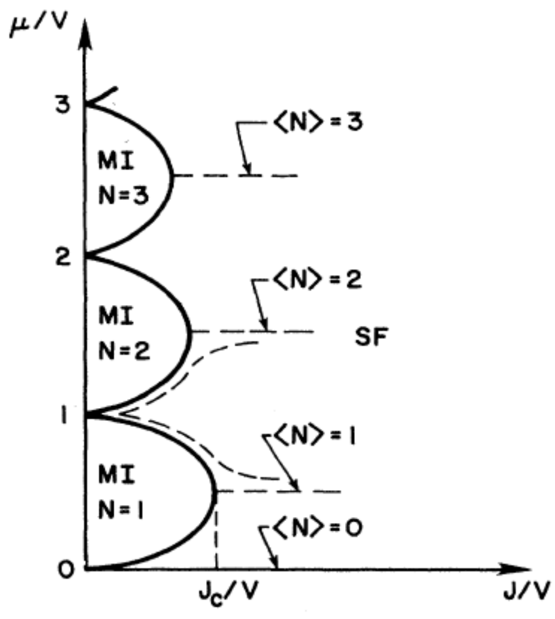
<u>More recent work</u>: M. Zwierlein group, J. Zhang group, P. Engels group,…

# Key issues

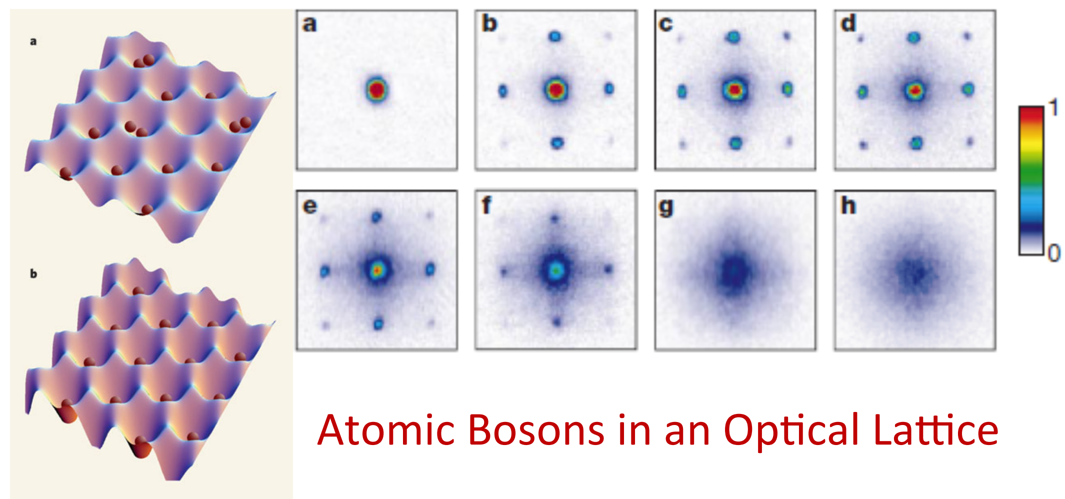
- Synthetic magnetic fields: **New strong correlation effects**?
- Synthetic spin-orbit coupling: **New strong correlation effects**?
- Currents induced by gauge fields: **How can we probe them**?

# One-Component Bose Hubbard model

$$\hat{H} = -\sum_{\langle i,j \rangle} \hat{a}_i^\dagger t_{ij} \hat{a}_j - \mu \sum_i \hat{a}_i^\dagger \hat{a}_i + \frac{U}{2} \sum_i \hat{a}_i^\dagger \hat{a}_i^\dagger \hat{a}_i \hat{a}_i$$




Fisher, Weichman, Grinstein, Fisher (PRB 1989)




Atomic Bosons in an Optical Lattice

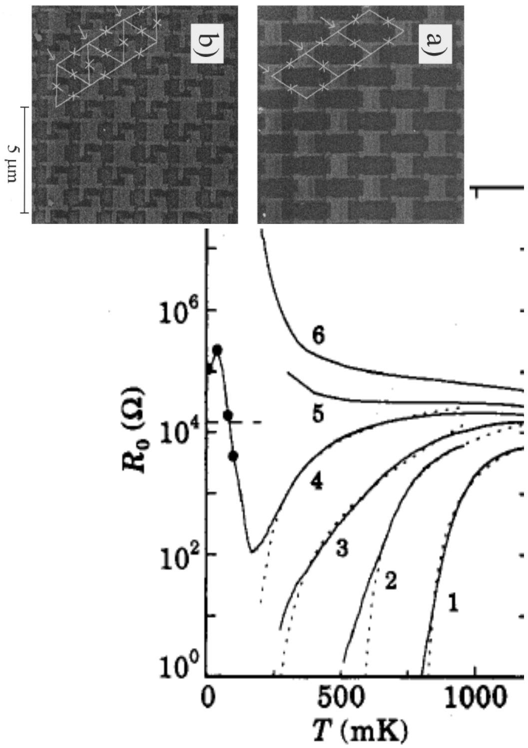
Proposal: D. Jaksch, et al (PRL 1998)
Experiment: M. Greiner, et al (Nature 2002)

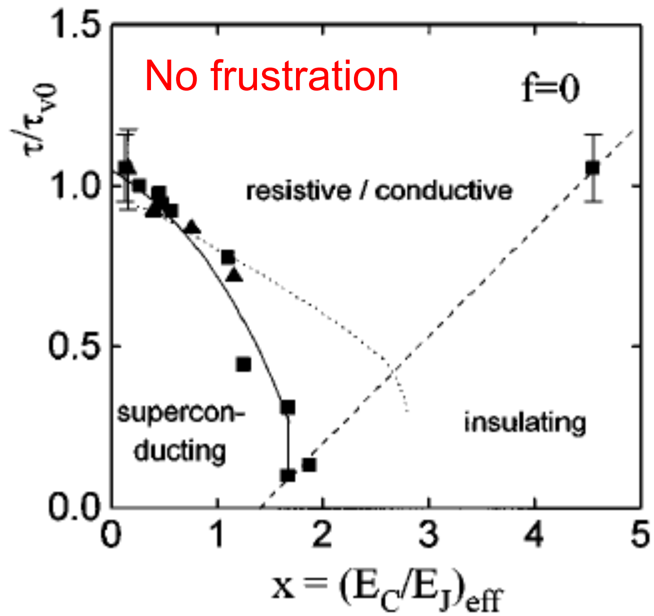
What happens in the presence of magnetic flux?

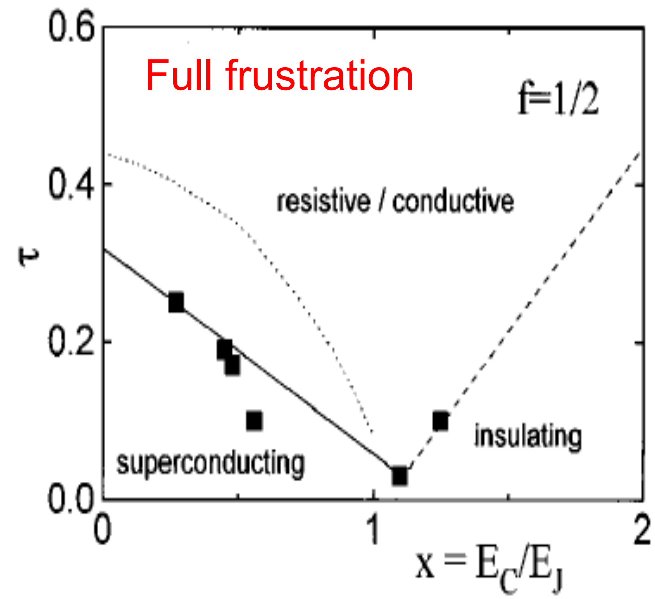
# Mott transition in Josephson junction arrays

Transport measurements on Josephson junction arrays

SC-insulator transition with zero and (hc/4e) flux per plaquette



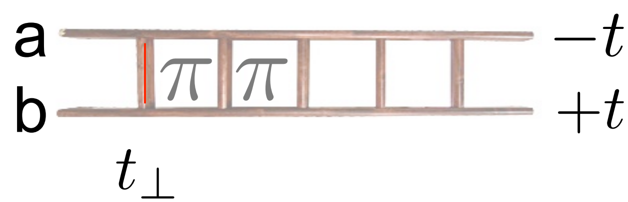

Theory: Teitel, Jayaprakash (PRB 1983)
Expt: Mooij group (EPL 1992, EPL 1996)

Hubbard U ~ Charging energy
Boson hopping ~ Josephson coupling
Total charge on each SC island is large: Map to a quantum XY model (rotors)
Trouble: Disorder and No Tunability

**Proposal to simulate this using cold atoms** - M. Polini, R. Fazio, A.H. MacDonald, M.P.Tosi (PRL, 2005)

# Bose Hubbard model in synthetic π-flux
# Carefully understand a simpler ladder version

How does kinetic frustration modify the physics?




Hubbard repulsion

$$H = -t\sum_x (a_x^\dagger a_{x+1} + a_{x+1}^\dagger a_x) + t\sum_x (b_x^\dagger b_{x+1} + b_{x+1}^\dagger b_x) - t_\perp \sum_x (a_x^\dagger b_x + b_x^\dagger a_x) + \frac{U}{2}\sum_x (n_{a,x}^2 + n_{b,x}^2)$$

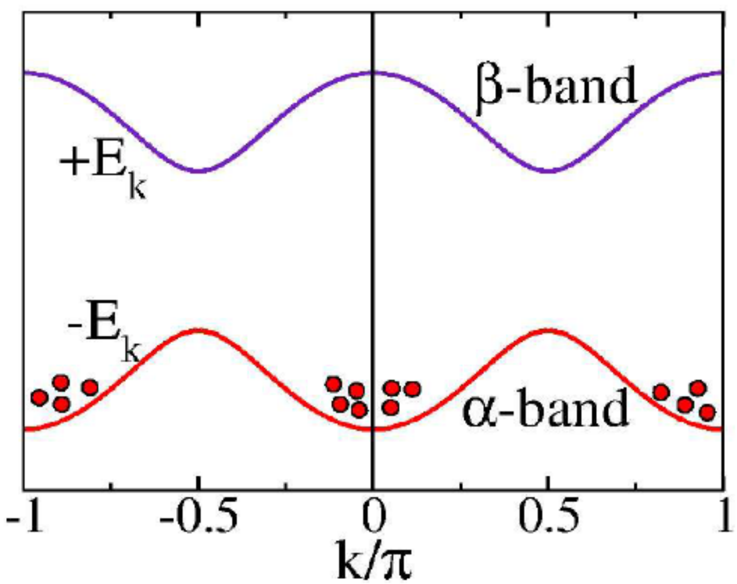
Intrachain hopping terms

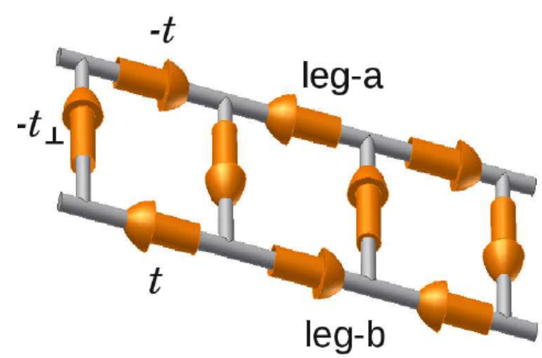
Interchain hopping

- “Fully frustrated” Josephson junction array including “charging energy”
- Ladder: Numerically exact phase diagram via DMRG and Monte Carlo

# Weak correlation limit

## Weak Correlations: Landau theory




$$E_{\rm low}^{\rm mft} = (-E_0-\mu)\sum_{i=0,\pi}|\varphi_i|^2 + U(u_0^4+v_0^4)\sum_{i=0,\pi}|\varphi_i|^4 + 8U\,u_0^2v_0^2|\varphi_0|^2|\varphi_\pi|^2$$

$$+\ 2U\,u_0^2v_0^2(\varphi_0^{*2}\varphi_\pi^2+\varphi_\pi^{*2}\varphi_0^2)$$

Umklapp-type term which favors relative +/- pi/2 phase and breaks time reversal spontaneously

# Mean field theory

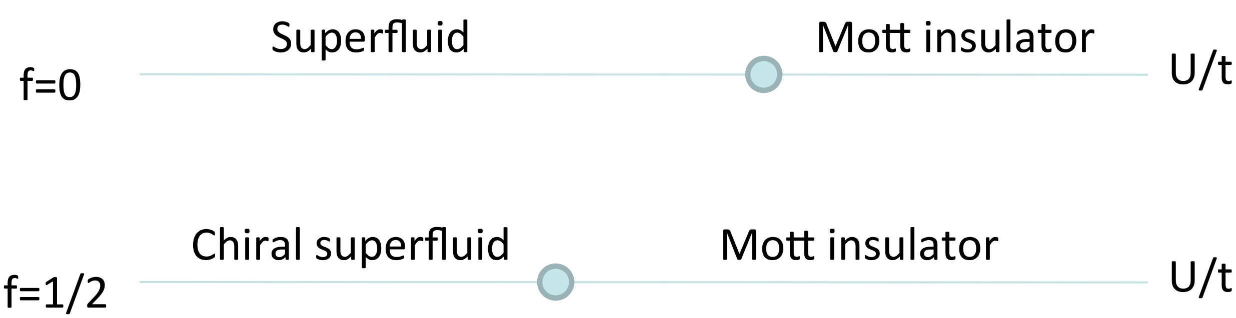
Generalize usual superfluid-Mott transition mean field theory (effective single-site approximation)

1. Continuous Chiral Superfluid to Mott transition
2. $U_c$ for the Mott transition suppressed

Ladder mean field theory

$$\frac{1}{\sqrt{4t^2+t_\perp^2}} = \frac{n}{\mu - U(n-1)} + \frac{n+1}{Un-\mu}$$

$$\frac{U^{\pi-\text{flux}}_{c,ladder}}{U^{0-\text{flux}}_{c,ladder}} = \frac{\sqrt{4t^2+t_\perp^2}}{2t+t_\perp} < 1$$




Recent 2D mean field study: Lim, Hemmerich, C. Morais-Smith, PRL (2008)

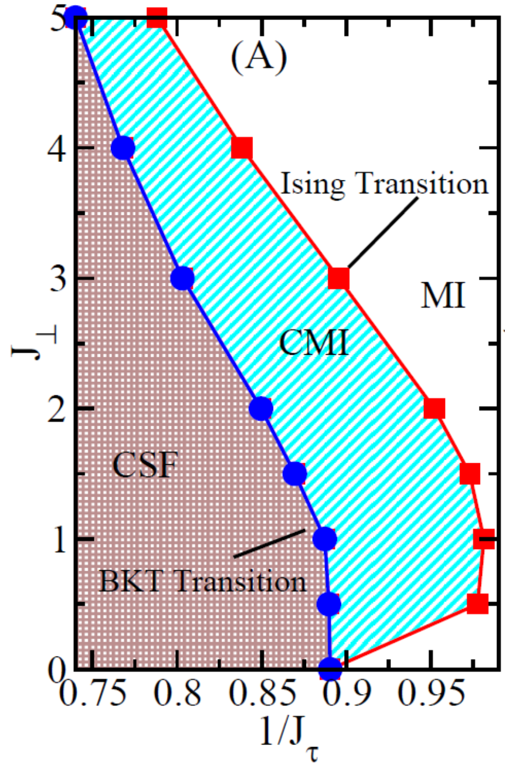
New physics beyond mean field: Critical theory involves 2 bosonic modes

We go **beyond mean field theory** using two numerical approaches

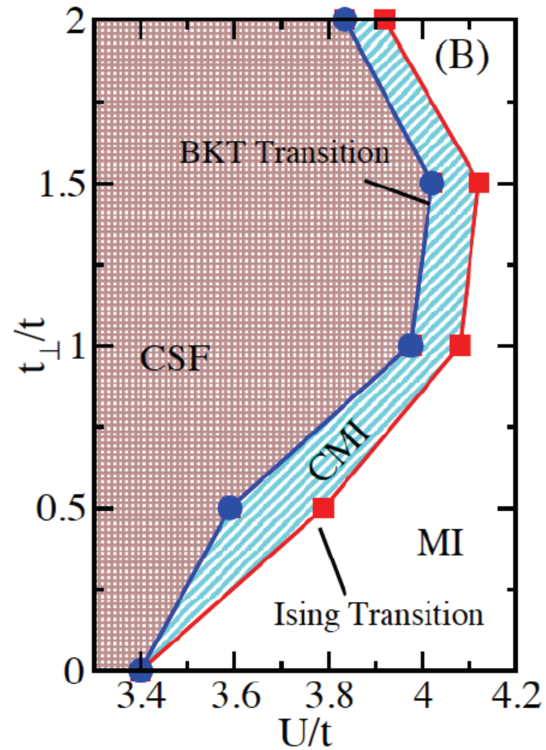
- Simulate1+1=2 dimensional frustrated classical XY model (Bilayer model)
- Directly simulate the ladder Hubbard model using DMRG methods

# Phase Diagram of Fully Frustrated Bose Hubbard Ladder

(d+1) classical XY model




DMRG result




**Three Phases**

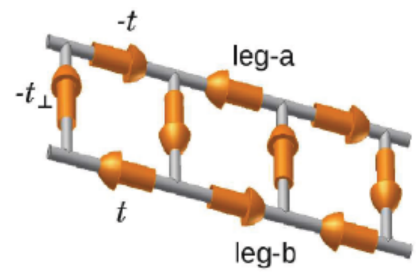
**Chiral Superfluid**: Luttinger superfluid, long range loop current order

**Conventional Mott Insulator:** charge gap, no superfluidity, no current order

**Chiral Mott Insulator: charge gap, no superfluidity, long range current order**

# Physical Pictures for the Chiral Mott Insulator

## Starting from the Superfluid




Chiral superfluid = Vortex Crystal

. Flux nucleates vortex or antivortex
. Vortex-vortex interaction is repulsive
. Equal number of V/AV
. "Antiferromagnetic" crystal

Regular Mott insulator = Vortex Superfluid (*D. Haldane; Halperin/Dasgupta; Fisher/Lee*)

. Dual - proliferated quantum phase slips

Chiral Mott insulator = Vortex Supersolid

. Defect in crystal: Extra vacancy/interstitial vortex/antivortex
. Proliferating and condensing dilute defects: Vortex superfluid
. Background current pattern preserved: Vortex crystal

# Physical Pictures for the Chiral Mott Insulator

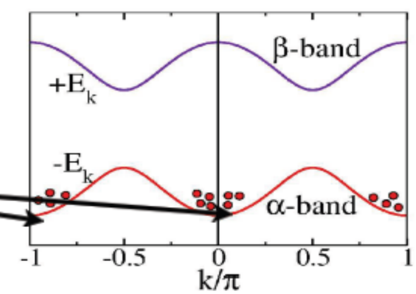
## Starting from the usual Mott insulator

**Excitations of a Conventional Mott insulator**

. Gapped Particles: "double occupancy"
. Gapped Holes: "vacancy"
. Dispersing particles/holes: Like a "semiconductor"

**Excitations of a Conventional Mott insulator with flux**

. Gapped Particles: "double occupancy"
. Gapped Holes: "vacancy"
. Dispersing particles/holes **with multiple minima**
Like a "semiconductor" with multiple valleys

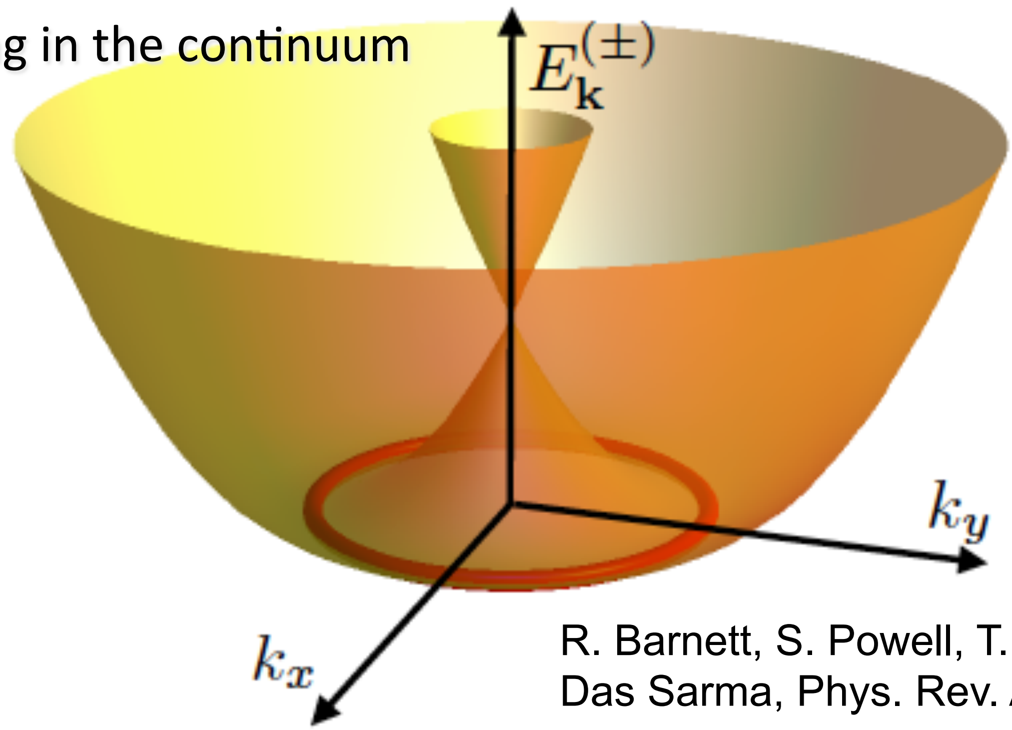



Semiconductors can have excitons and exciton condensation: Halperin-Rice

Chiral Mott insulator: Indirect exciton condensate

# Interacting bosons with synthetic spin-orbit Coupling

Rashba-Coupling in the continuum




R. Barnett, S. Powell, T. Graβ, M. Lewenstein, S. Das Sarma, Phys. Rev. A 85 023615 (2012)

Ring of low energy states:

"Quantum order by disorder" appears to select single-K condensate for simple interactions

Other work:

C. Wang, et al, PRL (2010); C.M.Jian, Hui Zhai, PRA (2011); Ozawa, Baym, PRL (2013), Zhou,Cui, PRL (2013), Ramachandran, Hu, Pu arXiv:1301.0800; K. Riedl, et al, arXiv:1304.5103

Mott transitions in a lattice:

T. Graβ, K. Saha, K. Sengupta, M. Lewenstein, PRA (2011)

S. Mandal, K. Saha, K. Sengupta, PRB (2012)

# Hubbard Model of Rashba Spin-Orbit Coupled Bosons

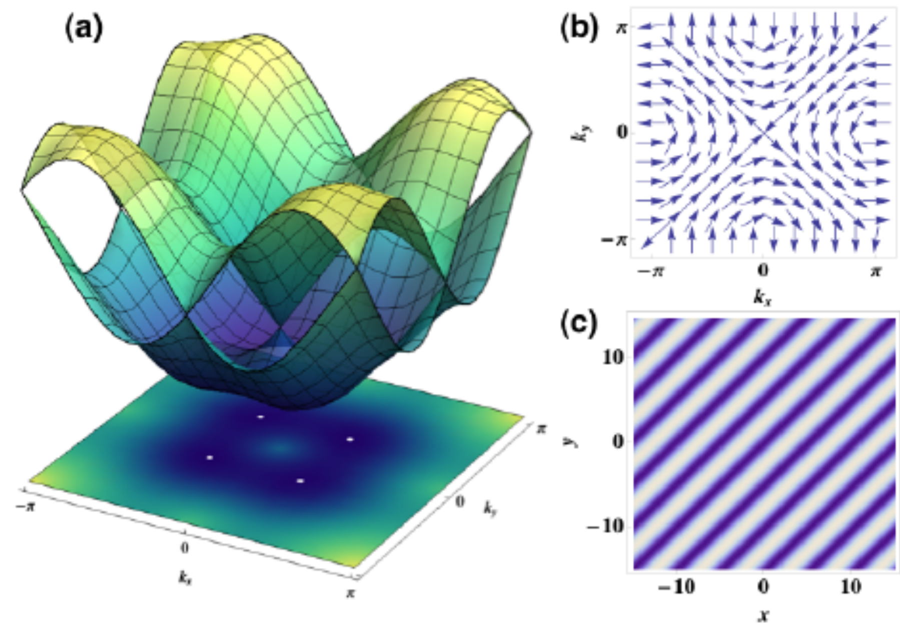
$$H = -t\sum_{\langle ij\rangle}(\psi_i^\dagger \mathcal{R}_{ij}\psi_j + \text{h.c.}) + \frac{1}{2}\sum_{i\sigma\sigma'} U_{\sigma\sigma'} a^\dagger_{i\sigma} a^\dagger_{i\sigma'} a_{i\sigma'} a_{i\sigma}$$

$$\mathcal{R}_{ij} \equiv \exp[i\vec{A}\cdot(\vec{r}_i - \vec{r}_j)]$$

$$\vec{A} = (\alpha\sigma_y, \beta\sigma_x, 0)$$

Spin-orbit coupling is like
a “non-abelian” gauge field

Lattice Rashba coupling: $\beta = -\alpha$
Lower degeneracy, more stable




Weak coupling

W.S. Cole, S. Zhang, A. Paramekanti, N. Trivedi, PRL 2012

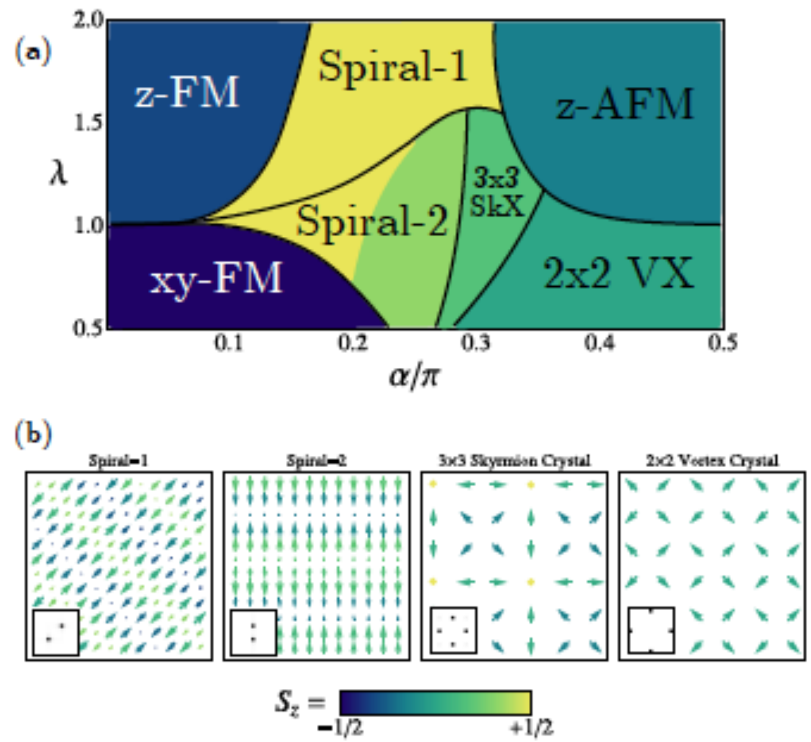
# Strong Coupling Mott insulator of Rashba-Coupled Bosons

$$H=-t\sum_{\langle ij\rangle}(\psi_i^\dagger \mathcal{R}_{ij}\psi_j+\text{h.c.})+\frac{1}{2}\sum_{i\sigma\sigma'}U_{\sigma\sigma'}a_{i\sigma}^\dagger a_{i\sigma'}^\dagger a_{i\sigma'}a_{i\sigma}$$

## Effective spin Hamiltonian at $O(t^2/U)$

$$H_{\text{spin}} = \sum_{i,\delta=\hat{x},\hat{y}} \left\{ \sum_{a=x,y,z} J_\delta^a S_i^a S_{i+\delta}^a + \vec{D}_\delta \cdot (\vec{S}_i \times \vec{S}_{i+\delta}) \right\}$$

| | |
|---|---|
| $J_{\hat{x}}^x = -\frac{4t^2}{\lambda U}\cos(2\alpha)$ | $J_{\hat{y}}^x = -\frac{4t^2}{\lambda U}$ |
| $J_{\hat{x}}^y = -\frac{4t^2}{\lambda U}$ | $J_{\hat{y}}^y = -\frac{4t^2}{\lambda U}\cos(2\alpha)$ |
| $J_{\hat{x}}^z = -\frac{4t^2}{\lambda U}(2\lambda-1)\cos(2\alpha)$ | $J_{\hat{y}}^z = -\frac{4t^2}{\lambda U}(2\lambda-1)\cos(2\alpha)$ |
| $\vec{D}_{\hat{x}} = -\frac{4t^2}{U}\sin(2\alpha)\hat{y}$ | $\vec{D}_{\hat{y}} = \frac{4t^2}{U}\sin(2\alpha)\hat{x}$ |




Rich classical magnetic phase diagram

W.S. Cole, S. Zhang, A. Paramekanti, N. Trivedi, PRL 2012
J. Radic, A. Di Ciolo, K. Sun, V. Galitski, PRL 2012
Z. Cai, X. Zhou, C. Wu, PRA 2012

# Magnetic Superfluids and Slave Boson Mean Field Theory

W.S. Cole, S. Zhang, A. Paramekanti, N. Trivedi, PRL 2012

$$H = -t\sum_{\langle ij\rangle}(\psi_i^\dagger \mathcal{R}_{ij}\psi_j + \text{h.c.}) + \frac{1}{2}\sum_{i\sigma\sigma'} U_{\sigma\sigma'} a_{i\sigma}^\dagger a_{i\sigma'}^\dagger a_{i\sigma'} a_{i\sigma}$$

Usual mean field theory: Decoupling of the hopping term

Magnetic superfluids with complicated current patterns – WHY??

Slave boson theory

Chargon → $b^\dagger_{\mathbf{r}}$, Spinon → $f^\dagger_{\mathbf{r},\sigma}$

$$a^\dagger_{\mathbf{r},\sigma} = \frac{1}{\sqrt{n^f_{\mathbf{r}}}} b^\dagger_{\mathbf{r}} f^\dagger_{\mathbf{r},\sigma}$$

Constraint $n_f = n_b$

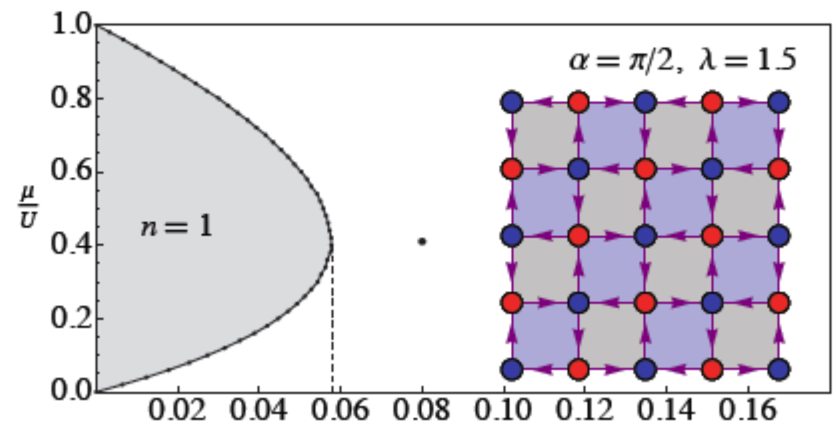
Spinon condensates

$$\vec{m} \sim (b^\dagger b)(z^*_\alpha \vec{\sigma}_{\alpha\beta} z_\beta)$$

$$H_b = -t\sum_{\mathbf{r}\delta}(b^\dagger_{\mathbf{r}} b_{\mathbf{r}+\delta} z^*_{\mathbf{r}\alpha} R^\delta_{\alpha\beta} z_{\mathbf{r}+\delta,\beta} + \text{h.c.}) + \frac{U}{2}\sum b^\dagger_{\mathbf{r}} b^\dagger_{\mathbf{r}} b_{\mathbf{r}} b_{\mathbf{r}}$$
$$+ (\lambda - 1)U\sum_{\mathbf{r}} |z_{\mathbf{r}\uparrow}|^2 |z_{\mathbf{r}\downarrow}|^2 b^\dagger_{\mathbf{r}} b^\dagger_{\mathbf{r}} b_{\mathbf{r}} b_{\mathbf{r}}$$

Magnetic ordering: “Chargons” see an **abelian gauge field**!

# Magnetic superfluid and Mott transition

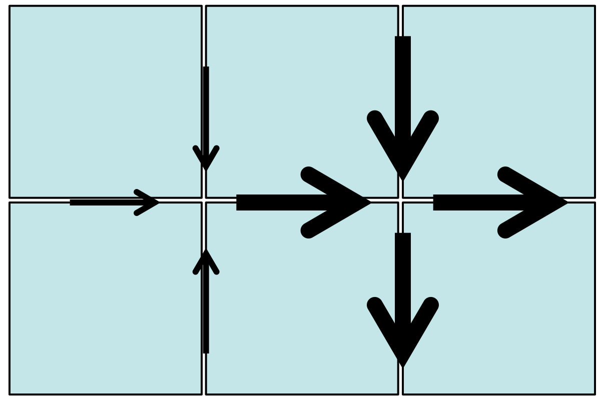



W.S. Cole, S. Zhang, A. Paramekanti, N. Trivedi, PRL 2012

- For zAFM: Going into the superfluid induces spontaneous loop currents
- Slave boson description shows abelian **π-flux** for chargons
- **zAFM Mott to zAFM superfluid is a two-component field theory**

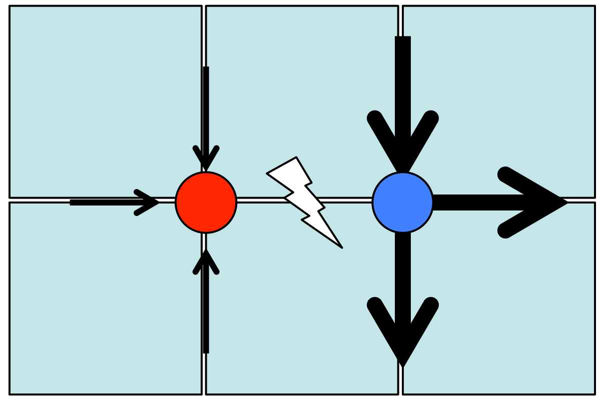
- Slave boson description also useful for analyzing SF-MI phase transitions

# Probing atom mass currents produced by synthetic gauge fields



In equilibrium, the currents satisfy a steady state condition: Current-In = Current-Out

# Quantum Quenches: Tool to Probe Atom Mass Currents

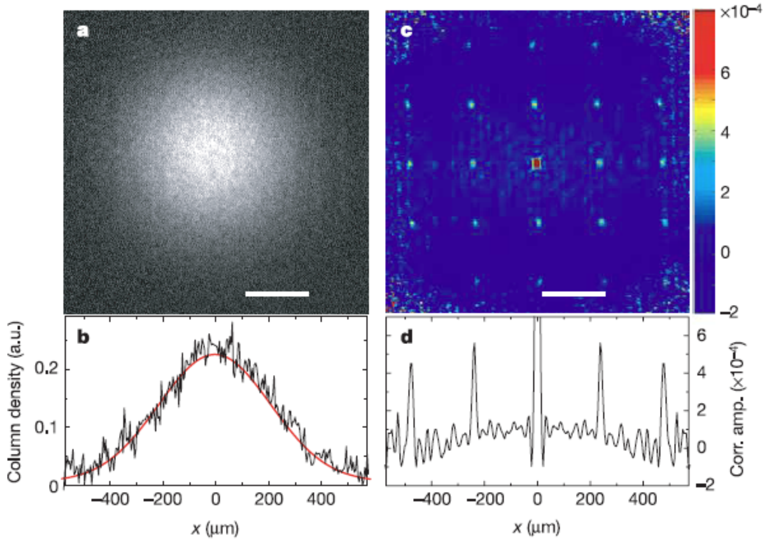


In equilibrium, the currents satisfy a steady state condition: Current-In = Current-Out

If we disrupt this condition, get current imbalance at the nodes

Continuity Equation tells us that this will lead to a density change: Measuring the induced densities after a short interval of time tells us about the underlying initial currents

# Experimental Probes of Atom Density




**Spatial quantum noise interferometry in expanding ultracold atom clouds**

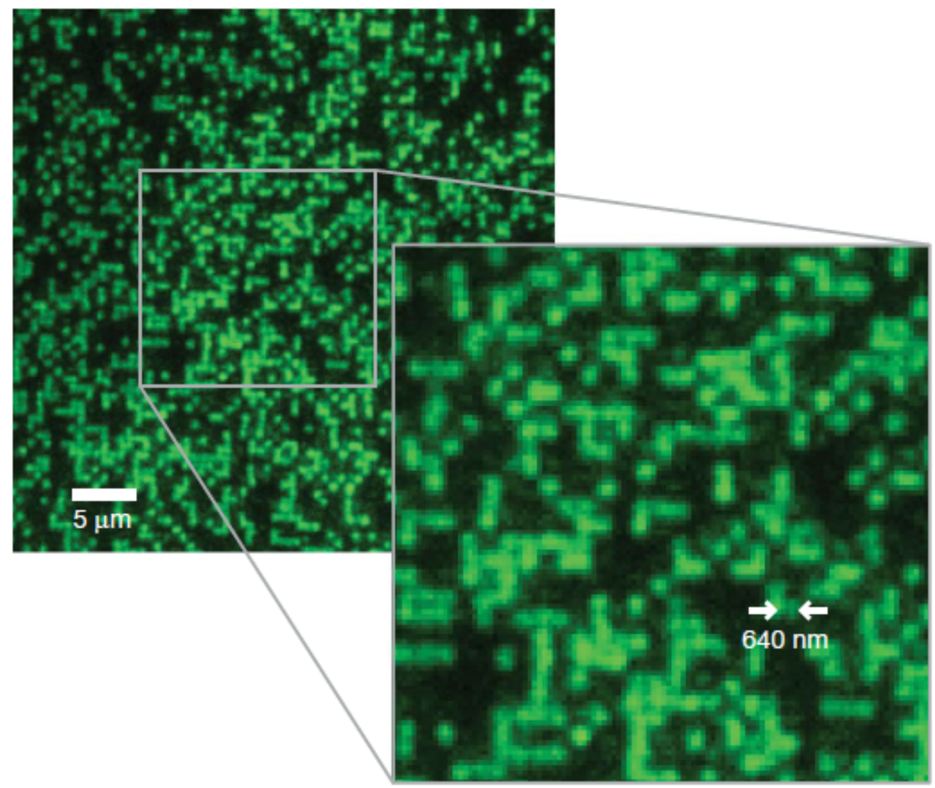

**Simon Fölling, Fabrice Gerbier, Artur Widera, Olaf Mandel, Tatjana Gericke & Immanuel Bloch**



Nature **434**, 481 (2005)


**A quantum gas microscope for detecting single atoms in a Hubbard-regime optical lattice**


Waseem S. Bakr[1], Jonathon I. Gillen[1], Amy Peng[1], Simon Fölling[1] & Markus Greiner[1]



nature Vol 462|5 November 2009


## Coherent Bragg scattering

*C. Weitenberg, et al (I. Bloch group)*
*Phys. Rev. Lett.* ***106****, 215301 (2011)*

# General scheme to probe currents

Imagine starting from a d-dimensional system

Let us quench the hopping in (d-1) “transverse dimensions”

1D continuity equation at short time:

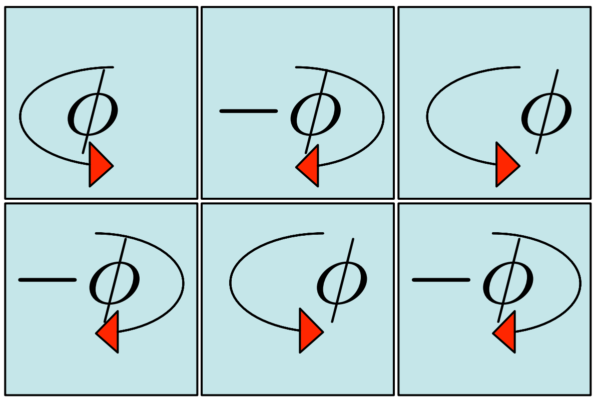
$$\delta n(\vec{q}, t = \epsilon) \approx -i q_x \epsilon J_x(\vec{q}, t = 0)$$

$$q \neq 0 \implies J_x(\vec{q}, t = 0) \approx i \frac{\delta n(\vec{q}, t=\epsilon)}{q_x \epsilon}$$

$$q = 0 \implies \text{Center of mass displacement}$$

# Checkerboard Staggered Flux Superfluid

$$H = -\sum_{\vec{r},\vec{r}'} J_{\vec{r},\vec{r}'} B^\dagger_{\vec{r}} B_{\vec{r}'} + \frac{U}{2}\sum_{\vec{r}} B^\dagger_{\vec{r}} B^\dagger_{\vec{r}} B_{\vec{r}} B_{\vec{r}}$$




$$J_{\mathbf{r},\mathbf{r}+\hat{x}} = J_x \ \text{and}\ J_{\mathbf{r},\mathbf{r}+\hat{y}} = J_y \exp(i(-1)^{x+y}\phi/2)$$

Let Jx=Jy initially

Imagine quenching Jx from initial value using GP equation to study dynamics

Low energy modes at k=(0,0) and k=(π,π)
Set condensate wavefunction: $\Psi(x,y,t)=A(t) + B(t)\,(-1)^{(x+y)}$

$$i\frac{dA}{dt} = \tilde{\epsilon}_0 A - i\gamma_0 B + U(A|A|^2 + 2A|B|^2 + B^2 A^*)$$
$$i\frac{dB}{dt} = i\gamma_0 A - \tilde{\epsilon}_0 B + U(B|B|^2 + 2B|A|^2 + A^2 B^*)$$

→

Staggered density

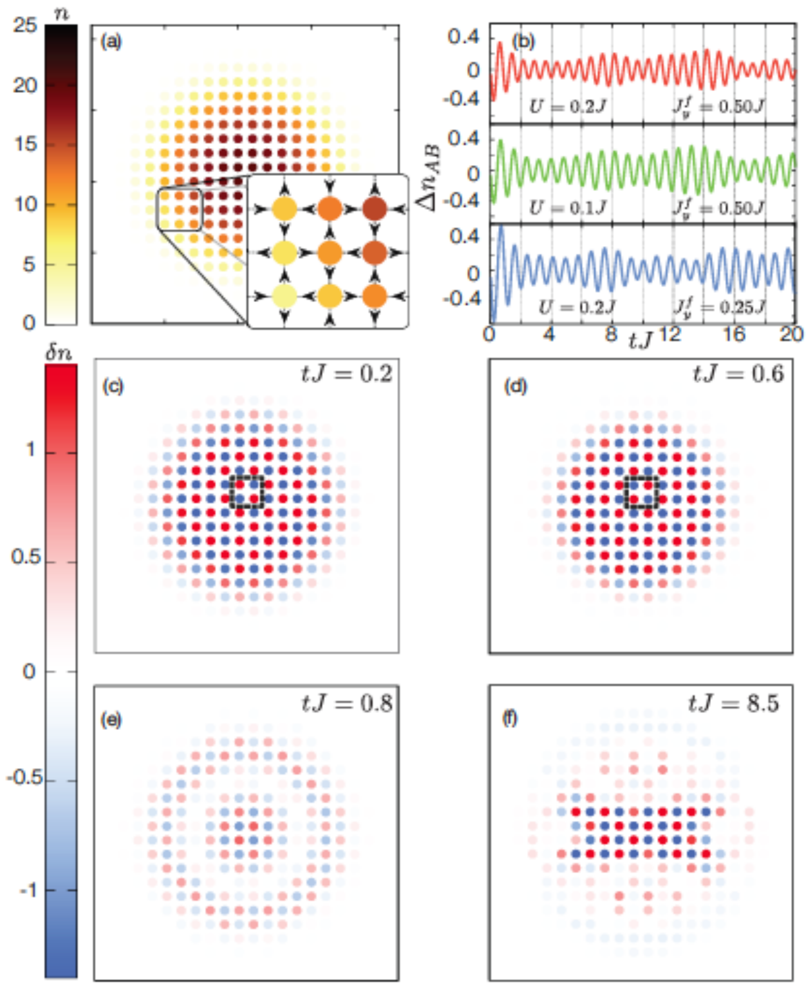
$$\Delta n(t) = 2\left(\frac{\delta J}{J}\right)\frac{\mathcal{I}}{\Omega}\sin(\Omega t)$$

M. Killi, A. Paramekanti, PRA (2012)

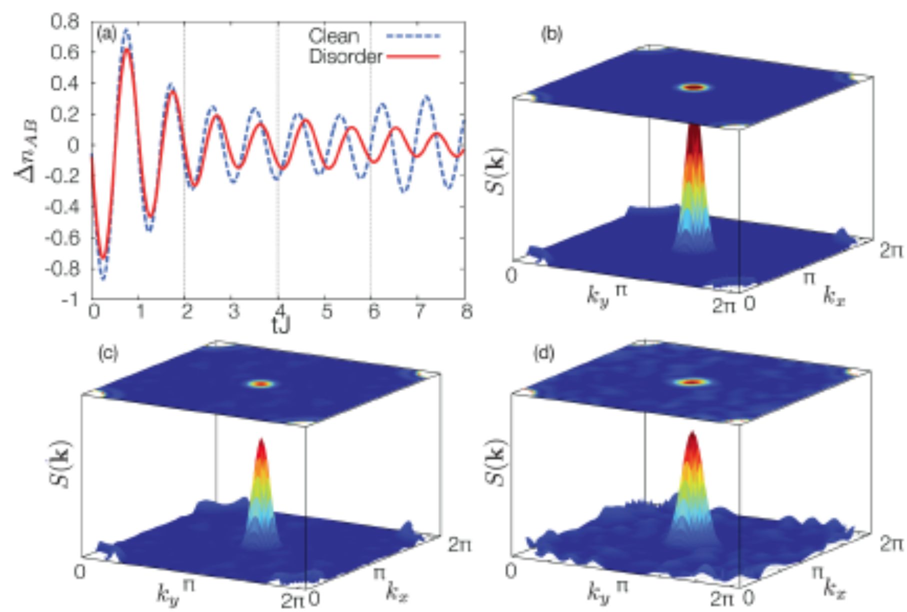
# Checkerboard Staggered Flux Superfluid

M. Killi, S. Trotzky, AP, PRA (2012)

Add harmonic trap / “fluctuations”



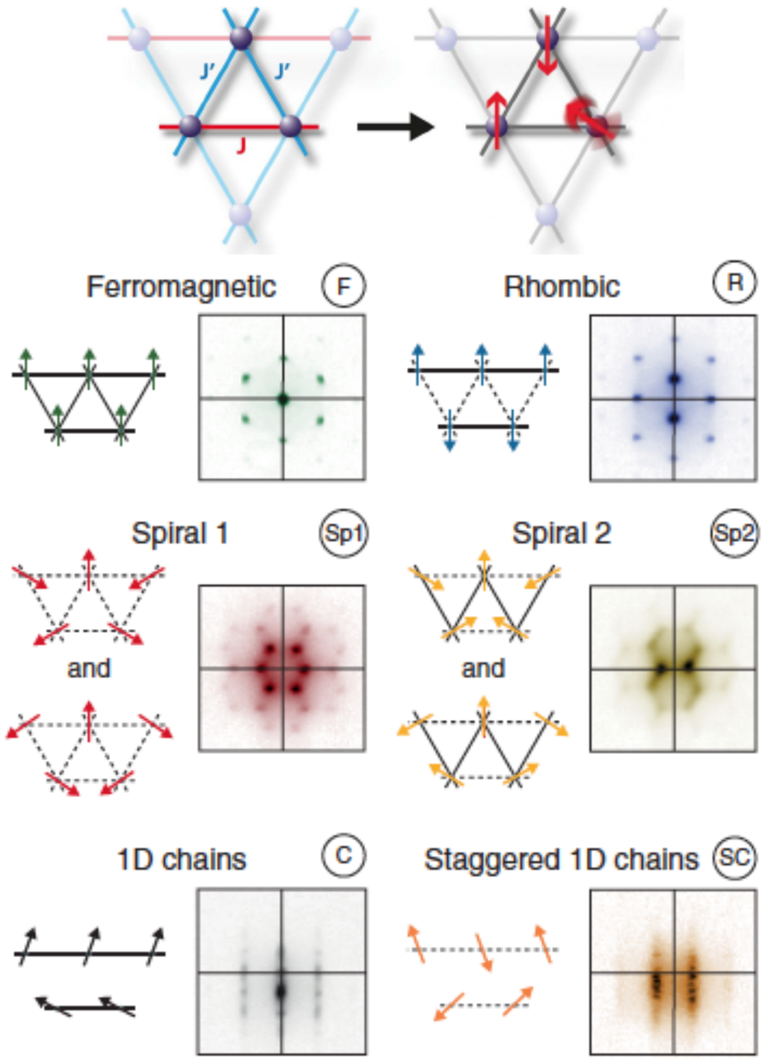

Real space



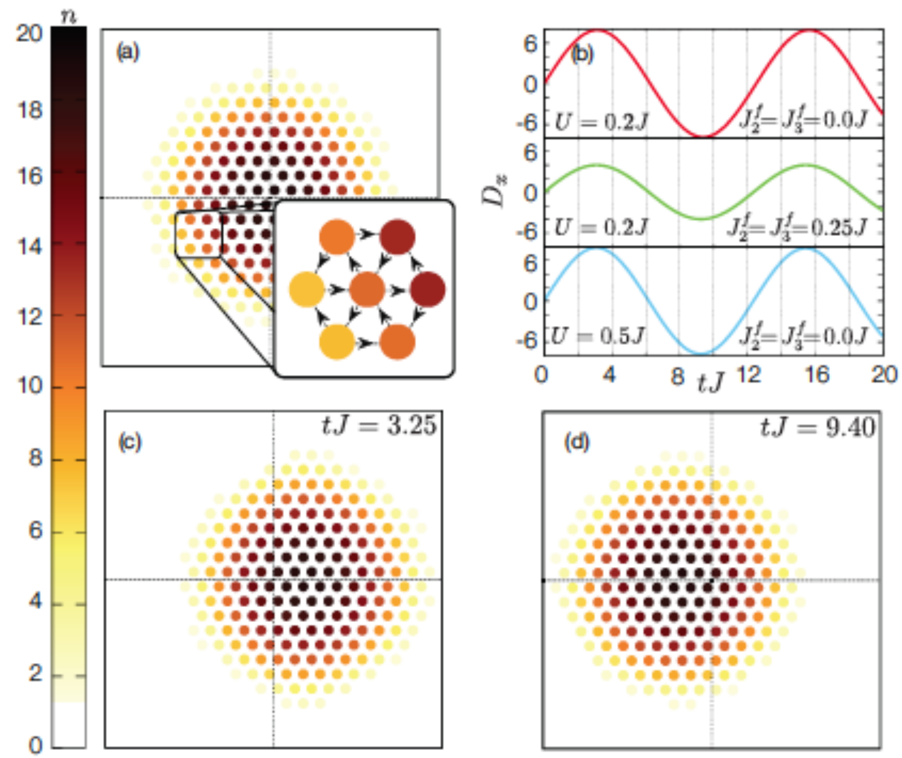

Momentum space

# Triangular lattice frustrated superfluid

M. Killi, S. Trotzky, AP, PRA (2012)




J. Struck, et al, Science (2011)
Sengstock group, Hamburg




Quench two directions
Get Dipole oscillations

M. Killi, S. Trotzky, AP, PRA (2012)

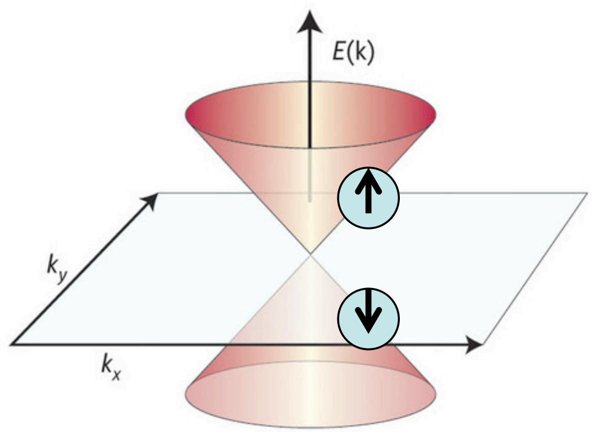
# Checkerboard Staggered Flux Fermi Gas

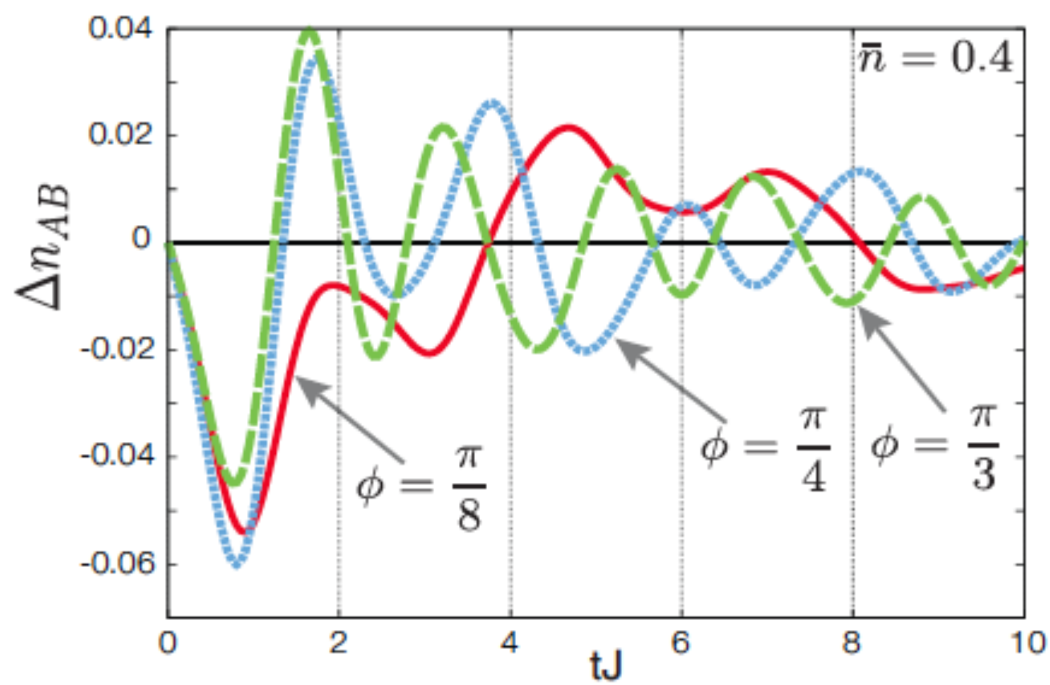
$$H = -\sum_{r,r'} J_{r,r'} C_r^\dagger C_{r'}$$

Fluxes ϕ, −ϕ

M. Killi, A. Paramekanti, PRA (2012)

$$H = \sum_{\mathbf{k}}{}' \begin{pmatrix} f_{\mathbf{k}}^\dagger & f_{\mathbf{k+Q}}^\dagger \end{pmatrix} (\varepsilon_{\mathbf{k}} \tau^z + \gamma_{\mathbf{k}} \tau^y) \begin{pmatrix} f_{\mathbf{k}} \\ f_{\mathbf{k+Q}} \end{pmatrix}$$




"Spin Precession Dynamics" when $J_y$ is quenched
More complex structure (like dHvA): But dominant frequency over range of fillings

$$\tilde{\Omega}^* \approx 2\sqrt{J^2 + (J_y^f)^2 - 2JJ_y^f \cos\frac{\phi}{2}}$$

# Lattice integer quantum Hall state of fermions

$$H = -\sum_{r,r'} J_{r,r'} C_r^\dagger C_{r'}$$

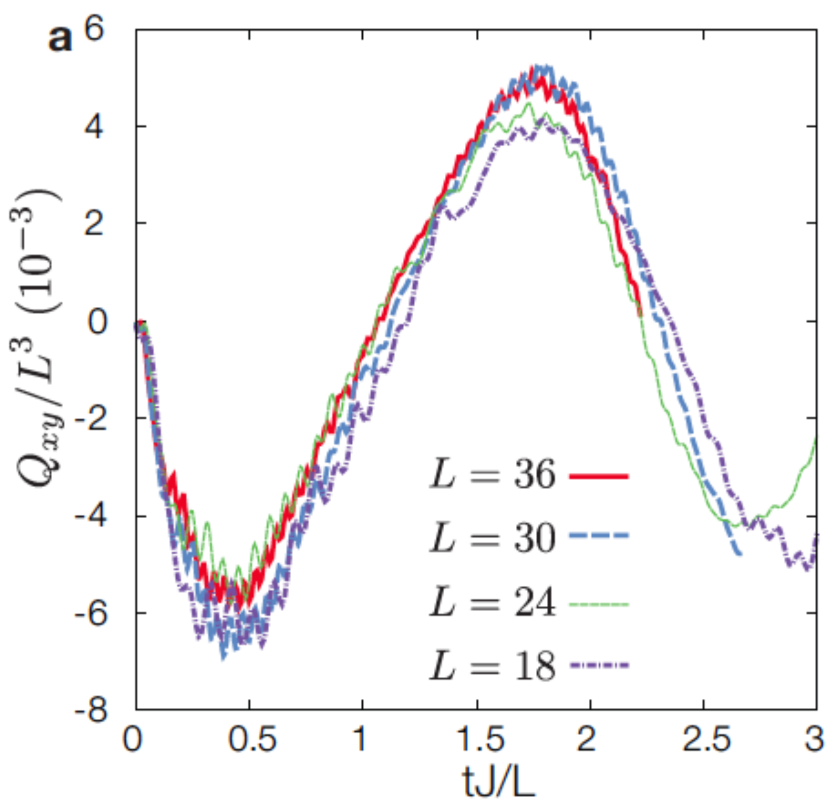
M. Killi, A. Paramekanti, PRA (2012)
M. Killi, S. Trotzky, AP, PRA (2012)

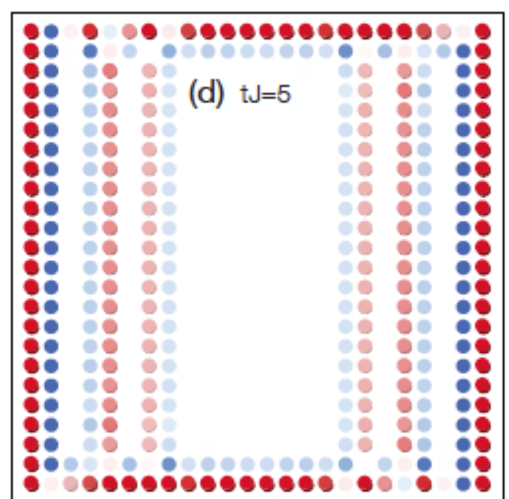
Hofstadter model with flux 2π/3 per plaquette

Study the dynamics of the quadrupole moment of the system with open boundary conditions

$$Q_{xy}(t) = \sum_{x,y} xy\delta n(x,y,t)$$

Motivated by: L. J. LeBlanc, et al, arXiv:1201.5857




“Quadrupolar current oscillations”: Dominated by Chiral Edge Mode

# Summary

- Magnetic flux introduces “frustration” and makes the system more susceptible to forming a Mott insulator
- On a ladder, Hubbard repulsion induces a “chiral Mott Insulator” state at intermediate correlation
- Rashba coupling and strong correlations induces various exotic magnetically ordered Mott insulators and superfluids
- Slave boson theory of strongly correlated magnetic superfluid – shows how magnetic orders imprint various “abelian fluxes” on charge degrees of freedom
- Quantum quench dynamics is an effective tool to uncover mass current patterns (Bosons/Fermions)

# Collaborators

**Ciaran Hickey**, **M. Killi**, I. Kivlichan, S. Trotzky (Toronto)
**William S. Cole**, S. Zhang, N. Trivedi (Ohio State University)
**Arya Dhar**, M. Maji, T. Mishra, R. V. Pai, S. Mukerjee (Bangalore)

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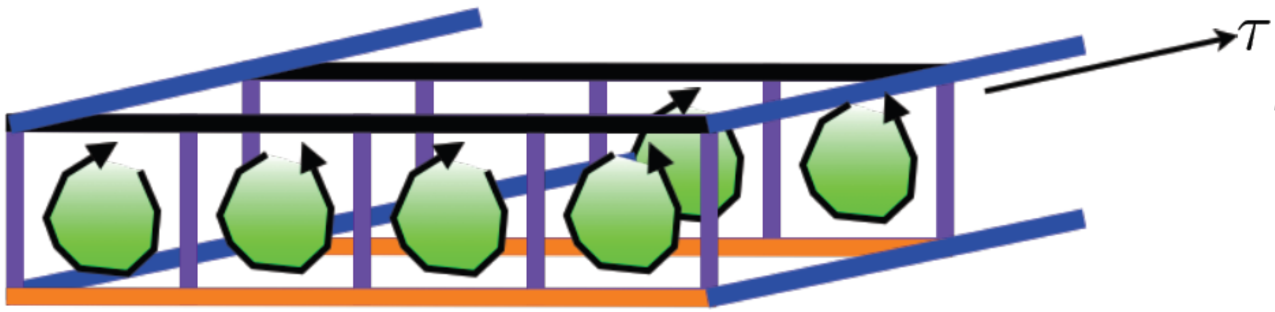
Chiral Mott story:
A.Dhar, M. Maji, T. Mishra, R. V. Pai, S. Mukerjee, and AP, Phys. Rev. A **85**, (R)041602 (2012)
A.Dhar, T. Mishra, M. Maji, R. V. Pai, S. Mukerjee, and AP, Phys. Rev. B **87**, 174501 (2013)

Spin orbit story:
W. S. Cole, S. Zhang, AP, and N. Trivedi, PRL **109**, 085302 (2012)
C. Hickey, et al (manuscript in preparation)

Quench story:
M. Killi and AP, PRA **85**, (R)061606 (2012)
M. Killi, S. Trotzky, AP, Phys. Rev. A **86**, 063632 (2012)

# Classical XY model and Monte Carlo study




$$S_{\rm cl}^{1+1} = -\sum_{x\tau}\left[J_{\|}\cos(\varphi^a_{x+1,\tau}-\varphi^a_{x,\tau}) - J_{\|}\cos(\varphi^b_{x+1,\tau}-\varphi^b_{x,\tau}) + J_{\perp}\cos(\varphi^a_{x,\tau}-\varphi^b_{x,\tau})\right]$$
$$- \; J_\tau \sum_{x\tau}\left[\cos(\varphi^a_{x,\tau+1}-\varphi^a_{x,\tau}) + \cos(\varphi^b_{x,\tau+1}-\varphi^b_{x,\tau})\right]$$

$2\epsilon\tilde{t} = J_{\|},\; 2\epsilon\tilde{t}_{\perp} = J_{\perp}$ : Spatial couplings are like interchain hopping amplitudes

$1/\epsilon U = J_\tau$ : Imaginary time coupling is like 1/U

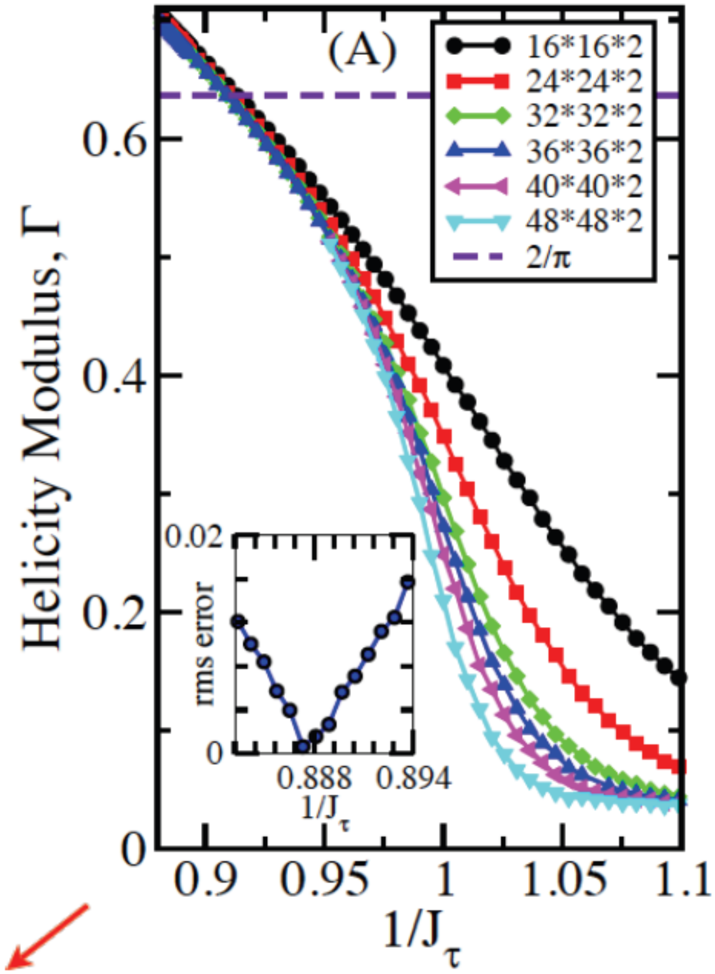
# Classical XY model and Monte Carlo study

To detect the vanishing of superfluidity
we compute the Helicity Modulus

Helicity modulus ~ superfluid density
How much free energy cost to twist the phase?
How much kinetic energy of superflow?

Looks like a Berezinskii Kosterlitz Thouless transition at which Helicity Modulus jumps

To locate the transition and check if it is of the BKT type, use finite size scaling form derived from RG equations



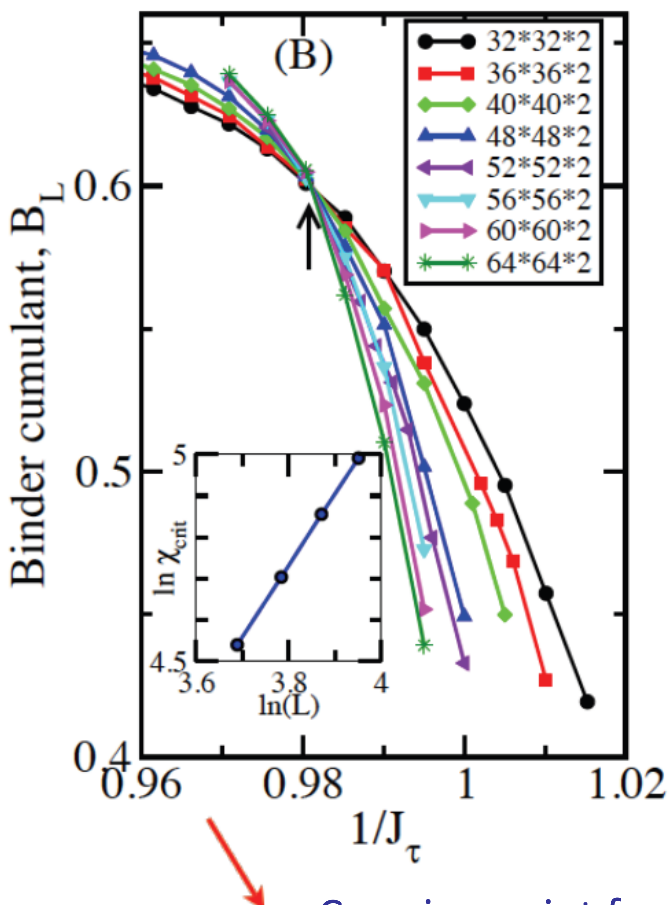

$$\Gamma(L) = A\left(1 + \frac{1}{2}\frac{1}{\log L + C}\right)$$

We find A $= 2/\pi$ as expected for a BKT transition

Weber/Minnhagen (PRB 1993)
Olsson (PRL 1995)
S. Mukerjee, et al (PRL 2006)

# Classical XY model and Monte Carlo study

To detect the staggered current order, we use the method of Binder cumulants




$$B_L = \left(1 - \frac{\langle m^4 \rangle_L}{3 \langle m^2 \rangle_L^2}\right)$$

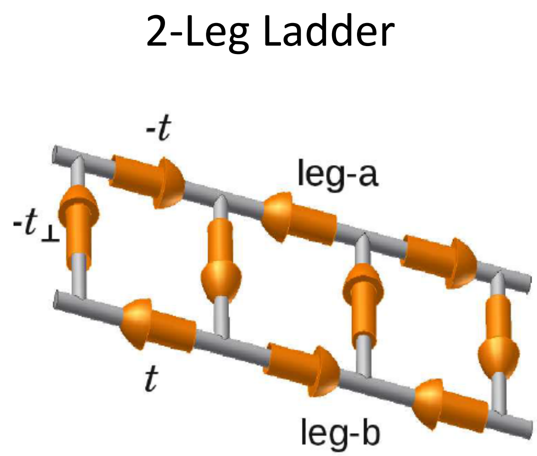
m: Staggered current order parameter

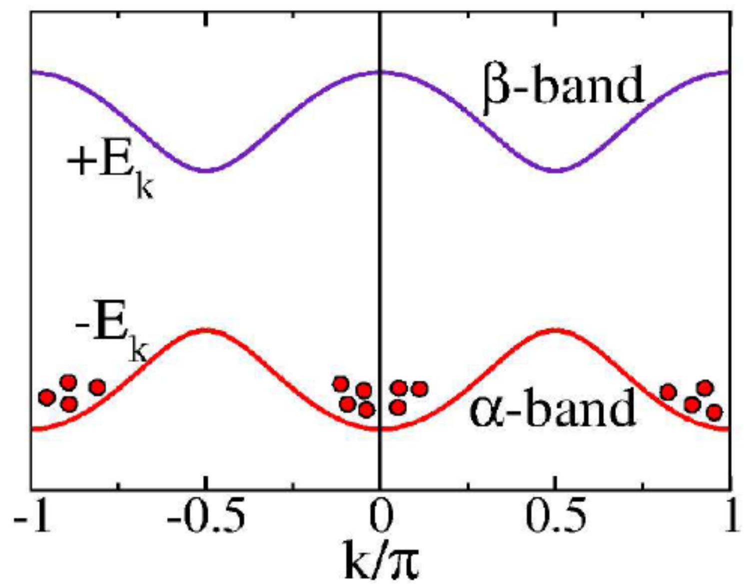
Basic idea: Fixed points have universal order parameter distributions

. Crossing point for all L indicates a critical point
. Susceptibility scaling consistent with 2D Ising transition

# What happens with half-flux-quantum and Hubbard repulsion?

**Weak Correlations: Landau theory**




Fixing the relative phase leads to

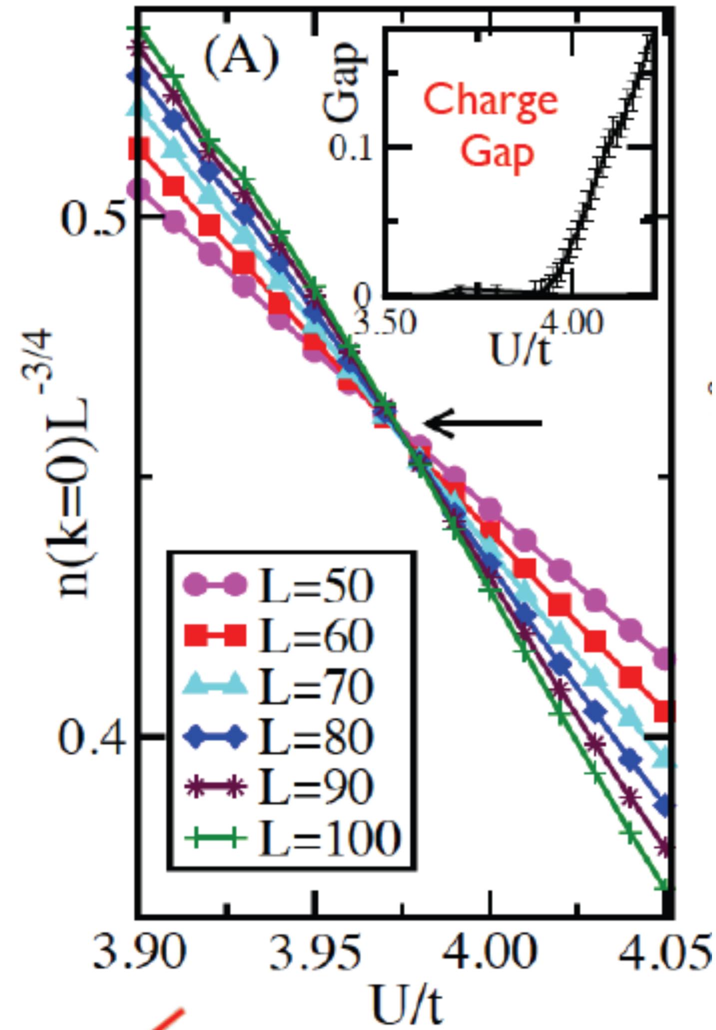
$$E_{\rm low}^{\rm mft} = (-E_0 - \mu) \sum_{i=0,\pi} |\varphi_i|^2 + U(u_0^4 + v_0^4)(|\varphi_0|^2 + |\varphi_\pi|^2)^2 - 2U(u_0^2 - v_0^2)^2 |\varphi_0|^2 |\varphi_\pi|^2$$

Favors equal amplitude condensate

# DMRG study of the Bose Hubbard Ladder

To detect the onset of insulating behavior, we compute the charge gap

To detect the vanishing of superfluidity, plot the scaled momentum distribution at k=0

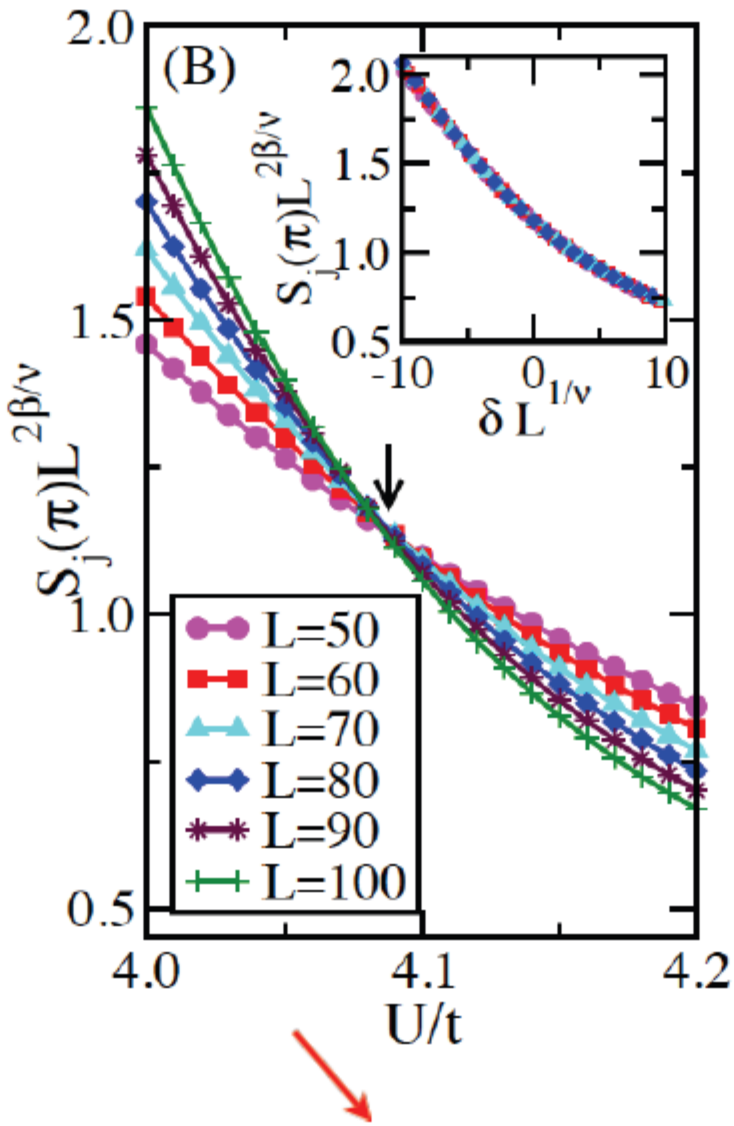



For a BKT transition, the power law decay of phase correlations at the transition is 1/r^{1/4}

This translates into a L^{3/4} divergence for n(k=0)

Onset of charge gap coincides with identification of BKT transition point

# DMRG study of the Bose Hubbard Ladder




To detect the staggered current order, look at the scaling of the structure factor

Using 2D Ising exponents, find the transition point where loop current order vanishes

Observe scaling collapse of the data for various system sizes

Find consistent crossing point and data collapse for 2D Ising exponents