Spin-Orbit Coupling and Artificial Gauge Fields

Topological Phases and Mott Physics

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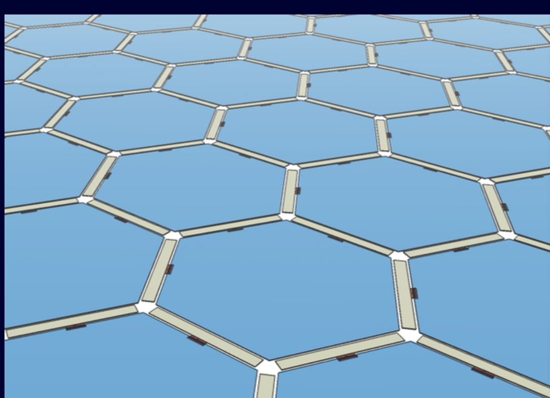
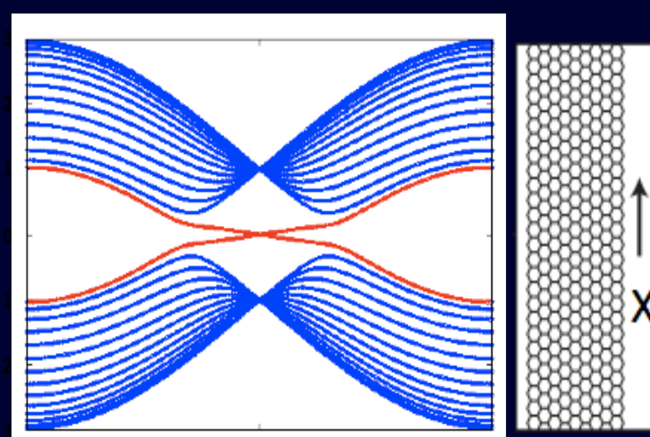
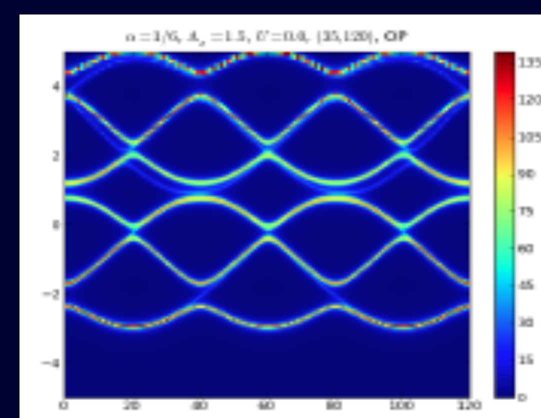
TRIESTE MAY 17th 2013

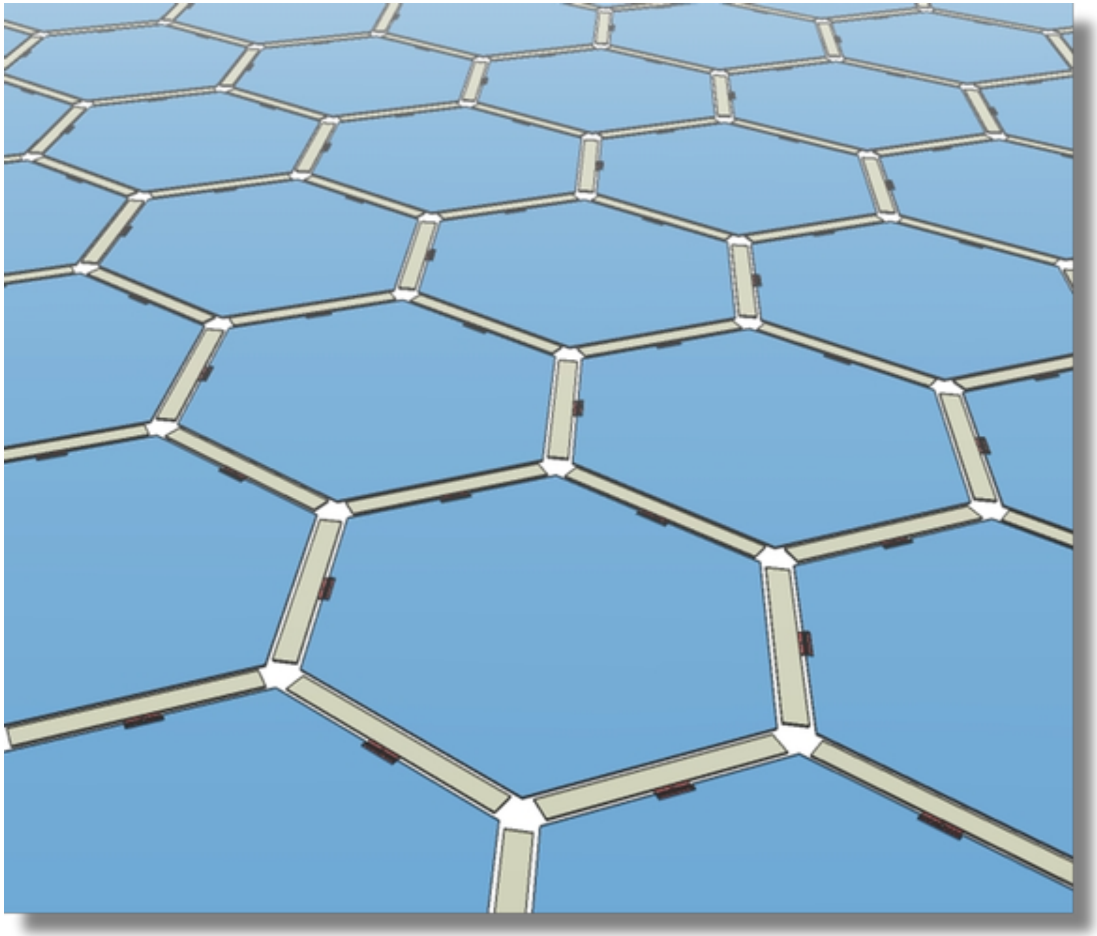






New phases of matter on the Honeycomb lattice

Example: Kitaev spin model



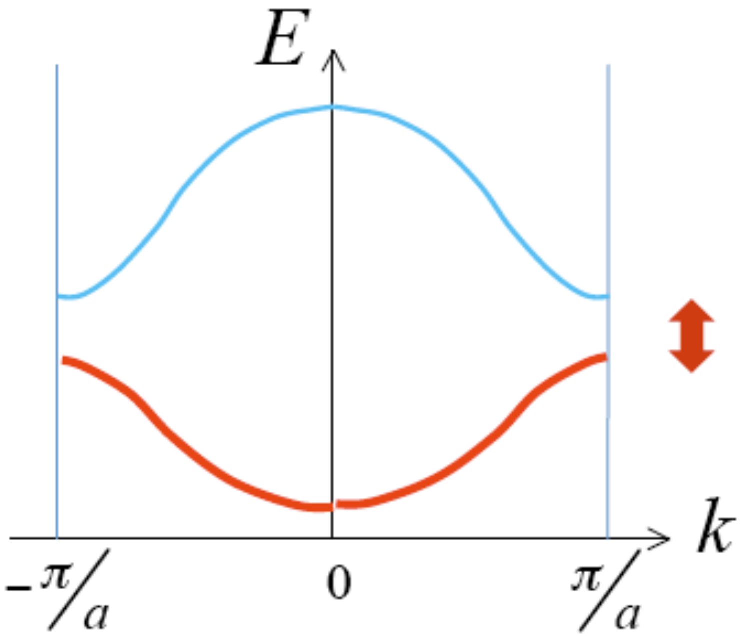
Topological Insulators
Interaction Effects
(Mott physics)
PART I

Artificial gauge fields:
Relation to **cold atoms**
PART II

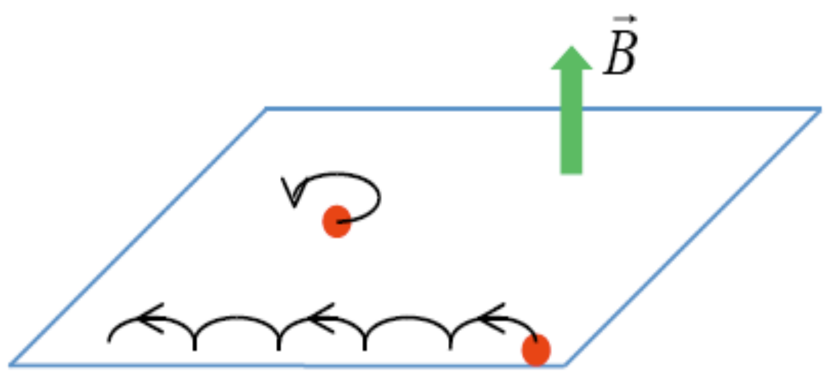
Photons & CQED:
Quantum Hall phases
Analogy to Mott physics
PART III

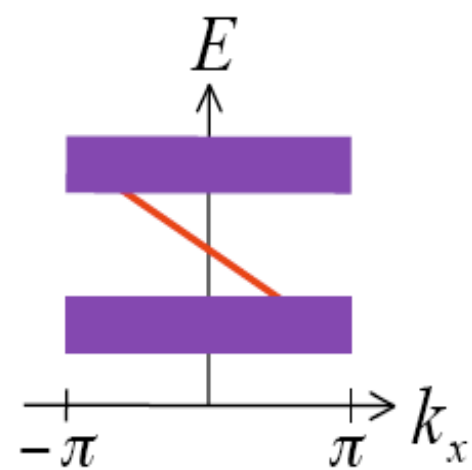
Starting point: "Honeycomb Lattice" but things will be more general

Topological Insulators: PART I

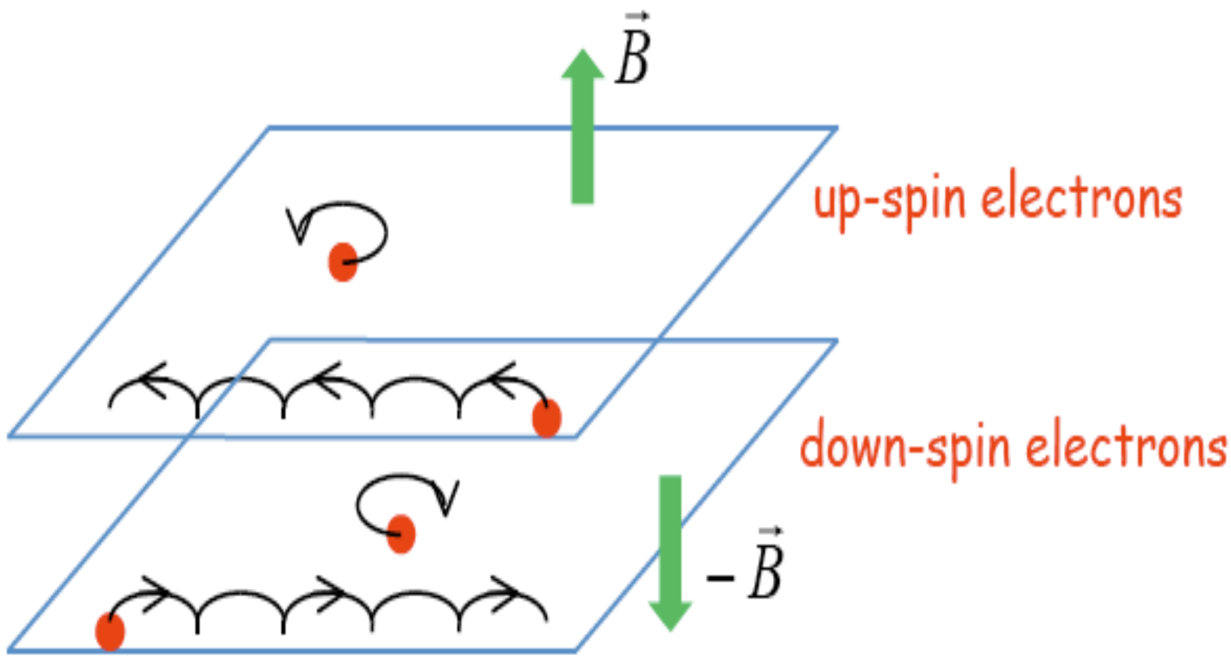


That's all? No





- Time-reversal invariant band insulator
- Strong spin-orbit interaction $\lambda\vec{L}\cdot\vec{\sigma}$
- Gapless helical edge mode (Kramers pair)



No spin rotation symmetry

Microscopic Description: Simple Standard Model, Kane-Mele

Time reversal invariant of **Haldane model (1988)**: Kane-Mele model

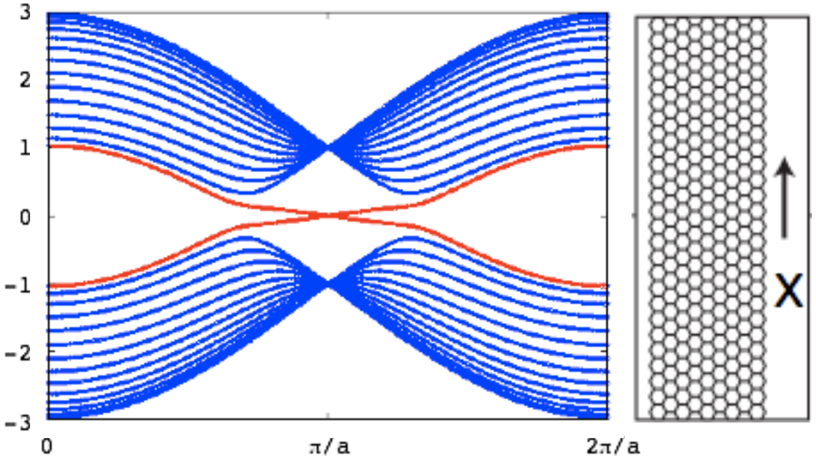
Kane & Mele, PRL 95, 226801 (2005)

see also: **Bernevig, Hughes, and Zhang, Science 314, 1757 (2006) + Molenkamp-experiments in three dimensions, experiments by M. Z. Hasan et al. (Bismuth materials)**

$$\mathcal{H} = -t \sum_{<ij>\sigma} c^\dagger_{i\sigma} c_{j\sigma} + i\lambda \sum_{\langle\langle ij \rangle\rangle} \sum_{\sigma\sigma'} \nu_{ij} \sigma^z_{\sigma\sigma'} c^\dagger_{i\sigma} c_{j\sigma'}$$

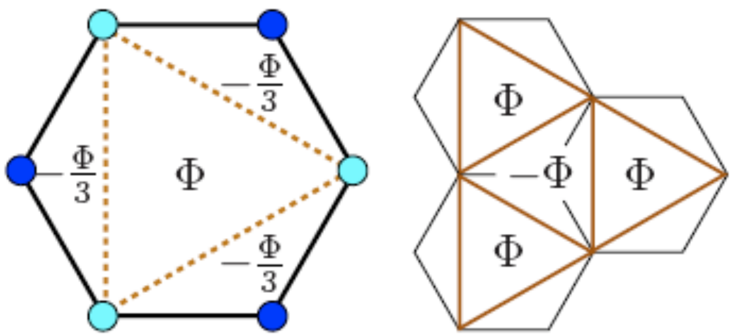
$\nu_{ij} = \pm 1$

strip geometry:

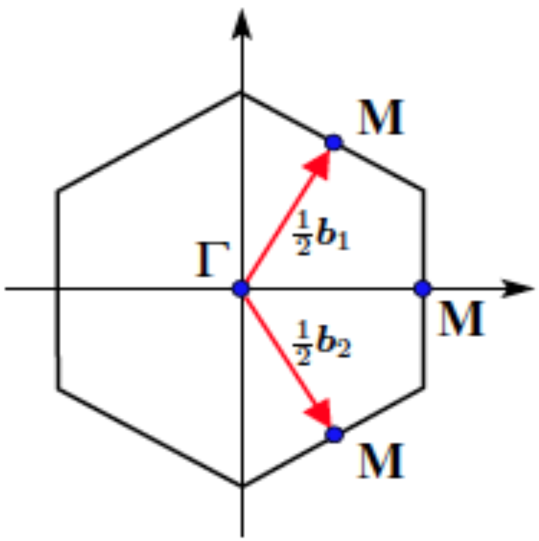


edge states: Kramer's pair

$$\mathcal{H} \propto \Psi^\dagger_k \sigma^z \tau^z \Psi_k$$



Z_2 Invariant: Spin Chern number



Time-reversal invariant
Points of the Brillouin zone

Following L. Fu and C. Kane:

$$\mathcal{H}_{\mathbf{k}} = \sum_{a=1}^{5} d_a(\mathrm{k})\Gamma^a$$

$$\mathcal{P} = \tau^x \otimes I = \Gamma^1$$

$$\mathcal{T} = i(I \otimes \sigma^y)K$$

Time-Reversal & Inversion Symmetry

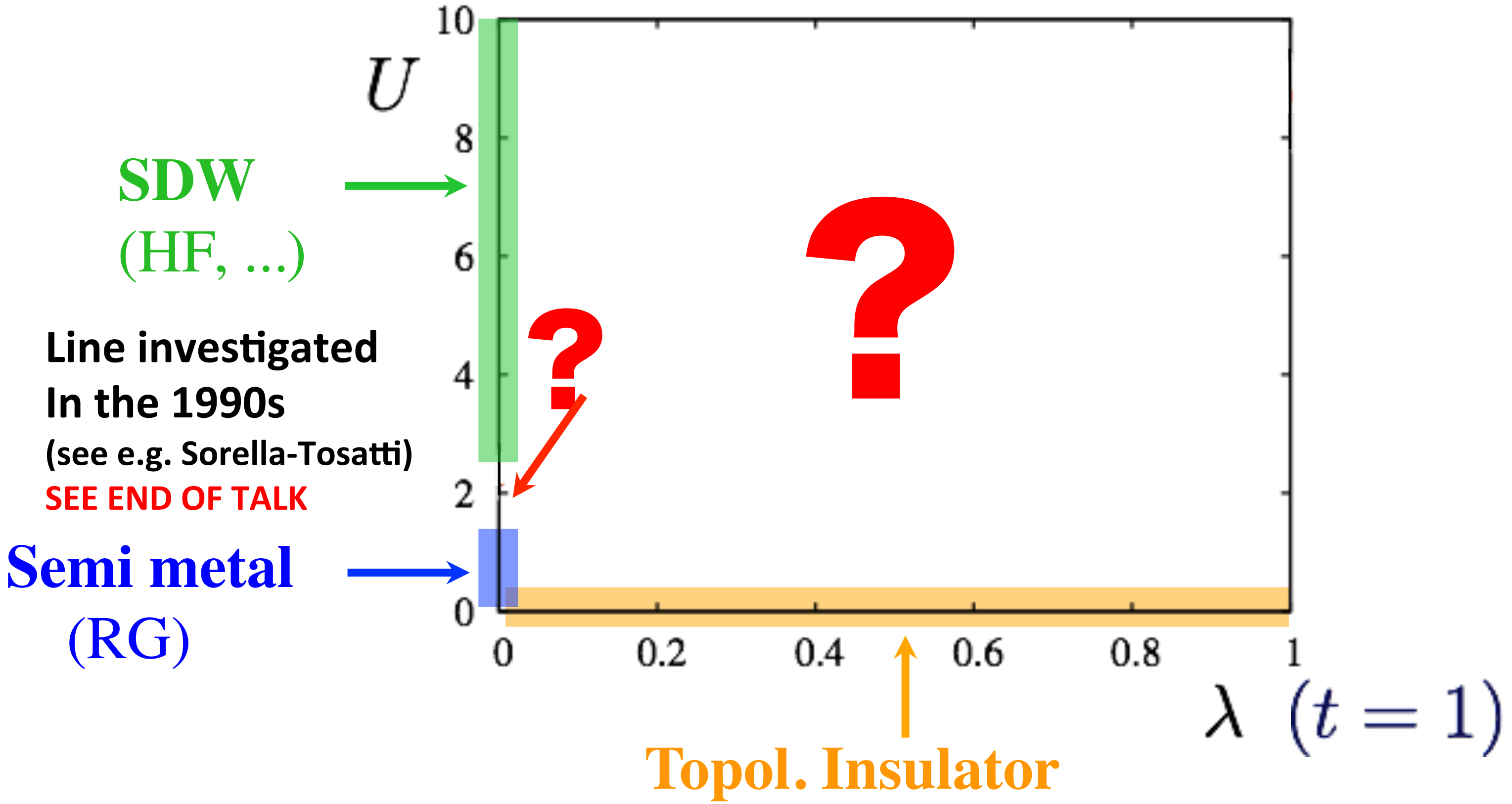
Z_2 invariant given by (here, ν=0 or 1):

$$(-1)^\nu = \prod_{i=1}^{4} -\mathrm{sign}[d_1(\mathbf{\Gamma}_i)]$$

Single-particle band structure: see also Balents & Moore
Efforts to define top. Invariants for interacting systems:
Qi-S.C. Zhang; V. Gurarie; Y. B. Kim (CDMFT); A. Kitaev; Savrasov (LDA+U)

Effect of electron-electron interaction?

$$\mathcal{H} = -t \sum_{<ij>\sigma} c^\dagger_{i\sigma} c_{j\sigma} + i\lambda \sum_{\ll ij \gg} \sum_{\sigma\sigma'} \nu_{ij} \sigma^z_{\sigma\sigma'} c^\dagger_{i\sigma} c_{j\sigma'} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

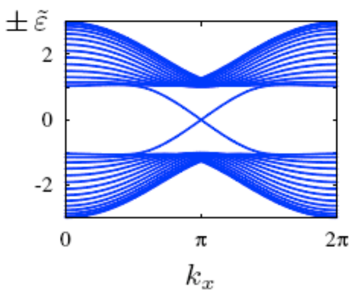


S. Rachel & KLH, PRB **82** 075106 2010

Many other related works: recent review M. Hohenadler & F. Assaad, arXiv: 2012

Presence of spin orbit coupling

- **Apply (Slave)-Rotor theory of Florens & Georges, PRB 70, 035114 (2004)**
- **See review E. Zhao & A. Paramekanti**
- **Rewrite fermions as rotors (charge degrees of freedom) and spinons**



$$c_{i\sigma} = e^{i\theta_i} f_{i\sigma}$$

- **Introduce constraint**

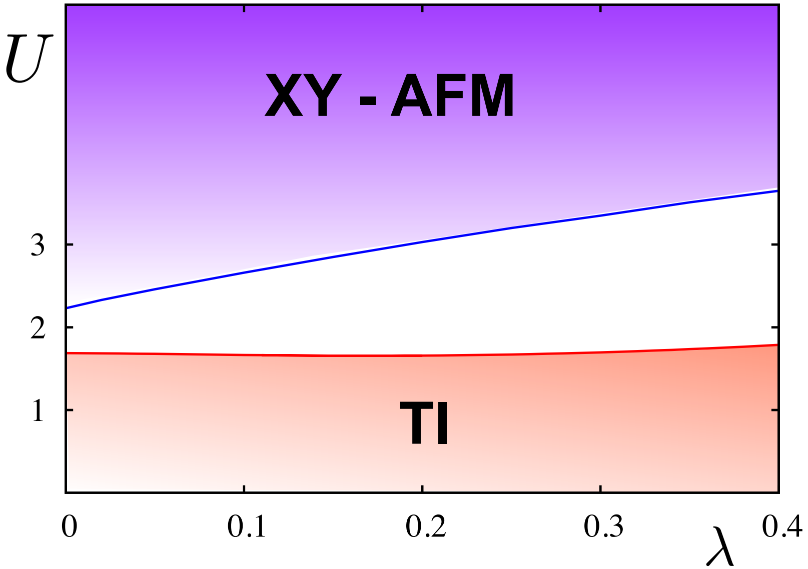
$$\sum_\sigma f^\dagger_{i\sigma} f_{i\sigma} + L_i = 1$$

$$\frac{U}{2}\sum_i \left(\sum_\sigma n_{i\sigma} - 1\right)^2 \longrightarrow \frac{U}{2}\sum_i L_i^2$$

- **Hubbard interaction simplifies**

$$L = (i/U)\partial_\tau \theta.$$

- **Interaction affects rotor only**
- **weak *U*: rotor condense,** $f_\sigma \propto c_\sigma$



S. Rachel & KLH, PRB 2010

[+ other theory arguments]

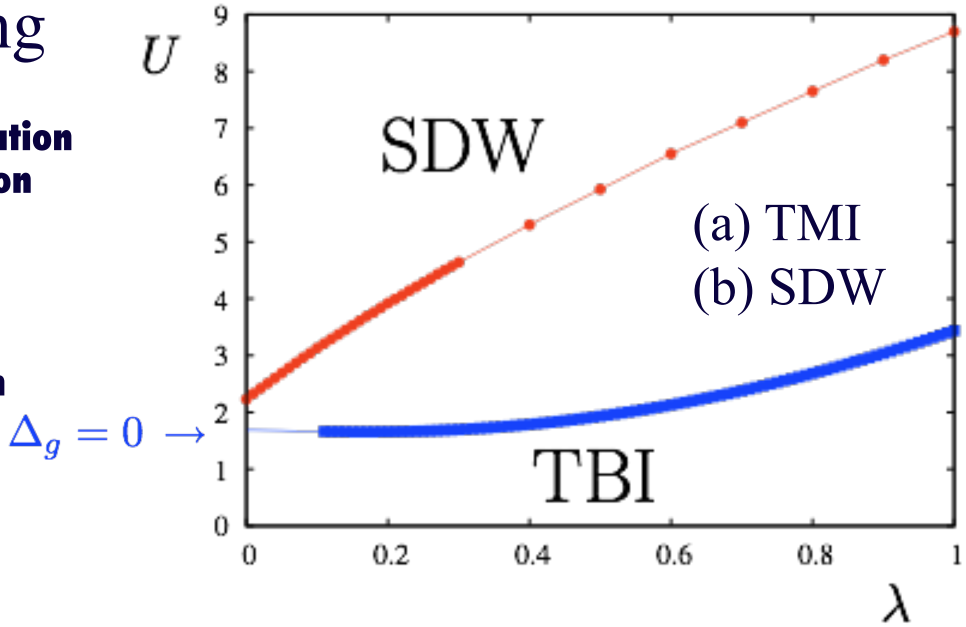
Presence of spin orbit coupling

- **More Slave-Rotor: use sigma-model representation**
- **At the Blue transition, "spin-charge" separation**

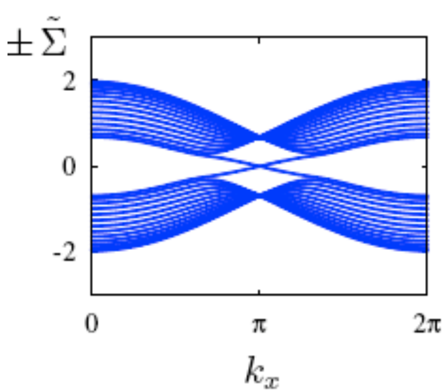
$$X = e^{i\theta} \qquad |X|^2 = 1$$

- **mean-field decoupling or Hubbard Stratonovich**
- **Gap of the rotor-field (zero at the transition)**

$$\Delta_g = 2\sqrt{U(\rho + \min \xi_k)}$$



S. Rachel & KLH, PRB 82, 075106 (2010); arXiv:1003.2238, 20 pages



See also S.S. Lee & P. Lee PRL 2005
Young, S. S. Lee, C. Kallin, PRB 2008
Pesin & Balents, Nat. Phys. 2010
Y.-B. Kim & et al. 2010 + many recent works

Mean-Field Solution allows TMI phase:

- Mott gap
- Spin degrees of freedom form a topological Kane-Mele phase

Analogue of S=1 spin Haldane chain (probed through thermal transport?)

2D: Direct Transition from TBI to XY

$$\mathcal{L}_{MF} = m \sum_{a=\pm} \left(f^\dagger_{\uparrow a} \tau^z f_{\uparrow a} - f^\dagger_{\downarrow a} \tau^z f_{\downarrow a} \right)$$

Monopole insertion = “spin flip” operator
Localized +2π flux of the gauge field implies that a single extra spin-up spinon will be Induced along with the gauge flux, while one spin-down spinon will be depleted

Fermions are gapped:

$$\mathcal{L}_{Maxwell} = (1/2e^2) \sum_{\mu} (\epsilon_{\mu\nu\lambda} \partial_\nu a_\lambda)^2$$

Monopoles only cost a finite action: monopole propagator is long-ranged

Here, this implies magnetic order in the XY plane: $\langle S^+ \rangle$ is finite

Polyakov’s gauge field argument: see also S. S. Lee & P. Lee; Y. Ran et al; M. Hermele…

Edge Theory & Mott Transition

C. Xu & J. Moore; C. Wu, A. Bernevig & S.-C. Zhang;...

$$H_0 = v_F \int dx \left(\psi^\dagger_{R\uparrow} i\partial_x \psi_{R\uparrow} - \psi^\dagger_{L\downarrow} i\partial_x \psi_{L\downarrow} \right)$$

$\psi^\dagger_{R\uparrow}\psi_{L\downarrow} + \text{h.c.}$ (**elastic**) Backscattering forbidden

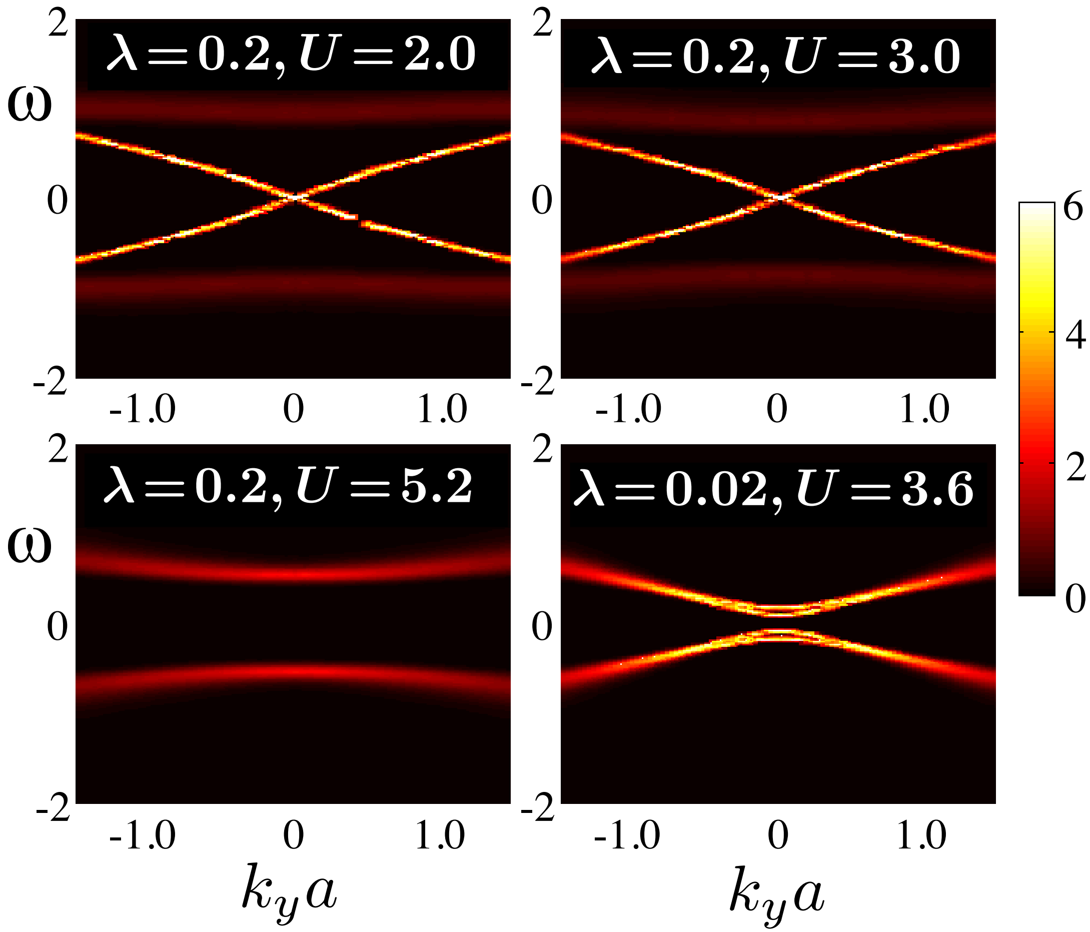
$$H_I = U \int dx \left(\psi^\dagger_{R\uparrow} \psi_{R\uparrow} \psi^\dagger_{L\downarrow} \psi_{L\downarrow} \right)$$

$$H = \int dx \frac{v}{2} \left[\frac{1}{K} \left(\partial_x \phi \right)^2 + K \left(\partial_x \theta \right)^2 \right] - \frac{U m \sin \sqrt{4\pi}\phi}{(\pi a)^2}$$

Minimal Model
for TBI/QSH phase

with $m = \langle \psi^\dagger_{R\uparrow} \psi_{L\downarrow} \rangle$

(No) Edge States in $A_s(k,\omega)$



CDMFT

Real-space version
QMC continous-time
Impurity solver

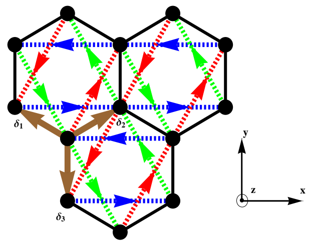
Some Reviews (not full list):
G. Kotliar et al, RMP 2006
T. Maier et al, RMP 2005
A.-M. Tremblay, B.-S. Kyung,
D. Senechal, 2006

DMFT:
Review A. Georges et al. RMP

Wei Wu, Stephan Rachel, Wu-Ming Liu and KLH, PRB **85**, 205102 (2012)

Connection to reality?

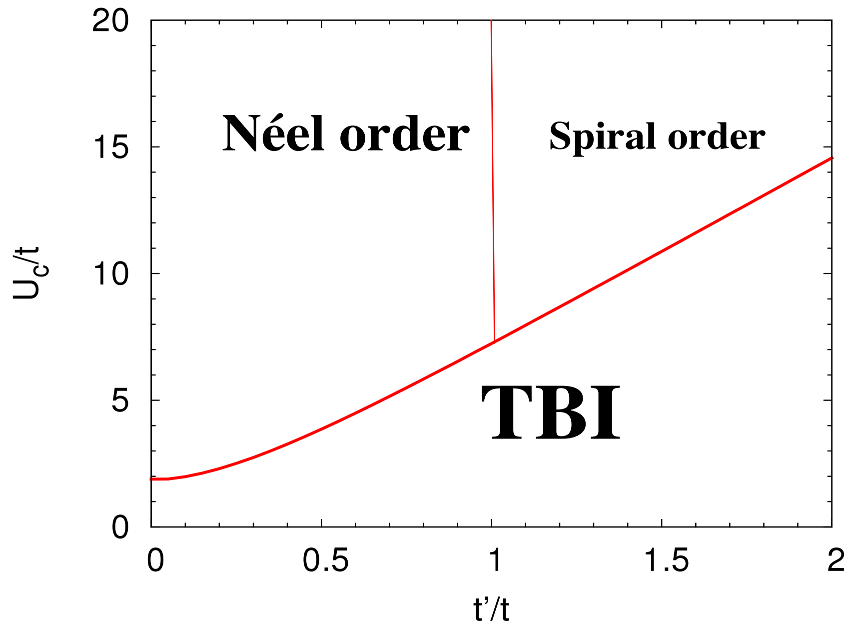
- Na_2IrO_3: honeycomb layers (thin films: arXiv:1303:5245)



Shitade et al. PRL **102** 256402 (2009); G. Jackeli & G. Khaliullin, PRL 102, 017205 (2009)

H.-C. Jiang, Z.-C. Gu, X.-L. Qi and S. Trebst, Phys. Rev. B 83, 245104 (2011);

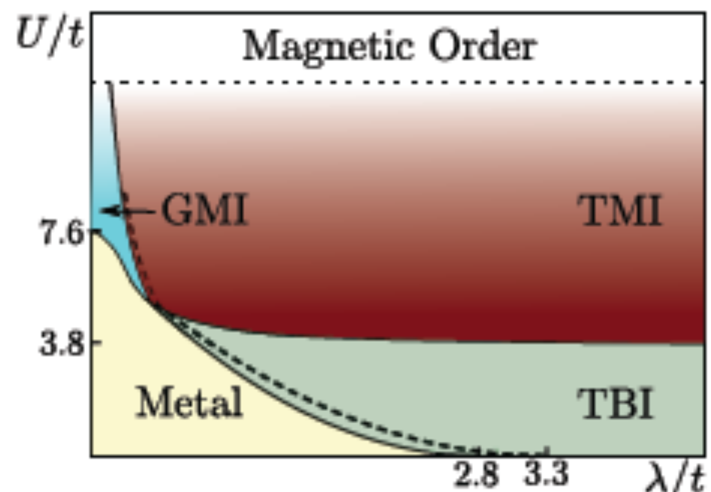
S. Bhattacharjee, Sung-Sik Lee and Yong-Baek Kim, New J. Phys. 14, 073015 (2012)



Tianhan Liu, Benoit Doucot, Karyn Le Hur, in preparation

A. Ruegg and G. Fiete, PRL 2012

J. Reuther, R. Thomale & S. Rachel, PRB 2012



D. Pesin & L. Balents, Nature Phys. 2010

Relevance for 3D pyrochlore Iridates (?)

Y. B. Kim and co-authors

Cold Atoms: Part II

A. L. Fetter RMP 2009; J. Dalibard, F. Gerbier, G. Juzeliunas, P. Ohberg RMP 2011; Bloch et al. Nature (2012); Juzeliunas & Spielman NJP (2012);…

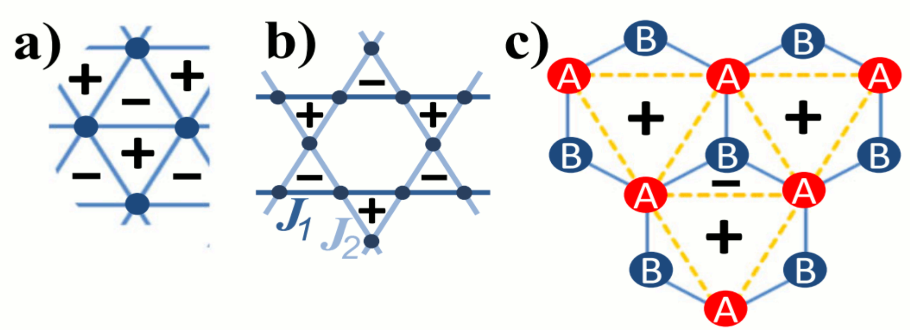
- Ways to implement magnetic fields & gauge fields

I. Spielman (NIST/Maryland) and collaborators: several papers

M. Aidelsburger et al. arXiv:1110.5314 (Muenich's group, PRL)

J. Struck et al. arXiv:1203.0049 (Hamburg's group)

One model by N. Goldman et al.
arXiv:1011.3909 (PRL 2010)



$$H_0 = -\sum_j \Big\{ t_x c^\dagger_{j+\hat{x}}\, e^{-i2\pi\gamma\sigma^x} c_j + t_y c^\dagger_{j+\hat{y}}\, e^{i2\pi\alpha x \sigma^z} c_j + \text{h.c.} \Big\} + \lambda_x \sum_j (-1)^x c^\dagger_j c_j \,,$$

D. Cocks, P. P. Orth, S. Rachel, M. Buchhold, K. Le Hur, W. Hofstetter, arXiv 2012 & PRL

Interaction Effects

Interacting spinful Hofstadter Problem

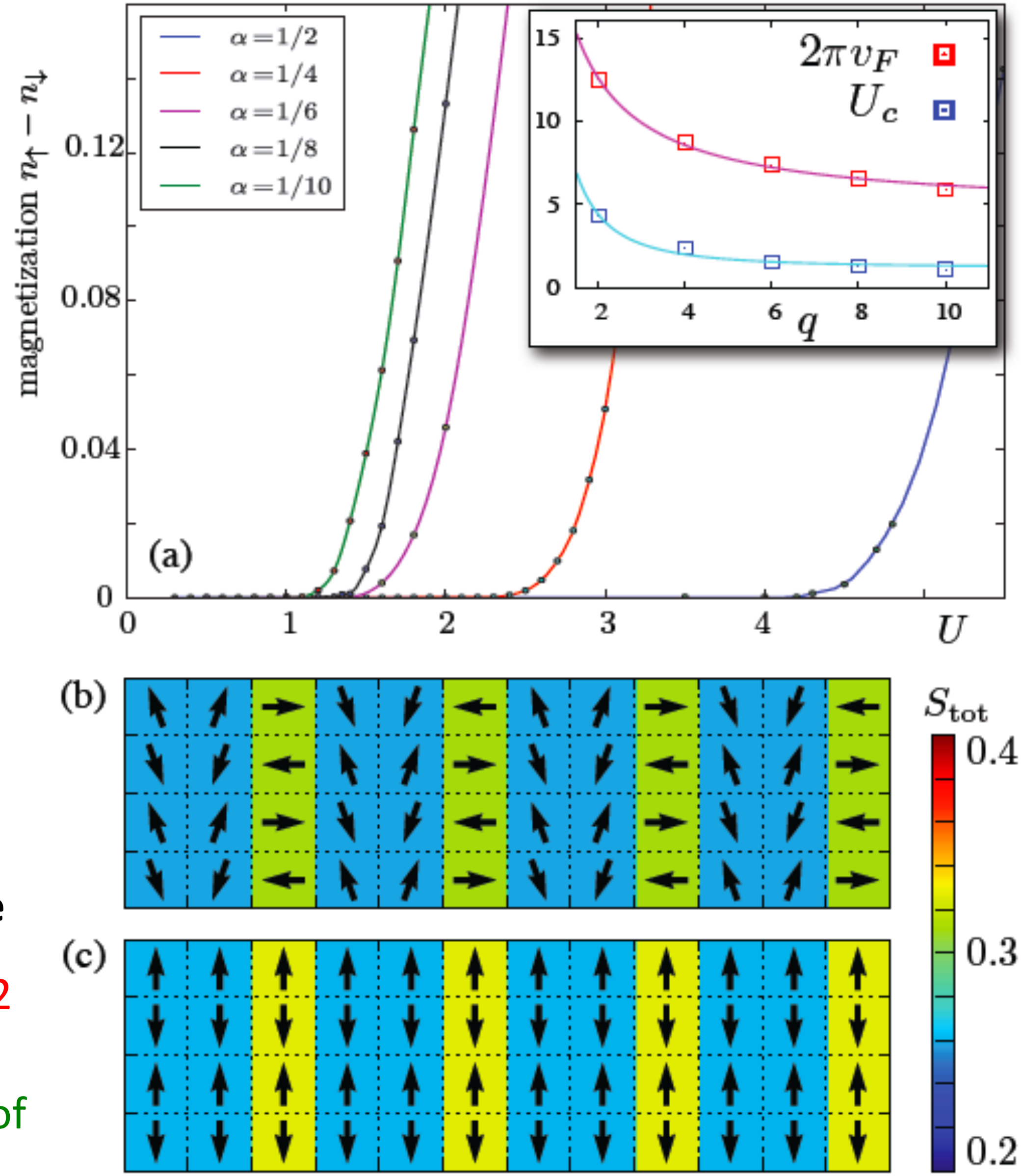
$\Upsilon = \lambda_x = 0$

At weak U and half-filling, semi-metal (SM), graphene
Number of Dirac points vary with $\alpha = 1/q$ (q even)

Application of I. Herbut's theory For transition SM to ordered state

The transition occurs for $U_c = 1/q^2$

Magnetism depends on the value of γ

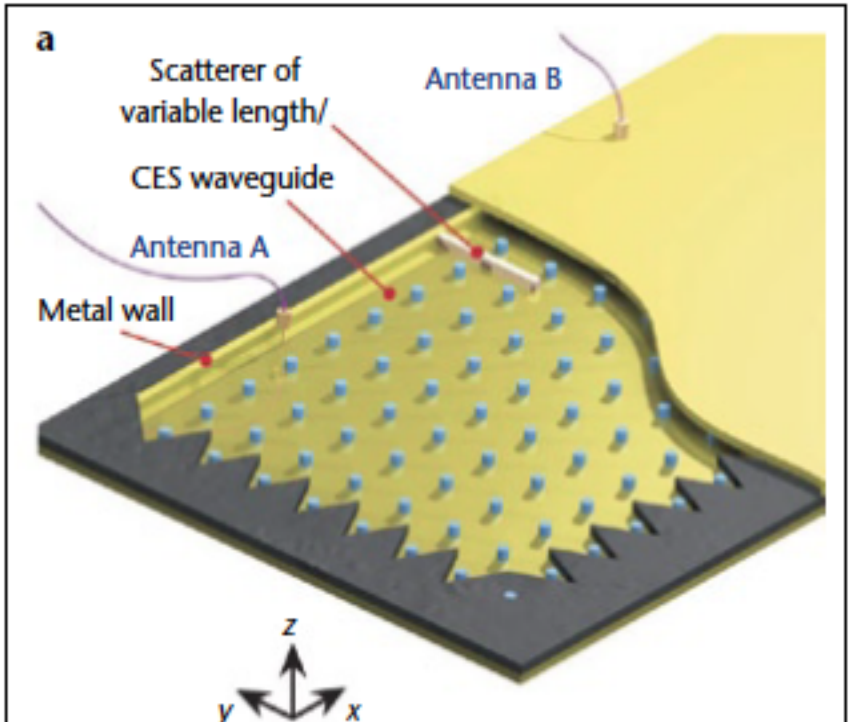


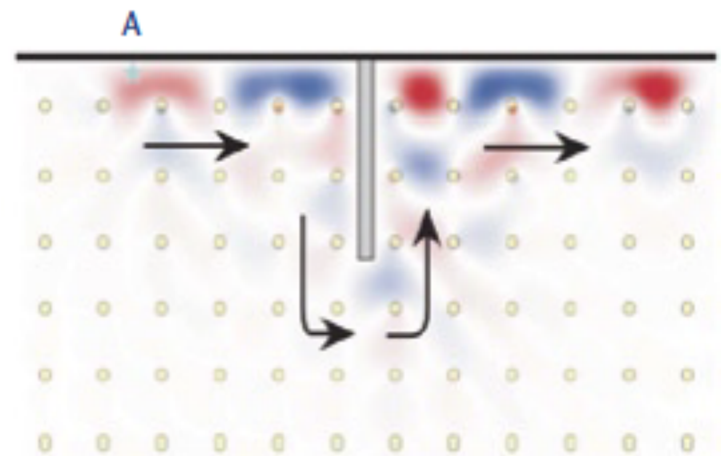
DMFT in Real space

PART III

One-Way Road in a Photonic Crystal

Chiral edge states channel light waves in one direction, like electrons in the quantum Hall effect





(a) A model of the photonic crystal. The distance between the ferrite rods is 4 cm.

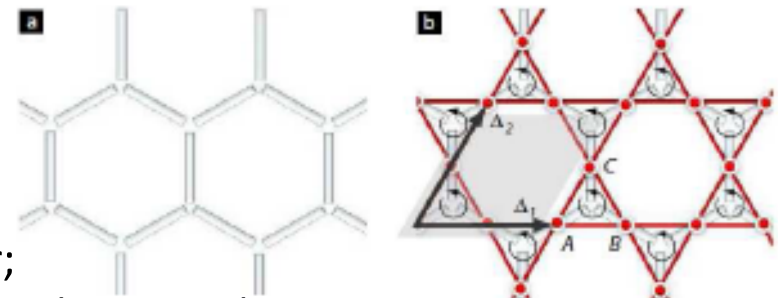
Realizations of AQH effect in Photonic crystals: following Haldane & Raghu
(lattice of rods and Faraday effect opens a gap breaking time-reversal symmetry)
Experiment: M. Soljacic et al. Nature **461**, 772 (2009);
Review by C. Ciuti and I. Carusotto, RMP

QSH phase experiments, M. Hafezi et al arXiv:1302.2153; M. C. Rechtsman et al. Nature 496, 196-200 (2013)

Breaking T-reversal symmetry

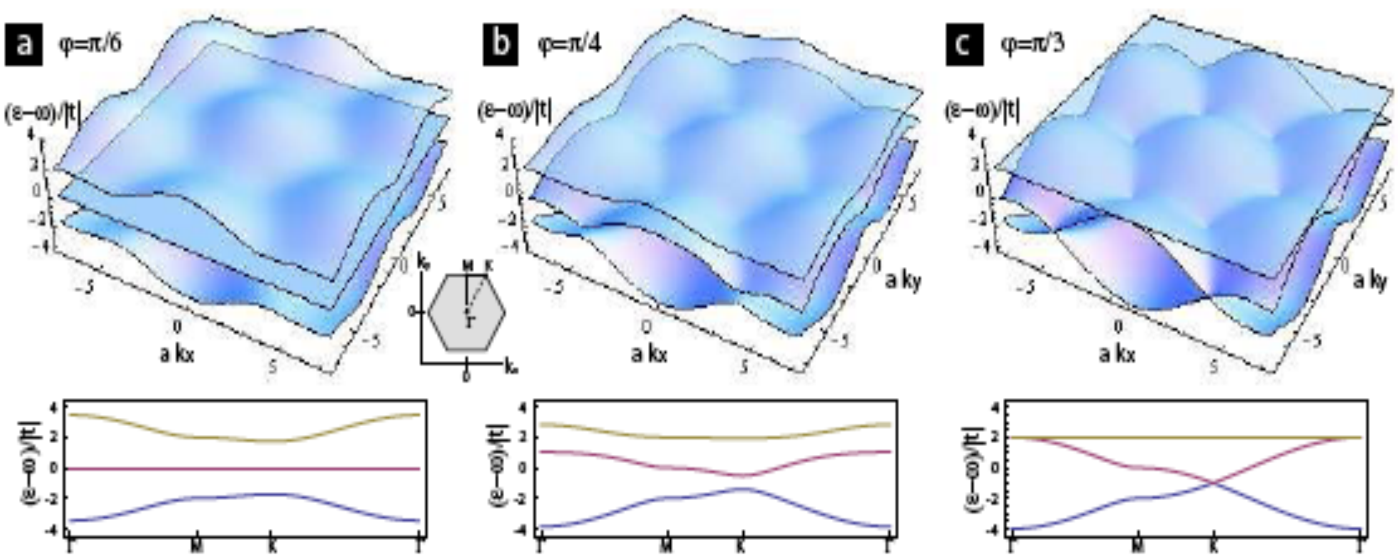
Photonic
Waveguides
with circulators

Analogy to cold
Atoms:
J. Dalibard & F. Gerbier;
K. Sengstock et al. ; I. Spielman et al.



J. Koch, A. Houck, KLH
and S. Girvin
PRA **82**, 043811 (2010)

A. Greentree & A. Martin,
Physics 3, **85** (2010)



Kagome lattice: why interesting…

Flat band (search for ferromagnetism)

A. Mielke; H. Tasaki; E. Lieb

Exotic Topological Phases:

H. M. Guo & M. Franz, PRB 2009

E. Tang, J.-W. Mei, X.-G. Wen, PRL 2011

N. Regnault and A. Bernevig, PRB 2012,…

Spin liquid search, classical degeneracies

Experimentally relevant: 2D Materials (Orsay; Princeton;…)

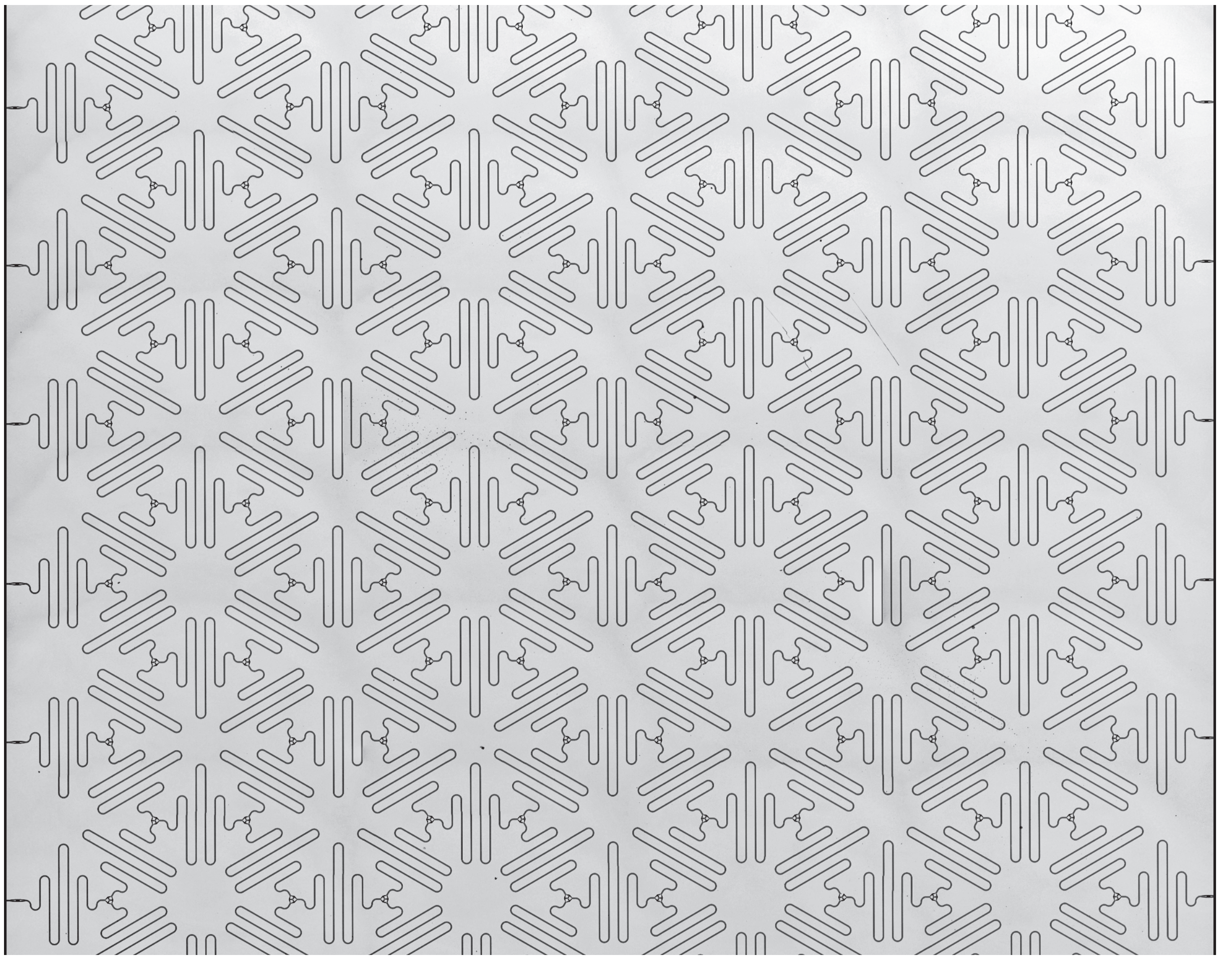
Cold atoms: Berkeley; see D. Stamper-Kurn group, 2011

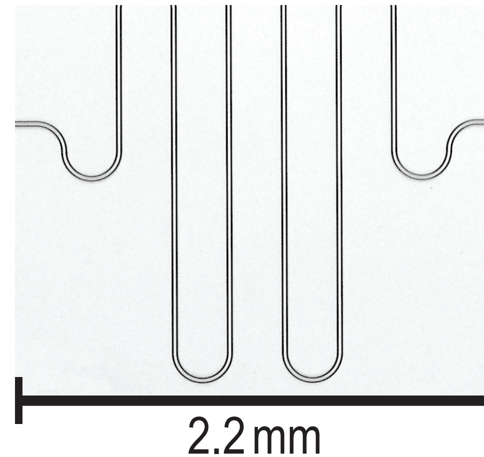
L. Balents, Nature 464, 199 (2010)

S. Yang, D. Huse and S. White, Science (2011)

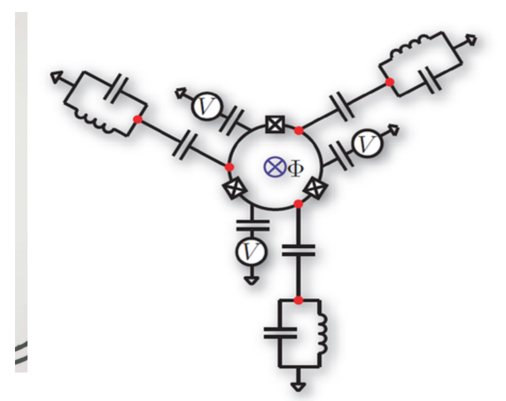
Work by Laura Messio, Claire Lhuillier, Bernard Bernu, G. Misguich…

D. Underwood, W. E. Shanks, J. Koch and A. Houck, PRA **86**, 023837 (2012)
A. Houck, H. Tureci, J. Koch, Nature Physics 2012





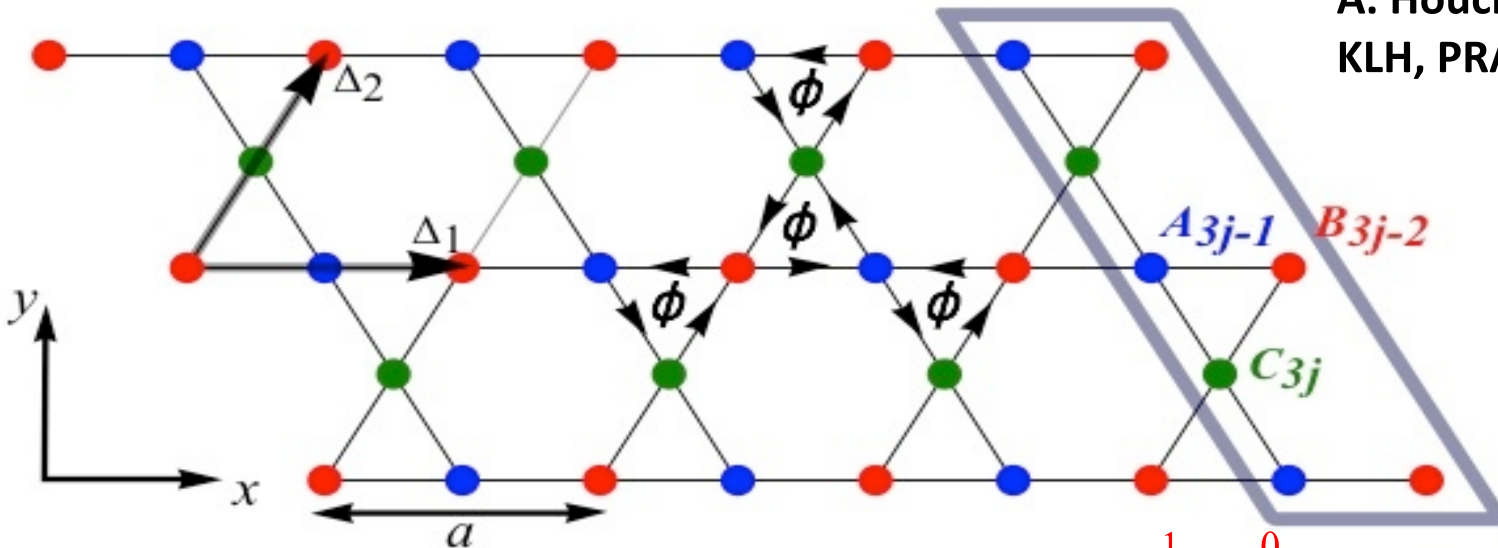
Realization of cQED networks



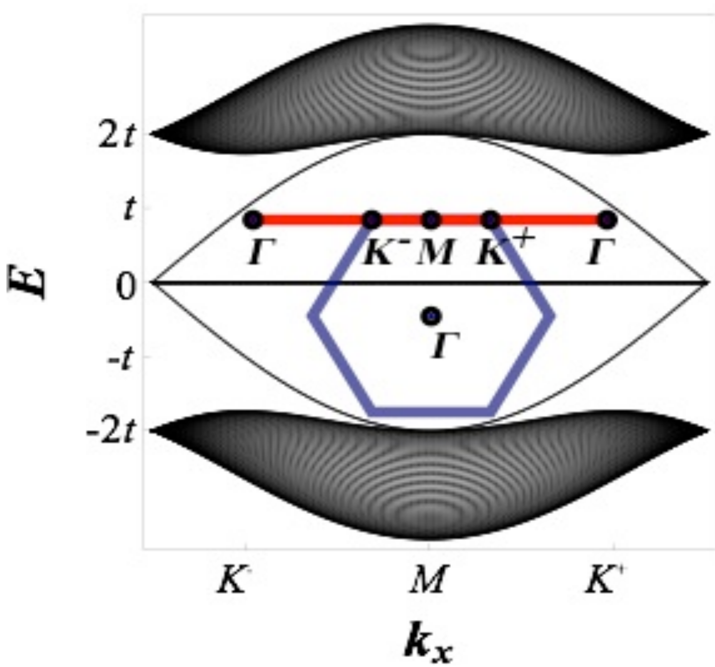
Idea to engineer gauge fields

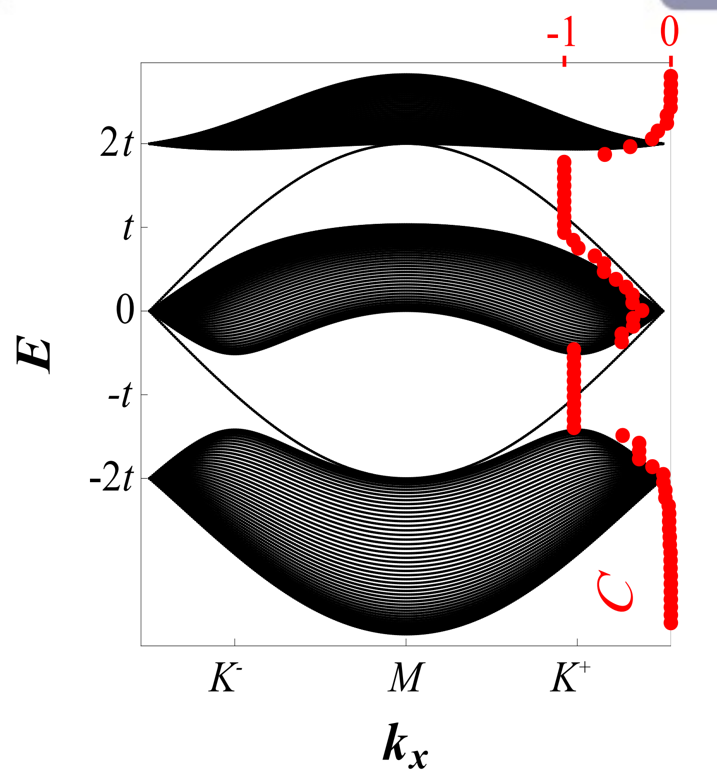
Topological Phases?

A. Petrescu,
A. Houck &
KLH, PRA 2012



$\Phi = \pi/6$





Karplus-Luttinger, 1954

D. Haldane, 2004

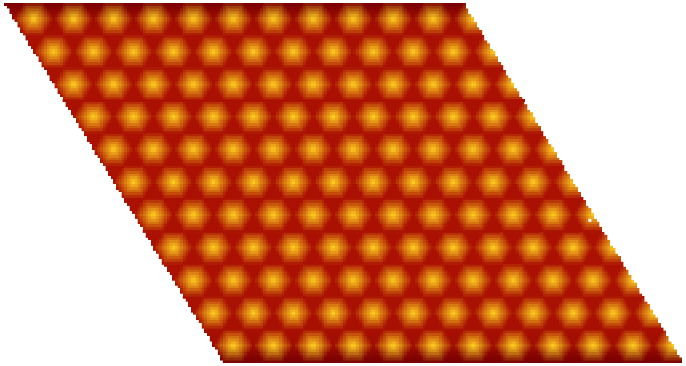
D. Bergman & G. Refael, 2010

J. Meyer & G. Refael, 2013

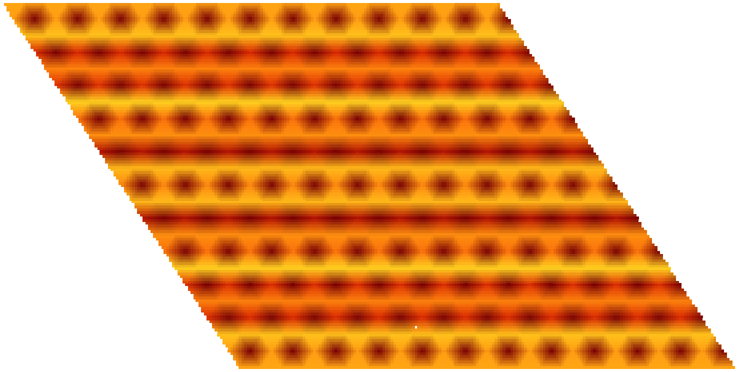
$\Phi = \pi/4$

LDOS

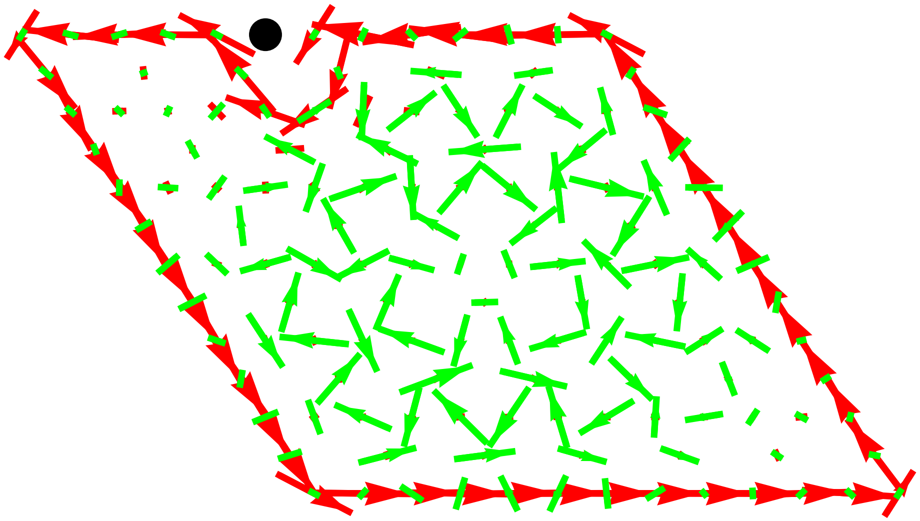
Φ = π/6

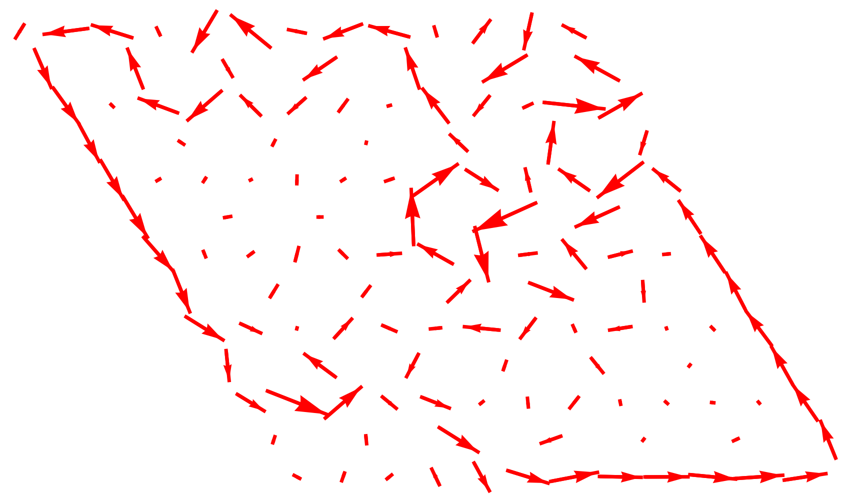


Φ = π/4



Quantum versus Anomalous Hall Effect of Light…





situation at Φ = π/6
disordered case

Red: situation at Φ = π/6
Green: situation at Φ = π/4

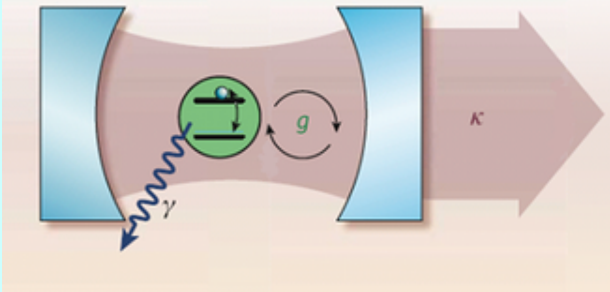
Chern number **non-quantized** for AHE and measurable…
Synthetic B-field: Loops in k space and interference experiment
See also related idea by D. Price and N. Cooper, PRA 2012

Cavity & Circuit QED: 1 cavity a lot of activity…

Coupling atoms to the EM field

- atoms can couple to the EM field via dipole moment
- coupling strength can be enhanced by confining field to a cavity

cavity QED: LKB ENS,
S. Haroche, J. M. Raimond, M. Brune

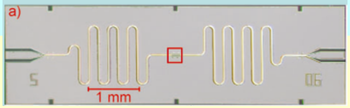


2g = vacuum Rabi frequency
γ = atomic relaxation rate
κ = photon escape rate

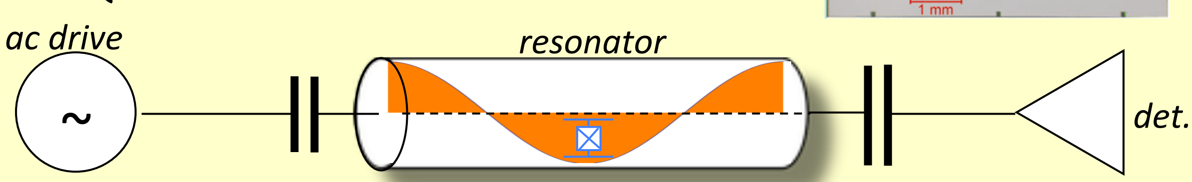
Jaynes-Cummings Hamiltonian

$$H = \tfrac{1}{2}\omega_a\sigma_z + \omega_r a^\dagger a + g\left(\sigma_- a^\dagger + \sigma_+ a\right) + \left(H_{\text{drive}} + H_{\text{baths}}\right)$$

- same concept works for superconducting qubits!

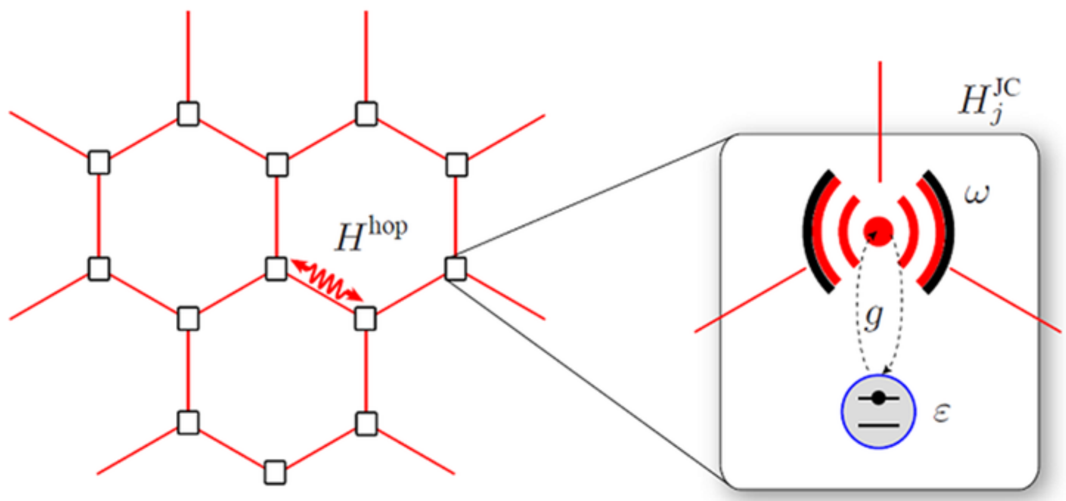


circuit QED



Reviews: J.M. Raimond et al., Rev. Mod. Phys. **73**, 565 (2001); R. J. Schoelkopf, S.M. Girvin, Nature **451**, 664 (2008)

The Jaynes-Cummings "Lattice" Model



Jaynes-Cummings model: 1963
(famous model in quantum optics)

Greentree et al., Nat. Phys. **2**, 856 (2006)
Angelakis et al., PRA **76**, 031805 (2007)
Jens Koch and KLH, PRA **80**, 023811 (2009)

Other groups: H. Tureci, R. Fazio, G. Blatter, S. Bose, Y. Yamamoto, P. Littlewood, M. Plenio, B. Simons, A. Sandvik,…

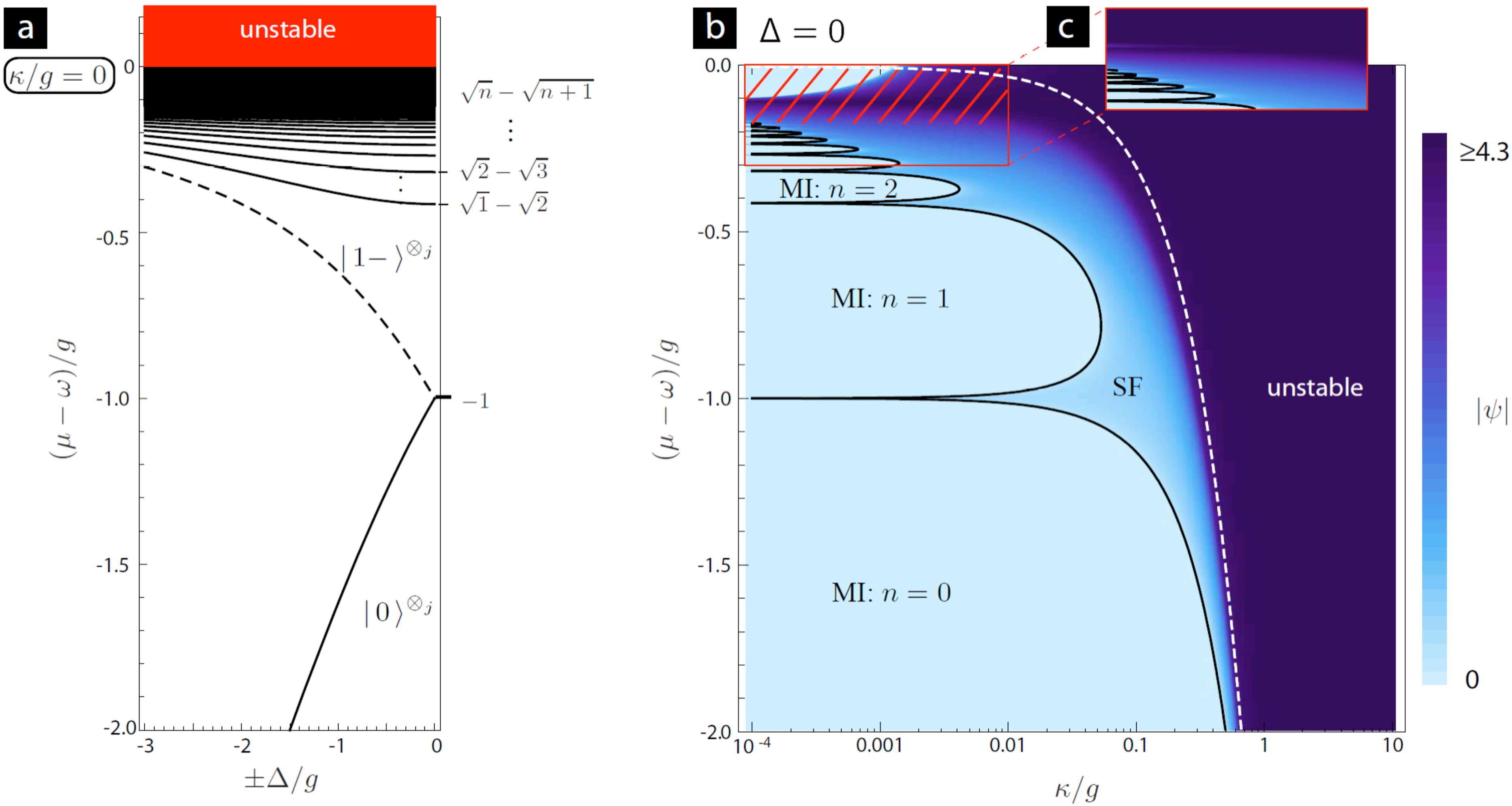
Jaynes-Cummings lattice model $H = \sum_j H_j^{\mathrm{JC}} + H^{\mathrm{hop}} - \mu N$ "chemical potential"

- *Jaynes-Cummings:* $H_j^{\mathrm{JC}} = \omega a_j^\dagger a_j + \varepsilon \sigma_j^+ \sigma_j^- + g(a_j^\dagger \sigma_j^- + \sigma_j^+ a_j)$
- *nearest-neighbor photon hopping:* $H^{\mathrm{hop}} = -\kappa \sum_{\langle i,j \rangle} (a_i^\dagger a_j + a_j^\dagger a_i)$
- *polariton number:* $N = \sum_j (a_j^\dagger a_j + \sigma_j^+ \sigma_j^-)$

Other models: Spins coupled to light, experiments at Institut d'Optique, Palaiseau
Y. Sortais, A. Fuhrmanek, R. Bourgain and A. Browaeys, PRA **85**, 035403 (2012)

Analogy to Mott Physics with photons

Greentree et al., Nat. Phys. **2**, 856 (2006)
Angelakis et al., PRA **76**, 031805 (2007)



Jens Koch and KLH, PRA **80**, 023811 (2009): **need to engineer μ**

Little Summary

Bloch Bands with non-trivial Chern numbers and Z_2 topological invariants

Applications: Materials, Cold Atoms, Photons

Gauge Theories: progress in theory and numerics

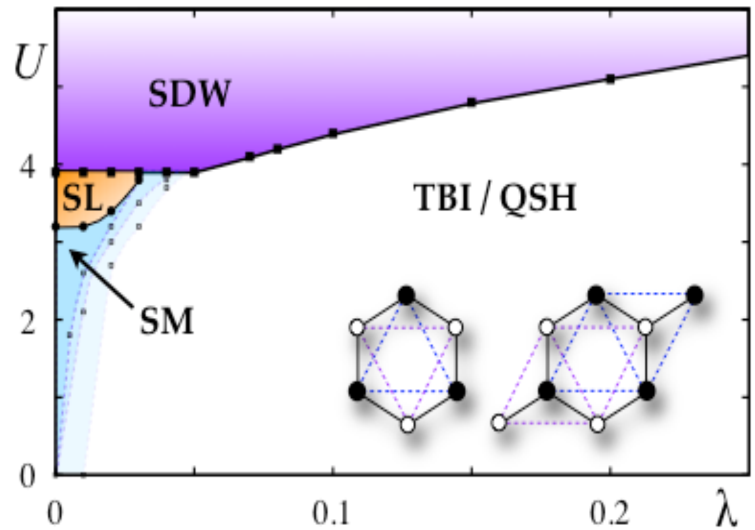
Finally, let's come back to the honeycomb lattice…

Phase Diagram: debate

Wei Wu,
Stephan Rachel,
Wu-Ming Liu
and KLH, PRB 2012

CDMFT

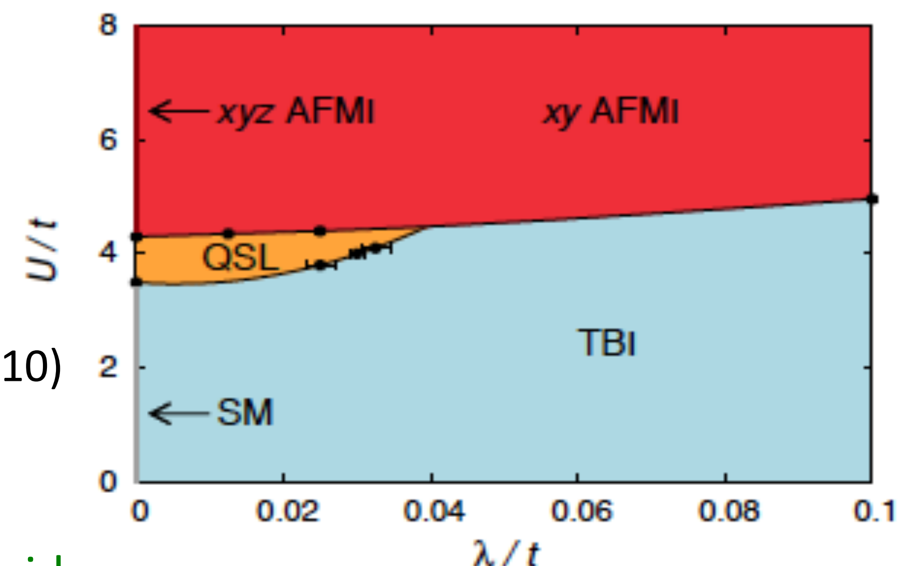
Real-space version
QMC continous-time
Impurity solver



3D XY
S. Rachel & KLH, 2010
Griset & C. Xu, 2011
D.-H. Lee, 2011

QMC

Z.Y. Meng et al.
Nature **464**, 847 (2010)



M. Hohenadler et al.
arXiv:1111.3949

Phys. Rev. Lett. **106**,
100403 (2011)

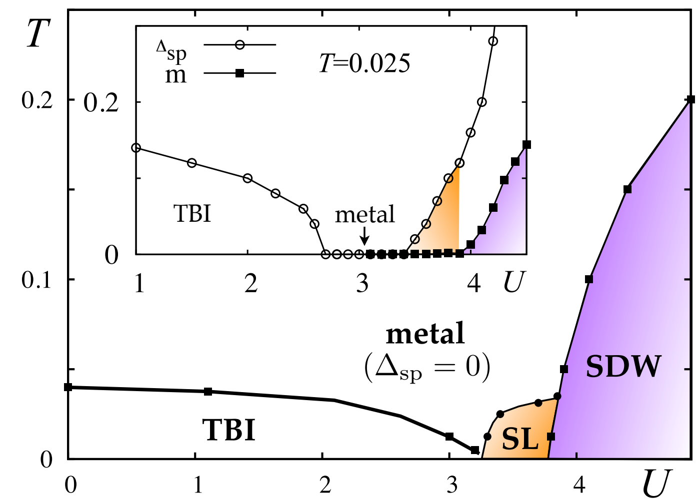
Absence of spin liquid:
S. Sorella et al. Scientific Reports 2012; S. R. Hassan & D. Senechal PRL 2013

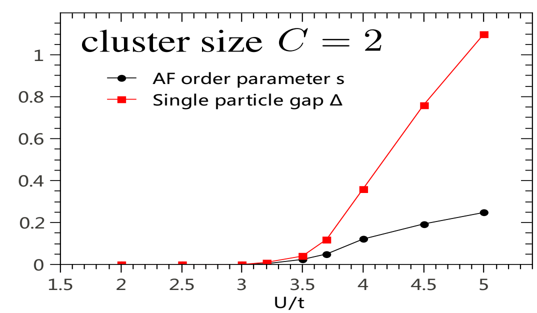
“Spin Liquid”: PI

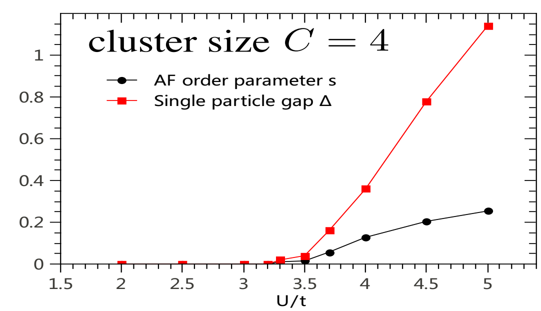
Numerical Facts: CDMFT

Single-particle gap

No long-range order

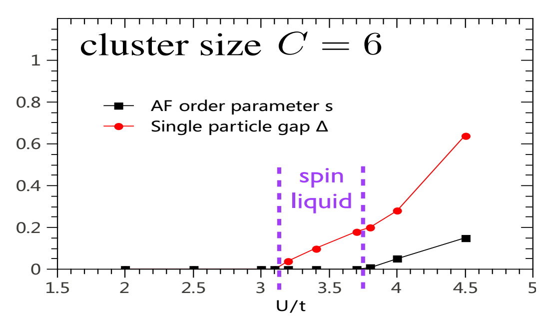


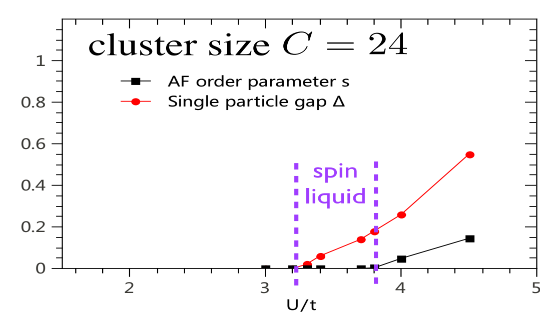




Wei Wu

CDMFT





Plaquette Model and PI: **Possible explanation**