The Abdus Salam International Centre for Theoretical Physics

2472-5

Advanced Workshop on Nonlinear Photonics, Disorder and Wave Turbulence

15 - 19 July 2013

Charge transport in graphene and light propagation in dielectric structures with metamaterials: a comparative study

Valentin Freilikher
Bar-Ilan University, Israel

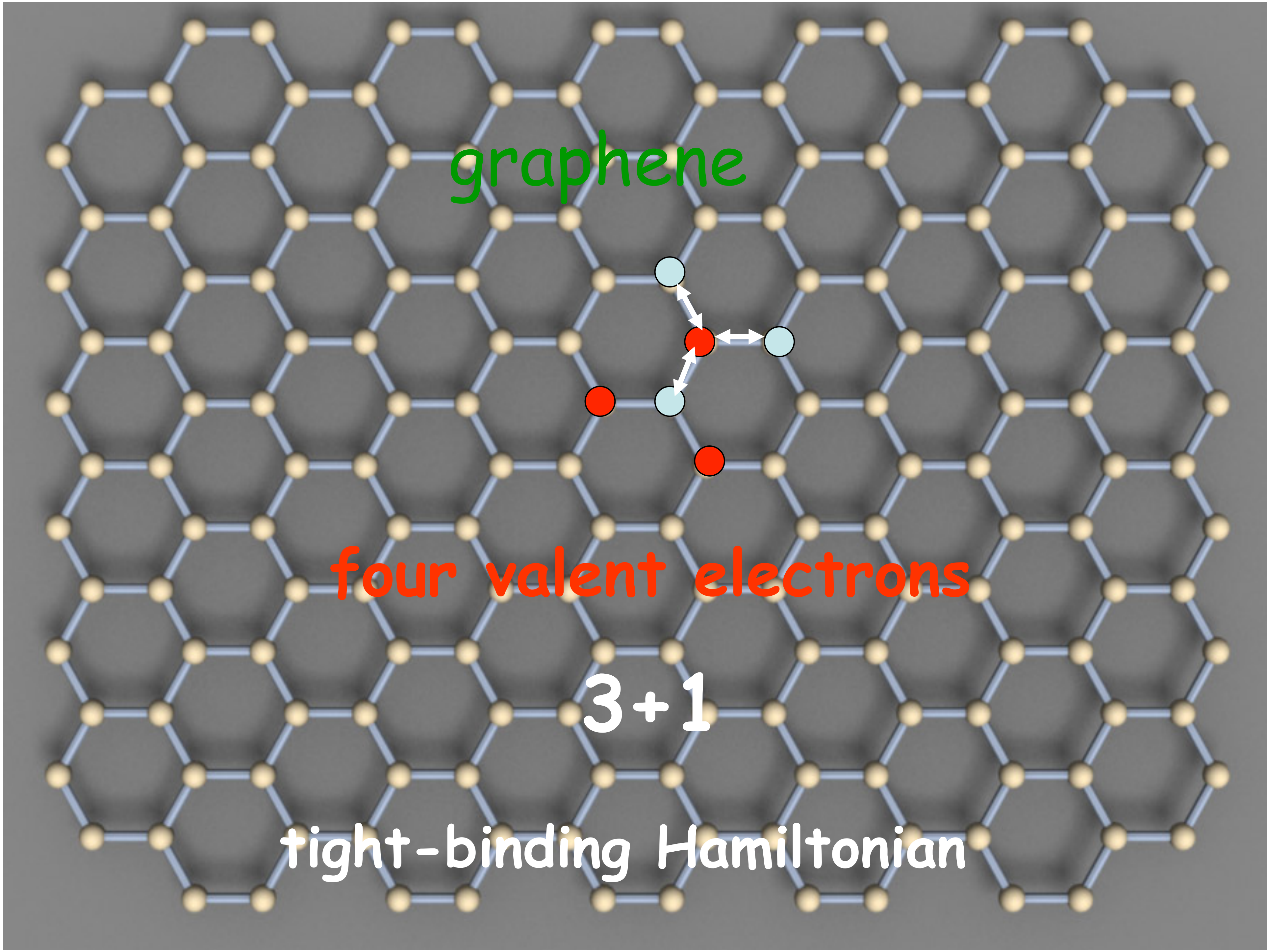
Charge transport in graphene and light propagation in dielectric structures with metamaterials: a comparative study

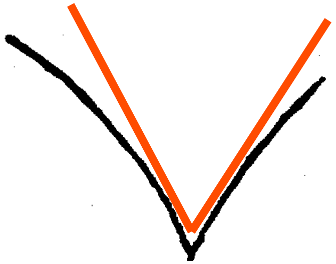
Valentin Freilikher

Bar-Ilan University

ISRAEL

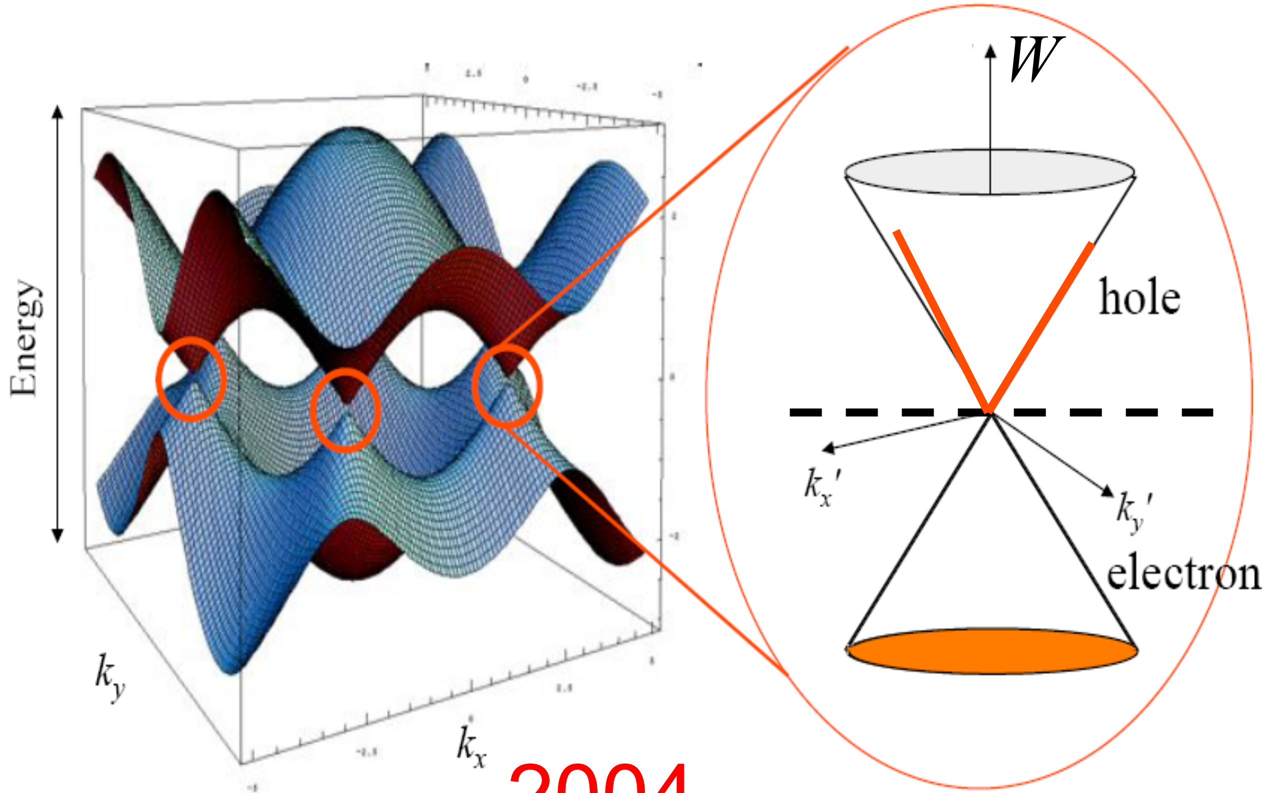
Yu. Bliokh, F. Nori


graphene
four valent electrons
3+1
tight-binding Hamiltonian



1947

Wallace



2004

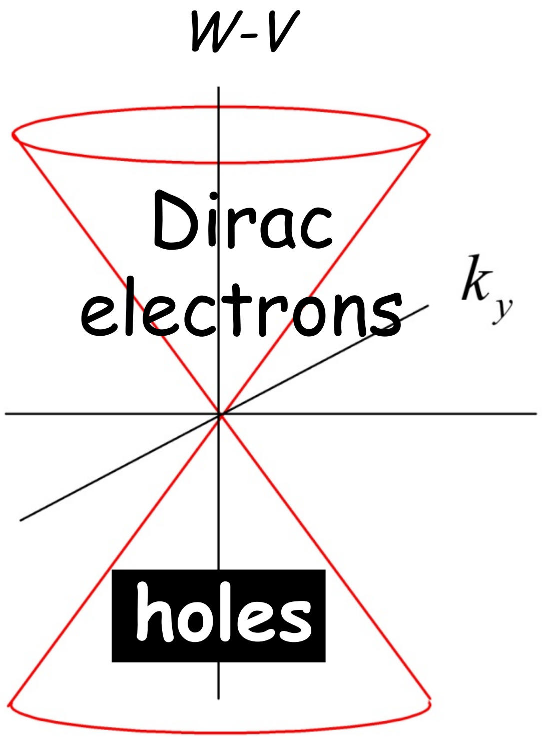
Dirac points – point-like transparency zones graphene

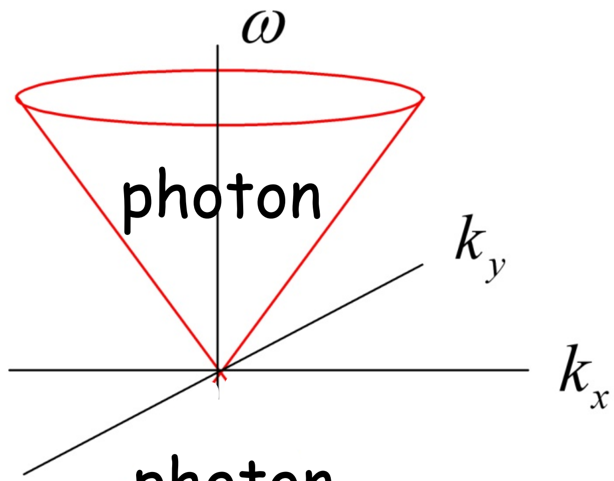
1947 - 2004

negative-index metamaterials

1945, 1964 - 1998

Dirac point





photon
is its own antiparticle

$$(W - V)^2 = h^2 v_F^2 (k_x^2 + k_y^2)$$

$$n^2 \omega^2 = c^2 (k_x^2 + k_y^2)$$

can a *bona fide* Dirac point (a sort of a optical particle – antiparticle pair) exist in dielectrics?

$$\psi = (\psi_A, \psi_B)^T$$

$$i\hbar \frac{\partial \psi}{\partial t} = \left(\hat{H}_0 + V\right)\psi$$

$$\hat{H}_0 = v_F \vec{\sigma}\,\vec{p}, \quad \vec{\sigma} = (\sigma_1, \sigma_2), \quad \vec{p} = -i\hbar\vec{\nabla}$$

Dirac equation for massless relativistic particles

Maxwell equations for electromagnetic waves

$$\frac{\partial \vec{E}}{\partial t} = \frac{c}{\varepsilon}\, curl\vec{H}$$

$$\frac{\partial \vec{H}}{\partial t} = -\frac{c}{\mu}\, curl\vec{E}$$

$$\left(\vec{E}, \vec{H}\right)$$

L. Silberstein, Ann. Phys. 22, 579 **(1907)**

Maiorana (1930

$$Z(x) = \sqrt{\mu / \varepsilon}$$

$$n(x) = \pm\sqrt{\varepsilon\mu}$$

$$\mathrm{E} = E_y - iE_x$$

$$\mathrm{H} = ZH_z$$

$$\psi_{A,B} \sim \exp\left(-i\frac{W}{\hbar}t + ik_y y\right)$$

$$\mathrm{E}, \mathrm{H} \sim \exp\left(-i\omega t + ik_y y\right)$$

$$\psi_A = -\frac{iv_F\hbar}{W - V}\left(\frac{d\psi_B}{dx} + k_y\psi_B\right)$$

$$\psi_B = -\frac{iv_F\hbar}{W - V}\left(\frac{d\psi_A}{dx} - k_y\psi_A\right)$$

$$\mathrm{H} = -\frac{i}{kn}\left(\frac{d\mathrm{E}}{dx} + k_y\mathrm{E}\right)$$

$$\mathrm{E} = -\frac{i}{kn}\left(\frac{d\mathrm{H}}{dx} - k_y\mathrm{H}\right)$$

Quasiparticles with the energy W

in the graphene sheet subjected

to the electrostatic potential $V(x)$

$\Updownarrow$

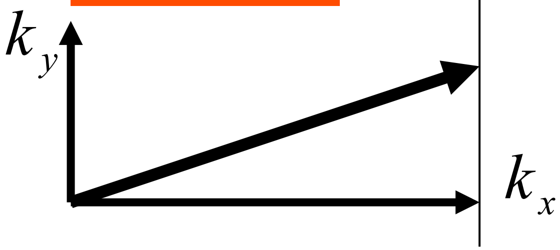
Electromagnetic waves with the frequency ω

in the dielectric with the refractive index $n(x) = \frac{c}{\omega}\frac{W - V(x)}{\hbar v_F}$

boundary conditions

$$Z^{(1)} = Z^{(2)}$$

$$\frac{H_z^{(1)}}{H_z^{(2)}} = 1; \frac{E_y^{(1)}}{E_y^{(2)}} = 1$$



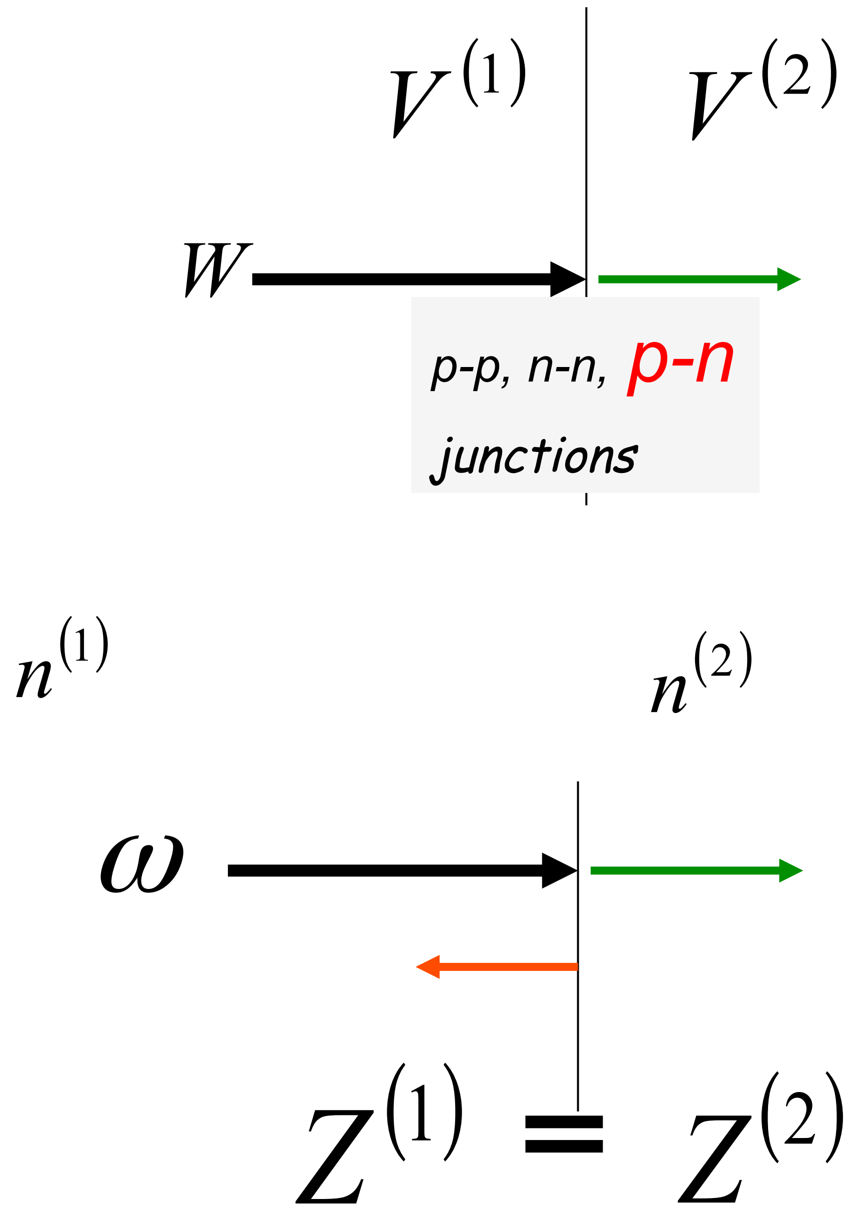
$$\frac{\psi_A^{(1)}}{\psi_A^{(2)}} = 1$$

$$\frac{\psi_B^{(1)}}{\psi_B^{(2)}} = 1$$

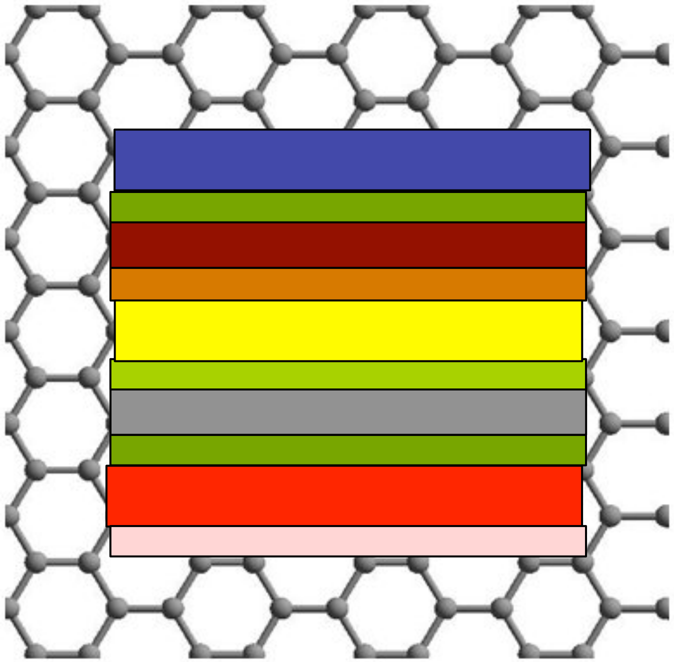
$$\frac{\mathbf{H}^{(1)}}{\mathbf{H}^{(2)}} = 1$$

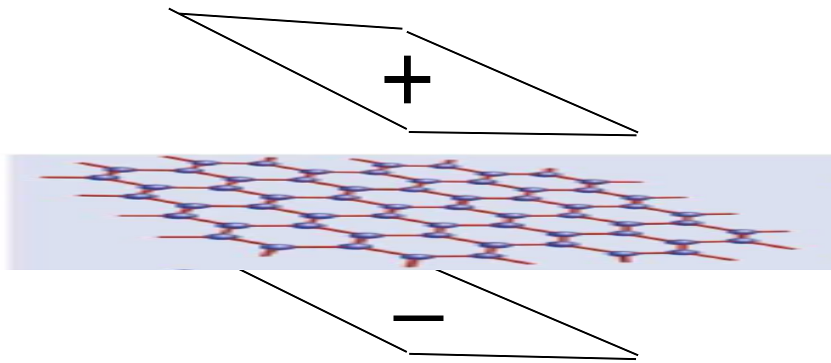
$$\frac{\mathbf{E}^{(1)}}{\mathbf{E}^{(2)}} = 1$$

$$k_y = 0$$

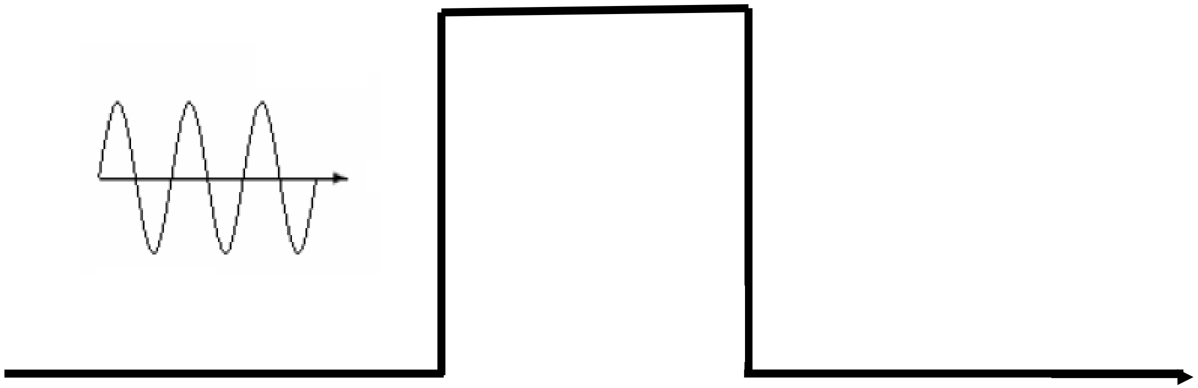


$V^{(1)}$
$V^{(2)}$
W
p-p, n-n, p-n
junctions
$n^{(1)}$
$n^{(2)}$
ω
$Z^{(1)} = Z^{(2)}$





Klein paradox



$$Z^{(1)} = Z^{(2)} = Z^{(3)}$$

$$R_{12} = R_{23} = 0$$

matching microwave elements

$$T_d = \frac{2\cos\theta_1 \cos\theta_2}{1+\cos(s_1\theta_1 + s_2\theta_2)}, \quad R_d = \frac{1-\cos(s_1\theta_1 - s_2\theta_2)}{1+\cos(s_1\theta_1 + s_2\theta_2)}$$

$$T_g = \frac{4\cos\theta_1 \cos\theta_2}{(\cos\theta_1 + \cos\theta_2)^2}, \quad R_g = \frac{(\cos\theta_1 - \cos\theta_2)^2}{(\cos\theta_1 + \cos\theta_2)^2}$$

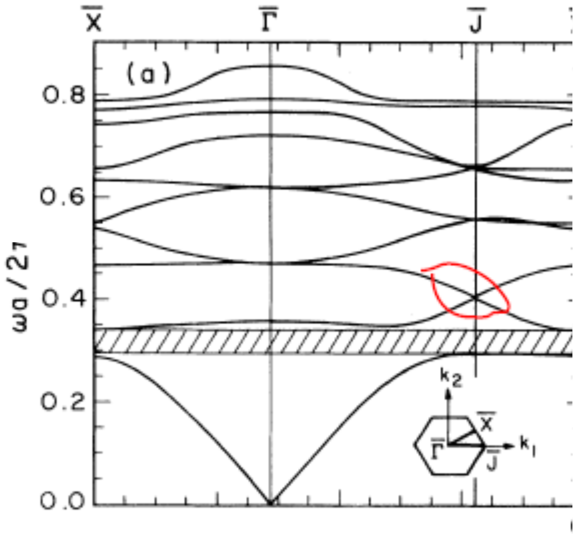
can a “genuine” Dirac point exist in dielectrics?

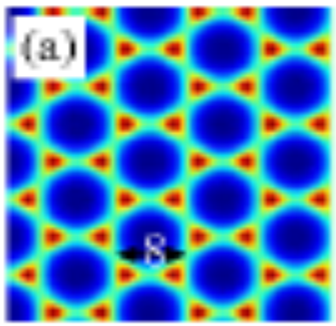
YES... in structured (periodic) systems

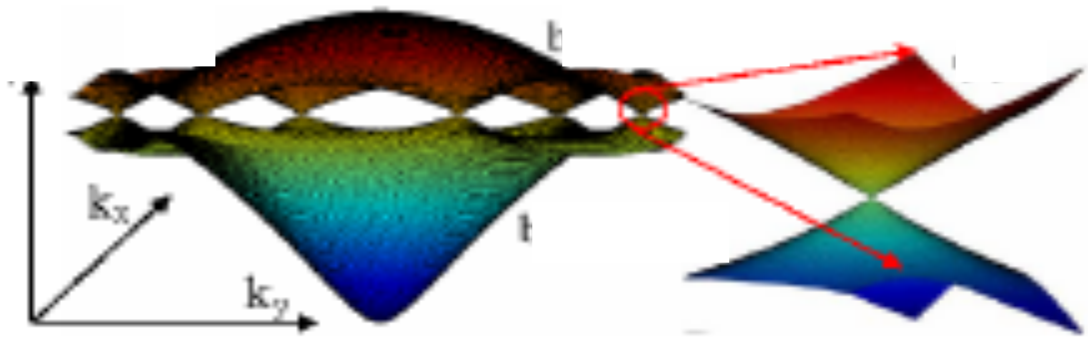
2-D photonic crystals

M. Plihal and A. A. Maradudin

1991



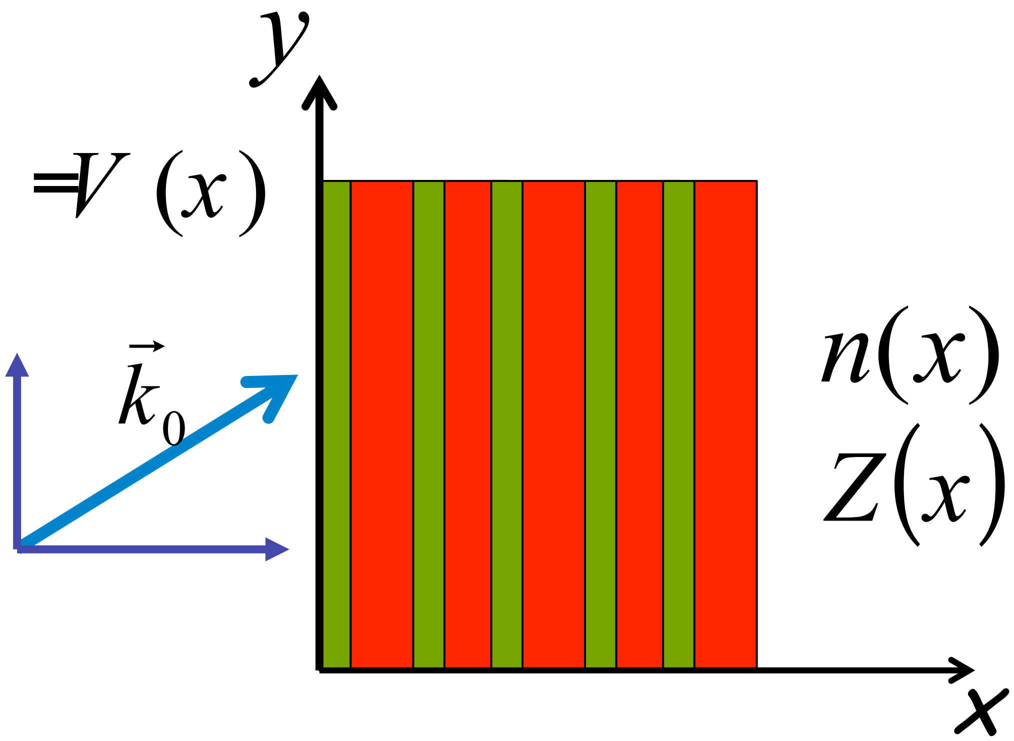




M. Segev 2007

K. Sakoda 2012

1-D (layered) structures

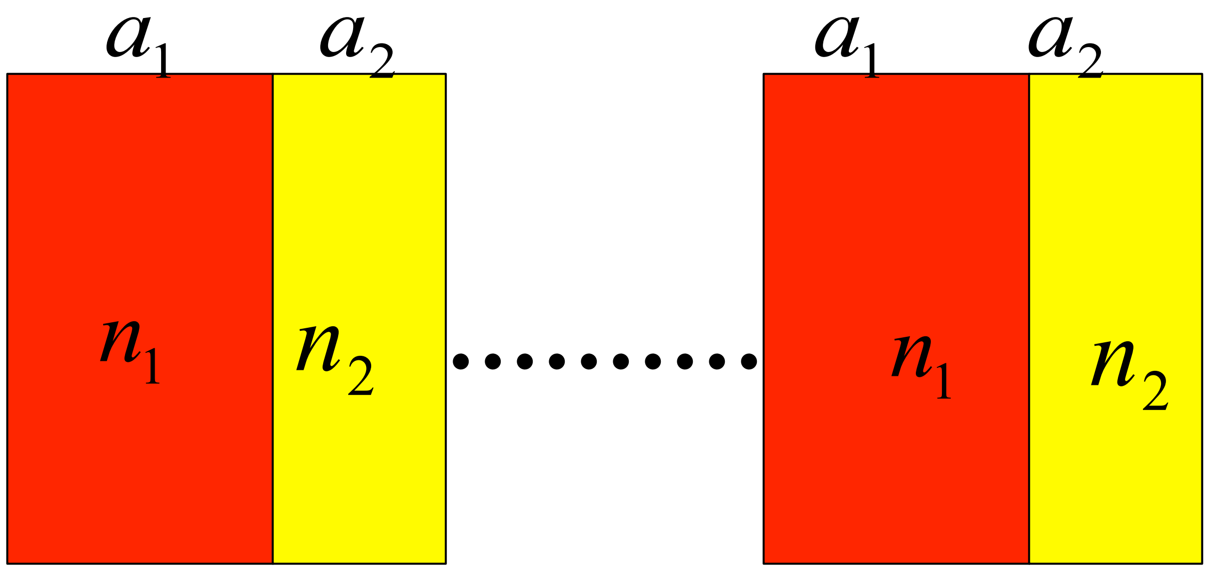


$$V = V(x)$$

$$n(x) = \sqrt{\varepsilon\mu}$$
$$Z(x) = \sqrt{\mu/\varepsilon}$$

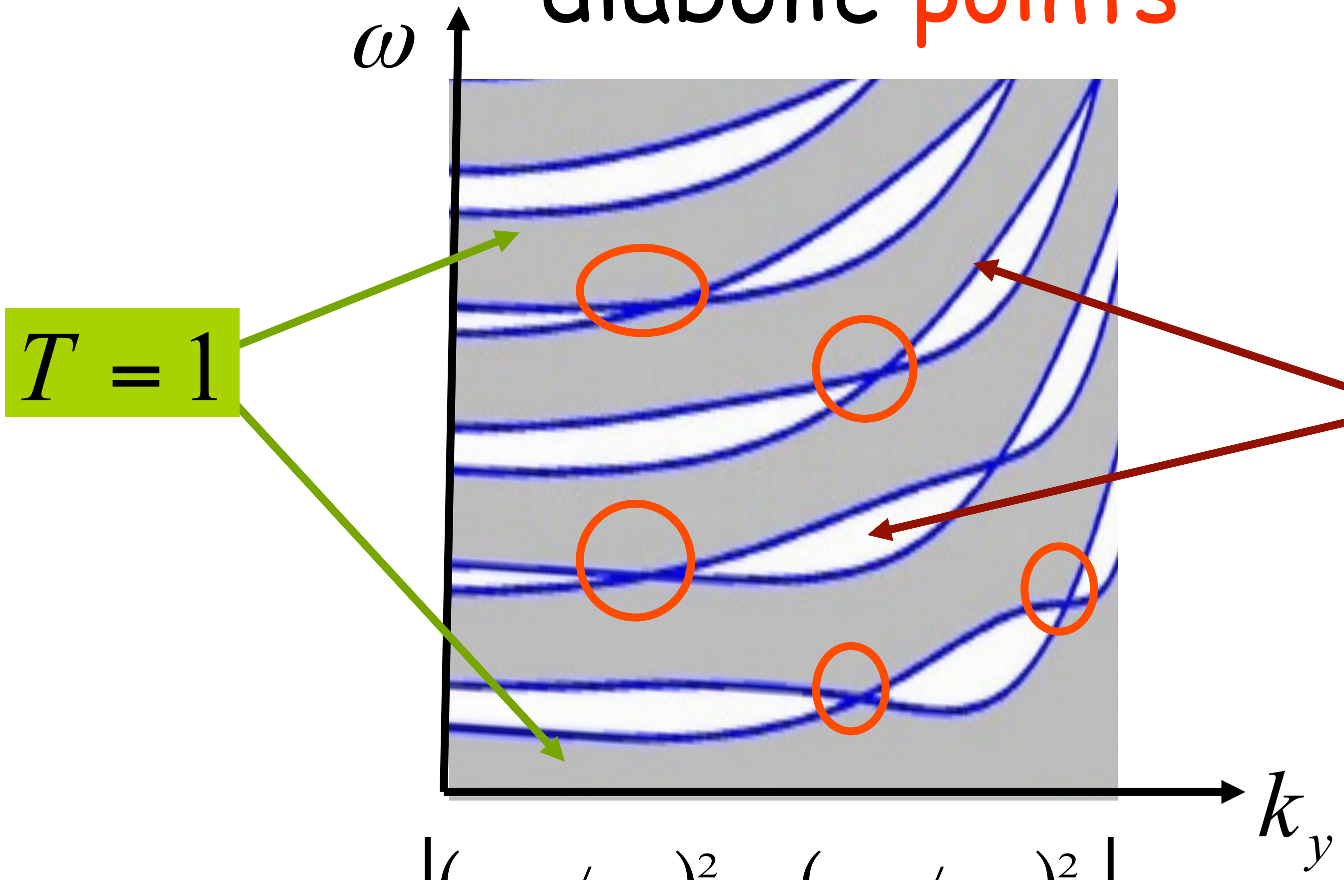
Quasiparticles with the energy ***W*** in a graphene super-latice created by a electrostatic potential ***V(x)***	Electromagnetic wave with the frequency ***ω*** in a dielectric sample with ***n(x)***

The image cannot be displayed. Your computer may not have enough memory to open the image, or the image may have been corrupted. Restart your computer, and then open the file again. If the red x still appears, you may have to delete the image and then insert it again.



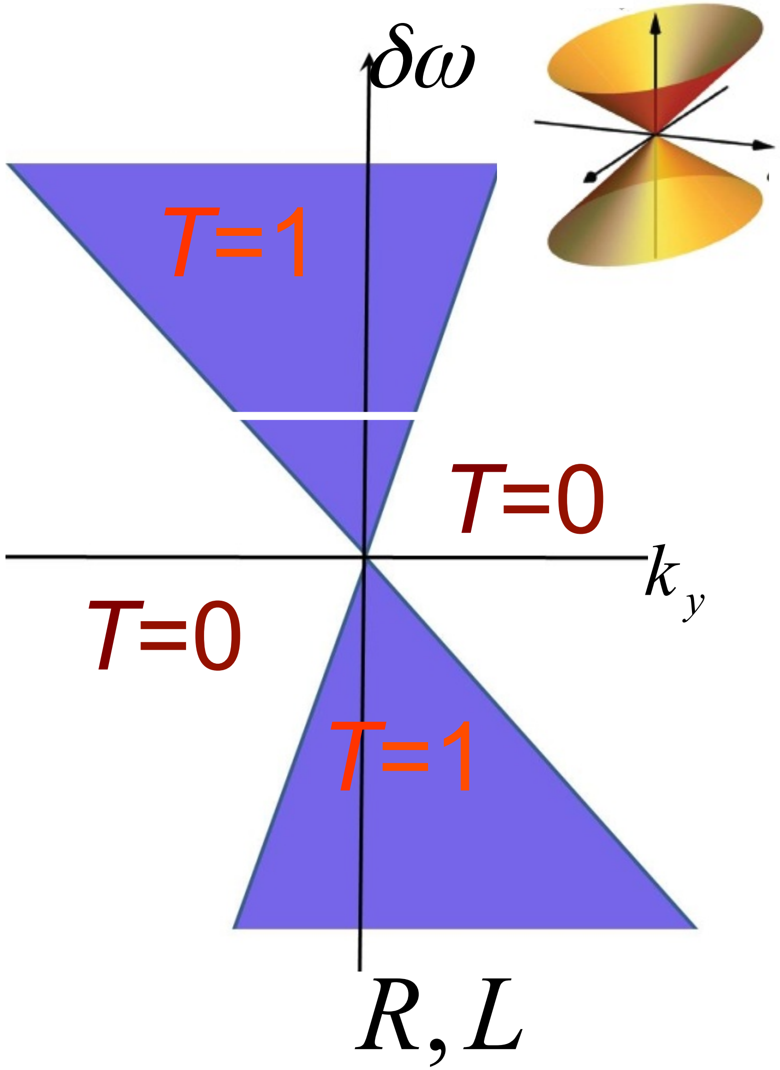
cone-like (linear) singularities

diabolic points



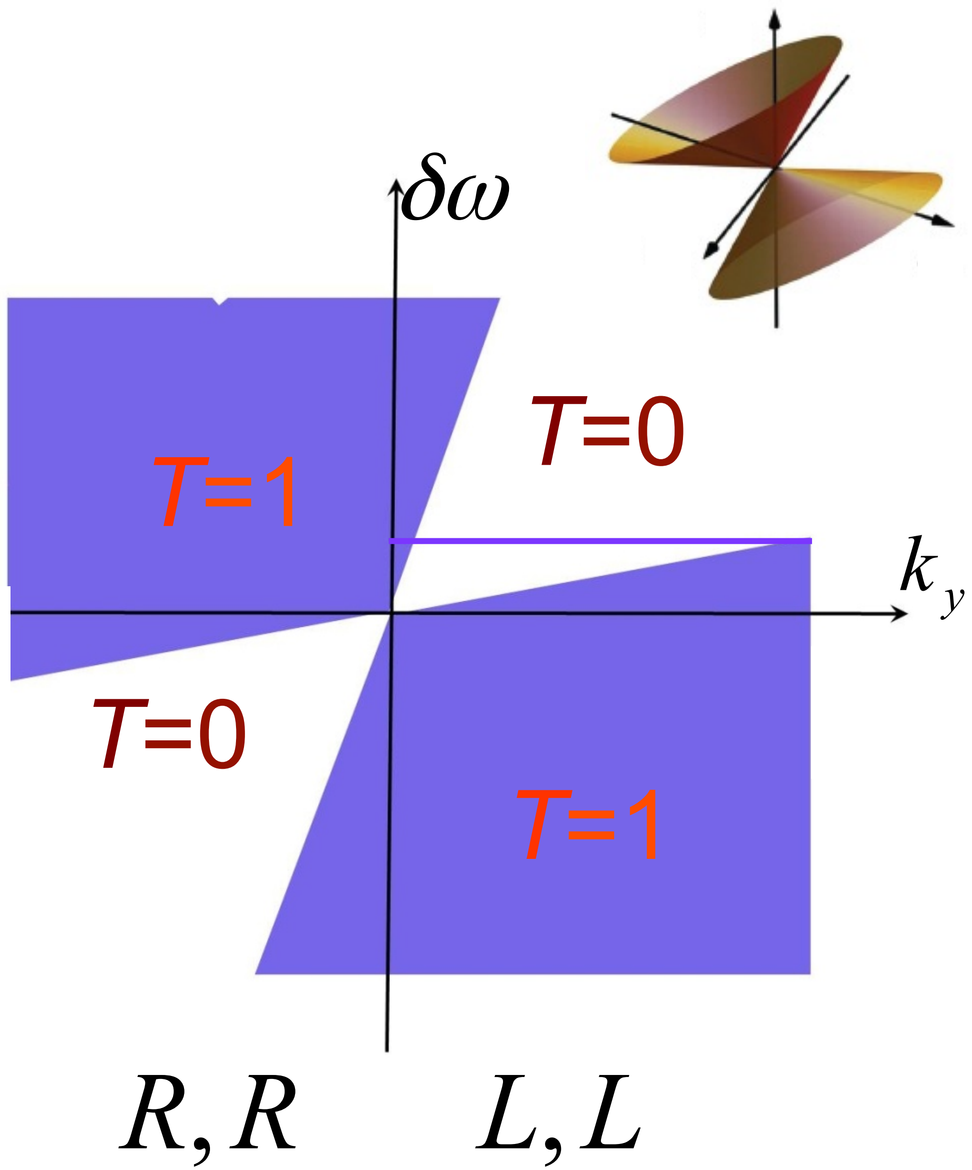
$$\omega_D^2 = \frac{\left\lfloor (p\pi / a_1)^2 - (q\pi / a_2)^2 \right\rfloor}{c^2\left(n_1^2 - n_2^2\right)}$$

$$k_{Dy}^2 = \frac{\left\lfloor n_2^2 (p\pi / a_1)^2 - n_1^2 (q\pi / a_2)^2 \right\rfloor}{{}^2\left(n_1^2 - n_2^2\right)} \qquad p, q = 1, 2, 3 \ldots$$



Dirac point

point-like transmission zone



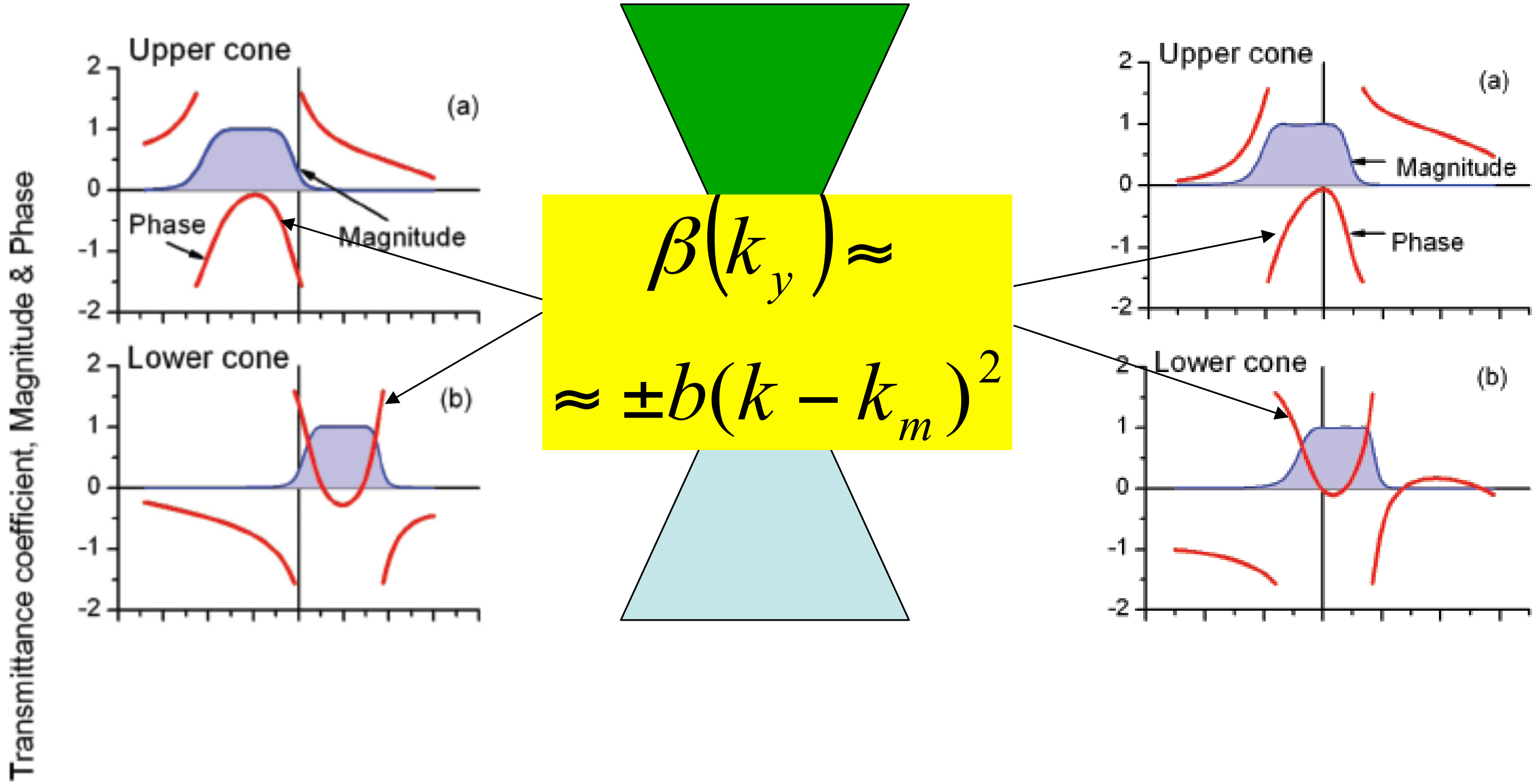
point-like gap

$$\left(\frac{d_1}{d_2}\right)^3\left(\frac{q}{p}\right)^2\left(\frac{\varepsilon_1}{|\varepsilon_2|}\right) < 1 < \left(\frac{d_1}{d_2}\right)\left(\frac{|\varepsilon_2|}{\varepsilon_1}\right)$$

$$\left(\frac{d_2}{d_1}\right)\left(\frac{p}{q}\right)\left(\frac{|n_2|}{n_1}\right) > 1,$$

transmission near Dirac points

monochromatic plane wave



$$\beta\left(k_y\right) \approx$$

$$\approx \pm b(k - k_m)^2$$

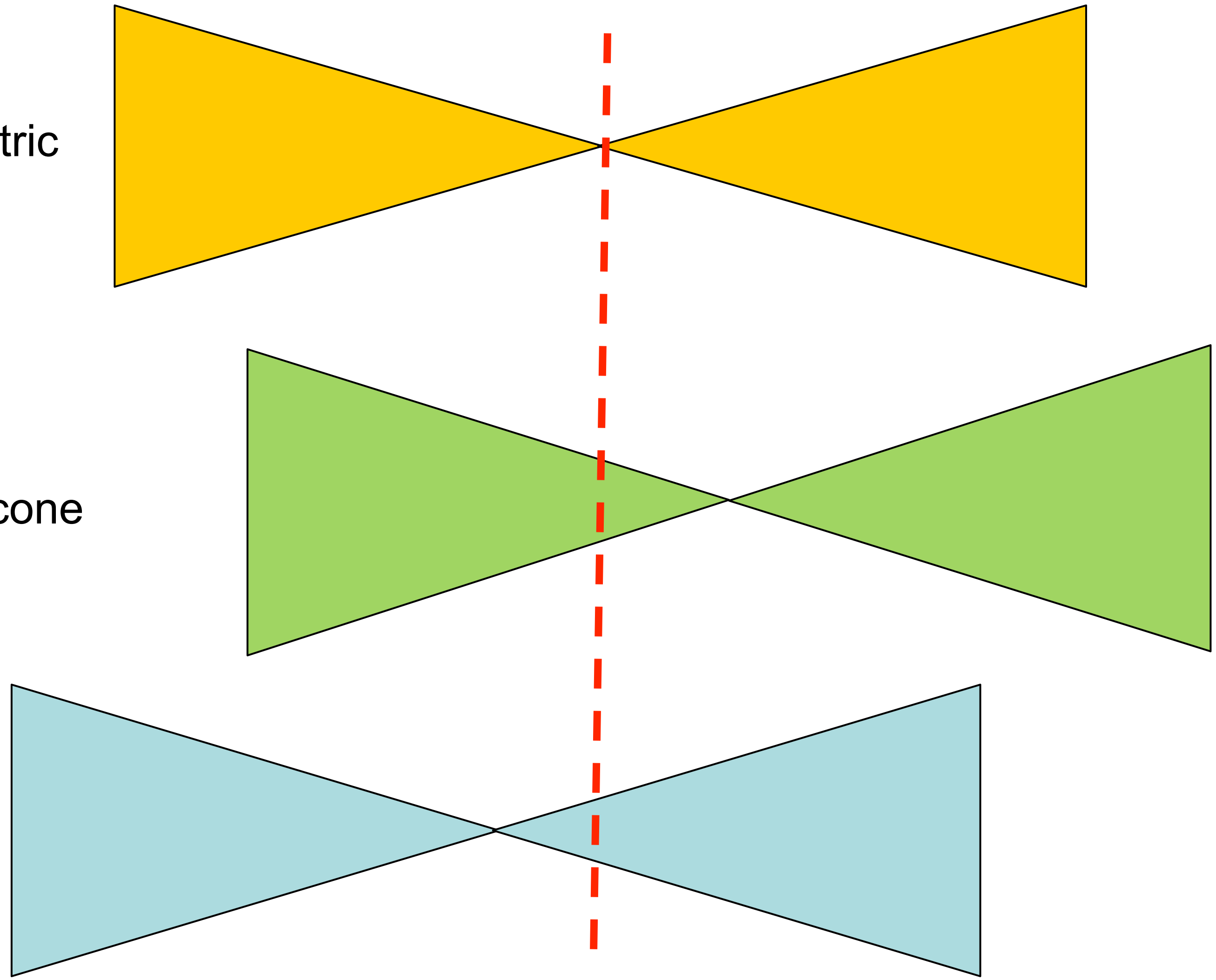
transmission near the Dirac points

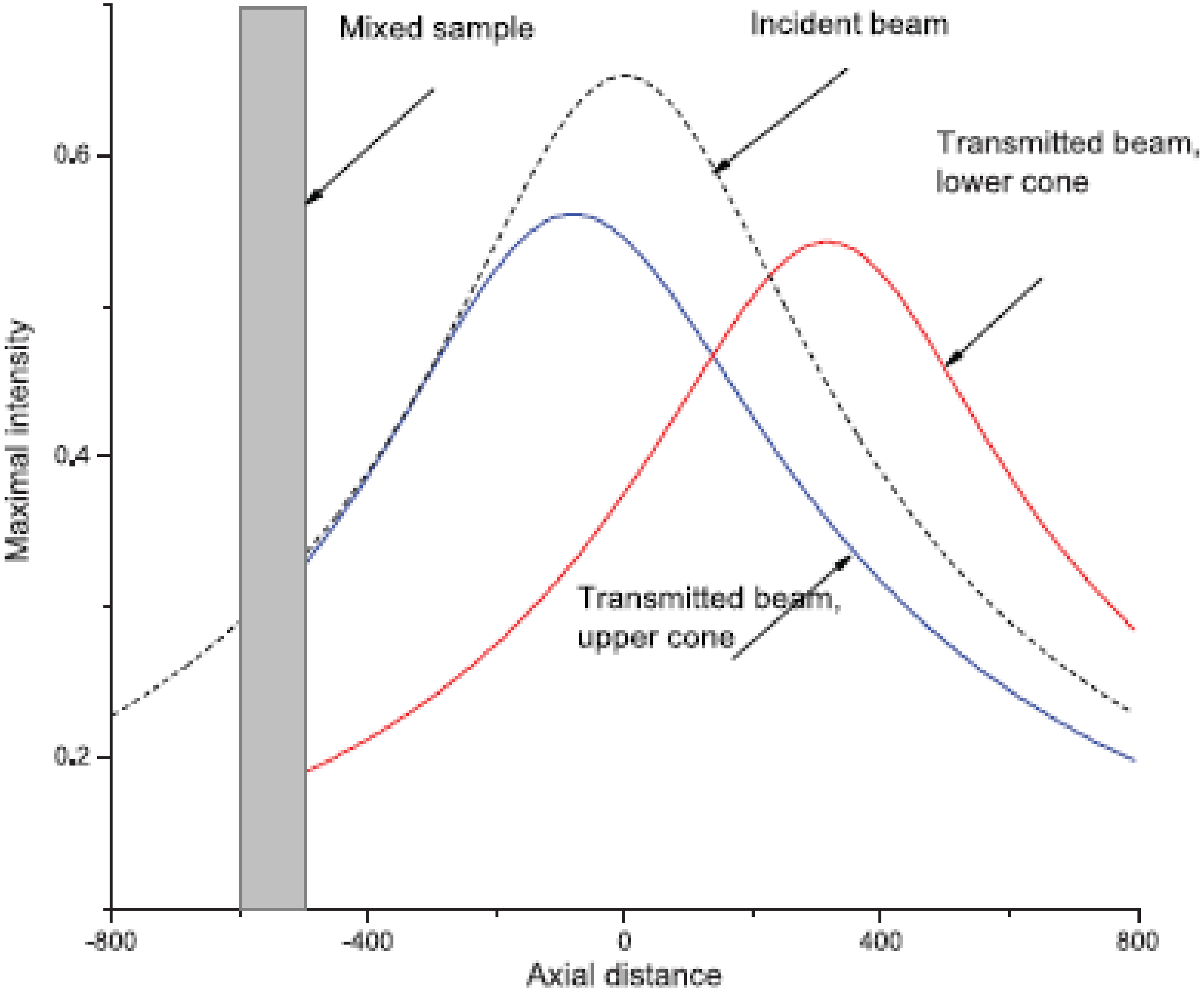
monochromatic converging beam

beam in a normal dielectric

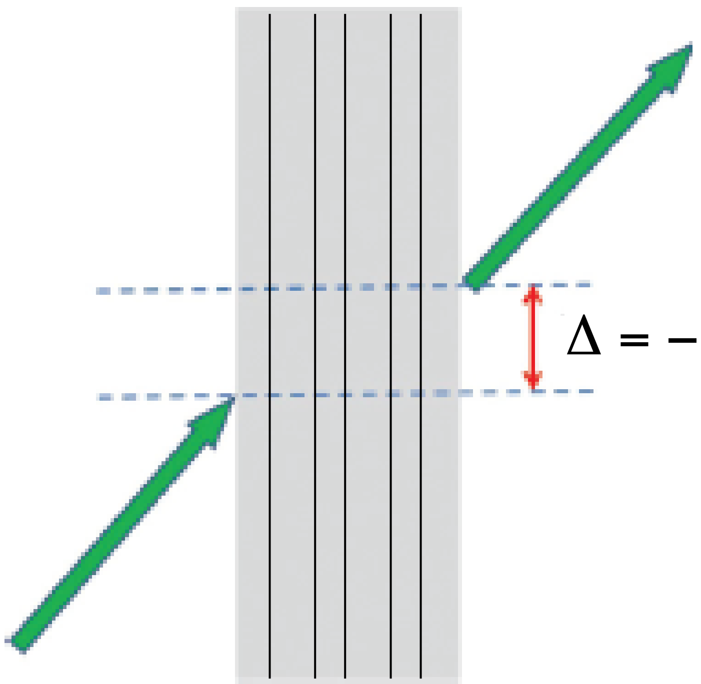
frequency in the lower cone

frequency in the the upper cone




Mixed sample
Incident beam
Transmitted beam,
lower cone
Transmitted beam,
upper cone
Maximal intensity
0.6
0.4
0.2
-800
-400
0
400
800
Axial distance

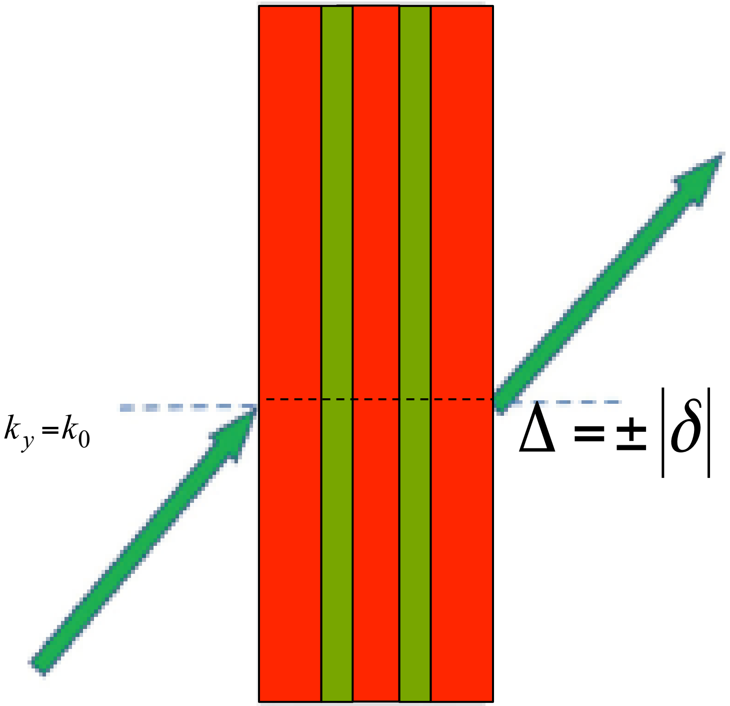
mono-type sample



$$\Delta = -\left.\frac{d\varphi_{tr}(k_y)}{dk_y}\right|_{k_y=k_0}$$

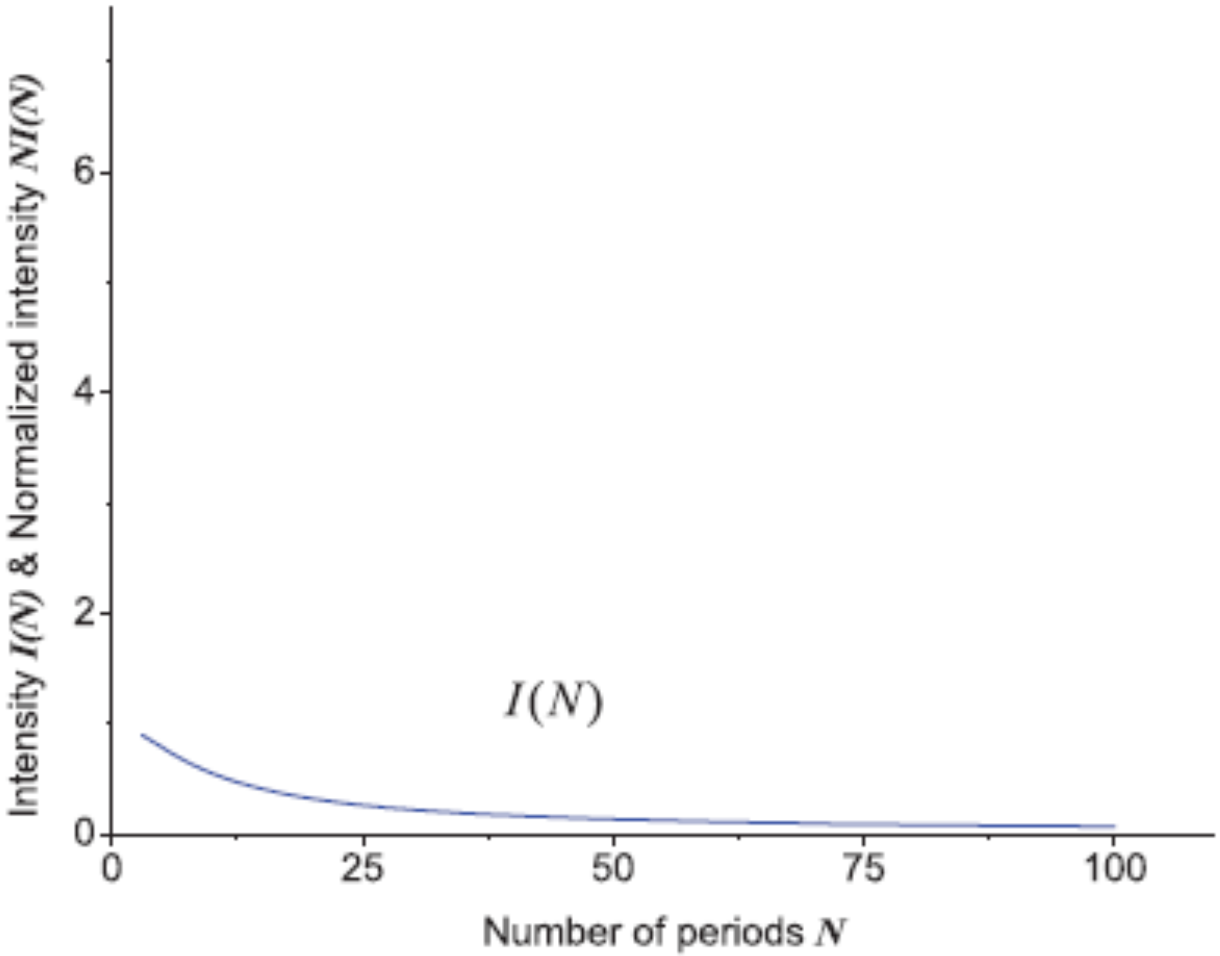
$$\varphi_{mt}(k_y) \approx a k_y$$

mixed sample

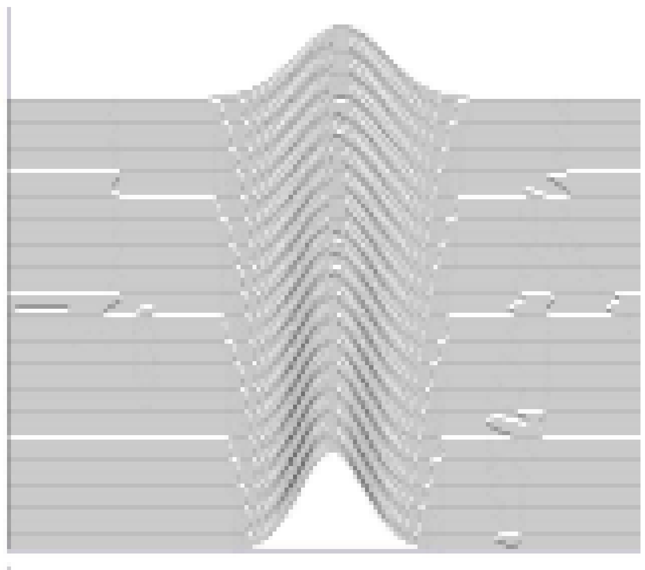


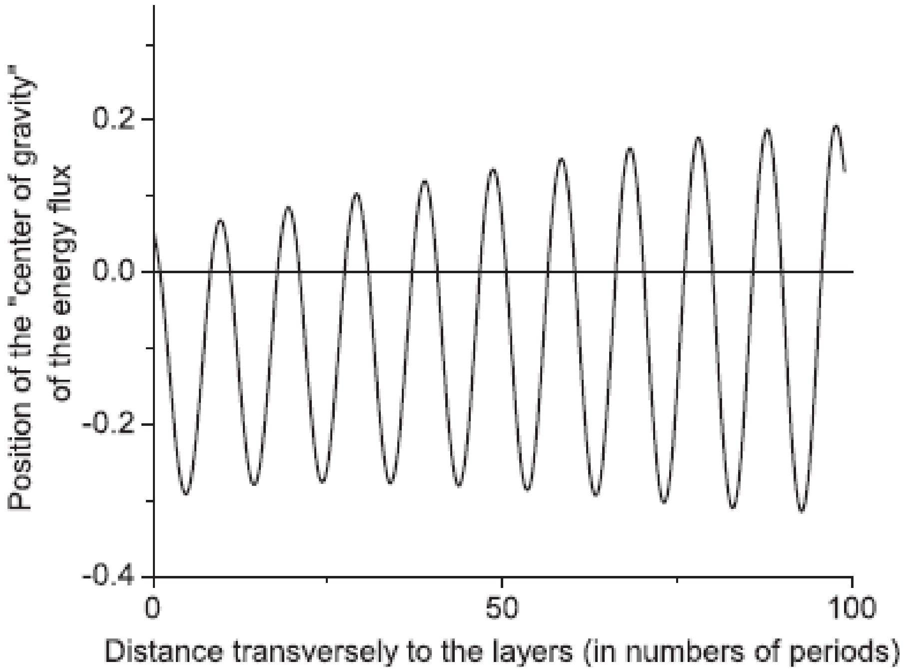
$$\Delta = \pm|\delta| << \Delta_{mt}$$

$$\varphi_{mx}(k_y) \approx \pm b(k_y - k_0)^2$$



diffusion-like (1/L) dependence of the intensity of the distance





trembling motion

spatial analog of the Zitterbewegung phenomenon

Charge transport in graphene superlatices is identical (at normal incidence) to the propagation of light through layered dielectric structures built of slabs with equal impedances.

There are two types of diabolic points in periodically-layered structures: point-like gaps, and point-like transparency zones (Dirac points).

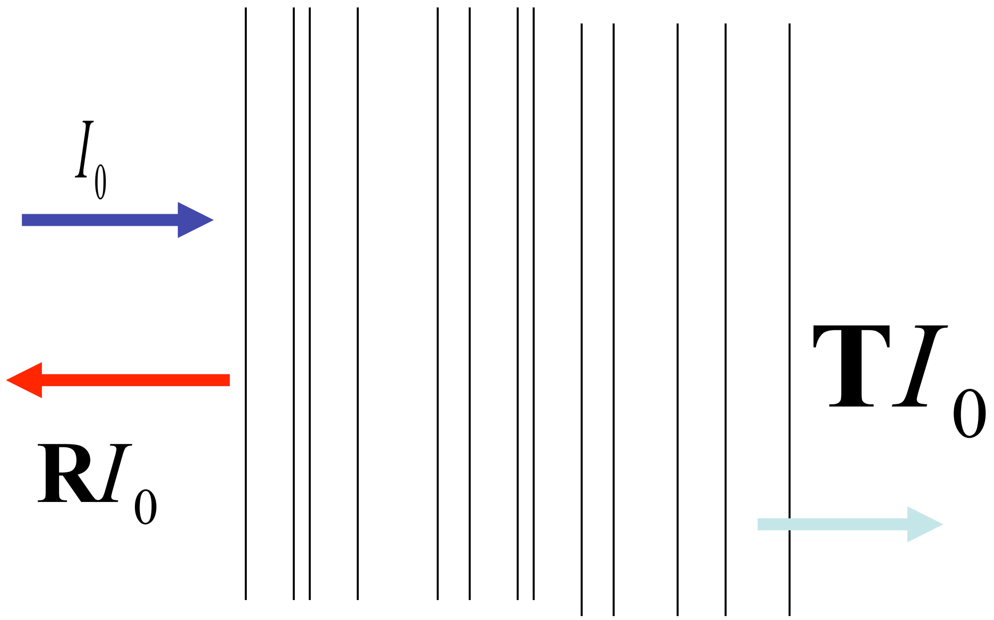
Dirac points exist only in samples built of alternating ***R – L*** elements.

In such samples, the transport properties of two Dirac cones are different. In particular:

(i) one focuses beams, another defocuses;

(ii) the transverse shifts of the beam have different signs and are anomalously small.

Randomly layered structures



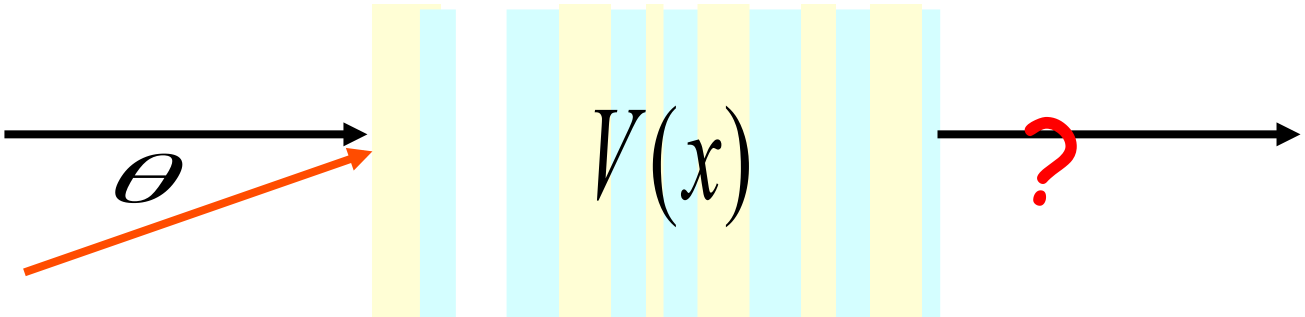
Open disordered system

$$< T(L) >= \exp(-\frac{L}{l_{loc}})$$

Randomly layered graphene

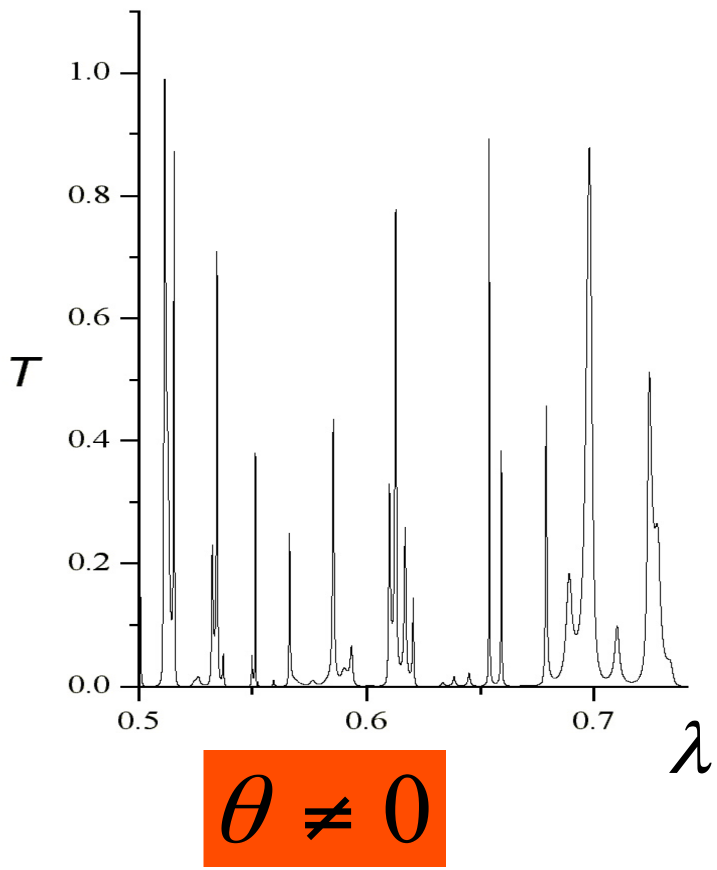
no backscattering in 1-D disordered graphene
super-latices

all states are delocalized, no matter how strong the disorder is

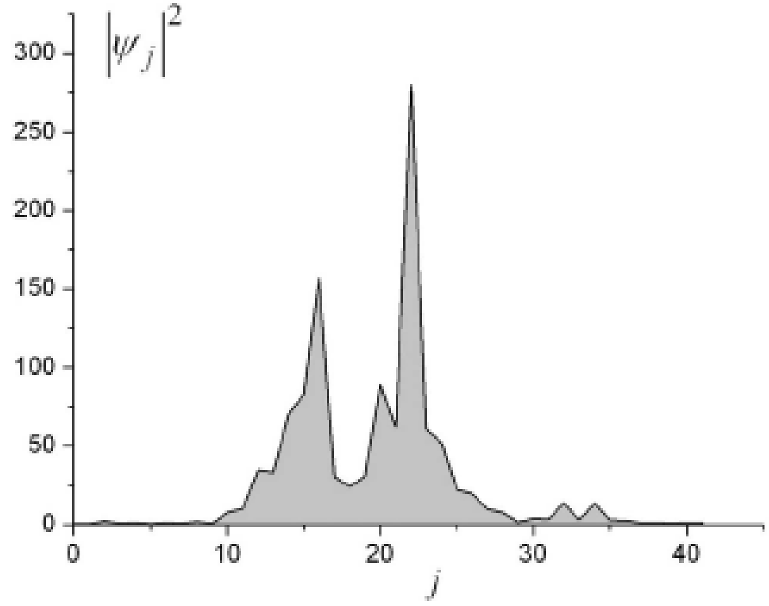


Klein paradox

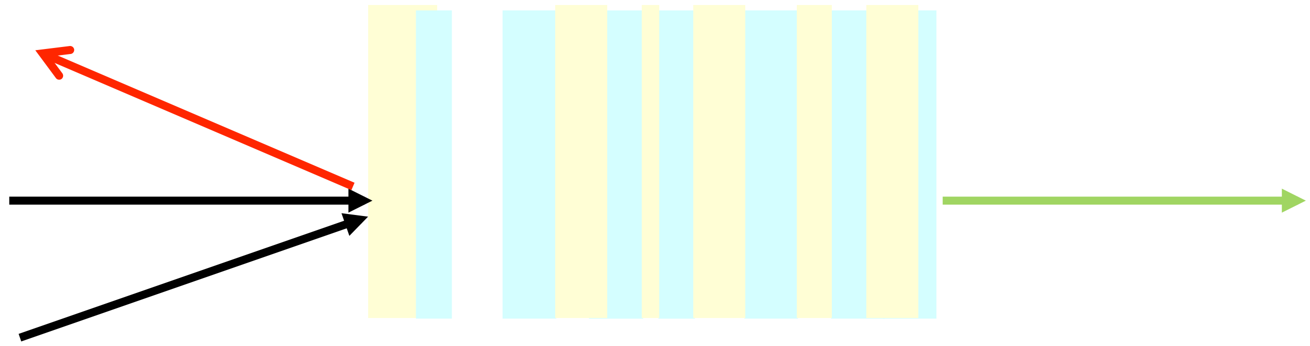
no localization in 1-d random graphene superlatice



T
1.0
0.8
0.6
0.4
0.2
0.0
0.5
0.6
0.7
λ
θ ≠ 0



|ψ_j|^2
300
250
200
150
100
50
0
0
10
20
30
40
j



$$Z^{(1)} = Z^{(2)} = Z^{(3)} = \ldots$$

$$R_{i,i+1}(\theta = 0) = 0$$

graphene in electric and magnetic fields

V_1	V_2
A_1	A_2