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Growing condensate in two-dimensional turbulence

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Growing condensate in two-dimensional turbulence

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We report a numerical study, supplemented by phenomenological explanations, of "energy condensation" in forced 2D turbulence in a biperiodic box. Condensation is a finite size effect which occurs after the standard inverse cascade reaches the size of the system. It leads to emergence of a coherent vortex dipole. We show that the time growth of the dipole is self-similar, and it contains most of the injected energy, thus resulting in an energy spectrum which is markedly steeper than the standard $k^{-5/3}$ one. Once the coherent component is subtracted, however, the remaining fluctuations have a spectrum close to k^{-1}. The fluctuations decay slowly as the coherent part grows.

A big difference between 2D and 3D turbulence is the generation of large scale structures from small scale motions. This occurs because, if pumped at intermediate scales, the 2D Navier–Stokes equations favor energy transfer to larger scales (Kraichnan 1967, Leith 1968, Batchelor 1969), a phenomenon known as an inverse cascade. The essential difference with 3D turbulence is the presence of a second inviscid invariant, in addition to energy, the enstrophy. Stirring the 2D flow leads to emergence of two cascades. Enstrophy cascades from the forcing scale, l, to smaller scales (direct cascade) while energy cascades from the forcing scale to larger scales (inverse cascade). Viscosity dissipates enstrophy at the Kolmogorov scale, η, which is much smaller than l when the Reynolds number is large.

The energy cascade is blocked by a frictional dissipation (usually due to friction between the fluid and substrate) after a transient in time quasi-stationary regime. Then a stationary turbulence is established. Applying Kolmogorov phenomenology KLB theory predicts an energy spectrum scaling as k^{-3} in the direct cascade, and as $k^{-5/3}$ in the inverse cascade. The KLB spectra imply that velocity fluctuations at a scale r are

$$\delta v_r \sim \epsilon^{1/3} l^{-2/3} r, \quad \delta v_r \sim (\epsilon r)^{1/3},$$

in the direct and inverse cascade ranges respectively. Here ϵ is energy flux and l is pumping length. KLB theory is confirmed by simulations (Boffetta, Celani, and Vergassola, 2000; Boffetta 2006) and experiments (Kellay and Goldburg 2002; Bruneau and Kellay, 2005), where a sufficient range of scales was available to form the cascades.

If the frictional dissipation is weak (the system size exceeds the friction length) then the energy accumulation at large scales should occur (Kraichnan), then the standard KLB theory does not apply. Simulations of Smith and Yakhot (1993, 1994) and Borue (1994) and experiments of Paret and Tabeling (1998), Jin and Dubin (2000) and Shats, Xia, and Punzmann (2005) show that large scale accumulation of energy is observed, indeed, if conditions permit the energy to reach the system size.

A traditional motivation for studying 2D turbulence is its structural and phenomenological similarity to quasi-geostrophic turbulence in planetary atmospheres. Recent interest specifically in the condensate state, however, was sparked by experimental (Shats, Xia, and Punzmann, 2005) and numerical (Molenaar, Clercx, and van Heijst 2004) observations of large scale coherent vortices associated with energy condensation in forced, bounded flows. Shats et. al. noted that 2D spectral condensation is connected to the L-H (confinement) transition in magnetically confined plasmas which is often described by quasi-2D dynamics.

We solved the incompressible forced Navier–Stokes equations with hyperviscous dissipation in 2D:

$$\begin{aligned} \partial_t \boldsymbol{u} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} + \boldsymbol{\nabla} p &= \nu \Delta^8 \boldsymbol{u} + \boldsymbol{f} \\ \boldsymbol{\nabla} \cdot \boldsymbol{u} &= 0. \end{aligned} \tag{1}$$

The domain is a doubly-periodic box of size, $L = 2\pi$. The forcing, $\boldsymbol{f}$, injects velocity fluctuations and energy at an intermediate scale l with energy injection rate ϵ. Let us stress that there is no term representing friction in our equations.

First one observes formation of the inverse cascade. One clearly sees a transition at t_* from the spectrum $k^{-5/3}$ to scaling steeper than $k^{-5/3}$, that is numerically close to k^{-3}. This exponent does not signify a cascade in the KLB sense. The energy flux to large scales remains constant with respect to k before and after t_*. By contrast, the k^{-3} enstrophy cascade of KLB involves fluctuating vortices across many scales. Fluctuations play a relatively minor role in the overall energy balance with the majority of the energy absorbed into the condensate. They contain more of the enstrophy, but they decay in amplitude as the condensate grows so that the flow becomes more and more coherent as time passes.

The total vorticity is equal to zero. Therefore the only possibility to store the energy at the box size is in spacial separation where domains with positive and negative vorticity appear. Coherent vortices are formed in the regions. The process can be illustrated by a sequence of snapshots for the vorticity field (red and blue colors designate different signs of the vorticity). The coherent part of the flow has almost no fluctuations and, if it is removed, the steeper than $k^{-5/3}$ scaling disappears entirely. The fluctuation spectrum appears to be close to k^{-1}. Thus the steep spectrum has no relation to fluctuations, it is a consequence of the Fourier transform of the condensate (coherent flow).

One observes $\propto \sqrt{t}$ growth of the maximum value of the coherent part of vorticity with time. Furthermore, simulations show the global growth $\propto \sqrt{t}$ of the coherent velocity profile. The law $\propto \sqrt{t}$ is naturally explained by the energy accumulation injected at the constant rate, ϵ, by forcing. In the hyperbolic region one estimates the coherent velocity as $\sqrt{\epsilon t}$. The mean velocity profile within the vortex is almost perfectly circular. To a good precision higher order harmonics are suppressed relative to the zeroth order one. The velocity profile deduced from the simulations fits, is $\propto r^{-\xi}$, where $\xi \approx 0.25$, in the range, $L \gg r \gg l$, and thus the vortex core is roughly l. We plot $r^{-1.25}$ profiles for two different forcing scales to check that the profile is insensitive to it.