The Rate of Convergence to the Semi-Circular Law and to the Marchenko – Pastur Law

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We explain some results obtained jointly with F. Götze.

Let $\mathbf{X} = (X_{jk})_{j,k=1}^n$ denote a Hermitian random matrix with entries X_{jk}, which are independent for $1 \le j \le k \le n$. We consider the rate of convergence of the empirical spectral distribution function of the matrix $\mathbf{X}$ to the semi-circular law assuming that $\mathbf{E}\,X_{jk} = 0$, $\mathbf{E}\,X_{jk}^2 = 1$ and that the distributions of the matrix elements X_{jk} have a uniform sub exponential decay in the sense that there exists a constant $\varkappa > 0$ such that for any $1 \le j \le k \le n$ and any $t \ge 1$ we have

$$\Pr\{|X_{jk}| > t\} \le C \exp\{-t^{\varkappa}\}.$$

It is shown that the Kolmogorov distance between the empirical spectral distribution of the Wigner matrix $\mathbf{W} = \frac{1}{\sqrt{n}}\mathbf{X}$ and the semicircular law is of order $O(n^{-1}\log^b n)$ with some positive constant $b > 0$.

Similar result is obtained for the Marchenko – Pastur Law. Let $\mathbf{X} = (X_{jk})_{1 \le j \le n; 1 \le k \le p}$ denote a rectangular $n \times p$ matrix with entries X_{jk} which are independent for $1 \le j \le n; 1 \le k \le p$. Consider the sample covariance matrix $\mathbf{V} = \frac{1}{p}\mathbf{X}^*\mathbf{X}$. Assuming that $\mathbf{E}\,X_{jk} = 0$, $\mathbf{E}\,X_{jk}^2 = 1$ and that the distributions of the matrix elements X_{jk} have a uniform sub exponential decay, we proved that Kolmogorov distance between the empirical spectral distribution of the matrix $\mathbf{V}$ is of order $O(n^{-1}\log^b n)$ with some positive constant $b > 0$.

The bound of Kolmogorov distance between the empirical distribution function of Wigner matrix and semi-circular distribution function partially improved one of the results Yau, Erdös and Yin [1], Theorem 2.1, claim i) and inequality (1.5). Let $\lambda_1 \le \ldots \le \lambda_n$ be

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eigenvalues of matrix $\mathbf{W}$ and let ξ_{kn}. The bound of Kolmogorov distance between empirical spectral distribution function and limit distribution function implies inequality

$$\Pr\{\max_{c_1 \log^b n \le k \le n - c_2 \log^b n} |\lambda_k - \xi_{kn}| \le \frac{c \log^b n}{n}\} \le \exp\{-c \log n \log\log n\}, \qquad (0.1)$$

where ξ_{kn} are quantiles of semicircular distribution, i.e. $G(\xi_{kn}) = \frac{k}{n}$, $k = 1, \ldots, n$ and $G(x)$ denotes the semicircular distribution function. We would like to explain our approach to investigation of asymptotic of resolvent matrix $\mathbf{R}(z) = (\mathbf{W} - z\mathbf{I})^{-1}$, $z = u + iv$, near the real line. In particular, we consider the bound of quantities like $\frac{1}{n^2}\sum_{j=1}^{n}\sum_{q\ne r\in\mathbb{T}_j} X_{qj}X_{rj}R^{(j)}_{rq}$, where $\mathbf{R}^{(j)}$ denotes the resolvent matrix of matrix $\mathbf{W}^{(j)}$ which obtained from the matrix $\mathbf{W}$ by deleting jth row and jth column. Such bounds play crucial role in the investigation of Stieltjes transform of spectral distribution function of random matrices.

Список литературы

[1] Erdös L., Yau H.-T., Yin J. *Rigidity of Eigenvalues of Generalized Wigner Matrices.* Preprint, arXiv:1007.4652