2572-4

Winter College on Optics: Fundamentals of Photonics - Theory, Devices and Applications

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Introduction to Waveguide Optics

Jiri Ctroky
Institute of Photonics and Electronics AS CR, v.v.i., Prague
Czech Republic

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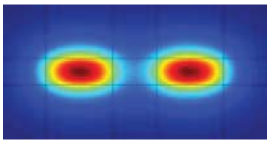
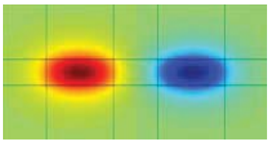
Jiří Čtyroký
ctyroky@ufe.cz

Institute of Photonics and Electronics AS CR, v.v.i.,
Prague, Czech Republic

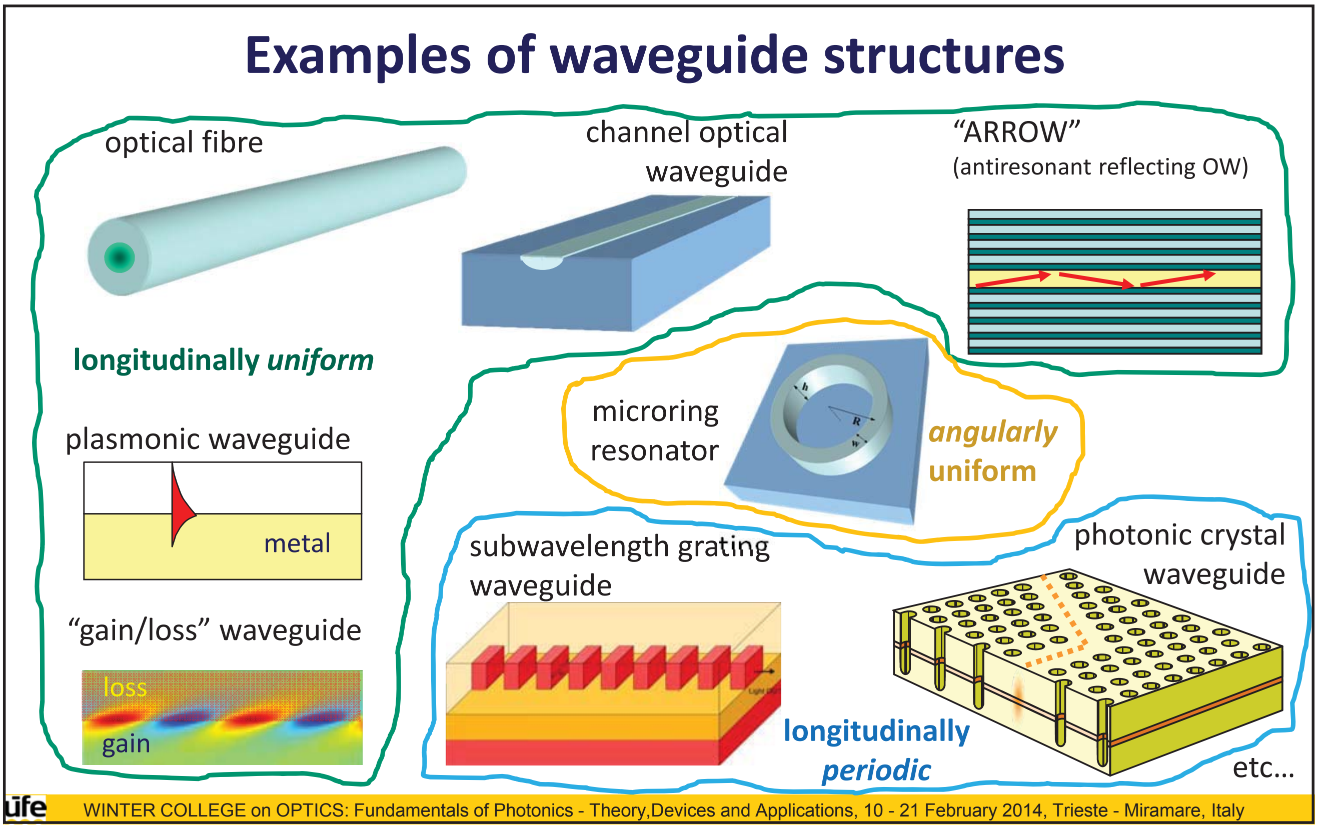




Where I come from…

Institute of Photonics and Electronics AS CR, v.v.i.

Academy of Sciences of the Czech Republic is a non-university institution for basic and applied research consisting of 54 independent institutes



Examples of waveguide structures

optical fibre

channel optical waveguide

"ARROW" (antiresonant reflecting OW)

longitudinally *uniform*

plasmonic waveguide

metal

microring resonator

***angularly* uniform**

"gain/loss" waveguide

loss

gain

subwavelength grating waveguide

photonic crystal waveguide

longitudinally *periodic*

etc…

Theoretical fundamentals of optical waveguides

Tentative list of topics

- Planar waveguides; waveguide modes, their properties. Guided and leaky modes. Other types of waveguiding, guiding by a single interface
- Waveguide bends, whispering-gallery modes, circular resonators
- Channel waveguides, approximate analytical methods.
- More complex waveguide structures. Fundamentals of a rigorous coupled-mode theory
- Introduction to modal methods; transfer matrix method, basics of the film mode matching
- Periodic media, Bloch modes, origin of the bandgap, SWG waveguides, (photonic crystals)
- "Canonical" waveguide structures: Y-junctions, directional coupler, two- and multimode interference couplers, microresonators
- Plasmonic waveguides and structures. Surface plasmon sensing. Hybrid dielectric-plasmonic slot waveguide, plasmonic devices
- Waveguide structures with loss and gain; asymmetric grating couplers, (*"PT-symmetric"* waveguide structures)
- …

Basic requirements: Theory of electromagnetic field, Maxwell equations

Basic math & phys background

Dielectric (possibly also metallic) non-magnetic linear source-free medium, time-harmonic dependence of electromagnetic field:

$$\mathscr{E}(\mathbf{r},t) = \mathrm{Re}\{\mathbf{E}(\mathbf{r})\exp(-i\omega t)\}$$

$$\mathscr{H}(\mathbf{r},t) = \mathrm{Re}\{\mathbf{H}(\mathbf{r})\exp(-i\omega t)\}$$

$$k_0^2 = \omega^2\mu_0\varepsilon_0 = \frac{2\pi}{\lambda}$$

$$k^2 = k_0\varepsilon = k_0 n^2$$

$$\frac{1}{v} = \frac{k}{\omega} = \frac{n}{c} \qquad \text{(phase velocity)}$$

$$\frac{1}{v_g} = \frac{dk}{d\omega} = \frac{n_g}{c} \qquad \text{(group velocity)}$$

$$n_g = \frac{d(\omega n)}{d\omega} = n + \omega\frac{dn}{d\omega} \qquad \text{group index}$$

$$= n - \lambda\frac{dn}{d\lambda}, \qquad \text{typically larger than } n$$

$$\nabla\times\mathbf{E} = i\omega\mu_0\mathbf{H}, \qquad \nabla\cdot\mathbf{D} = 0$$

$$\nabla\times\mathbf{H} = -i\omega\varepsilon_0 n^2\mathbf{E}, \qquad \nabla\cdot\mathbf{B} = 0$$

$$\mathbf{D} = \varepsilon_0 n^2\mathbf{E}$$

$$\mathbf{B} = \mu_0\mathbf{H}$$

Plane-wave solution:

$$\mathbf{E} = \mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r}} = \mathbf{E}_0 e^{i\mathbf{k}'\cdot\mathbf{r}} e^{-\mathbf{k}''\cdot\mathbf{r}}$$

$$\mathbf{H} = \mathbf{H}_0 e^{i\mathbf{k}\cdot\mathbf{r}} = \mathbf{E}_0 e^{i\mathbf{k}'\cdot\mathbf{r}} e^{-\mathbf{k}''\cdot\mathbf{r}}$$

$$\mathbf{k} = \mathbf{k}' + i\,\mathbf{k}'', \qquad n^2 = \varepsilon' + i\varepsilon''$$

$$\mathbf{k}'^2 - \mathbf{k}''^2 = k_0^2\varepsilon'$$

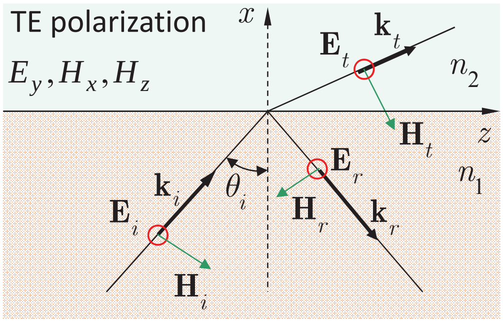
$$2\mathbf{k}'\cdot\mathbf{k}'' = k_0^2\varepsilon''$$

complex
wave vector

Field at the interface between two media

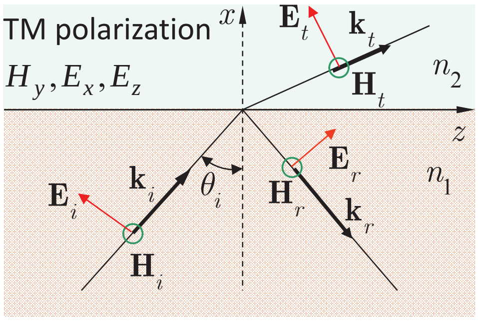
Plane wave incident on a planar interface

$$\mathbf{E}_i = \mathbf{E}_{i0}\,\mathrm{e}^{i\mathbf{k}_i\cdot\mathbf{r}}, \mathbf{E}_r = \mathbf{E}_{r0}\,\mathrm{e}^{i\mathbf{k}_r\cdot\mathbf{r}}, \mathbf{E}_t = \mathbf{E}_{t0}\,\mathrm{e}^{i\mathbf{k}_t\cdot\mathbf{r}}, \qquad \mathbf{H}_i = \mathbf{H}_{i0}\,\mathrm{e}^{i\mathbf{k}_i\cdot\mathbf{r}}, \mathbf{H}_r = \mathbf{H}_{r0}\,\mathrm{e}^{i\mathbf{k}_r\cdot\mathbf{r}}, \mathbf{H}_t = \mathbf{H}_{t0}\,\mathrm{e}^{i\mathbf{k}_t\cdot\mathbf{r}}$$


TE polarization

E_y, H_x, H_z

$n_1 > n_2$


TM polarization

H_y, E_x, E_z

$$\mathbf{k}_{i,r} = k_0\left(\pm\gamma_1\mathbf{x}^0 + N_{i,r}\mathbf{z}^0\right), \ \mathbf{k}_t = k_0\left(\gamma_2\mathbf{x}^0 + N_t\mathbf{z}^0\right), \ \gamma_1^2 + N_{i,r}^2 = n_1^2, \ \ \gamma_t^2 + N_t^2 = n_2^2$$

Field continuity conditions at $x = 0$:

$$E_{i0}e^{i\mathbf{k}_i\cdot\mathbf{z}^0 z} + E_{r0}e^{i\mathbf{k}_r\cdot\mathbf{z}^0 z} = E_{t0}e^{i\mathbf{k}_r\cdot\mathbf{z}^0 z}, \qquad H_{i0}e^{i\mathbf{k}_i\cdot\mathbf{z}^0 z} + H_{r0}e^{i\mathbf{k}_r\cdot\mathbf{z}^0 z} = H_{t0}e^{i\mathbf{k}_r\cdot\mathbf{z}^0 z}$$

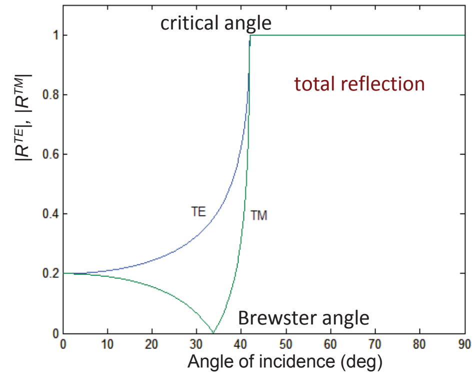
$$\boxed{N_i = N_r = N_t = N = n_1\sin\theta_i}$$

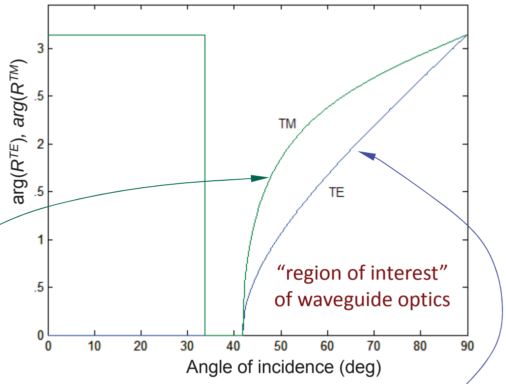
$$\gamma_1 = \sqrt{n_1^2 - N^2}, \quad \gamma_2 = \sqrt{n_2^2 - N^2}$$

Fresnel coefficients

$$R^{TE}=\frac{E_r}{E_i}=\frac{\gamma_1-\gamma_2}{\gamma_1+\gamma_2}$$
$$=\frac{\sqrt{n_1^2-N^2}-\sqrt{n_2^2-N^2}}{\sqrt{n_1^2-N^2}+\sqrt{n_2^2-N^2}}$$

$$R^{TM}=\frac{H_r}{H_i}=\frac{\gamma_1/n_1^2-\gamma_2/n_2^2}{\gamma_1/n_1^2+\gamma_2/n_2^2}$$
$$=\frac{n_2^2\sqrt{n_1^2-N^2}-n_1^2\sqrt{n_2^2-N^2}}{n_2^2\sqrt{n_1^2-N^2}+n_1^2\sqrt{n_2^2-N^2}}$$

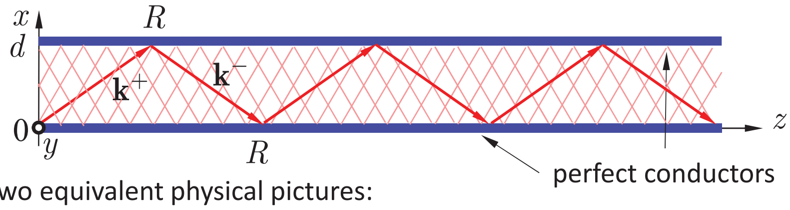




$$R^{TM}=e^{i\Phi^{TM}};$$
$$\Phi^{TM}=-2\arctan\left[\left(\frac{n_1}{n_2}\right)^2\sqrt{\frac{N^2-n_2^2}{n_1^2-N^2}}\right].$$

$$R^{TE}=e^{i\Phi^{TE}};$$
$$\Phi^{TE}=-2\arctan\sqrt{\frac{N^2-n_2^2}{n_1^2-N^2}}.$$

The simplest waveguide: two (perfect) conductors



$$\mathbf{k}^{\pm}=k_0(\pm\gamma\mathbf{x}^0+N\mathbf{z}^0),$$
$$R^{TE}=\frac{E_{y,ref}}{E_{y,inc}}=-1$$
$$R^{TM}=\frac{H_{y,ref}}{H_{y,inc}}=1$$

Two equivalent physical pictures:
1. a series of successive reflections of a plane wave
2. two plane waves propagating upwards and downwards

In both cases, the "nonzero wave" in the waveguide can exist only under the "*condition of transverse resonance*"

(for TM only)

$$\boxed{R^2\exp(2ik_0\gamma d)=1,}\quad\text{or}\quad 2k_0\gamma d=2m\pi,\quad m=0,\ 1,\ 2,\ \ldots$$

Waves can thus propagate only as discrete *waveguide modes* with *propagation constants*

$$\beta_m^{TE}=\beta_m^{TM}=k_0^2N_m^2=\sqrt{k^2-k_0^2\gamma_m^2}=\sqrt{k^2-(m\pi/d)^2}$$

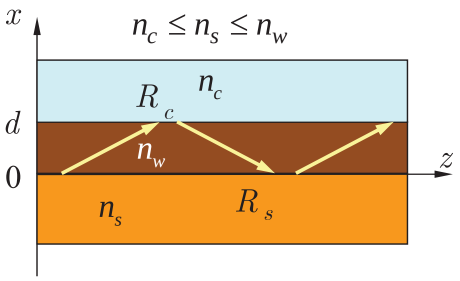
… *polarization degeneracy* (except for *m* = 0)

for $m>2d/\lambda$ …*evanescent modes*

The transverse field distributions have the forms

$$E_y(x,z)=E_0\sin(m\pi x/d)\exp(i\beta_m z)$$
$$H_y(x,z)=H_0\cos(m\pi x/d)\exp(i\beta_m z)$$

Dielectric slab waveguide



Total reflection at both interfaces: $n_w > N > n_{c,s}$

$$\gamma_w = \sqrt{n_w^2 - N^2}, \quad \gamma_{c,s} = i\sqrt{N^2 - n_{c,s}^2}$$

Condition of transverse resonance:

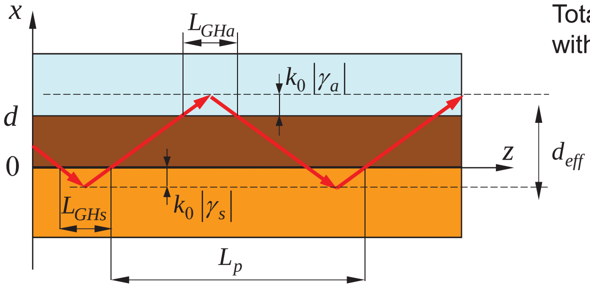
$R_c R_s \exp(2ik_0\gamma_w d) = 1,$ or

$$\Phi_{tot}(N) = k_0\gamma_w d + \frac{1}{2}\arg R_s + \frac{1}{2}\arg R_c = \pi m$$

$$\boxed{k_0 d\sqrt{n_w^2 - N_m^2} = \arctan\left[\left(\frac{n_w}{n_s}\right)^{\nu}\sqrt{\frac{N_m^2 - n_s^2}{n_w^2 - N_m^2}}\right] + \arctan\left[\left(\frac{n_w}{n_c}\right)^{\nu}\sqrt{\frac{N_m^2 - n_C^2}{n_w^2 - N_m^2}}\right] + m\pi,} \quad \boxed{\nu = \begin{cases} 0\ (\mathrm{TE}) \\ 2\ (\mathrm{TM}) \end{cases}}$$

Dispersion equation for N_m shows *polarization birefringence*, $N_m^{TE} > N_m^{TM}$

Effective thickness & “period of propagation”



Total internal reflection is linked up with the *Goos-Hänchen shift,* $L_{GH} = -\frac{d\Phi}{d\beta} = -\frac{1}{k_0}\frac{d(\arg R)}{dN}$

$$L_{GHs,c}^{TE} = \frac{2}{k_0}\frac{d}{dN}\left[\arctan\sqrt{\frac{N^2 - n_{s,c}^2}{n_w^2 - N^2}}\right] = \frac{2N}{k_0\sqrt{(N^2 - n_{s,c}^2)\left(n_w^2 - N^2\right)}},$$

$$L_{GHs,c}^{TM} = \frac{2}{k_0}\frac{d}{dN}\left[\arctan\frac{n_w^2}{n_{s,c}^2}\sqrt{\frac{N^2 - n_{s,c}^2}{n_w^2 - N^2}}\right] =$$

$$= \frac{2N}{k_0\sqrt{\left(N^2 - n_{s,c}^2\right)\left(n_w^2 - N^2\right)}}\frac{n_w^2 n_{s,c}^2(n_w^2 - n_{s,c}^2)}{n_{s,c}^4\left(n_w^2 - N^2\right) + n_w^4\left(N^2 - n_{s,c}^2\right)}.$$

$d_{eff}^{TE} = d + k_0\left(|\gamma_s| + |\gamma_c|\right)$ (and a similar, somewhat more complicated expression for TM) – *effective thickness*

$$L_p = -\frac{d\Phi_{tot}}{d\beta} = -\frac{2}{k_0}\frac{d\left(k_0\gamma_w d\right)}{dN} + L_{GSs} + L_{GSc} = 2d\frac{N}{\sqrt{n_w^2 - N^2}} + L_{GSs} + L_{GSc}$$ … *“period of propagation”*

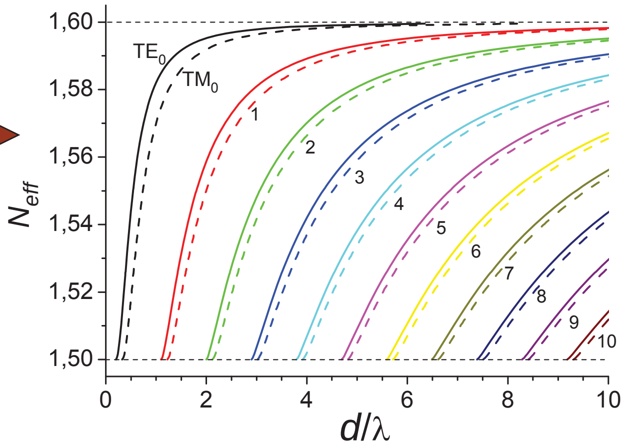
These *“ray-optic concepts”* are useful also for inherently “wave-optic” phenomenon of optical waveguiding.

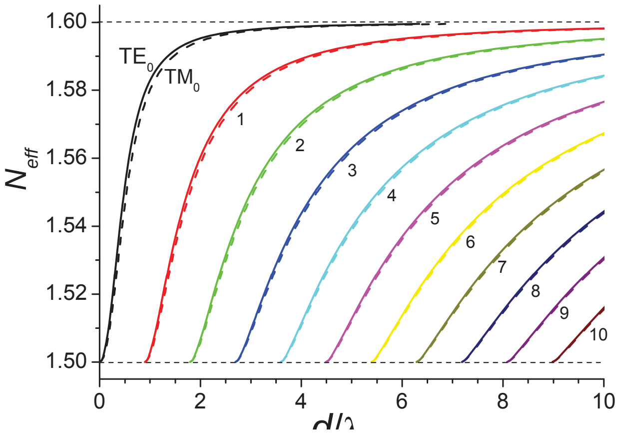
Dispersion diagram (examples)

Asymmetric waveguide

$$n_c = 1 < n_s = 1.5 < n_w = 1.6$$

All modes exhibit cut-off



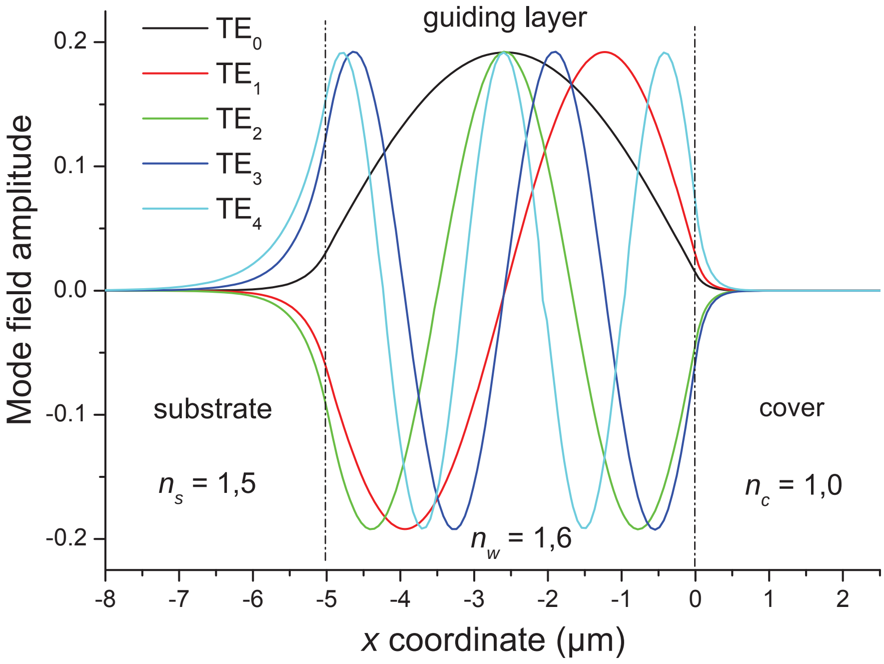


Symmetric waveguide

$$n_c = n_s = 1.5 < n_w = 1.6$$

Number of TE and TM modes identical, TE$_0$ and TM$_0$ *always exist*

Distribution of modes of a slab waveguide



Electromagnetic theory of a slab waveguide

Planar waveguide as a structure with 1D permittivty distribution: $\varepsilon(x)$; $\frac{\partial}{\partial y}\equiv 0$

1. TE polarization: E_y, H_x, H_z

2. TM polarization: H_y, E_x, E_z

Eigenmode field: $E_y(x,z)=E_y(x)\mathrm{e}^{i\beta z}$, etc.

$$H_z(x)=-\frac{i}{\omega\mu_0}\frac{dE_y(x)}{dx},$$
$$H_x(x)=\frac{\beta}{\omega\mu_0}E_y(x),$$
$$\frac{dH_z(x)}{dx}-i\beta H_x(x)=i\omega\varepsilon_0 n^2(x)E_y(x)$$

$$E_z(x)=\frac{i}{\omega\varepsilon_0 n^2(x)}\frac{dH_y(x)}{dx},$$
$$E_x(x)=\frac{\beta}{\omega\varepsilon_0 n^2(x)}H_y(x),$$
$$\frac{dE_z(x)}{dx}-i\beta E_x(x)=-i\omega\mu_0 H_y(x)$$

$$\left\{\frac{d^2}{dx^2}+k_0^2n^2(x)\right\}E_y(x)=\beta^2E_y(x)$$

$$\left\{n^2(x)\frac{d}{dx}\left[\frac{1}{n^2(x)}\frac{d}{dx}\right]+k_0^2n^2(x)\right\}H_y(x)=\beta^2H_y(x)$$

Eigenvalue equations for eigenfunctions $E_y(x)$ or $H_y(x)$ and eigenvalues β^2

Analogy between a planar waveguide and a potential well in quantum mechanics

Field equation for a TE mode in a planar waveguide

Schrödinger equation for a particle in a potential well

$$\frac{1}{k_0^2}\frac{d^2E_y}{dx^2}+n^2(x)E_y=N^2E_y \quad\Leftrightarrow\quad -\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2}+V(x)\psi(x)=W\psi(x)$$

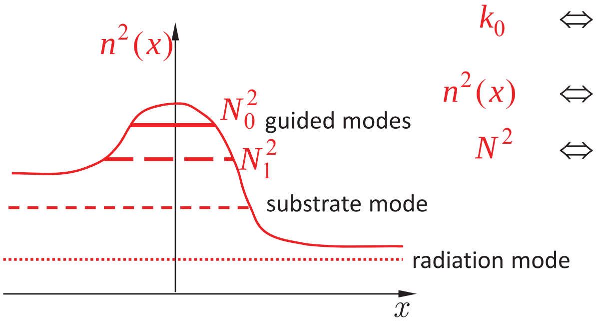
$$E_y(x)\quad\Leftrightarrow\quad \psi(x)$$

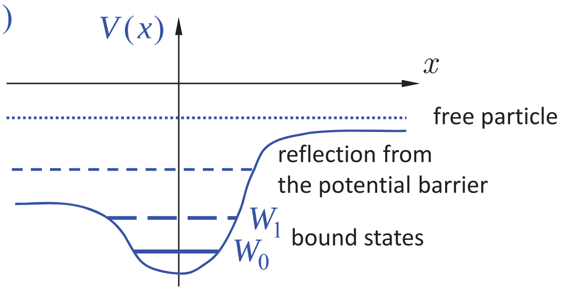
$$k_0\quad\Leftrightarrow\quad \frac{\sqrt{2m}}{\hbar}$$

$$n^2(x)\quad\Leftrightarrow\quad -V(x)$$

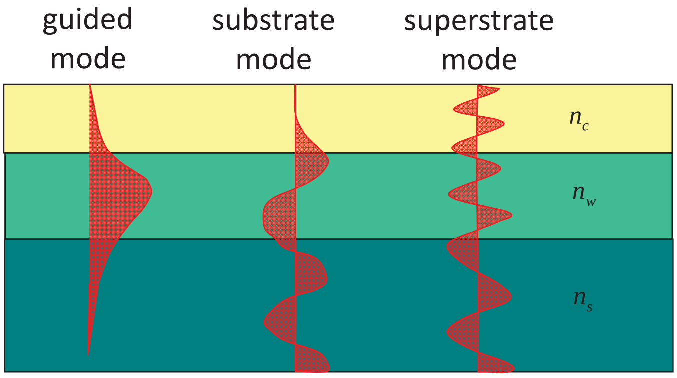
$$N^2\quad\Leftrightarrow\quad -W$$

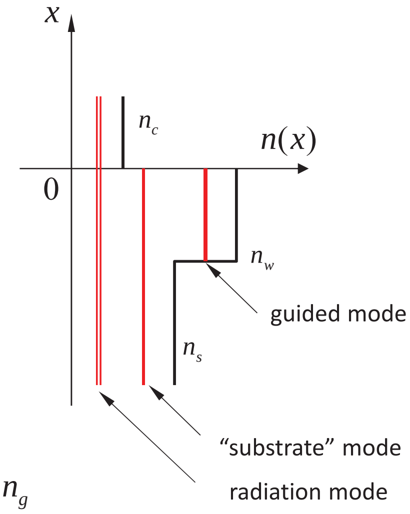
There is not such an exact analogy for TM polarization, but its behaviour is very similar





Guiding of optical radiation in a dielectric waveguide





Guided modes: $k_0 n_c \le k_0 n_s < \beta < k_0 n_g$

Radiation (substrate) modes: $k_0 n_c < \beta < k_0 n_s < k_0 n_g$

Radiation modes (superstrate): $\beta < k_0 n_c \le k_0 n_s < k_0 n_g$

Orthogonality of eigenmodes

It can be shown that *fields of guided modes* (from the discrete spectrum) are *orthogonal,*

$$\frac{1}{2}\int_{-\infty}^{\infty} \mathbf{E}_m(x)\times\mathbf{H}_n(x)\cdot\mathbf{z}^0 dx = \frac{\beta_m}{|\beta_m|}\,\delta_{mn}, \qquad \beta_m = k_0 N_m.$$

For *radiation and evanescent modes*, the orthogonality condition sounds

$$\frac{1}{2}\int_{-\infty}^{\infty} \mathbf{E}(x,\beta)\times\mathbf{H}_n(x,\beta')\cdot\mathbf{z}^0 dx = \frac{\beta}{|\beta|}\,\delta(\beta-\beta')$$

(in the sense
of the principal value
of the integral)

Radiation and evanescent modes are always *orthogonal* to discrete *guided modes:*

$$\frac{1}{2}\int_{-\infty}^{\infty} \mathbf{E}(x,\beta)\times\mathbf{H}_n(x)\cdot\mathbf{z}^0 dx = 0,$$

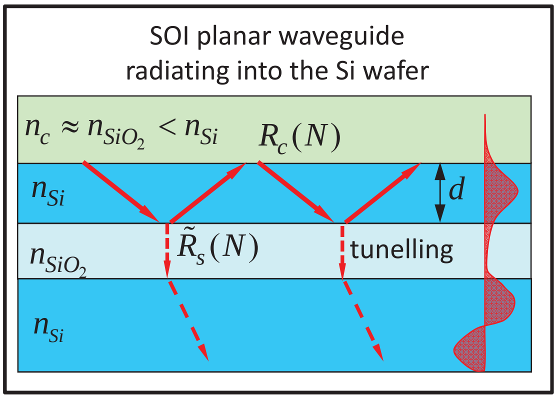
For lossless waveguides, $\mathbf{E}_\perp$ and $\mathbf{H}_\perp$ of guided modes are real, and thus

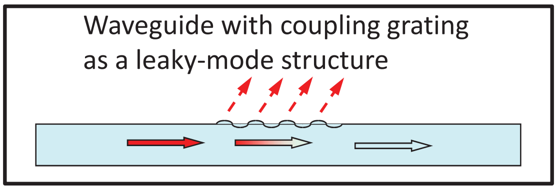
$$\frac{1}{2}\int_{-\infty}^{\infty} \mathbf{E}_m(x)\times\mathbf{H}_n^*(x)\cdot\mathbf{z}^0 dx = \frac{\beta_m}{|\beta_m|}\,\delta_{mn}.$$

(Only) *in this case*, eigenmodes are also *power-orthogonal:*
power carried by a superposition of (guided and non-evanescent radiation) modes
is given by the sum of powers of individual modes

Leaky modes

Leaky modes are not “true” eigenmodes of the waveguide structure but often allow for a simple description and physically understandable interpretation of wave propagation in waveguide structures with (weak) radiation loss


SOI planar waveguide
radiating into the Si wafer


Waveguide with coupling grating
as a leaky-mode structure

$\tilde{R}_s(N)$ ...“frustrated” reflection coefficient , $|\tilde{R}_s| < 1$

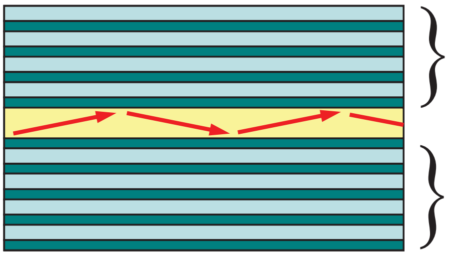
“Standard” dispersion relation
(transverse resonance condition)

$$R_c(N)\tilde{R}_s(N)e^{2ik_0 d\sqrt{n_{Si}^2 - N^2}} = 1$$

$$k_0 d\sqrt{n_{Si}^2 - N^2} = -\frac{1}{2}\arg\left[R_c(N)\right] - \frac{1}{2}\arg\left[\tilde{R}_s(N)\right] + \frac{i}{2}\ln\left|\tilde{R}_s(N)\right| + m\pi$$

SOI waveguide supports *only leaky modes* with complex effective indices, $N = N' + iN''$

Another type of waveguiding – ARROW


Bragg grating (“1D photonic crystal”)

waveguiding layer (“defect in the photonic crystal”)

Bragg grating (“1D photonic crystal”)

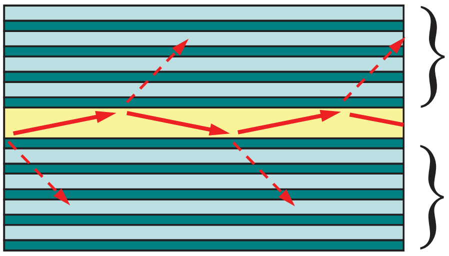
Antiresonant **r**eflecting **o**ptical **w**aveguide – **ARROW**)

Differences between an ARROW and a conventional index-guiding waveguide

1. refractive index of the guiding layer can be lower than those of the grating
2. the gratings must operate in the Bragg regime
   (the 1D photonic crystal has to exhibit a bandgap)
3. theoretically, ARROW with lower index guiding layer supports only *leaky modes*;
   the number of periods has to be sufficient to suppress power leakage
   through the grating

M. A. Duguay et al., *Appl. Phys. Lett*. vol. 49, 13, 1986.

Another type of waveguiding – ARROW



Bragg grating (“1D photonic crystal”)

waveguiding layer (“defect in the photonic crystal”)

Bragg grating (“1D photonic crystal”)

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Guiding by a *single interface*

Are two interfaces essential for waveguiding? (Typically yes, but…)

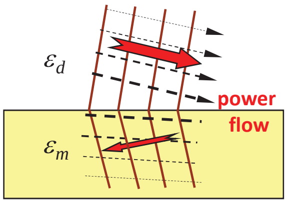
Pole of the reflection coefficient: possible only for TM polarization

$$\varepsilon_2\gamma_1 + \varepsilon_1\gamma_2 = 0;\;\; \gamma_1 = \sqrt{\varepsilon_1 - N^2},\;\; \gamma_2 = \sqrt{\varepsilon_2 - N^2} \;\;\Rightarrow\;\; \boxed{N = \sqrt{\frac{\varepsilon_1\varepsilon_2}{\varepsilon_1 + \varepsilon_2}}.}$$

Wave is *confined* if $\operatorname{Im}\{\gamma_1\} > 0,\ \operatorname{Im}\{\gamma_2\} > 0.$

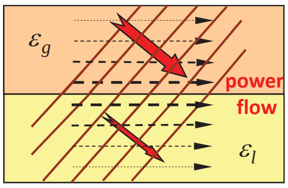
1. ***Surface plasmon****-polariton* at dielectric-metal interface

$\operatorname{Re}\{\varepsilon_m\} < 0,\ \ \operatorname{Im}\{\varepsilon_m\} > 0$ Supported if $\varepsilon_d < |\varepsilon_m|$



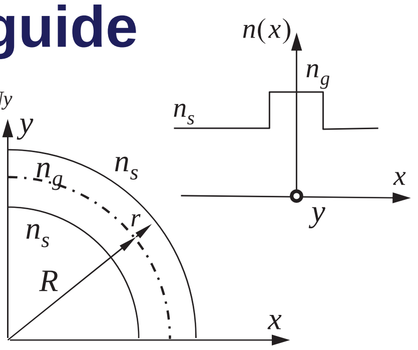
2. Interface between media with a ***balance of loss and gain***

$\varepsilon_g,\ \varepsilon_l$ complex, $\varepsilon_g \approx \varepsilon_l^*$ $\;\; N = \sqrt{\dfrac{\varepsilon_g\varepsilon_l}{\varepsilon_g + \varepsilon_l}} = \dfrac{|\varepsilon_l|}{\sqrt{2\operatorname{Re}\{\varepsilon_l\}}} \approx \sqrt{\dfrac{|\varepsilon_l|}{2}}$



3. Zenneck wave at the interface of a lossy and lossless media (1907)

Circularly bent waveguide



Straight waveguide: $\frac{\partial^2 E}{\partial x^2}+\frac{\partial^2 E}{\partial y^2}+k_0^2 n^2(x)E=0, \quad E(x,y)=E(x)e^{ik_0 Ny}$

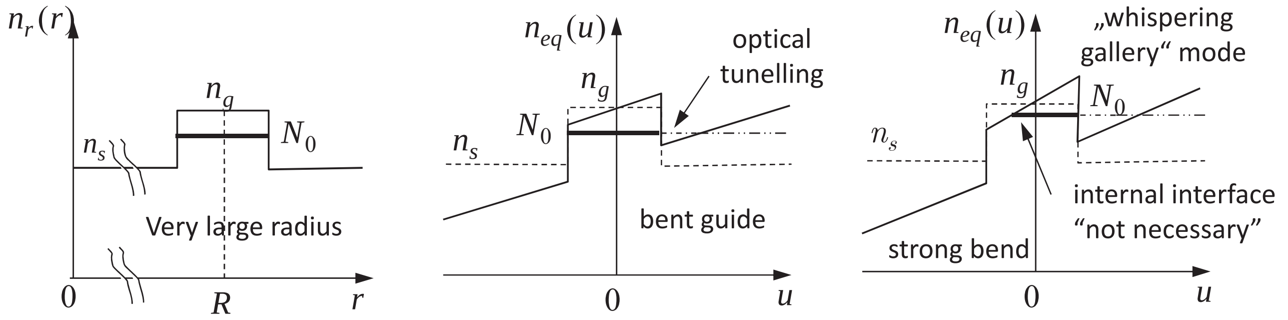
Bent waveguide: $\frac{\partial^2 E}{\partial x^2}+\frac{\partial^2 E}{\partial y^2}+k_0^2 n^2\left(r-R\right)E=0,\ n_r(r)=n(r-R)$

Conformal mapping: $z=x+iy=re^{i\varphi}, \quad w=u+iv=R\ln\left(z/R\right)$

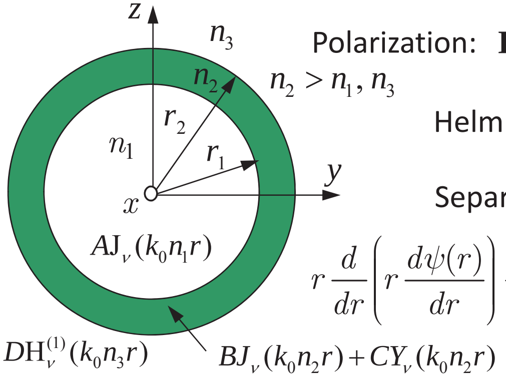
Transformed equation: $u=R\ln\left(r/R\right)\approx r-R, \qquad v=R\varphi$

$$\frac{\partial^2 E}{\partial u^2}+\frac{\partial^2 E}{\partial v^2}+k_0^2 n_{eq}^2(u)E=0, \quad n_{eq}(u)=e^{u/R}n_r\left(Re^{u/R}\right)\approx\left(1+\frac{u}{R}\right)n(u), \quad E(u,v)=E(u)e^{i\beta_b v}=E(u)e^{ik_0 N_b R\varphi}$$

M. Heiblum and J. H. Harris, *IEEE JQE,* vol. QE-11, pp. 75-83, 1975.



Ring resonator, or bent waveguide?



Polarization: $\mathbf{E}\parallel\mathbf{x}^0;\ \frac{\partial}{\partial x}\equiv 0, \quad \mathbf{E}\left(r,\varphi\right)=E_x\left(r,\varphi\right)\mathbf{x}^0$

Helmholz equation: $\Delta_\perp E_x + k_0^2 n^2(r)E_x = 0,$

Separation of variables: $\boxed{E_x(r,\varphi)=\psi(r)\exp\left(i\nu\varphi\right)}$

$$r\frac{d}{dr}\left(r\frac{d\psi(r)}{dr}\right)+\left(k_0^2 n^2 r^2-\nu^2\right)\psi(r)=0$$

angular propagation constant

Bessel equation

Field continuity conditions: $\psi(r),\ \frac{d\psi}{dr}\sim H_\varphi$ continuous at $r_1,\ r_2$:

$$\begin{pmatrix} n_1 J_\nu'(k_0 n_1 r_1) & -n_2 J_\nu'(k_0 n_2 r_1) & -n_2 Y_\nu'(k_0 n_2 r_1) & 0 \\ J_\nu(k_0 n_1 r_1) & -J_\nu(k_0 n_2 r_1) & -Y_\nu(k_0 n_2 r_1) & 0 \\ 0 & -n_2 J_\nu'(k_0 n_2 r_2) & -n_2 Y_\nu'(k_0 n_2 r_2) & n_3 H_\nu^{(1)\prime}(k_0 n_3 r_2) \\ 0 & -J_\nu(k_0 n_2 r_2) & -Y_\nu(k_0 n_2 r_2) & H_\nu^{(1)}(k_0 n_3 r_2) \end{pmatrix}\cdot\begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

(We are not going *to solve* this equation!)

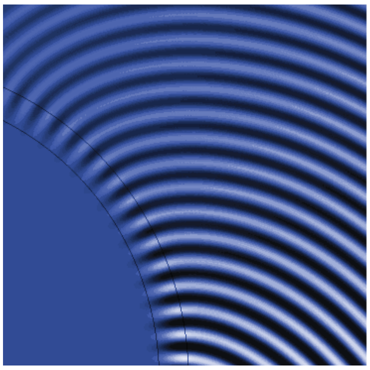
Ring resonator, or bent waveguide?

Dispersion equation: $\boxed{\det(\cdot) = \Phi(\nu,\omega) = 0\ ;}$ Frequency introduced "on purpose"

We have *two* basic *possibilities* how to proceed:

1. fix ω and seek for *(complex) azimuthal propagation constant* ν *for a bent waveguide,* or
2. fix ν (as an integer) and seek for *(complex) resonant frequency* ω of a *ring resonator*.

$$\exp(i\nu\varphi) = \exp(i\nu'\varphi)\exp(-\nu''\varphi) \qquad \omega = \omega_0\left[1 - i/(2Q)\right]$$



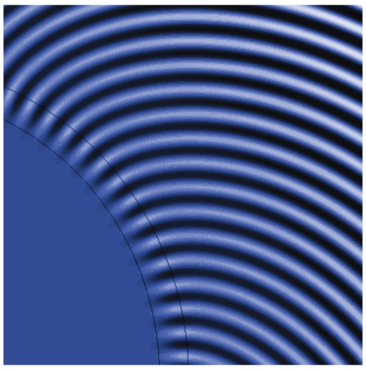
bent waveguide

$n_1 = n_3 = 1.6,$

$n_3 = 1.7,$

$r_1 = 10\ \mu\text{m},$

$r_2 = 11\ \mu\text{m}$



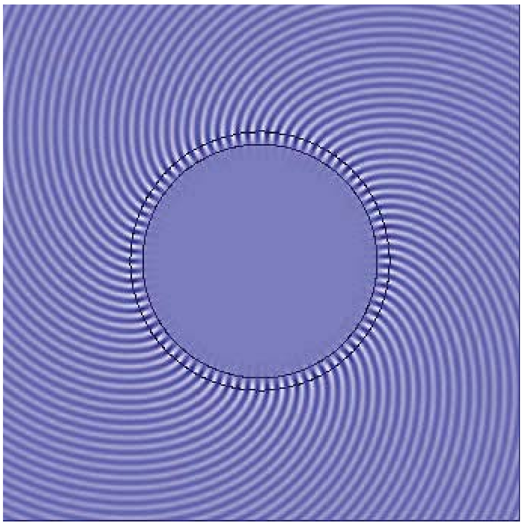
ring resonator

Examples of field distributions

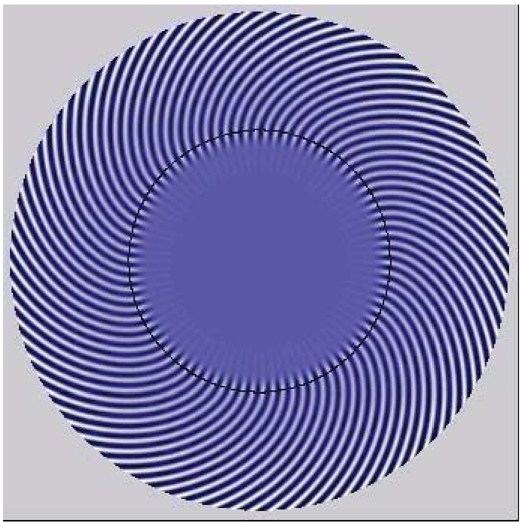
$n_s = 1.6, \quad n_w = 1.7, \quad r = 10\ \mu\text{m}, \quad \lambda = 1.55\ \mu\text{m}$

(Lossy) resonators

ring

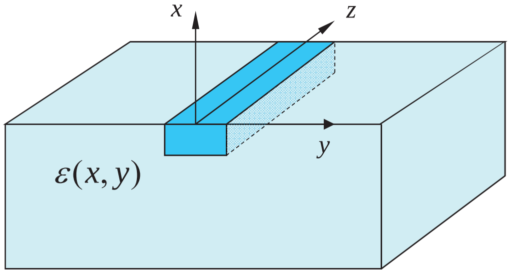


disk



Eigenmodes of dielectric resonators are *leaky modes*!

Channel waveguides



General considerations: vector equation

$$\nabla\times\nabla\times\mathbf{E}-k_0^2\varepsilon(x,y)\mathbf{E}=\mathbf{0}$$

$$\nabla\cdot(\varepsilon\mathbf{E})=0 \;\Rightarrow\; \nabla\cdot\mathbf{E}=-\frac{1}{\varepsilon}\nabla\varepsilon\cdot\mathbf{E}=-\nabla_\perp(\ln\varepsilon)\cdot\mathbf{E}$$

$$\Delta\mathbf{E}+\nabla\left[\nabla_\perp(\ln\varepsilon)\cdot\mathbf{E}\right]+k_0^2\varepsilon\mathbf{E}=\mathbf{0}$$

Let's separate transversal and longitudinal field components *of an eigenmode*:

$$\mathbf{E}=\mathbf{e}(x,y)e^{i\beta z}=\mathbf{e}_\perp(x,y)e^{i\beta z}+\mathbf{e}_z(x,y)e^{i\beta z},\qquad \nabla=\nabla_\perp+\mathbf{z}^0\frac{\partial}{\partial z},\;\; \Delta=\Delta_\perp+\frac{\partial^2}{\partial z^2}$$

We obtain a 2D eigenvalue equation

$$\boxed{\Delta_\perp\mathbf{e}_\perp(x,y)+\nabla_\perp\left[\nabla_\perp(\ln\varepsilon)\cdot\mathbf{e}_\perp\right]+k_0^2\varepsilon(x,y)\mathbf{e}_\perp=\beta^2\mathbf{e}_\perp},\quad \mathbf{e}_z=\frac{i}{\beta}\mathbf{z}^0\left[\nabla_\perp\varepsilon+\nabla_\perp\right]\cdot\mathbf{e}_\perp$$

Modes of channel waveguides are **hybrid** – *all field components are generally nonzero*

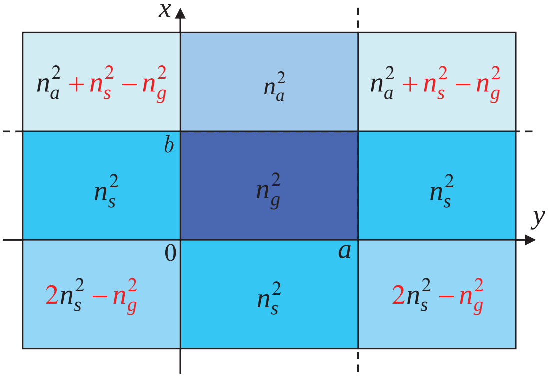
Weakly guiding waveguide: term $\nabla_\perp\left[\nabla_\perp(\ln\varepsilon)\cdot\mathbf{e}_\perp\right]$ is negligibly small; then

$$\boxed{\Delta_\perp\mathbf{e}_\perp(x,y)+k_0^2\varepsilon(x,y)\mathbf{e}_\perp=\beta^2\mathbf{e}_\perp}$$ …essentially *scalar equation*, $\mathbf{e}_z$ small.

E.A.J. Marcatili, Bell System Tech Journal 48, 2071-2102, 1969

Marcatili method – separation of variables

A very simple approximate mode solving method for 2D waveguides



$$n_x^2=\begin{cases}n_a^2, & x>b\\ n_g^2, & 0<x<b,\\ n_s^2, & x<0\end{cases}\qquad n_y^2=\begin{cases}n_s^2, & y<0\\ n_g^2, & 0<y<a,\\ n_s^2, & y>a\end{cases}$$

Approximation of weak guiding,

$$\Delta_\perp e(x,y)+k_0^2\left[n^2(x,y)-N^2\right]e(x,y)=0$$

Simple solution if the profile were separable:

$$n^2(x,y)\overset{!}{=}n_x^2(x)+n_y^2(y)+const$$

Then $e(x,y)=e_x(x)e_y(y)$, and

$$\frac{d^2e_x(x)}{dx^2}+k_0^2\left[n_x^2-N_x^2\right]e_x(x)=0,$$

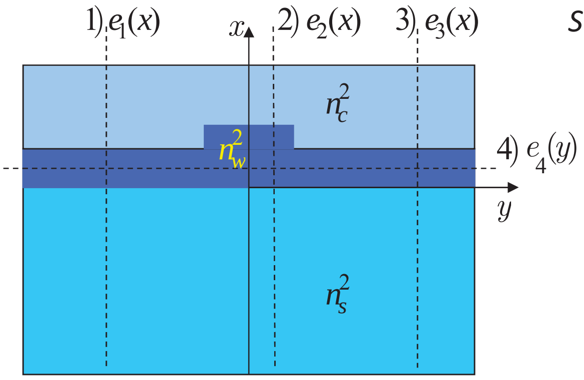
$$\frac{d^2e_y(x)}{dy^2}+k_0^2\left[n_y^2-N_y^2\right]e_y(x)=0,$$

$$N^2=N_x^2+N_y^2+const$$

Subtracting $n_g^2-n_s^2$ from the permittivity in the corners makes the profile separable, $const=-n_g^2$

The task is reduced to solving 2 simple 1D equations, but results close to cut-off are questionable.

Effective-index method for 2D profile



Semi-intuitive method – reduction to a 1D problem

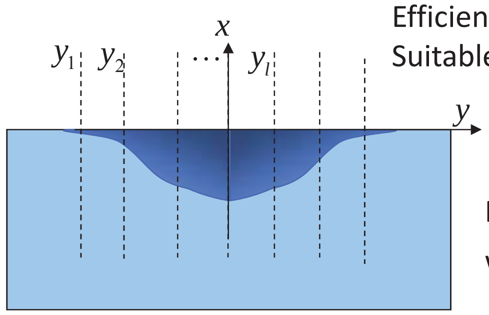
1) planar waveguide with a *vertical profile,* effective index N_1 and mode field $e_1(x)$
2) planar waveguide with a *vertical profile,* effective index N_2 and mode field $e_2(x)$
3) planar waveguide with a *vertical profile,* effective index N_3 and mode field $e_3(x)$
4) „planar" waveguide with a *lateral profile* N_1 , N_2, N_3 and the field $e_4(y)$

The total field is approximately given by the product $e(x,y) \cong e_2(x)e_4(x)$.

Advantage: simplicity, clear physical interpretation.
Disadvantage: inaccurate near cut-off, accuracy difficult to assess;

Well-applicable to shallow ridges and diffused channel waveguides,
in many other cases very useful as a first guess. Commonly used method.

Effective-index method for diffused channel waveguides



Efficient approximate method based on physical intuition.
Suitable for graded-index waveguides and complex structures

Weak guiding, lateral (y) dependence weaker*:*

$$\Delta_\perp e(x,y) + k_0^2\left[n^2(x,y) - N^2\right]e(x,y) = 0$$

Non-separable, but let us try $e(x,y) \cong e_x(x;y)e_y(y)$

where "the strong" *y*-dependence is concentrated in $e_y(y)$:

y is *a parameter*

$$\frac{d^2e_x(x;y)}{dx^2} + k_0^2\left[n^2(x;y) - N_x^2(y)\right]e_x(x;y) = 0$$

Then we solve this equation for several values of y and get $N_x^2(y)$ as an *effective lateral profile.*
In the next step, we solve the "lateral equation" to get N and $e_y(y)$

$$\frac{d^2e_y(y)}{dy^2} + k_0^2\left[N_x^2(y) - N^2\right]e_y(y) = 0.$$

What if there is no guided mode for some y? Take the substrate value n_s instead of $N_x(y)$

More complex waveguide structures: “rigorous” formulation of the coupled-mode theory

Wave propagation in the permittivity distribution $\varepsilon(x,y,z)$ can be analyzed using the completeness and orthogonality of the eigenmodes of a waveguide with the permittivity profile $\varepsilon^{(0)}(x,y)$:

Eigenmode fields: $\mathbf{E}_\mu(x,y,z)=A_\mu\mathbf{e}_\mu(x,y)e^{i\beta_\mu z},\quad \mathbf{H}_\mu(x,y,z)=A_\mu\mathbf{h}_\mu(x,y)e^{i\beta_\mu z}$

Mode orthogonality: $\frac{1}{2}\iint_S \mathbf{e}_\mu\times\mathbf{h}_\nu\cdot d\mathbf{S}=\frac{1}{2}\iint_S \mathbf{e}_{\mu\perp}\times\mathbf{h}_{\nu\perp}\cdot d\mathbf{S}=\frac{\beta_\mu}{|\beta_\mu|}\delta_{\mu\nu}$

Wave in $\varepsilon(x,y,z)$ can be expressed as

$$\mathbf{E}_\perp(x,z,y)=\sum_\mu\left[a_\mu(z)\mathbf{e}_{\mu\perp}(x,y)+b_\mu(z)\mathbf{e}_{\mu\perp}(x,y)\right],\quad E_z(x,y,z)=\sum_\mu\left[a_\mu(z)e_{\mu z}(x,y)-b_\mu(z)e_{\mu z}(x,y)\right],$$

$$\mathbf{H}_\perp(x,z,y)=\sum_\mu\left[a_\mu(z)\mathbf{h}_{\mu\perp}(x,y)-b_\mu(z)\mathbf{h}_{\mu\perp}(x,y)\right],\ H_z(x,z,y)=\sum_\mu\left[a_\mu(z)h_{\mu z}(x,y)+b_\mu(z)h_{\mu z}(x,y)\right].$$

From Maxwell equations we get the set of first order linear equations for complex amplitudes,

$$\frac{da_\mu(z)}{dz}=i\beta_\mu a_\mu(z)+\sum_\nu\left[K_{\mu\nu}^{++}(z)a_\nu(z)+K_{\mu\nu}^{+-}(z)b_\nu(z)\right],$$

$$\frac{db_\mu(z)}{dz}=-i\beta_\mu b_\mu(z)+\sum_\nu\left[K_{\mu\nu}^{-+}(z)a_\nu(z)+K_{\mu\nu}^{--}(z)b_\nu(z)\right].$$

D. Marcuse, *Theory of dielectric optical waveguides, 2nd ed. Academic Press, 1991*

Slowly varying amplitudes

The coupling constants are given by overlap integrals

$$K_{\mu\nu}^{pq}=pK_{\mu\nu}+qk_{\mu\nu},\qquad p,\ q=\begin{cases}1 \text{ for } +\\ -1 \text{ for } -\end{cases},$$

$$K_{\mu\nu}(z)=\frac{i\omega\varepsilon_0}{4}\frac{|\beta_\mu|}{\beta_\mu}\iint_S\left[\varepsilon(x,z,y)-\varepsilon^{(0)}(x,y)\right]\mathbf{e}_{\mu\perp}\cdot\mathbf{e}_{\nu\perp}dxdy,$$

$$k_{\mu\nu}(z)=\frac{i\omega\varepsilon_0}{4}\frac{|\beta_\mu|}{\beta_\mu^*}\iint_S\frac{\varepsilon^{(0)}(x,y)}{\varepsilon(x,y,z)}\left[\varepsilon(x,z,y)-\varepsilon^{(0)}(x,y)\right]\mathbf{e}_{\mu z}\cdot\mathbf{e}_{\nu z}dxdy,$$

Introducing *slowly varying amplitudes* $A_\mu(z)=a_\mu(z)e^{-i\beta_\mu z},\quad B_\mu(z)=b_\mu(z)e^{i\beta_\mu z}$ we obtain

$$\frac{dA_\mu}{dz}=\sum_\nu\left[K_{\mu\nu}^{++}(z)e^{-i(\beta_\mu-\beta_\nu)z}A_\nu(z)+K_{\mu\nu}^{+-}(z)e^{-i(\beta_\mu+\beta_\nu)z}B_\nu(z)\right],$$

$$\frac{dB_\mu}{dz}=\sum_\nu\left[K_{\mu\nu}^{-+}(z)e^{i(\beta_\mu+\beta_\nu)z}A_\nu(z)+K_{\mu\nu}^{--}(z)e^{i(\beta_\mu-\beta_\nu)z}B_\nu(z)\right].$$

First-order (Born) approximation

Let us formally integrate the set of equations:

$$\int_0^z \frac{dA_\mu}{dz}dz \approx \int_0^z \sum_\nu \left[K_{\mu\nu}^{++}(z)e^{-i(\beta_\mu-\beta_\nu)z}A_\nu(z)+K_{\mu\nu}^{+-}(z)e^{-i(\beta_\mu+\beta_\nu)z}B_\nu(z)\right]dz,$$

$$\int_0^z \frac{dB_\mu}{dz}dz \approx \int_0^z \sum_\nu \left[K_{\mu\nu}^{-+}(z)e^{i(\beta_\mu+\beta_\nu)z}A_\nu(z)+K_{\mu\nu}^{--}(z)e^{i(\beta_\mu-\beta_\nu)z}B_\nu(z)\right]dz,$$

Supposing slow variation of amplitudes and for small *z*

$$A_\mu(z) \approx A_\mu(0)+\sum_\nu \left[A_\nu(0)\int_0^z K_{\mu\nu}^{++}(z)e^{-i(\beta_\mu-\beta_\nu)z}dz+B_\nu(0)\int_0^z K_{\mu\nu}^{+-}(z)e^{-i(\beta_\mu+\beta_\nu)z}dz\right],$$

$$B_\mu(z) \approx B_\mu(0)+\sum_\nu \left[A_\nu(0)\int_0^z K_{\mu\nu}^{-+}(z)e^{i(\beta_\mu+\beta_\nu)z}dz+B_\nu(0)\int_0^z K_{\mu\nu}^{--}(z)e^{i(\beta_\mu-\beta_\nu)z}dz\right].$$

The integrals are “importantly non-zero” only if the integrands do not rapidly oscillate. Thus, *if K is slowly vaying, the modes with close propagation constants are strongly coupled . Coupling of modes with significantly different β’s requires that this difference is compensated by rapidly varying K(z).*

Two simple applications

1. Coupling of two forward modes

Let us preserve only “slow” terms satisfying the phase-matching condition:

$$\frac{dA_\mu}{dz} \approx K_{\mu\nu}^{++}(z)e^{-i(\beta_\mu-\beta_\nu)z}A_\nu(z)$$

For slowly-varying amplitudes w can approximately take $A_\nu(z) \approx A_\nu(0).$

Next, we will apply the Taylor expansion for $\beta_\mu-\beta_\nu$ and take only the first two terms:

$$\beta_\mu-\beta_\nu \approx \beta_\mu(\omega_0)-\beta_\nu(\omega_0)+\frac{d}{d\omega}\left[\beta_\mu(\omega)-\beta_\nu(\omega)\right](\omega-\omega_0)=\beta_\mu(\omega)-\beta_\nu(\omega)+\frac{N_{\mu g}-N_{\nu g}}{c}(\omega-\omega_0).$$

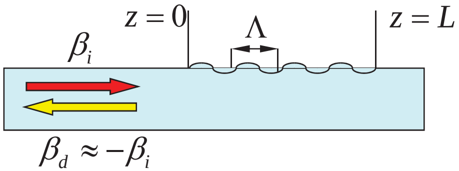
We define the “transmission coefficient” from mode ν to mode μ and get

$$T(z)=\frac{A_\mu(z)}{A_\nu(0)} \approx \int_0^z K_{\mu\nu}^{++}(z')e^{-i(\beta_\mu-\beta_\nu)z'}dz' \approx \int_0^z K_{\mu\nu}^{++}(z')e^{-i\left[\frac{N_{\mu g}-N_{\nu g}}{c}(\omega-\omega_0)\right]z'}dz'.$$

The spectral dependence of the “transmission coefficient” is approximately given by the Fourier expansion of the spatial dependence of the coupling constant.

Waveguide Bragg grating mirror

2. Coupling of forward and backward modes by a waveguide grating



$$K=\frac{2\pi}{\Lambda},\quad \beta_i+\beta_d-K\simeq 0_i \quad\Rightarrow\quad K\simeq 2\beta_i,\ \ \Lambda\simeq\frac{\lambda}{2N}$$

Coupled equations:
$$\boxed{\begin{aligned} &dA_i/dz=i\kappa^* e^{-i\Delta\beta z}B_d(z), &&\Delta\beta=\beta_i+\beta_d-K\\ &dB_d/dz=-i\kappa\, e^{i\Delta\beta z}A_i(z), &&\kappa=iK^{++}_{d,i,1}.\end{aligned}}$$

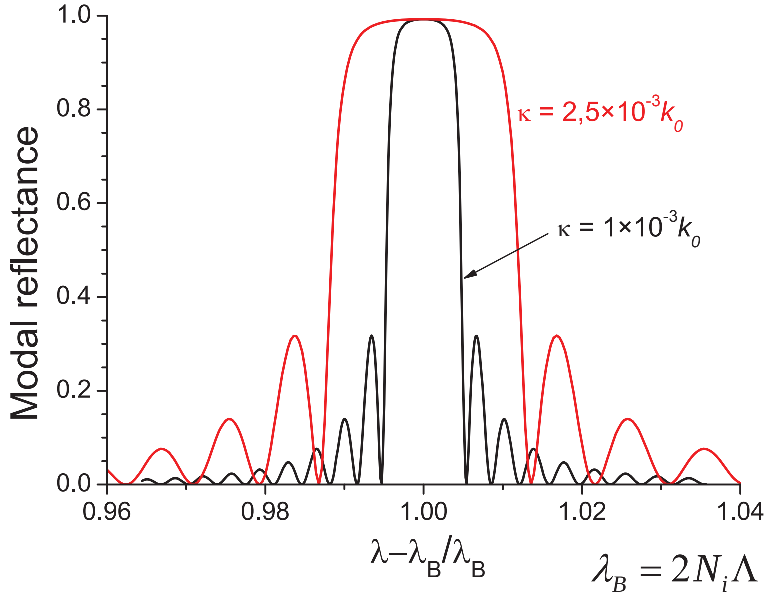
Boundary conditions:
$$\boxed{A_i\left(0\right)=A_{i0},\ B_d\left(L\right)=0.}$$

Solution:
$$\boxed{\begin{aligned} &A_i(z)=\delta A_{i,0}\left[\delta\cosh\delta z-i\left(\Delta\beta/2\right)\sinh\delta z\right]^{-1}, \qquad \delta=\sqrt{\left|\kappa\right|^2-\left(\Delta\beta/2\right)^2}.\\ &B_d(z)=i\kappa^* A_{i,0}e^{-i\frac{\Delta\beta}{2}z}\left[\delta\coth\delta z-i\frac{\Delta\beta}{2}\right]^{-1}\end{aligned}}$$

Grating reflectance:
$$\boxed{\left|R\right|^2=\left|\frac{B_d(0)}{A_{i0}}\right|^2=\left|\frac{\kappa\sinh\delta L}{\delta\cosh\delta L-i\left(\Delta\beta/2\right)\sinh\delta L}\right|^2}$$

For $\Delta\beta=0,\ \left|R^2\right|=\tanh^2\left|\kappa\right|L.$

Spectral dependence of the reflectance



$$\lambda_B=2N_i\Lambda$$

Numerical methods for modelling waveguide structures

Three main classes of modelling methods:

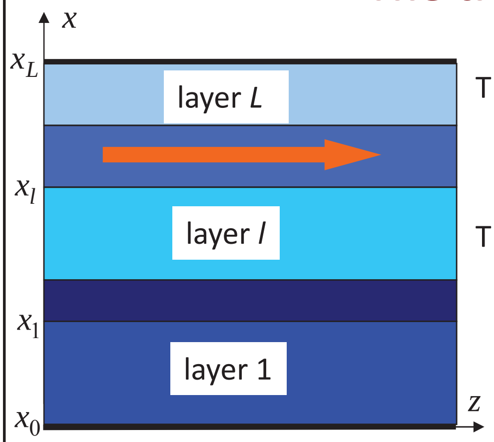
1. Frequency-domain **mode solvers** for calculation of eigenmodes and propagation constants of straight and bent uniform waveguides; 1D, 2D, approximate, “rigorous”;
   scalar, semivectorial, full-vector; modal methods (Fourier modal method),
   discretization-based methods like FD, FE. BE, etc.

2. Frequency-domain **“beam propagation” methods (BPM);** in principle, scattering methods calculating optical field distribution within a waveguide structure for a given excitation field;
   modal, FFT-BPM, FD-BPM, FE-BPM; 2D, 3D, scalar, full-vector;
   unidirectional, bi-directional (in fact, omnidirectional), etc.

3. **Time-domain methods (FDTD , FETD,…)**: numerical model of optical field generated by a given distribution of sources. Essentially, direct numerical solution of Maxwell equations.

… and many other special methods

The transfer matrix method



From Maxwell equations applied to a layer *l* we get

TE

$$\begin{pmatrix} E_y(x_l) \\ H_z(x_l) \end{pmatrix} = \begin{pmatrix} \cos\varphi_l & -i\dfrac{Z_0}{\gamma_l}\sin\varphi_l \\ -iY_0\gamma_l\sin\varphi_l & \cos\varphi_j \end{pmatrix} \cdot \begin{pmatrix} E_y(x_{l-1}) \\ H_z(x_{l-1}) \end{pmatrix}$$

TM

$$\begin{pmatrix} H_y(x_l) \\ E_z(x_l) \end{pmatrix} = \begin{pmatrix} \cos\varphi_l & iY_0\dfrac{n^2}{\gamma_l}\sin\varphi_l \\ iZ_0\dfrac{\gamma_l}{n^2}\sin\varphi_l & \cos\varphi_l \end{pmatrix} \cdot \begin{pmatrix} H_y(x_{l-1}) \\ E_z(x_{l-1}) \end{pmatrix}$$

Matrices are even functions of γ_l! $\varphi_l = k_0\gamma_l(x_l - x_{l-1}), \quad \gamma_l = \sqrt{n_l^2 - N^2}, \quad Z_0 = Y_0^{-1} = \sqrt{\mu_0/\varepsilon_0}$

Transformation over the whole multilayer structure:

$$\begin{pmatrix} f(x_L) \\ g(x_L) \end{pmatrix} = \mathbf{M} \cdot \begin{pmatrix} f(x_0) \\ g(x_0) \end{pmatrix}, \quad \mathbf{M} = \prod_{l=1}^{L} \mathbf{M}_l.$$

For $f(x_0) = f(x_L) = 0$,

$$\begin{pmatrix} 0 \\ g(x_L) \end{pmatrix} = \mathbf{M} \cdot \begin{pmatrix} 0 \\ g(x_0) \end{pmatrix} \Rightarrow \boxed{M_{12}\left(N^2\right) = 0,}$$

dispersion equation

$$g(x_L) = M_{22}\,g(x_0).$$

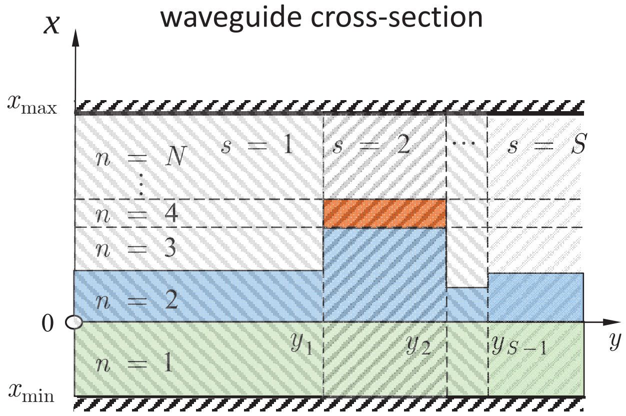
2D vectorial mode solver

Method of Lines (MoL) - requires 1D discretization, FD method

school of prof. Reinhold Pregla, Fern-Universität Hagen, Germany

Film mode matching (FMM)

(straight guides: Sudbø 1993, 1994, bent guides Prkna 2004)



Transversal refractive index distribution piecewise constant

- Subdivide the cross-section into laterally uniform “slices”; each slice represents a multilayer
- Find TE and TM modes of each slice
- Express total field as superposition of slice modes
- Match fields at the boundaries of slices

Stable (impedance or scattering matrix) formalism; fully vectorial solution

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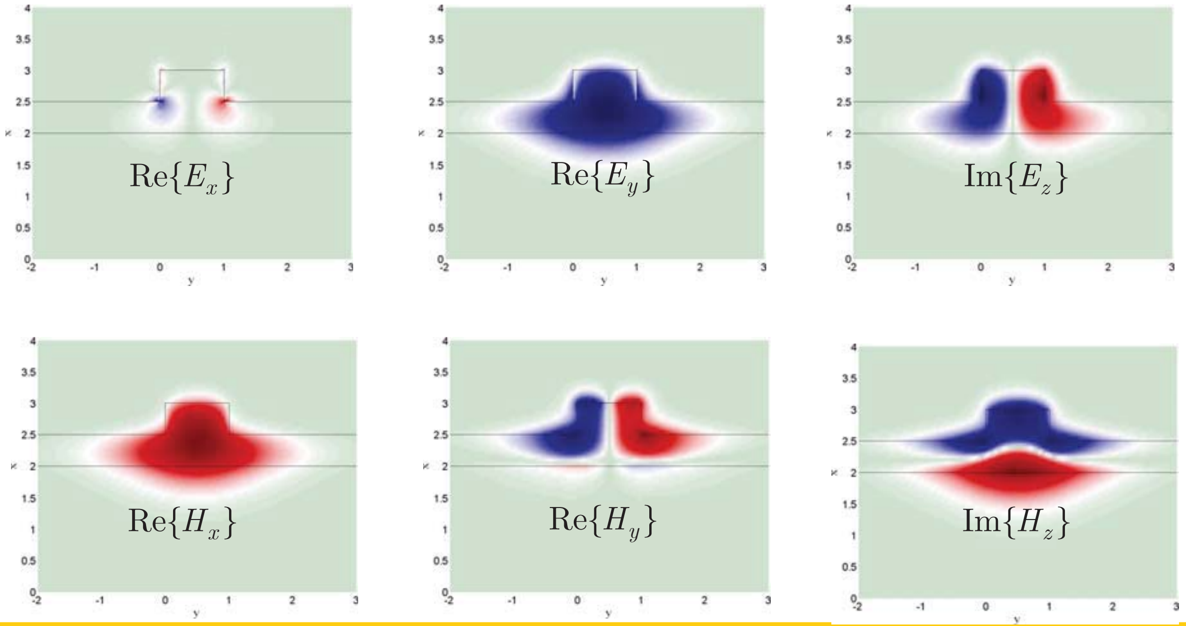
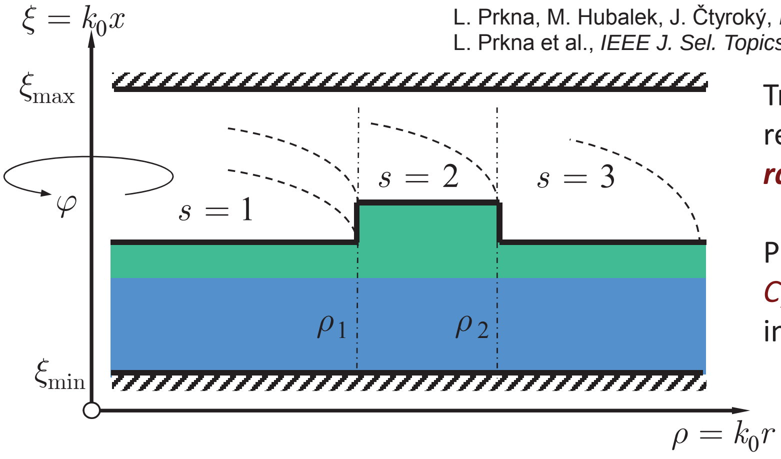
An example: quasi-TE mode of a rib waveguide

$n_g = 2.2,\ n_s = 1.9,\ n_c = 1,$ thickness = height $= 0.5\ \mu\text{m}$



Film mode matching for circularly bent waveguides

L. Prkna, M. Hubalek, J. Čtyroký, *IEEE Phot. Technol. Lett. vol.* 16, 2057-2059 (2004).
L. Prkna et al., *IEEE J. Sel. Topics in Quantum Electr.* vol. 11, 217-223 (2005)



Treatment analogous to the rectangular case;
radial instead of ***lateral*** dependence.

Problem:
Cylindrical functions of complex order
instead of trigonometric functions.

1. subdivision of the structure into radially uniform “slices”, each “slice” forms a multilayer;
2. in each “slice”, mode field is expanded into TE and TM modes of a multilayer
3. field matching at the interfaces between “slices”.

- *No* (or minimum) *discretization*
- Field within the slice *described analytically*

High-contrast SOI microresonator

$R = 2\ \mu\text{m}$

$h = 360\ \text{nm}$

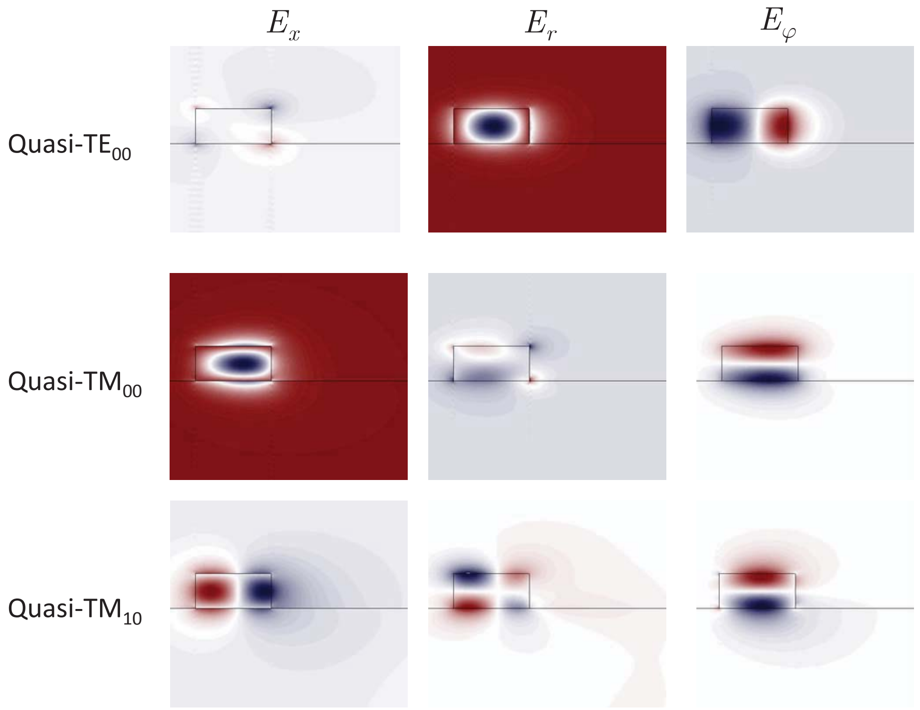
$w = 500\ \text{nm}$

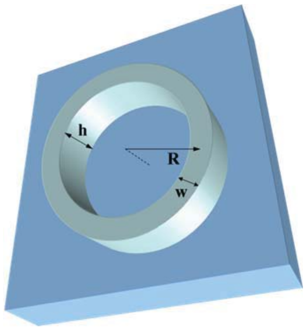
$n_{\text{Si}} = 3.48$

$n_{\text{SiO}_2} = 1.45$

$n_c = 1$

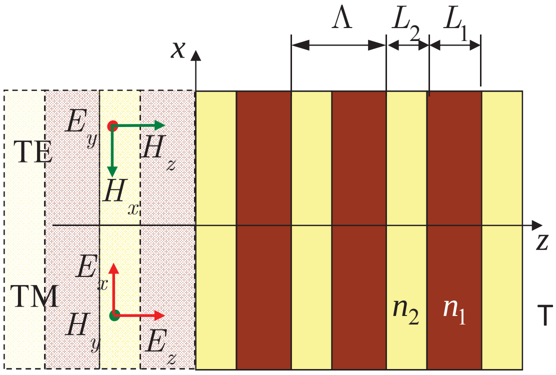
$\lambda = 1.55\ \mu\text{m},$





Wave propagation in a periodic structure

Photonic analogy of the “Kronig – Penney” model of an electronic crystal



$\gamma = n_1 \sin\theta_1 = n_2 \sin\theta_2$ *tangential* propagation constant (*given* θ_1 or θ_2)

$N_l^2 = \sqrt{n_l^2 - \gamma^2},\quad l = 1,2.$ *longitudinal* prop. constant

$\varphi_l = k_0 N_l L_l$

$$\nu = \begin{cases} 0, & \text{TE} \\ 2, & \text{TM} \end{cases}$$

TE

$$\begin{pmatrix} E_y(L_l) \\ H_x(L_l) \end{pmatrix} = \overbrace{\begin{pmatrix} \cos\varphi_l & -i\dfrac{Z_0}{N_l}\sin\varphi_l \\ -iY_0 N_l \sin\varphi_l & \cos\varphi_j \end{pmatrix}}^{\mathbf{M}_l} \cdot \begin{pmatrix} E_y(0) \\ H_z(0) \end{pmatrix}$$

TM

$$\begin{pmatrix} H_y(L_l) \\ E_x(L_l) \end{pmatrix} = \begin{pmatrix} \cos\varphi_l & iY_0\dfrac{n^2}{N_l}\sin\varphi_l \\ iZ_0\dfrac{N_l}{n^2}\sin\varphi_l & \cos\varphi_j \end{pmatrix} \cdot \begin{pmatrix} H_y(0) \\ E_x(0) \end{pmatrix}$$

Electromagnetic Floquet – Bloch modes

Transmission through layers 1 and 2 is described by matrices $\mathbf{M}_1$ and $\mathbf{M}_2$

$$\nu = \begin{cases} 0, & \text{TE} \\ 2, & \text{TM} \end{cases}$$

The matrix for transitin over a single period is evidently ${}^{\Lambda}\mathbf{M} = \mathbf{M}_2 \cdot \mathbf{M}_1$

$${}^{\Lambda}\mathbf{M} = \begin{pmatrix} \cos\varphi_1\cos\varphi_2 - \dfrac{n_2^\nu N_1}{n_1^\nu N_2}\sin\varphi_1\sin\varphi_2 & iY_0\left(\dfrac{n_1^\nu}{N_1}\sin\varphi_1\cos\varphi_2 + \dfrac{n_2^\nu}{N_2}\sin\varphi_2\cos\varphi_1\right) \\ iZ_0\left(\dfrac{N_2}{n_2^\nu}\sin\varphi_2\cos\varphi_1 + \dfrac{N_1}{n_1^\nu}\sin\varphi_1\cos\varphi_2\right) & \cos\varphi_1\cos\varphi_2 - \dfrac{n_1^\nu N_2}{n_2^\nu N_1}\sin\varphi_1\sin\varphi_2 \end{pmatrix}$$

Floquet-Bloch mode is determined as an eigenfunction and eigenvector of ${}^{\Lambda}\mathbf{M}$,

$$\boxed{{}^{\Lambda}\mathbf{M}^{TE} \cdot \begin{pmatrix} E_{y1}^F(0) \\ H_{x1}^F(0) \end{pmatrix} = s\begin{pmatrix} E_{y1}^F(0) \\ H_{x1}^F(0) \end{pmatrix},}\quad \text{or} \quad \boxed{{}^{\Lambda}\mathbf{M}^{TM} \cdot \begin{pmatrix} H_{y1}^F(0) \\ E_{x1}^F(0) \end{pmatrix} = s\begin{pmatrix} H_{y1}^F(0) \\ E_{x1}^F(0) \end{pmatrix},}\quad \boxed{s = \exp(i\beta^F\Lambda),}$$

β^F is determined up to an additive constant $K = 2\pi/\Lambda$,

β^F is the prop. const. of the FB mode.

it is sufficient to determine β^F in the interval $\boxed{-K/2 < \beta^F \le K/2}$ $\Rightarrow$ *first Brillouin zone.*

Eigenvalues and the photonic bandgap

Let us denote $\Lambda = L_1 + L_2, \quad \varphi_1 = k_0 N_1 L_1, \quad \varphi_2 = k_0 N_2 L_2, \quad \rho = \dfrac{n_1^{\nu} N_2}{n_2^{\nu} N_1}$

eigenvalues of the matrix ${}^{\Lambda}\mathbf{M}$ are then

$$\boxed{s = \cos\varphi_1 \cos\varphi_2 - \frac{1}{2}\left(\rho^2 + \frac{1}{\rho^2}\right)\sin\varphi_1 \sin\varphi_2 \pm \sqrt{\left[\cos\varphi_1 \cos\varphi_2 - \frac{1}{2}\left(\rho^2 + \frac{1}{\rho^2}\right)\sin\varphi_1 \sin\varphi_2\right]^2 - 1}\,.}$$

FB mode is propagating only if $|s| = 1$, *i.e.*, if

$$\boxed{\left|\cos\varphi_1 \cos\varphi_2 - \frac{1}{2}\left(\rho^2 + \frac{1}{\rho^2}\right)\sin\varphi_1 \sin\varphi_2\right| \le 1.}$$

outside the bandgap, β^F real

The normalized wavenumber can be explicitly expressed as

$$\beta^{F\prime} = \frac{\beta^F}{K/2} = \frac{1}{\pi}\arccos\left[\cos\left(\frac{\omega}{c}N_1 L_1\right)\cos\left(\frac{\omega}{c}N_2 L_2\right) - \frac{1}{2}\left(\rho^2 + \frac{1}{\rho^2}\right)\sin\left(\frac{\omega}{c}N_1 L_1\right)\sin\left(\frac{\omega}{c}N_2 L_2\right)\right].$$

If $\boxed{\left|\cos\varphi_1 \cos\varphi_2 - \frac{1}{2}\left(\rho^2 + \frac{1}{\rho^2}\right)\sin\varphi_1 \sin\varphi_2\right| > 1,}$

within the bandgap β^F complex

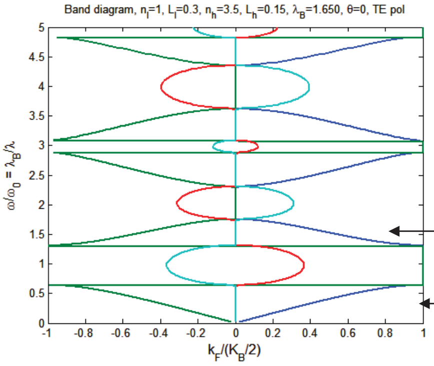
β^F is complex, the wave is attenuated, and the **photonic bandgap** is created.

Band diagrams of a "1D crystal"

$n_1 = 1$

$n_2 = 3.5$

$\theta_1 = 0^{\circ}$



$n_1 = 1$

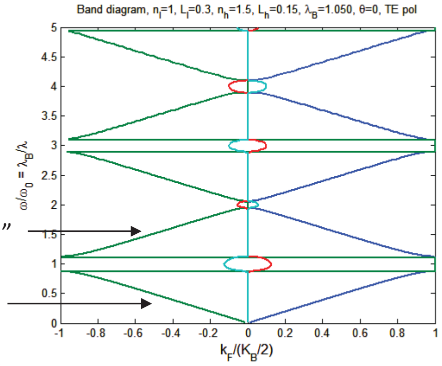
$n_2 = 1.5$

$\theta_1 = 0^{\circ}$

"conduction"

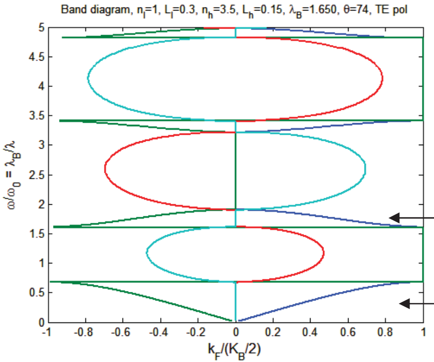
band

"valence"



TE

$\theta_1 = 74^{\circ}$



TM

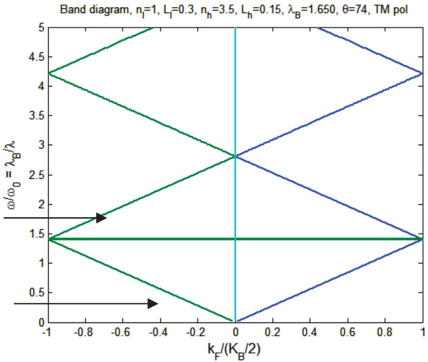
$\theta_1 = 74^{\circ}$

Brewster angle

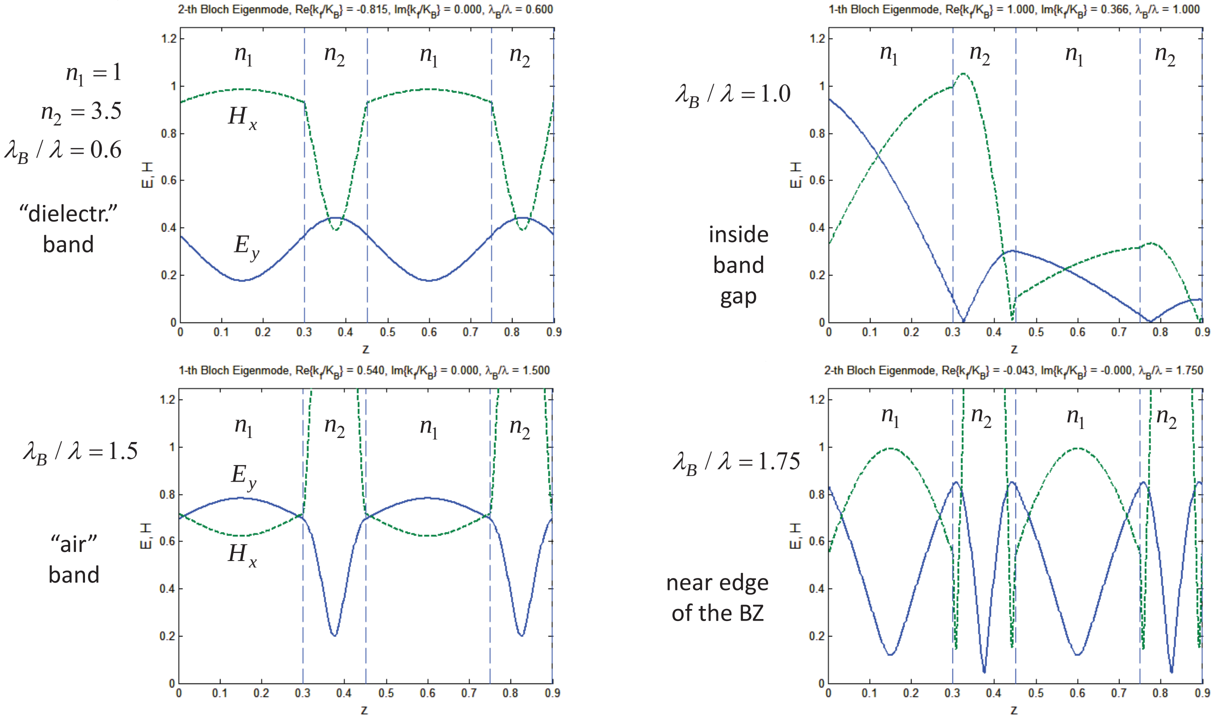
"air"

band

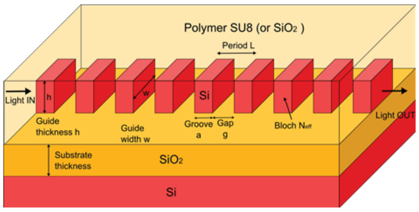
"dielectric"



Electromagnetic Floquet – Bloch modes



3D analogue – subwavelength grating waveguides



Propagating modes in SWGW are Bloch modes

Propagation constant and the effective refractive index of the *m*-th **Bloch mode**:

$$\Gamma_{mm} = \exp\left(i\beta_{B,m}\Lambda\right), \qquad \boxed{\beta_{B,m} = -\frac{i}{\Lambda}\ln\Gamma_{mm} = \frac{2\pi}{\lambda}N_{B,m}}$$

Grating constant of the SWGW: $K = \dfrac{2\pi}{\Lambda}$

"First Brillouin zone" of the SWGW as a 1D photonic crystal: $|\beta_B| < K/2 = \pi/\Lambda$

For lossless propagation, $\boxed{n_s < N_B < \dfrac{\lambda}{2\Lambda} = n_{BZ}}$

Group effective index: $\boxed{N_{B,g} = N_B - \lambda\, dN_B/d\lambda}$

In analogy with the behaviour of photonic crystals we expect that $N_{B,g} \to \infty$ as $N_B \to \dfrac{\lambda}{2\Lambda}$ (the "bandgap edge")

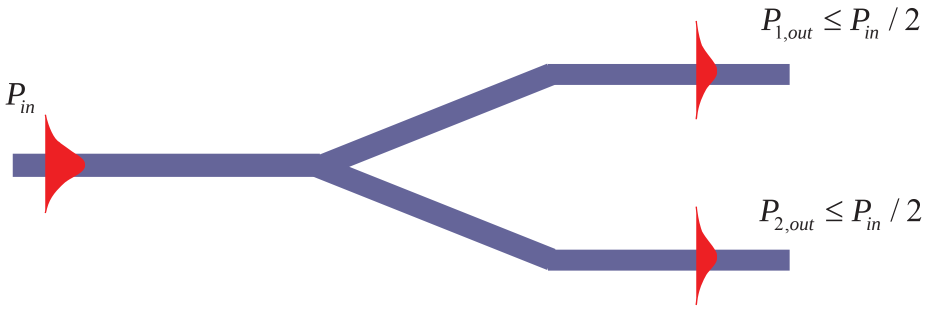
Region of N_B "close to" $\dfrac{\lambda}{2\Lambda}$ represents the **slow light region**

“Canonic” (elementary) waveguide devices

Elementary waveguide structures

Symmetric Y-junction (1×2 power splitter)

1. Excitation into the single mode common arm



Power is equally divided between the two output arms due to symmetry

Basic waveguide structures

Symmetric Y-junction excited in the opposite direction

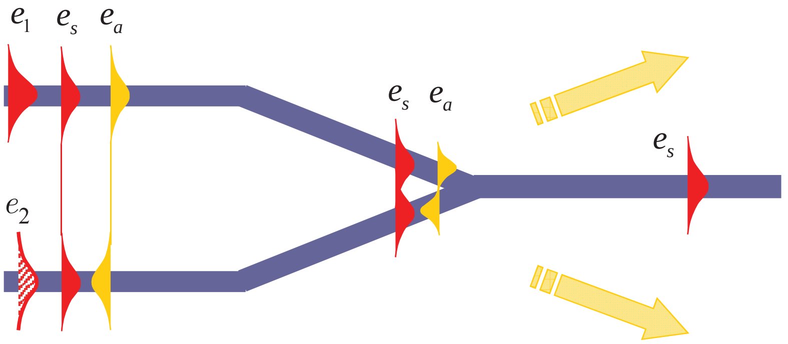
2. Excitation into a single arm

$$e_1 \approx \frac{1}{\sqrt{2}}(e_s + e_a),$$

$$e_2 \approx \frac{1}{\sqrt{2}}(e_s - e_a),$$

$$e_s \approx \frac{1}{\sqrt{2}}(e_1 + e_2),$$

$$e_a \approx \frac{1}{\sqrt{2}}(e_1 - e_2).$$



The antisymmetric mode canot propagate in the single-mode output arm and its power is radiated into the substrate

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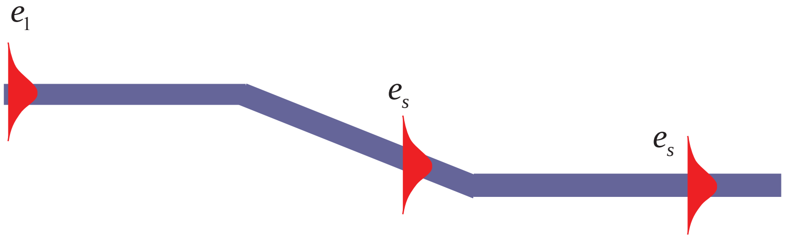
Basic waveguide structures

Symmetric Y-junction excited in the opposite direction

2. Excitation into a single arm

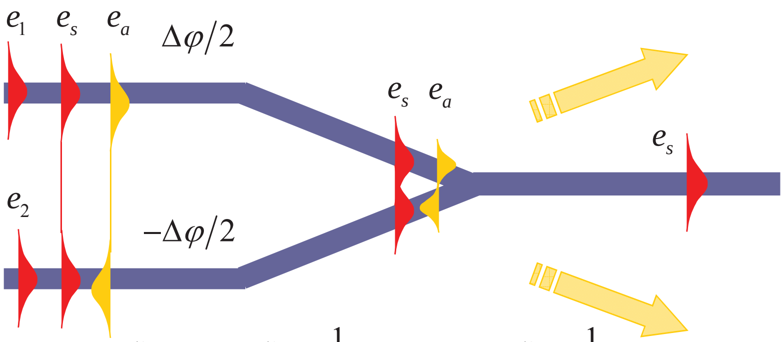


Without the second arm, the transmittance is ≤ 100 %

Basic waveguide structures

Symmetric Y-junction excited in the opposite direction

3. Excitation into both arms with an arbitrary phase shift between the arms



$$E_{in} = e_1 + e_2$$

$$e_1 \approx \frac{1}{\sqrt{2}}\left(e_s + e_a\right),$$

$$e_2 \approx \frac{1}{\sqrt{2}}\left(e_s - e_a\right),$$

$$e_s \approx \frac{1}{\sqrt{2}}\left(e_1 + e_2\right),$$

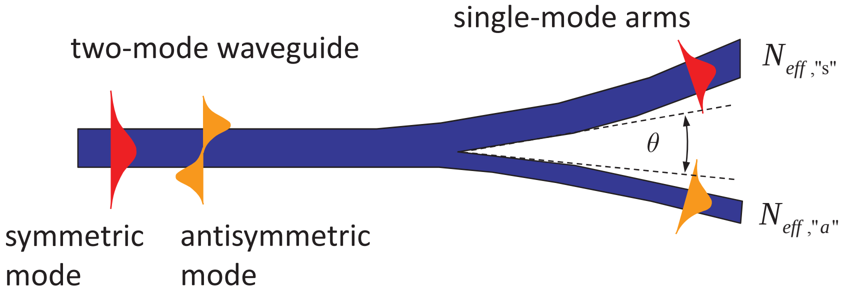
$$e_a \approx \frac{1}{\sqrt{2}}\left(e_1 - e_2\right).$$

$$E_{out} \cong e_1 e^{i\Delta\varphi/2} + e_2 e^{-i\Delta\varphi/2} = \frac{1}{\sqrt{2}}\left(e_s + e_a\right)e^{i\Delta\varphi/2} + \frac{1}{\sqrt{2}}\left(e_s - e_a\right)e^{-i\Delta\varphi/2} =$$

$$= \cancel{\sqrt{2}e_s\cos\frac{\Delta\varphi}{2}} + \sqrt{2}\,i\,e_a\sin\frac{\Delta\varphi}{2} \rightarrow \left(e_1 + e_2\right)\cos\frac{\Delta\varphi}{2} = E_{in}\cos\frac{\Delta\varphi}{2}$$

$$\boxed{P_{out} \le P_{in}\cos^2\frac{\Delta\varphi}{2}}$$

Asymmetric Y-junction as a mode splitter



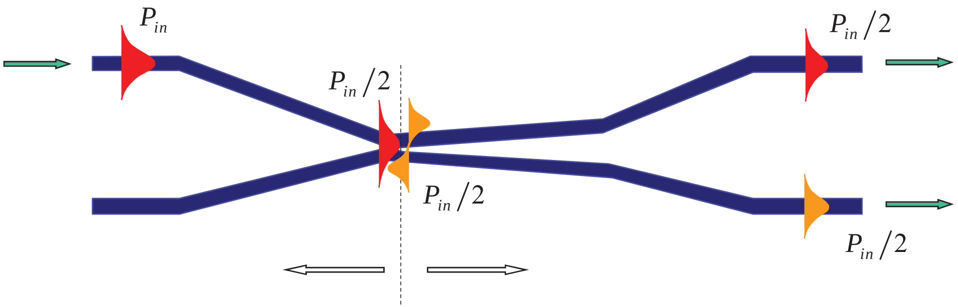
Adiabatic splitter: negligibly small mode coupling along the propagation length. The fundamental mode of the two-mode section remains "fundamental", etc.

Criterion of asymmetry:

$$\frac{\Delta N_{eff}}{\sqrt{n_s^2 - \bar{N}_{eff}^2}\;\theta}\begin{cases} >1, & \Rightarrow \text{asymmetric Y, mode splitter} \\ <0.1, & \Rightarrow \text{symmetric Y, power splitter} \end{cases}$$

Since ΔN_{eff} cannot be made too large, the decisive role is overtaken by θ.
Typically, for $\theta \le 0.2^\circ$ and $\Delta N_{eff} > 0$, the Y-junction behaves "asymmetrically",
for $\theta \ge 1^\circ$ and $\Delta N_{eff} \approx 0$, the Y-junction behaves as power splitter.

Application: spectrally independent 2×2 (3-dB) splitter



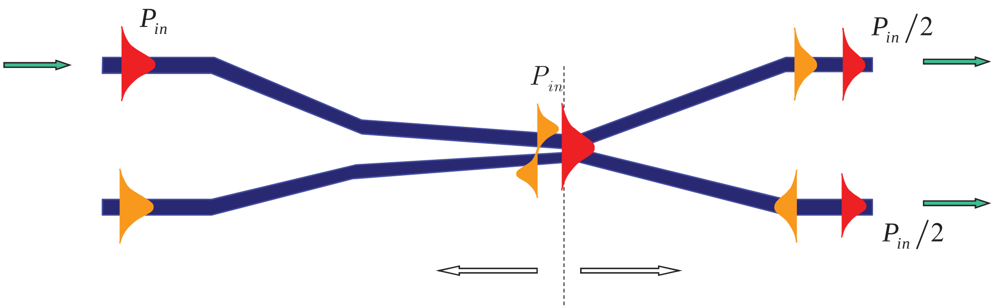
Symmetric Y-junction
(power divider)

Asymmetric Y-junction
(mode splitter)

Spectral bandwidth is limited by the requirements regarding the number of modes;
the input and output ports must be single-mode, the central junction double-moded.
The bandwidth 1.2 – 1.6 μm is rather easily attainable.

Application: spectrally independent 2×2 (3-dB) splitter

Reversed direction of propagation

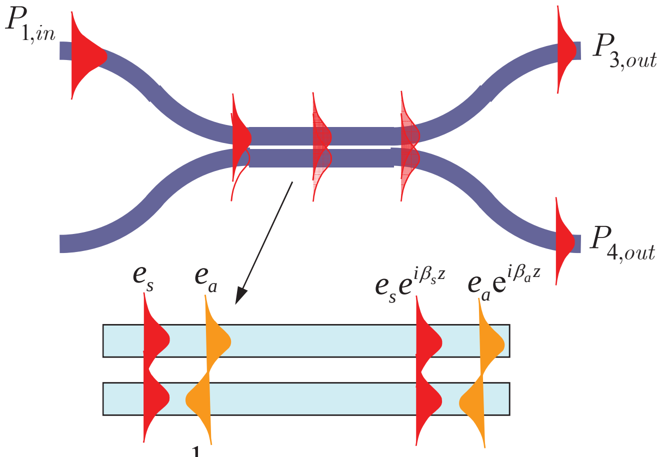


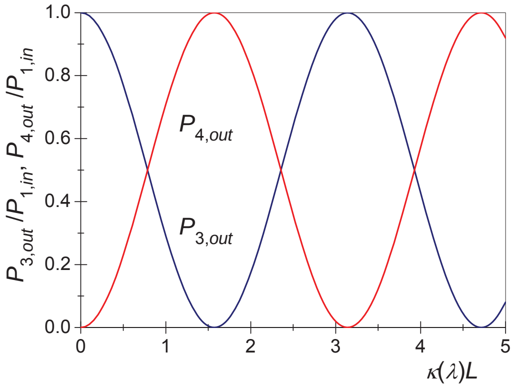
AsymmetricY-junction
(mode splitter)

Symmetric Y-junction
(power divider)

Spectral bandwidth is limited by the requirements regarding the number of modes;
the input and output ports must be single-mode, the central junction double-moded.
The bandwidth 1.2 – 1.6 μm is attainable.

Directional coupler





$$E(0) = e_1 = \frac{1}{\sqrt{2}}(e_s + e_a),$$

$$E(z) = \frac{1}{\sqrt{2}}\left(e_s e^{i\beta_s z} + e_a e^{i\beta_a z}\right) = \frac{1}{2}\left[\left(e_1 + e_2\right)e^{i\beta_s z} + \left(e_1 - e_2\right)e^{i\beta_a z}\right]$$

$$\approx \mathrm{e}^{i(\beta_s+\beta_a)z/2}\left(e_1 \cos\kappa z + i e_2 \sin\kappa z\right).$$

$$P_{3,out} = P_{1,in}\cos^2\left(\kappa z\right),$$

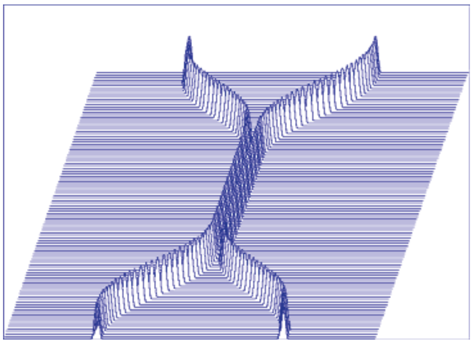
$$P_{4,out} = P_{1,in}\sin^2\left(\kappa z\right),$$

$$\kappa = (\beta_s - \beta_a)/2,$$

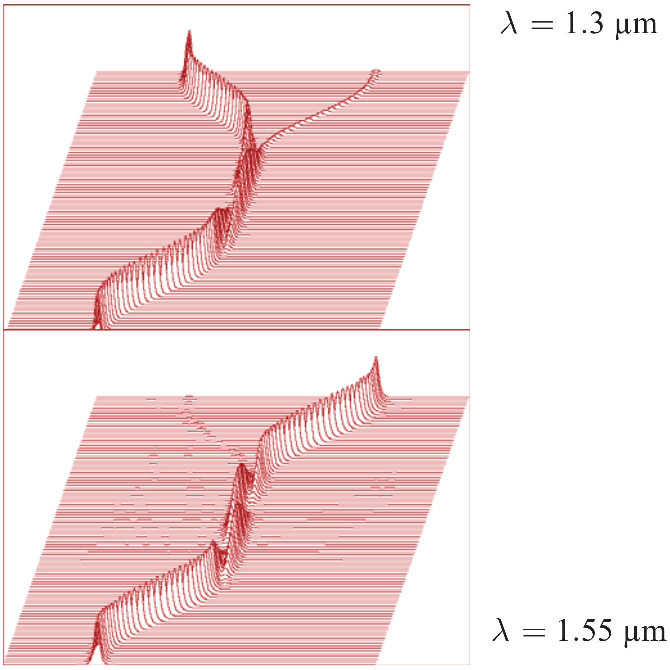
$$L_c = \pi/(\beta_s - \beta_a).$$

Spectral dependence of the directional coupler

Refractive index profile (eff.index)

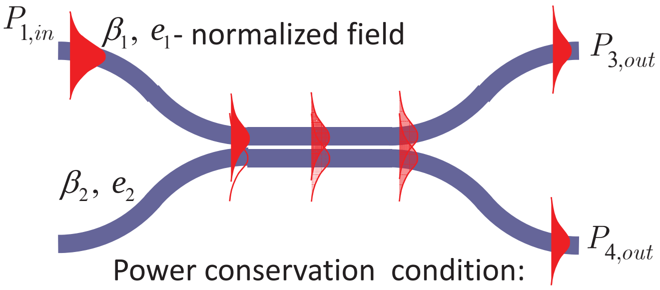


Distribution of optical intensity



Directional coupler can be used as a (coarse) wavelength demux

Coupled-mode theory of the directional coupler



Power conservation condition:

$$\boxed{\frac{d}{dz}\left(|a_1|^2+|a_2|^2\right)=0 \quad \Rightarrow \quad \kappa_{12}=\kappa_{21}^{*};}$$

Without lost of generality we choose $\kappa_{12}=\kappa_{21}=\kappa.$

Approximate description of the field:

$$E(x,y,z)\approx a_1(z)e_1(x,y)+a_2(z)e_2(x,y)$$

Coupled-mode equations:

$$\boxed{\begin{aligned}\frac{da_1(z)}{dz}&=i\beta_1 a_1(z)+i\kappa_{12}a_2(z),\\ \frac{da_2(z)}{dz}&=i\beta_2 a_2(z)+i\kappa_{21}a_1(z);\end{aligned}}$$

Boundary conditions: $a_1(0)=1,$
$a_2(0)=0.$

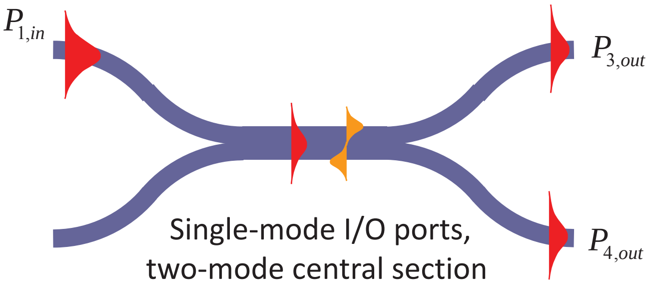
Solution:

$$a_1(z)=a_1(0)e^{i\frac{\beta_1+\beta_2}{2}z}\left[\cos\delta z-i\left(\frac{\Delta\beta}{2}\right)\sin\delta z\right],\ \Delta\beta=\beta_2-\beta_1.$$

$$a_2(z)=ia_1(0)\frac{\kappa}{\delta}e^{i\frac{\beta_1+\beta_2}{2}z}\sin\delta z,\quad \delta=\sqrt{\left(\frac{\Delta\beta}{2}\right)^2+\kappa^2},$$

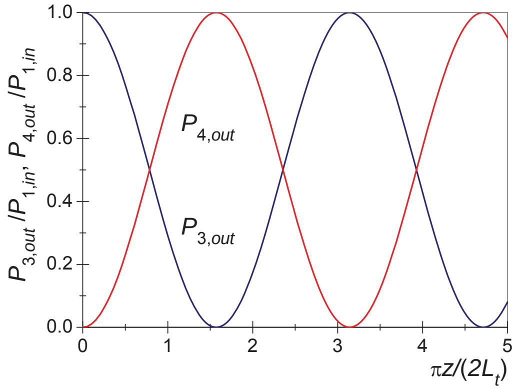
For $\Delta\beta$ = 0 $\begin{cases} P_{3,ou}=P_{1,in}\cos^2(\kappa z),\\ P_{4,out}=P_{1,in}\sin^2(\kappa z),\\ \kappa=(\beta_2-\beta_2)/2.\end{cases}$

Two-mode interference coupler



Single-mode I/O ports,
two-mode central section

Similar to directional coupler,
shorter "transfer length" L_t



$$E(0)=e_1=\frac{1}{\sqrt{2}}(e_s+e_a),$$

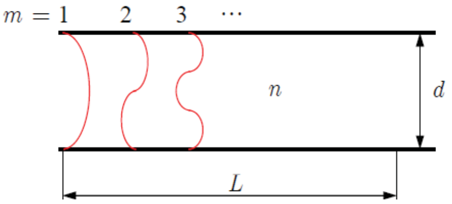
$$E(z)=\frac{1}{\sqrt{2}}\left(e_s e^{i\beta_s z}+e_a e^{i\beta_a z}\right)=\frac{1}{2}\left[(e_1+e_2)e^{i\beta_s z}+(e_1-e_2)e^{i\beta_a z}\right]$$

$$\approx e^{i(\beta_s+\beta_a)z/2}\left(e_1\cos\frac{\pi z}{2L_t}+ie_2\sin\frac{\pi z}{2L_t}\right).$$

$$P_{3,out}=P_{1,in}\cos^2(\kappa L),$$
$$P_{4,out}=P_{1,in}\sin^2(\kappa L),$$
$$\kappa=(\beta_s-\beta_a)/2,$$
$$L_t=\pi/(\beta_s-\beta_a).$$

Imaging properties of an optical waveguide

Two-conductor waveguide:



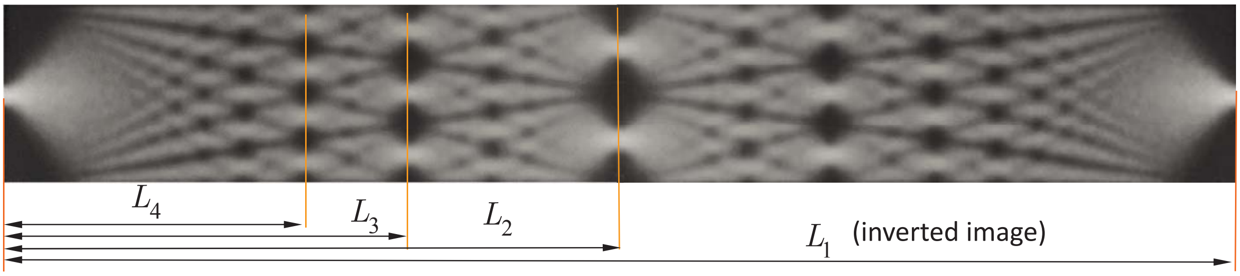
Propagation constants *in a multimode waveguide*:

$$\beta_m = \sqrt{k_0^2 n^2 - \left(\frac{m\pi}{d}\right)^2} \approx k_0 n \left[1 - \frac{1}{2}\left(\frac{m\pi}{k_0 n d}\right)^2\right], \quad m \ll M,$$

$$m = (0), 1, 2, \ldots, M, \; M = \left[\frac{k_0 n d}{\pi}\right] = \left[\frac{2nd}{\lambda}\right].$$

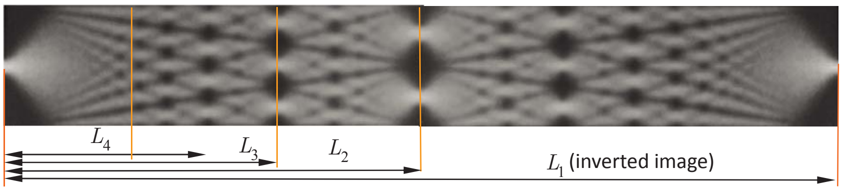
If $\frac{\pi L}{2k_0 n d^2} = 2$, i.e., $L \approx \frac{4k_0 n d^2}{\pi} = \frac{8nd^2}{\lambda} \approx 4Md$, then $\beta_m L \approx k_0 n L - \frac{1}{2}\frac{m^2\pi^2 L}{k_0 n d^2} = k_0 n L - 2m^2\pi.$

After propagation along a distance *L* , all modes meet (approximately) with the same phase. The field distribution at *z* = 0 is then reproduced at *z* = *L*. At the halfway, the “image” is reversed.



O. Bryngdahl and W.-H. Lee, *J. Opt. Soc. Am. vol. 68, pp. 310-315, 1978.*

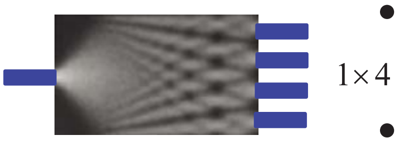
Multimode interference coupler (MMI)

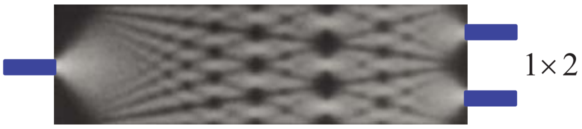


Advantage:
splitters with small footprint;

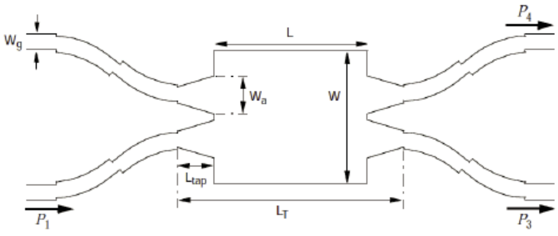
Problems:

- $\beta(m)$ dependence in dielectric waveguides is “less parabolic” $\Rightarrow$ phase error for higher-order modes; error is larger for higher refractive-index contrast;
- transversal spatial resolution depends on the number of guided modes with small phase errors



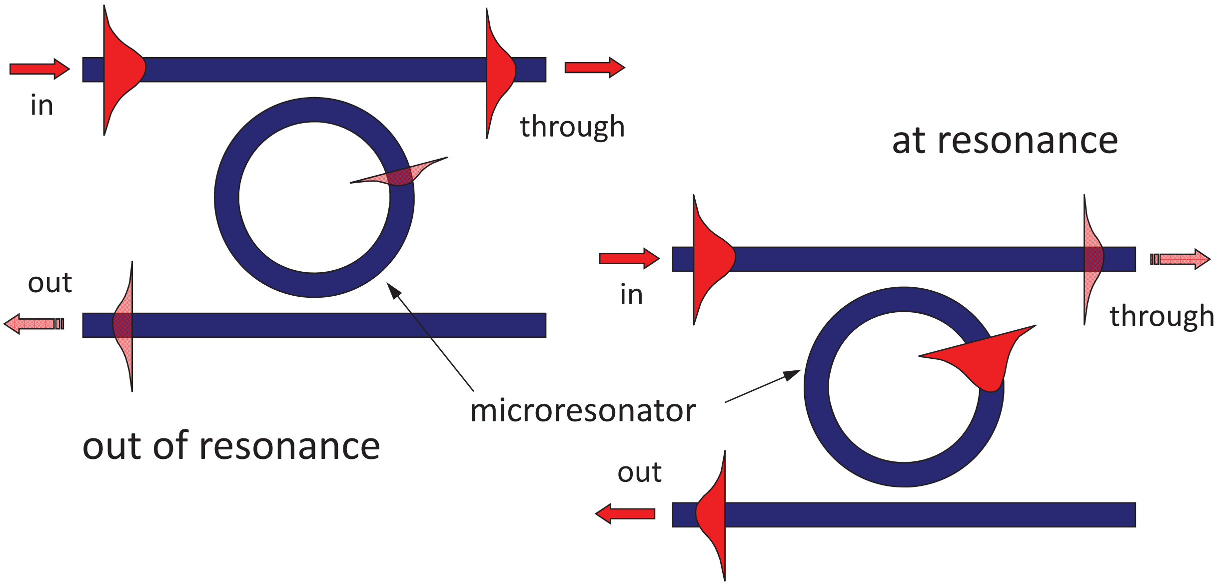


Solution:



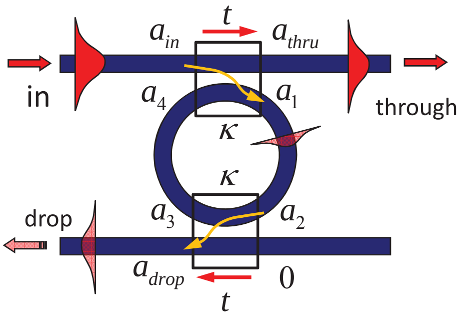
M.T.Hill, J. Lightwave Technol. **21**, 2305-2313, 2003

Microring resonators



A. Driessen et al.:*"Microresonators as building blocks for VLSI photonics," AIP Proceedings Vol.709, 2003.*

Basic theory of microring resonators



(Lossless) couplers: $t^2+\kappa^2=1;$

$$\boxed{\begin{pmatrix} a_{thru} \\ a_1 \end{pmatrix} = \begin{pmatrix} t & i\kappa \\ i\kappa & t \end{pmatrix} \cdot \begin{pmatrix} a_{in} \\ a_4 \end{pmatrix}, \quad \begin{pmatrix} a_{drop} \\ a_3 \end{pmatrix} = \begin{pmatrix} t & i\kappa \\ i\kappa & t \end{pmatrix} \cdot \begin{pmatrix} 0 \\ a_2 \end{pmatrix}}$$

Ring with the perimeter d and the (complex) propagation constant β_r :

$$a_2 = e^{i\beta_r d/2} a_1, \quad a_4 = e^{i\beta_r d/2} a_3,$$

After elementary manipulations we obtain

$$a_{thru} = \frac{t\left(1-e^{i\beta_r d}\right)}{1-t^2 e^{i\beta_r d}} a_{in}, \quad a_{drop} = -\frac{\kappa^2 e^{i\beta_r d/2}}{1-t^2 e^{i\beta_r d}} a_{in},$$

$$a_3 = \frac{i\kappa t e^{i\beta_r d/2}}{1-t^2 e^{i\beta_r d}} a_{in}, \qquad a_1 = \frac{i\kappa}{1-t^2 e^{i\beta_r d}} a_{in}.$$

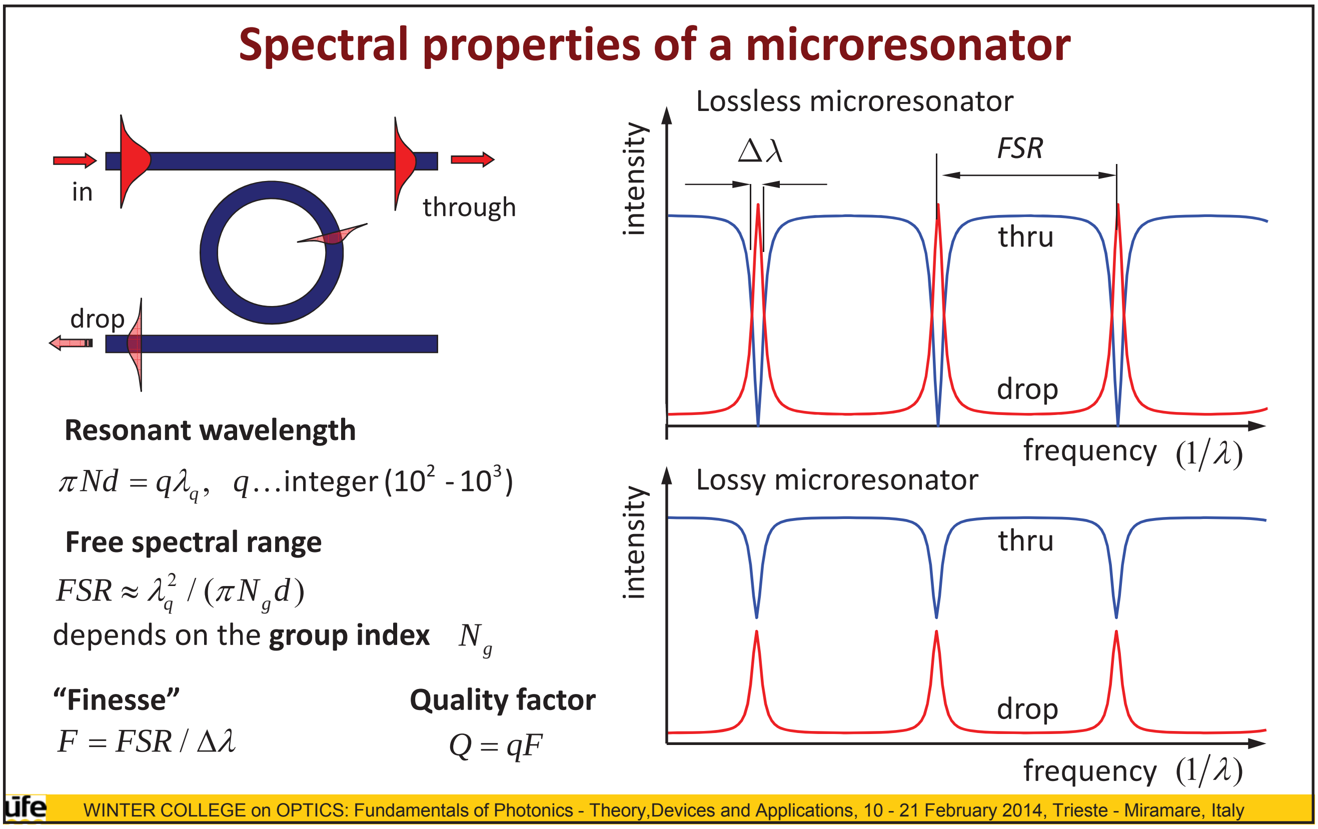
At resonance, $\beta_r d = 2q\pi$, $q\ldots$integer,

$$\boxed{a_{thru} = 0, \quad a_{drop} = -a_{in},} \quad a_1 = \frac{i\kappa}{1-t^2} a_{in},$$

Off resonance, $\beta_r d = 2q\pi + 1$,

$$\boxed{\begin{array}{l} a_{thru} = \dfrac{2t}{1+t^2} a_{in}, \quad a_{drop} = -\dfrac{i\kappa^2}{1+t^2} a_{in}, \\ \left|a_{thru}\right|^2 + \left|a_{drop}\right|^2 = \left|a_{in}\right|^2 . \end{array}}$$

For minimum cross-talk, $\left|a_{drop}\right| \ll \left|a_{in}\right|$, $\boxed{\kappa \ll 1.}$



Spectral properties of a microresonator

Resonant wavelength

$\pi N d = q\lambda_q$, $q\ldots$integer $(10^2$ - $10^3)$

Free spectral range

$FSR \approx \lambda_q^2 / (\pi N_g d)$

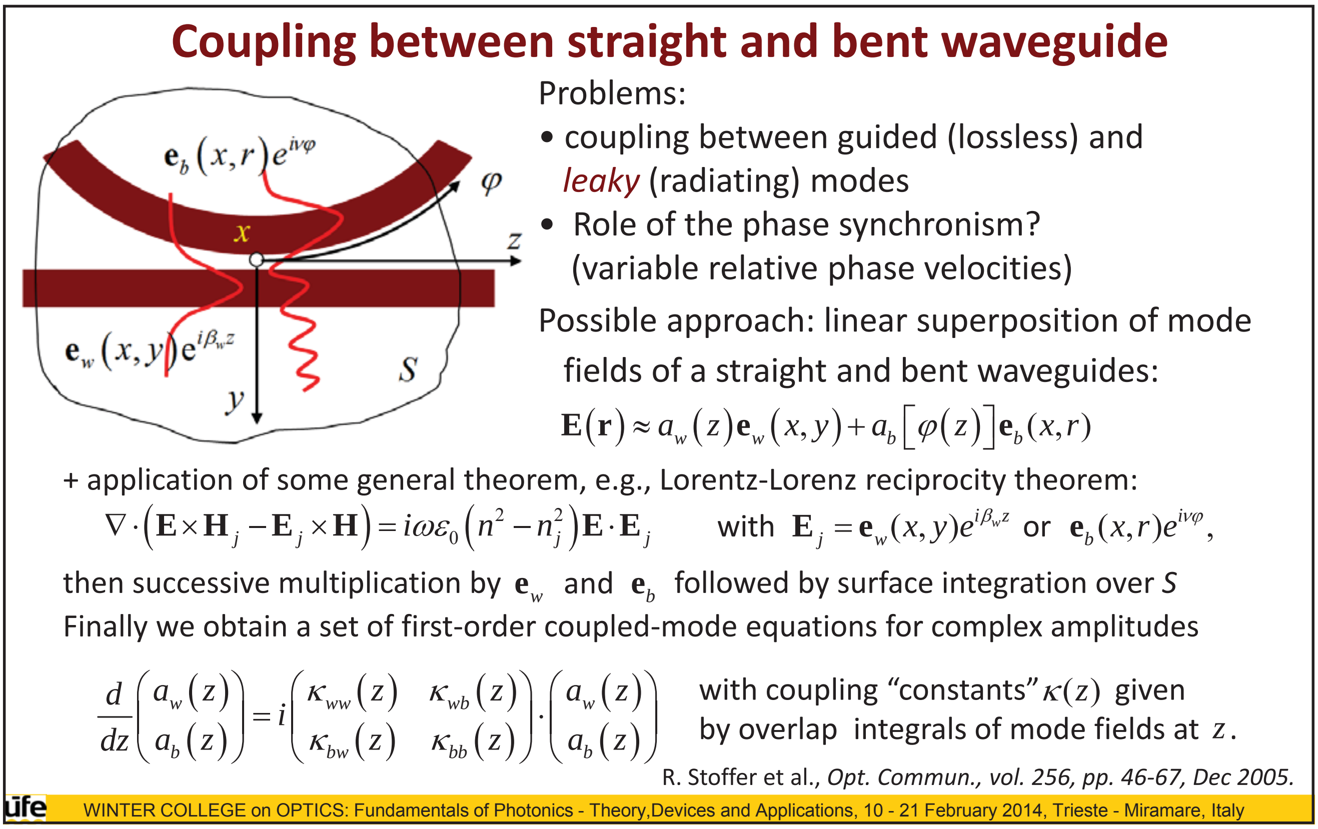
depends on the **group index** N_g

"Finesse"

$F = FSR / \Delta\lambda$

Quality factor

$Q = qF$



Coupling between straight and bent waveguide

Problems:

- coupling between guided (lossless) and *leaky* (radiating) modes
- Role of the phase synchronism? (variable relative phase velocities)

Possible approach: linear superposition of mode fields of a straight and bent waveguides:

$$\mathbf{E}(\mathbf{r}) \approx a_w(z)\mathbf{e}_w(x,y) + a_b\left[\varphi(z)\right]\mathbf{e}_b(x,r)$$

+ application of some general theorem, e.g., Lorentz-Lorenz reciprocity theorem:

$$\nabla\cdot\left(\mathbf{E}\times\mathbf{H}_j - \mathbf{E}_j\times\mathbf{H}\right) = i\omega\varepsilon_0\left(n^2 - n_j^2\right)\mathbf{E}\cdot\mathbf{E}_j$$

with $\mathbf{E}_j = \mathbf{e}_w(x,y)e^{i\beta_w z}$ or $\mathbf{e}_b(x,r)e^{i\nu\varphi}$,

then successive multiplication by $\mathbf{e}_w$ and $\mathbf{e}_b$ followed by surface integration over *S*

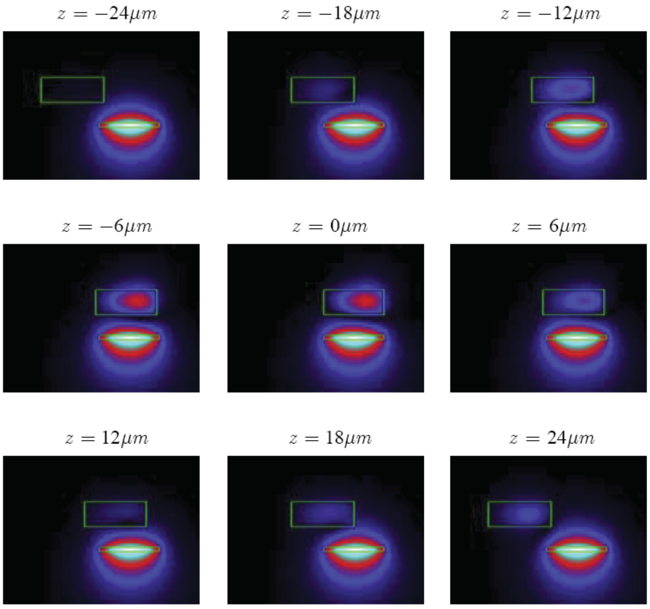
Finally we obtain a set of first-order coupled-mode equations for complex amplitudes

$$\frac{d}{dz}\begin{pmatrix} a_w(z) \\ a_b(z) \end{pmatrix} = i\begin{pmatrix} \kappa_{ww}(z) & \kappa_{wb}(z) \\ \kappa_{bw}(z) & \kappa_{bb}(z) \end{pmatrix}\cdot\begin{pmatrix} a_w(z) \\ a_b(z) \end{pmatrix}$$

with coupling "constants" $\kappa(z)$ given by overlap integrals of mode fields at z.

R. Stoffer et al., *Opt. Commun., vol. 256, pp. 46-67, Dec 2005.*

Some results of the 3D CMT for the coupling of straight and bent waveguides



Cooperation within 6FP “NAIS”,
University of Twente & IPE AS CR

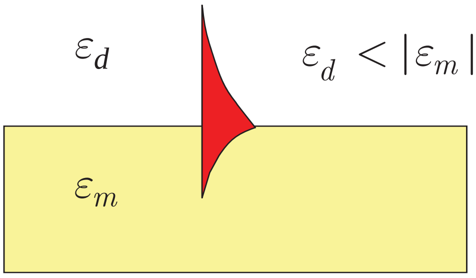
R. Stoffer, K. R. Hiremath, M. Hammer,
L. Prkna, and J. Čtyroký,
"Cylindrical integrated optical microresonators: Modeling by 3-D vectorial coupled mode theory,“ *Optics Communications, vol. 256, pp. 46-67, Dec 2005.*

Surface plasmons in guided-wave optics

Surface plasmon(-polariton)

Mutually coupled electromagnetic and charge wave
localized at the interface between a dielectric and a metal

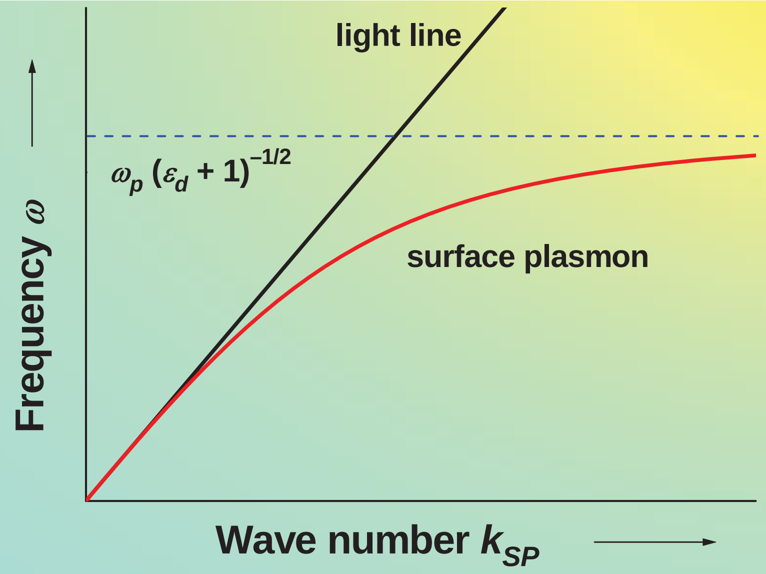
$$\boxed{N_{SP} = \sqrt{\frac{\varepsilon_d \varepsilon_m}{\varepsilon_d + \varepsilon_m}}}$$



$\varepsilon_d < |\varepsilon_m|$

Surface plasmon is a *slow wave*

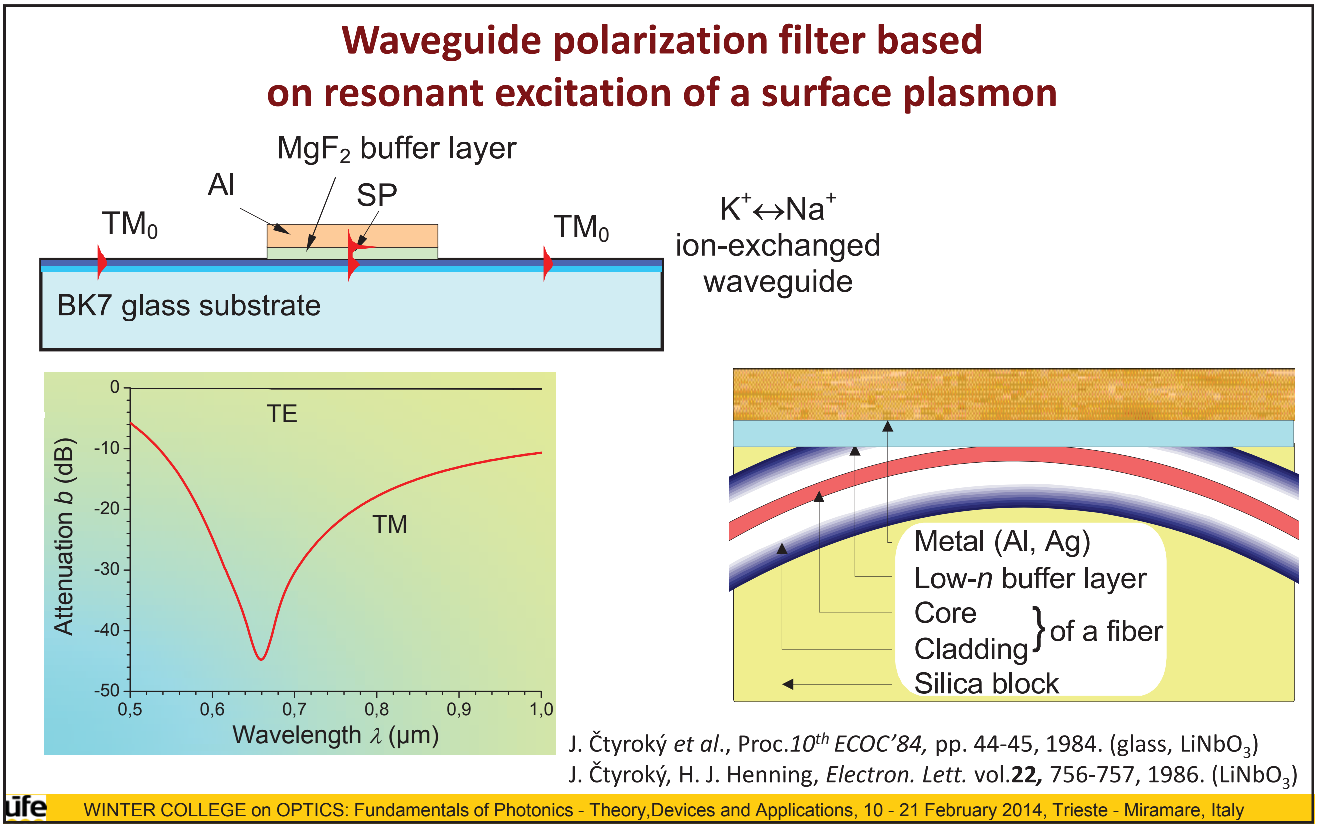
Lossless approximation: $\gamma = 0, \quad \omega < \omega_p / \sqrt{(\varepsilon_d + 1)}$ $\quad k_{SP} = k_0 N_{SP} = \frac{\omega}{c}\sqrt{\frac{\varepsilon_m \varepsilon_d}{\varepsilon_m + \varepsilon_d}} \approx \frac{\omega}{c}\underbrace{\sqrt{\frac{|\varepsilon_m|\varepsilon_d}{|\varepsilon_m| - \varepsilon_d}}}_{\text{factor } >1}$



$\mathrm{Re}\{N_{SP}\} > \sqrt{\varepsilon_d} = n_d$

SP is a **slow wave**

It cannot be excited
from the dielectric!!

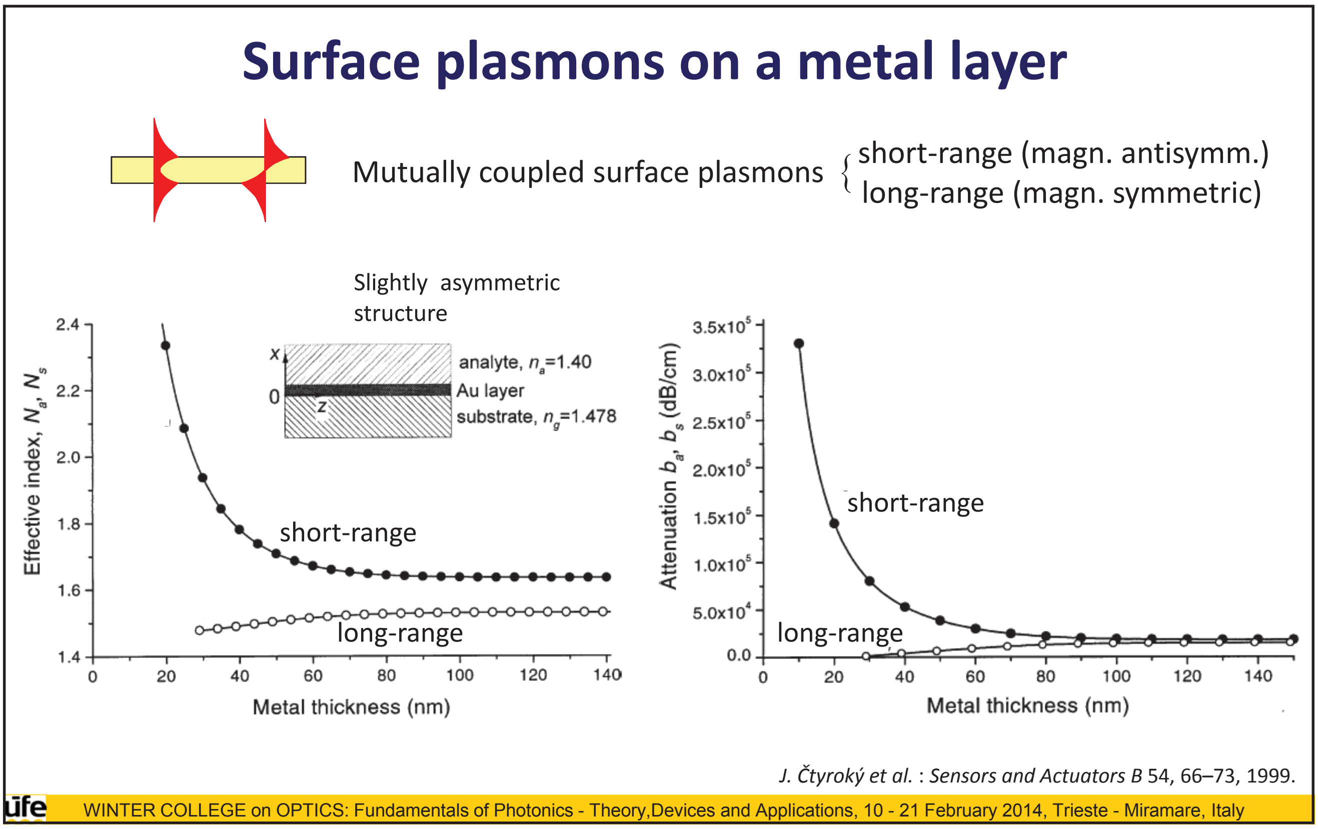


Waveguide polarization filter based on resonant excitation of a surface plasmon

K$^+$↔Na$^+$ ion-exchanged waveguide

Metal (Al, Ag)
Low-*n* buffer layer
Core
Cladding } of a fiber
Silica block

J. Čtyroký *et al*., Proc.*10th ECOC'84,* pp. 44-45, 1984. (glass, LiNbO$_3$)
J. Čtyroký, H. J. Henning, *Electron. Lett.* vol.**22,** 756-757, 1986. (LiNbO$_3$)

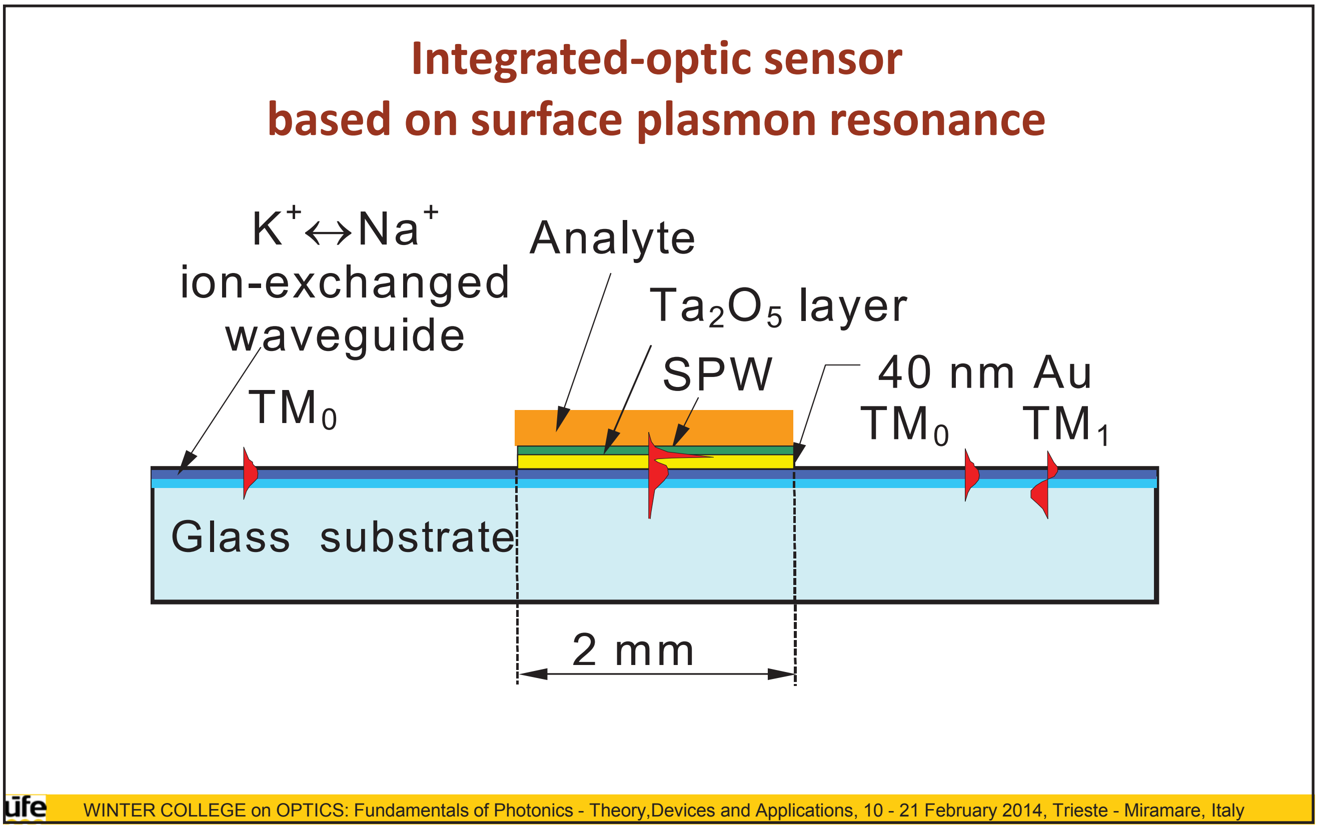


Surface plasmons on a metal layer

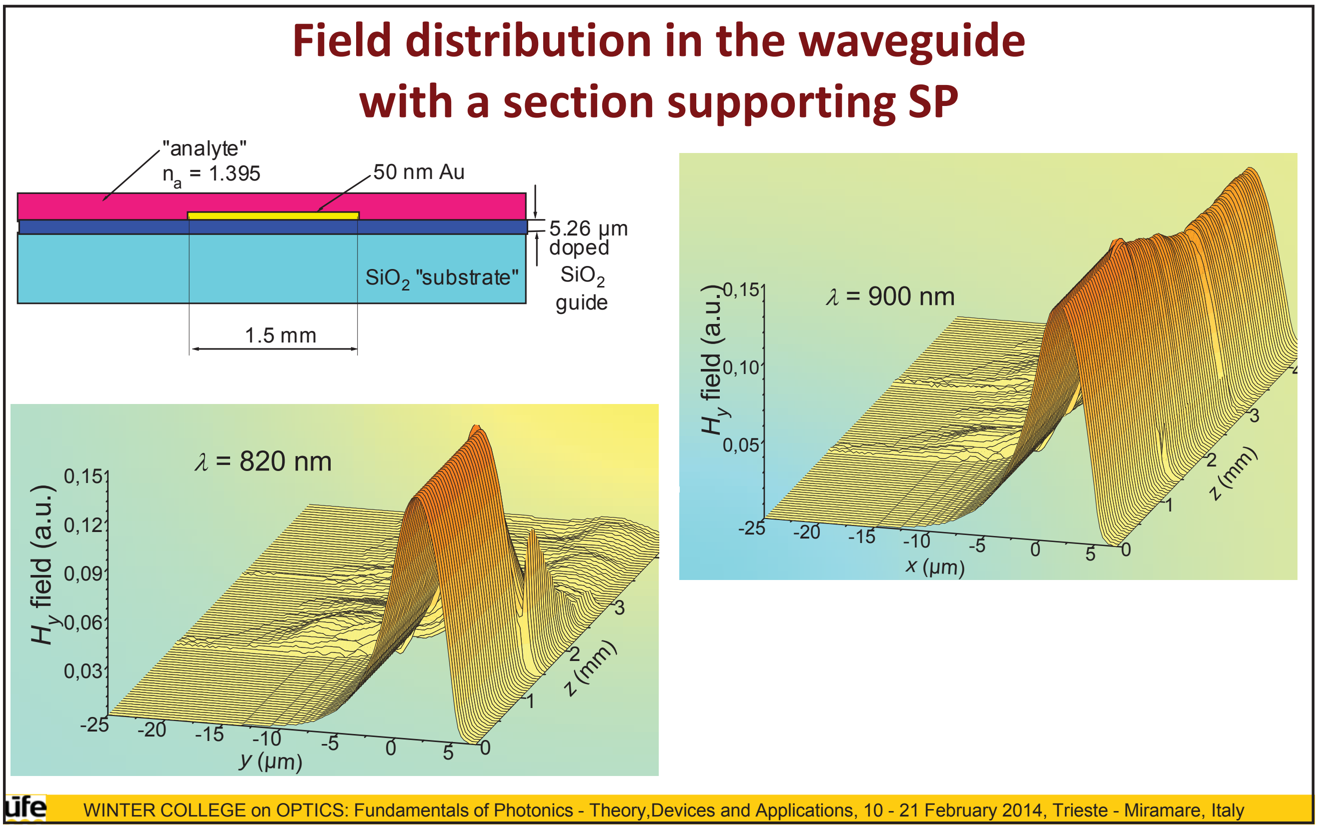
Mutually coupled surface plasmons { short-range (magn. antisymm.)
long-range (magn. symmetric)

Slightly asymmetric structure

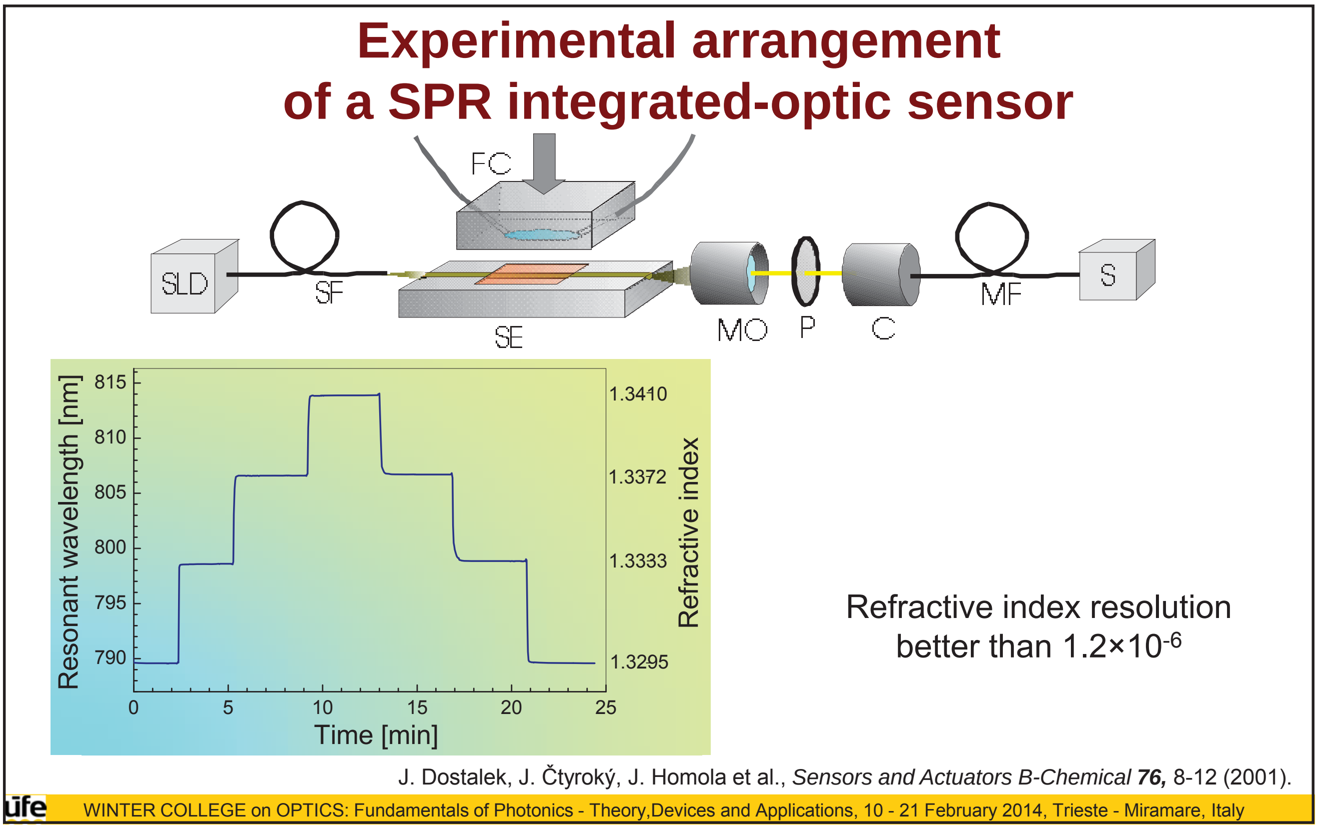
J. Čtyroký et al. : Sensors and Actuators B 54, 66–73, 1999.



Integrated-optic sensor based on surface plasmon resonance



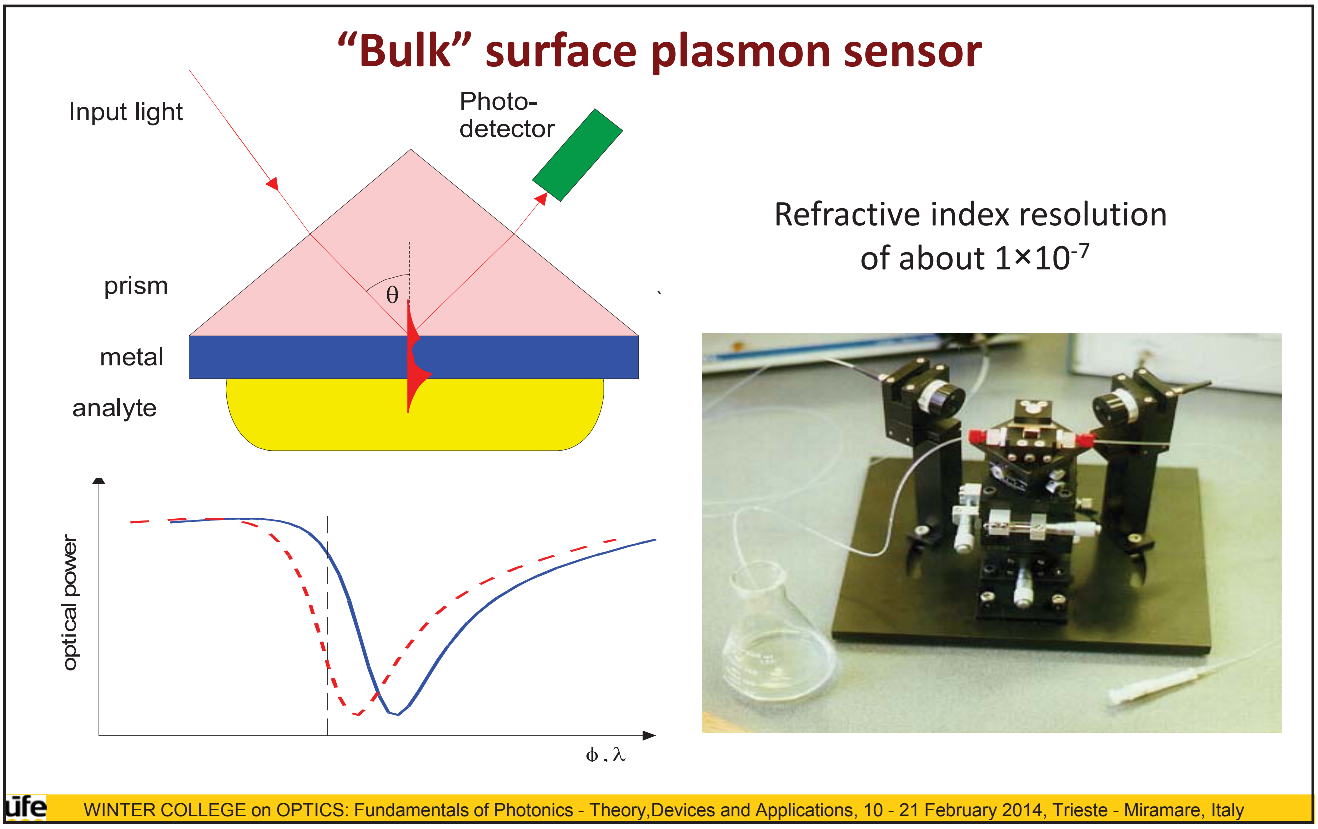
Field distribution in the waveguide with a section supporting SP



Experimental arrangement of a SPR integrated-optic sensor

Refractive index resolution better than 1.2×10^{-6}

J. Dostalek, J. Čtyroký, J. Homola et al., *Sensors and Actuators B-Chemical* ***76,*** 8-12 (2001).



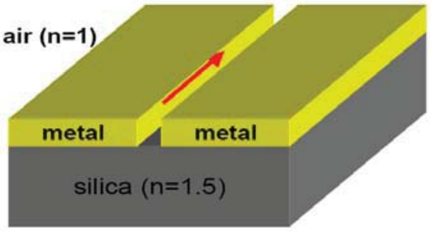
"Bulk" surface plasmon sensor

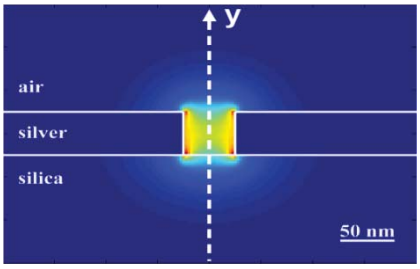
Refractive index resolution of about 1×10^{-7}

“Plasmonics”

(“photonics” using surface plasmons instead of photonics)

2D guiding of surface plasmons

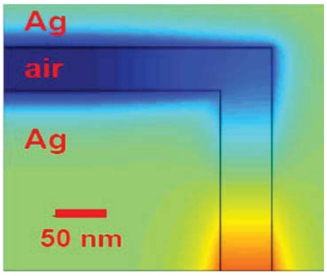




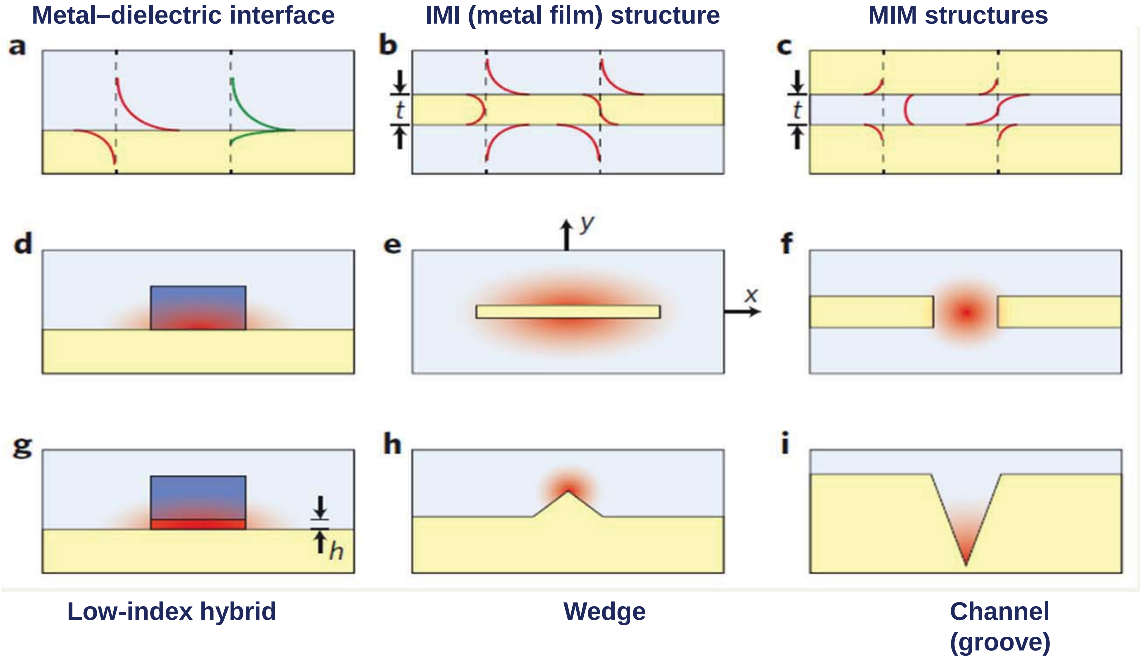
SP enables localization of radiation well below the diffraction limit.

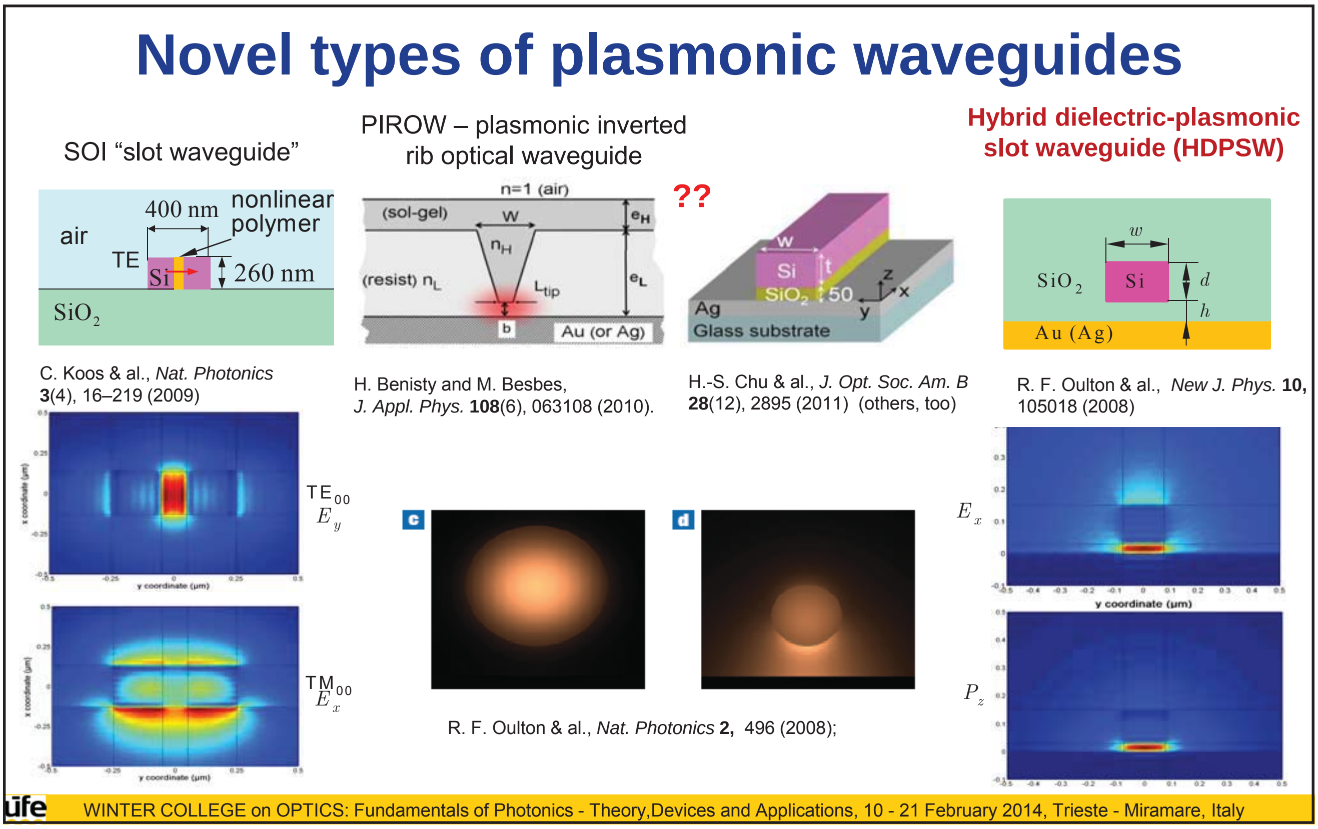
However, strong attenutation due to ohmic loss in metal allows for propagation at very short distances of the orders of 1-100 μm

90° bend



Plasmonic waveguides





Novel types of plasmonic waveguides

SOI "slot waveguide"

PIROW – plasmonic inverted rib optical waveguide

Hybrid dielectric-plasmonic slot waveguide (HDPSW)

C. Koos & al., *Nat. Photonics* **3**(4), 16–219 (2009)

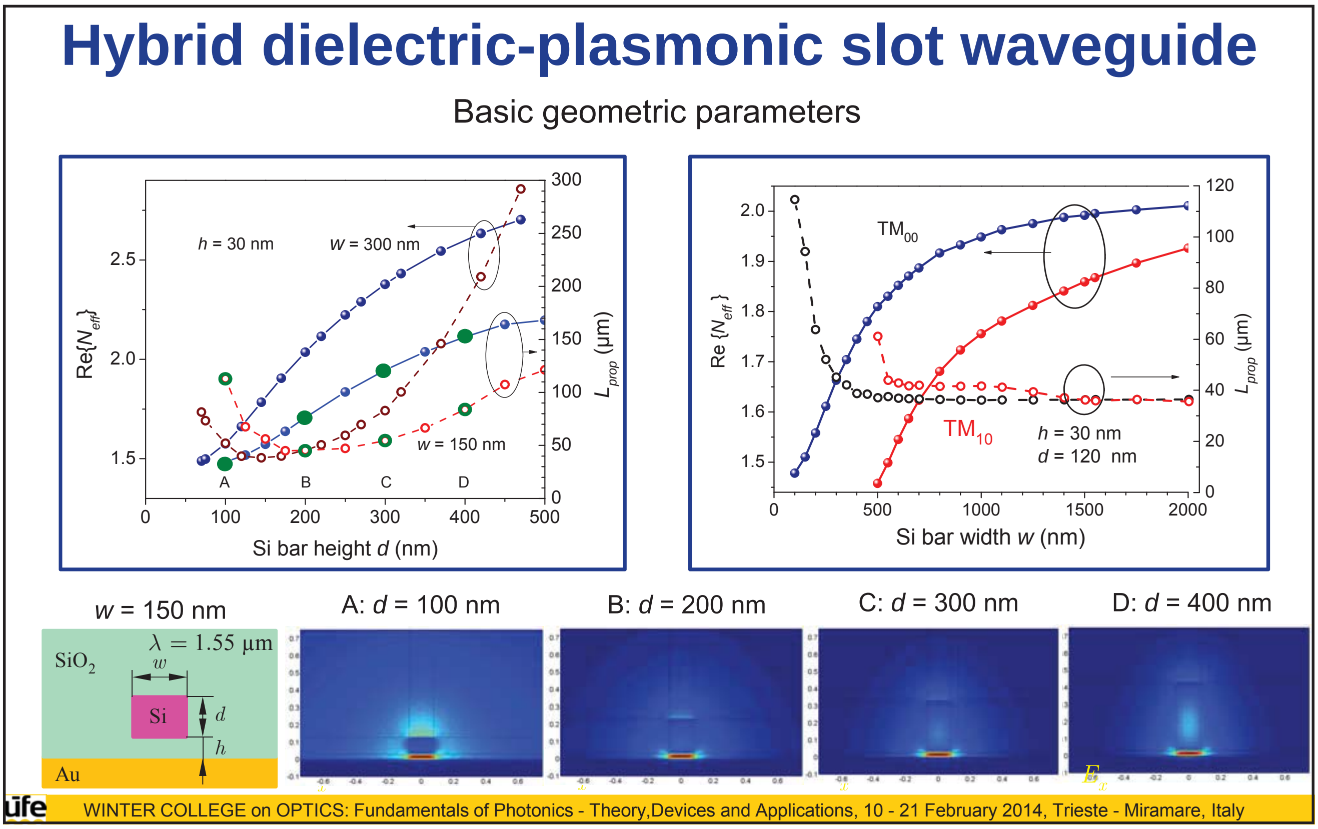
H. Benisty and M. Besbes, *J. Appl. Phys.* **108**(6), 063108 (2010).

H.-S. Chu & al., *J. Opt. Soc. Am. B* **28**(12), 2895 (2011) (others, too)

R. F. Oulton & al., *New J. Phys.* **10,** 105018 (2008)

R. F. Oulton & al., *Nat. Photonics* **2,** 496 (2008);

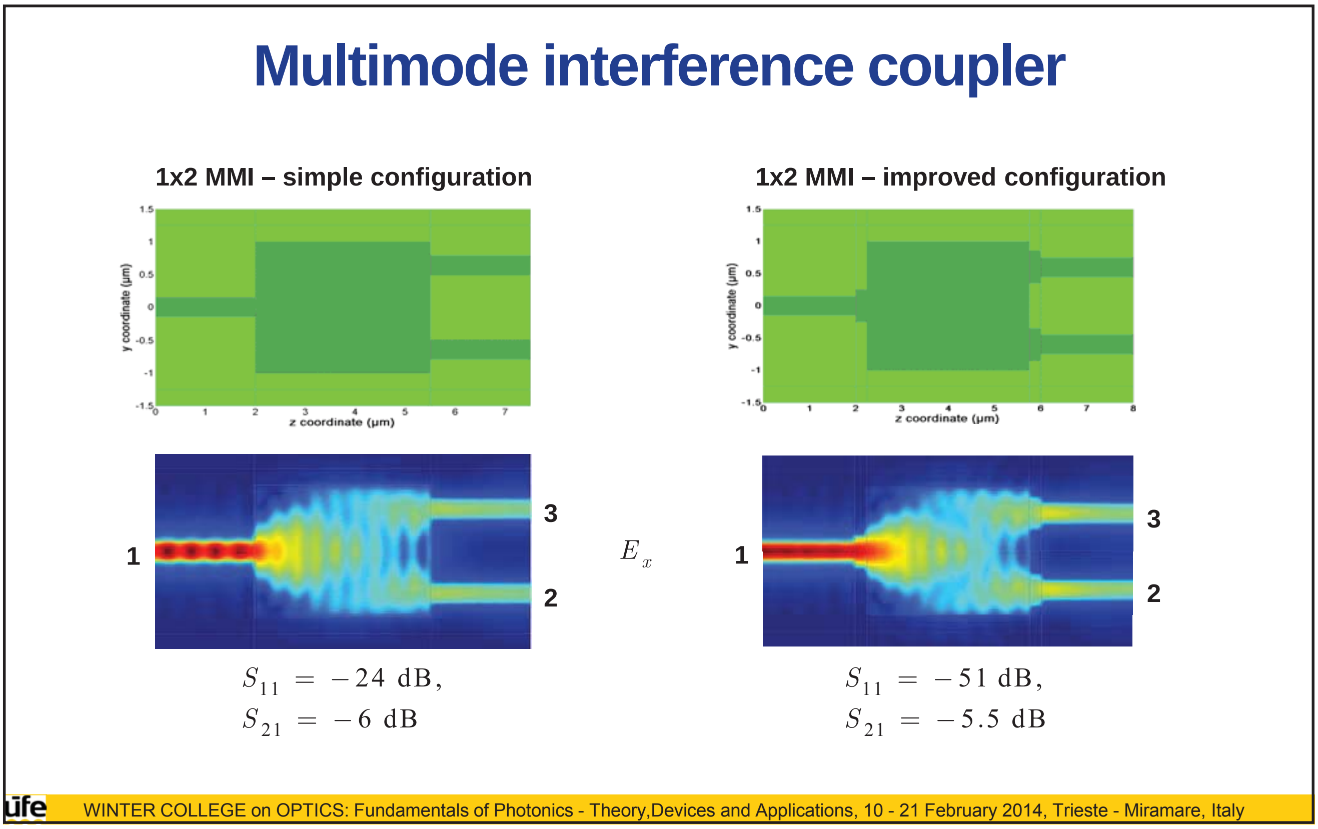
WINTER COLLEGE on OPTICS: Fundamentals of Photonics - Theory,Devices and Applications, 10 - 21 February 2014, Trieste - Miramare, Italy



Hybrid dielectric-plasmonic slot waveguide

Basic geometric parameters

WINTER COLLEGE on OPTICS: Fundamentals of Photonics - Theory,Devices and Applications, 10 - 21 February 2014, Trieste - Miramare, Italy



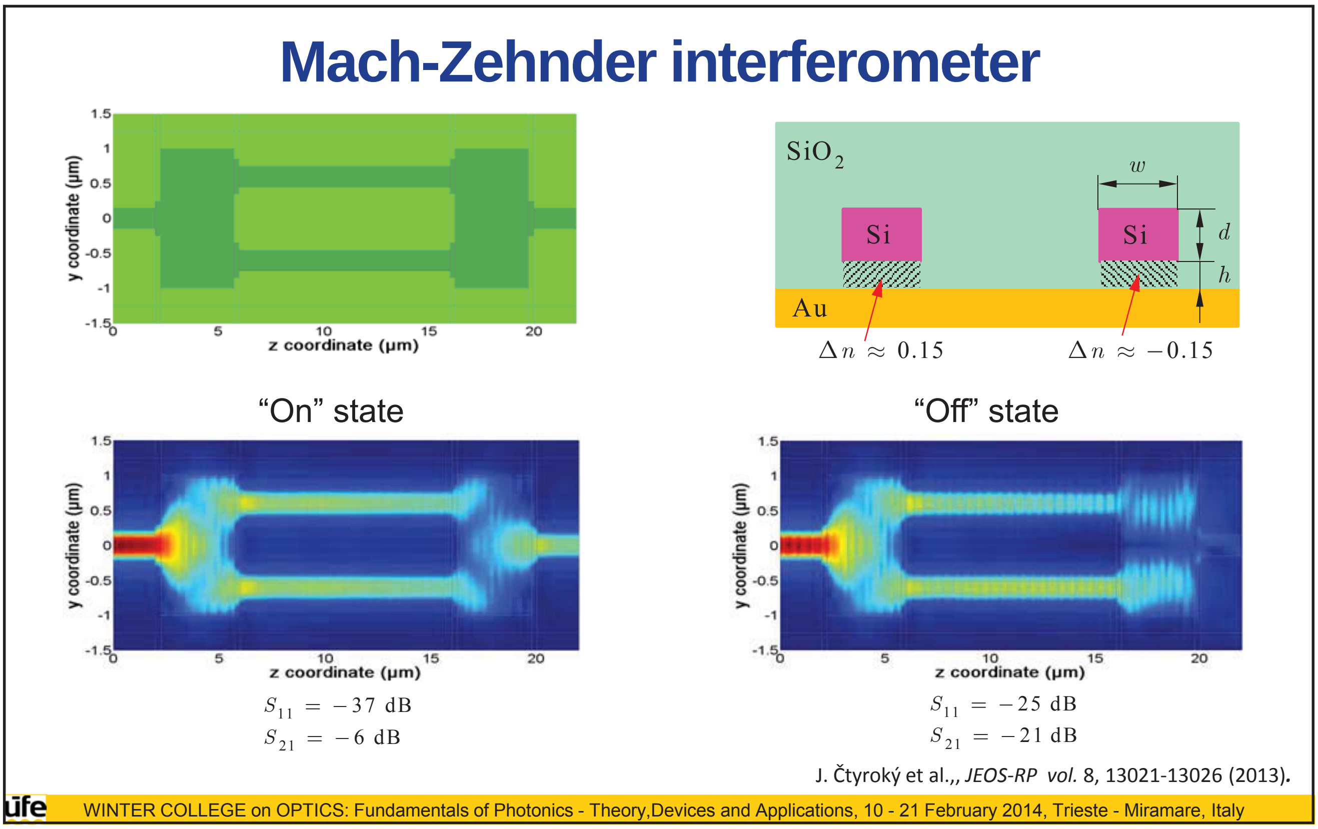
Multimode interference coupler

1x2 MMI – simple configuration

1x2 MMI – improved configuration

$$S_{11} = -24 \text{ dB}, \quad S_{21} = -6 \text{ dB}$$

$$S_{11} = -51 \text{ dB}, \quad S_{21} = -5.5 \text{ dB}$$



Mach-Zehnder interferometer

"On" state

"Off" state

$$S_{11} = -37 \text{ dB}, \quad S_{21} = -6 \text{ dB}$$

$$S_{11} = -25 \text{ dB}, \quad S_{21} = -21 \text{ dB}$$

J. Čtyroký et al.,, *JEOS-RP vol.* 8, 13021-13026 (2013).

Waveguide structures with loss and gain

Asymmetric complex grating-assisted coupler

Basic theory:

L. Poladian, *Phys. Rev. E,* 54, 2963-2975, (1996).
M. Greenberg, M. Orenstein, *Opt. Express*, 12, 4013-4018, 2004.
M. Greenberg, M. Orenstein, *Opt. Lett.*, 29, pp. 451-453, 2004.
M. Greenberg, M. Orenstein, *IEEE JQE*, 41, 1013-1023, 2005.

Application proposals:

M. Greenberg and M. Orenstein, *PTL*, 17, 1450-1452, 2005.
M. Kulishov et al, *Optics Express*, 13, 3567-3578, 2005.

Photonic analogues of quantum-mechanical “*PT*-symmetric” systems

First works:

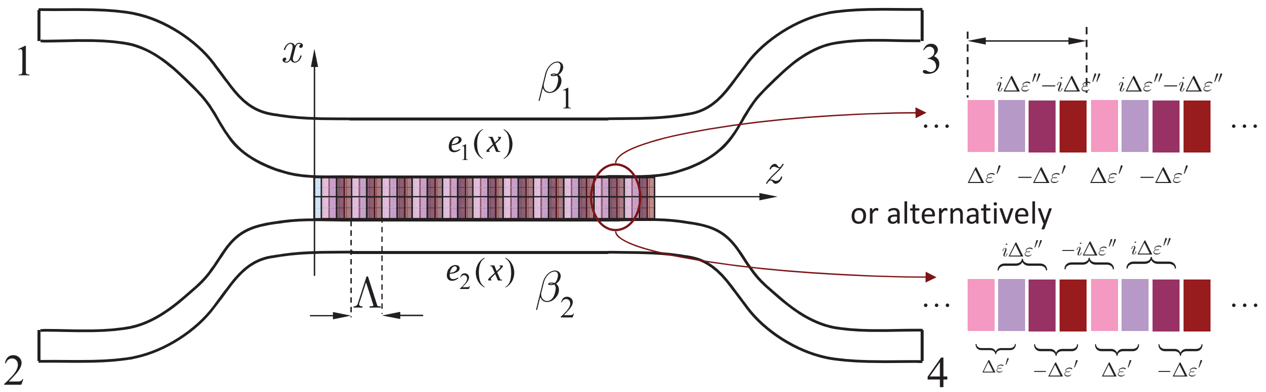
H.-P. Nolting, M. Sztefka, J. Čtyroký, Proc. of IPR, Boston, 76-79, 1996.
G. Guekos, Ed., Photonic Devices for telecommunications, Springer, 1998, pp. 76-78. (“COST 240 Book”)

From the recent avalanche of papers:

R. El-Ganainy et al., Optics Letters, vol. 32, pp. 2632-2634, 2007.
K. G. Makris et al., Phys. Rev. Lett. vol. 100, pp. 103904(1-4), 2008.
J. Čtyroký et al., Optics Express, vol. 18, pp. 21585-21593, 2010.
C. E. Rüter et al., Nature Physics, vol. 6, pp. 192-195, 2010.
H. Benisty et al., Optics Express, vol. 19, pp. 18004-18019, Sep 2011.
A. A. Sukhorukov et al., Optics Letters, vol. 37, pp. 2148-2150, 2012.
J. Čtyroký, Opt. Quantum Electron. vol.46, 465-475, 2014.

…and many others…

Asymmetric complex grating-assisted coupler



Grating-assisted directional coupler using ***asymmetric complex grating***

$$E_y(x,z) \approx A_1(z)e_1(x)\exp(i\beta_1 z) + A_2(z)e_2(x)\exp(i\beta_2 z);$$

complex, periodic in *z*

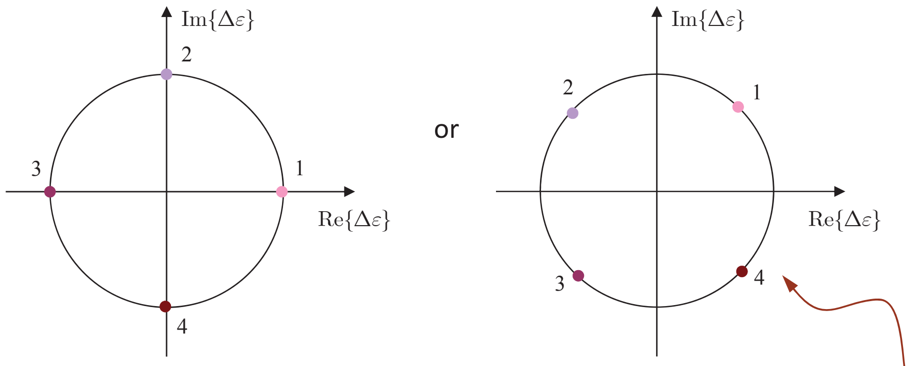
$$\frac{dA_1(z)}{dz} \cong i\kappa_{11}(z)A_1(z) + i\kappa_{12}(z)e^{-i(\beta_1-\beta_2)z}A_2(z), \qquad \kappa_{mn}(z) = \frac{k_0}{2}\iint_S \Delta\varepsilon(x,z)e_m(x)e_n(x)\,dS$$

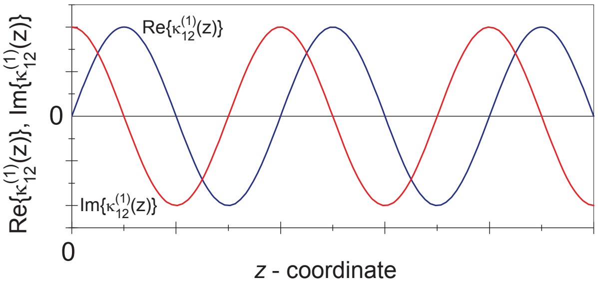
$$\frac{dA_2(z)}{dz} \cong i\kappa_{21}(z)e^{i(\beta_1-\beta_2)z}A_1(z) + i\kappa_{22}(z)A_2(z), \qquad = \kappa_{mn}^{(1)}e^{iKz} + \kappa_{mn}^{(2)}e^{2iKz} + \ldots, \quad K = 2\pi/\Lambda$$

Fourier expansion contains only *positive exponentials* (“SSB modulation”)

Asymmetric complex grating

Complex permittivity perturbation in individual grating segments:





The second option seems technologically simpler since it requires only *two different values* of $\Delta\varepsilon'$ and $\Delta\varepsilon''$

Some more coupled-mode theory…

Let us consider the following ideal case of the grating at synchronism,

$$\kappa_{11}(z)=0,\quad \kappa_{12}(z)=\kappa_{12}^{(1)}\exp(iKz),$$
$$\kappa_{21}(z)=0,\quad \kappa_{22}(z)=0,$$
$$\Delta\beta=K-(\beta_1-\beta_2)=0.$$

Then, the coupled equations read

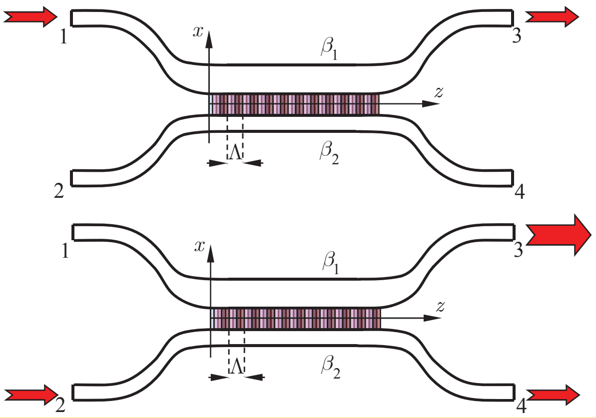
$$\frac{dA_1(z)}{dz}\cong i\kappa_{12}^{(1)}A_2(z),$$
$$\frac{dA_2(z)}{dz}\cong 0.$$

For $A_1(0)\neq 0,\quad A_2(0)=0$
we get the solution

$$A_1(z)=A_1(0)=const.,\qquad P_1(z)=P_1(0),$$
$$A_2(z)=0,\qquad P_2=0.$$

for $A_1(0)=0,\quad A_2(0)\neq 0$

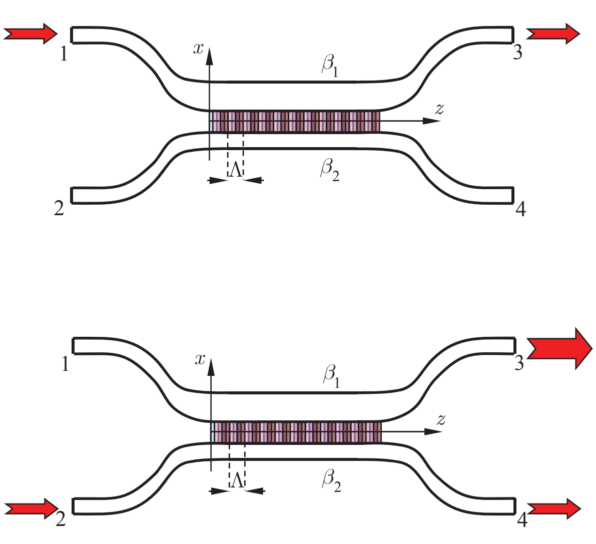
$$A_1(z)=i\kappa_{12}^{(1)}A_2(0)z,\qquad P_1(z)=\left|\kappa_{12}^{(1)}\right|^2 P_2(0)z^2,$$
$$A_2(z)=A_2(0)=const.,\quad P_2(z)=P_2(0)=const.$$



ACGC is a reciprocal device!

Forward propagation

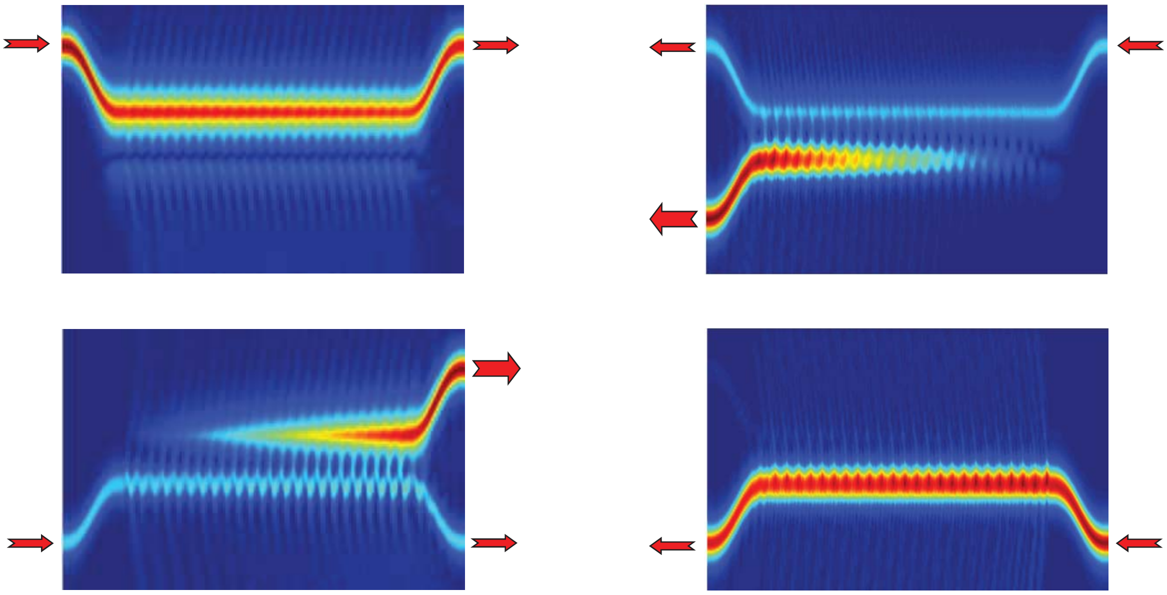
Backward propagation



Numerical modelling (in-house 2D Fourier Modal Method)

Forward propagation

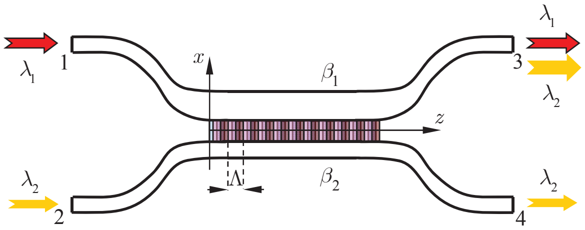
Backward propagation



Straightforward ACGC applications

Wideband ADD multiplexor

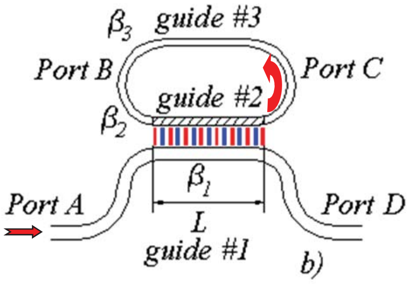
M. Greenberg and M. Orenstein,
PTL **17**, 1450-1452, 2005



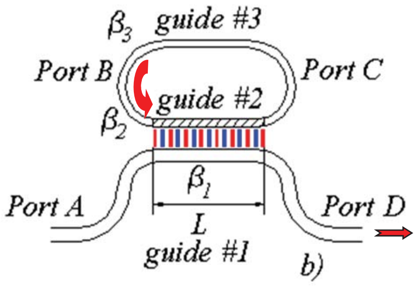
Light trapping in a ring resonator
(a “dynamic memory cell”)

M. Kulishov *et al.*, *OE* **13**, 3567-3578, 2005

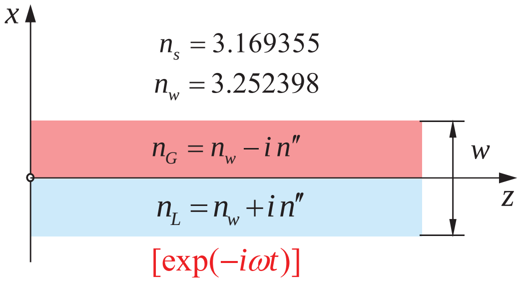
grating “switched on”:



grating “switched off”:



Coupled waveguides with loss & gain: historical remarks

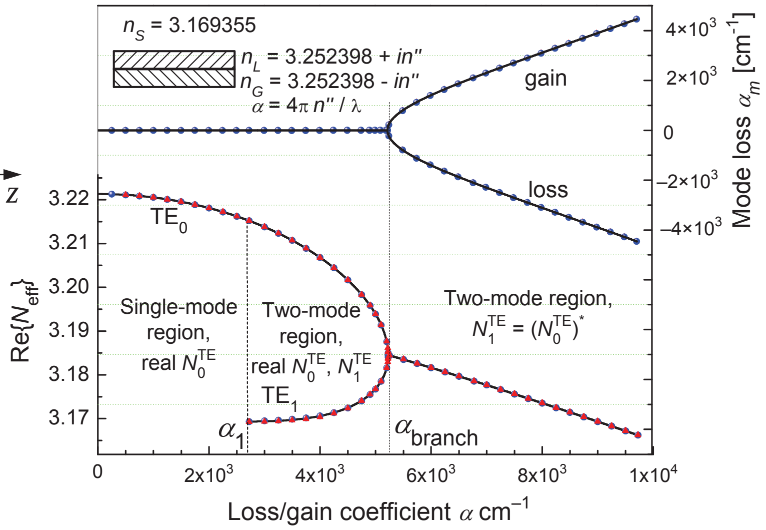


$w = 1\ \mu\text{m},$

$n'' = \dfrac{\lambda}{4\pi}\alpha \times 10^{-4}\ [-;\ \mu\text{m, cm}^{-1}],$

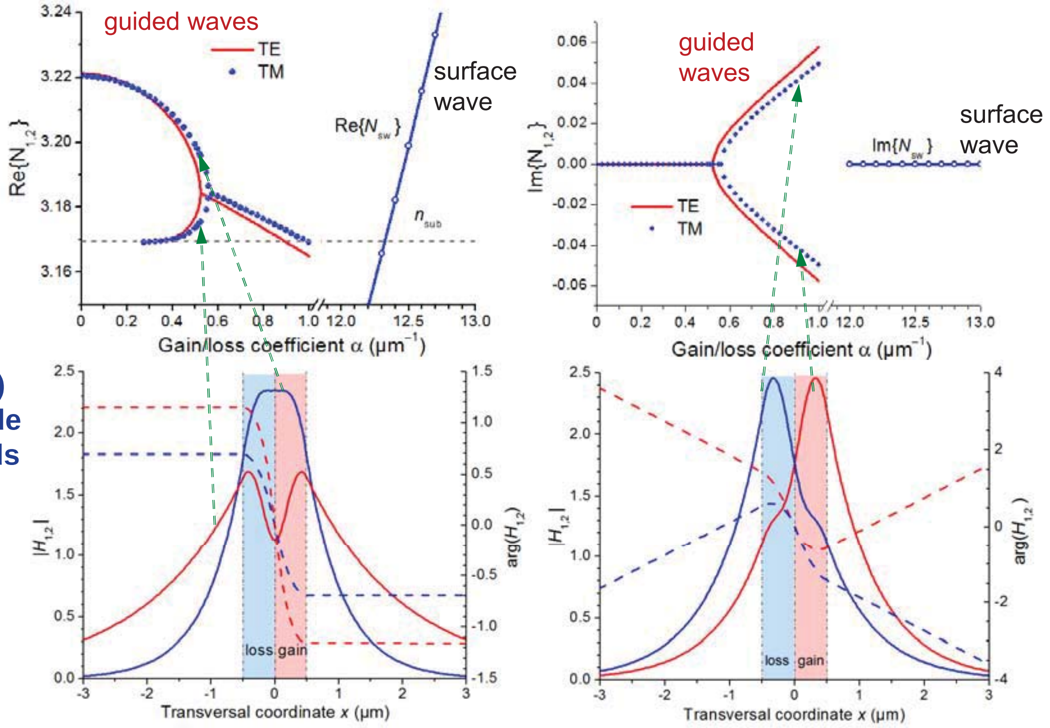
$\lambda = 1.55\ \mu\text{m},$

$\alpha\ldots$ "loss/gain coefficient" [cm^{-1}]



H.-P. Nolting, M. Sztefka, J. Čtyroký, Proc. of IPR, Boston, 76-79, 1996.
G. Guekos, Ed., Photonic Devices for telecommunications, Springer, 1998, pp. 76-78. (“COST 240 Book”)

“Rigorous” 2D analysis (planar waveguides)



(TM) mode fields

Formal analogy between a photonic waveguide and quantum-mechanical potential well

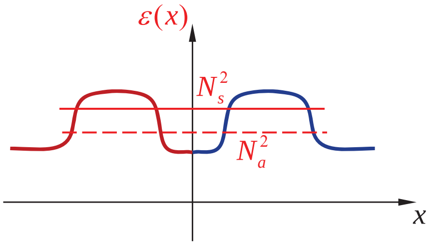
Eigenmode equation for TE modes of a planar waveguide

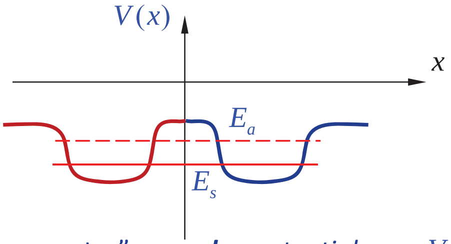
$$\frac{1}{k_0^2}\frac{d^2E(x)}{dx^2}+\varepsilon(x)E(x)=N^2E(x)$$

Schrödinger equation for a particle in a 1D potential well

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2}+V(x)\psi(x)=E\psi(x)$$

mode field distribution	$E(x)$	$\Leftrightarrow$	$\psi(x)$	wave function
wave number	k_0	$\Leftrightarrow$	$\sqrt{2m}/\hbar$	mass; Planck constant
relative permittivity profile	$\varepsilon(x)$	$\Leftrightarrow$	$-V(x)$	potential
effective refractive index	N^2	$\Leftrightarrow$	$-E$	particle energy





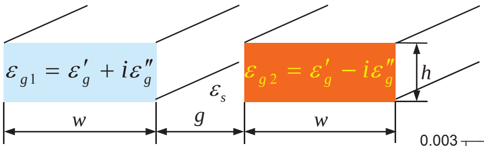
Loss/gain structure: $\varepsilon(-x)=\varepsilon^*(x)$

“*PT* symmetry”: ***complex*** *potential*, $V(-x)=V^*(x)$

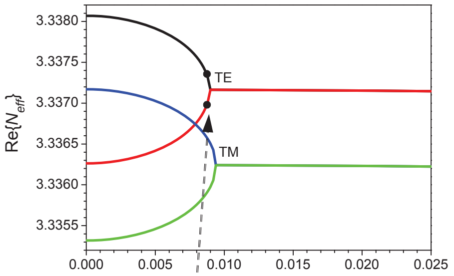
Coupled waveguides with loss/gain

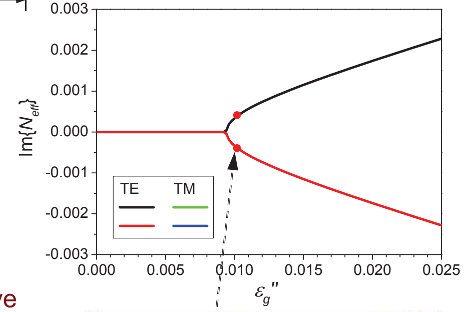
Balanced loss/gain "switching":

$$\varepsilon(-x,y)=\varepsilon^*(x,y)$$

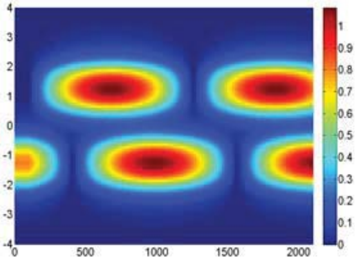


$\varepsilon_s=10.89,\quad \varepsilon_g'=11.56$
$w=1.5\ \mu\text{m},\quad h=0.75\ \mu\text{m},$
$g=1\ \mu\text{m},\quad \lambda=1.55\ \mu\text{m}.$





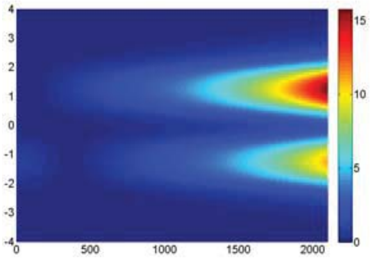
Below exceptional point



Above exceptional point

gain channel

loss channel

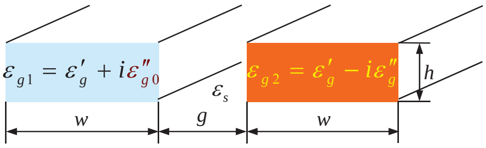


C. E. Rüter *et al.* "Observation of parity–time symmetry in optics," *Nature Physics*, vol. 6, pp. 192-195, 2010.

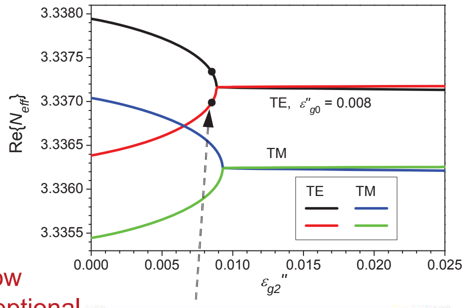
Coupled waveguides with loss/gain

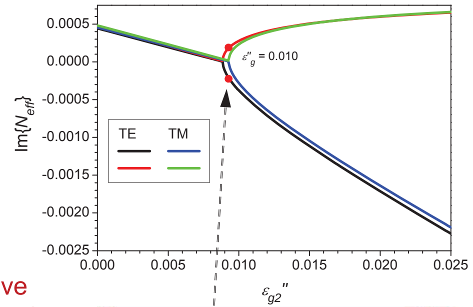
Fixed loss/variable gain switching:

$$\varepsilon(-x,y)\neq\varepsilon^*(x,y)$$

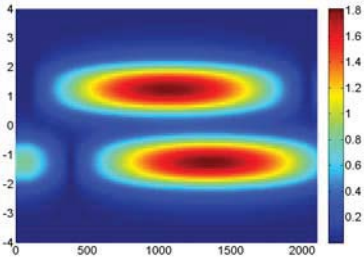


$\varepsilon_s=10.89,\quad \varepsilon_g'=11.56$
$w=1.5\ \mu\text{m},\quad h=0.75\ \mu\text{m},$
$g=1\ \mu\text{m},\quad \lambda=1.55\ \mu\text{m}.$





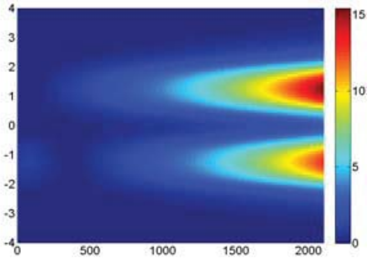
Below exceptional point

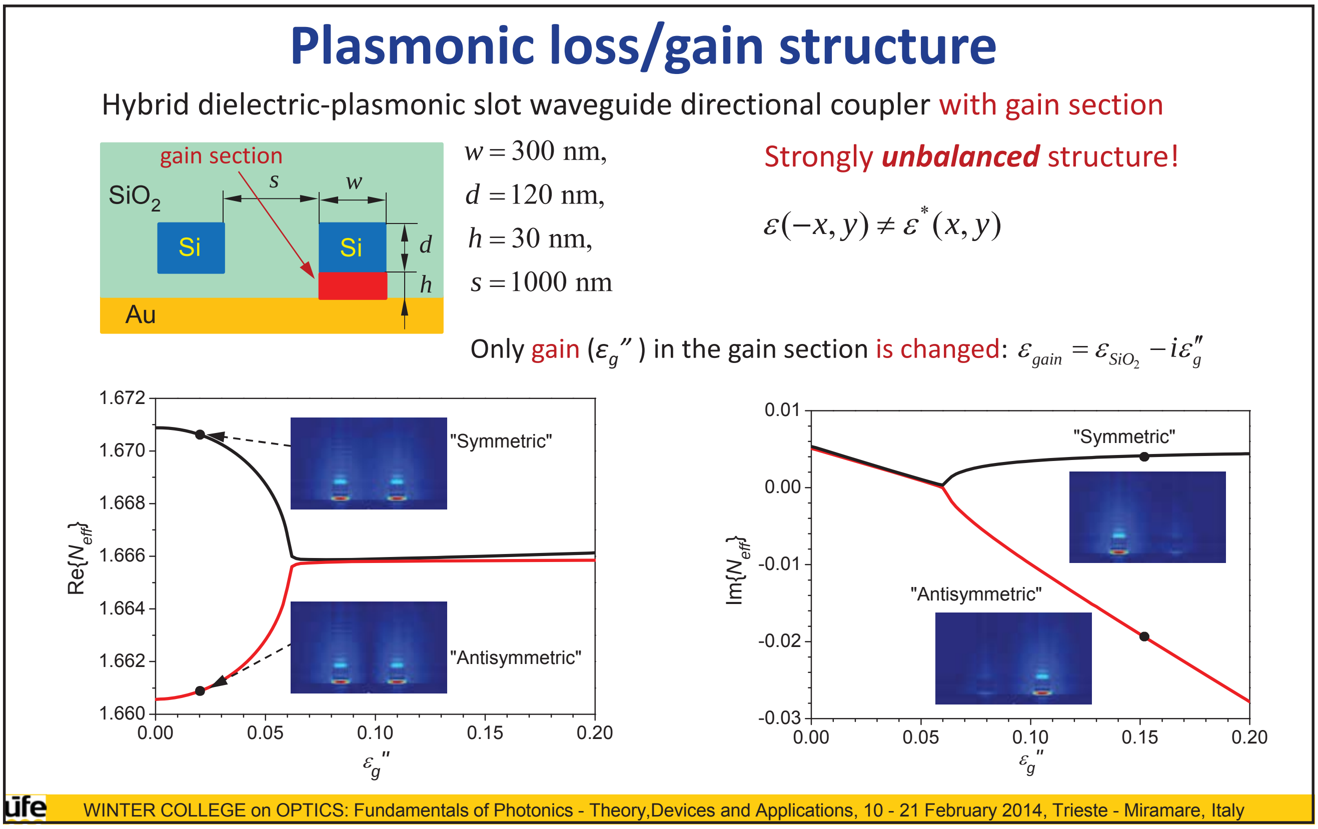


Above exceptional point

gain channel

loss channel





Plasmonic loss/gain structure

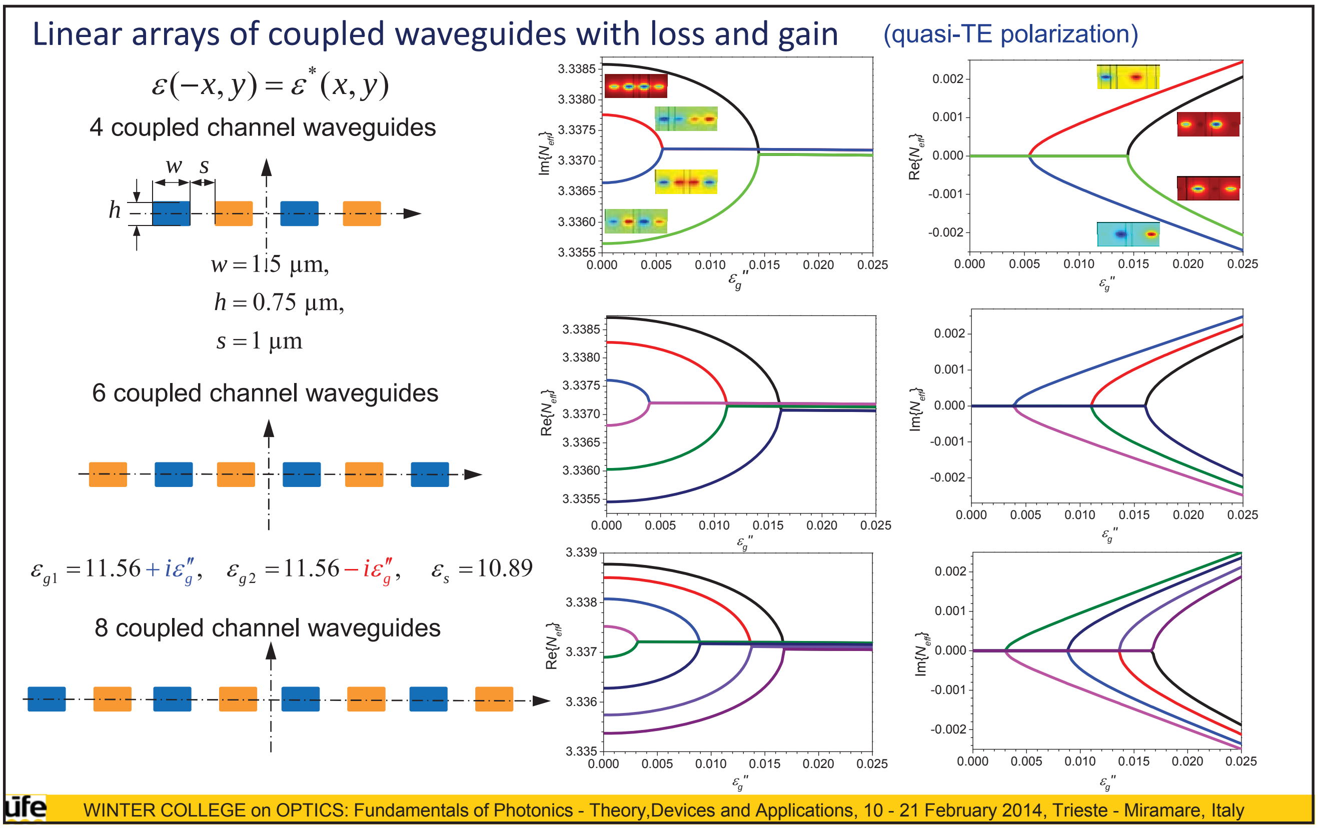
Hybrid dielectric-plasmonic slot waveguide directional coupler with gain section

$w = 300$ nm,
$d = 120$ nm,
$h = 30$ nm,
$s = 1000$ nm

Strongly ***unbalanced*** structure!

$$\varepsilon(-x, y) \neq \varepsilon^*(x, y)$$

Only gain (ε_g'') in the gain section is changed: $\varepsilon_{gain} = \varepsilon_{SiO_2} - i\varepsilon_g''$



Linear arrays of coupled waveguides with loss and gain

(quasi-TE polarization)

$$\varepsilon(-x, y) = \varepsilon^*(x, y)$$

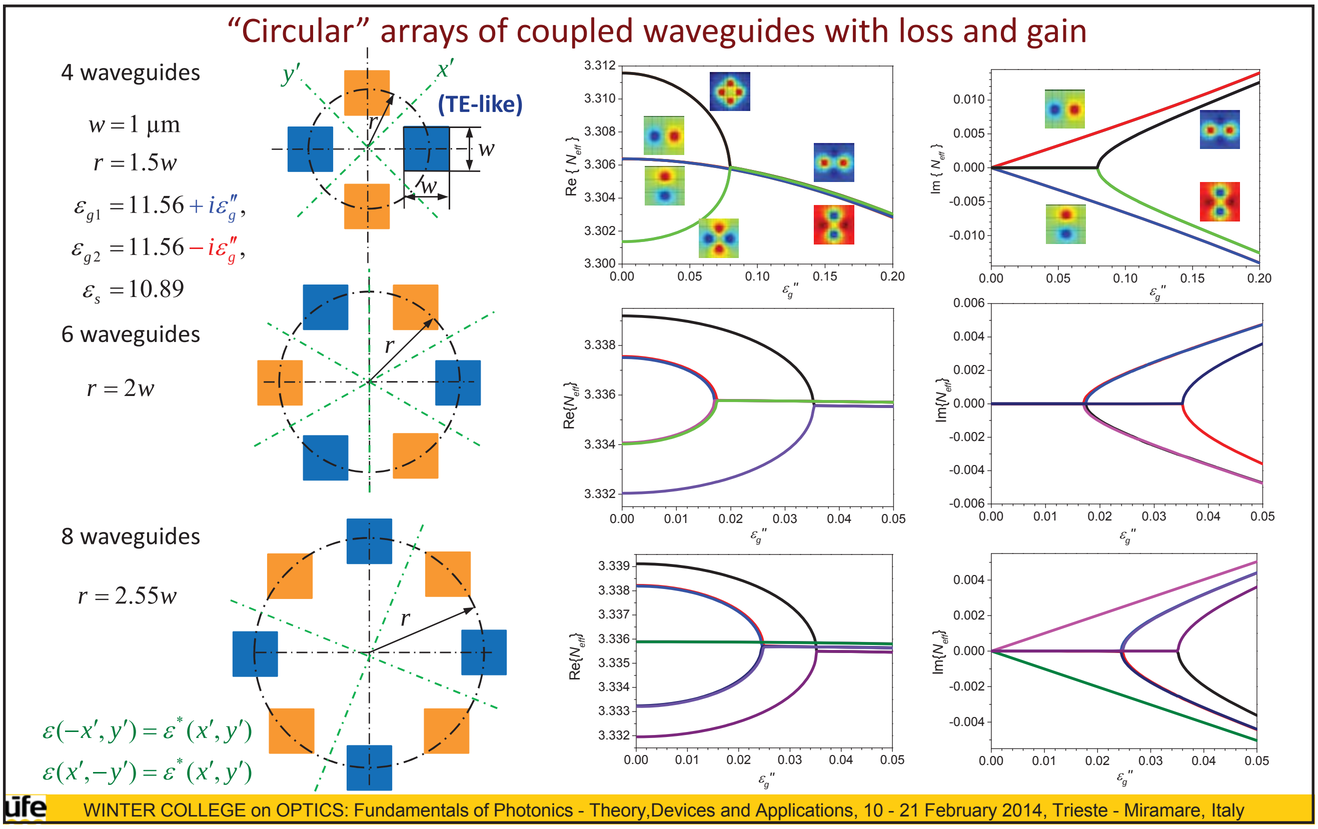
4 coupled channel waveguides

$w = 1.5$ µm,
$h = 0.75$ µm,
$s = 1$ µm

6 coupled channel waveguides

$$\varepsilon_{g1} = 11.56 + i\varepsilon_g'', \quad \varepsilon_{g2} = 11.56 - i\varepsilon_g'', \quad \varepsilon_s = 10.89$$

8 coupled channel waveguides



“Circular” arrays of coupled waveguides with loss and gain

4 waveguides

$w = 1\ \mu\text{m}$

$r = 1.5w$

$\varepsilon_{g1} = 11.56 + i\varepsilon_g''$,

$\varepsilon_{g2} = 11.56 - i\varepsilon_g''$,

$\varepsilon_s = 10.89$

6 waveguides

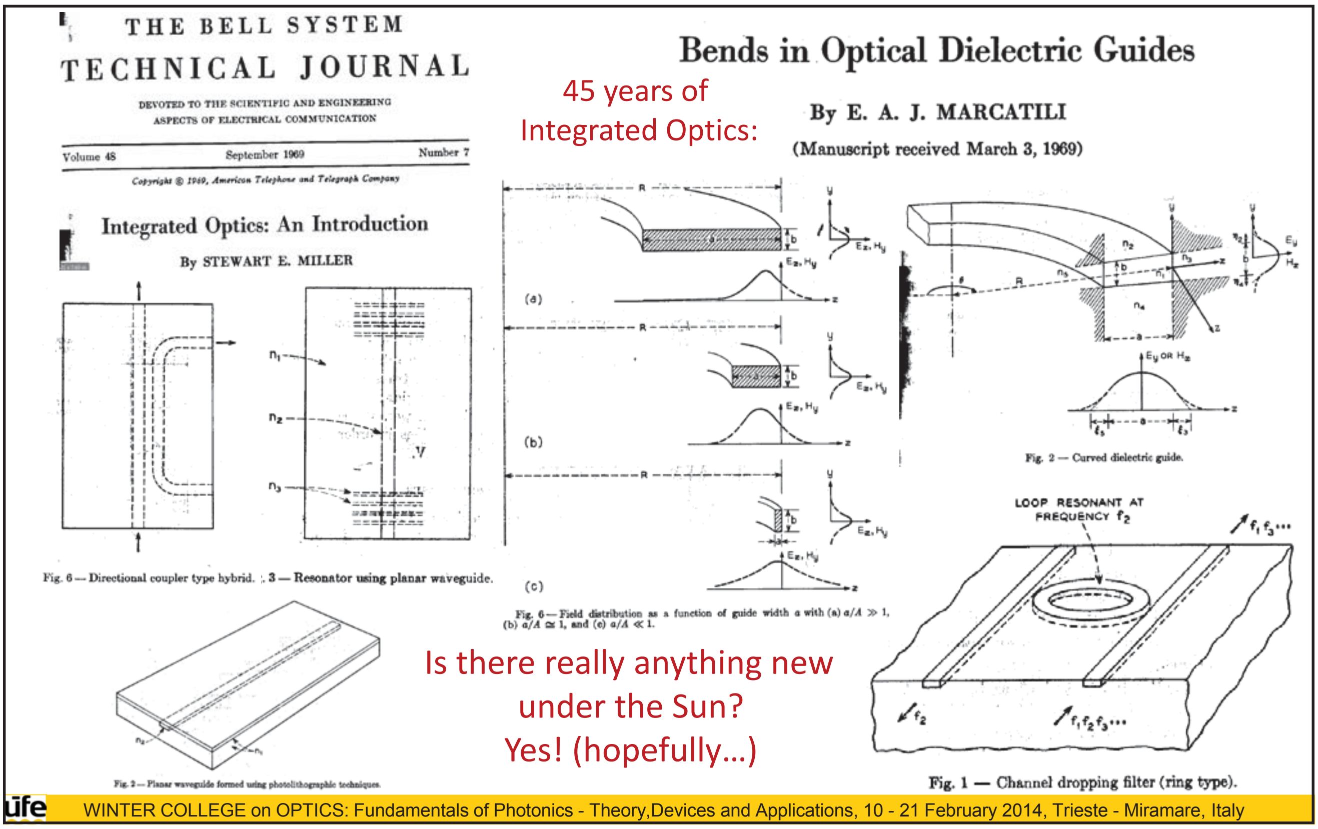
$r = 2w$

8 waveguides

$r = 2.55w$

$\varepsilon(-x', y') = \varepsilon^*(x', y')$

$\varepsilon(x', -y') = \varepsilon^*(x', y')$



THE BELL SYSTEM

TECHNICAL JOURNAL

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Volume 48 September 1969 Number 7

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Integrated Optics: An Introduction

By STEWART E. MILLER

Fig. 6 — Directional coupler type hybrid. ; 3 — Resonator using planar waveguide.

Fig. 2 — Planar waveguide formed using photolithographic techniques.

Bends in Optical Dielectric Guides

By E. A. J. MARCATILI

(Manuscript received March 3, 1969)

45 years of Integrated Optics:

Fig. 6 — Field distribution as a function of guide width a with (a) a/A ≫ 1, (b) a/A ≃ 1, and (c) a/A ≪ 1.

Fig. 2 — Curved dielectric guide.

Fig. 1 — Channel dropping filter (ring type).

Is there really anything new under the Sun?
Yes! (hopefully…)