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Winter College on Optics: Fundamentals of Photonics - Theory, Devices and Applications

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Introduction to Quantum Photonics

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Introduction to QUANTUM PHOTONICS

Light → wave : Huygens, Young, Arago-(Biot)
Light → particle : Newton, Einstein

Q.E.D. Dirac, Fermi, Feynman, ...

light = photons with dual nature $\begin{cases} \text{particle} \\ \text{wave} \end{cases}$

Usual language : 2nd quantization (here Coulomb gauge)

ground state → vacuum (non-degenerate)

each e.m. mode @ $(\vec{k}, \vec{\varepsilon})$ (in free space)

wavevector, polarization

→ harmonic oscillator mode $\hat{a}_{k\varepsilon}$, $\hat{a}^{+}_{k\varepsilon}$

$$[\hat{a}_{k\varepsilon}, \hat{a}^{+}_{k'\varepsilon'}] = \delta_{kk'}\,\delta_{\varepsilon\varepsilon'}$$

$$H = E_{vac} + \sum_{k\varepsilon} \hbar\omega_{k\varepsilon}\, \hat{a}^{+}_{k\varepsilon}\, \hat{a}_{k\varepsilon}$$

Photon "number" $N_{k\varepsilon} = a^{+}_{k\varepsilon}\, a_{k\varepsilon}$ → values in $\mathbb{N}$.

"number of photons in mode (k, ε)"

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Single-mode cavity

↳ isolate a mode from the rest (all other ones very detuned ...)

$$H = \hbar\omega_0\, \hat{a}^+ \hat{a}$$

most interesting states:

* Fock state $|n\rangle = \frac{(\hat{a}^+)^n}{\sqrt{n!}} |vac\rangle$

↳ eigenstate of $\hat{N}$ with n

* coherent state $|coh: \alpha\rangle = e^{-|\alpha|^2/2} \sum_n \frac{\alpha^n}{n!} (a^+)^n |vac\rangle$

↳ eigenstate of $\hat{a}$

↳ minimum uncertainty in $\hat{E}$ and $\hat{B}$

$[\hat{E} \propto \hat{a} + \hat{a}^+ \quad , \quad \hat{B} \propto \hat{a} - \hat{a}^+]$

Free space

* Fock state $A_\phi^+ |vac\rangle$ with 1 photon in $\phi(n, \varepsilon)$

with $\hat{A}_\phi^+ = \sum_{n,\varepsilon} \phi(n,\varepsilon)\, \hat{a}^+_{n,\varepsilon}$

* coherent state $e^{-\|\phi\|^2/2}\, e^{\sum_{n\varepsilon} \phi(n,\varepsilon)\, \hat{a}^+(n,\varepsilon)} |vac\rangle$

↳ NOTE: state factorized over modes

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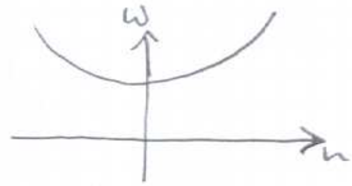
example : plane wave $\phi(k,\varepsilon) = \delta(k-k_0)\,\delta_{\varepsilon\varepsilon_0}$

In photonics ⟶ waveguides

$$\omega_k \cong \omega_{co} + \frac{\hbar}{2m^*} k^2$$



dispersion law (neglect polariz.)

$$H = \hbar\omega_k\, \hat{a}_k^+ \hat{a}_k$$

define real space "photon field" $\hat{\psi}(x) = \int \frac{dk}{2\pi} e^{ikx} \hat{a}_k$

$$\left[\hat{\psi}(x), \hat{\psi}^+(x')\right] = \delta(x-x') \quad \left[\underline{\text{NOTE}} = \delta^{\perp}_{3D}(r,r') \text{ non local}\right]$$

Coherent state : $|coh : \underset{\sim}{\phi(x)}\rangle = e^{-\|\phi\|^2/2}\, e^{\int dx\, \phi(x) \hat{\psi}^+(x)} |vac\rangle$

↳ all photons share same $\phi(x)$

Evolution under H preserves coherent state nature and

$$i\frac{\partial}{\partial t}\phi(x,t) = \omega_{co}\,\phi(x,t) - \frac{\hbar}{2m^*}\frac{\partial^2}{\partial x^2}\phi(x,t)$$

Schrödinger equation

Extracting $E(x,t)$, $B(x,t)$ from $\phi(x,t)$

↳ satisfy Maxwell's eqs.

for $\langle\hat{E}(x,t)\rangle$, $\langle\hat{B}(x,t)\rangle$

So : Coherent states closest quantum states to classical physics.

Photodetection signal

$$P(x,t) \propto \langle \hat{E}^{+}(x)\,\hat{E}(x)\rangle_t \longrightarrow \text{probability of click @ } (x,t)$$

$$P_2(x_1,t_1;\,x_2,t_2) \propto \langle \hat{E}^{+}(x_1,t_1)\,\hat{E}^{+}(x_2,t_2)\,\hat{E}(x_2,t_2)\,\hat{E}(x_1,t_1)\rangle$$

(for $t_2 > t_1$)

→ probability of 2 clicks @ (x_1,t_1) AND (x_2,t_2)

$$\hat{E}(x,t) \propto \hat{a}(x) \longrightarrow \text{click } \underline{\text{absorbs a photon}}$$

Single photon pulse

$$|1:\phi(x)\rangle = \hat{A}_\phi^{+}|\text{vac}\rangle$$

Under H: $\phi(x)$ evolves according to Schrodinger

Young double-slit experiment → fringes also for single photons

But ... $P_2 = 0$ never 2 clicks on the same wavepacket.

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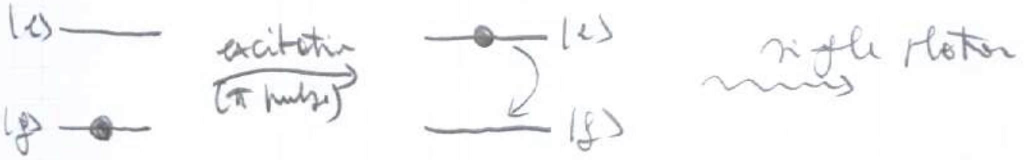
How to create single photon pulse?



2 level atom

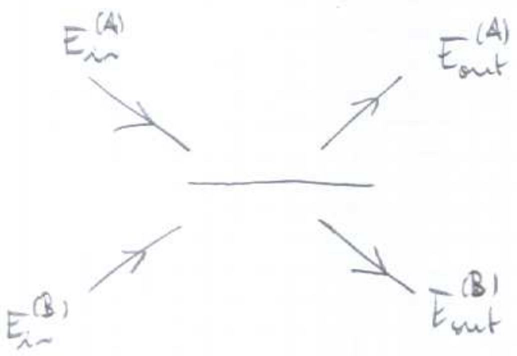
Excitation @ t_0 :
$$\phi(x, t>t_0) \propto \Theta\left(c(t-t_0) - |x-x_0|\right) \cdot e^{+\gamma|x-x_0|/c} \, e^{-\gamma(t-t_0)} \cdot e^{i\omega_{eg}|x-x_0|/c} \, e^{-i\omega_{eg}(t-t_0)}$$

Expanding spherical wavepacket @ c.

$\hookrightarrow$ frequency ω_{eg}, wavevector $k_{eg} = \omega_{eg}/c$

Quantum photonics

i) beam splitter



classical scattering matrix

$$\begin{pmatrix} E_{out}^{(A)} \\ E_{out}^{(B)} \end{pmatrix} = \underbrace{\begin{pmatrix} r & it \\ it & r \end{pmatrix}} \begin{pmatrix} E_{in}^{(A)} \\ E_{in}^{(B)} \end{pmatrix}$$

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classical / coherent incident beams :

(for linear device coherent state = classical)

$$\left|E^{(A)}_{out}\right|^2 = r^2 \left|E^{(A)}_{in}\right|^2 + t^2 \left|E^{(B)}_{in}\right|^2 + \underbrace{\left(i r t \; E^{(B)}_{in} E^{(A)*}_{in} + c.c\right)}$$

fringes depending relative phase of $E^{(A)}_{in}$ and $E^{(B)}_{in}$.

For single photon :

initial state $\propto E^{(A)}_{in} \, |A\rangle + E^{(B)}_{in} \, |B\rangle$

probability of exit in (A) = ?

$|A\rangle \longrightarrow |A\rangle \qquad a = r$

$|B\rangle \longrightarrow |A\rangle \qquad a = it$

$P((A)) \propto \underline{\left|E^{(A)}_{in} \cdot r + E^{(B)}_{in} \cdot (it)\right|^2}$ recovers classical result !

Feynman's sum over paths.

What about 2 incident single photons in (A) and (B) ;

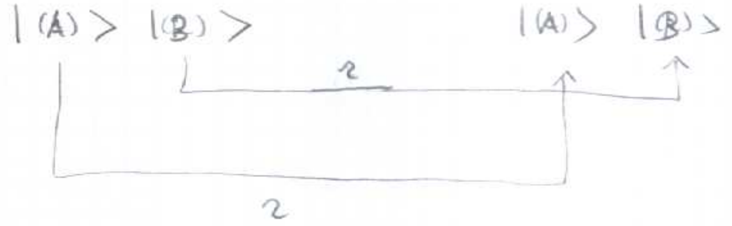
initial state $|A\rangle|B\rangle = |i\rangle$

final state $|(A)\rangle|(B)\rangle = |f\rangle$

[Note : $|(A)\rangle = |1:A\rangle \ldots$]

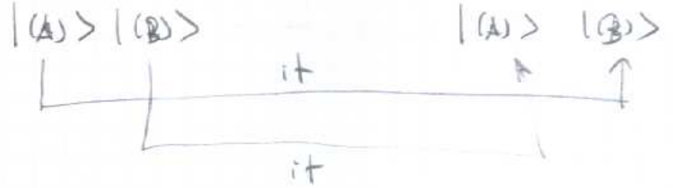
$|i\rangle \longrightarrow |f\rangle$

<u>two</u> possible paths



Amplitude $r \cdot r$

and (exchange of 2 photons → (+1) bosonic)



Amplitude $(it)(it) = -t^2$

Total amplitude $a = r^2 - t^2$ can <u>vanish</u> for 50/50 B.S.

In this case: final states $\begin{cases} |2:(A)\rangle \\ |2:(B)\rangle \end{cases}$

<u>"PHOTON BUNCHING"</u>

More standard derivation <u>(input-output)</u> theory:

$$\begin{pmatrix} \hat{a}^{(A)}_{out} \\ \hat{a}^{(B)}_{out} \end{pmatrix} = S \begin{pmatrix} \hat{a}^{(A)}_{in} \\ \hat{a}^{(B)}_{in} \end{pmatrix}$$

initial state $\left(E^{(A)}_{in}\, a^{(A)+}_{in} + E^{(B)}_{in}\, a^{(B)+}_{in}\right)|vac\rangle$

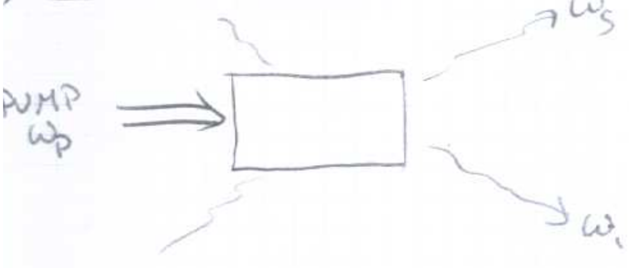
final state $\langle a^{(A)+}_{out}\, a^{(A)}_{out} \rangle = \langle \left(r\, a^{(A)+}_{in} - it\, a^{(B)+}_{in}\right)\left(r\, a^{(A)}_{in} + it\, a^{(B)}_{in}\right)\rangle =$

$$= r^2 |E^{(A)}_{in}|^2 + t^2 |E^{(B)}_{in}|^2 + \left(irt\, E^{(A)*}_{in} E^{(B)}_{in} + c.c.\right)$$

↳ only involves $\langle a^{+} a \rangle$ → Fock = coherent

<u>Difference</u> appears in $\langle a^{(B)+}_{out}\, a^{(B)}_{out}\, a^{(A)+}_{out}\, a^{(A)}_{out} \rangle$ = joint photodetection signal

) <u>Parametric amplifier</u>



$$H_{int} = g\left(\hat{a}_s^{+}\hat{a}_i^{+} a_p + h.c.\right)$$

$$\hat{a}_i(t+\Delta t) \simeq \underbrace{\hat{a}_i(t)}_{\text{output}} - i g\, \hat{a}_p\, \underbrace{\hat{a}_s^{+}}_{\text{input}}$$

$\hat{a}_p \sim \alpha_p$ classical

$$\begin{pmatrix} a_s \\ a_i \\ a_s^+ \\ a_i^+ \end{pmatrix}_{out} \simeq \begin{pmatrix} 1 & 0 & 0 & -ig\alpha_p \\ 0 & 1 & -ig\alpha_p & 0 \\ 0 & ig\alpha_p^* & 1 & 0 \\ ig\alpha_p^* & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a_s \\ a_i \\ a_s^+ \\ a_i^+ \end{pmatrix}_{in}$$

also terms connecting a_s to a_i^+ ...

Even for vacuum input $\langle a_s^+ a_s \rangle_{in} = \langle a_i^+ a_i \rangle_{in} = 0$

$$\langle a_s^+ a_s \rangle_{out} = \langle (\cancel{a_s^+} + i g \alpha_p^* a_I)(\cancel{a_s} - i g \alpha_p a_I^+) \rangle =$$

$$= g^2 |\alpha_p|^2 > 0.$$

$$\langle a_s^+ a_I^+ a_I a_s \rangle \overset{\text{Wick}}{=} \langle a_s^+ a_s \rangle \langle a_I^+ a_I \rangle + \langle a_s^+ a_I^+ \rangle \langle a_I a_s \rangle + \underbrace{\langle a_s^+ \cancel{a_I} \rangle}_{0} \underbrace{\langle a_I^+ \cancel{a_s} \rangle}_{0} =$$

$$= \left(g^2|\alpha_p|^2\right)^2 + (i g \alpha_p^*)(-i g \alpha_p) = \left(g^2|\alpha_p|^2\right)^2 + g^2|\alpha_p|^2$$

$$g^{(2)}(I,S) = \frac{\langle a_s^+ a_I^+ a_I a_s \rangle}{\langle a_s^+ a_s \rangle \langle a_I^+ a_I \rangle} = 1 + \frac{1}{g^2|\alpha_p|^2} > 1 \qquad \text{BUNCHING}$$

The same parametric amplifier mechanism:

↳ creation of entangled photon pairs

↳ Hawking radiation for black holes.

↳ fundamental quantum mechanics experiments (Bell inequalities ...)

i) Optical nonlinearity

cavity frequency $\omega_{cav} = \frac{\pi c}{L n}$

optical nonlinearity $n = n_0 + \chi^{(3)} I$

$$\Rightarrow \omega_{cav} = \frac{\pi c}{L n} \simeq \frac{\pi c}{L n_0}\left(1 - \frac{\chi^{(3)}}{n_0} I\right) =$$

$$= \omega_0 - \frac{\pi c \chi^{(3)}}{L n_0^2} I .$$

$$H = \hbar \omega_0 a^+ a + \frac{\omega_{nl}}{2} a^+ a^+ a a$$

$$i \dot{a} = (\omega_0 + \omega_{nl} \underbrace{a^+ a}_{I}) a$$

$$\hookrightarrow \omega_{nl} \simeq - \frac{\pi c}{L n_0^2} \chi^{(3)}$$

two regimes:

i) MF ω_{nl} (nonlinear shift / photon) $\ll \gamma$:

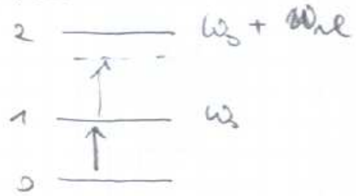
$\rightarrow$ many photons needed for observable nonlinear effect $\rightarrow$ classical NLO.

optical bistability $\omega_p > \omega_0$

optical limiting $\omega_p < \omega_0$

ii) Quantum $\omega_{nl} \gg \gamma$ $\rightarrow$ single photon triggers nonlinearity.

for $\omega_L = \omega_0$ $\longrightarrow$ PHOTON BLOCKADE.



$\Longrightarrow$ ANTI-BUNCHED LIGHT as TLA.

More material on my website:

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$\downarrow$

Quantum optics notes

BEC notes