



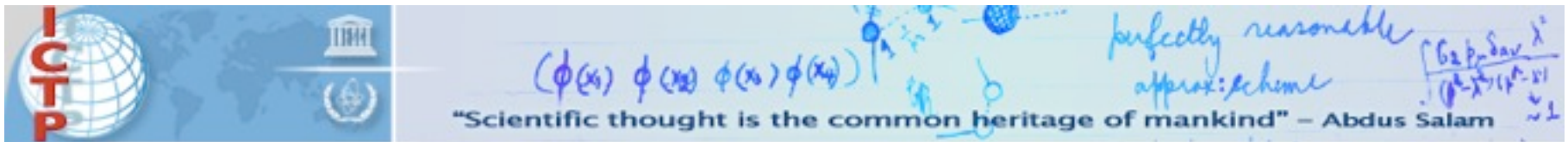
ulm university

universität
uulm

Quantum Simulation of (non-)abelian gauge theories

Enrique Rico Ortega
16 May 2013

Workshop on UltraCold Atoms
& Gauge Theories



The team

- **Albert Einstein Center - Bern University**



D. Banerjee



M. Bögli



P. Stebler



P. Widmer



U.-J. Wiese

- **Complutense University
Madrid**



M. Müller

- **Technical University
Vienna**

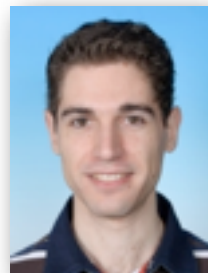


P. Rabl

- **IQOQI - Innsbruck University**



M. Dalmonte



D. Marcos



P. Zoller

The references

Atomic Quantum Simulation of Dynamical Gauge Fields Coupled to Fermionic Matter: From String breaking to Evolution after a Quench

Phys. Rev. Lett. 109, 175302 (2012)

**Atomic Quantum Simulation of $U(N)$
and $SU(N)$ non-abelian Gauge Theories**

arXiv:1211.2242 (2012)

**Quantum Simulation of Dynamical Lattice Gauge
Field Theories with Superconducting Q-bits**

Recent related works on dynamical gauge fields

H. Büchler, M. Hermele, S. Huber, M.P.A. Fisher, P. Zoller, Atomic quantum simulator for lattice gauge theories and ring exchange models, *Phys. Rev. Lett.* 95, 40402 (2005).

H. Weimer, M. Müller, I. Lesanovsky, P. Zoller, H.P. Büchler, A [digital] open system Rydberg quantum simulator, *Nat. Phys.* 6, 382 (2010).

J.I. Cirac, P. Maraner, J.K. Pachos, Cold Atom Simulation of Interacting Relativistic Quantum Field Theories, *Phys. Rev. Lett.* 105, 190403 (2010).

E. Kapit, E. Mueller, Optical-lattice Hamiltonians for relativistic quantum electrodynamics, *Phys. Rev. A* 83, (2011).

E. Zohar, J.I. Cirac, B. Reznik, Simulating Compact Quantum Electrodynamics with ultracold atoms: Probing confinement and nonperturbative effects, *Phys. Rev. Lett.* 109, 125302 (2012).

L. Tagliacozzo, A. Celi, A. Zamora, M. Lewenstein, Optical Abelian Lattice Gauge Theories, *Ann. Phys.* 330, 160-191 (2013).

**Barcelona, Bern, Innsbruck, Leeds, Madrid,
Munich, New York, Stuttgart, Tel Aviv, ...**

... and on dynamical non-abelian gauge fields

E. Zohar, J.I. Cirac, B. Reznik, A cold-atom quantum simulator for SU(2) Yang-Mills lattice gauge theory, arXiv:1211.2241 (2012).

L. Tagliacozzo, A. Celi, P. Orland, M. Lewenstein, Simulations of non-Abelian gauge theories with optical lattices, arXiv:1211.2704 (2012).

**Barcelona, Bern, Innsbruck, Leeds, Madrid,
Munich, New York, Stuttgart, Tel Aviv, ...**

Why quantum simulate Gauge theories?

The study of Gauge theories
is the study of Nature.

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is the study of Nature.

Gauge symmetry as a fundamental principle

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Gauge symmetry as an emergent phenomenon

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Gauge symmetry as a fundamental principle

Gauge symmetry as an emergent phenomenon

Gauge symmetry as a resource

Gauge symmetry as a fundamental principle

Standard model: for every force there is a gauge boson,

Gauge symmetry as a fundamental principle



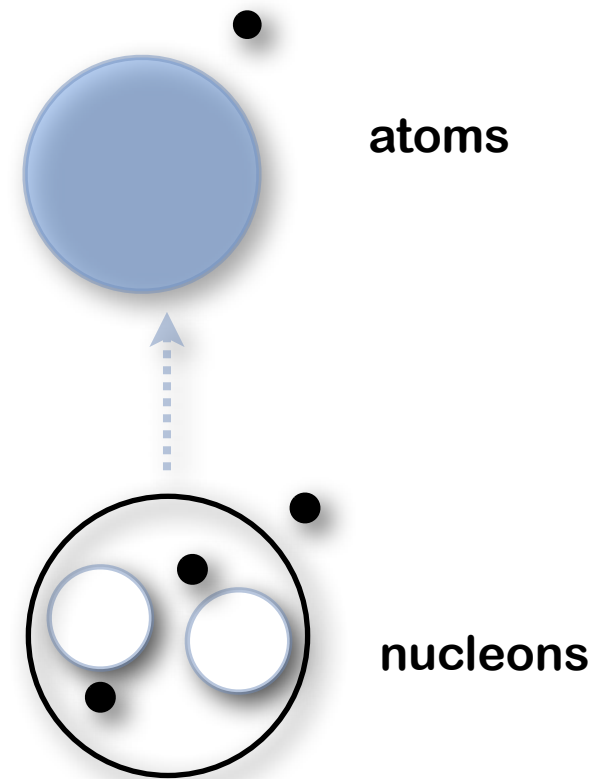
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- The photon is the “carrier” of the electromagnetic force.

Gauge symmetry as a fundamental principle

Standard model: for every force there is a gauge boson,

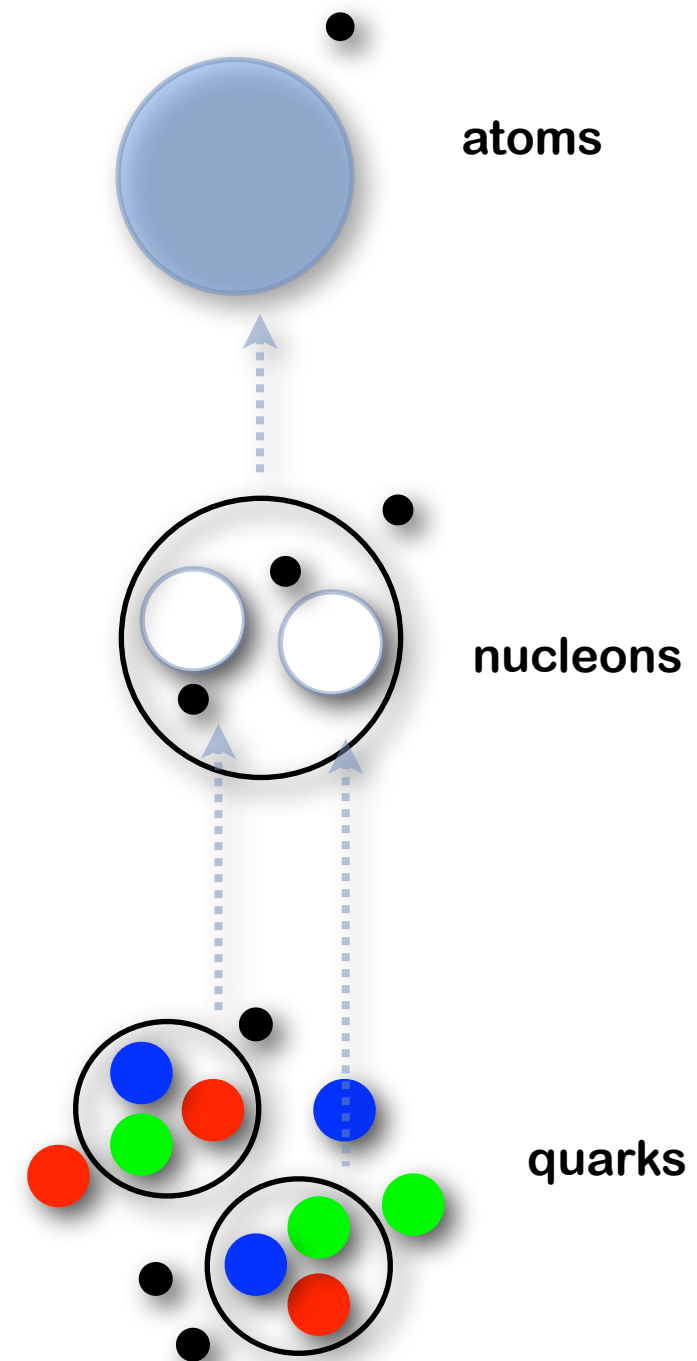
- The photon is the “carrier” of the electromagnetic force.
- The W^+ , W^- and Z^0 are the “carriers” of the weak force.



Gauge symmetry as a fundamental principle

Standard model: for every force there is a gauge boson,

- The photon is the “carrier” of the electromagnetic force.
- The W^+ , W^- and Z^0 are the “carriers” of the weak force.
- The gluons are the “carriers” of the strong force.

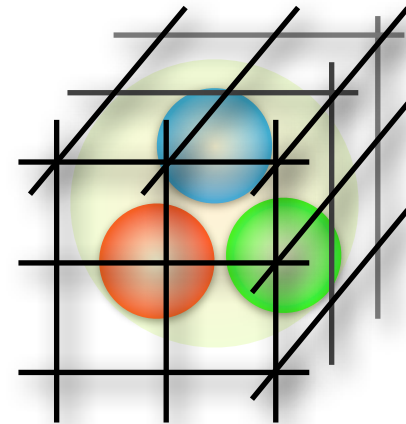


Gauge symmetry as a fundamental principle

Gauge theories on a discrete lattice structure.

K. Wilson, Phys. Rev. D
(1974)

Non-perturbative approach
to fundamental theories of
matter, e.g. Q.C.D.



Gauge symmetry as a fundamental principle

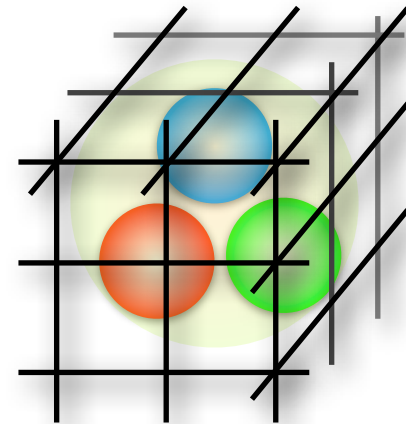
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K. Wilson, Phys. Rev. D
(1974)

**Monte Carlo simulation =
Classical Statistical Mechanics**

$$\begin{aligned}\langle O \rangle &= \frac{1}{Z} \int \mathcal{D}[\psi, U] e^{-S[\psi, U]} O[\psi, U] \\ &\sim \frac{1}{N} \sum_{n=1}^N e^{-S[\psi_n, U_n]} O[\psi_n, U_n] \\ &\sim \frac{1}{N} \sum_{P[U_n] \propto e^{-S[\psi_n, U_n]}} O[\psi_n, U_n]\end{aligned}$$

**Non-perturbative approach
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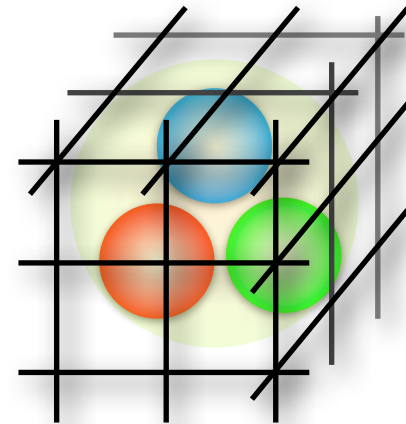
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$$\sim \frac{1}{N} \sum_{P[U_n] \propto e^{-S[\psi_n, U_n]}} O[\psi_n, U_n]$$

**Non-perturbative approach
to fundamental theories of
matter, e.g. Q.C.D.**



Classical simulations achievements:

- evidence of quark-gluon plasma
- estimate of the entire hadronic spectrum

Gauge symmetry as an emergent phenomenon

Gauge bosons can appear as collective fluctuations of a strongly correlated many-body quantum system

F. Wegner, J. Math. Phys. (1971)
J.B. Kogut, Rev. Mod. Phys. (1979)
A. Kitaev, Ann. Phys. (2003)
P.A. Lee, N. Nagaosa, X.G. Wen,
Rev. Mod. Phys. (2006)

Gauge symmetry as an emergent phenomenon

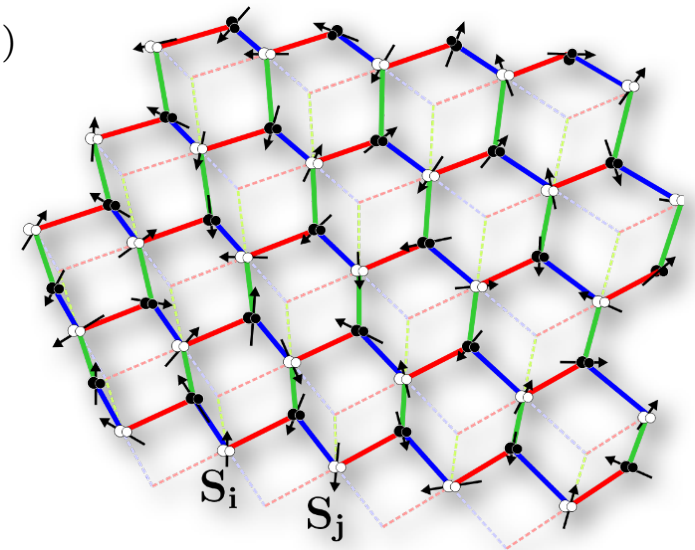
Gauge bosons can appear as collective fluctuations of a strongly correlated many-body quantum system

Ex.- Kitaev model (Z_2 gauge theory)

$$H = J_1 \sum_{1\langle i,j \rangle} S_i^{(1)} S_j^{(1)} + J_2 \sum_{2\langle i,j \rangle} S_i^{(2)} S_j^{(2)} + J_3 \sum_{3\langle i,j \rangle} S_i^{(3)} S_j^{(3)}$$
$$\rightarrow \frac{J_1^2 J_2^2}{16 |J_3|^3} \left(\sum_{\text{vertex}} XXXX + \sum_{\text{plaq}} ZZZZ \right)$$

$$|J_3| \gg \{|J_1|, |J_2|\}$$

F. Wegner, J. Math. Phys. (1971)
J.B. Kogut, Rev. Mod. Phys. (1979)
A. Kitaev, Ann. Phys. (2003)
P.A. Lee, N. Nagaosa, X.G. Wen, Rev. Mod. Phys. (2006)



Gauge symmetry as an emergent phenomenon

Some questions:

- (i) fractionalization,
- (ii) confinement-deconfinement quantum phase transition,
- (iii) spin-liquid physics...

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Volume 94B, number 2

PHYSICS LETTERS

28 July 1980

DYNAMICAL STABILITY OF LOCAL GAUGE SYMMETRY

Creation of Light From Chaos

D. FOERSTER H.B. NIELSEN M. NINOMIYA

And God said “Let there be light”, and there was light – Genesis 1–3

We show that the large distance behavior of gauge theories is stable, within certain limits, with respect to addition of gauge noninvariant interactions at small distances.

PHYSICAL REVIEW B **68**, 115413 (2003)

Artificial light and quantum order in systems of screened dipoles

Xiao-Gang Wen*

Gauge symmetry as a resource

Topological quantum computation:
Deconfined phases of gauge models
may have excitations with non-abelian
statistics and degenerate ground states.

A. Kitaev, Ann. Phys. (2003)

M.H. Freedman, A. Kitaev, M.J. Larsen, Z. Wang,
Bull. Amer. Math. Soc. (2003)

C. Nayak, S.H. Simon, A. Stern, M. Freedman, S. Das Sarma, Rev. Mod. Phys. (2008)

Some questions:

- (i) new materials,
- (ii) how to create and manipulate quasi-particles.

Why quantum simulate Gauge theories?

Problems not solvable
on a classical machines

**Various flavors of sign problems
in strongly correlated systems**

**Real time evolution:
Heavy ion experiments
(collisions)**

**Why quantum
simulate Gauge
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Why quantum simulate Gauge theories?

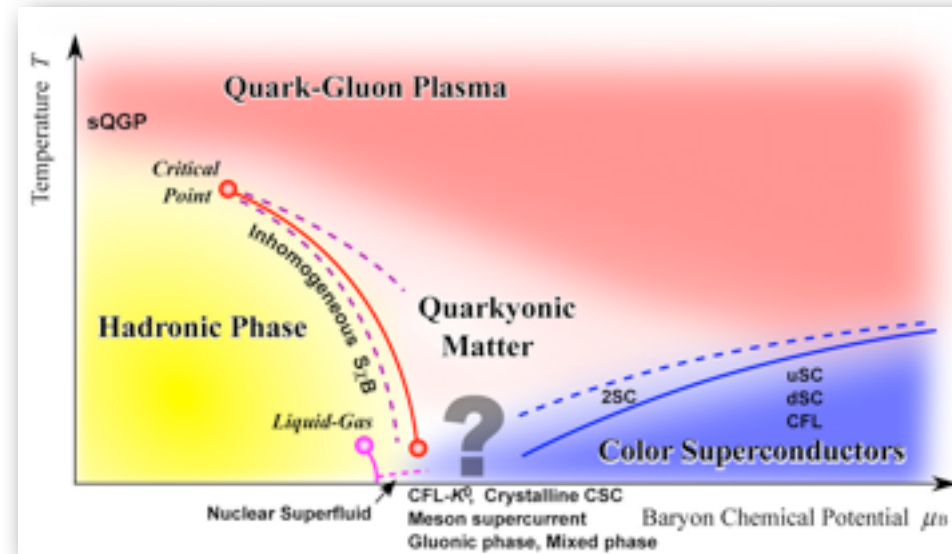
Real time evolution:
Heavy ion experiments
(collisions)

QCD with finite density of fermions:
Dense nuclear matter, color
superconductivity
(phase diagram of QCD)

S. Hands, Contemp. Phys. (2001)

M.G. Alford, A. Schmitt, K. Rajagopal, T. Schäfer,
Rev. Mod. Phys. (2008)

K. Fukushima, T. Hatsuda, Rep. Prog. Phys. (2011)



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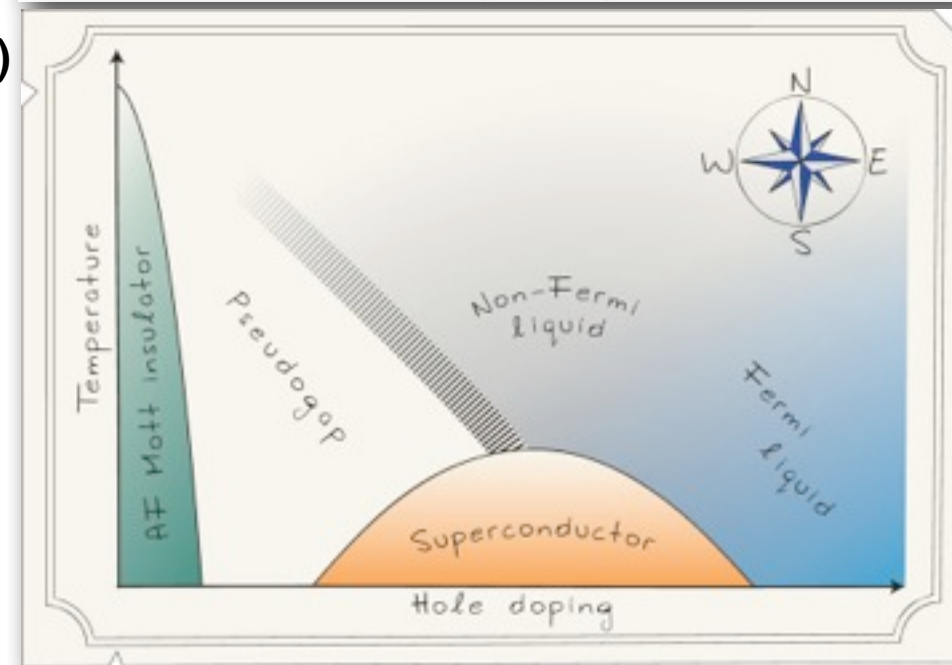
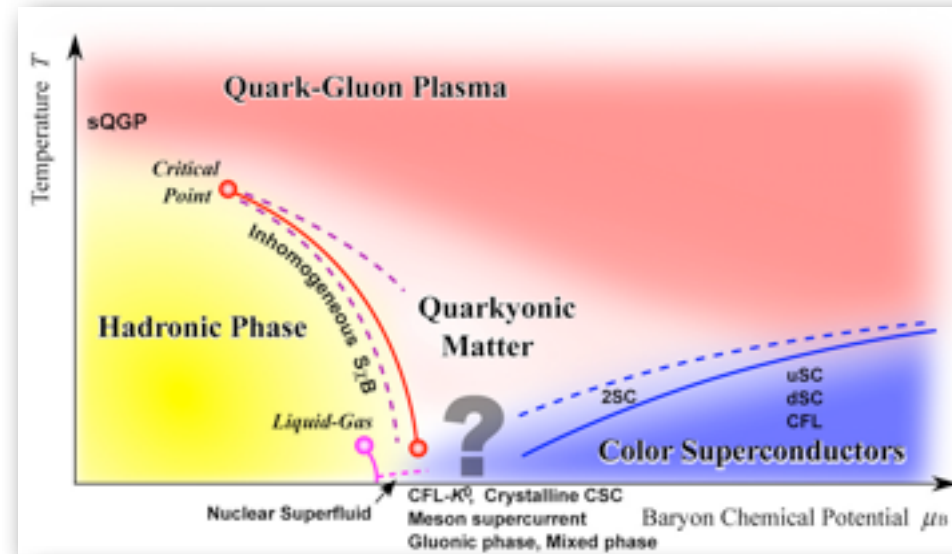
K. Fukushima, T. Hatsuda, Rep. Prog. Phys. (2011)

Frustrated spin models:
Spin liquid physics, RVB states
(High T_c superconductivity?)

E. Dagotto, Science (2005)

M.R. Norman, D. Pines, C. Kallin,
Adv. Phys. (2005)

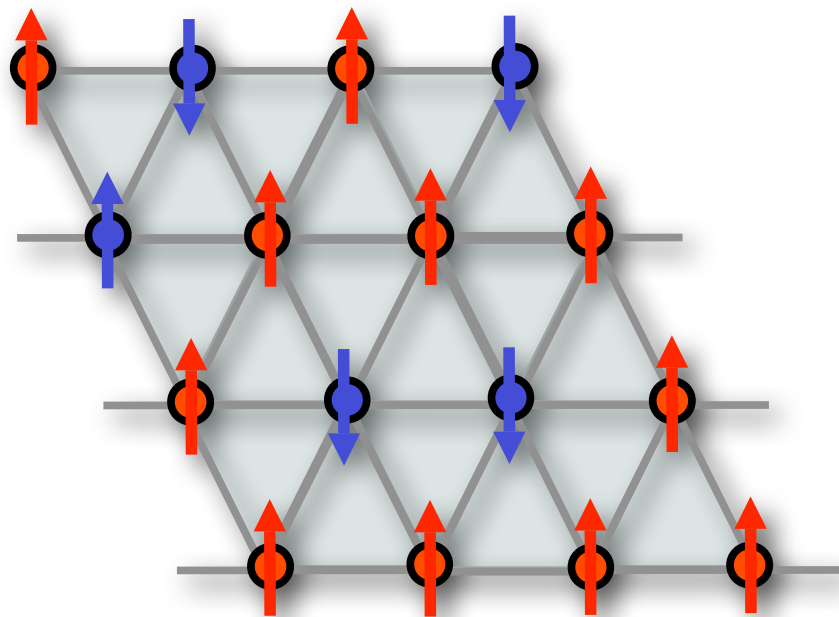
P. Wahl, Nat. Phys. (2012)



Why quantum simulate Gauge theories?

Why quantum simulate Gauge theories?

Feynman: “It is difficult to simulate quantum physics on a classical computer”



R.P. Feynman, Int. J. Theor. Phys.
(1982)

Entanglement

$$|\psi\rangle = c_1 |\uparrow\uparrow \cdots \uparrow\rangle + c_2 |\uparrow\uparrow \cdots \downarrow\rangle + \cdots + c_{2^N} |\downarrow\downarrow \cdots \downarrow\rangle$$

Huge

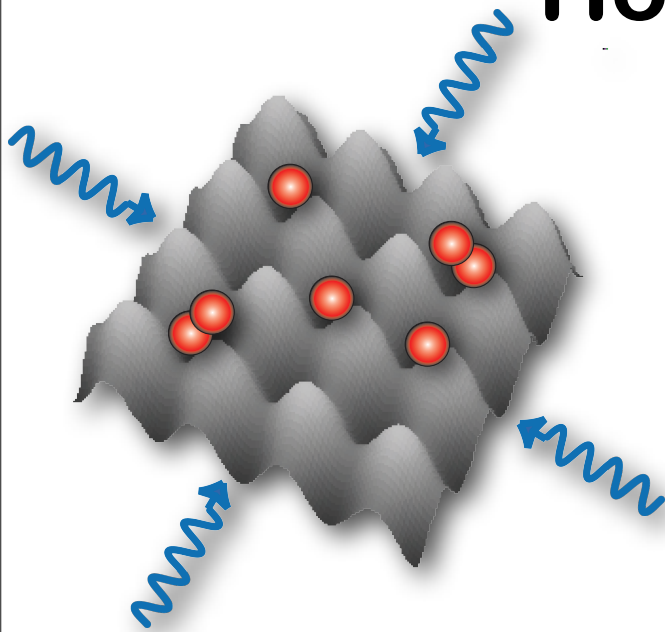
Why quantum simulate Gauge theories?

Feynman's universal quantum simulator:
controlled quantum device which
efficiently reproduces the dynamics of
any other many-particle quantum system.

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Feynman's universal quantum simulator:
controlled quantum device which
efficiently reproduces the dynamics of
any other many-particle quantum system.

How?... cold atoms, ions, photons,
superconducting circuit, etc.



J.I. Cirac, P. Zoller
I. Bloch, J. Dalibard, S. Nascimbène
R. Blatt, C.F. Roos,
A. Aspuru-Guzik, P. Walther
A.A. Hock, H.E. Türeci, J. Koch
Nature Physics Insight -
Quantum Simulation (2012)

Why quantum simulate Gauge theories?

NEED

Design a controlled microscopic quantum simulator for lattice gauge theories.

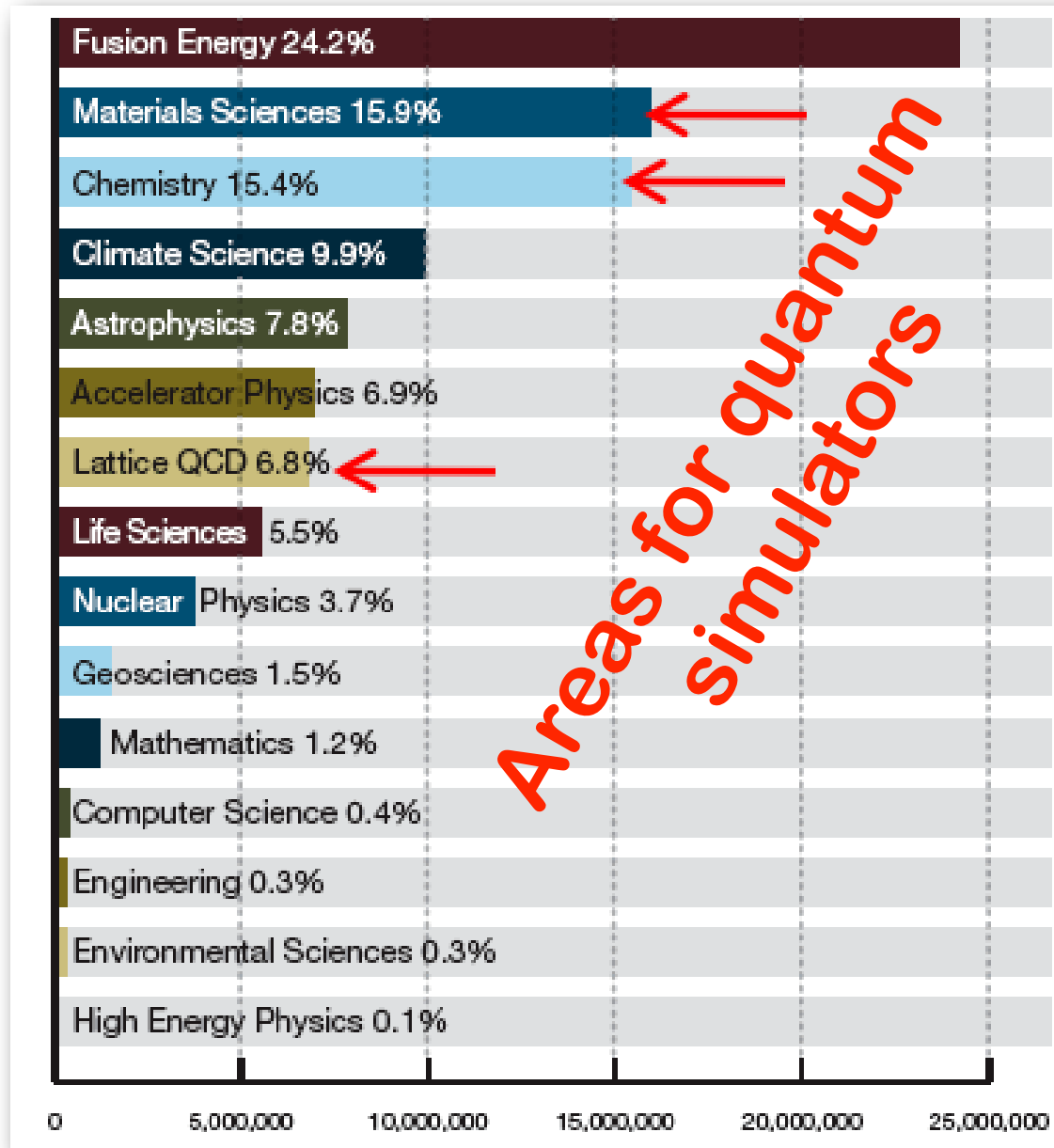
AIM

Investigate relevant phenomena, e.g., characterize the phase diagram and dynamics of strongly coupled lattice gauge models.

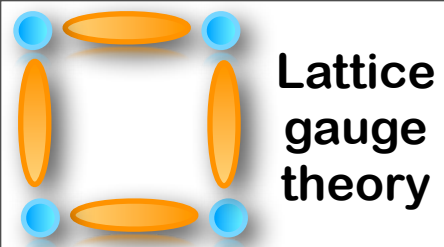
Why quantum simulate Gauge theories?

Use of DoE supercomputers by area


(from a talk by Alán Aspuru-Guzik)




Areas for quantum simulators

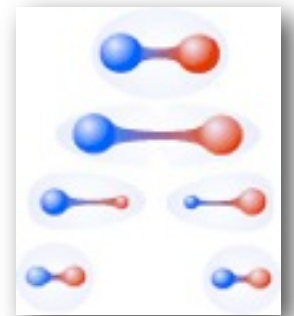


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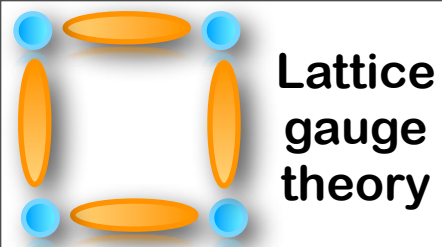
 ψ_x : fermion

 $U_{x,x+1}$: boson


- **Hamiltonian formulation of lattice gauge theories.** [degrees of freedom, symmetry generators, dynamics]




String breaking

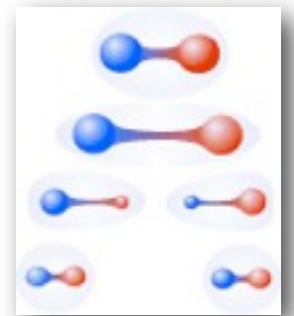


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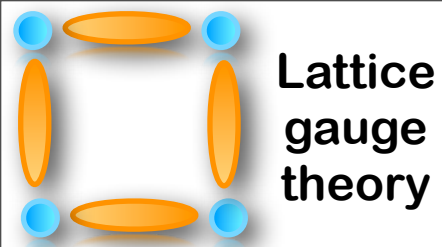
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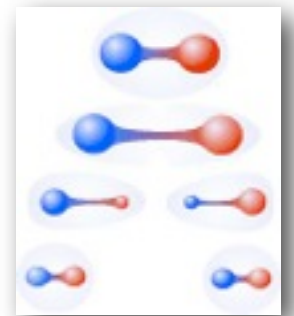


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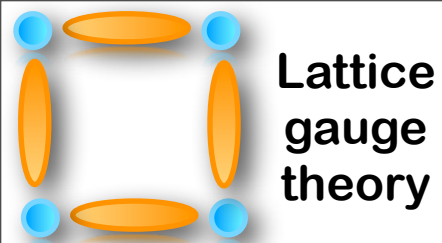
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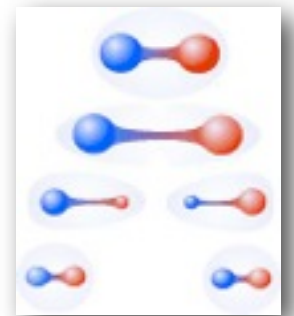


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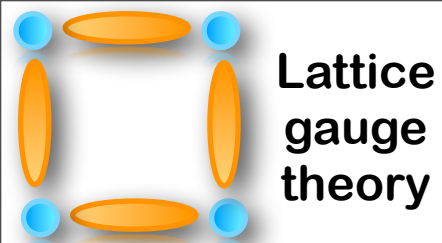
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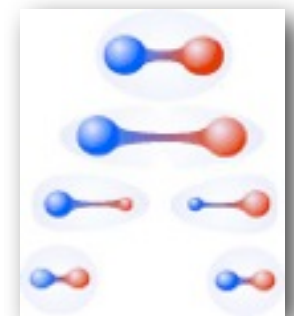


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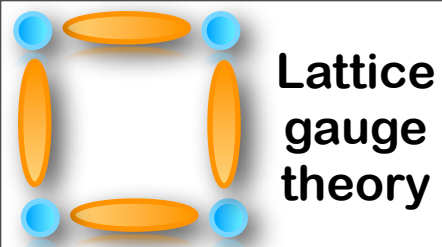
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- **Observability of interesting phenomena**




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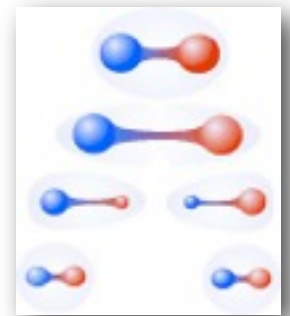


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- **Wilson lattice gauge formulation vs. Quantum link models.** [connections with condensed matter]
- **Phenomenology.** [confinement, string-breaking, quark-gluon plasma]
- **Implementation of quantum link models.** [Bose-fermi mixture, fermionic alkaline earth atoms, superconducting q-bits]
- **Observability of interesting phenomena**
- **Conclusions & Outlook**



String breaking

Hamiltonian formulation of lattice gauge theories

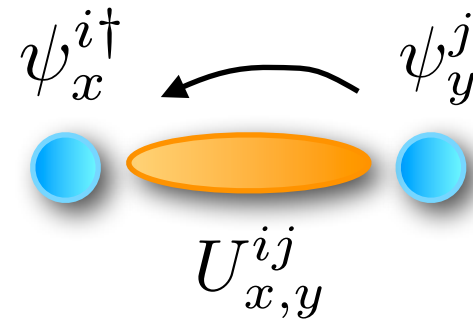
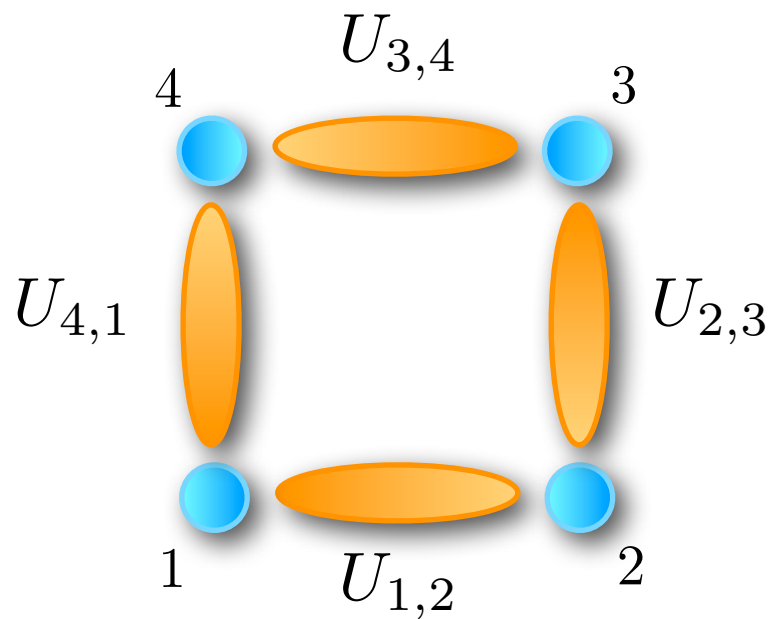
A gauge invariant model is defined by:

Set of local dynamical operators acting on the
vertices (matter fields) and on the links (gauge fields)

Hamiltonian formulation of lattice gauge theories

A gauge invariant model is defined by:

Set of local dynamical operators acting on the vertices (matter fields) and on the links (gauge fields)



(operator acting on a Hilbert space)

J.B. Kogut, L. Susskind, PRD (1975)
 ref. Creutz and Montvay/Muenster books
 J.B. Kogut, Rev. Mod. Phys. (1979)

$$i = \begin{cases} 1 & : U(1) \\ \uparrow\downarrow & : U(2) \\ brg & : U(3) \end{cases}$$

Hamiltonian formulation of lattice gauge theories

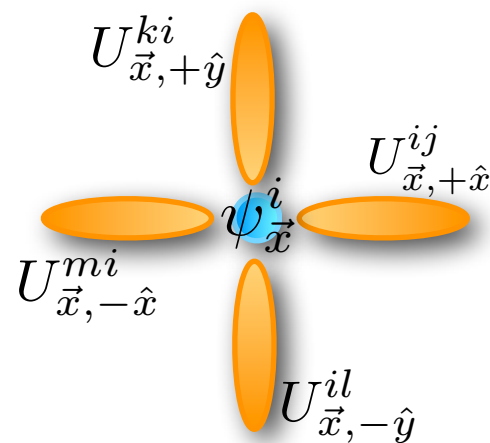
Gauge invariant quantum Hamiltonian:

$$\left[H, \vec{G} \right] = 0 \quad \forall x$$

Local conserved quantities
Gauge (local) symmetries

Hamiltonian formulation of lattice gauge theories

Set of local generators of gauge transformations



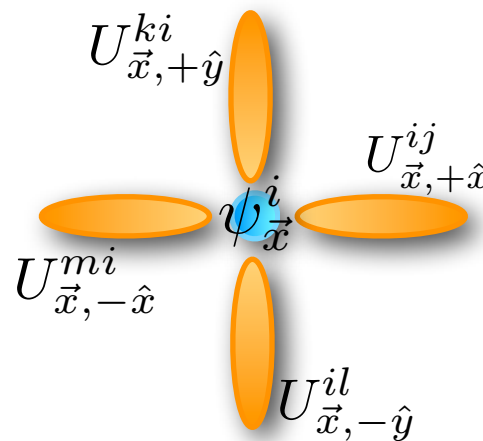
Hamiltonian formulation of lattice gauge theories

Set of local generators of gauge transformations

Generators of the local symmetry:

$$e^{i \sum_z \vec{\theta}_z \vec{G}_z} \psi_x^i e^{-i \sum_z \vec{\theta}_z \vec{G}_z} = \sum_j \Omega_x^{ij} \psi_x^j$$

$$e^{i \sum_z \vec{\theta}_z \vec{G}_z} U_{x,y}^{ij} e^{-i \sum_z \vec{\theta}_z \vec{G}_z} = \sum_{k,l} \Omega_x^{ik} U_{x,y}^{kl} \Omega_y^{jl*}$$



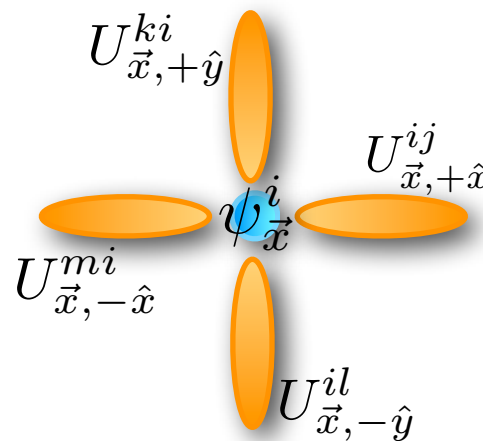
Hamiltonian formulation of lattice gauge theories

Set of local generators of gauge transformations

Define the Hilbert space:

$$\vec{G}_x |\text{physical}\rangle = 0$$

$$\sum_x \left(\vec{G}_x \right)^2 = \begin{pmatrix} \begin{bmatrix} 0 \end{bmatrix} & & & \\ & \begin{bmatrix} 1 \end{bmatrix} & & \\ & & \ddots & \\ & & & \ddots \end{pmatrix}$$



Block-diagonal Hilbert space

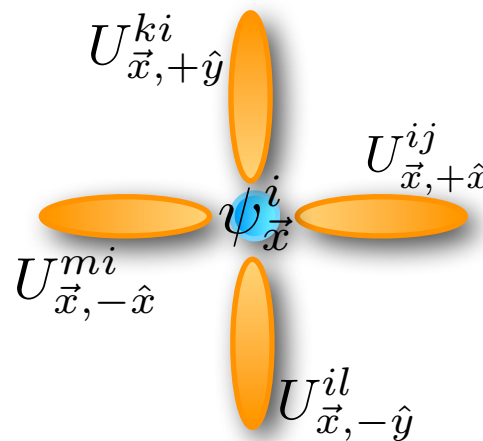
Hamiltonian formulation of lattice gauge theories

Set of local generators of gauge transformations

ex.- U(1) group

$$G_x = \underbrace{\psi_x^\dagger \psi_x}_{\text{matter}} - \sum_{\hat{\mu}} \underbrace{E_{x, x+\hat{\mu}} - E_{x-\hat{\mu}, x}}_{\text{electric field}}$$

$$\left[\rho - \vec{\nabla} \cdot \vec{E} \right]_{\text{phys}} = 0 \quad : \text{Gauss' law}$$



Wilson formulation: continuum valued operator, infinite-dimensional local Hilbert space

ex.- U(1) group

$$U_{x,y} \rightarrow e^{i\phi_{x,y}} \quad E_{x,y} \rightarrow -i \frac{\partial}{\partial \phi_{x,y}}$$

Implementation in AMO setup very challenging

E. Kapit, E. Mueller, Phys. Rev. A (2011)
E. Zohar, B. Reznik, Phys. Rev. Lett. (2011)

Quantum link formulation: gauge fields span a finite- dimensional local Hilbert space

D. Horn, Phys. Lett. B (1981)

P. Orland, D. Röhrlich, Nucl. Phys. B (1990)

S. Chandrasekharan, U.-J. Wiese, Nucl. Phys. B (1997)

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S. Chandrasekharan, U.-J. Wiese, Nucl. Phys. B (1997)

Q.C.D. can be formulated as a non-abelian quantum link model

R. Brower, S. Chandrasekharan, U.-J. Wiese, Phys. Rev. D (1999)

R. Brower, S. Chandrasekharan, S. Riederer, U.-J. Wiese, Nucl. Phys. B (2004)

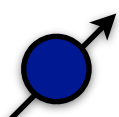
Quantum Link models

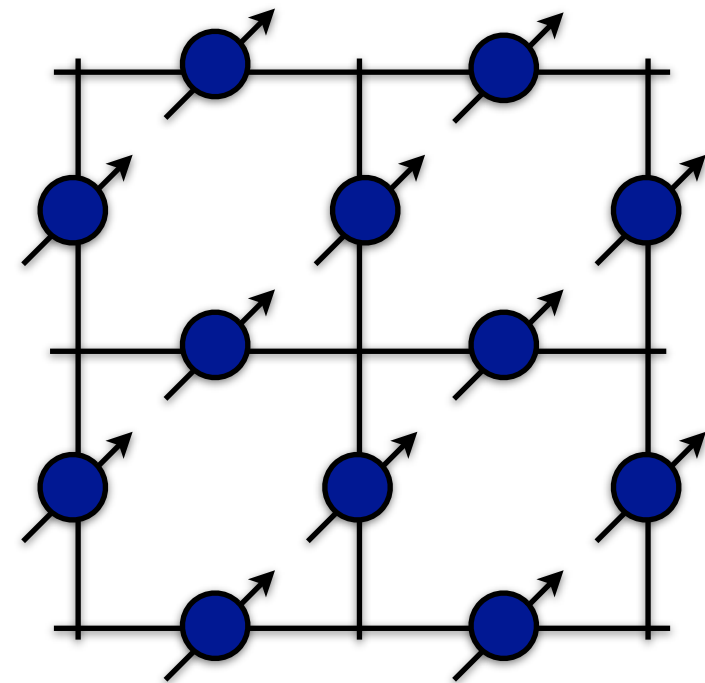
Connections with Condensed Matter

(U(1) gauge theory-Quantum Spin Ice model)

Local degrees of freedom.-

Quantum two level system living on the link


$$\{ \sigma_{x,y}^{(3)}, \sigma_{x,y}^{(+)}, \sigma_{x,y}^{(-)} \}$$



L. Balents, Nature (2010)

C. L. Henley, Ann. Rev. Cond. Matt. Phys. (2010)

C. Castelnovo, R. Moessner, and S.L. Sondhi,
Ann. Rev. Cond. Matt. Phys. (2012)

Quantum Link models

Connections with Condensed Matter

(U(1) gauge theory-Quantum Spin Ice model)

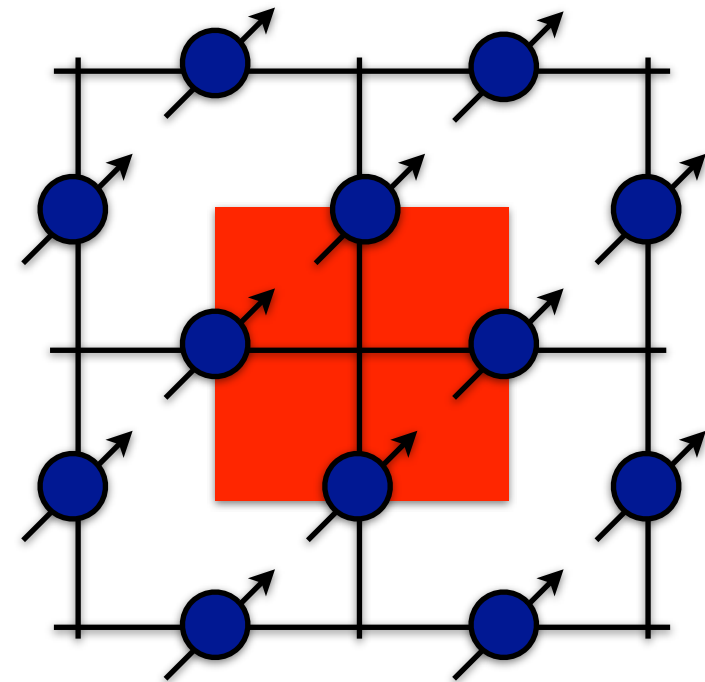
Local generator of gauge transformations.-

Local generator around every vertex

$$\exp \left[i \frac{\theta_{\text{vert}}}{2} G_{\text{vert}} \right] \sigma_{1,2}^{(+)} \exp \left[-i \frac{\theta_{\text{vert}}}{2} G_{\text{vert}} \right] = e^{i\theta_{\text{vert}}} \sigma_{1,2}^{(+)}$$

$$G_{\text{vert}} = \sigma_{1,2}^{(3)} + \sigma_{2,3}^{(3)} + \sigma_{3,4}^{(3)} + \sigma_{4,1}^{(3)}$$

U(1) gauge transformation



Quantum Link models

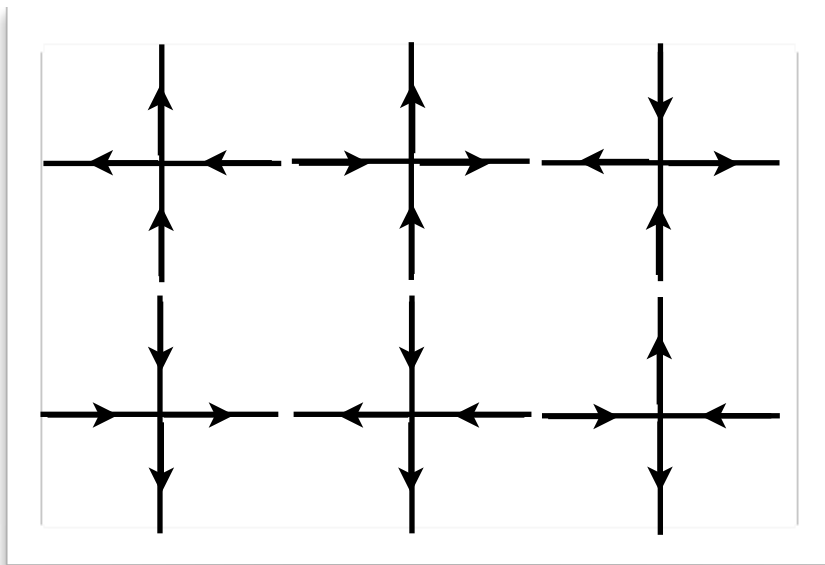
Connections with Condensed Matter

(U(1) gauge theory-Quantum Spin Ice model)

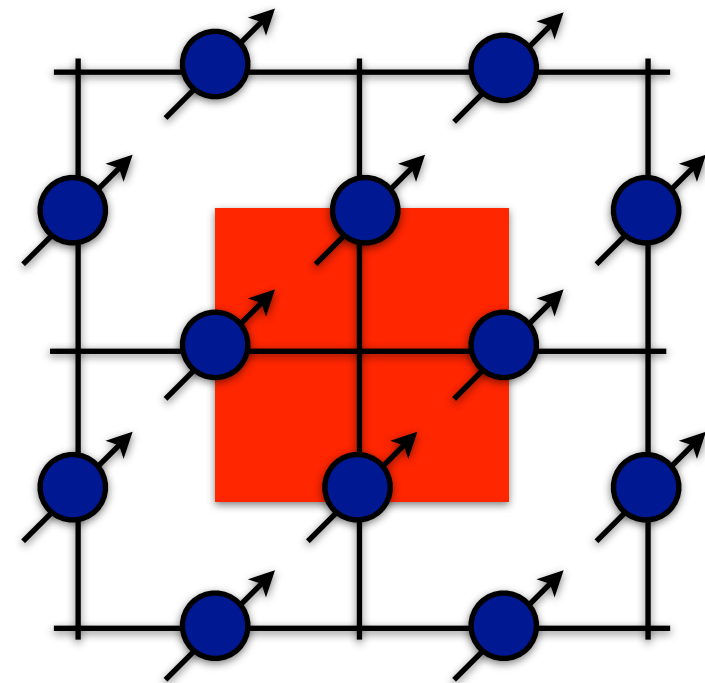
Local generator of gauge transformations.-

“Physical” Hilbert space (Gauss’ law)

$$G_{\text{vert}} |\text{phys}\rangle = 0$$



6-vertex model:
zero magnetization subspace



Quantum Link models

Connections with Condensed Matter

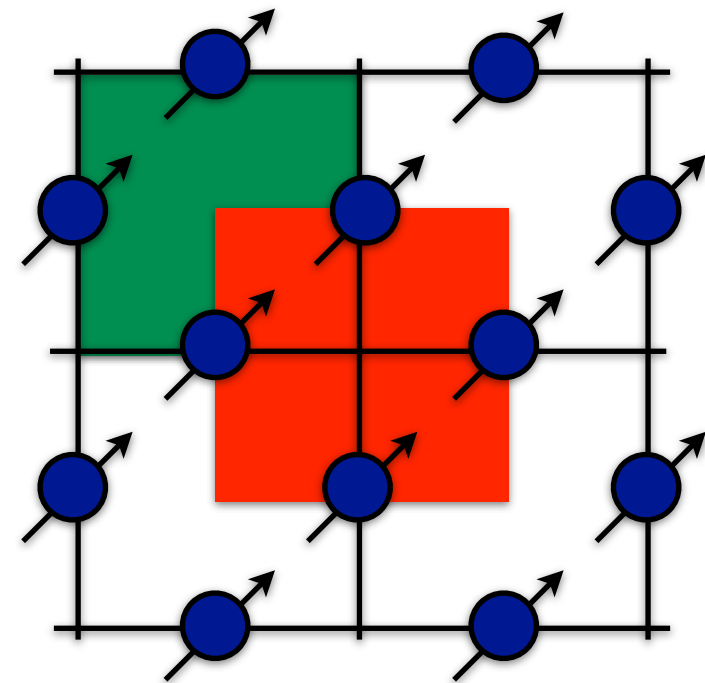
(U(1) gauge theory-Quantum Spin Ice model)

Gauge invariant Hamiltonian.-

$$H = - \sum_{\text{plaq}} \left[\sigma_{1,2}^+ \sigma_{2,3}^- \sigma_{3,4}^+ \sigma_{4,1}^- + \sigma_{1,2}^- \sigma_{2,3}^+ \sigma_{3,4}^- \sigma_{4,1}^+ \right]$$

magnetic term

$$[H, G_{\text{vert}}] = 0, \quad \forall \text{ vertex}$$



Quantum Link models

Local degrees of freedom.-

Quantum link carrying an electric flux

$$U_{x,y} \equiv S_{x,y}^+$$

$$E_{x,y} \equiv S_{x,y}^{(3)}$$

	Spin-1/2:	Spin-1:	
$E=1/2$	\rightarrow	$\rightarrow\rightarrow$	$E=+1$
$E=-1/2$	\leftarrow	$\leftarrow\leftarrow$	$E=-1$
		---	$E=0, \text{ no flux}$

T. Calarco, M. Dalmonte, S. Montangero, T. Pichler,
E. Rico, P. Zoller (in preparation-2013)

Quantum Link models

Gauge invariant Hamiltonian.-

$$H = \frac{g^2}{2} \sum_{\langle x,y \rangle} [E_{x,y}]^2 - \frac{1}{4g^2} \sum_{\text{plaq}} \left[U_{1,2}^\dagger U_{2,3} U_{3,4}^\dagger U_{4,1} + U_{1,2} U_{2,3}^\dagger U_{3,4} U_{4,1}^\dagger \right]$$

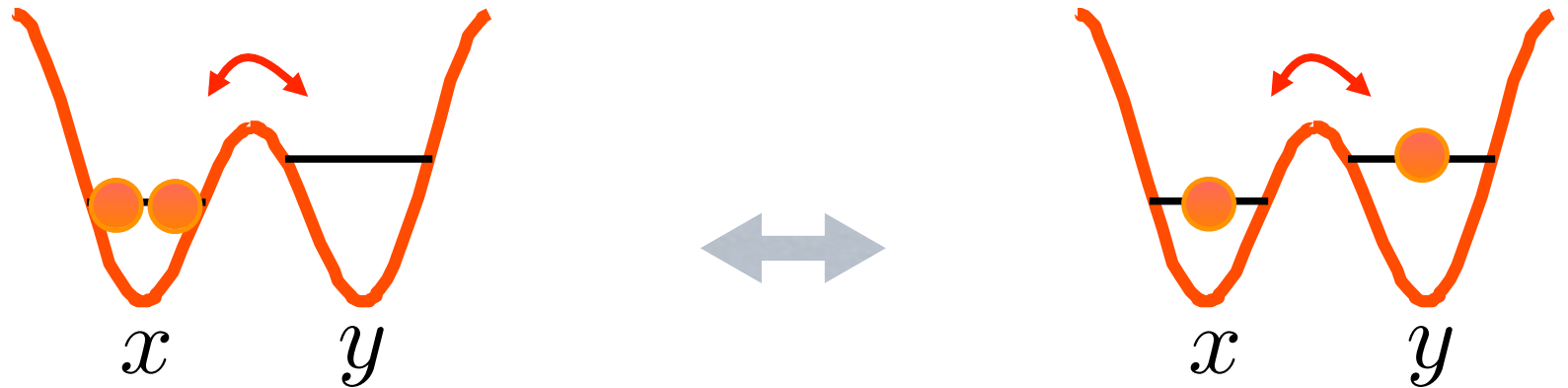
$S_{x,y}^{(3)}$ $S_{x,y}^-$ $S_{x,y}^+$

↓ ↓ ↓

Electric term **Magnetic term**

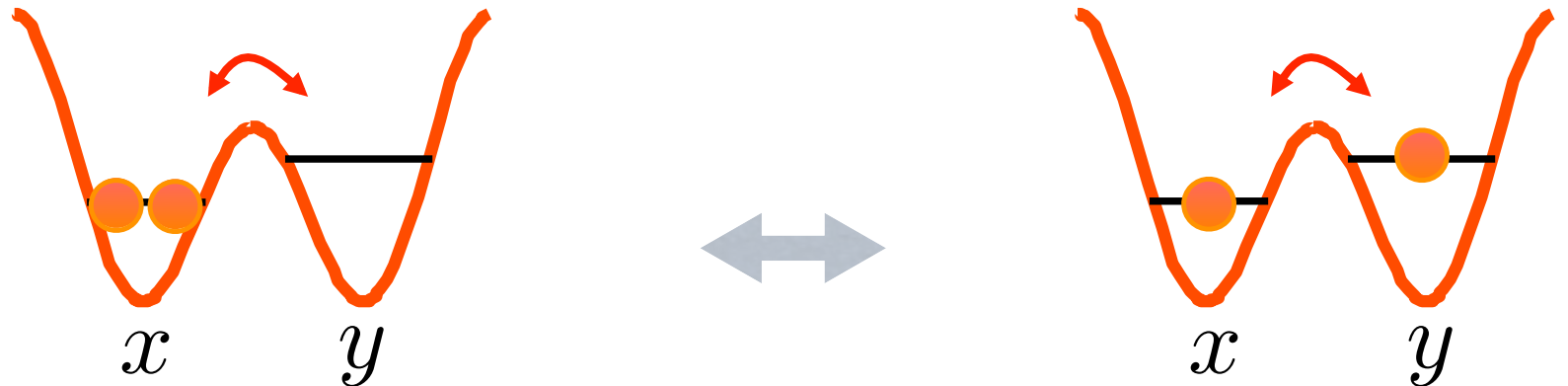
Quantum Link models

Rishon (Schwinger) representation



Quantum Link models

Rishon (Schwinger) representation



Link operator $U_{x,y} \equiv S_{x,y}^+ = c_y^\dagger c_x$

Electric field
[U(1) generator] $E_{x,y} \equiv S_{x,y}^{(3)} = \frac{1}{2} [c_y^\dagger c_y - c_x^\dagger c_x]$

$$\{c_x, c_y^\dagger\} = \delta_{x,y}$$

Schwinger fermions (rishons)

$$[c_x, c_y^\dagger] = \delta_{x,y}$$

Schwinger bosons

Quantum Link models

Rishon (Schwinger) representation

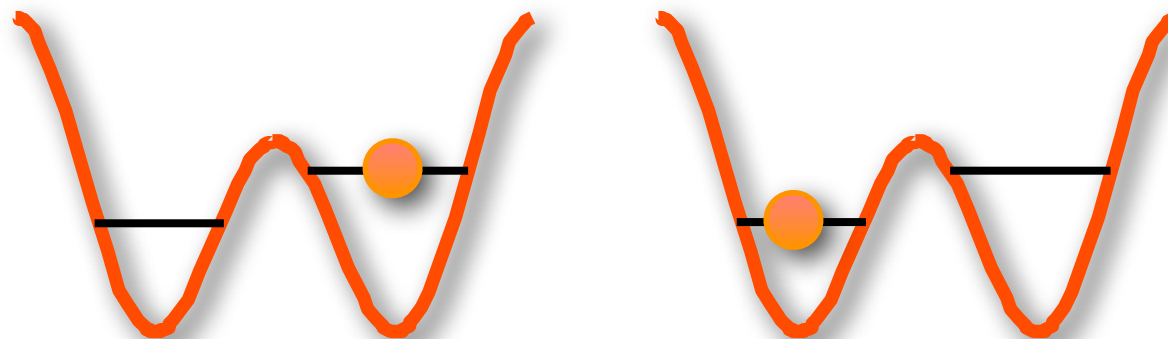
Spin representation:

$$N_{x,y} = c_y^\dagger c_y + c_x^\dagger c_x \quad \left[\vec{S}_{x,y} \right]^2 \equiv \frac{N_{x,y}}{2} \left[\frac{N_{x,y}}{2} + 1 \right]$$

Spin-1/2:

$E=1/2 \rightarrow$

$E=-1/2 \leftarrow$



Quantum Link models

Rishon (Schwinger) representation

Spin representation:

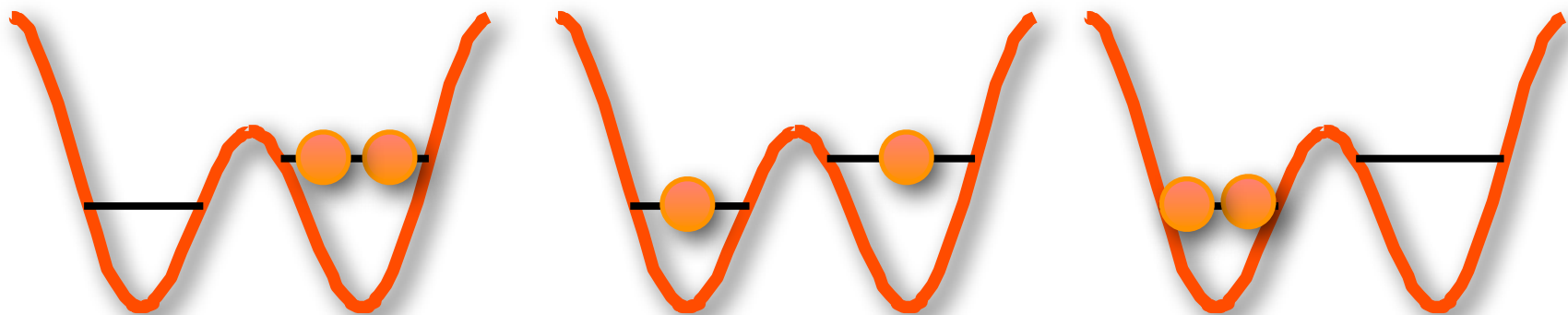
$$N_{x,y} = c_y^\dagger c_y + c_x^\dagger c_x \quad \left[\vec{S}_{x,y} \right]^2 \equiv \frac{N_{x,y}}{2} \left[\frac{N_{x,y}}{2} + 1 \right]$$

Spin-1:

$E=+1 \rightarrow$

$E=0 \text{ — }$

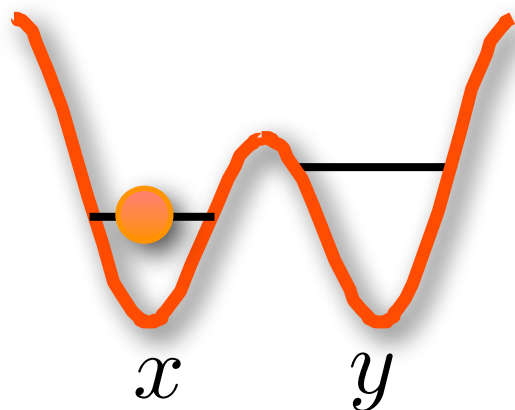
$E=-1 \leftarrow$



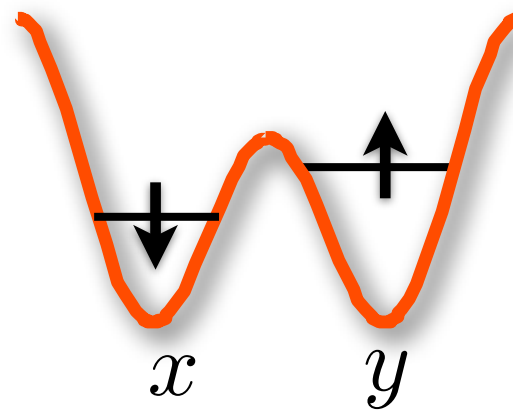
Non-abelian quantum link models

Rishon (Schwinger) representation with internal degrees of freedom

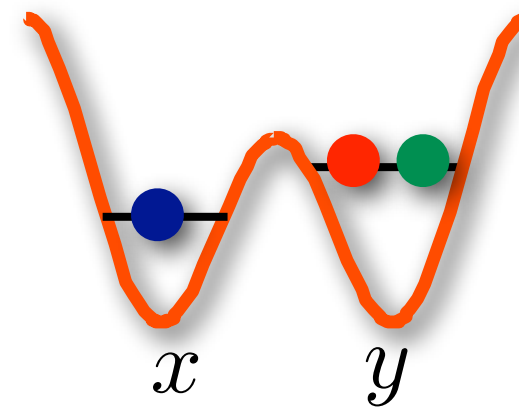
$$i = \begin{cases} 1 & : U(1) \\ \uparrow\downarrow & : U(2) \\ \text{brg} & : U(3) \end{cases}$$



U(1) group



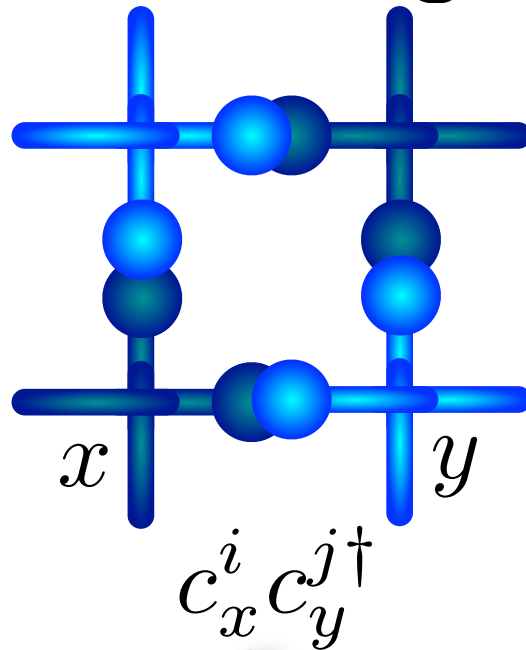
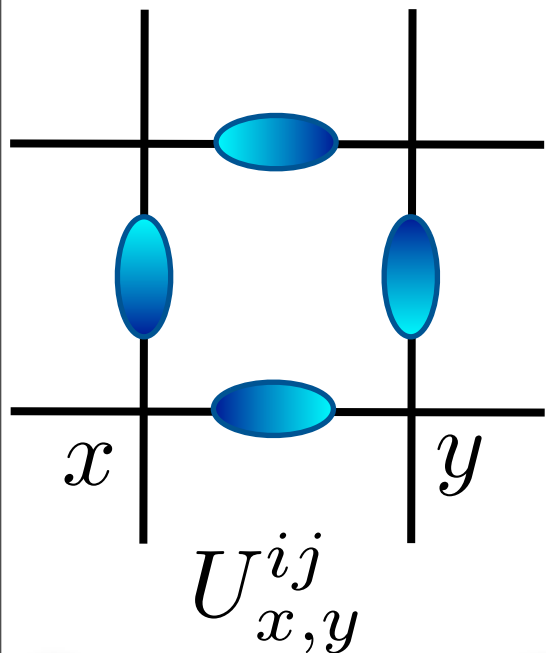
U(2) group



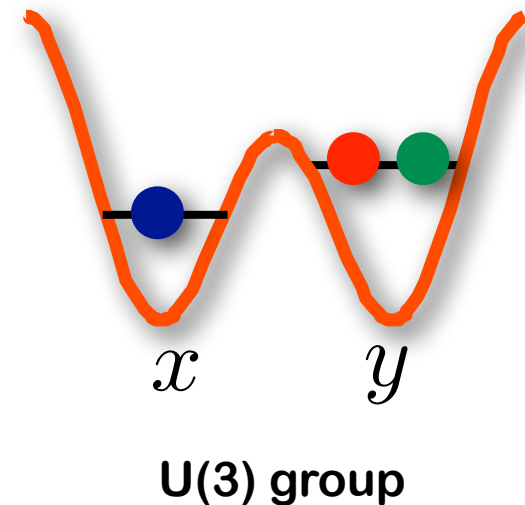
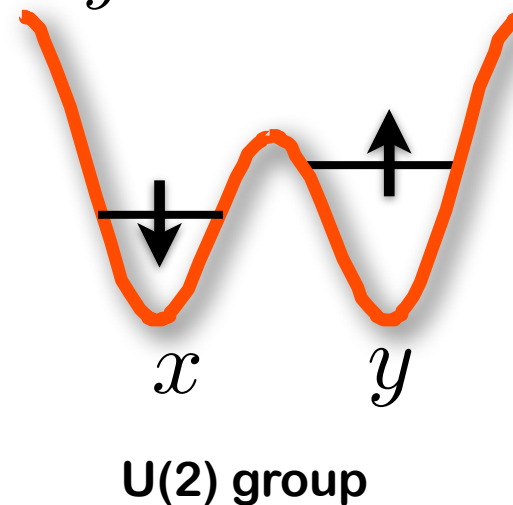
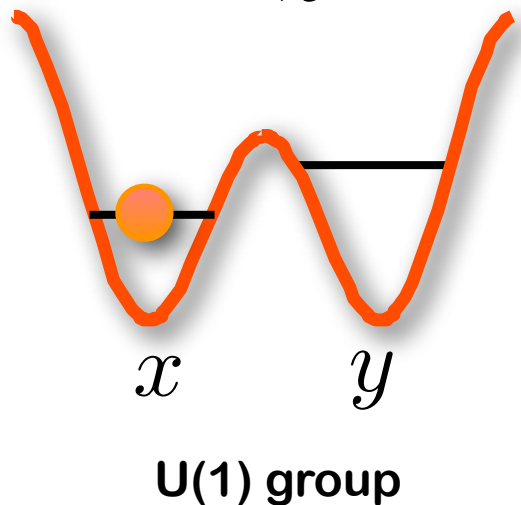
U(3) group

Non-abelian quantum link models

Rishon (Schwinger) representation with internal degrees of freedom



$$i = \begin{cases} 1 & : U(1) \\ \uparrow\downarrow & : U(2) \\ brg & : U(3) \end{cases}$$



Non-abelian quantum link models

Rishon (Schwinger) representation with internal degrees of freedom

Local degrees of freedom.-

Link operator

$$U_{x,y}^{ij} \equiv c_x^i c_y^{j\dagger}$$

Electric field [U(1) generator]

$$E_{x,y} \equiv \frac{1}{2} [c_y^{i\dagger} c_y^i - c_x^{i\dagger} c_x^i]$$

Representation [occupation]

$$N_{x,y} = c_y^{i\dagger} c_y^i + c_x^{i\dagger} c_x^i$$

Non-abelian quantum link models

Rishon (Schwinger) representation with internal degrees of freedom

Local degrees of freedom.-

Non-abelian electric fields [SU(N) generators]

Left generators

$$L_{x,y}^a = c_x^{i\dagger} \lambda_{ij}^a c_x^i$$

Right generators

$$R_{x,y}^a = c_y^{i\dagger} \lambda_{ij}^a c_y^i$$

$$\exp \left[i\theta_x^a L_{x,y}^a \right] U_{x,y} \exp \left[-i\theta_x^a L_{x,y}^a \right] = \exp \left[-i\theta_x^a \lambda^a \right] U_{x,y}$$

$$\exp \left[i\theta_y^a R_{x,y}^a \right] U_{x,y} \exp \left[-i\theta_y^a R_{x,y}^a \right] = U_{x,y} \exp \left[i\theta_y^a \lambda^a \right]$$

Non-abelian quantum link models

Rishon (Schwinger) representation
with internal degrees of freedom

Local generators.-

$$G_x = - \sum_k \left(E_{x, x+\hat{k}} - E_{x-\hat{k}, x} \right)$$

$$G_x^a = \sum_k \left(L_{x, x+\hat{k}}^a + R_{x-\hat{k}, x}^a \right)$$

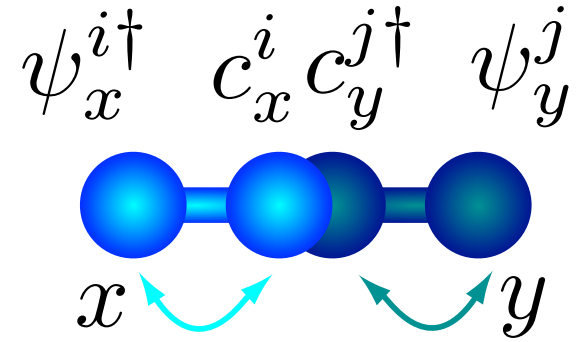
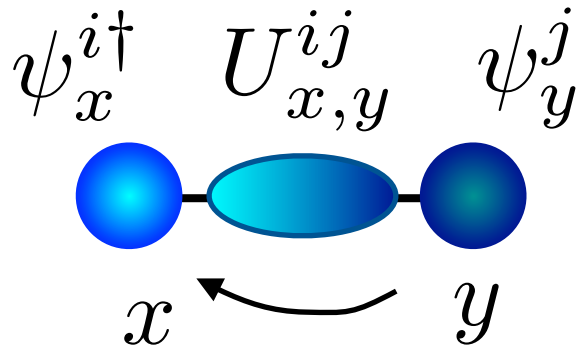
Non-abelian quantum link models

Rishon (Schwinger) representation
with internal degrees of freedom

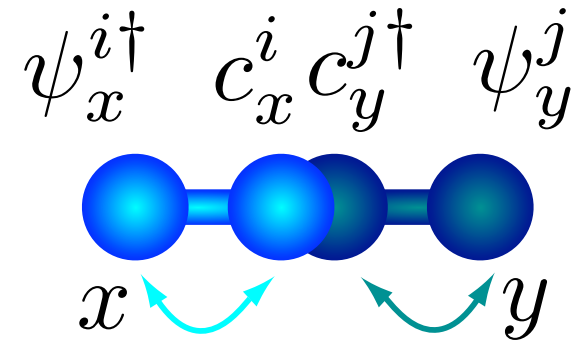
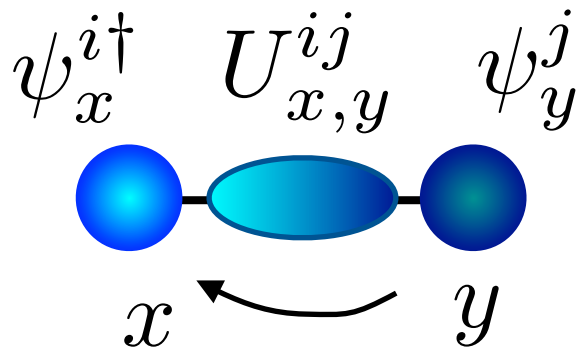
Hamiltonian.-

$$H = \frac{g'^2}{2} \sum_{\langle x,y \rangle} (E_{x,y})^2 + \frac{g^2}{2} \sum_{\langle x,y \rangle} \left[\left(\vec{L}_{x,y} \right)^2 + \left(\vec{R}_{x,y} \right)^2 \right] - \frac{1}{4g^2} \sum_{\text{plaq}} \left[U_{1,2}^\dagger U_{2,3} U_{3,4}^\dagger U_{4,1} + U_{1,2} U_{2,3}^\dagger U_{3,4} U_{4,1}^\dagger \right]$$

Non-abelian quantum link models with matter



Non-abelian quantum link models with matter



$$H = -t \sum_{\langle x,y \rangle, i,j} (\psi_x^{i\dagger} U_{x,y}^{ij} \psi_y^j + \text{h.c.}) + \dots$$

$$= -t \sum_{\langle x,y \rangle} \left[\left(\sum_i \psi_x^{i\dagger} c_x^i \right) \left(\sum_j c_y^{j\dagger} \psi_y^j \right) + \text{h.c.} \right] + \dots$$

Matter - gauge interaction
= hopping of fermions mediated by a quantum link
= correlated hopping of fermions and rishons

Non-abelian quantum link models with matter

Local generators.-

U(1) generator

$$G_x = \psi_x^{i\dagger} \psi_x^i - \sum_k \left(E_{x, x+\hat{k}} - E_{x-\hat{k}, x} \right)$$

SU(N) generator

$$G_x^a = \psi_x^{i\dagger} \lambda_{ij}^a \psi_x^j + \sum_k \left(L_{x, x+\hat{k}}^a + R_{x-\hat{k}, x}^a \right)$$

“Physical” Hilbert space

$$\vec{G}_x |\text{phys}\rangle = 0 \quad \forall x$$

Non-abelian quantum link models with matter

Hamiltonian.-

$$[H, G_x] = [H, G_x^a] = 0 \quad \forall x$$

Non-abelian quantum link models

Strong coupling Hamiltonian with staggered fermions

$$H = \frac{g'^2}{2} \sum_{\langle x,y \rangle} (E_{x,y})^2 + \frac{g^2}{2} \sum_{\langle x,y \rangle} \left[\left(\vec{L}_{x,y} \right)^2 + \left(\vec{R}_{x,y} \right)^2 \right] - t \sum_{\langle x,y \rangle, i,j} \left(\psi_x^{i\dagger} U_{x,y}^{ij} \psi_y^j + \text{h.c.} \right) + m \sum_{x,i} (-1)^x \psi_x^{i\dagger} \psi_x^i$$

Electric field

Non-abelian
electric field

Matter-gauge
interaction

Staggered mass

Non-abelian quantum link models

Strong coupling Hamiltonian with staggered fermions

$$H = \frac{g'^2}{2} \sum_{\langle x,y \rangle} (E_{x,y})^2 + \frac{g^2}{2} \sum_{\langle x,y \rangle} \left[(\vec{L}_{x,y})^2 + (\vec{R}_{x,y})^2 \right] - t \sum_{\langle x,y \rangle, i, j} (\psi_x^{i\dagger} U_{x,y}^{ij} \psi_y^j + \text{h.c.}) + m \sum_{x,i} (-1)^x \psi_x^{i\dagger} \psi_x^i$$

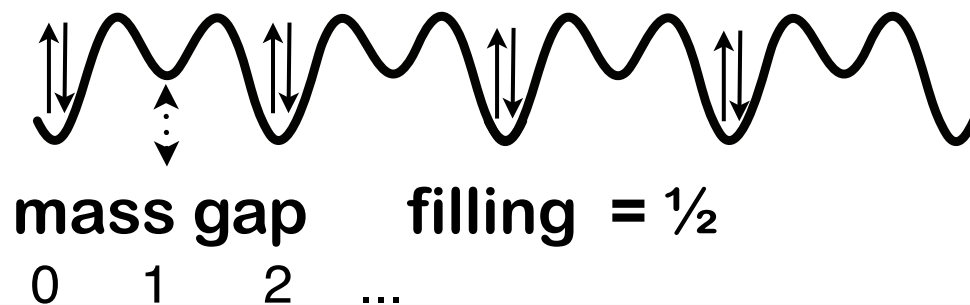
Electric field

Non-abelian
electric field

Matter-gauge
interaction

Staggered mass

Staggered fermions:



energy

empty

mass gap

filled
fermi sea

Phenomenology

Confinement and string breaking: QED in 1+1D (Schwinger model)

Gauss' law $G_x = \psi_x^\dagger \psi_x + \frac{(-1)^x - 1}{2} - (E_{x,x+1} - E_{x-1,x})$

$$G_x |\text{phys}\rangle = 0 \Leftrightarrow \rho - \vec{\nabla} \cdot \vec{E} = 0$$

Spin-1 representation

$|0\rangle$ ○

$|1\rangle$ ●

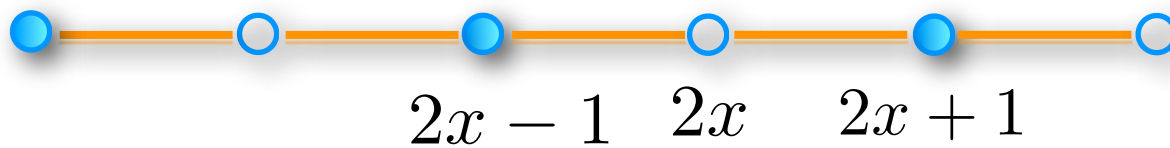


Phenomenology

Confinement and string breaking: QED in 1+1D (Schwinger model)

Vacuum state

$$H = \frac{g^2}{2} \sum_{\langle x,y \rangle} \left(S_{x,y}^{(3)} \right)^2 + m \sum_x (-1)^x \psi_x^\dagger \psi_x$$



Creating a quark - antiquark pair:

$$\psi_{2x}^\dagger S_{2x,2x+1}^+ \psi_{2x+1}$$

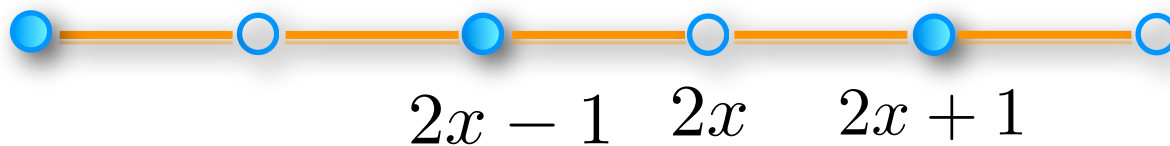


Phenomenology

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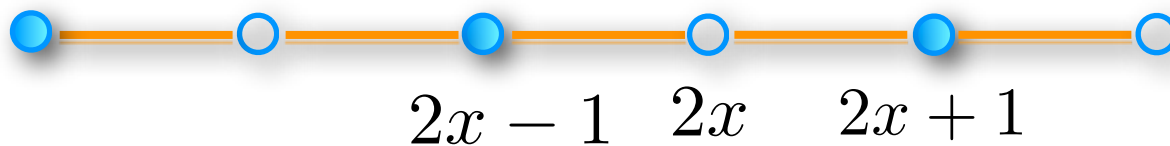


Phenomenology

Confinement and string breaking: QED in 1+1D (Schwinger model)

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Phenomenology

**Confinement and string breaking:
QED in 1+1D (Schwinger model)**

Phenomenology

Confinement and string breaking: QED in 1+1D (Schwinger model)



Phenomenology

Confinement and string breaking: QED in 1+1D (Schwinger model)



Confinement

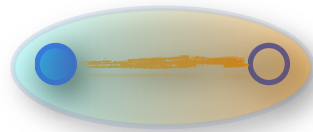
$$E_{\text{string}} - E_0 = \frac{g^2}{2} (L - 1)$$

Phenomenology

**Confinement and string breaking:
QED in 1+1D (Schwinger model)**

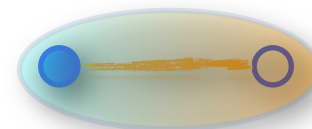
Phenomenology

Confinement and string breaking: QED in 1+1D (Schwinger model)



meson

**String breaking
and hadronization**

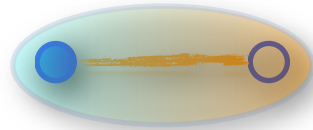


meson

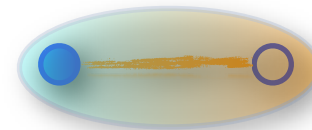
$$E_{\text{meson}} - E_0 = g^2 + 2m$$

Phenomenology

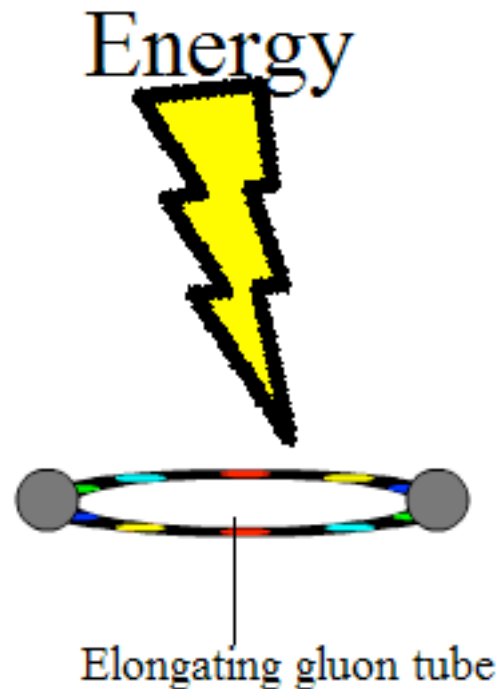
Confinement and string breaking: QED in 1+1D (Schwinger model)



meson



meson



$$L_c = 3 + \frac{4m}{g^2}$$

Quantum Chromodynamics: Confinement under normal conditions

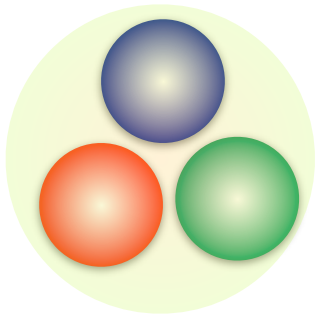
Quarks and gluons carry a color charge ψ_x^i $i =$ 

Quantum Chromodynamics: Confinement under normal conditions

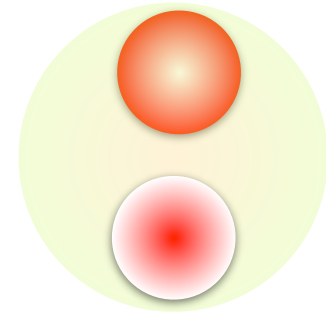
Quarks and gluons carry a color charge ψ_x^i $i =$   

Quarks are confined into color-neutral (color singlet)
bound states (hadrons)

qqq baryons: proton, neutron, ...



$q\bar{q}$ mesons: pions (lightest), kaon, rho, ...

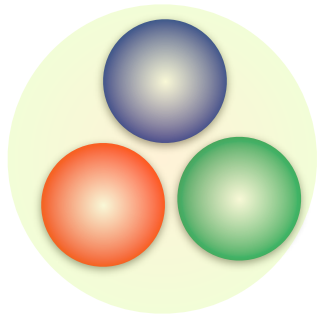


Quantum Chromodynamics: Confinement under normal conditions

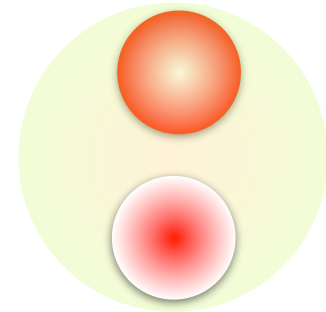
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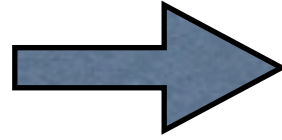


Quarks interact by exchanging gluons

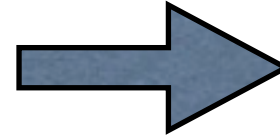
$$\psi_x^{i\dagger} U_{x,y}^{ij} \psi_y^j$$

QCD under extreme conditions

Compress or
heat baryons

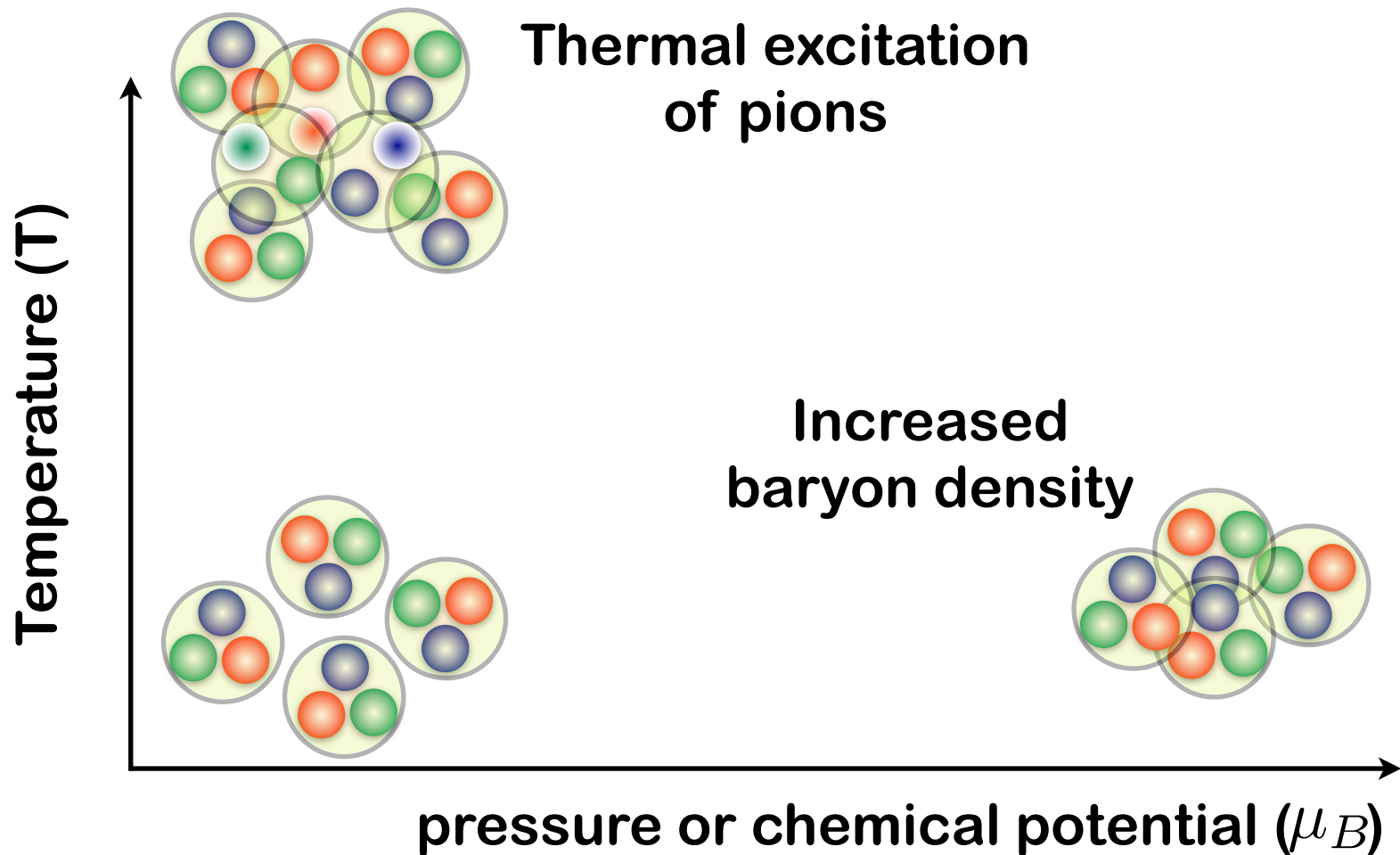


Hadrons
overlap



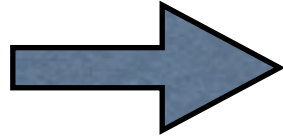
Confinement
is "lost"

Expect interesting/unusual behaviour

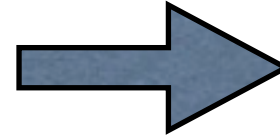


QCD under extreme conditions

Compress or heat baryons

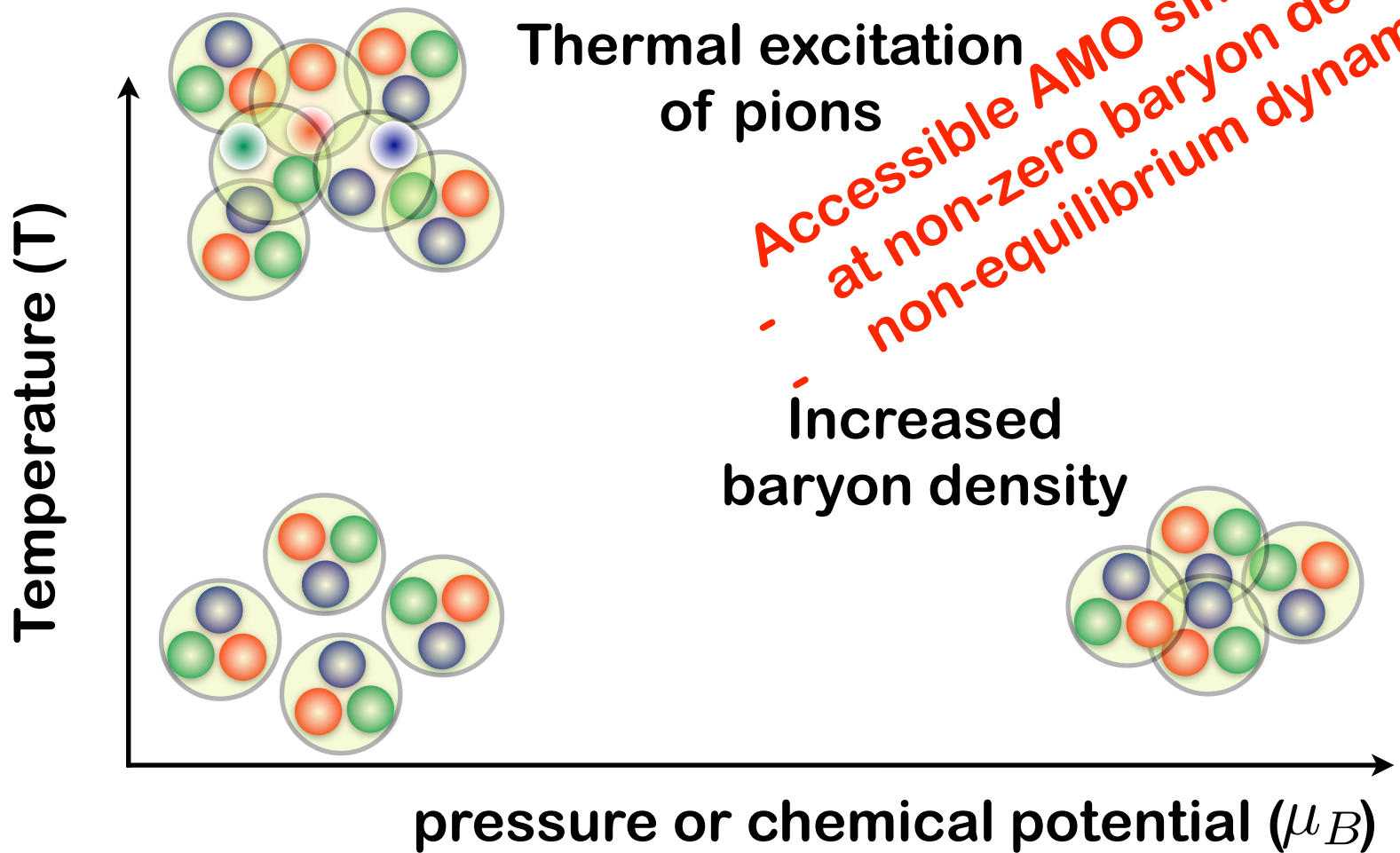


Hadrons overlap



Confinement is "lost"

Expect interesting/unusual behaviour



Implementation of (non-)abelian quantum link models

Strong coupling Hamiltonian with staggered fermions

$$H = \frac{g'^2}{2} \sum_{\langle x,y \rangle} (E_{x,y})^2 + \frac{g^2}{2} \sum_{\langle x,y \rangle} \left[(\vec{L}_{x,y})^2 + (\vec{R}_{x,y})^2 \right] - t \sum_{\langle x,y \rangle, i,j} (\psi_x^{i\dagger} U_{x,y}^{ij} \psi_y^j + \text{h.c.}) + m \sum_{x,i} (-1)^x \psi_x^{i\dagger} \psi_x^i$$

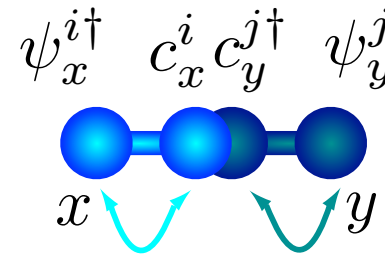
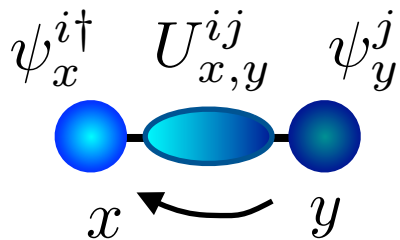
Electric field **Non-abelian electric field** **Matter-gauge interaction** **Staggered mass**
(on-site interaction) (on-site interaction) (...?...)

Implementation of (non-)abelian quantum link models

Strong coupling Hamiltonian with staggered fermions

$$H = \frac{g'^2}{2} \sum_{\langle x,y \rangle} (E_{x,y})^2 + \frac{g^2}{2} \sum_{\langle x,y \rangle} \left[(\vec{L}_{x,y})^2 + (\vec{R}_{x,y})^2 \right] - t \sum_{\langle x,y \rangle, i,j} (\psi_x^{i\dagger} U_{x,y}^{ij} \psi_y^j + \text{h.c.}) + m \sum_{x,i} (-1)^x \psi_x^{i\dagger} \psi_x^i$$

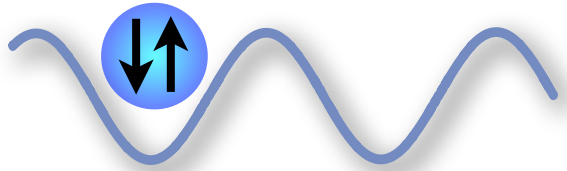
Electric field (on-site interaction)
 Non-abelian electric field (on-site interaction)
Matter-gauge interaction (...?..)
Staggered mass (lattice potential)



$$H = -t \sum_{\langle x,y \rangle, i,j} (\psi_x^{i\dagger} U_{x,y}^{ij} \psi_y^j + \text{h.c.}) + \dots = -t \sum_{\langle x,y \rangle} \left[\left(\sum_i \psi_x^{i\dagger} c_x^i \right) \left(\sum_j c_y^{j\dagger} \psi_y^j \right) + \text{h.c.} \right] + \dots$$

Matter - gauge interaction
 = hopping of fermions mediated by a quantum link
 = correlated hopping of fermions and rishons

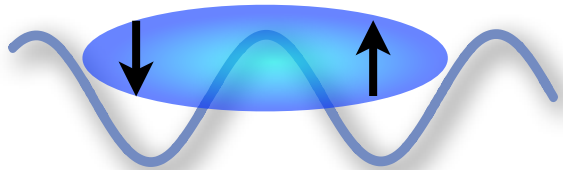
Implementation of (non-)abelian quantum link models



$$H_{\text{hop}} = -\tilde{t} \sum_i \psi_x^{i\dagger} c_x^i + \text{h.c.}$$

(Color singlet) hopping
fermion-rishon

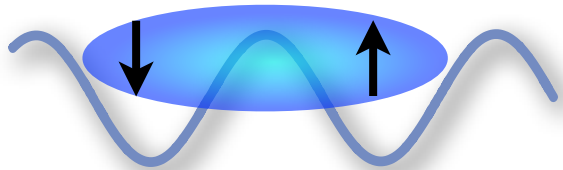
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Implementation of (non-)abelian quantum link models



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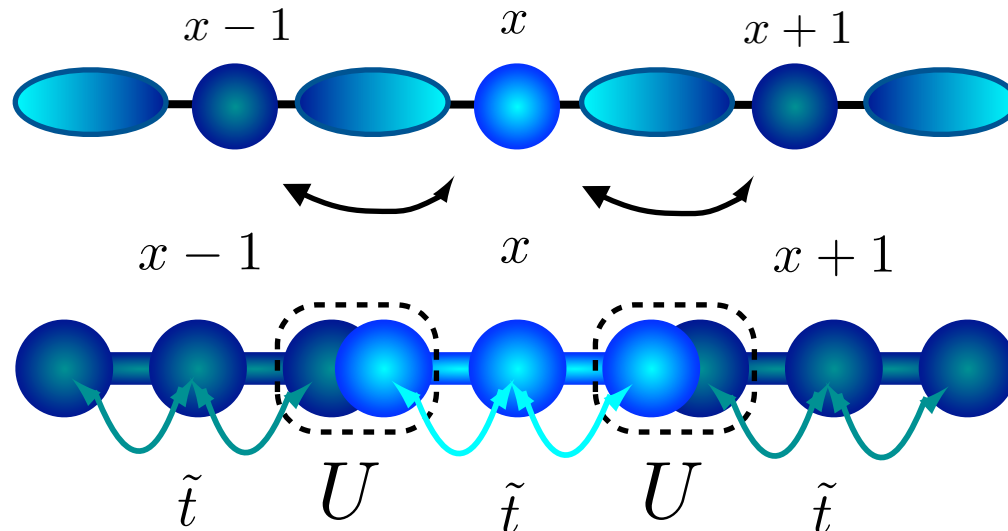
$$\frac{1}{\sqrt{2}} (|\uparrow_\psi \downarrow_c\rangle - |\downarrow_\psi \uparrow_c\rangle)$$

(Color singlet) hopping fermion-rishon

The building block is already gauge invariant (summation over internal degrees of freedom)

Action of the hopping fermion-rishon swaps the local singlet to nearest-neighbor ones

Implementation of (non-)abelian quantum link models



(Color-singlet interaction/constraint) number of rishon per link

$$H_U = U [N_{x,y} - n]^2 = U \left[\sum_i (c_x^{i\dagger} c_x^i + c_y^{i\dagger} c_y^i) - n \right]^2$$

Constraint: on-site $SU(2N)$ interaction

Implementation of (non-)abelian quantum link models

Implementation: fermionic alkaline earth atoms

hydrogen 1 H 1.0079	Implementation: fermionic alkaline earth atoms																helium 2 He 4.0026						
lithium 3 Li 6.941	beryllium 4 Be 9.0122																	boron 5 B 10.811	carbon 6 C 12.011	nitrogen 7 N 14.007	oxygen 8 O 15.999	fluorine 9 F 18.998	neon 10 Ne 20.180
sodium 11 Na 22.990	magnesium 12 Mg 24.305																	aluminum 13 Al 26.982	silicon 14 Si 28.086	phosphorus 15 P 30.974	sulfur 16 S 32.065	chlorine 17 Cl 35.453	argon 18 Ar 39.948
potassium 19 K 39.098	calcium 20 Ca 40.078	scandium 21 Sc 44.956	titanium 22 Ti 47.867	vanadium 23 V 50.942	chromium 24 Cr 51.996	manganese 25 Mn 54.938	iron 26 Fe 55.845	cobalt 27 Co 58.933	nickel 28 Ni 58.693	copper 29 Cu 63.546	zinc 30 Zn 65.39	gallium 31 Ga 69.723	germanium 32 Ge 72.61	arsenic 33 As 74.922	selenium 34 Se 78.96	bromine 35 Br 79.904	krypton 36 Kr 83.80						
rubidium 37 Rb 85.468	strontium 38 Sr 87.62	yttrium 39 Y 88.906	zirconium 40 Zr 91.224	niobium 41 Nb 92.906	molybdenum 42 Mo 95.94	technetium 43 Tc [98]	ruthenium 44 Ru 101.07	rhodium 45 Rh 102.91	palladium 46 Pd 106.42	silver 47 Ag 107.87	cadmium 48 Cd 112.41	indium 49 In 114.82	tin 50 Sn 118.71	antimony 51 Sb 121.76	tellurium 52 Te 127.60	iodine 53 I 126.90	xenon 54 Xe 131.29						
caesium 55 Cs 132.91	barium 56 Ba 137.33	57-70 *	lutetium 71 Lu 174.97	hafnium 72 Hf 178.49	tantalum 73 Ta 180.95	tungsten 74 W 183.84	rhenium 75 Re 186.21	osmium 76 Os 190.23	iridium 77 Ir 192.22	platinum 78 Pt 195.08	gold 79 Au 196.97	mercury 80 Hg 200.59	thallium 81 Tl 204.38	lead 82 Pb 207.2	bismuth 83 Bi 208.98	polonium 84 Po [209]	astatine 85 At [210]	radon 86 Rn [222]					
francium 87 Fr [223]	radium 88 Ra [226]	89-102 **	lawrencium 103 Lr [262]	rutherfordium 104 Rf [261]	dubnium 105 Db [262]	seaborgium 106 Sg [266]	bohrium 107 Bh [264]	hassium 108 Hs [269]	meitnerium 109 Mt [268]	unnilium 110 Uun [271]	ununium 111 Uuu [272]	ununbium 112 Uub [277]	ununquadium 114 Uuq [289]										

Strontium / Ytterbium

* Lanthanide series

** Actinide series

lanthanum 57 La 138.91	cerium 58 Ce 140.12	praseodymium 59 Pr 140.91	neodymium 60 Nd 144.24	promethium 61 Pm [145]	samarium 62 Sm 150.36	europium 63 Eu 151.96	gadolinium 64 Gd 157.25	terbium 65 Tb 158.93	dysprosium 66 Dy 162.50	holmium 67 Ho 164.93	erbium 68 Er 167.26	thulium 69 Tm 168.93	ytterbium 70 Yb 173.04
actinium 89 Ac [227]	thorium 90 Th 232.04	protactinium 91 Pa 231.04	uranium 92 U 238.03	neptunium 93 Np [237]	plutonium 94 Pu [244]	americium 95 Am [243]	curium 96 Cm [247]	berkelium 97 Bk [247]	californium 98 Cf [251]	einsteinium 99 Es [252]	fermium 100 Fm [257]	mendelevium 101 Md [258]	nobelium 102 No [259]

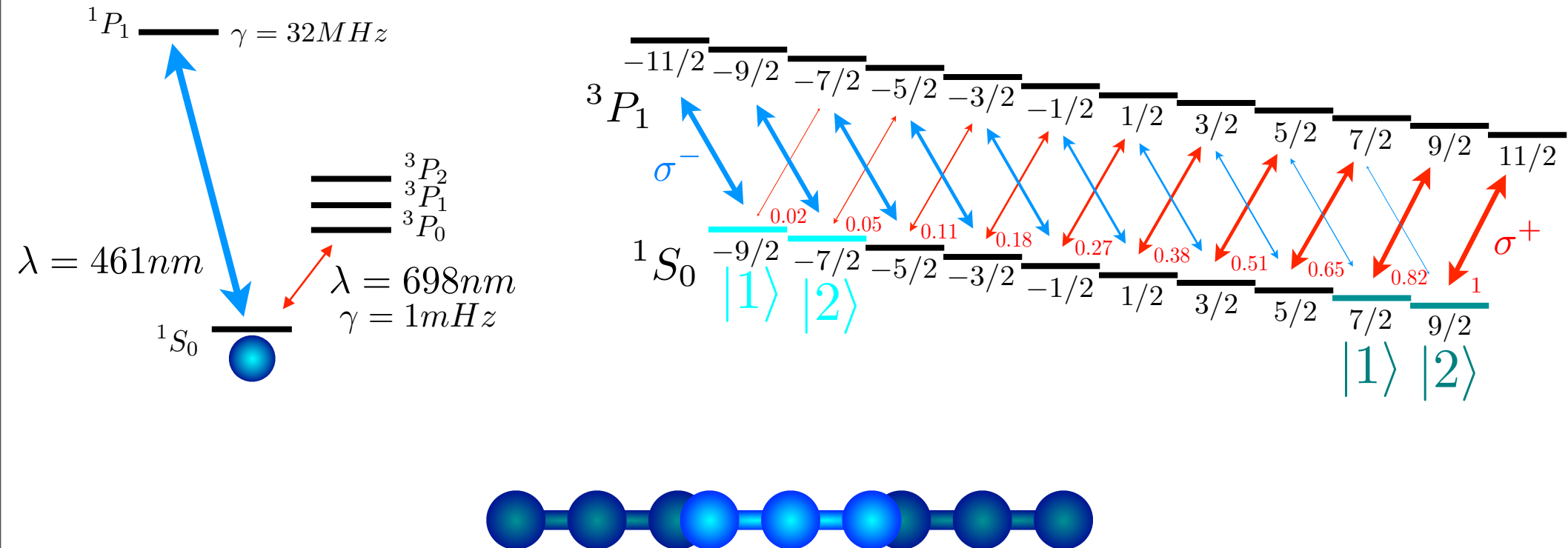
$$^{87}\text{Sr}(I = 9/2)$$

$$^{173}\text{Yb}(I = 5/2)$$

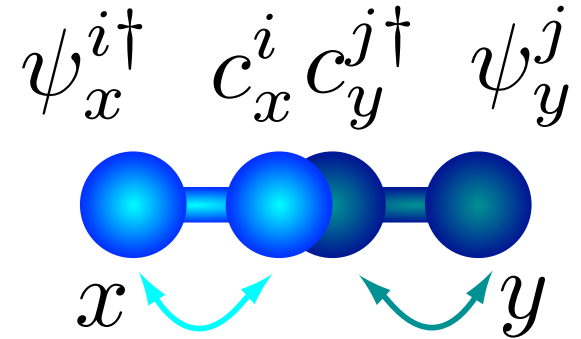
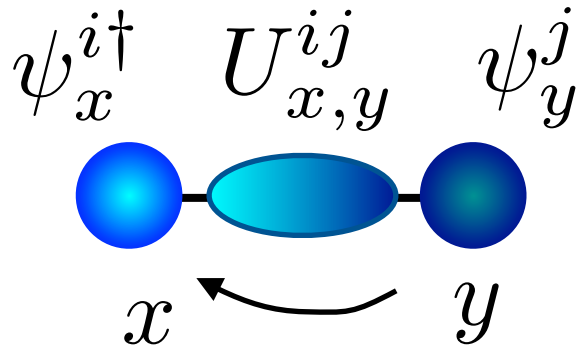
- i) fermionic alkaline earths have nuclear spin $I > 0$
- ii) scattering independent of the nuclear spin

Implementation of (non-)abelian quantum link models

Ground state hyperfine Zeeman levels encode the color degrees of freedom

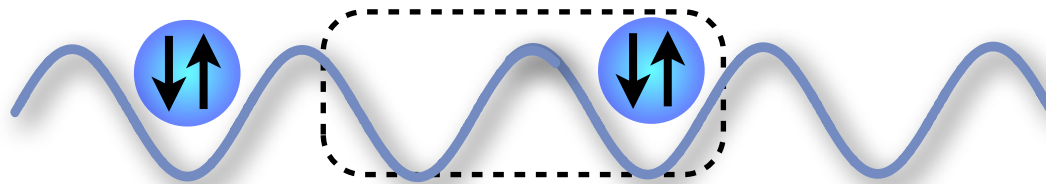


Implementation of (non-)abelian quantum link models

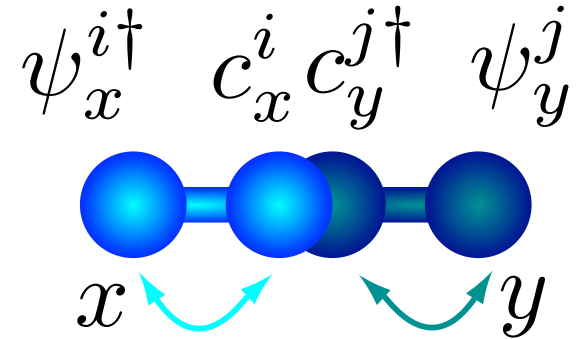
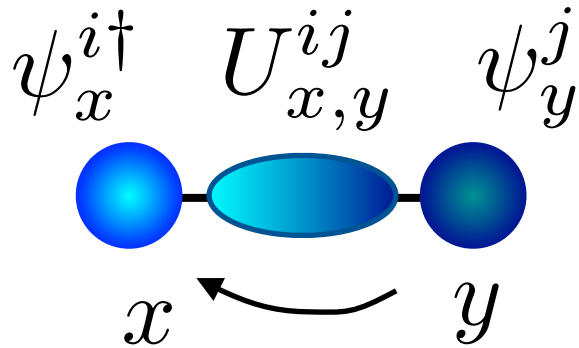


At second order in perturbation theory (t/U):

$$H_{\text{micro}} = U \left[\sum_i (c_x^{i\dagger} c_x^i + c_y^{i\dagger} c_y^i) - n \right]^2 - \tilde{t} \sum_i (\psi_x^{i\dagger} c_x^i + c_y^{i\dagger} \psi_y^i) + \text{h.c.}$$

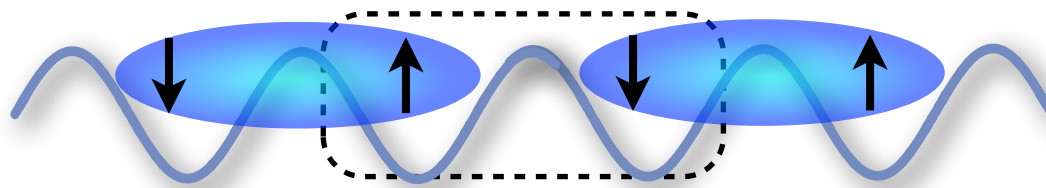


Implementation of (non-)abelian quantum link models



At second order in perturbation theory (t/U):

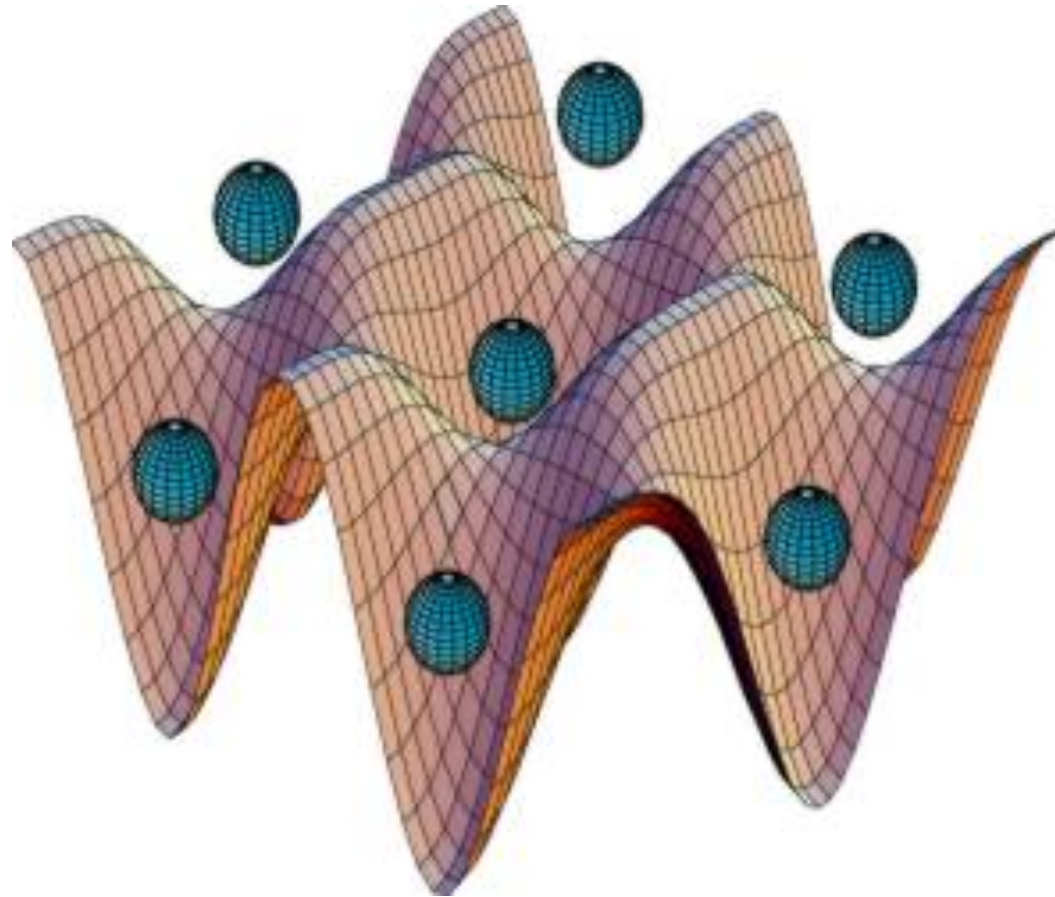
$$H_{\text{micro}} = U \left[\sum_i (c_x^{i\dagger} c_x^i + c_y^{i\dagger} c_y^i) - n \right]^2 - \tilde{t} \sum_i (\psi_x^{i\dagger} c_x^i + c_y^{i\dagger} \psi_y^i) + \text{h.c.}$$



$$H_{\text{eff}} = -t \sum_{\langle x,y \rangle} \left[\left(\sum_i \psi_x^{i\dagger} c_x^i \right) \left(\sum_j c_y^{j\dagger} \psi_y^j \right) + \text{h.c.} \right] + \dots$$

Observability of phenomena

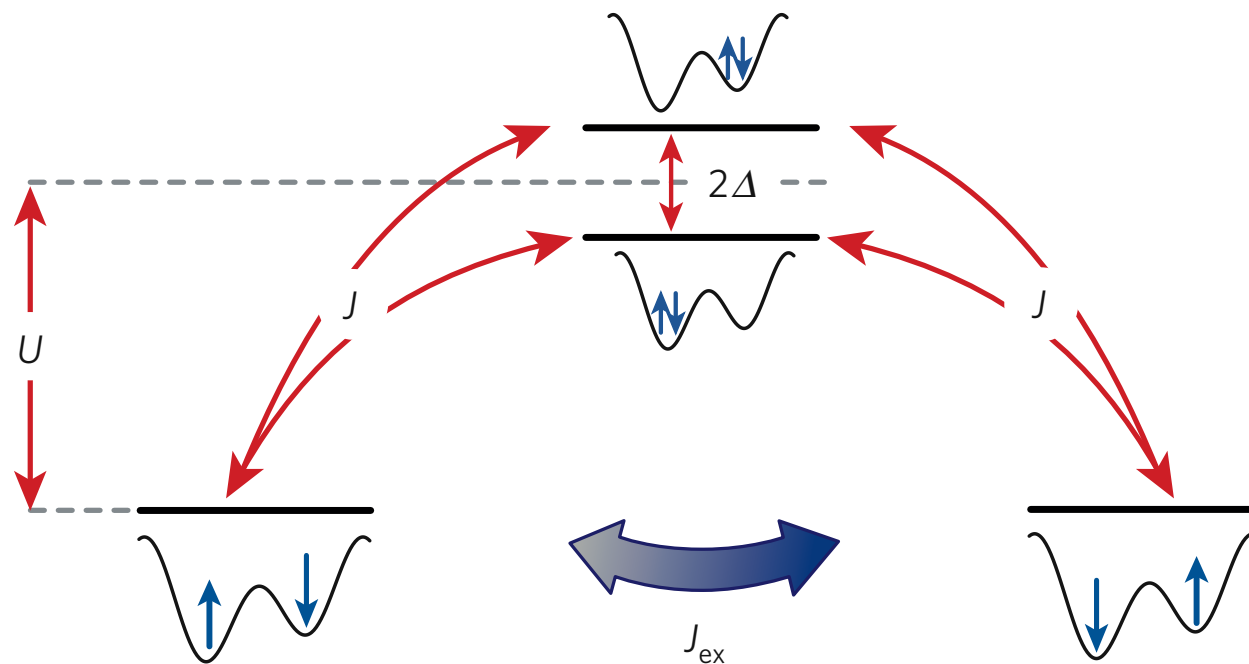
Preparation of many
body states
(Mott phase)



Greiner et al. (2002)
Joerdens et al. (2008)
Schneider et al. (2008)

Observability of phenomena

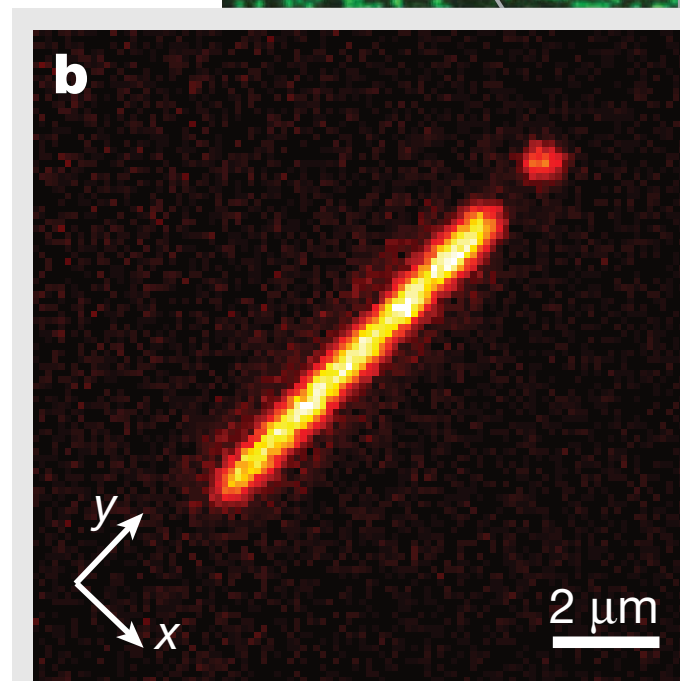
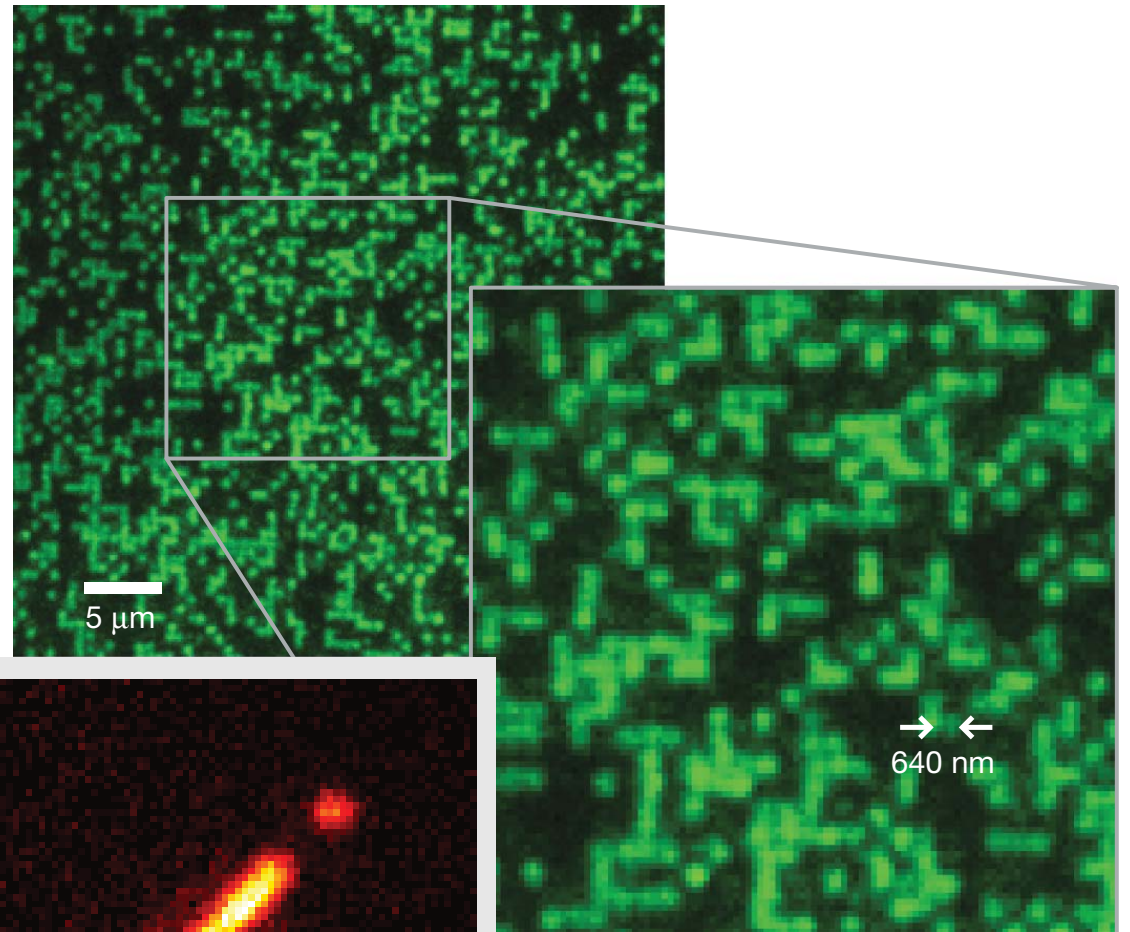
Evolution (Super-exchange)



Anderlini et al. (2007)
Trotzky et al. (2008)

Observability of phenomena

Detection
(Single-site fluorescence)



Bakr et al. (2010)
Weitenberg et al. (2011)

Conclusions

**Simpler atomic/molecular/solid state implementations
(not in the talk: QLM with magnetic atoms/polar molecules!)?**

Superconducting Qubits (D. Marcos,...), Dipoles and Rydberg (A. Glaetzle, ...)

**Connection with gauge magnets and spin liquids (in
principle accessible within this toolbox)**

**Finite-temperature confinement/deconfinement phase transition,
deconfined criticality in ‘feasible quantum link’?**

Still very far away from QCD (even with the SU(3)): next steps?

The team

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P. Widmer



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M. Müller

- **Technical University
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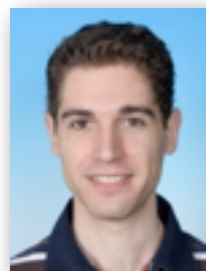


P. Rabl

- **IQOQI - Innsbruck University**



M. Dalmonte



D. Marcos



P. Zoller