The Abdus Salam International Centre for Theoretical Physics

2472-14

Advanced Workshop on Nonlinear Photonics, Disorder and Wave Turbulence

15 - 19 July 2013

Description of fiber laser spectra by wave kinetic theory

E.V. Podivilov

Institute of Automation and Electrometry SB RAS, Novosibirsk, Russia
and
Novosibirsk State University, Novosibirsk, Russia

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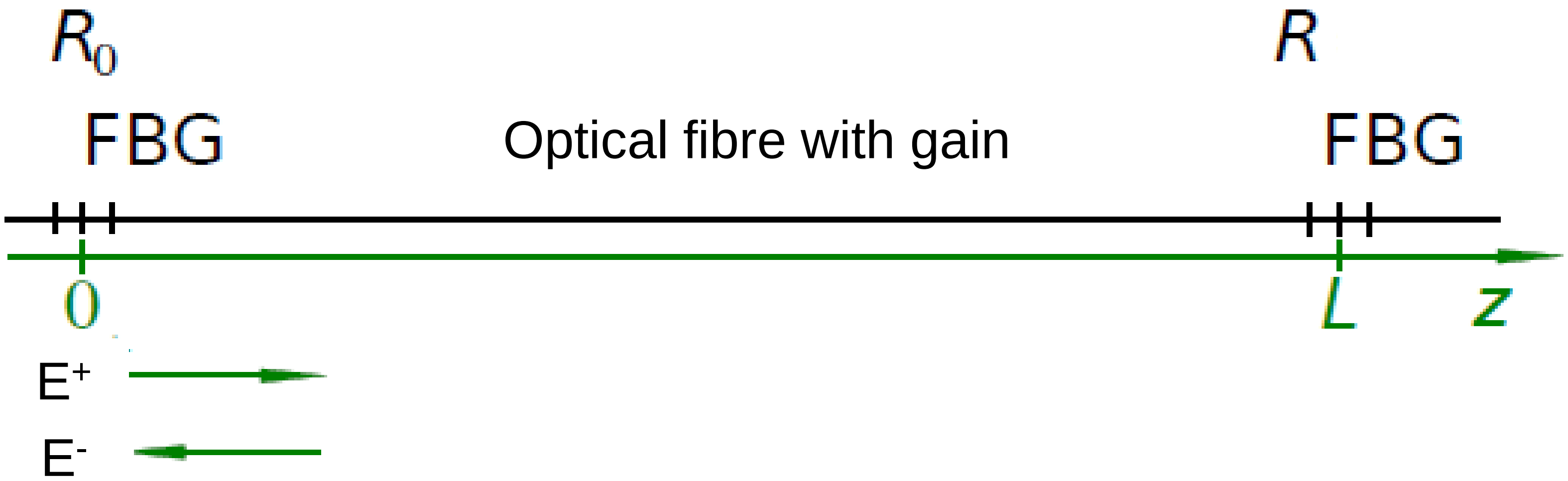
E.V. Podivilov[1,2], S. A. Babin[1,2]

1 - Institute of Automation and Electrometry SB RAS, Novosibirsk, Russia
2- Novosibirsk State University, Novosibirsk, Russia

Simplified fiber laser setup



$R(\omega) = R_0 \exp(-\omega^2/\Delta^2_{FBG})$
ω – detuning form the line center,
Δ_{FBG} – FBG half-width

$\Delta_{FBG} << kc = \omega_0$

$$E(z,t) = \left(E^+(z,t)e^{ik(ct-z)} + E^-(z,t)e^{ik(ct+z)}\right) + c.c.,$$

1-D NSU for Envelope

$$\pm\frac{\partial E^{\pm}}{\partial z}+\frac{1}{c}\frac{\partial E^{\pm}}{\partial t}-\frac{g[P]}{2}E^{\pm}=i\beta\frac{\partial^2 E^{\pm}}{\partial t^2}-i\gamma P^{\pm}E^{\pm}$$

gain dispersion 4WM

Boundary conditions at FBGs

$$E^{+}(0,\omega)=r_{left}(\omega)E^{-}(0,\omega),$$
$$E^{-}(L,\omega)=r_{right}(\omega)E^{+}(L,\omega)$$

$$|r(\omega)|^2=R(\omega)$$
$$|E^{\pm}(z,t)|^2=P^{\pm}(z,t)$$

Balance Equation for Laser Power

Average Power $P^{\pm}(z) = < |E^{\pm}(z,t)|^2 >$

$$\pm \frac{dP^{\pm}}{dz} \simeq g[P]P^{\pm}$$

$$P^{+}(0) = R_{eff}(0)P^{-}(0), \quad P^{-}(L) = R_{eff}(L)P^{+}(L)$$

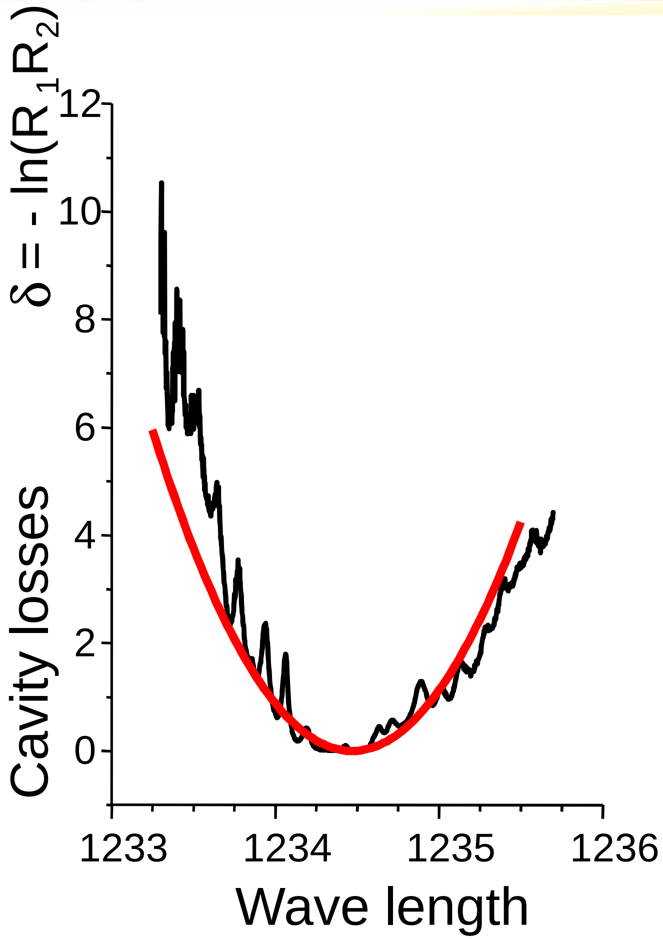
Gain – loss balance $2\int_0^L dz g[P] = -\ln(R_{eff}(0)R_{eff}(L))$

Gain saturation

$$g = \frac{g_0}{1 + P/P_s}$$

→ power distribution P(z) in laser cavity

1-point correlation function



$$R_j(\omega) = R_j \exp(-\omega^2/\Delta^2_{FBG})$$

$$R_{eff} = \frac{\int d\omega R(\omega) I(\omega)}{P}$$

$$P = \int d\omega I(\omega)$$

Spectral density

$$I(\omega) = \int dt K(t) \exp(i\omega t)$$

1-point CF

$$K(t) = < E(\tau) E^*(\tau - t)) >_\tau$$

Dispersionless limit for YDFL spectra

Self phase modulation

Assumption: random modes with Gaussian δ–correlated statistics

Short cavity L < 10 m and narrow spectral width Δ_{SP} < 0.1 nm

Babenko, V.A. et. al., *Sov. J. Quant. Electron.* **3**(2), 19 (1973).

Manassah, J.T., *Opt. Lett.* **16**(21), 1638 (1991).
Kuznetsov, A.G. et. al. *J. Opt. Soc. Am. B* **29** (2012)

$$K^-(\tau) = G\frac{K^+(\tau)}{\left[1+\nu^2\left(K^+(0)^2 - K^+(\tau)^2\right)\right]^2},$$

$K^\pm(\tau = 0) = P^\pm(z = 0)$,

$G = \exp\left(2\int_0^L g_S dx\right)$ is round trip amplification
$\nu = \gamma\left[\int_0^L \exp\left(\int_0^z g_s dz'\right) dz + \sqrt{G}\int_0^L \exp\left(\int_z^L g_s dz'\right) dz\right]$
is round trip nonlinearity

Analytical model for short cavity

Weak nonlinearity

$$K^{-}(\tau) = G\frac{K^{+}(\tau)}{\left[1+\nu^{2}\left(K^{+}(0)^{2}-K^{+}(\tau)^{2}\right)\right]^{2}},$$

Nonlinear the phase incursion is: $\phi_{NL} = \nu K^{+}(0) \approx \gamma 2L\left\langle P\right\rangle$,

where $\langle P\rangle = \int_0^L (P^{+}(x)+P^{-}(x))dx/(2L)$
is average intracavity power,
$\gamma \approx 4\ \mathrm{W}^{-1}\mathrm{km}^{-1}$ is Kerr coefficient.
Therefore $\phi_{NL} \ \leq\ 1$ at $\langle P\rangle \leq 10$ W.

Analytical model for YDFL spectra

Generation spectrum shape

Taylor series expansion in the case of weak nonlinearity:

$$K^-(\tau) \approx GK^+(\tau)\left[1 - 2\nu^2\left(K^+(0)^2 - K^+(\tau)^2\right) + \mathcal{O}\left(\phi_{NL}^4\right)\right].$$

Spectra of counter propagating waves are related to each other due to reflection from FBG:

$$I^+(\omega) = R(\omega)I^-(\omega), \qquad R_0 I^-(\omega) \approx \left(1 + \frac{\omega^2}{\Delta_{FBG}^2}\right) I^+(\omega)$$

Closed equation for correlation function $K^+(\tau)$:

$$\left(\frac{1}{\Delta_{FBG}^2}\frac{d^2K^+(\tau)}{d\tau^2} - K^+(\tau)\right) + R_0 G\left(1 - 2\nu^2 K^+(0)^2\right)K^+(\tau) + \left(2\nu^2 R_0 G\right)\cdot K^+(\tau)^3 = 0.$$

Solution has hyperbolic secant shape:

$$K^+(\tau) = \frac{P^+(0)}{\cosh\left((\pi/2)\tau\Delta\right)} \qquad I^+(\omega) = \frac{2P^+(0)}{\Delta\cosh\left(\omega/\Delta\right)}$$

Analytical model for YDFL spectra

Generation spectrum width

S.I. Kablukov, E.P. Zlobina, E.V. Podivilov,
S.A. Babin, Opt. Lett., 2012

Generation spectrum half-width and reflected wave power are related to each other by equation:

$$\Delta = (2/\pi)\Delta_{FBG}\nu P^{+}(0)$$

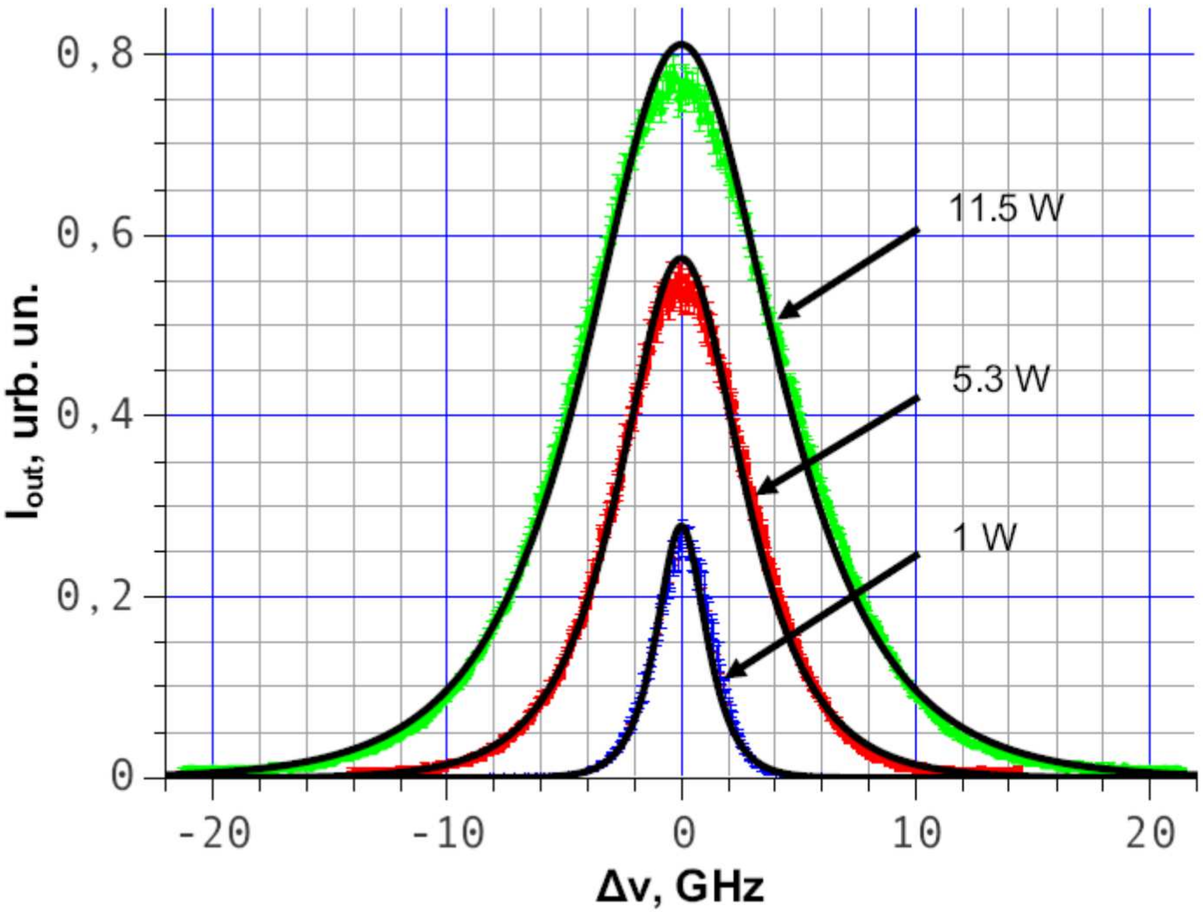
Taking into account fiber and cavity parameters one can find dependence of the half-width on generation output power:

$$\Delta = (4/\pi)\left(\Delta_{FBG}\gamma L/\ln\left(1/R_0\right)\right)P_{out}$$

Thereby the half-width is linear function of the laser power P_{out}.

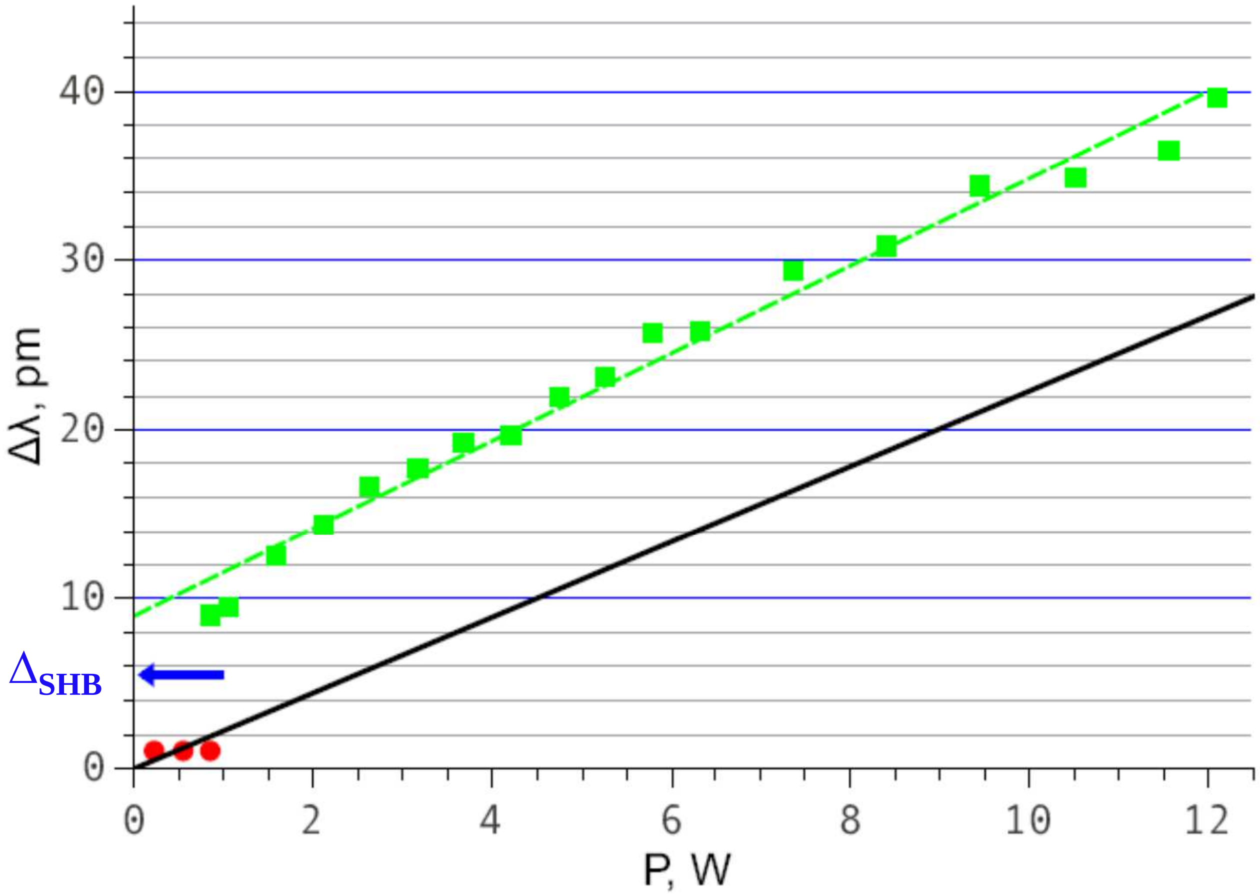
Experimental results

Output generation spectra



Generation linewidth in CW regime

FWHM generation linewidth



Cavity mode approach for Raman fiber laser

Low gain $g_R P$ = 1-10 km^{-1} , cavity length L>100м,
mode spacing is Δ = c/2nL<1 MHz, N~10^6 modes

$$E^{\pm}(z,t) = \frac{1}{\sqrt{2}} \sum_n E_n(t) \exp\left(in\Delta t \mp i\kappa z n\right) \exp(-i\nu_n t),$$

Dispersion and SPM frequency shift $\nu_n = \beta c(\Delta n)^2 - \gamma c I$

NSU in mode representation:

$$\tau_{rt} \frac{dE_n}{dt} - \frac{1}{2}(g - \delta_n) E_n(t) = -\frac{i}{2}\gamma L \sum_{l \neq 0} E_{n-l}(t) \sum_{m \neq 0} E_{n-m}(t) E^*_{n-m-l}(t) \exp(2i\beta m l \Delta^2 c t),$$

Cavity mode approach for Raman fiber laser

Assumption: random modes with Gaussian δ–correlated statistics

$$K_n(t) = \langle E_n(T) E_n^*(T-t) \rangle_T = I_n \exp(-t/\tau)$$

Wave kinetic equation for spectral density $I(\Omega) = \langle I(\Delta n)/\Delta \rangle_n$

$$\tau_{rt} \frac{dI(\Omega)}{dt} = [g - \delta(\Omega)] I(\Omega) + S_{FWM}(\Omega) = 0,$$

$$S_{FWM}(\Omega) = -\delta_{NL} I(\Omega) + (\gamma L)^2 \int \frac{I(\Omega-\Omega_1) I(\Omega-\Omega_2) I(\Omega-\Omega_1-\Omega_2)}{(3\tau_{rt}/\tau) \cdot [1 + (4\tau L\beta/3\tau_{rt})^2 \Omega_1^2 \Omega_2^2]} d\Omega_1 d\Omega_2,$$

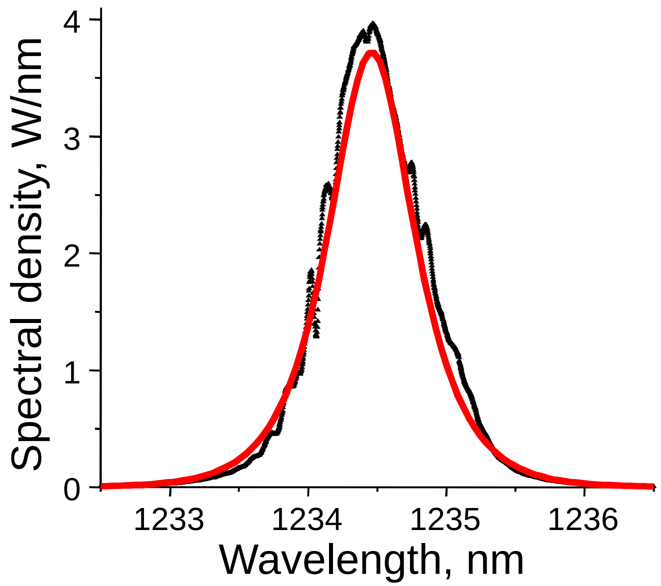
$$\delta_{NL} = (\gamma L)^2 \int \frac{[I(\Omega-\Omega_1) + I(\Omega-\Omega_2)] I(\Omega-\Omega_1-\Omega_2) - I(\Omega-\Omega_1) I(\Omega-\Omega_2)}{(3\tau_{rt}/\tau) \cdot [1 + (4\tau L\beta/3\tau_{rt})^2 \Omega_1^2 \Omega_2^2]} d\Omega_1 d\Omega_2,$$

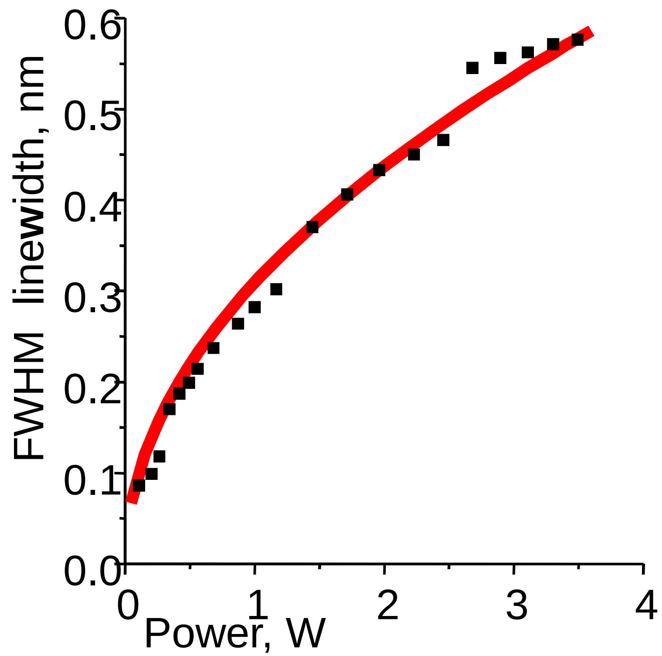
Assumption for correlation time : $\frac{\tau_{rt}}{\tau} \simeq \delta_{NL}/2,$

Comparison with experiment for RFL

$$I(\Omega) = \frac{2P}{\pi\,\Gamma \cosh(2\Omega/\Gamma)}$$

$$\Gamma = \frac{2}{\pi}\Delta_{FBG}\frac{\sqrt{2\sqrt{2/3}\gamma PL}}{[1+(4\beta L\Delta_{FBG}^2/3)^2]^{1/4}}$$





Babin S.A., Churkin D.V., Ismagulov A.E., Kablukov S.I., Podivilov E.V. **Opt. Lett. 2006; JOSA B 2007.**

Square – root power broadening

Conclusion

- **Multimode spectrum with random phases**
  takes hyperbolic secant shape with **exponential tails**
  both for YDFL and Raman lasers,
  whereas power broadening law is different – **linear** or **square root**, corresponding to dephasing at FBGs or at propagation
- **The exp – shape of tails is defined by Kerr nonlinearity:**
  different treatments (weak wave turbulence=FWM+dispersion for RFL or **SPM** for YDFL) converge at *D*->0, $\tau \to 2nL/c$,

- **Assumptions:**
- **Reflection from FBG randomize phase correlations induced by 4WM. Mode correlation time in short cavity is equal to roundtrip time.**
- **Mode correlation time in long cavity is inversely proportional to intra cavity laser power.**