Simulation of dynamic abelian and non-abelian gauge theories with ultracold atoms

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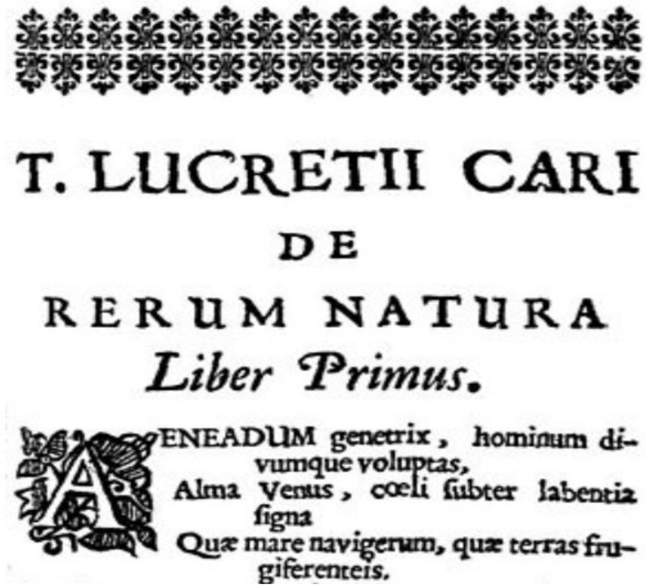
Joint work with

Erez Zohar, Tel Aviv University
J. Ignacio Cirac, MPQ

Trieste, May 16, 2013


T. LUCRETII CARI

DE

RERUM NATURA

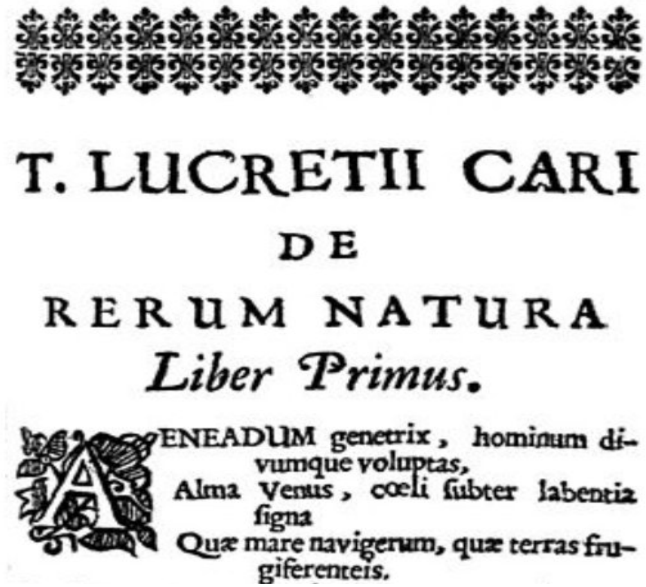
Liber Primus.

AENEADUM genetrix, hominum divumque voluptas,
Alma Venus, coeli ſubter labentia ſigna
Quæ mare navigerum, quæ terras frugiferenteis.

"On the Nature of the Universe".

Lucretius **(~50 BC)** :

…"**So far as it goes, a small thing may give analogy of great things, and show the tracks of knowledge**"



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Why are there Analogies between Condensed Matter and Particle Theory?

Frank Wilczek

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A.E. ?: "… **everything should be as simple as it can be, but not simpler.**"

List of relevant publications

1. **Confinement and Lattice Quantum-Electrodynamic Electric Flux Tubes Simulated with Ultracold Atoms**
   Erez Zohar, BR
   Phys. Rev. Lett. 107, 275301 (2011)
   Preprint: arXiv: 1108.1562v2 [quant-ph]
2. **Simulating Compact Quantum Electrodynamics with ultracold atoms: Probing confinement and nonperturbative effects**
   Erez Zohar, J. Ignacio Cirac, BR
   Phys. Rev. Lett. 109, 125302 (2012)
   Preprint: arXiv: 1204.6574 [quant-ph]
3. **Topological Wilson-loop area law manifested using a superposition of loops**
   Erez Zohar, BR
   New J. Phys. 15 (2013) 043041
   Preprint: arXiv:1208.1012 [quant-ph]
4. **Simulating 2+1d Lattice QED with dynamical matter using ultracold atoms**
   Erez Zohar, J. Ignacio Cirac, BR
   Phys. Rev. Lett. 110, 055302 (2013)
   Preprint: arXiv:1208.4299 [quant-ph]
5. **Cold-atom quantum simulator for SU(2) Yang-Mills lattice gauge theory**
   Erez Zohar, J. Ignacio Cirac, BR
   Phys. Rev. Lett. 110, 125304 (2013)
   Preprint: arXiv:1211.2241 [quant-ph]
6. **Quantum simulations of gauge theories with ultracold atoms: local gauge invariance from angular momentum conservation**
   Erez Zohar, J. Ignacio Cirac, BR
   Preprint: arXiv:1303.5040 [quant-ph]

Talk outline

- HEP and Lattice gauge theory – overview
- Confinement in 1+1 – a simple realization
- Further examples: non-abelian and higher dimensional cQED and Z_N and Yang-Mills gauge theories.
- Preparing and measuring; confinement, Wilson loops.

- Outlook and Summary

HEP vs. cond. mat.

- Experimentally more challenging compared with cond. mat. simulations. (For reasons to be explained).

- Could be used to explore otherwise <u>experimentally</u> and <u>computationally inaccessible</u> effects, or directly unobservable phenomena at short scales.

- There are different proposals with different levels of difficulty. Starting with relatively simple models, that can be developed to study complex challenging problems.

  Note also recent works by :

  P. Zoller group M. Lewenstein group E. Mueller group.

Requirements: HEP models

Fermion fields : = Matter

Bosonic, Gauge fields:= Interaction mediators

Symmetries:

- local gauge invariance = "charge" conservation

  (exact, or low energy, effective)

- Relativistic invariance = causal structure,

  (In the continuum limit.)

Structure of HEP models

- Matter particles:= fermions (+Higgs)
  (Charges: electric, color , flavor. Spin. Mass)

- Gauge fields:= mediate the Interactions

Electromagnetic:	massless photon,	(1),	U(1)
Weak interaction :	massive Z, W's ,	(3),	SU(2)
Strong interaction :	massless Gluons ,	(8),	SU(3)

Structure of HEP models

The abelian/non-abelian local symmetries much <u>determines</u> the Gauge field physical properties and dynamics.

It also much constrains the allowed possible interactions between Gauge fields and Matter-field.

U(1) => linear Maxwell theory

SU(N) => non-linear Yang-Mills theory

Abelian U(1): QED

$$\alpha_{QED} \ll 1, \quad V_{QED}(r) \propto \frac{1}{r}$$

We (ordinarily) don't need second quantization to understand the structure of atoms. $m_e c^2 \gg E_{Rydberg} \simeq \alpha_{QED}^2\, m_e c^2$

In HEP scattering, perturbation theory (Feynman diagrams)

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In HEP scattering, perturbation theory (Feynman diagrams) Works well.

✓

(e.g. in the computation of g-2)

Non-abelian: QCD

$$\alpha_{QCD} > 1 \,, V_{QCD}(r) \propto r$$

non-perturbative confinement effect!

=> <u>structure of Hadrons</u>:   quark pairs form Mesons ,
quark triplets form Baryons.

Color Electric flux-tubes: “a non-abelian Meissner effect”.

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Gauge fields

Abelian Fields **Maxwell theory**	**Non-Abelian fields** **Yang-Mills theory**
Massless	Massless
Long-range forces	**<u>Confinement</u>**
Chargeless	Carry charge
Linear dynamics	Self interacting & NL

Lattice gauge theory

- The standard avenue: $Z_{lattice}$ – computed using Monte Carlo “sampling”

(Other methods exist: non-abelian bosonization (1+1); instanton semi-classical summation, large N perturbation, variational methods…)

… but:

- Limited applicability with too many quarks . �
  Computationally hard “sign problem”.

➔ (e.g. color superconductivity, Quark-Gluon plasma.)

- Quark confinement with <u>dynamic</u> matter never proven.

First step:

- Confinement in <u>Abelian</u> lattice models!

- <u>Toy models</u> with <u>“QCD-like properties”</u> that capture the essential physics mechanism of confinement.

Confinement in abelian compact QED lattice models

- 1+1D: "Schwinger's model" manifests confinement. (analytic and lattice results available).

- 2+1D: confinement for <u>all values</u> of coupling constant, a non-perturbative mechanism (Polyakov).

  (For T > 0: there is a phase transition also in 2+1 D.)

- 3+1D: <u>phase transition</u> between strong coupling confinement phase, and weak coupling coulomb phase.

- Z_N discrete gauge symmetry: in d+1, d>1. Electric and magnetic confinement & deconfinement

QED in 1+1 : Schwinger's model

- No magnetic fields: EM has no dynamics of its own. Non trivial dynamics obtained by coupling to dynamical charge sources.

- Schwinger: e^+e^- form bound states. (analytic and lattice results available.)

- Non-abelian extension: in 1+1: $\mathrm{Q}CD_2$ version, but were <u>not</u> completely solved. Only in the large-N limit.

1+1, U(1) gauge theory

Start with a hopping fermionic Hamiltonian, in 1 spatial direction

$$H = \sum_n M_n \psi_n^\dagger \psi_n + \alpha_n \left(\psi_n^\dagger \psi_{n+1} + H.c. \right)$$

This Hamiltonian is invariant to global gauge transformations,

$$\psi_n \longrightarrow e^{-i\Lambda} \psi_n \quad ; \quad \psi_n^\dagger \longrightarrow e^{i\Lambda} \psi_n^\dagger$$

1+1, U(1) gauge theory

Promote the gauge transformation to be local:

$$\psi_n \longrightarrow e^{-i\Lambda_n}\psi_n \quad ; \quad \psi_n^\dagger \longrightarrow e^{i\Lambda_n}\psi_n^\dagger$$

Then, in order to make the Hamiltonian gauge invariant, add unitary operators, U_n,

$$H = \sum_n M_n \psi_n^\dagger \psi_n + \alpha_n\left(\psi_n^\dagger U_n \psi_{n+1} + H.c.\right)$$

$$U_n = e^{i\theta_n} \quad ; \quad \theta_n \longrightarrow \theta_n + \Lambda_{n+1} - \Lambda_n$$

Dynamic 1+1 U(1) gauge theory

Add *dynamics* to the gauge field:

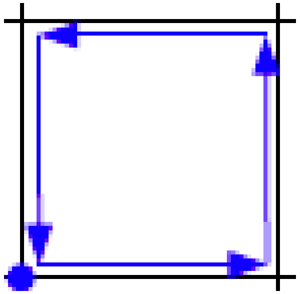
$$H_E = \frac{g^2}{2} \sum_n L_{n,z}^2$$

Where L_n is the angular momentum operator conjugate to θ_n, representing the (integer) electric field.

d+1 (d>1) : plaquette terms

In more spatial dimensions, one should consider also gauge-gauge interactions, which involve <u>plaquette terms</u>:

$$-\frac{1}{g^2}\sum_{m,n}\cos\left(\theta^1_{m,n}+\theta^2_{m+1,n}-\theta^1_{m,n+1}-\theta^2_{m,n}\right)$$



In the continuum limit, this corresponds to $(\nabla\times\boldsymbol{A})^2$ - gauge invariant magnetic energy term.

Figure from ref [6]

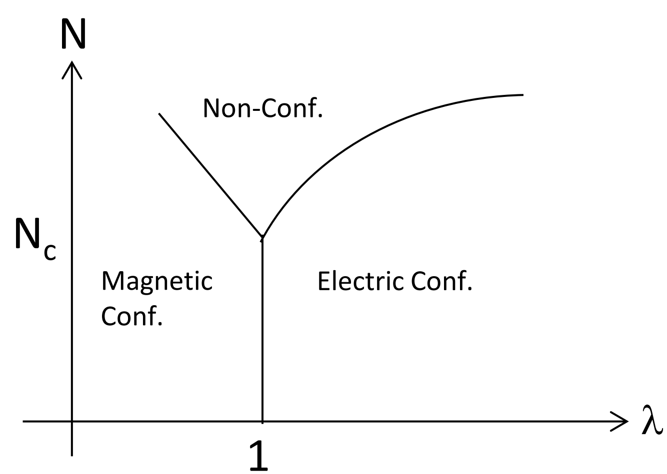
Z_N Gauge theory

- Abelian *discrete* gauge theory: the gauge field degrees of freedom operate in a finite Hilbert space
- Three phases in 3+1 dimensions

$$P^N = Q^N = 1 \quad ; \quad P^\dagger Q P = e^{i\delta} Q$$

$$P^\dagger P = Q^\dagger Q = 1 \qquad \delta = \frac{2\pi}{N}$$

$$H = H_E + H_B = \\ -\frac{1}{2}\lambda \sum_{\mathbf{n},k} \left(P_{\mathbf{n},k} + P^\dagger_{\mathbf{n},k} \right) \\ -\frac{1}{2}\sum_{\mathbf{n}} \left(Q_{\mathbf{n},1} Q_{\mathbf{n}+\hat{1},2} Q^\dagger_{\mathbf{n}+\hat{2},1} Q^\dagger_{\mathbf{n},2} + h.c. \right)$$



Adapted from Horn et. al., PRD 19, 3715, 1979

SU(2) in 1+1-d

- Confinement of “color” charges.
- Non-abelian Schwinger “QCD_2”

- Hamiltonian:

$$H = \sum_n \left(\frac{g^2}{2} \mathbf{L}_n^2 + m(-1)^n \psi_n^\dagger \psi_n + i\beta \left(\psi_n^\dagger U_n \psi_{n+1} - h.c. \right) \right)$$

on each link – an SU(2) element U_n ;
conjugate to the group’s left and right generators, $\{L_{n,a}\}$ and $\{R_{n,a}\}$, satisfying (separately) SU(2) algebras

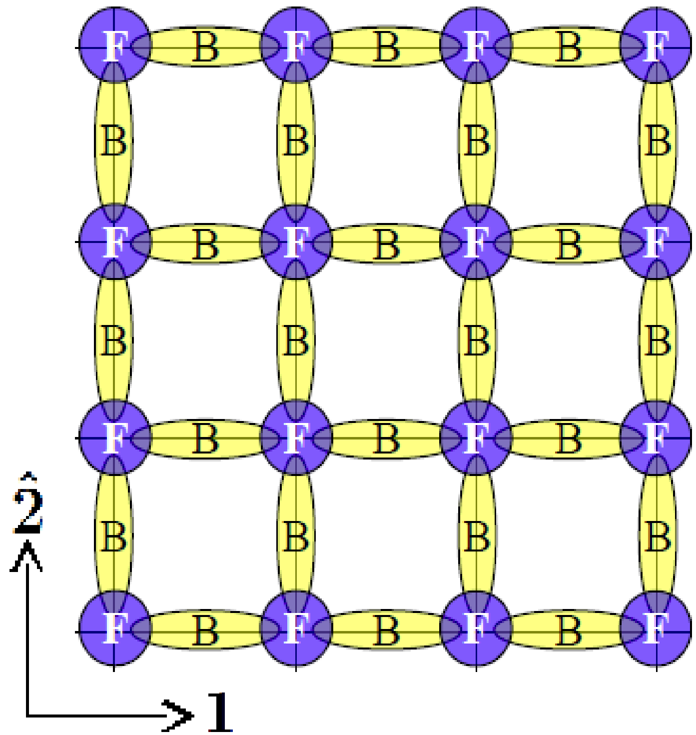
Non-Abelian plaquettes

In this case, the plaquette terms take the form

$$-\frac{1}{2g^2}\left(\mathrm{Tr}_{\square}\left(UUU^{\dagger}U^{\dagger}\right)+H.c.\right)$$

- This gives the nonlinear Yang-Mills model.

Atomic analog simulator

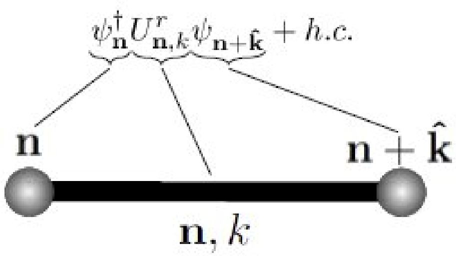


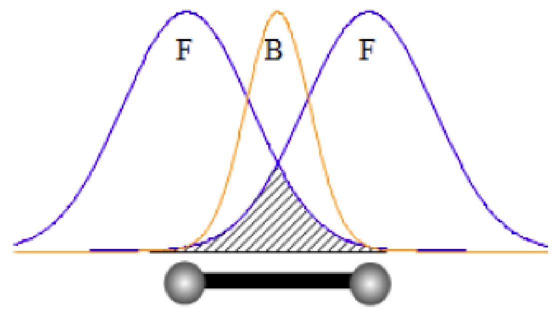
Gauge fields: $A \rightarrow \theta$, $E \rightarrow L$

Matter fields: $\psi, \psi^\dagger$

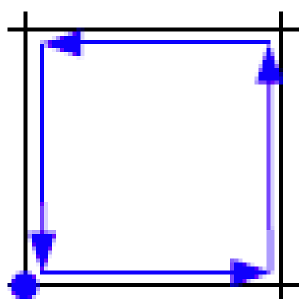
Figure from ref [6]

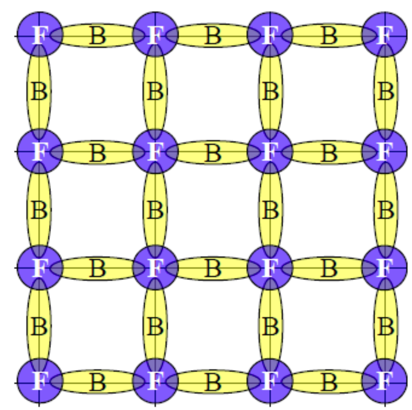
General idea of our work





- *Links: realized by atomic scattering : gauge invariance is* ***<u>fundamental</u>***





- Plaquette interactions are realized from gauge invariant building blocks, and virtual loop contributions of ancillary fermions.

Figures from ref [6]

Realization of the elementary interactions

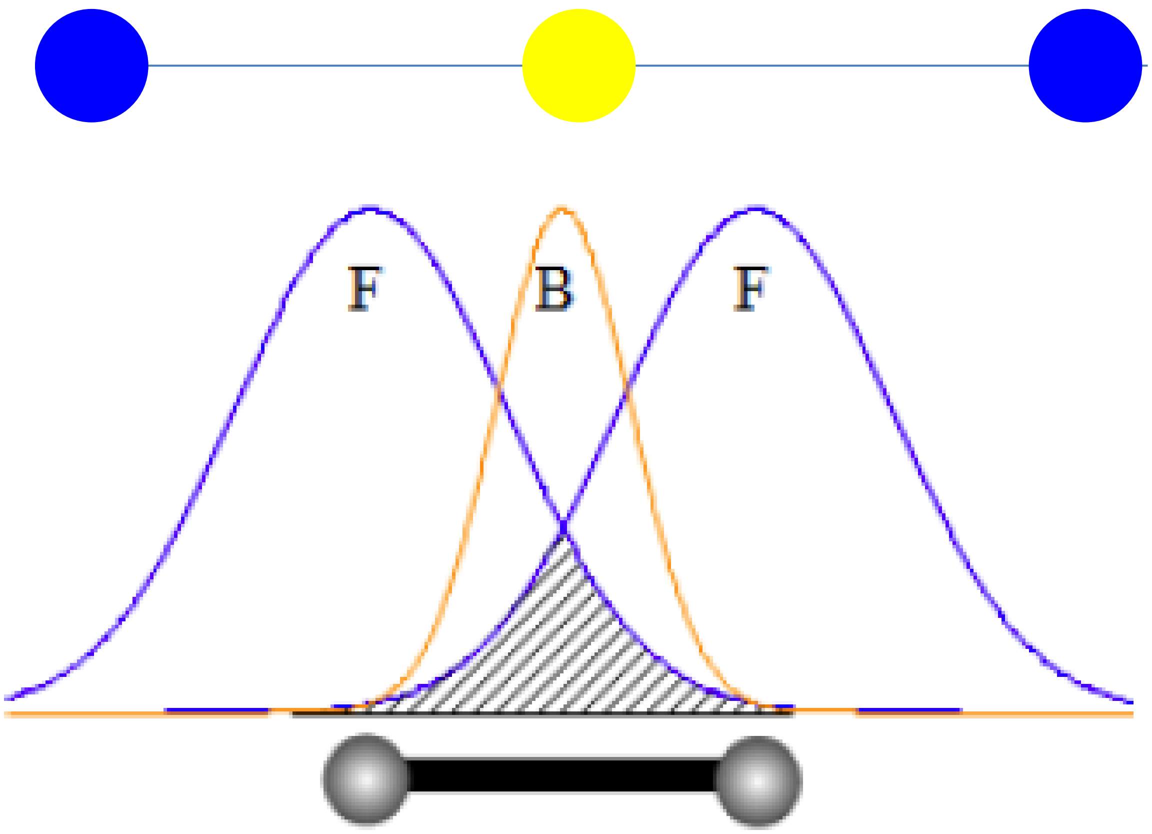


Figure from ref [6]

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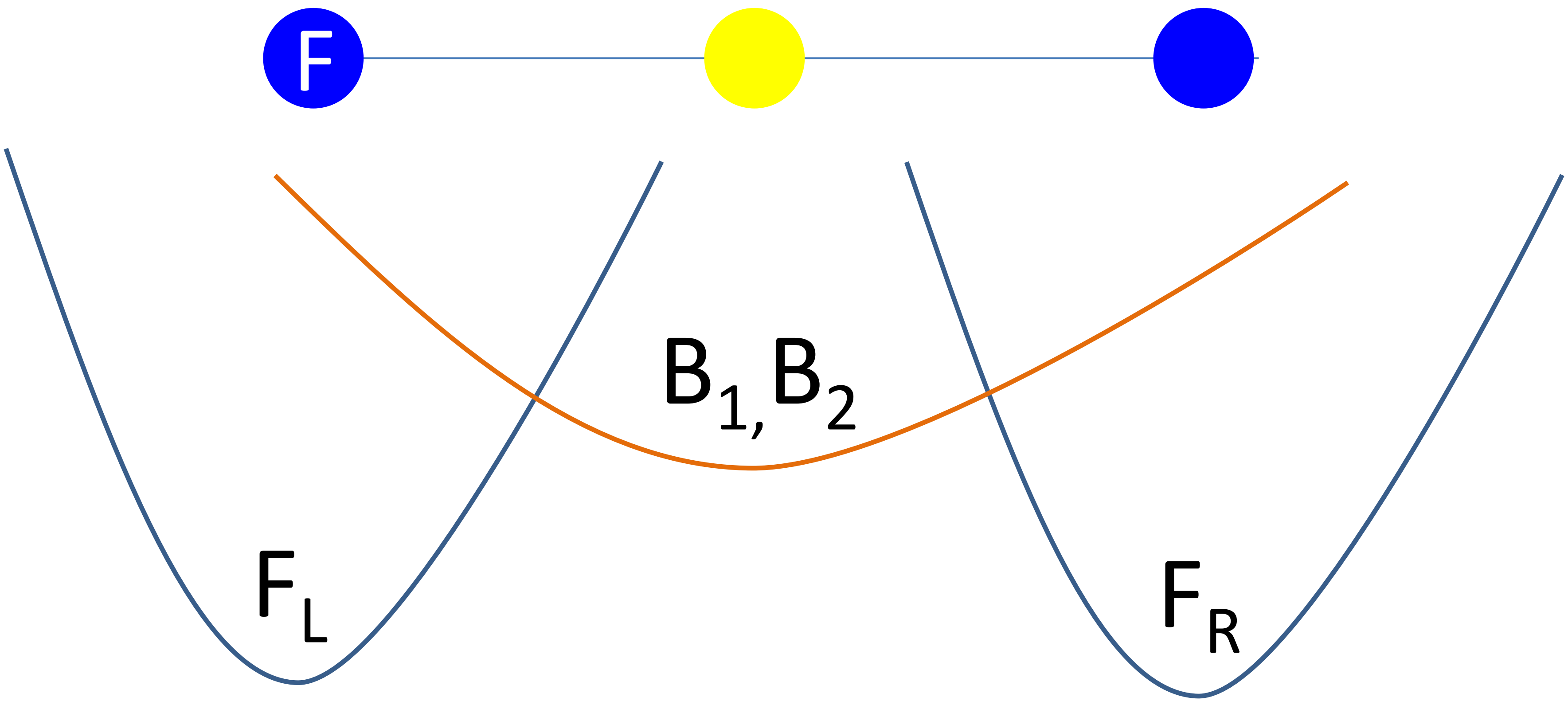
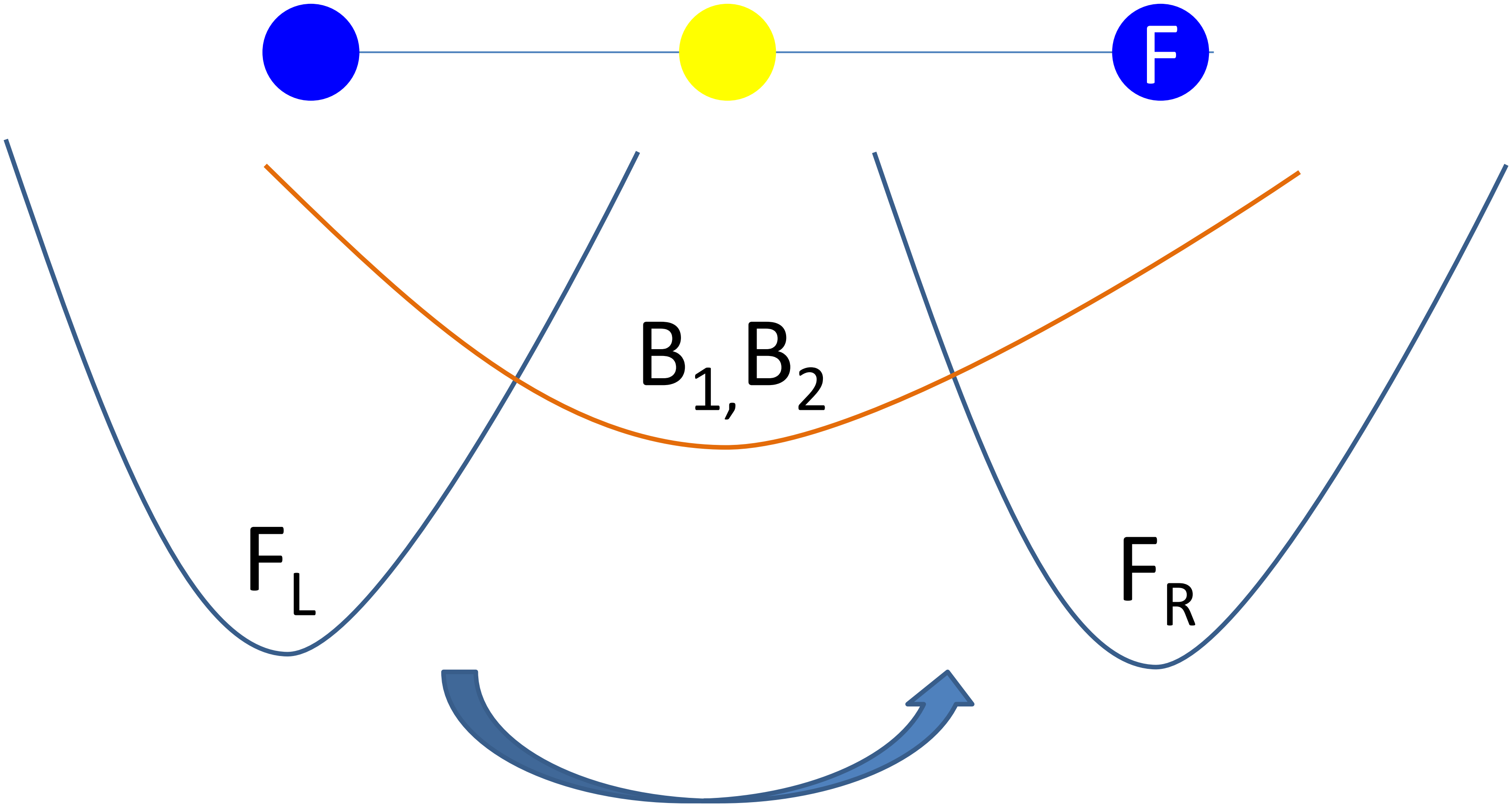


Realization of the elementary interactions

$$L \to L - 1$$



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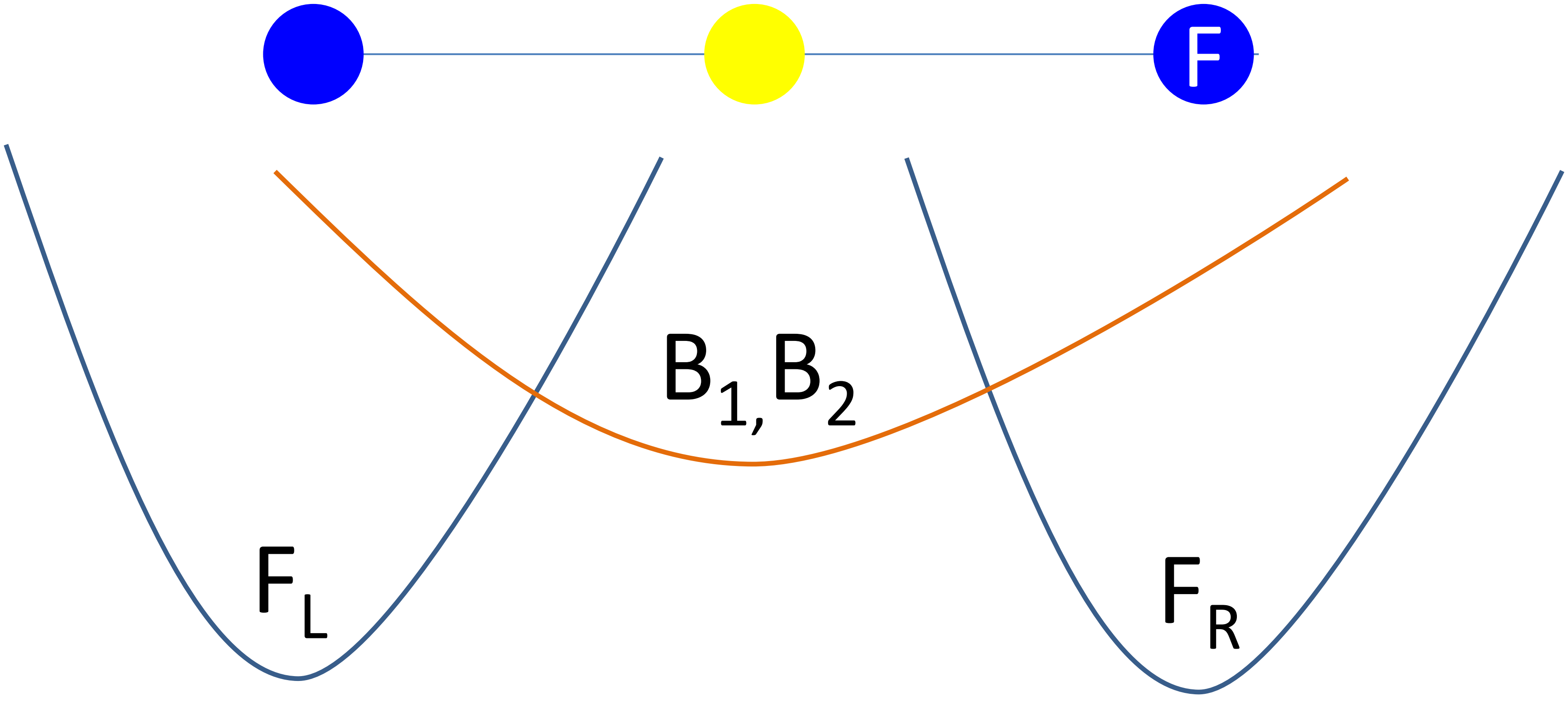

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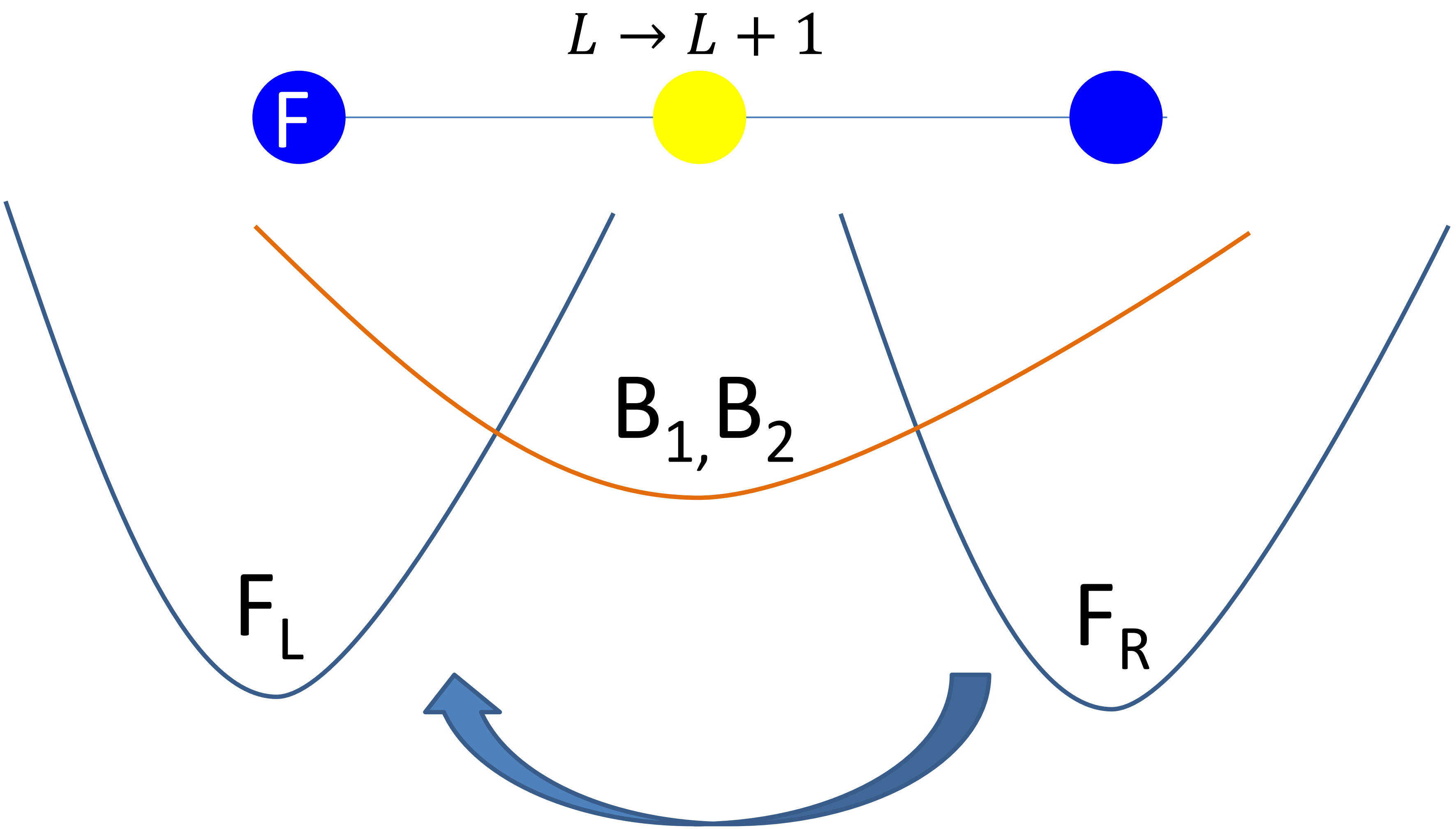
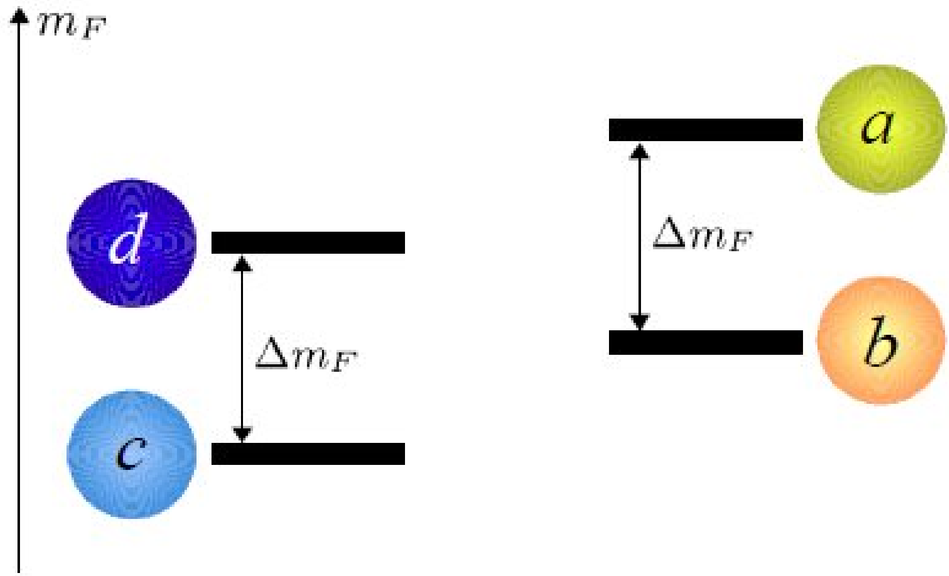
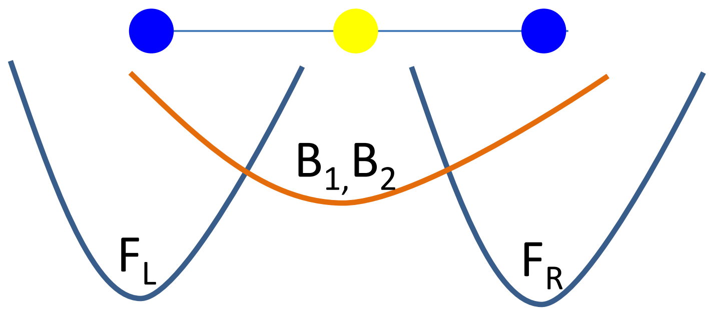




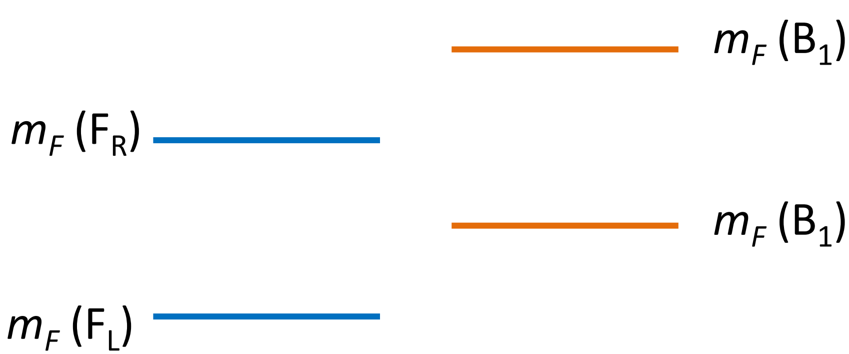
$$m_F\,(a) + m_F\,(c) = m_F\,(b) + m_F\,(d)$$

Figure from ref [6]

Angular Momentum conservation ↔ Local gauge invariance



$$\psi_L^\dagger b_1^\dagger b_2 \psi_R + \psi_R^\dagger b_2^\dagger b_1 \psi_L$$



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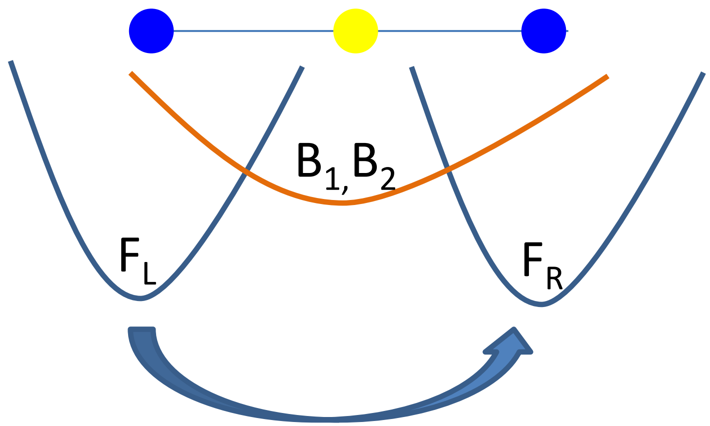

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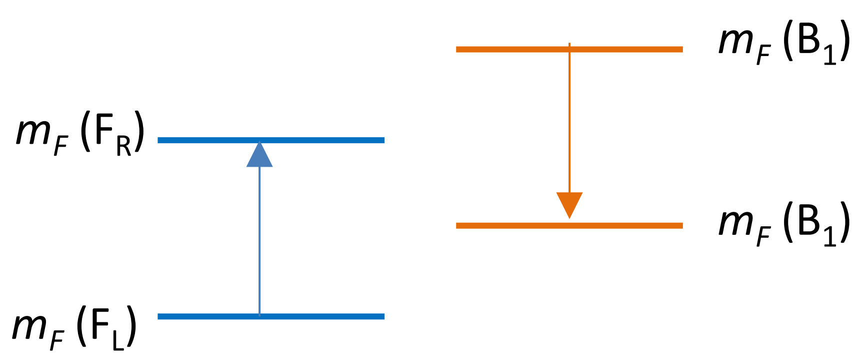

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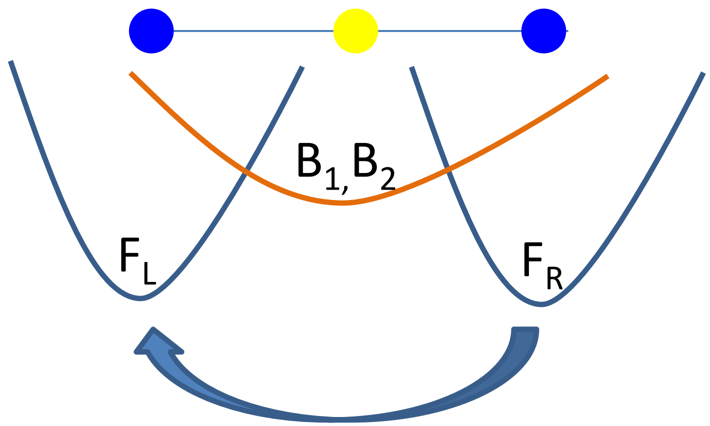

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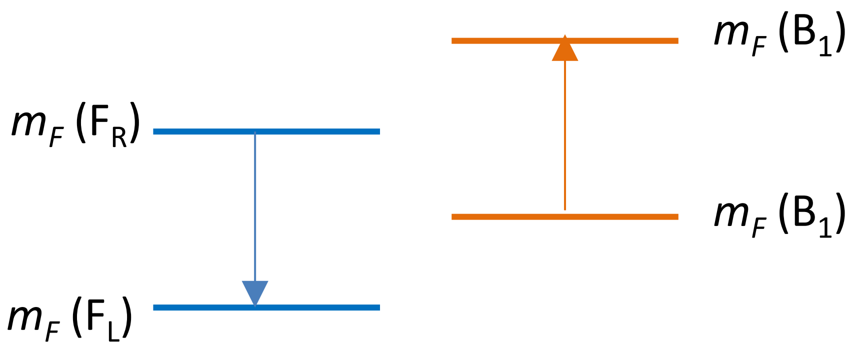


Gauge U(1) bosons: Schwinger algebra

$$L_+ = b_1^\dagger b_2 \,;\, L_- = b_2^\dagger b_1$$

$$L_z = \frac{1}{2}(N_1 - N_2) \quad ;l = \frac{1}{2}(N_1 + N_2)$$

Gauge bosons: Schwinger algebra

$$L_+ = b_1^\dagger b_2\,;\, L_- = b_2^\dagger b_1$$

$$L_z = \frac{1}{2}(N_1 - N_2) \quad ; \quad l = \frac{1}{2}(N_1 + N_2)$$

and thus what we have is

$$\psi_L^\dagger b_1^\dagger b_2 \psi_R + \psi_R^\dagger b_2^\dagger b_1 \psi_L$$

$$\Downarrow$$

$$\psi_L^\dagger L_+ \psi_R + \psi_R^\dagger L_- \psi_L$$

Gauge bosons: Schwinger algebra

$$\psi_L^\dagger L_+ \psi_R + \psi_R^\dagger L_- \psi_L$$

For large l , $m \ll l$

$$L_+ = b_1^\dagger b_2 \sim e^{i(\phi_1 - \phi_2)} \equiv e^{i\theta} = U$$

$$\boxed{\psi_L^\dagger U \psi_R + \psi_R^\dagger U^\dagger \psi_L}$$

Qualitatively similar results can be obtained with just two bosons on the link.

Electric dynamic term

$$E^2 = L_z^2 = \frac{1}{4}(N_1 - N_2)^2$$

$$= \frac{1}{4}\left(b_1^\dagger b_1\right)^2 + \frac{1}{4}\left(b_2^\dagger b_2\right)^2 - \frac{1}{2} b_1^\dagger b_1 b_2^\dagger b_2$$

$$\Rightarrow \quad H_E = \frac{g^2}{2}\sum_n L_{nz}^2$$

$E = N_1 - N_2$ *conjugate to* $\theta = \phi_1 - \phi_2$

Finally: cQED; U(1) in 1+1

The Schwinger model:

$$H = M\sum_n (-1)^n \psi_n^\dagger \psi_n + \alpha\left(\psi_n^\dagger U_n \psi_{n+1} + H.c.\right)$$

$$+\frac{g^2}{2}\sum_n L_{nz}^2$$

F-B scattering: link interaction

B-B Scattering: EM kinetic energy

Mass and Charge

Real Content	"Mass"	"Charge"	Simulated Content
$\lvert 0\rangle_C \lvert 1\rangle_D$	0	0	Empty vertex
$\lvert 0\rangle_C \lvert 0\rangle_D$	M	-1	$\bar{q}$
$\lvert 1\rangle_C \lvert 1\rangle_D$	M	$+1$	q
$\lvert 1\rangle_C \lvert 0\rangle_D$	$2M$	0	$q\bar{q}$

- Even n (particles):

$Q = N = \psi^{\dagger}\psi$

0 atoms: zero mass, zero charge

1 atom: M, Q=1

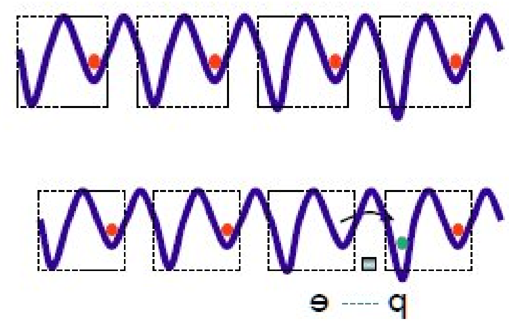
- Odd n (anti-particles):

$Q = N - 1$

1 atom : zero mass, zero charge ("Dirac sea")

0 atoms: mass M (relative to –M), charge Q=-1

note: mass measured relative to –M

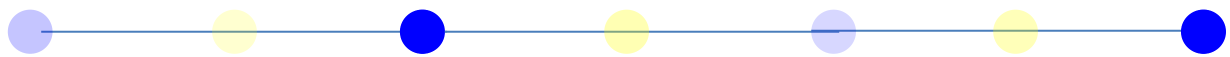


Quark confinement

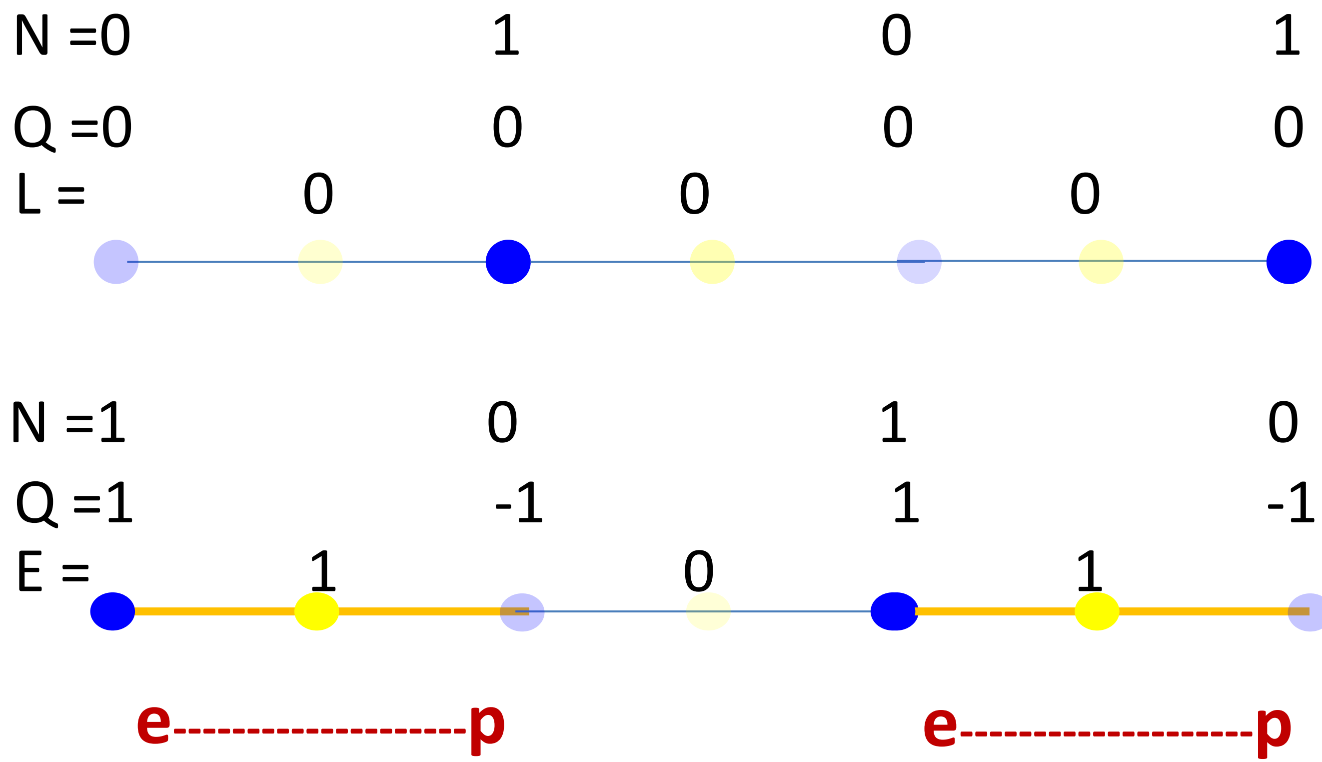
N =0 1 0 1

Q =0 0 0 0

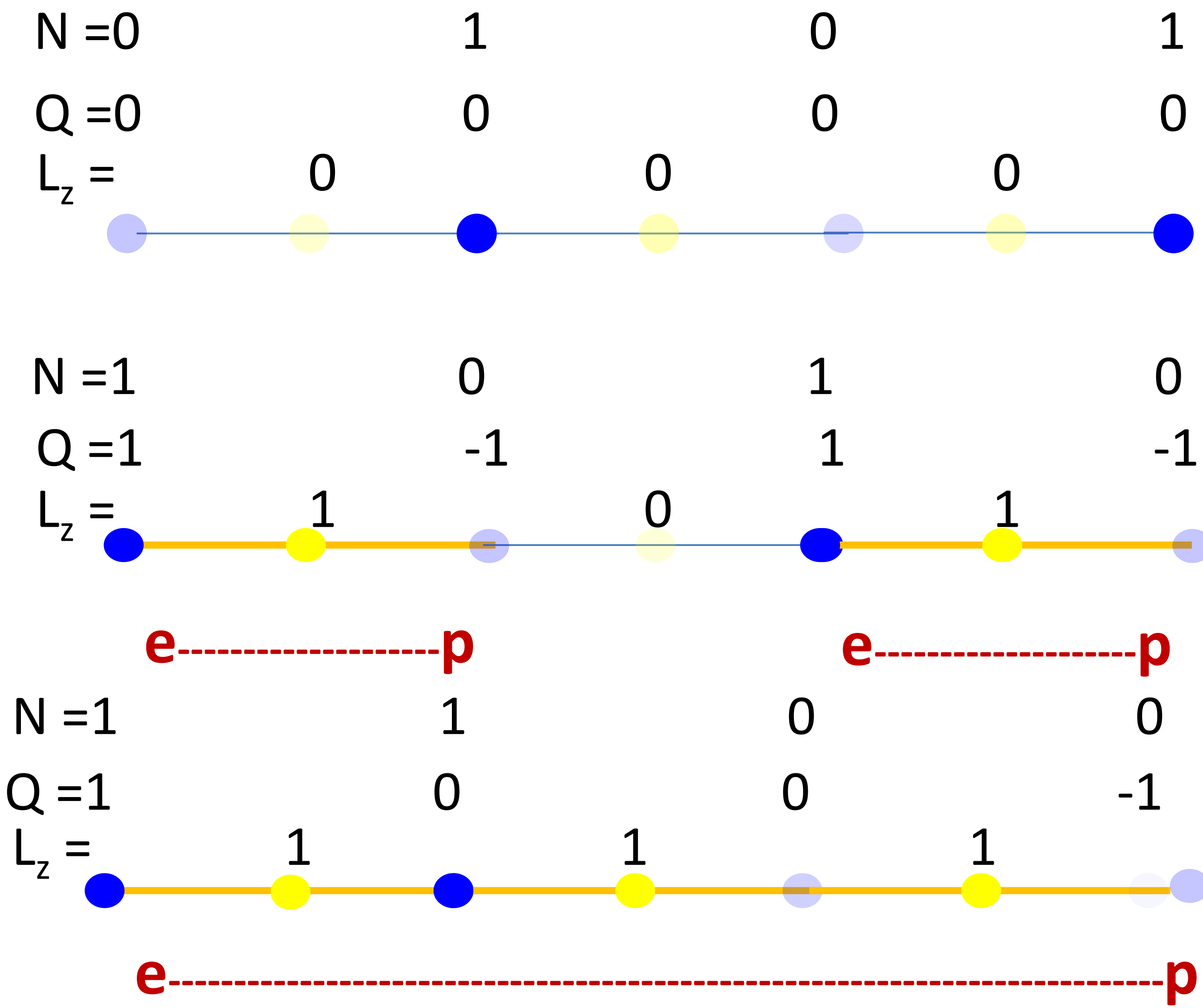
E = 0 0 0



Quark confinement

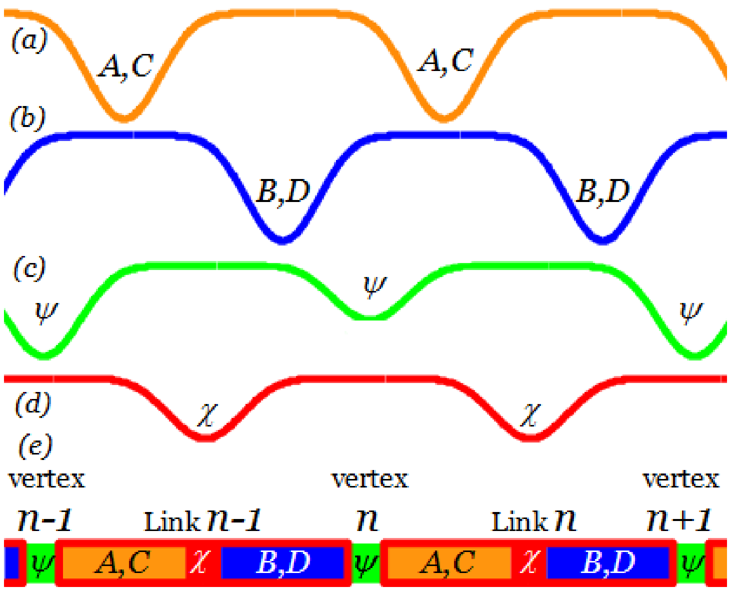


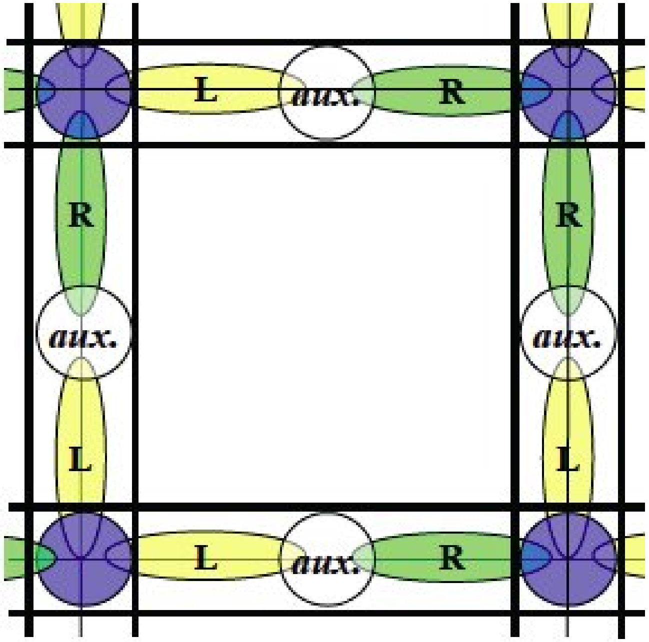
Quark confinement



SU(2) simulation

- Gauge field SU(2): 4 bosons on each link





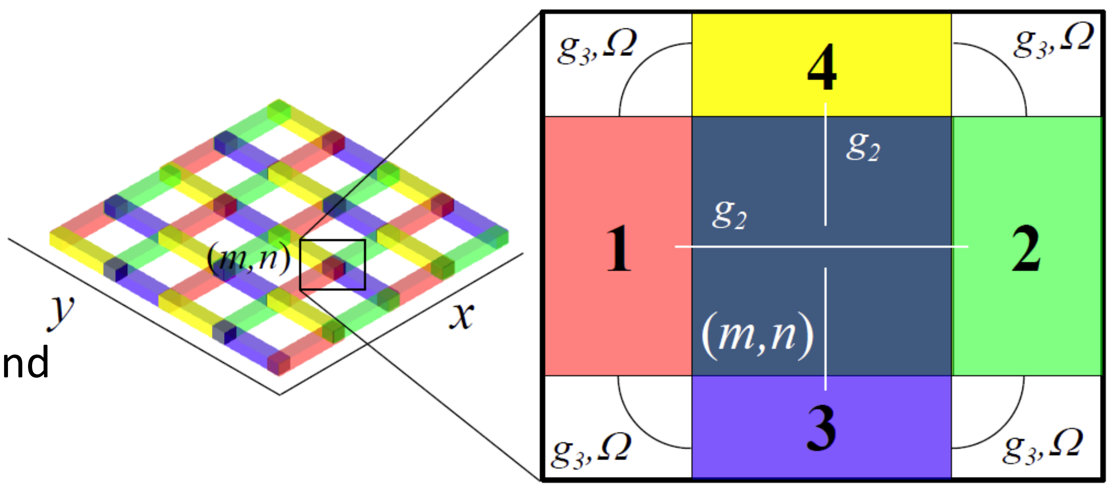
E. Zohar, J. Ignacio Cirac, BR,
Phys. Rev. Lett. 110, 125304 (2013)
E. Zohar, I. J. Cirac, BR, arXiv:1303.5040

Realization of plaquettes

Method 1: (effective gauge invariance)

Unlike in 1+1 dimensional case, here one has to obtain plaquette terms using effective methods.

For example – imposing Gauss's law as a constraint.



The abelian Kogut-Susskind model.

E. Zohar, BR, Phys. Rev. Lett. 107, 275301 (2011)

The “Loop Method”

E. Zohar, I. J. Cirac, BR, arXiv:1303.5040

1d elementary link interactions – **already gauge invariant building blocks** for effective plaquettes

Auxiliary fermions close the plaquettes

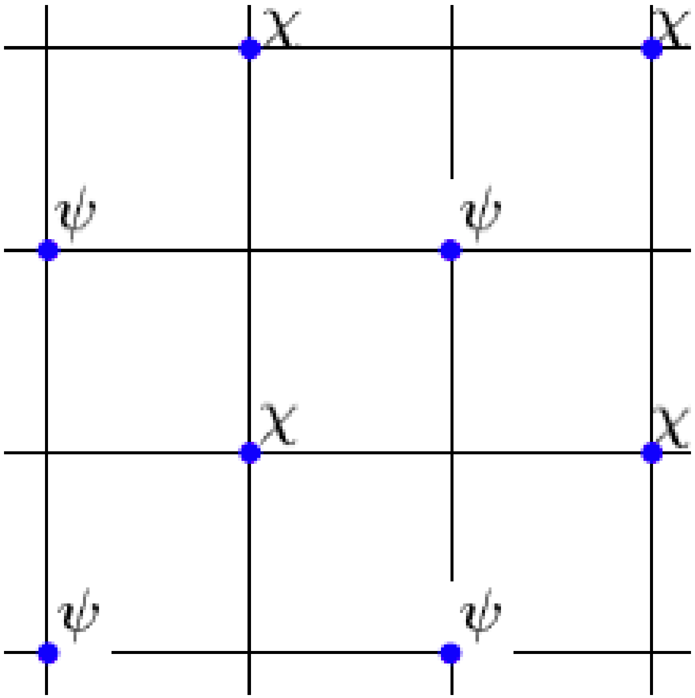


Figure from ref [6]

The “Loop Method”

1d elementary link interactions – **already gauge invariant building blocks** for effective plaquettes

Auxiliary fermions close
 the plaquettes!

$$-\frac{4\epsilon^4}{\lambda^3}\sum_{n}\cos\left(\phi_{n,1}+\phi_{n+\hat{1},2}-\phi_{n+\hat{2},1}-\phi_{n,2}\right)$$

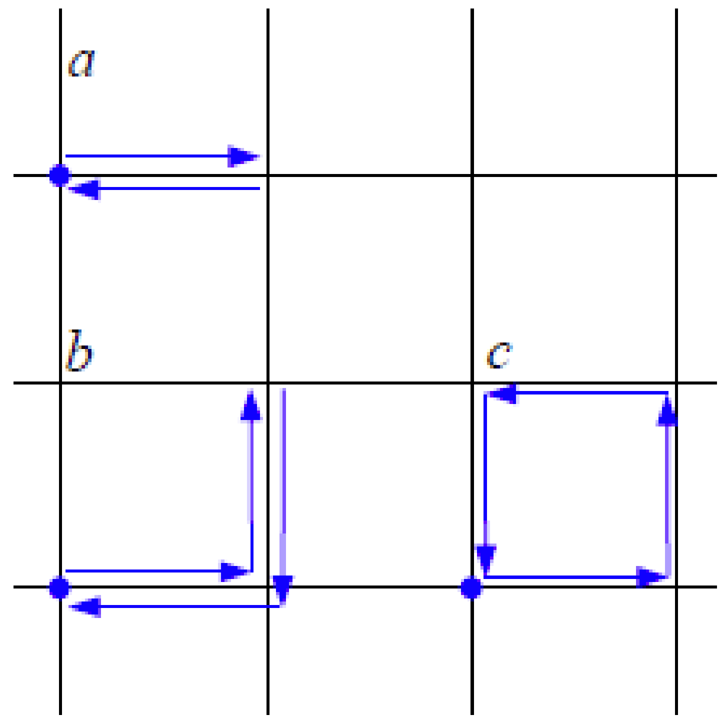


Figure from ref [6]

Simulating Z_N Gauge theory

E. Zohar, I. J. Cirac, BR, arXiv:1303.5040

- Finite Hilbert spaces on links: one can realize *unitary* operators in the elementary link interactions, obtained using hybridized levels
- In a pure gauge theory, plaquettes are obtained similarly, using the "loop method"

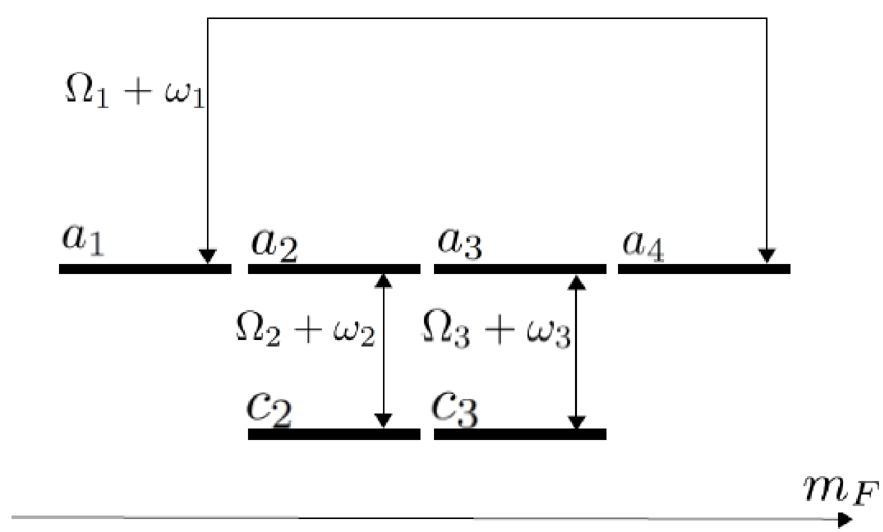
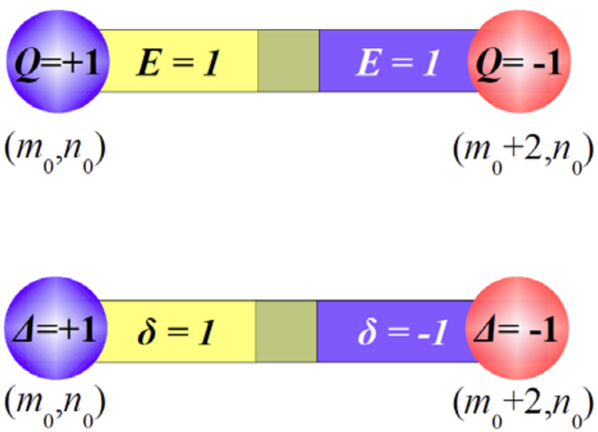


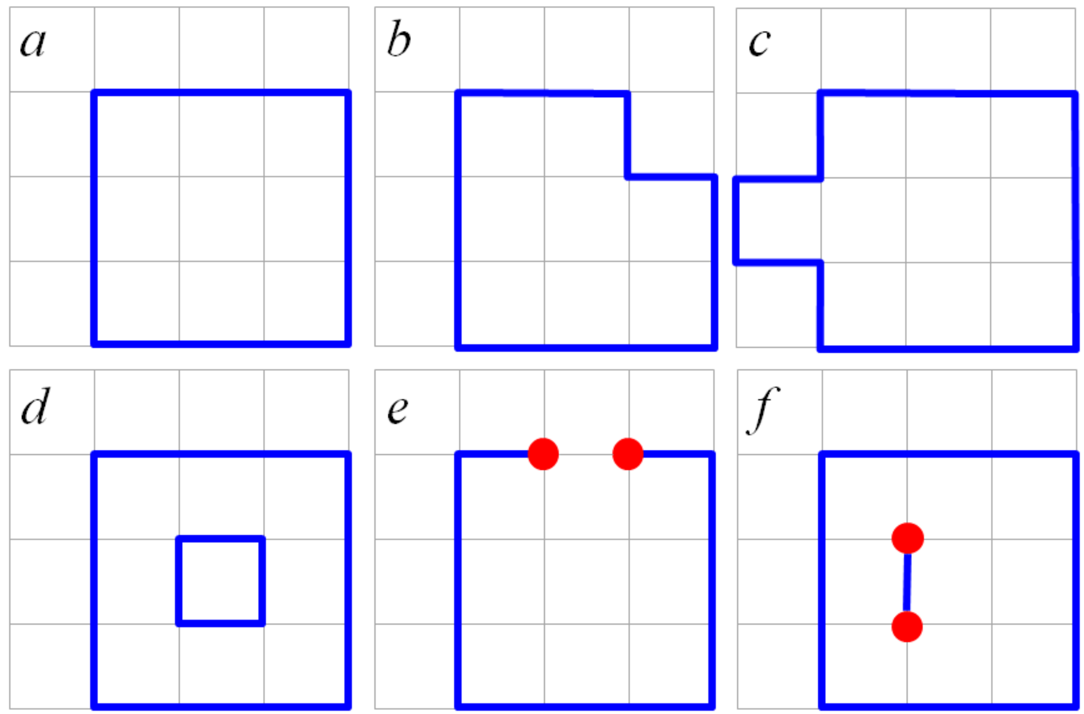
Figure from ref [6]

Confinement, flux breaking & glueballs



Electric flux tubes

E. Zohar, BR,
Phys. Rev. Lett. 107, 275301 (2011).



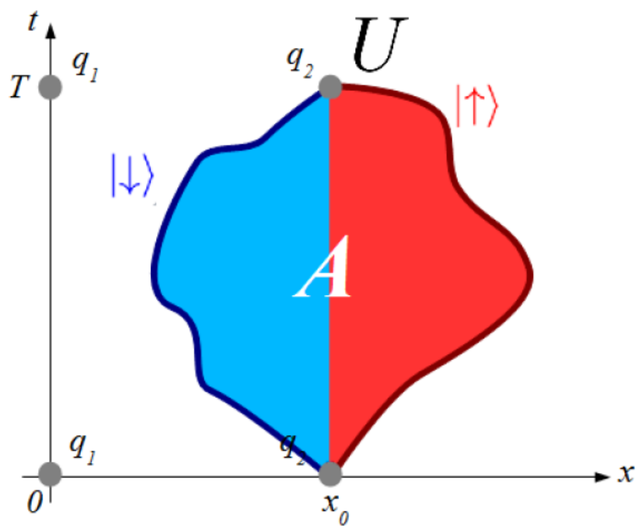
Flux loops deforming and breaking effects

E. Zohar, J. I. Cirac, BR,
Phys. Rev. Lett. 110, 055302 (2013)

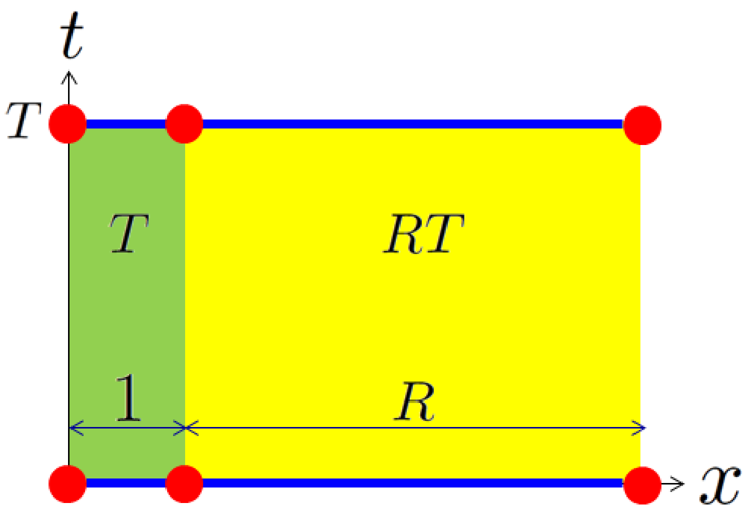
Wilson loop measurments

$$W(C) = P\left(e^{i\oint_C A_\mu dx^\mu}\right)$$

Detecting Wilson Loop's area law by interference of "Mesons".



E. Zohar , BR, New J. Phys. 15 (2013) 043041



E. Zohar, J. Ignacio Cirac, BR, PRL (2013).

Our current situation

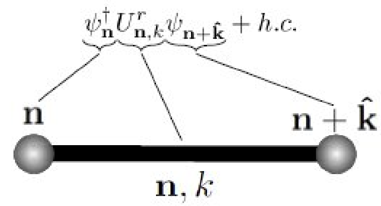
Theory	1+1 Pure	1+1 with matter	d+1 Pure	d+1 with matter
U(1) - cQED	Trivial	K.S. and truncated ✓	K.S. and truncated ✓	K.S. and truncated ✓
SU(2) – Yang Mills	Trivial	Full Simulation ✓	Strong limit Simulation ✓ ✗	Strong limit Simulation ✓ ✗
Z_N	Trivial	✗	✓	✗

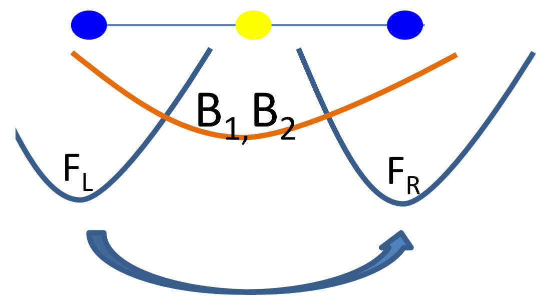
K.S. = Kogut Susskind Hamiltonian LGT

Summary

- Gauge invariant links

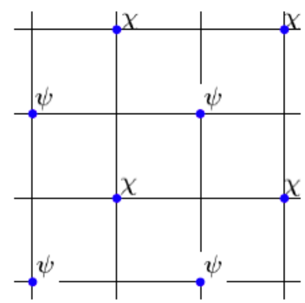
But given natural interactions:

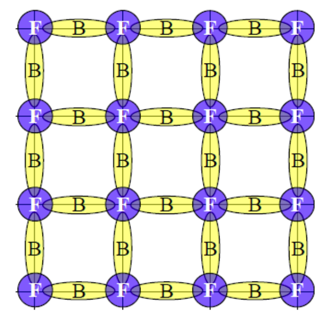




- We presented the loop method

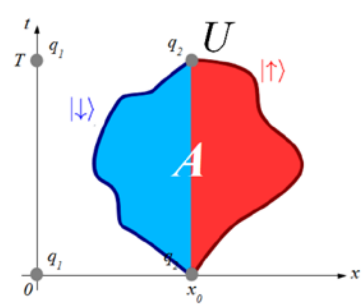
For generating plaquette interactions
In 2+1 and higher D.

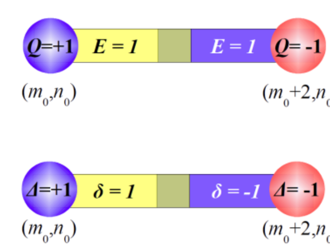




- Probing of Confinement, string tension pair

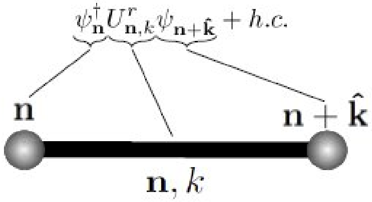
Production, string tension, gluons,
Wilson loops, non perturbative effects'
Phase transitions….

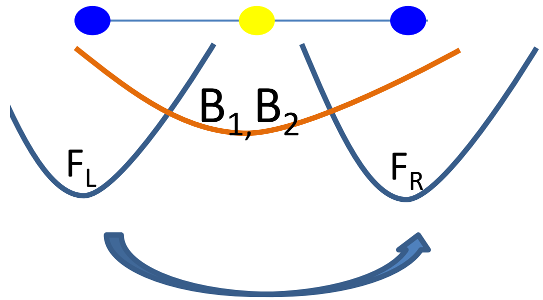


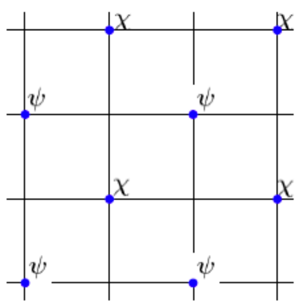


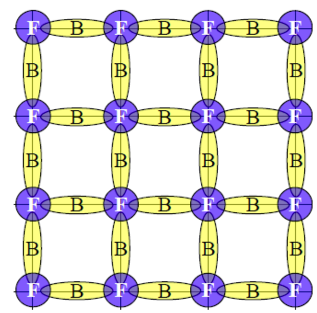
Figures from refs. [1,3,6]

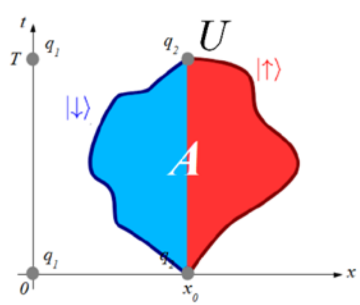
Thank you for your attention !

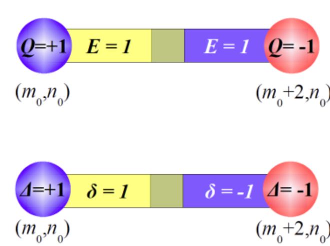












Figures from refs. [1,3,6]