



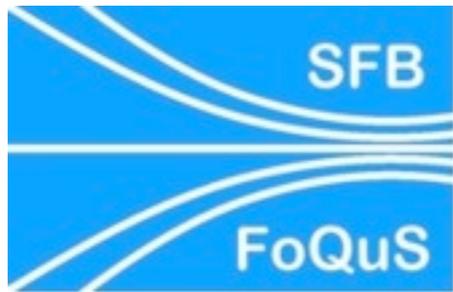
UNIVERSITY OF INNSBRUCK



IQOQI  
AUSTRIAN ACADEMY OF SCIENCES

# An AMO Toolbox for Majorana Fermions in Optical Lattices

Christina Kraus  
University of Innsbruck/IQOQI



Foundations and Applications of Quantum Science

EU AQUTE



Nanodesigning of atomic and molecular quantum matter



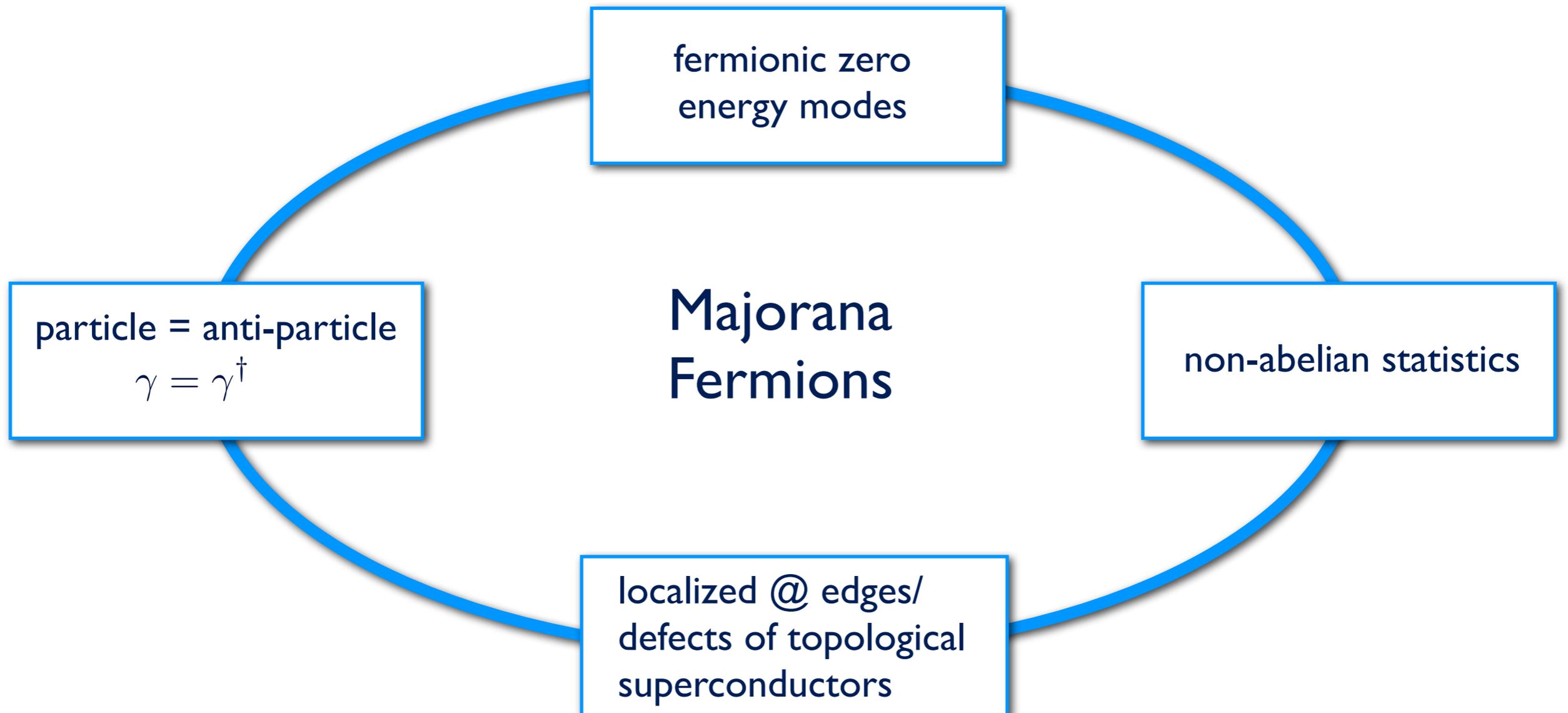
# Outline

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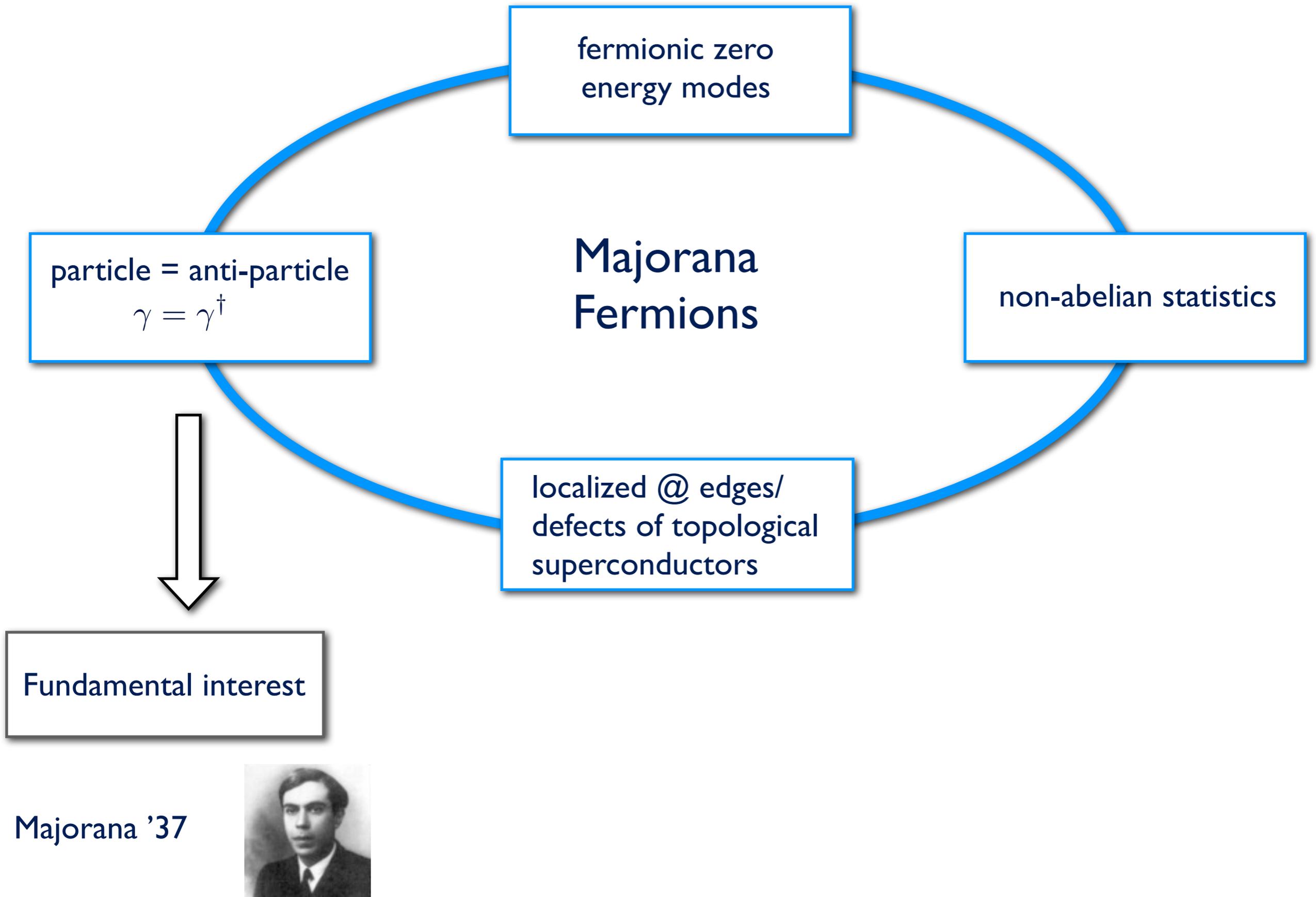
1. Introduction to Majorana Fermions
2. An AMO toolbox for Majorana fermions in the Kitaev wire
3. Majorana physics in the double wire system with pair hopping
4. Outlook

# Majorana Fermions are/have...

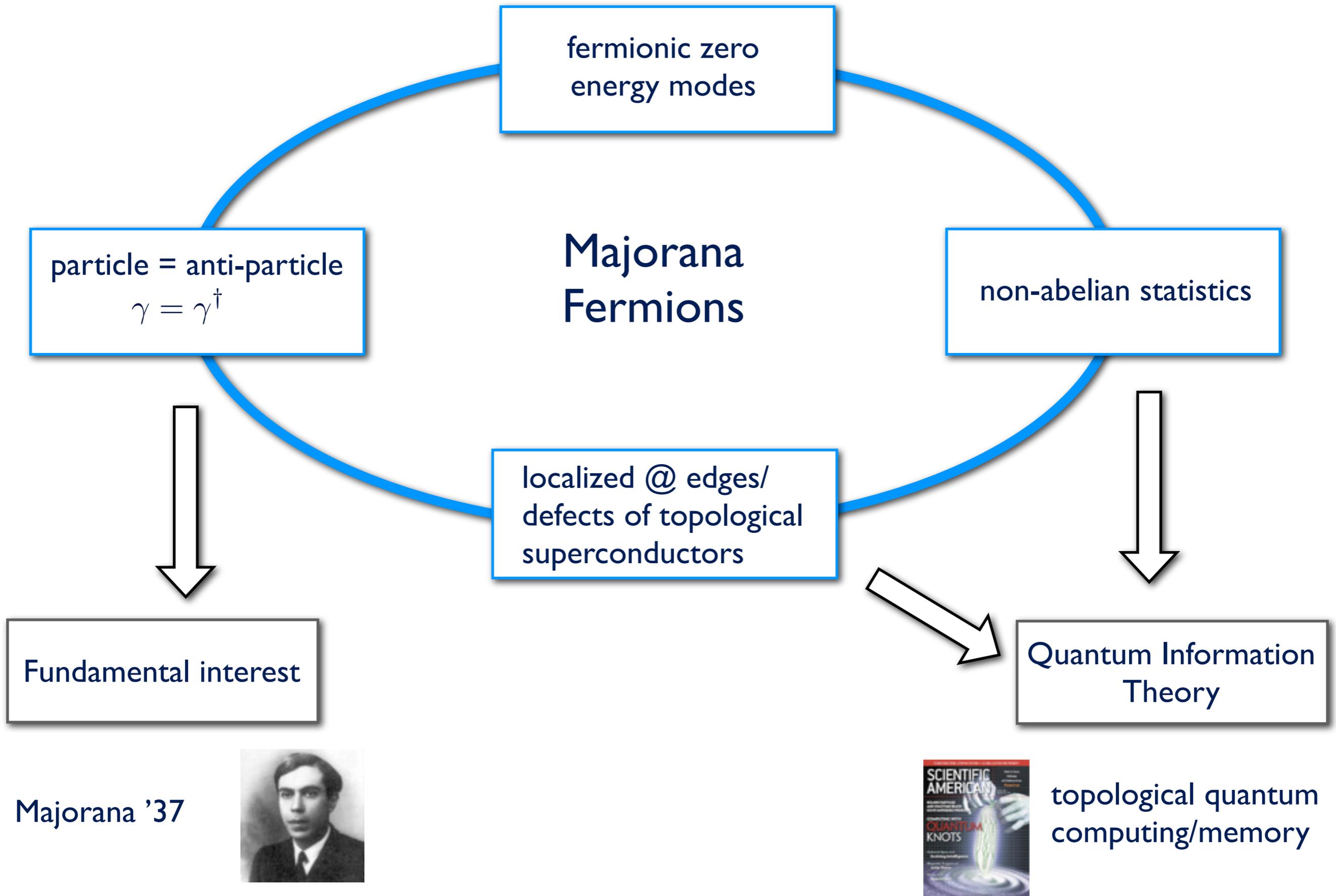
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# Solid state physics

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PHYSICAL REVIEW LETTERS

week ending  
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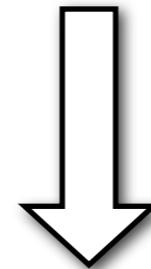
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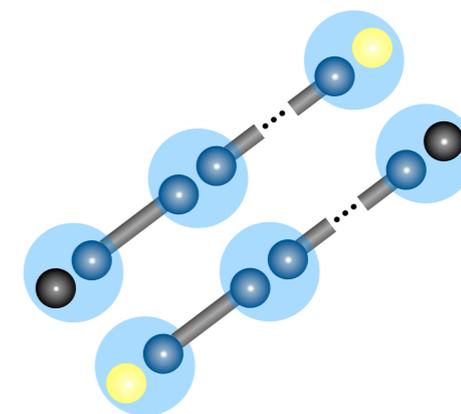
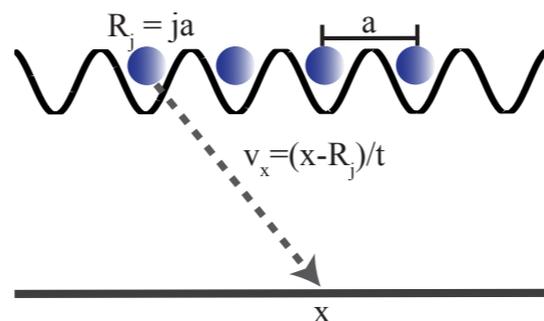
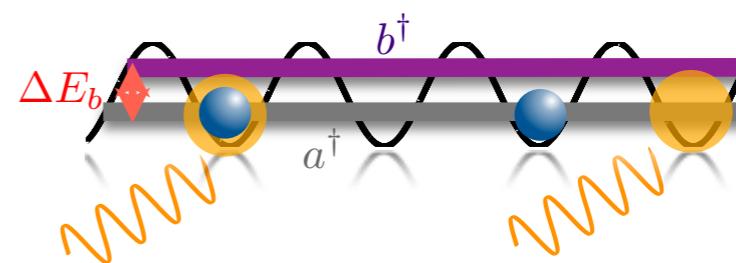
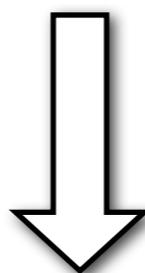
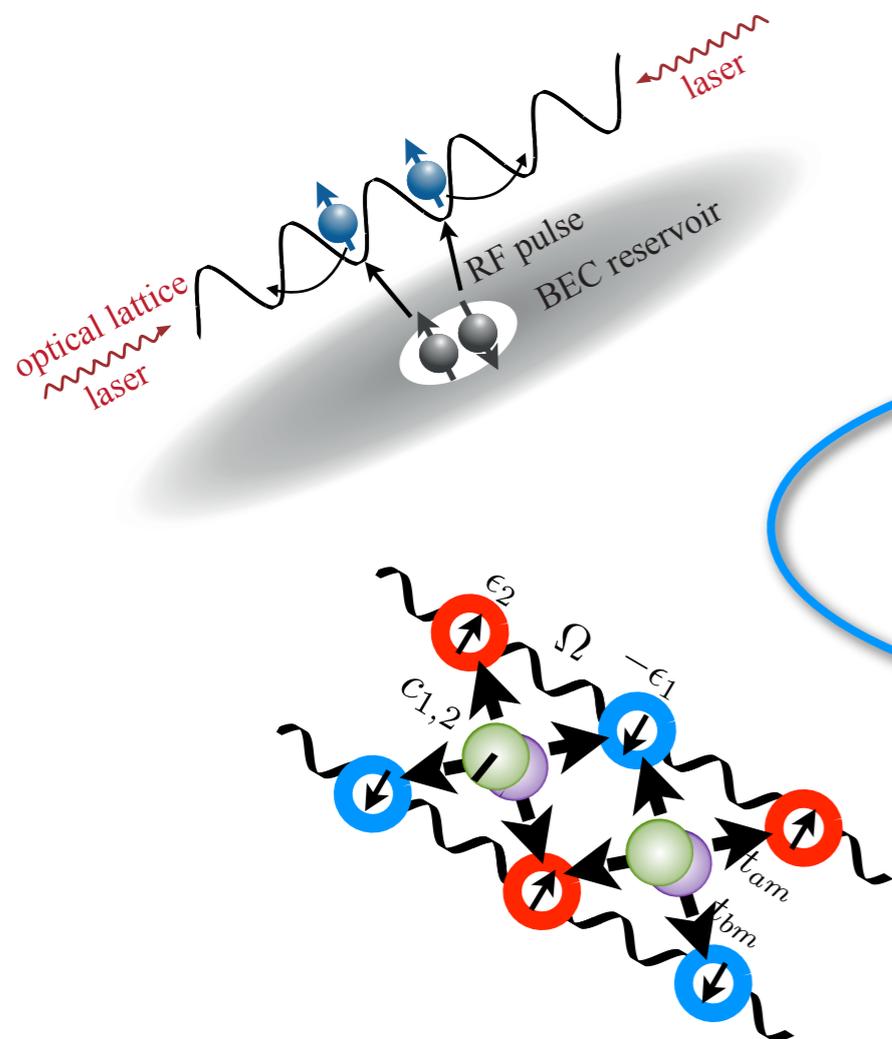
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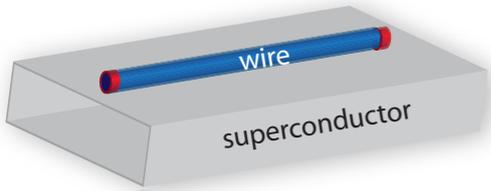
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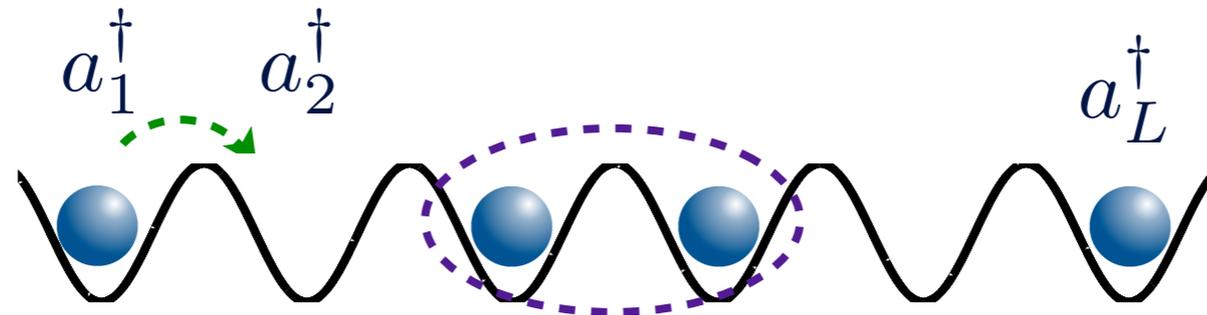


# Majorana Fermions in the Kitaev Wire

$$H = -J \sum_{j=1}^{L-1} a_j^\dagger a_{j+1} + h.c. + \sum_{j=1}^{L-1} \Delta a_j a_{j+1} + h.c. - \mu \sum_{j=1}^L a_j^\dagger a_j$$



A. Kitaev  
Physics-Uspekhi, 44, 131 (2001)



## Bogoliubov transformation

$$J = |\Delta|, \mu = 0$$

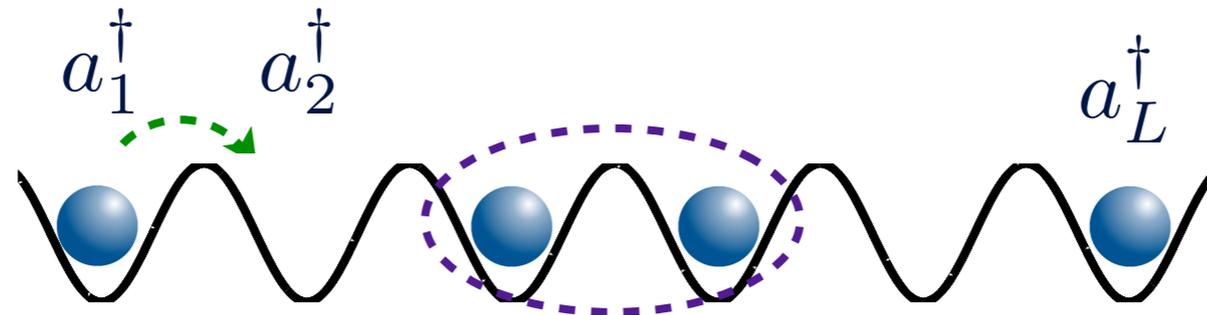
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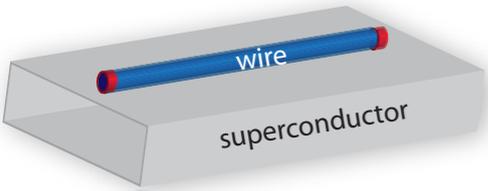
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$$c_{2j} = (-i)(a_j^\dagger - a_j)$$

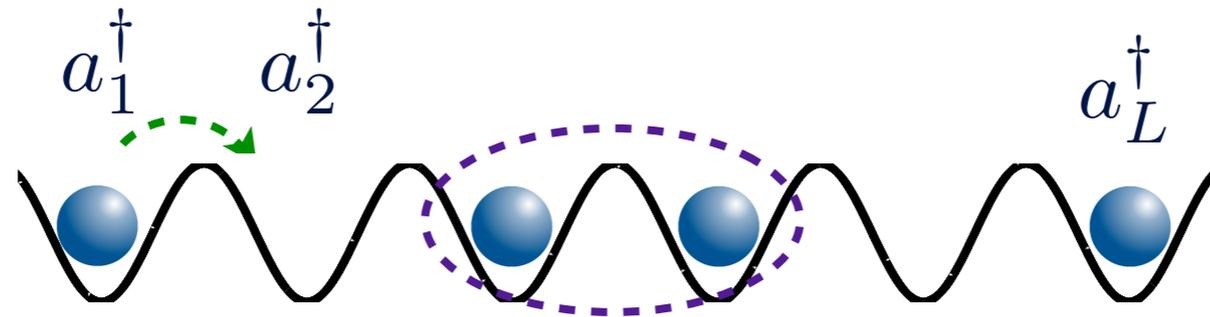
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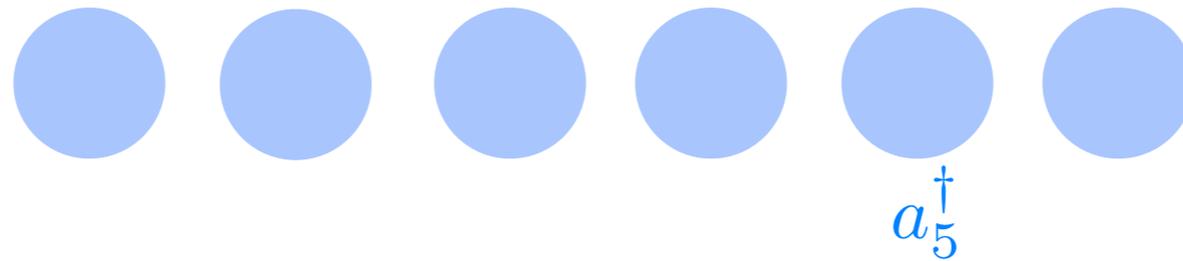
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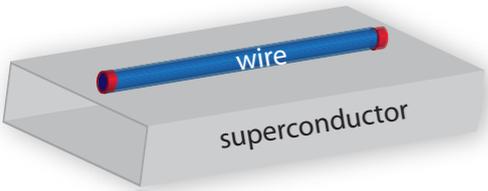
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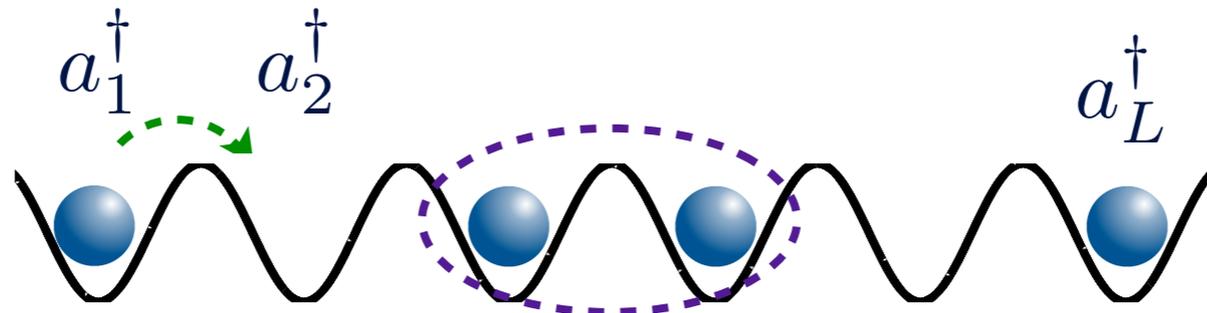


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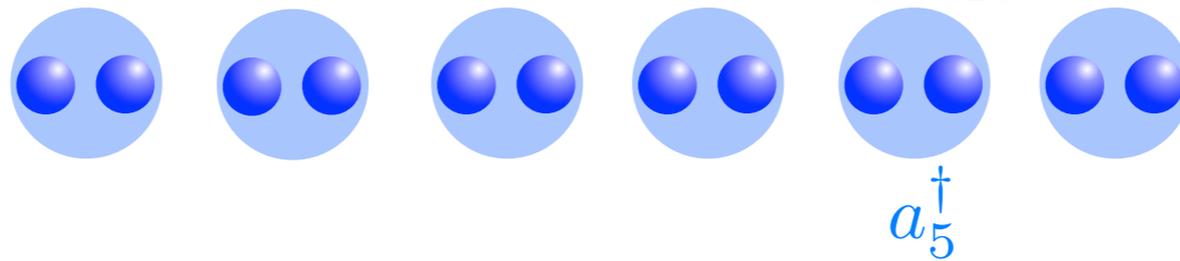
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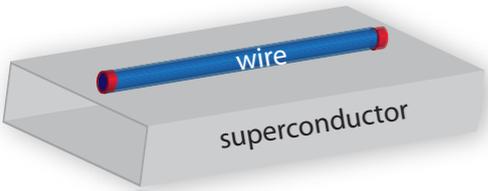
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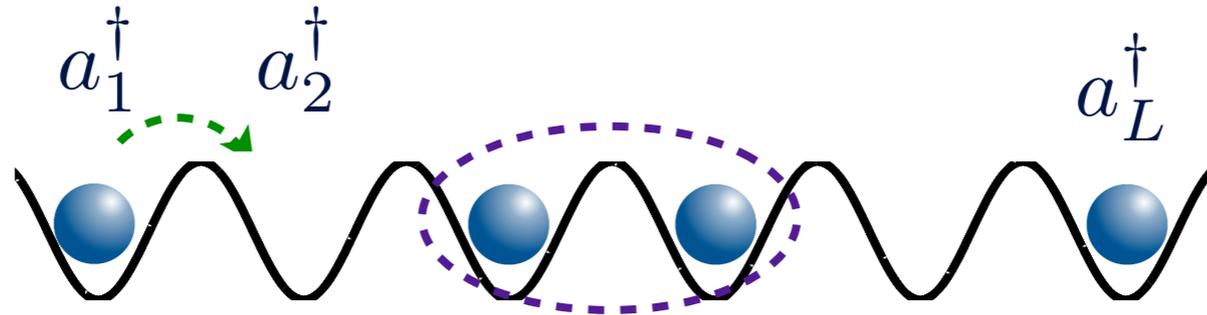


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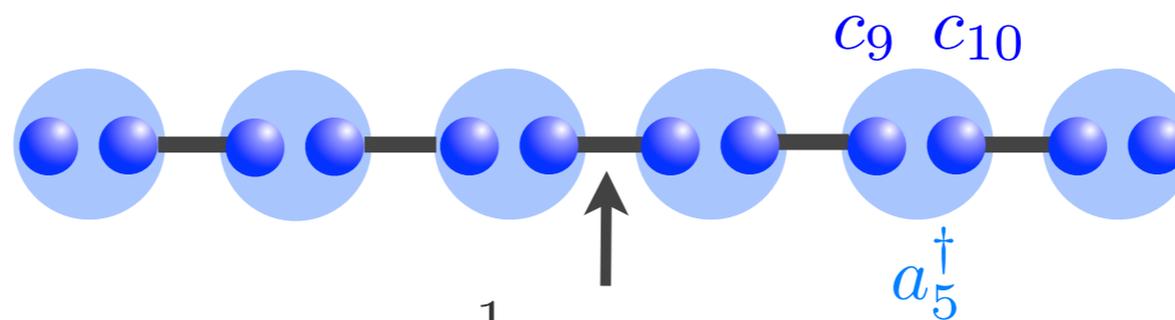
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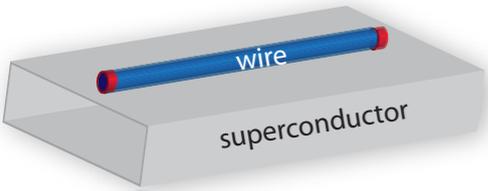
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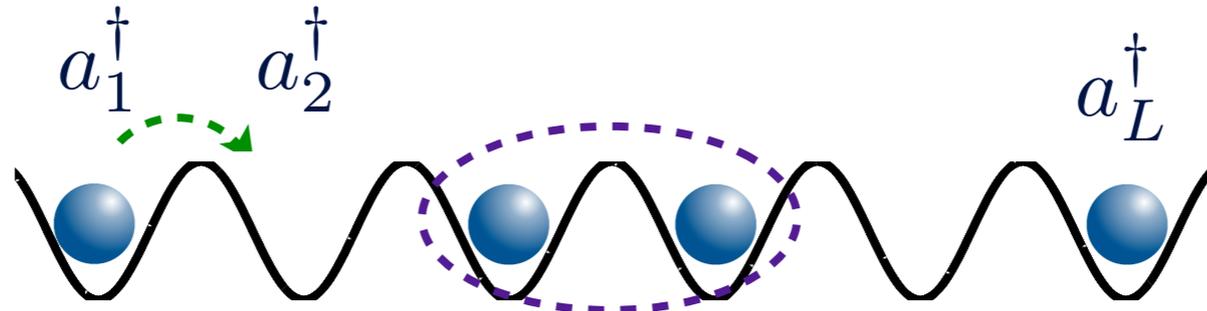
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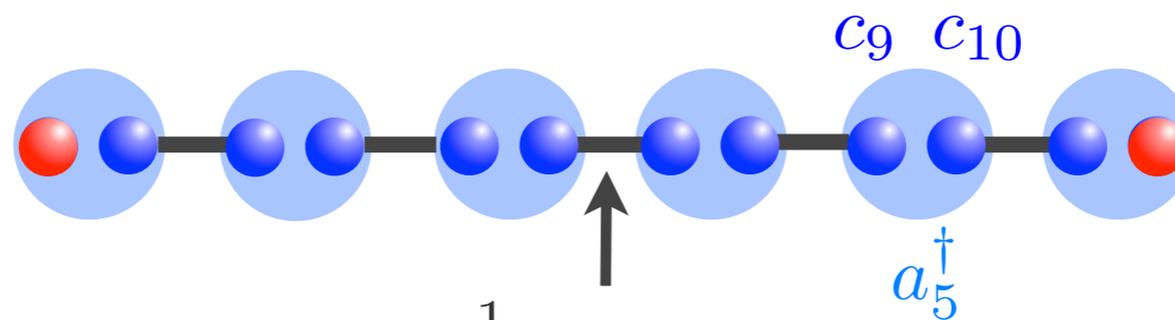
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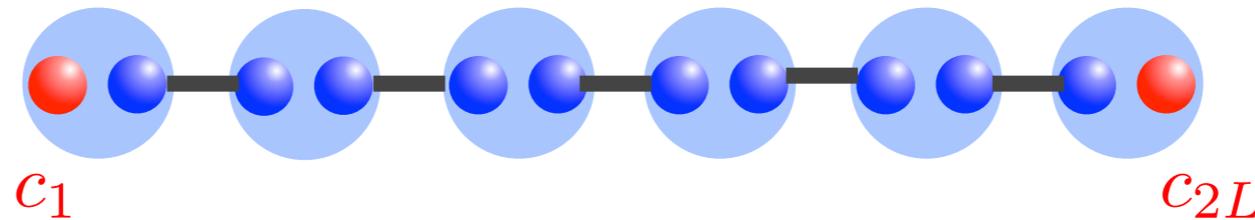
$$c = c^\dagger \quad c^2 = 1$$



$$\tilde{a}_j^\dagger = \frac{1}{2}(c_{2j} + ic_{2j+1})$$

# Properties of Majorana Fermions

$$H = \sum_{j=1}^{L-1} -J a_j^\dagger a_{j+1} + \Delta a_j a_{j+1} + h.c. - \mu \sum_{j=1}^L a_j^\dagger a_j$$

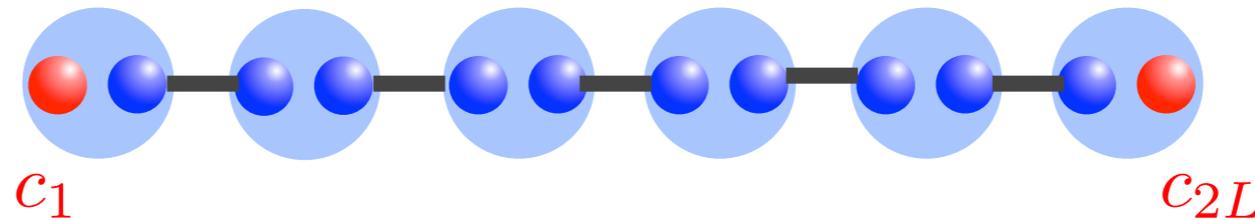


# Properties of Majorana Fermions

Two unpaired  
Majorana fermions

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$$c^2 = 1$$

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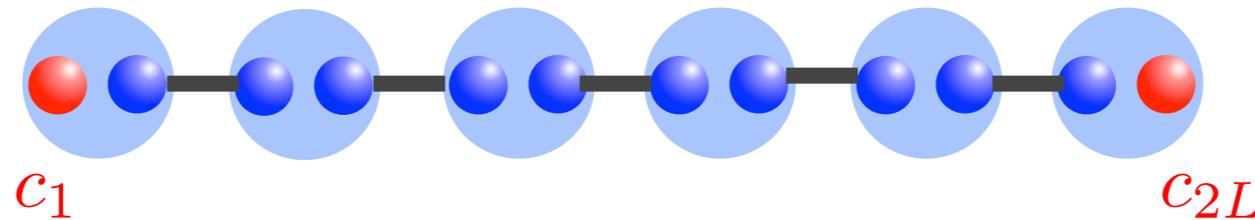
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Non-local zero  
Energy Mode

$$\tilde{a}_L^\dagger = \frac{1}{2}(c_{2L} + ic_1)$$

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# Properties of Majorana Fermions

Two unpaired  
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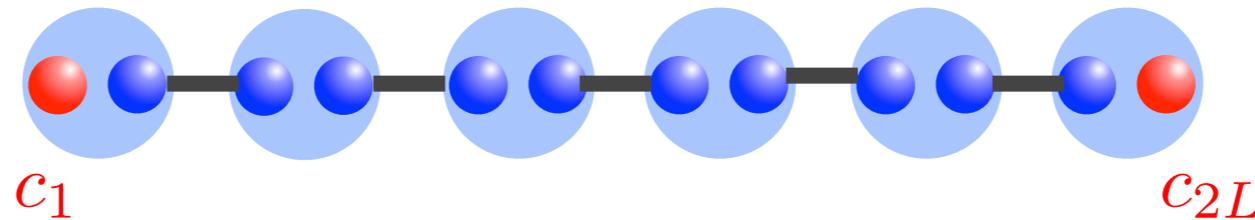
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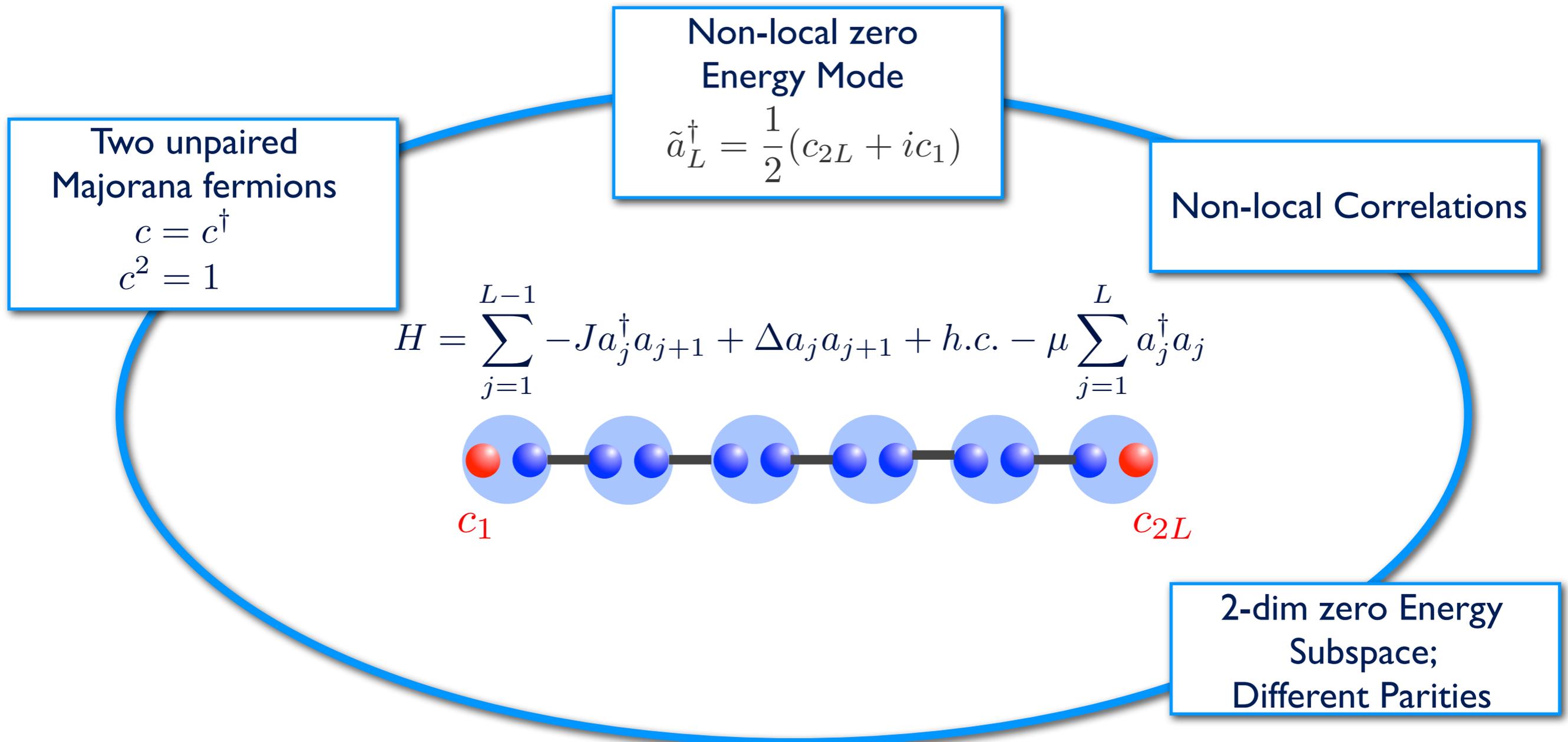
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Non-local Correlations

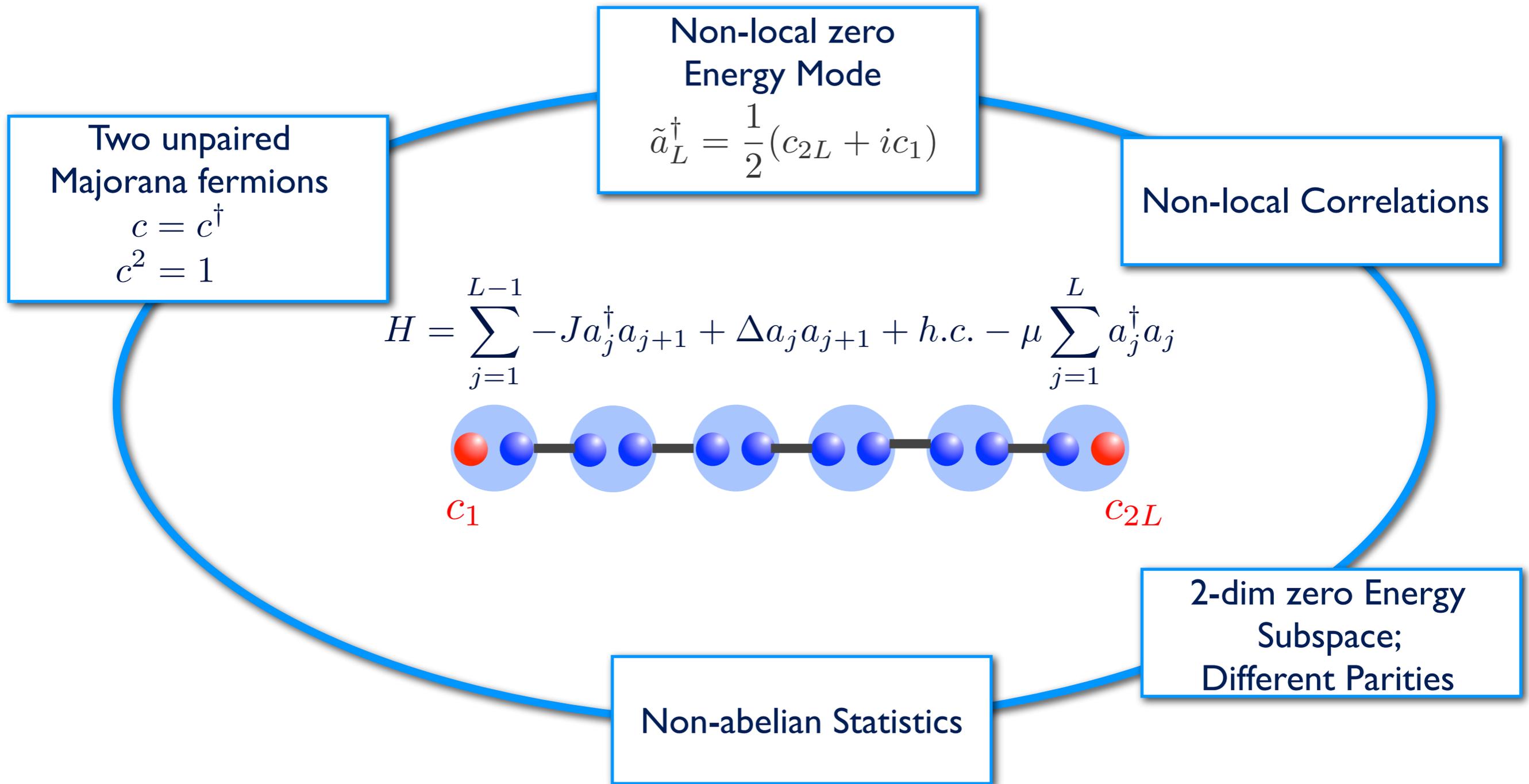
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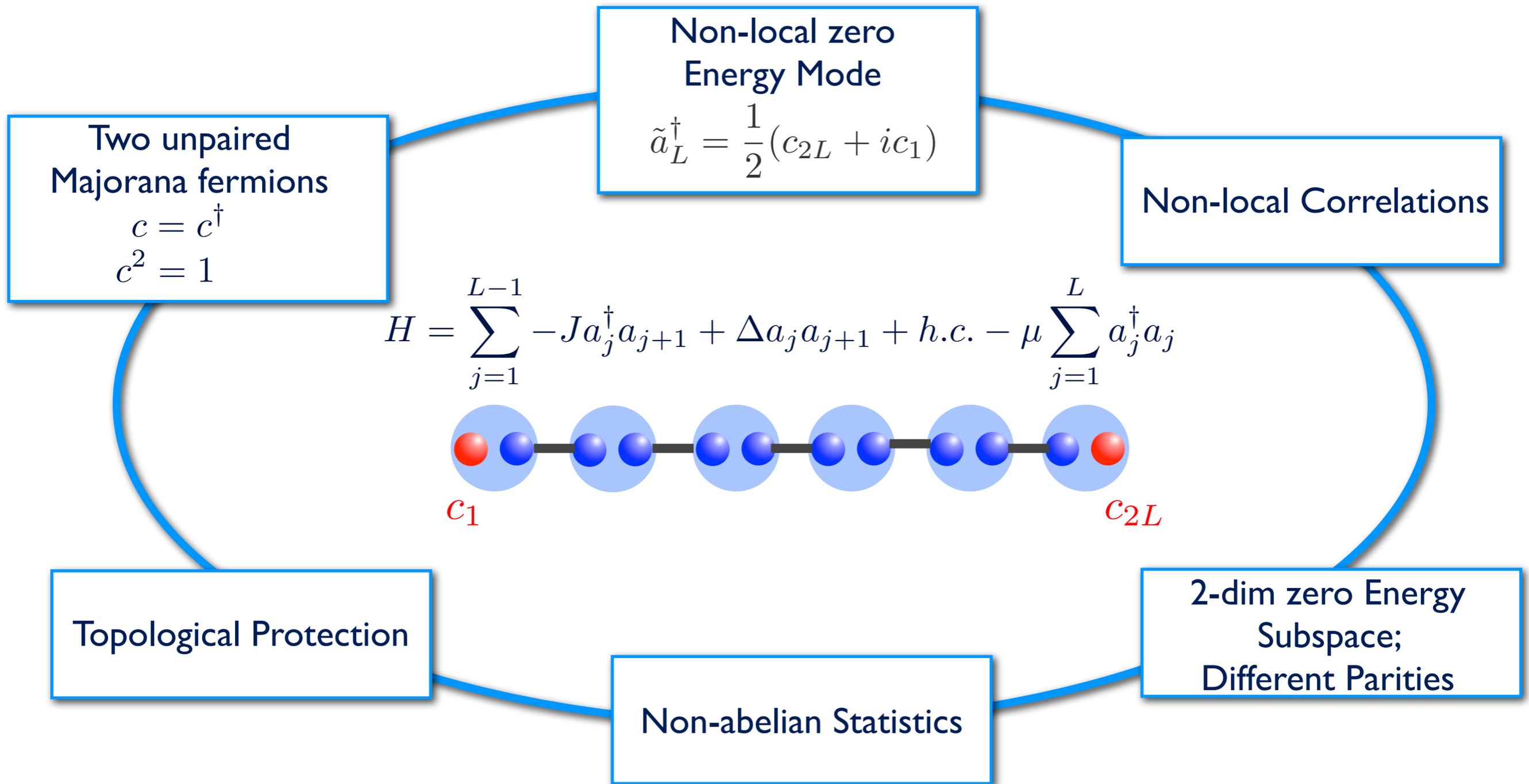
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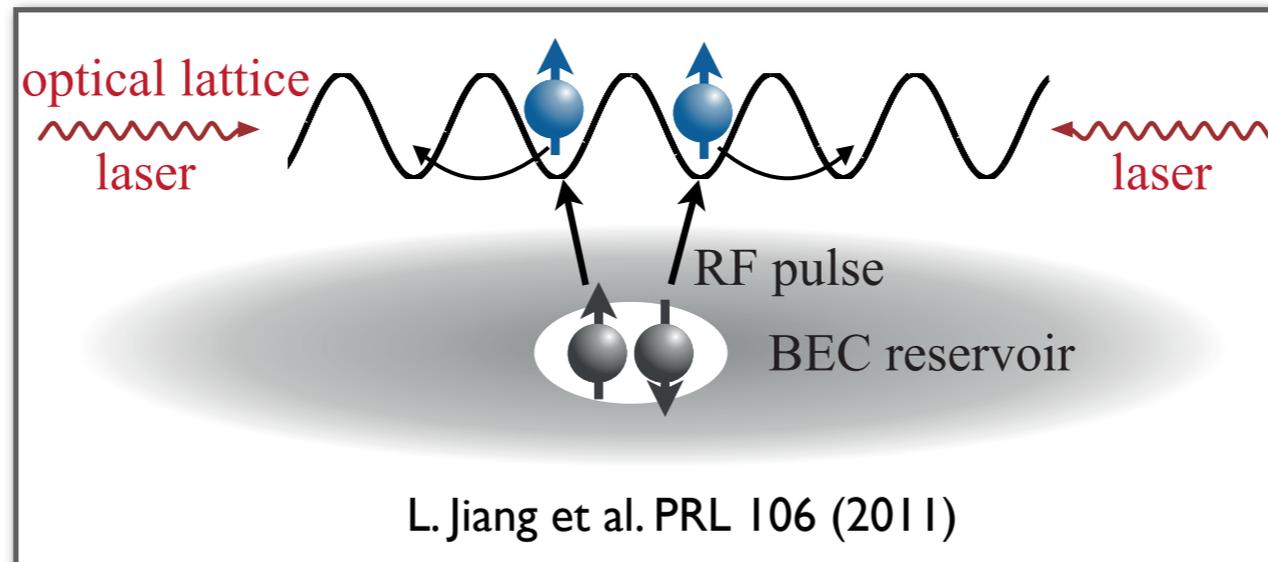


# Properties of Majorana Fermions



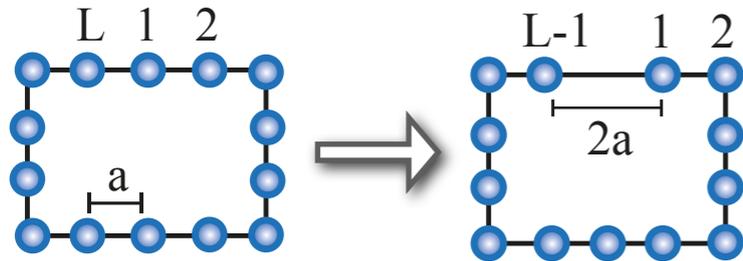
# Exploring Majorana Physics in the Kitaev Wire using AMO Tools

# Majorana Physics in an AMO Realization of the Kitaev Wire



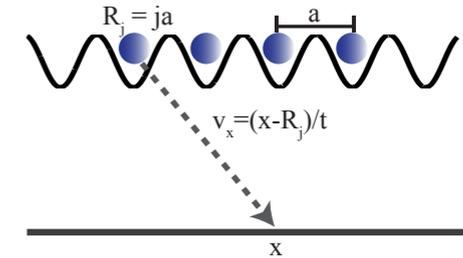
# Majorana Physics in an AMO Realization of the Kitaev Wire

Preparation of ground state with a definite parity

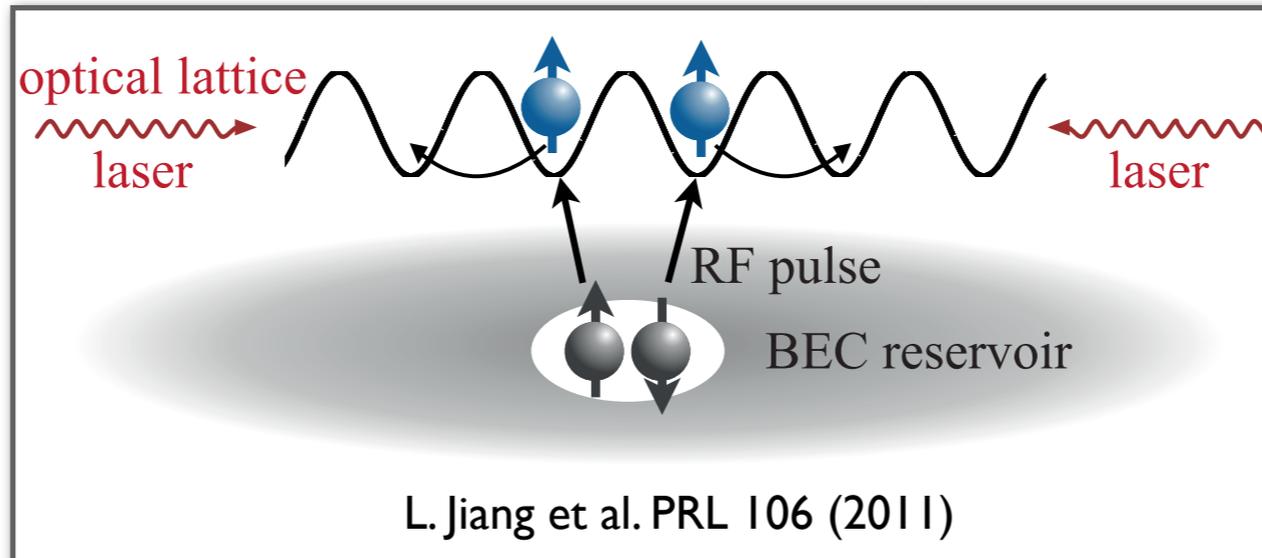


Detection with AMO tools

- Time-of-flight imaging
- Spectroscopy



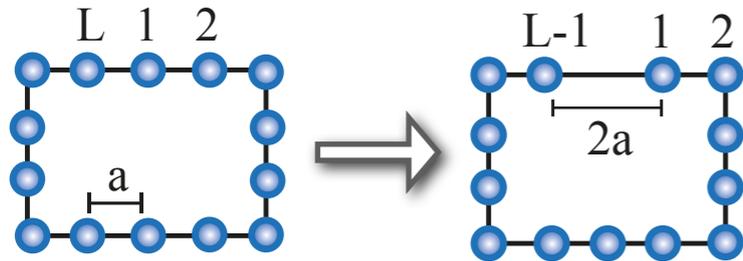
C.K, S. Diehl, P. Zoller, M. Baranov, NJP 2012



L. Jiang et al. PRL 106 (2011)

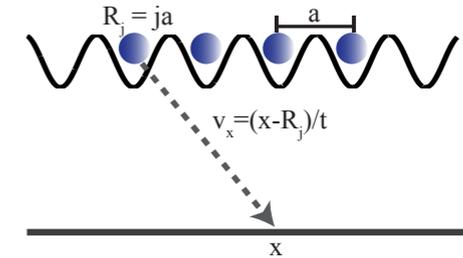
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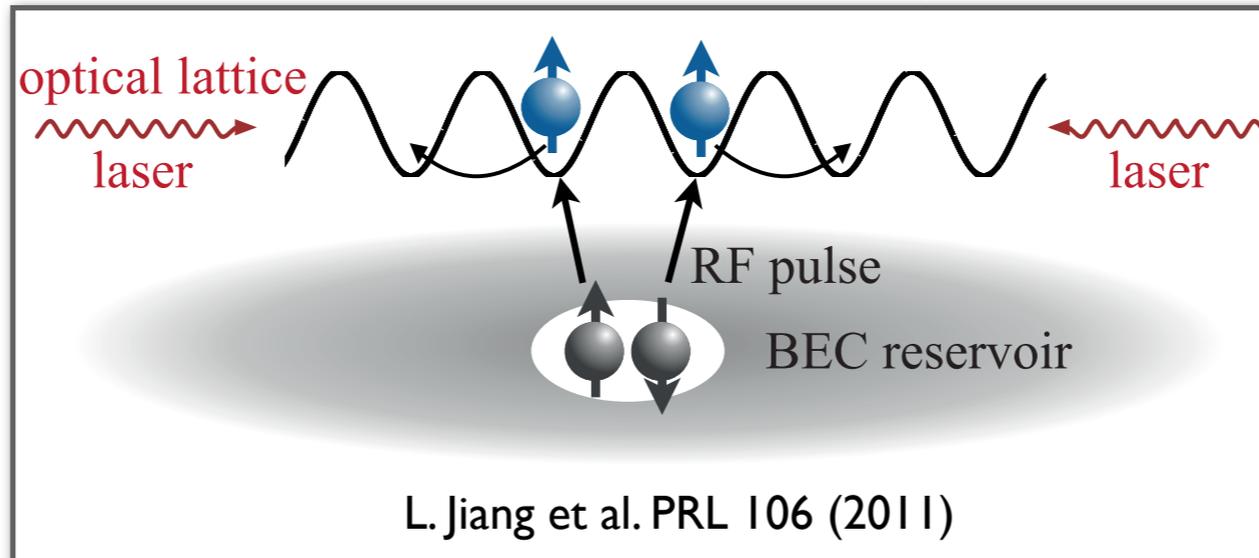


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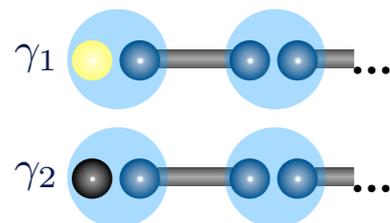
C.K, S. Diehl, P. Zoller, M. Baranov, NJP 2012



L. Jiang et al. PRL 106 (2011)

## Efficient braiding of MFs

- local site addressing
- 4 sites, 4 time steps

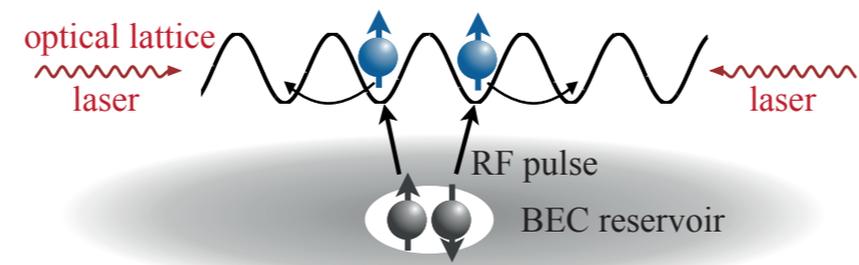


Implementation of Deutsch-Jozsa algorithm for 2 qubits via braiding only

(Braiding of MFs not universal!)

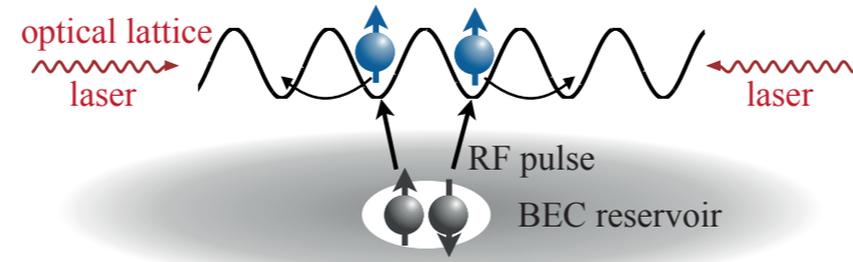
C.K, P. Zoller, M. Baranov, arxiv: 1302.1824

## Realization of the Kitaev wire...

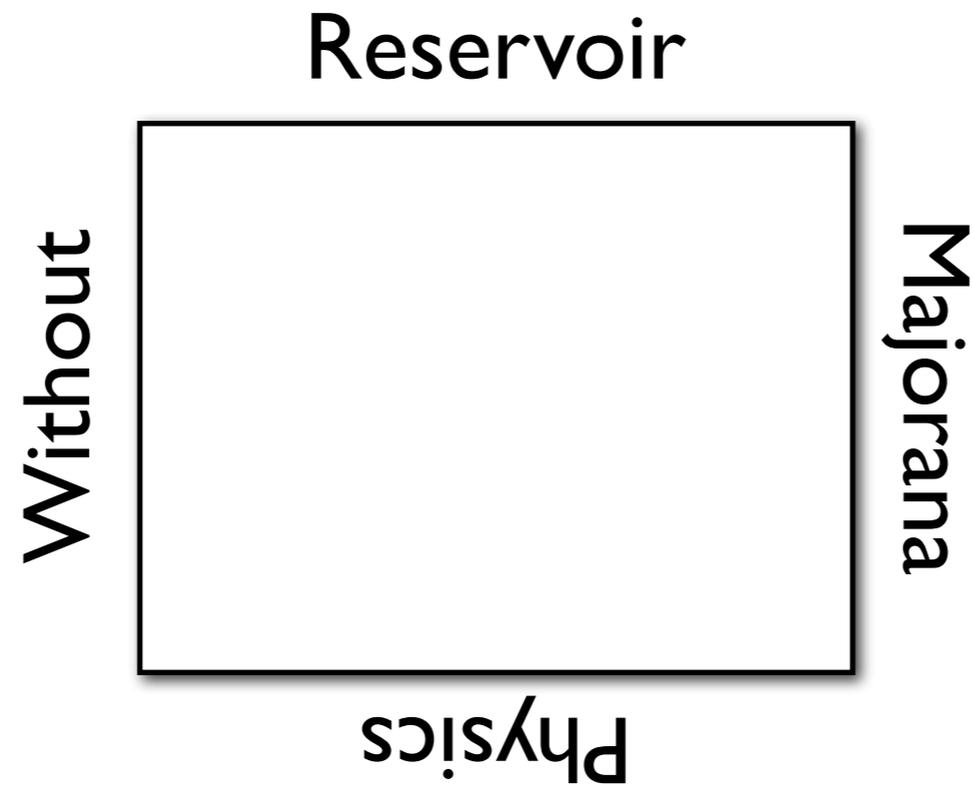


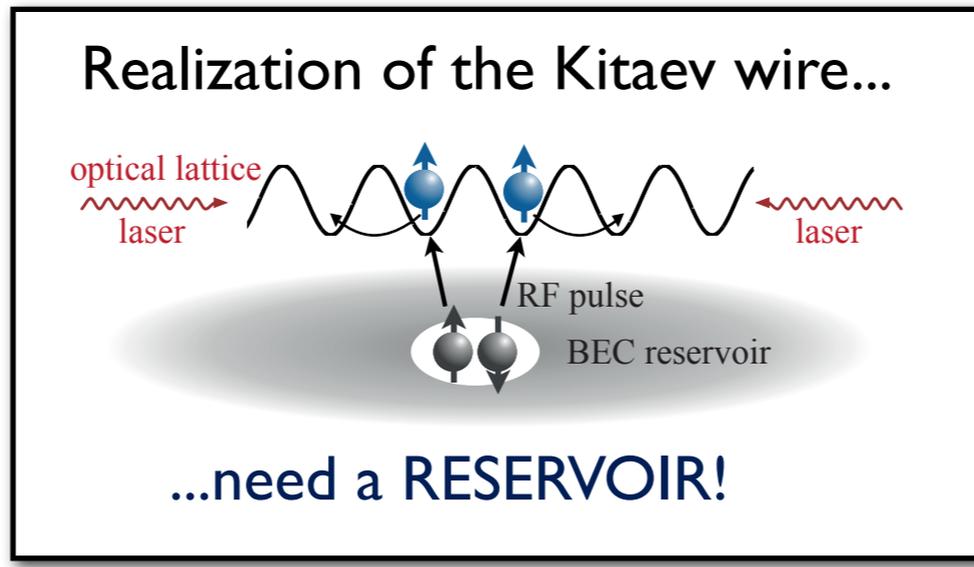
...need a RESERVOIR!

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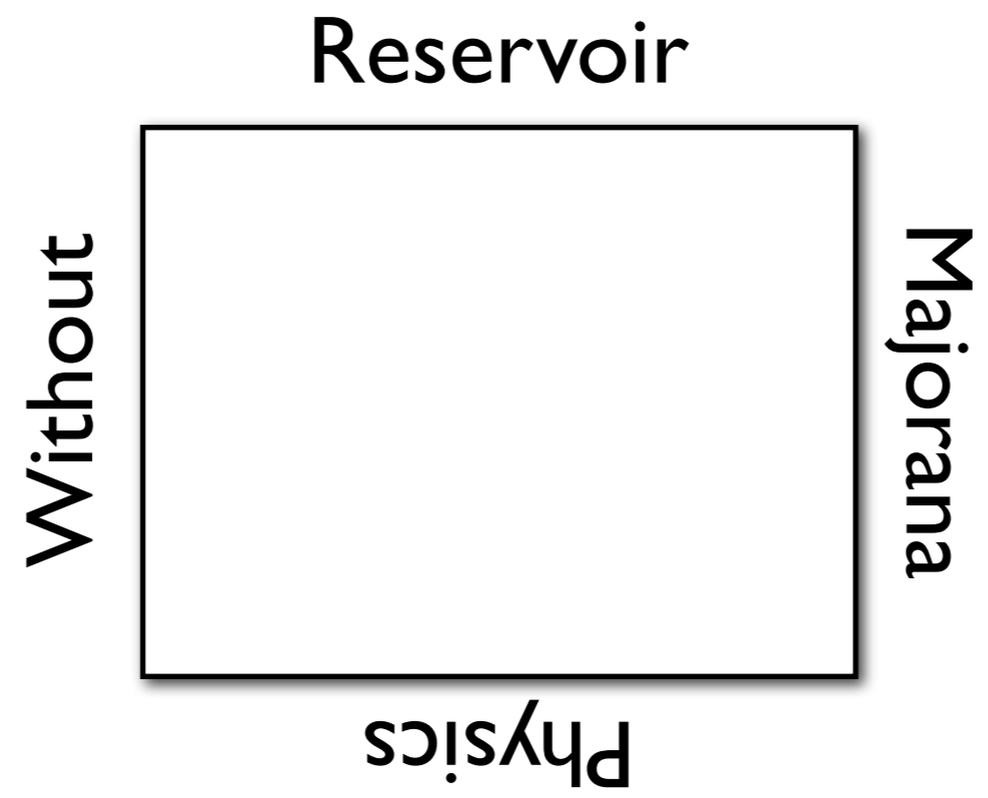


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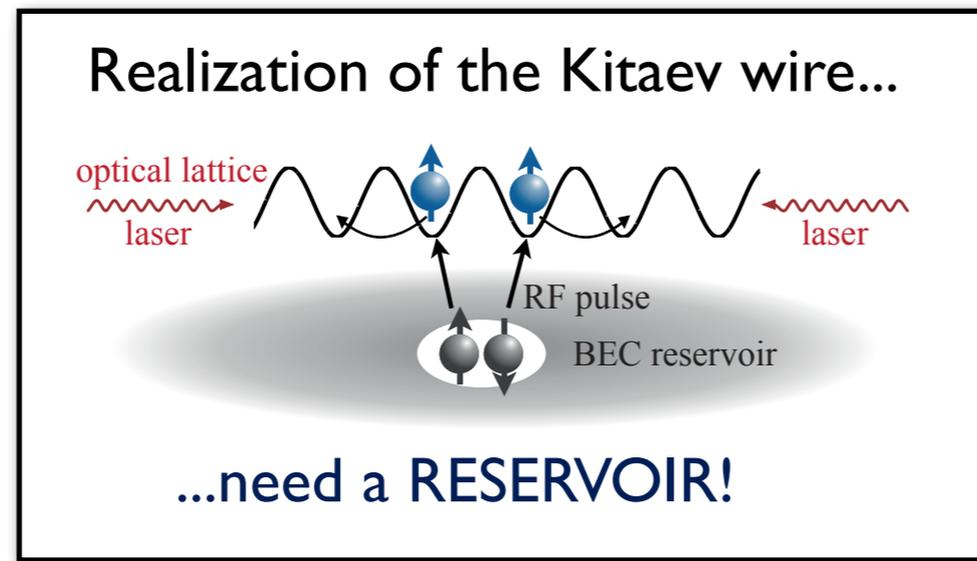




# Majorana Physics without Reservoir

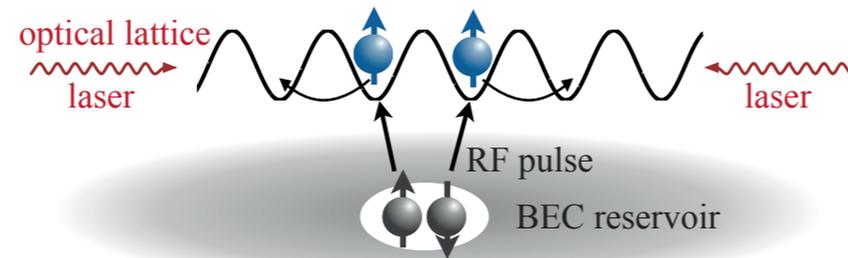


# An alternative Route towards Majorana Physics



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...need a RESERVOIR!

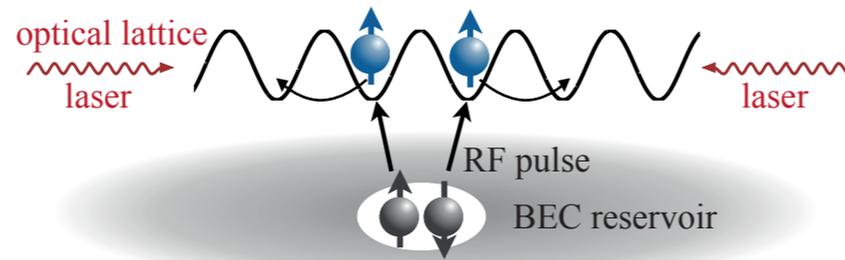
...BUT...

Signatures that MFs can emerge in a purely number conserving setting!

M. Cheng, H.H. Tu, PRB 84 (2011)  
L. Fidkowski, R. Lutchyn, C. Nayak, M. Fisher, PRB 84 (2011)  
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AMO realization of an  
INTERACTING system without  
RESERVOIR supporting Majoranas!

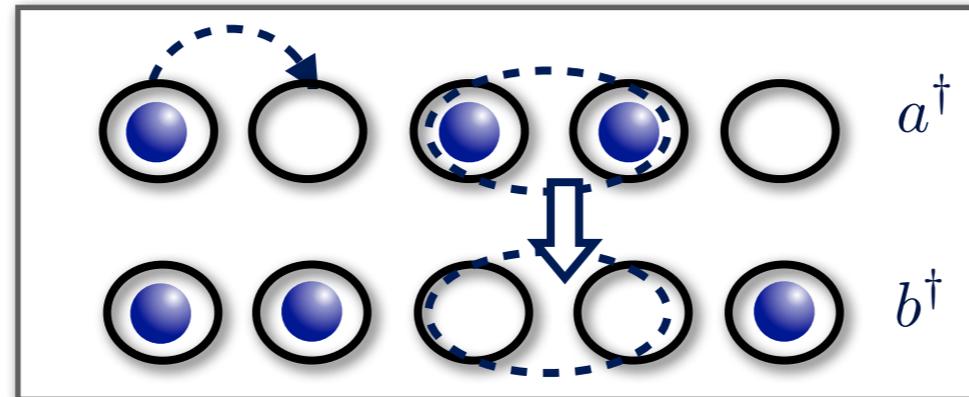
# Majorana Fermions in a Double Wire with Pair Hopping

## Role of the reservoir

- creates  $\Delta a_j^\dagger a_{j+1}^\dagger + h.c.$
- $U(1) \rightarrow \mathbb{Z}_2$



“minimal reservoir?”



SYSTEM  
↕  
BATH

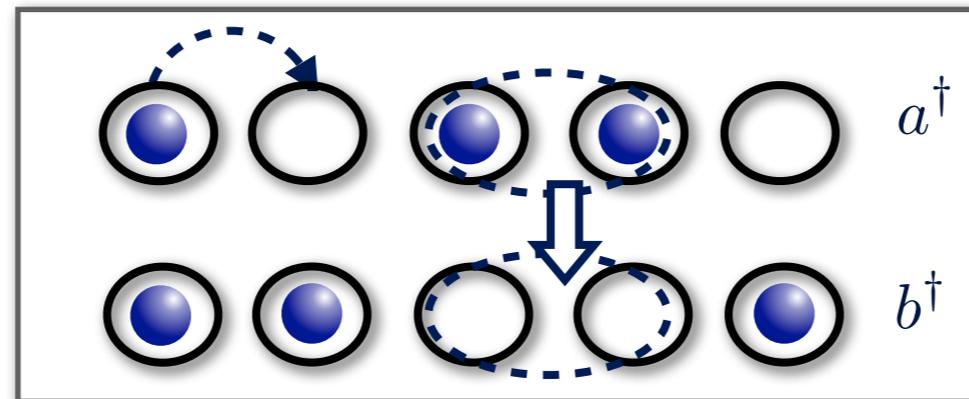
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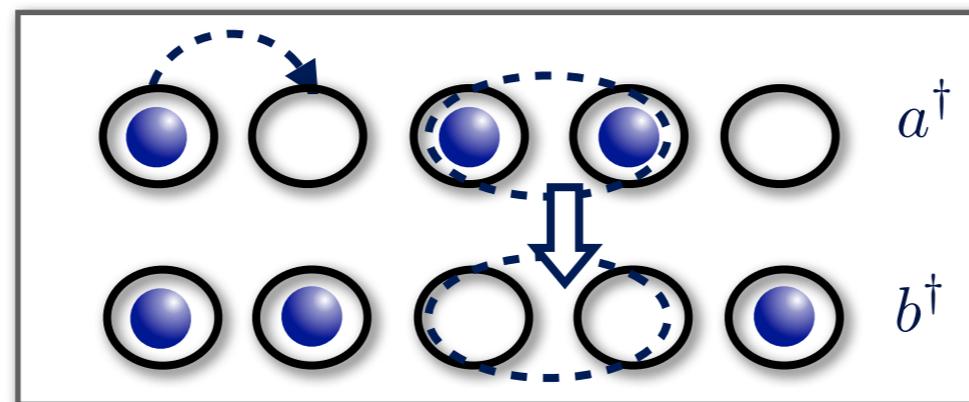
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“minimal reservoir?”



SYSTEM  
 $\updownarrow$   
 BATH

“One wires constitutes the reservoir for the other wire.”

$$H = \sum_{i=1}^{L-1} -t \left( a_i^\dagger a_{i+1} + b_i^\dagger b_{i+1} + h.c. \right) + W \left( a_i^\dagger a_{i+1}^\dagger b_{i+1} b_i + h.c. \right)$$

hopping
pair hopping

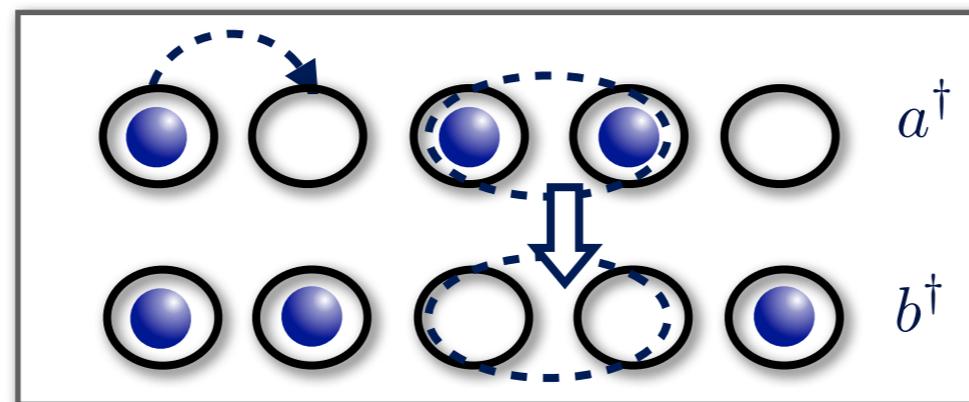
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hopping pair hopping

# We investigate....

---

Bosonization

Are there Majoranas in the  
INTERACTING double wire system ?

DMRG

Engineering of  
the pair hopping

# We investigate....

---

Low energy field theory  
maps to Kitaev wire

Bosonization

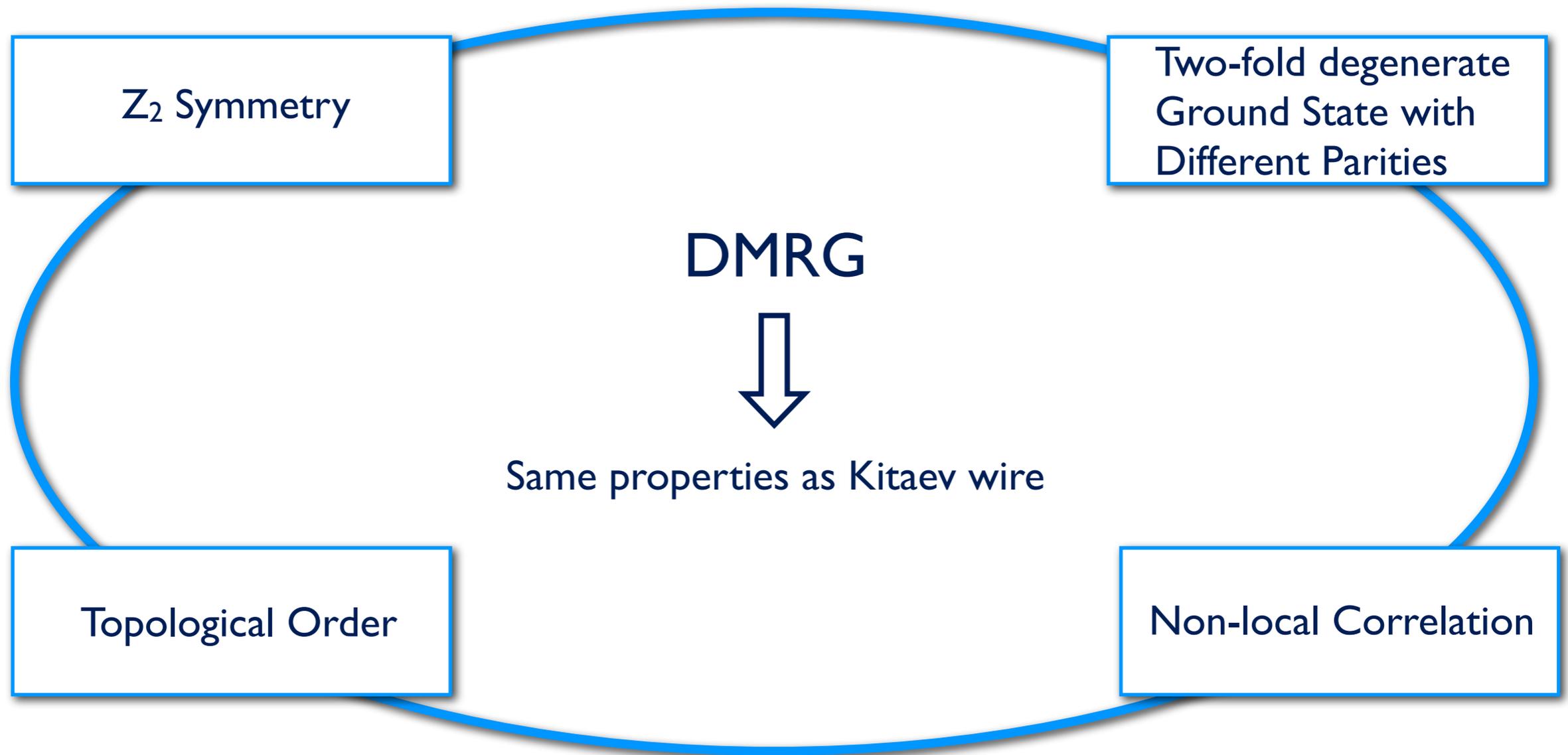
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DMRG

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# DMRG Study

---



# DMRG Study

pair hopping:  $U(1) \rightarrow \mathbb{Z}_2$

$\mathbb{Z}_2$  Symmetry

Two-fold degenerate  
Ground State with  
Different Parities

DMRG



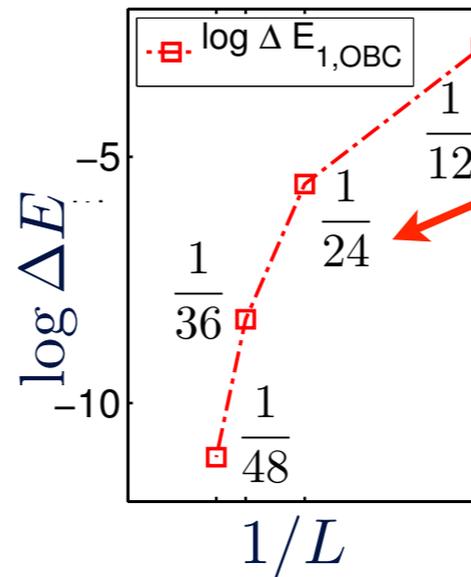
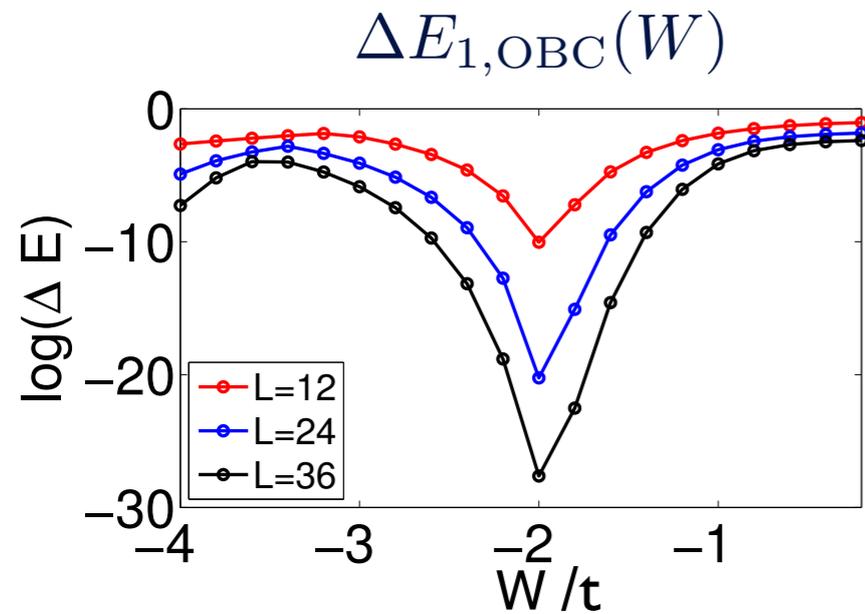
Same properties as Kitaev wire

Topological Order

Non-local Correlation

# Gaps and non-local Correlations

Energy gap  $\Delta E_1 = E_1 - E_0$  for open (OPB) and periodic (PBC) boundary conditions



$$\Delta E_{1,\text{OBC}} \simeq e^{-\alpha L}$$

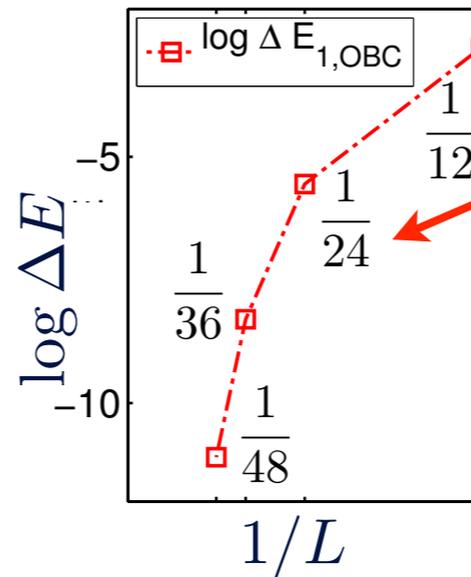
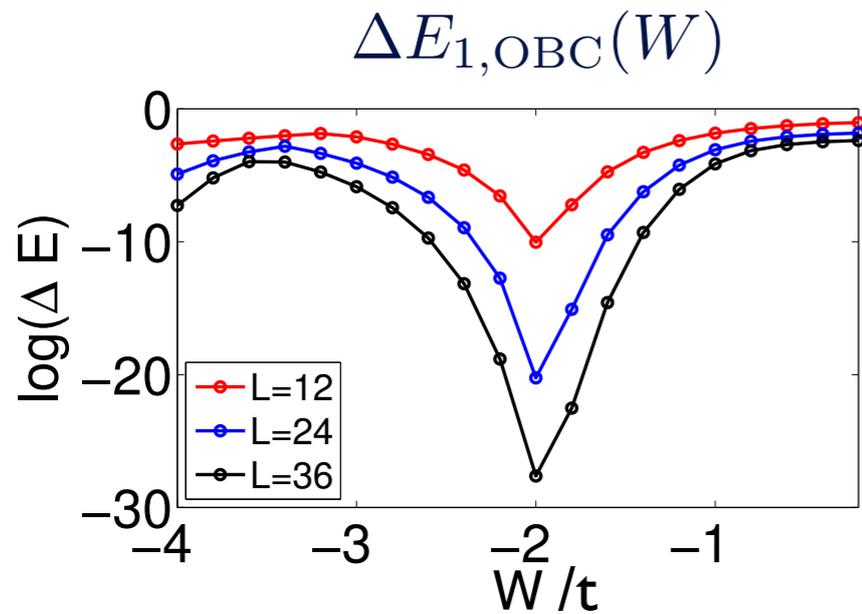
$$\Delta E_{1,\text{PBC}} \simeq 1/L^\beta$$

parities:  $|e, e\rangle, |o, o\rangle$

$$W/t = -1.8, n = 1/3$$

# Gaps and non-local Correlations

Energy gap  $\Delta E_1 = E_1 - E_0$  for open (OPB) and periodic (PBC) boundary conditions



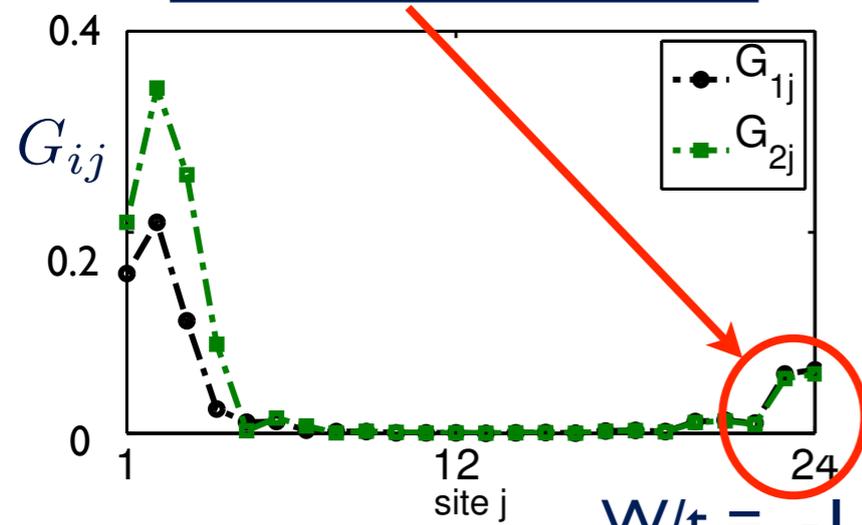
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## Non-local correlations



$$G_{ij} = \langle a_i^\dagger a_j \rangle$$

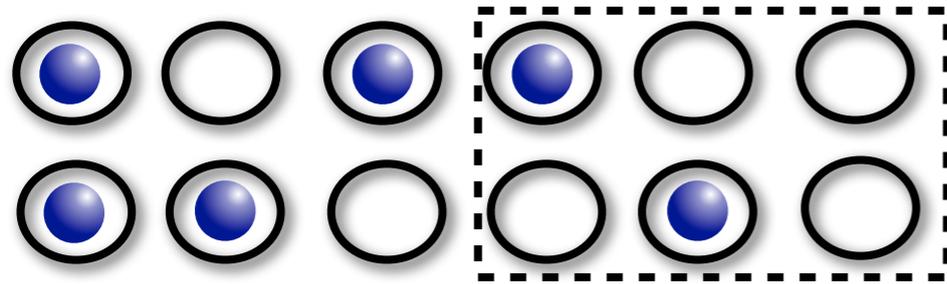
$$W/t = -1.8, n = 1/3, L = 24$$

- ✓ Twofold degenerate ground state with different parities on the wires
- ✓ Non-local correlations

# Topological Order

Topologically ordered system  $\leftrightarrow$  Degeneracy of the entanglement spectrum

A. Turner, F. Pollmann, E. Berg, PRB 83 (2011)



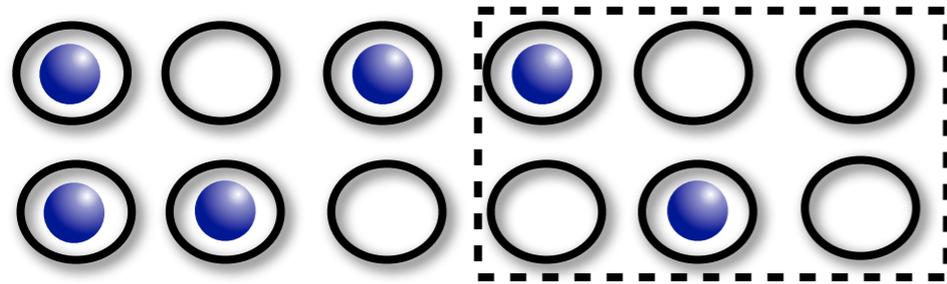
$$\rho_B = \text{tr}_A \rho = \sum_{j,N} \lambda_j^{(N)} \rho_j^{(N)}$$

$\lambda_j^{(N)}$  come in degenerate pairs: topological phase

# Topological Order

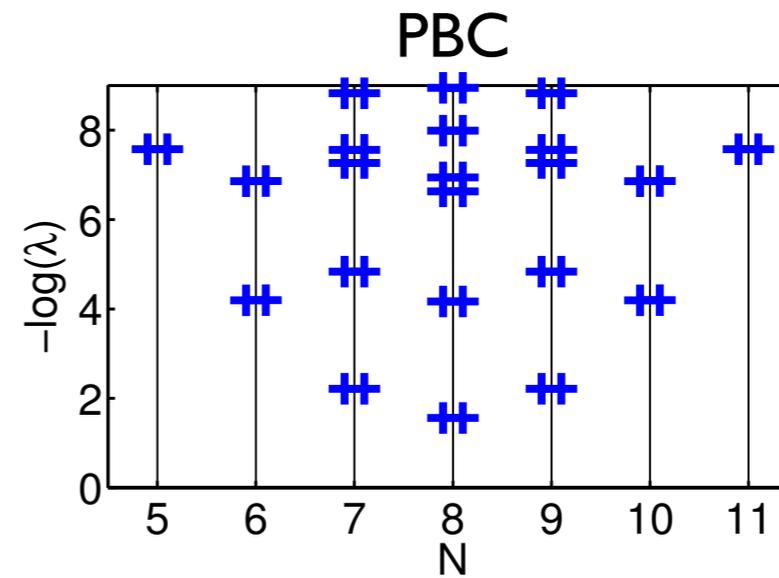
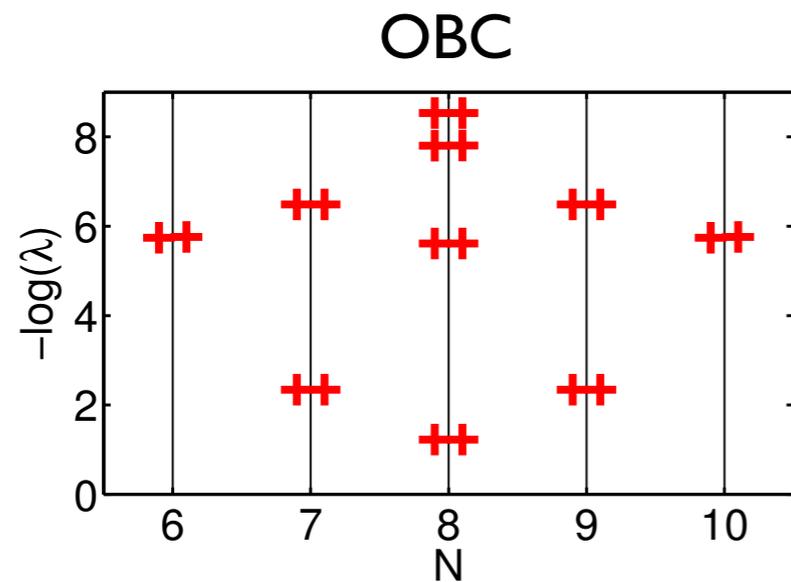
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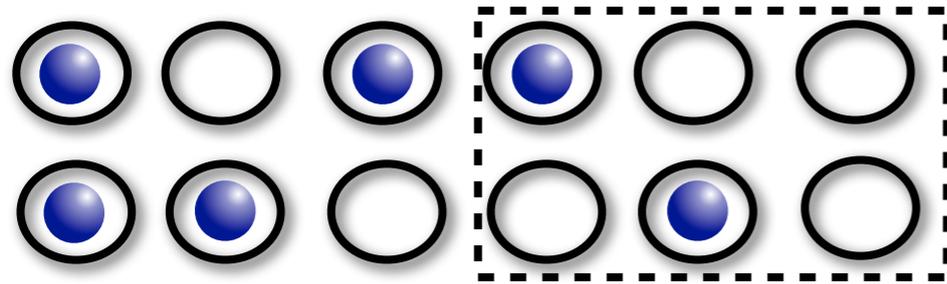
$W/t = -1.8,$   
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- Spectrum doubly degenerate for OBC and PBC
- Structure of the low lying part independent of boundary conditions

# Topological Order

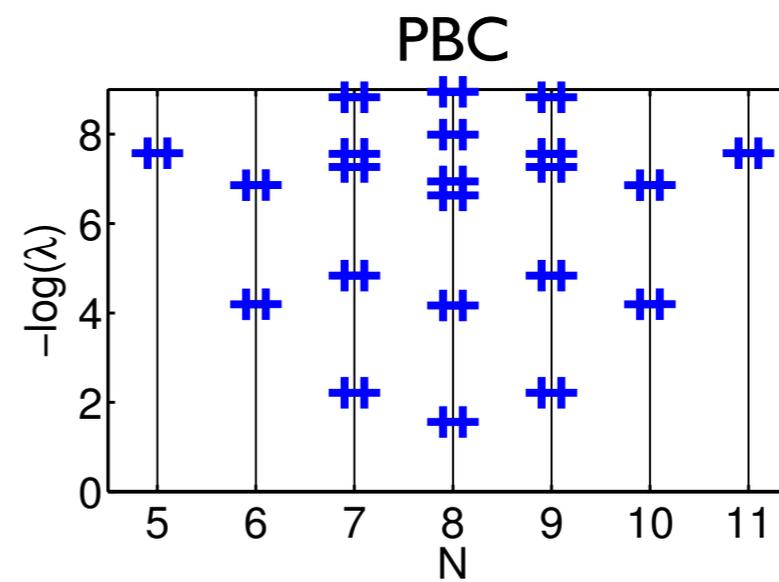
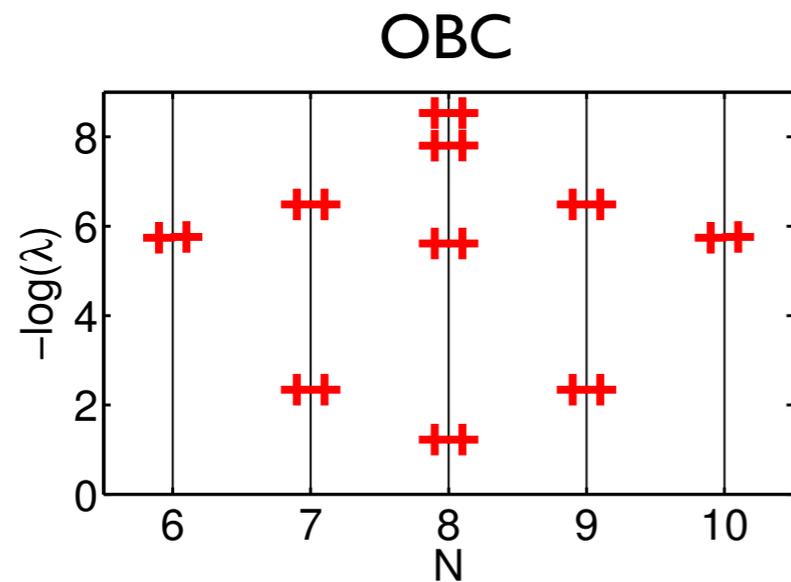
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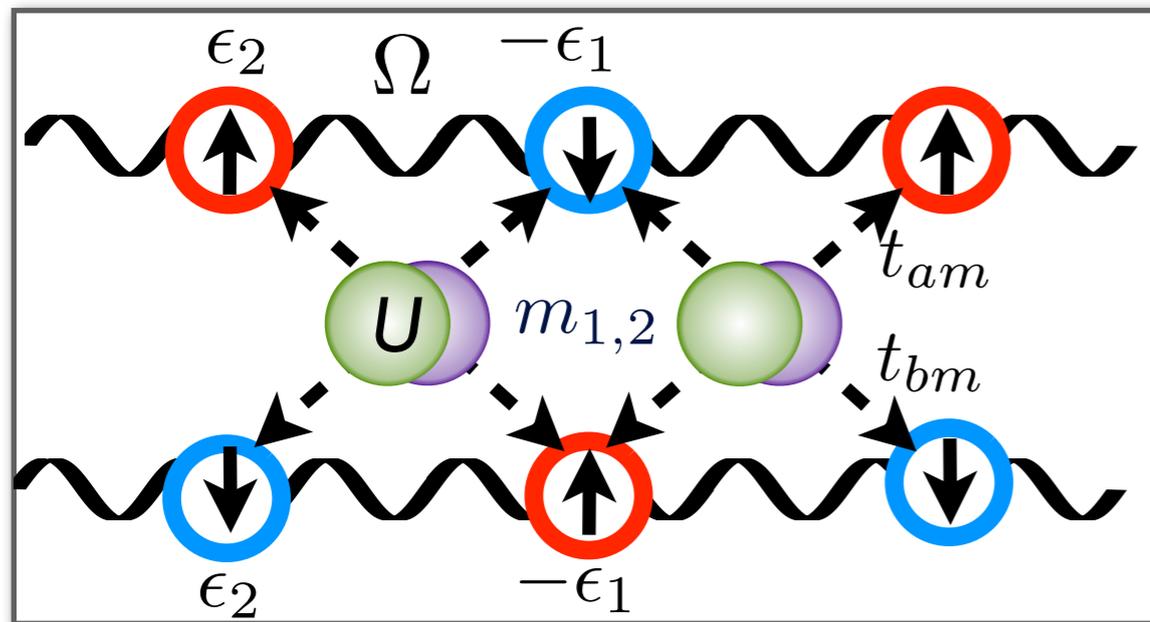


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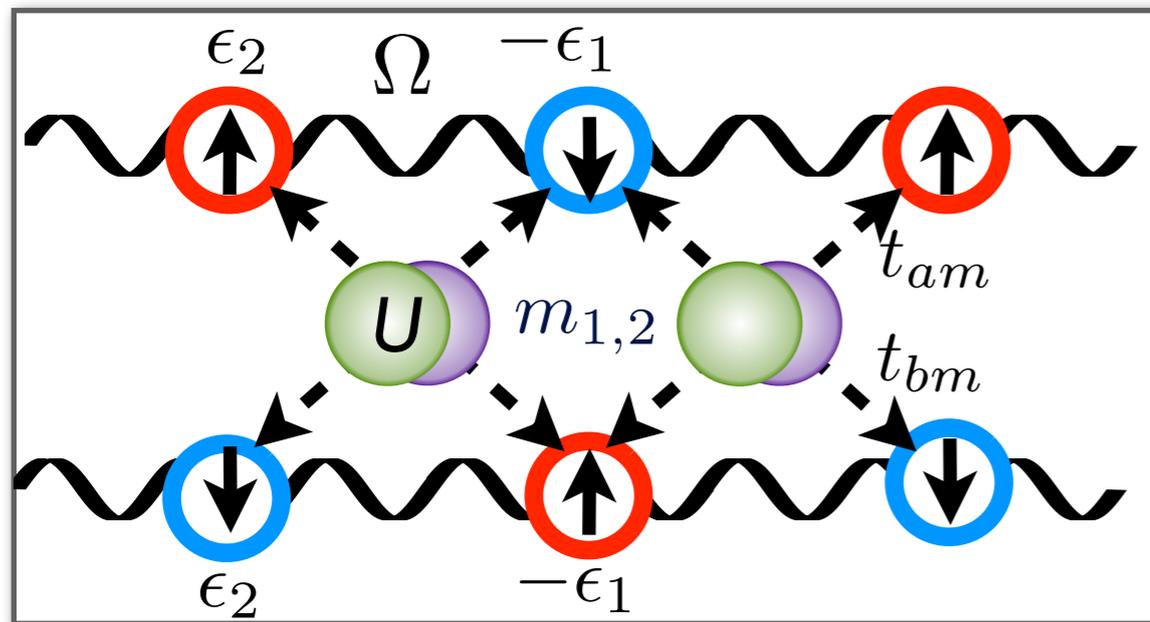
✓ Double wire is in a topological phase.

# Engineering of the Pair Hopping

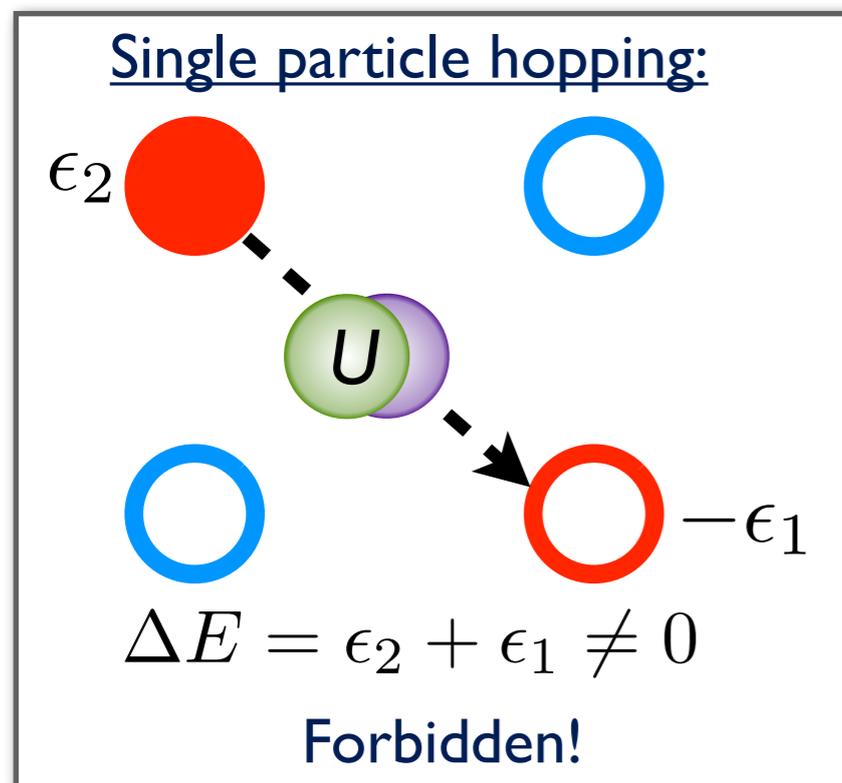


- Raman assisted tunneling along the wires
- Central site: two levels, interaction  $U$
- Energy offsets allow pair hopping and forbid single particle hopping

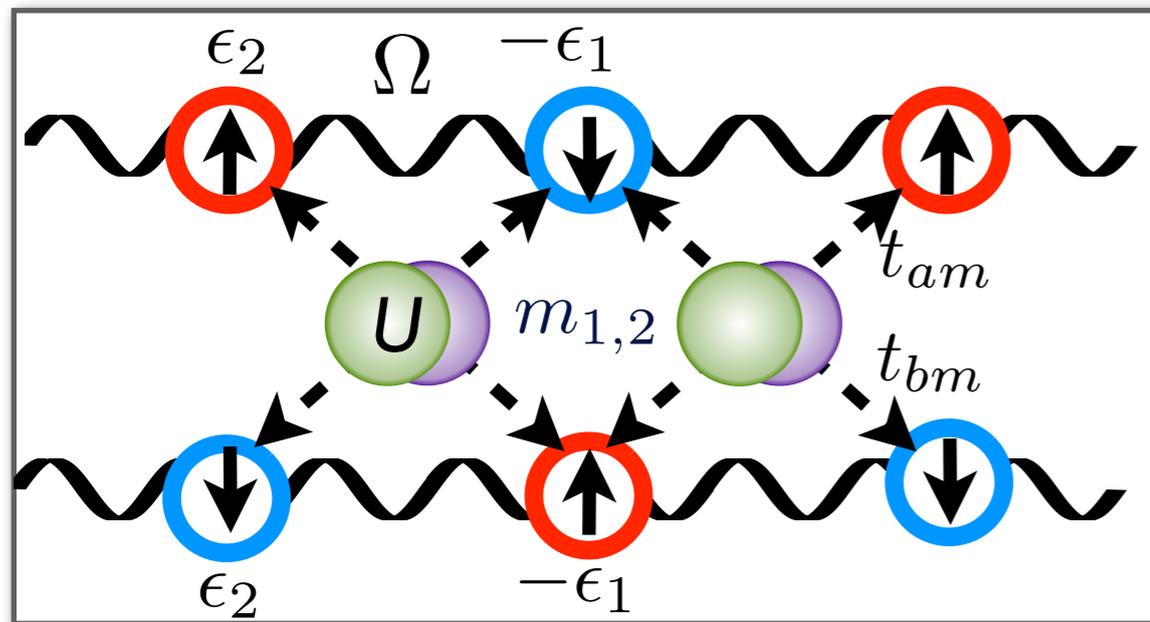
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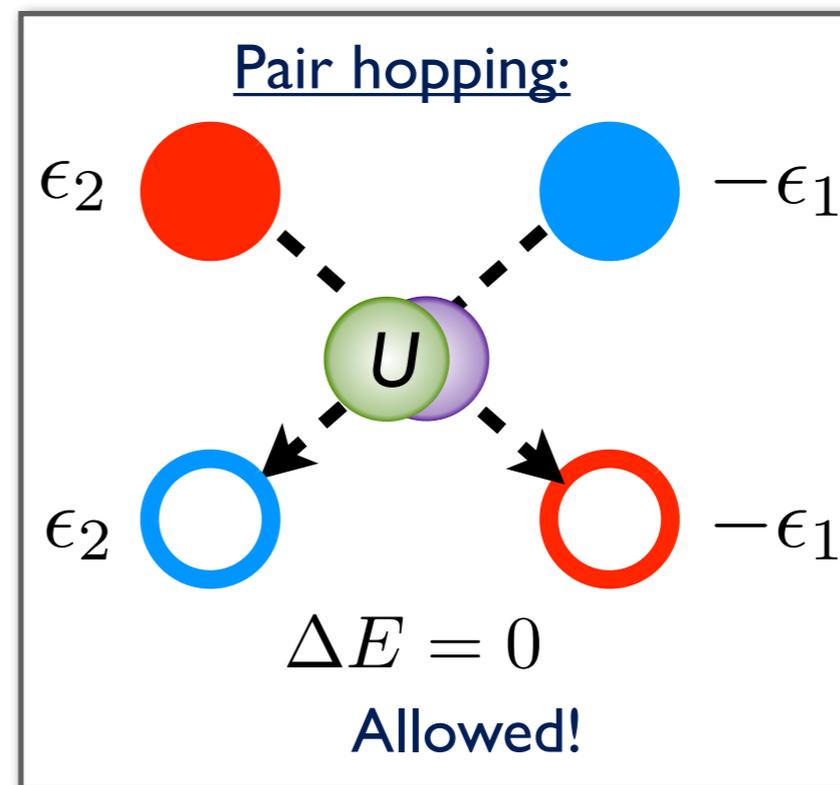
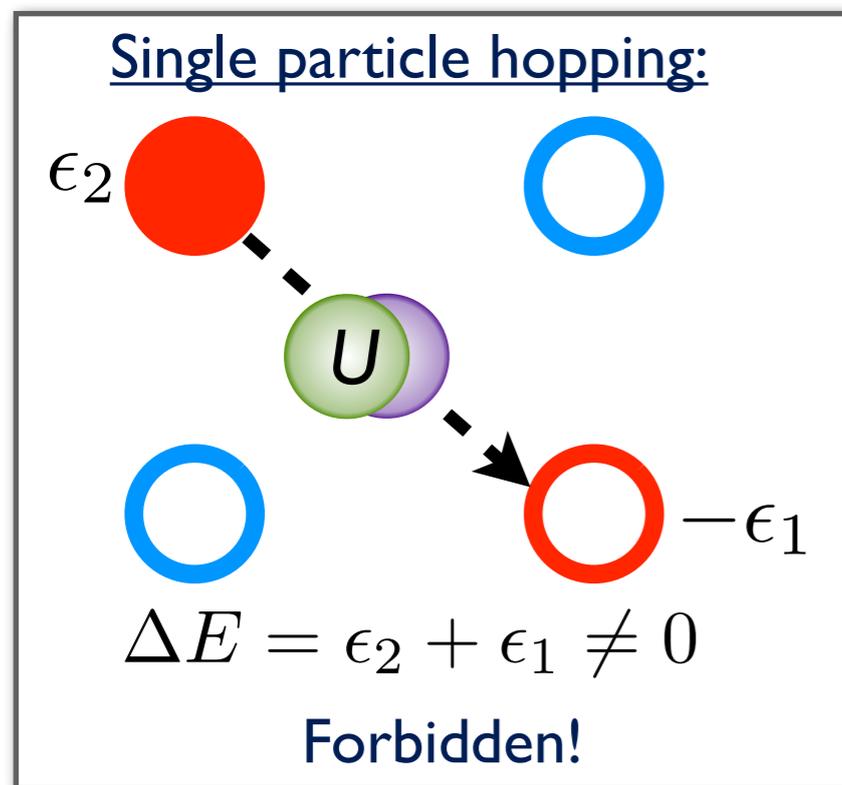
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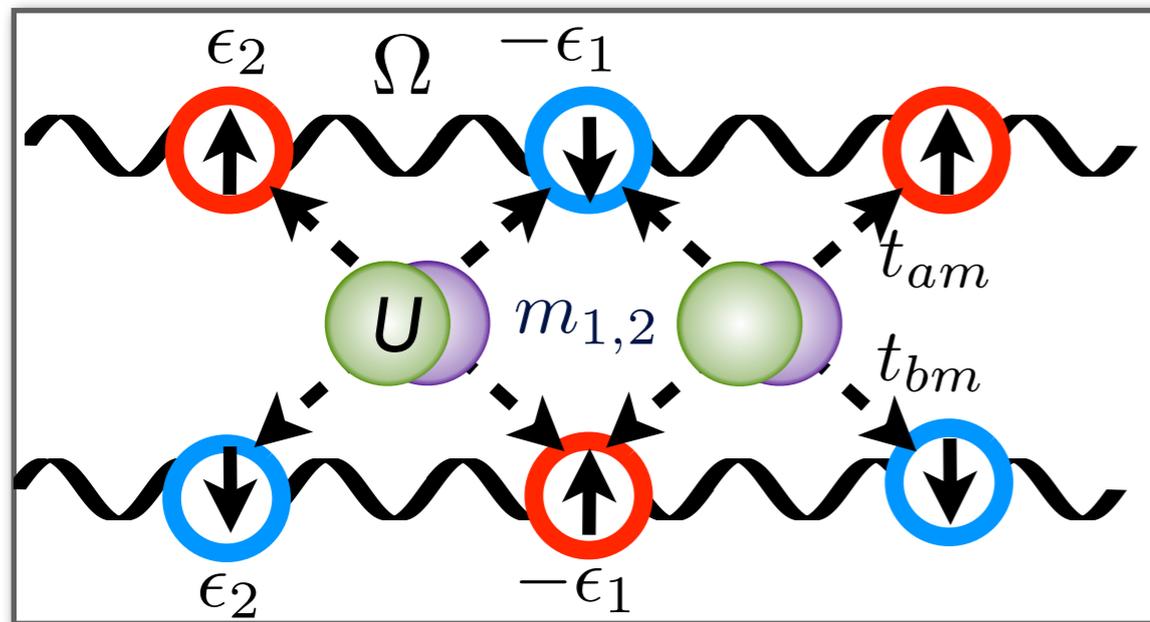
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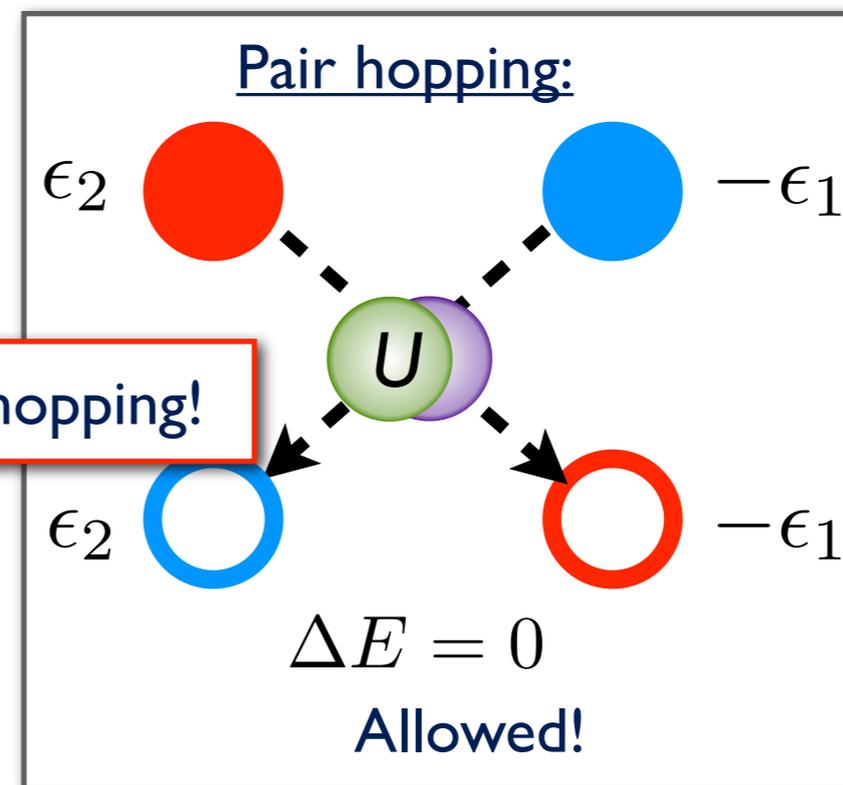
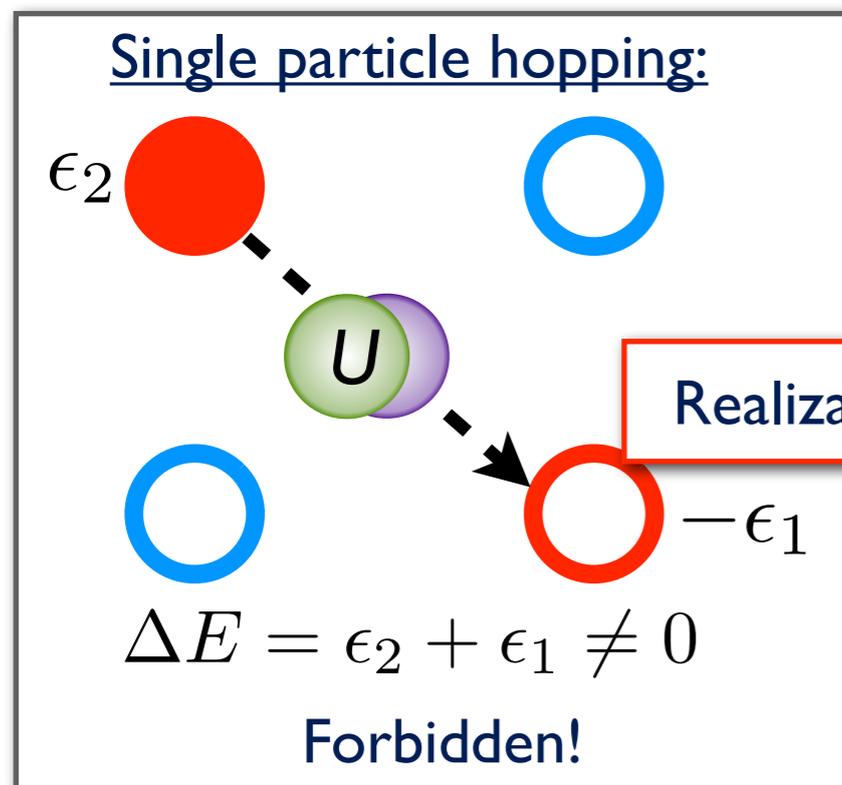
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Realization of the pair hopping!

# Effects of Perturbations

## AMO Realization

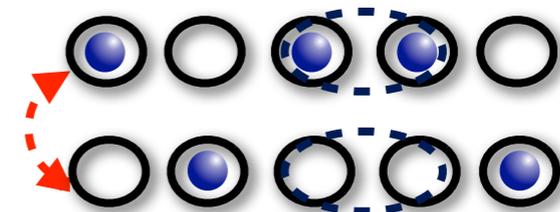
- Single particle intra-chain hopping
- Assisted single particle hopping
- Density-density interactions

## Perturbations

## Topological Order

- Random local potentials
- Randomness in  $W$

## Hopping between the chains



...breaks  $Z_2$  symmetry

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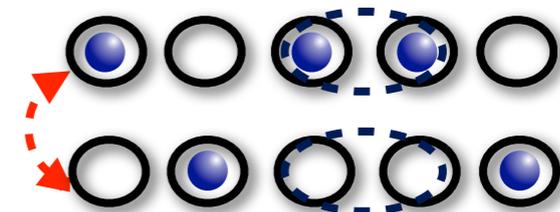
## Perturbations

No qualitative effect!  
(at least in the experimentally relevant regime)

## Topological Order

- Random local potentials
- Randomness in  $W$

## Hopping between the chains

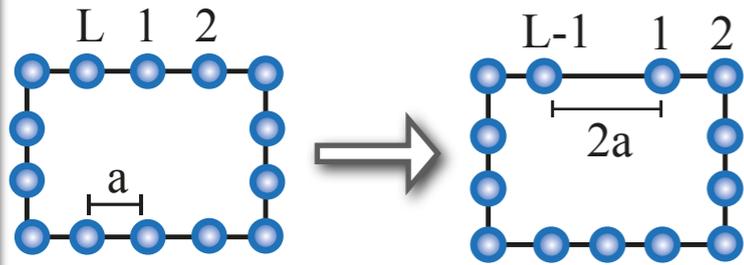


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# Summary and Outlook

## Majorana Physics in the Kitaev Wire

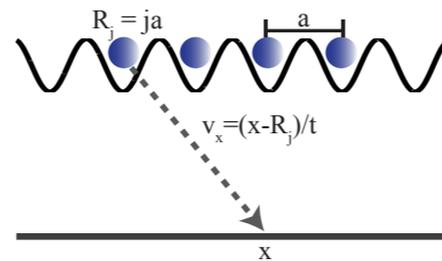
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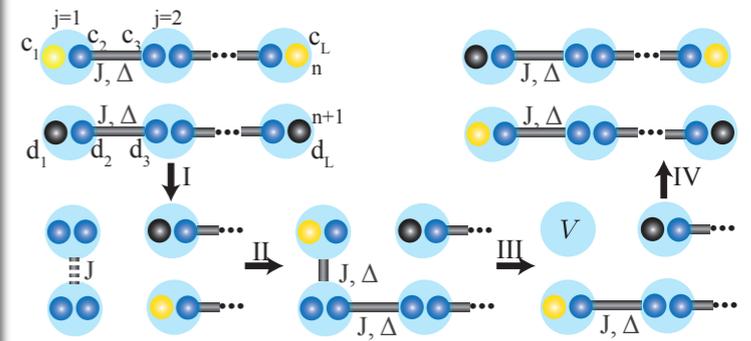
C.K, S. Diehl, P. Zoller, M. Baranov (NJP '12)

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- Spectroscopy



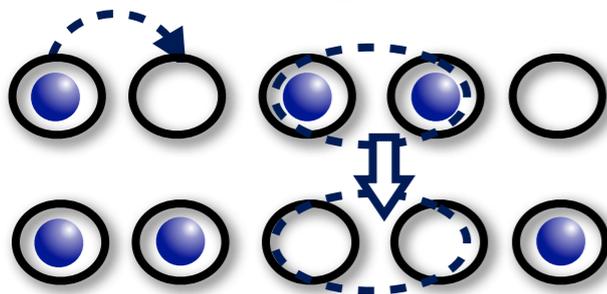
### Braiding of MFs and TQC



C.K, P. Zoller, M. Baranov, arxiv, '13

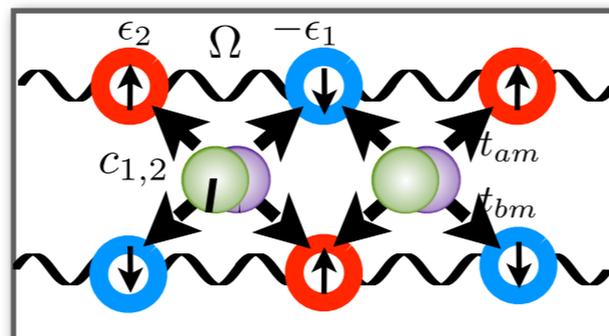
## Majorana Physics in a double wire with pair hopping

### MFs in an interacting number conserving system



C.K, M. Dalmonte, M. Baranov,  
A. Läuchli, P. Zoller, arxiv '13

### Engineering of the pair hopping



## Outlook

- Hybrid system for TQC
- Braiding in the double wire system
- Further applications of pair hopping