INO
ISTITUTO NAZIONALE
DI OTTICA

BEC

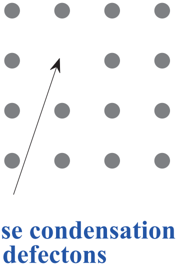
UNIVERSITA DEGLI STUDI
DI TRENTO

Superstripes and the excitation spectrum of a spin-orbit-coupled BEC

Yun Li, Giovanni I. Martone, Lev P. Pitaevskii,
and Sandro Stringari

Trieste, 2013 May

Breaking of two symmetries



A. F. Andreev and I. M. Lifshitz
JETP 29, 1107 (1969)

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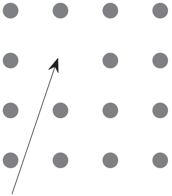

Bose condensation of defectons

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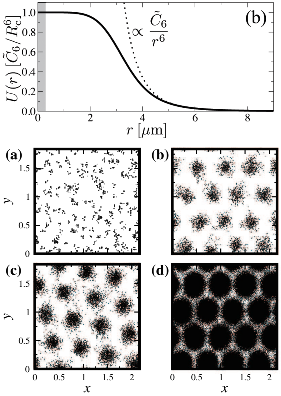
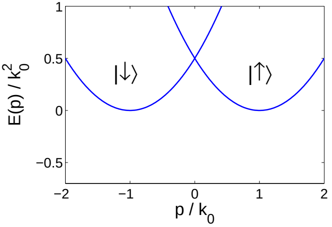
Soft-core interactions

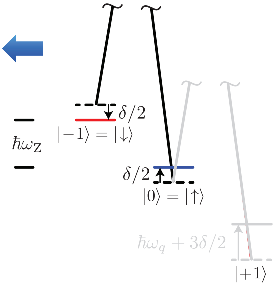


N. Henkel, et al., PRL 104, 195302 (2010)
F. Cinti, et al., PRL 105, 135301 (2010)

Spin-orbit coupled BEC (single particle picture)

bulk, no interactions, $\Omega = 0$





Single-particle Hamiltonian

$$h_0 = \frac{1}{2}\left[(p_x - k_0\sigma_z)^2 + p_\perp^2\right] + \frac{\Omega}{2}\sigma_x + \frac{\delta}{2}\sigma_z \qquad \Longleftarrow \; U^{-1}\, h_0^{\mathrm{lab}}\, U$$

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Equal Rashba and Dresselhaus couplings

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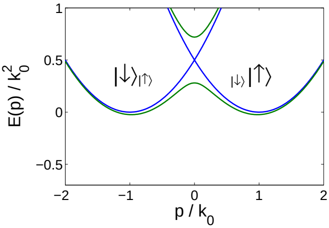

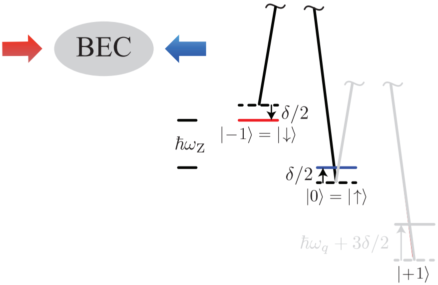

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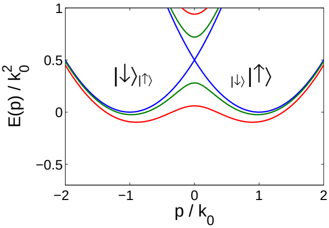

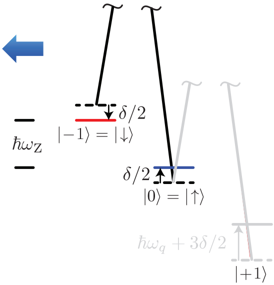

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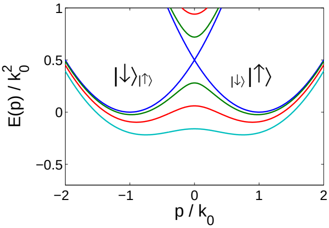

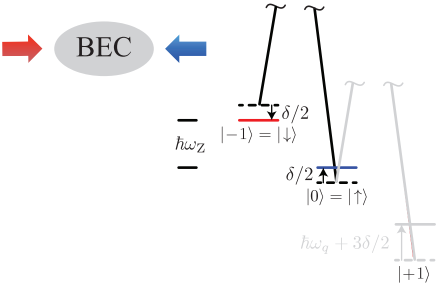

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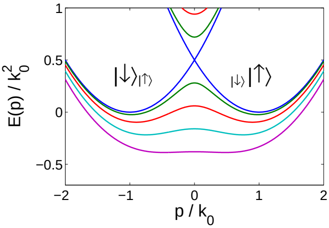

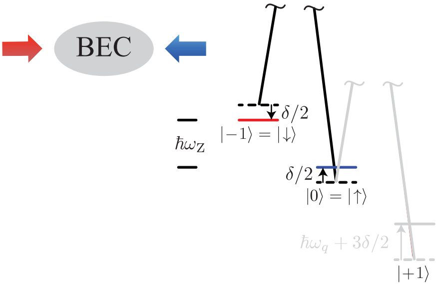

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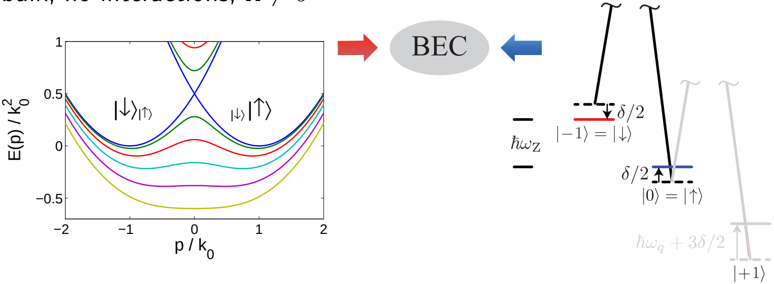

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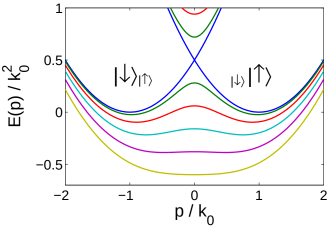

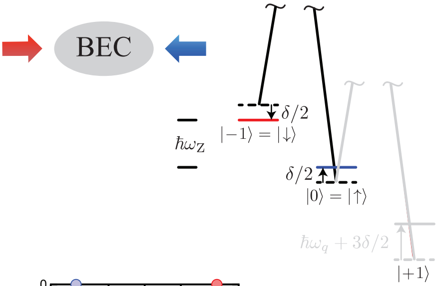

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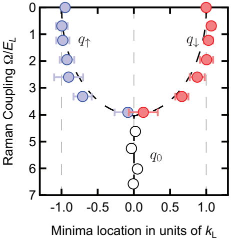

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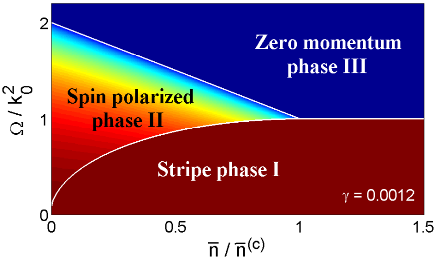
Lin et al, Nature 83, 1523 (2010)

Many-body ground state

Three quantum phases $\gamma = (g - g_{\uparrow\downarrow})/(g + g_{\uparrow\downarrow}) > 0$, $n^{(c)} = k_0^2/(2\gamma g)$



(I). $k_1 \neq 0$, $C_+ = C_-$, $\langle \sigma_z \rangle = 0$

(II). $k_1 \neq 0$, $C_- = 0$ or $C_+ = 0$, $\langle \sigma_z \rangle \neq 0$

(III). $k_1 = 0$, $\langle \sigma_z \rangle = 0$

$$\psi = \begin{pmatrix} \psi_\uparrow \\ \psi_\downarrow \end{pmatrix} = \sqrt{\bar{n}} \left[C_+ \begin{pmatrix} \cos\theta \\ -\sin\theta \end{pmatrix} e^{ik_1 x} + C_- \begin{pmatrix} \sin\theta \\ -\cos\theta \end{pmatrix} e^{-ik_1 x} \right]$$

$$\cos 2\theta = \frac{k_1}{k_0}, \quad \langle \sigma_z \rangle = \frac{k_1}{k_0} \left(|C_+|^2 - |C_-|^2 \right)$$

LY, Pitaevskii, Stringari, PRL 108, 225301 (2012)

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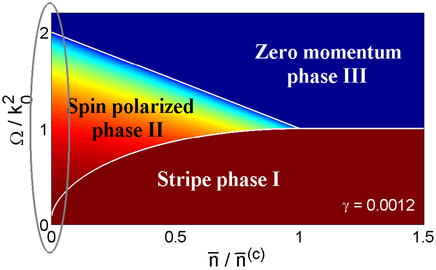

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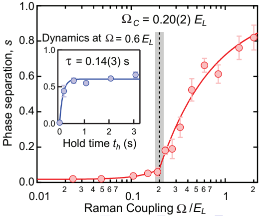
$$\Omega_{cr}^{(I\text{-}II)} = 2k_0^2\sqrt{2\gamma/(1+2\gamma)}$$ small for ^{87}Rb

$$\Omega_{cr}^{(II\text{-}III)} = 2k_0^2$$



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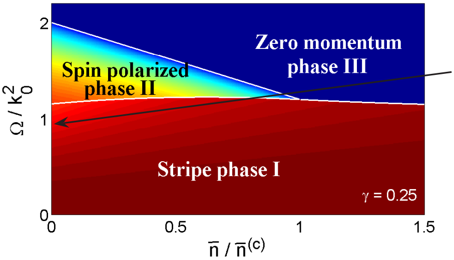

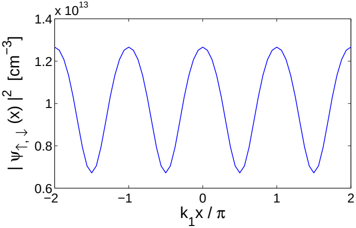

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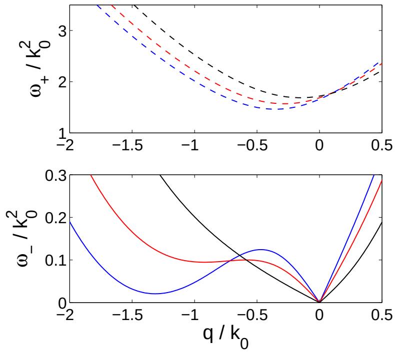
$\Omega_{\rm cr}^{(\text{II-III})} = 2k_0^2$

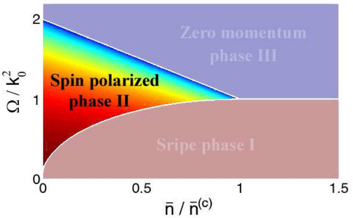
To increase the effect of the contrast, choose larger values of γ

Excitation spectrum in phase II



$$G_1/k_0^2 = 0.5,\ G_2/k_0^2 = 0.12$$

$$\Omega/k_0^2 = 1.22,\ 1.33,\ 1.46$$



- Despite spinor nature, occurrence of Raman coupling gives rise to a single gapless branch
- Emergence of a roton minimum at finite q: a tendency of the system towards crystallization

Martone, LY, Pitaevskii and Stringari, PRA 86, 063621(2012)

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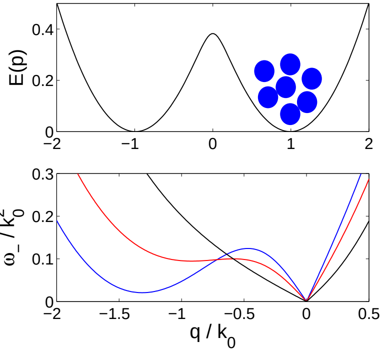

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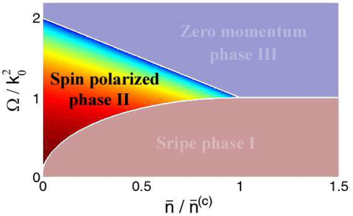

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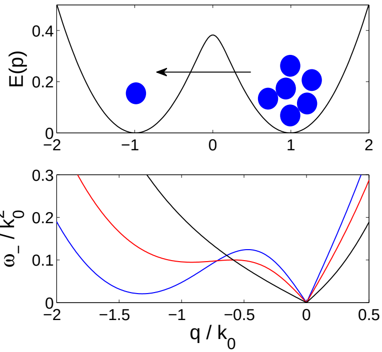

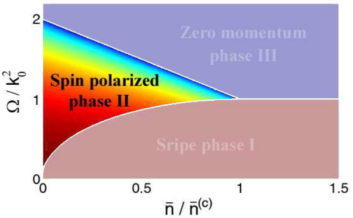

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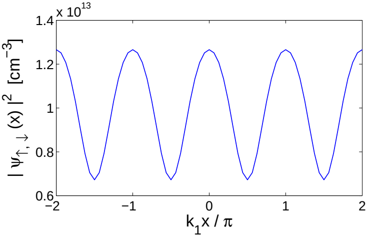
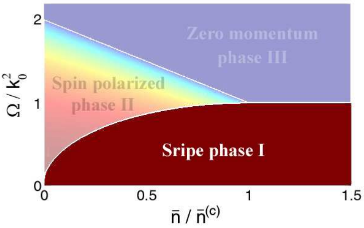
Excitation spectrum in phase I

Equilibrium state





$$G_1/k_0^2 = 0.3,\ G_2/k_0^2 = 0.08,\ \Omega/k_0^2 = 1$$

- **Translational invariance symmetry breaking**
- $U(1)$ **symmetry breaking**

$$\begin{pmatrix}\psi_{0\uparrow}\\ \psi_{0\downarrow}\end{pmatrix} = \sum_{\bar{K}} \begin{pmatrix} a_{-k_1+\bar{K}} \\ -b_{-k_1+\bar{K}}\end{pmatrix} e^{i(\bar{K}-k_1)x},$$

$\bar{K}$ is the reciprocal lattice vector

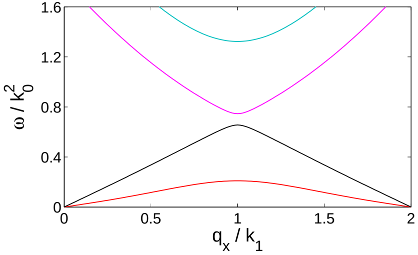
Excitation spectrum in phase I

Bogoliubov + **Bloch** theory

$$\begin{pmatrix}\psi_\uparrow\\ \psi_\downarrow\end{pmatrix} = e^{-i\mu t}\left[\begin{pmatrix}\psi_{0\uparrow}\\ \psi_{0\downarrow}\end{pmatrix} + \begin{pmatrix}u_\uparrow(\mathbf{r})\\ u_\downarrow(\mathbf{r})\end{pmatrix} e^{-i\omega t} + \begin{pmatrix}v^*_\uparrow(\mathbf{r})\\ v^*_\downarrow(\mathbf{r})\end{pmatrix} e^{i\omega t}\right]$$

$$u_{\mathbf{q}\uparrow,\downarrow}(\mathbf{r}) = e^{-ik_1 x}\sum_{\bar{K}} U_{\mathbf{q}\uparrow,\downarrow\bar{K}}\, e^{i\mathbf{q}\cdot\mathbf{r}+i\bar{K}x}$$

$$v_{\mathbf{q}\uparrow,\downarrow}(\mathbf{r}) = e^{ik_1 x}\sum_{\bar{K}} V_{\mathbf{q}\uparrow,\downarrow\bar{K}}\, e^{i\mathbf{q}\cdot\mathbf{r}-i\bar{K}x}$$



- Emergence of **two gapless** bands
- **Vanishing** of the frequency at the edge of the Brillouin zone
- **Divergent** behavior of static structure factor in density channel

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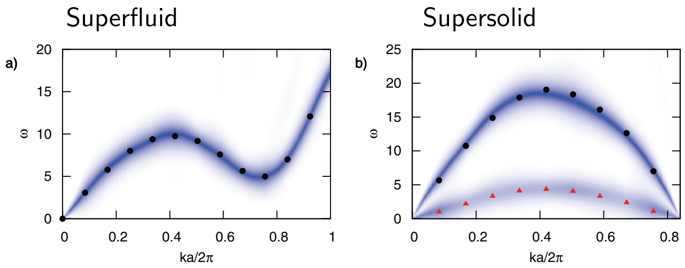
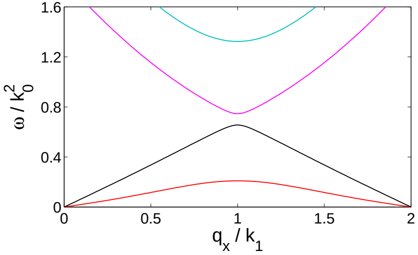


Soft-core interactions

S. Saccani et al., PRL 108, 175301 (2012)



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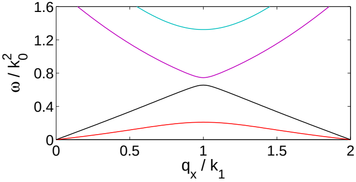

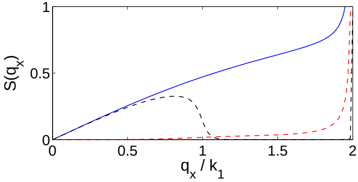

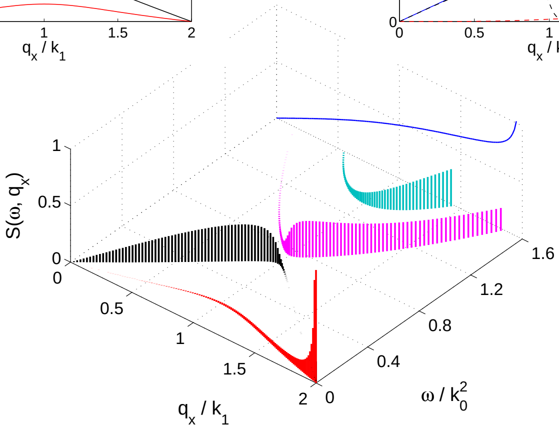

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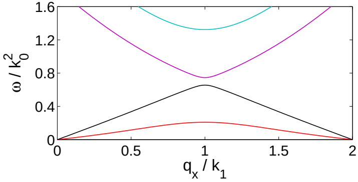
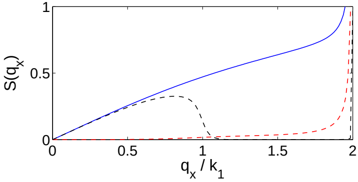
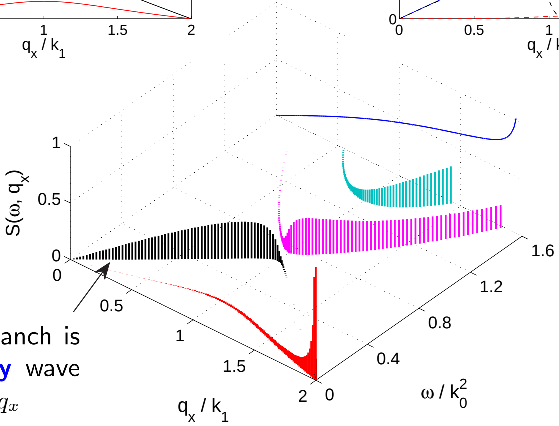
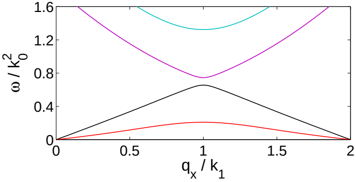


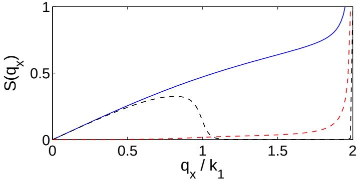


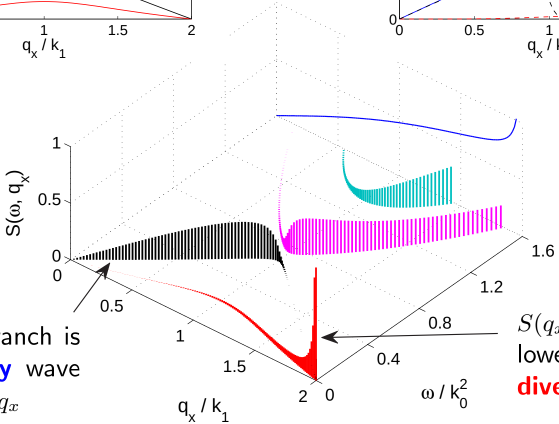


Upper branch is a **density** wave at small q_x

Density structure factor



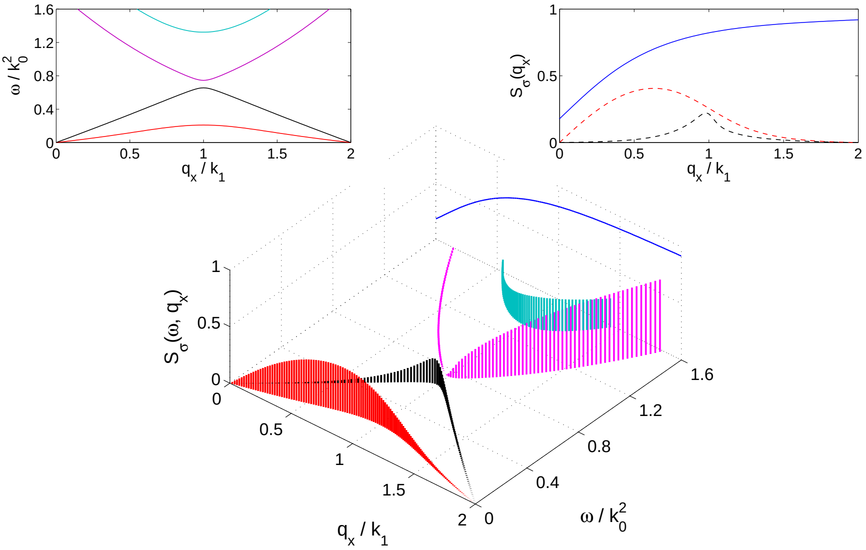




Upper branch is a **density** wave at small q_x

$S(q_x)$ for the lower branch **diverges**

Spin structure factor



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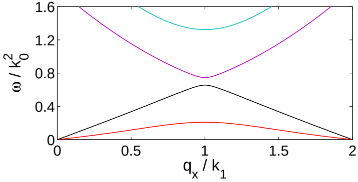
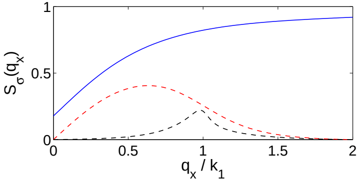
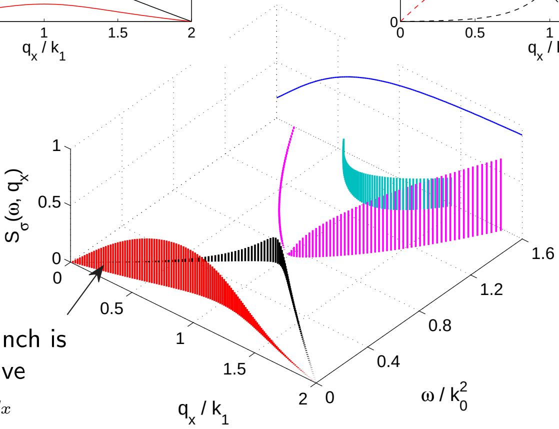






Lower branch is
a **spin** wave
at small q_x

Sum rule approach

Define q_x-component **density** operator F and $(q_x - q_B)$-component **momentum density** operator G

$$F = \sum_j e^{iq_x x_j}\,, \quad \langle\,[F,\, G]\,\rangle = q_x N \langle e^{iq_B x}\rangle$$

$$G = \sum_j \left[p_{xj}\, e^{-i(q_x - q_B)x_j} + e^{-i(q_x - q_B)x_j} p_{xj}\right]/2$$

$p-$th moments:

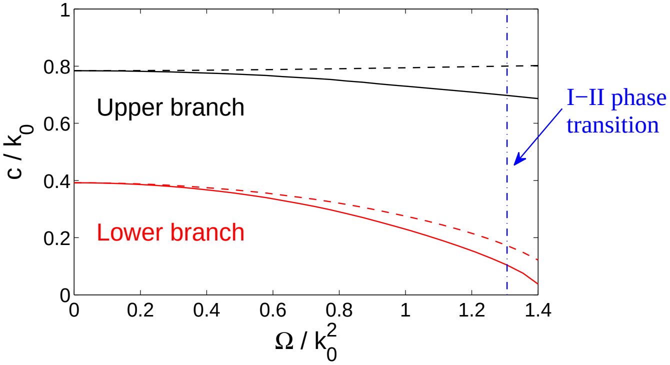
$$m_p(\mathcal{O}) = \sum_\ell (|\langle 0|\mathcal{O}|\ell\rangle|^2 + |\langle 0|\mathcal{O}^\dagger|\ell\rangle|^2)\omega_{\ell 0}^p$$

$q_x \to q_B$	F	G
m_1	q_x^2	$(q_x - q_B)^2$
m_0	$S(q_x)$	$\lvert q_x - q_B\rvert$
m_{-1}	$\chi(q_x)$	

Bogoliubov inequality: $m_{-1}(F)m_1(G) \geq |\langle\,[F, G]\,\rangle|^2 \quad \chi(q_x) \propto 1/(q_x - q_B)^2$

Uncertainty inequality: $m_0(F)m_0(G) \geq |\langle\,[F, G]\,\rangle|^2 \quad S(q_x) \propto 1/|q_x - q_B|$

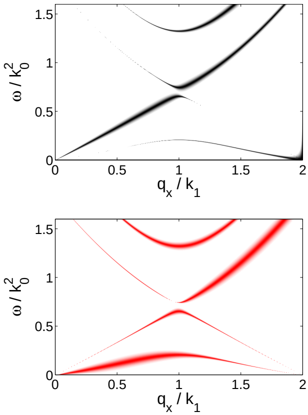
Sound velocity



- $c_{\perp}^{(\mathrm{d},\,\mathrm{s})} > c_{x}^{(\mathrm{d},\,\mathrm{s})}$, inertia of flow caused by stripes
- $c_{\perp}^{(\mathrm{d})} \simeq \sqrt{(g + g_{\uparrow\downarrow})\bar{n}/2}$, well reproduced by usual Bogoliubov sound velocity

Conclusions

- Excitation spectrum in the stripe phase: double gapless band structure
- At small wave vector the lower and upper branches have, respectively, a spin and density nature
- Close to the first Brillouin zone the lower branch acquires an important density character, $S(q_x)$ diverges



LY, Martone, Pitaevskii, Stringari
arXiv: 1303.6903

Thank you !