Limiting spectral distribution of patterned random matrices

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We present a unified approach to establishing limiting spectral distribution (LSD) of patterned matrices via the moment method. This allows us to demonstrate relatively short proofs for the LSDs of common matrices (Wigner, Toeplitz, Hankel, Reverse Circulant, Symmetric Circulant) and provide insight into the nature of different LSDs and their interrelations.

The method is applicable to matrices with appropriate dependent entries, banded matrices (including triangular matrices) and matrices of the form $A_p=\frac{1}{n}XX^\prime$ where X is a $p\times n$ matrix with real entries and $p\to \infty$ with $n=n(p)\to\infty$ and $p/n \to y$ with $0\leq y <\infty$. The sample variance covariance matrix being a particular example of the latter. We can also establish the existence of the LSD of the sample autocovariance matrix by this method. Other matrices that can be handled are skew symmetric patterned matrices and Hadamard product of independent patterned matrices.

Joint convergence of several copies of different patterned matrices may also be established by this approach.

This approach raises interesting questions about the class of patterns for which LSD exists and the nature of the possible limits. In many cases the LSDs are not known in any explicit forms and so deriving probabilistic properties of the limit are also interesting issues.