The Altshuler-Shklovskii Formulas for Random Band Matrices

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We consider the spectral statistics of large random band matrices on mesoscopic energy scales. We show that the variance and the two-point correlation function are governed by a universal power law behaviour that differs from the Wigner-Dyson-Mehta statistics. This law had been predicted in the physics literature by Altshuler and Shklovskii, and describes the eigenvalue density correlations in general metallic samples with weak disorder. Our result rigorously establishes the Altshuler--Shklovskii formulas for band matrices. In two dimensions, where the leading term vanishes owing to an arithmetic cancellation, we identify the first non-vanishing term and show that it differs substantially from the prediction of Kravtsov and Lerner. We also introduce a family of random band matrices that interpolates between real symmetric (\beta=1) and complex Hermitian (\beta=2) models, and track the transition for the mesoscopic density-density correlation. Finally, we prove that the two-point function completely describes the mesoscopic eigenvalue statistics by proving a central limit theorem for mesoscopic eigenvalue densities.