GDIS 2014 – the second week

Speaker	Title and abstract
Simonetta Abenda	**Grassmannians and multi-soliton solutions to KP-II: an algebraic geometric approach** In this talk I shall present some results in collaboration with P. G. Grinevich (Moscow, Russia) An (N,M)-soliton solution to KP-II is a real bounded regular solution u(x;y;t) which has M line soliton solutions in asymptotics in the (x,y)-plane whose directions are invariant w.r.t. to t. These solutions are defined as a torus orbit on the Grassmannian manifold Gr(N,M). They may be classified in terms of the matroid strata in the totally non-negative part of Gr(N,M) and to each such point there is associated a real and totally non-negative matrix A in reduced echelon form. In this talk we address the classication problem of such multi-solitonic solutions from another point of view: we associate to a generic point in the totally non negative part of Gr(N,M) a compatible set of (N+1) divisors sitting on a m-curve (perturbation of the rational curve associated to the multi-solitonic solution) and give an explicit representation of the matrix A in terms of such system of divisors.
Sergey Agafonov	**On symmetries of singular implicit ODEs** We study implicit ODEs, cubic in derivative, with infinitesimal symmetry at singular points. Cartan showed that even at regular points the existence of nontrivial symmetry imposes restrictions on the ODE. Namely, this algebra has the maximal possible dimension 3 iff the web of solutions is flat. For cubic ODEs with flat 3-web of solutions we establish sufficient conditions for the existence of non-trivial symmetries at singular points and show that under natural assumptions such a symmetry is semi-simple, i.e. is a scaling is some coordinates. We use this symmetry to find first integrals of the ODE.
Alain Albouy	**The Neumann potential on the sphere and the geodesics on an ellipsoid** Knoerrer discovered in 1981 that the Gauss map sends any geodesic on an ellipsoid onto a trajectory of a quadratic (Neumann's) potential on the unit sphere, after a change of time. We will examine this result and discuss the following points. 1) Knoerrer's change of time is associated through projective dynamics to the normalization of the normal vector of the ellipsoid. 2) Suppose one does not know that Neumann potential is integrable. One could think of the Gauss map as a way to discover its first integrals. This order of presentation does not work as we cannot establish the property of the Gauss map without at the same time discovering a first integral. 3) Looking for a proof of Knoerrer's result is one of the fastest way to discover the first integrals of both problems.

Ivan Bizyaev	**On the nonintegrability and obstructions to the Hamiltonization of the nonholonomic Chaplygin top** This work continues a cycle of investigations of the problem of an inhomogeneous ball having a displaced center of mass and rolling without slipping and spinning. We consider in detail a particular case of the system studied in [1], where one of the quadratic integrals reduces to the linear one. It is shown that this system has no invariant measure, as before. However, the flow of the system is a superposition of two conformally Hamiltonian vector fields. [1] A. V. Bolsinov, A. V. Borisov, I. S. Mamaev, Rolling of a Ball without Spinning on a Plane: the Absence of an Invariant Measure in a System with a Complete Set of Integrals. Regul. Chaotic Dyn., 2012, 17, 6, 571–579;
Sergey Bolotin	**Billiards in the three body problem** We consider the plane three body problem with two of the masses much smaller than the third one. Periodic solutions with near collisions of the small bodies were named by Poincar\'e second species periodic solutions. The description of such solutions is reduced to a billiard type system with discrete Lagrangian determined by the classical Lambert's problem. In the limit of many revolutions between near collisions the billiard system admits relatively simple description.
Alexey Borisov & Ivan Mamaev	**Topological methods in nonholonomic mechanics** We consider systems that can be represented in Hamiltonian form, which can turn out to be a priori nonobvious. The resulting Poisson structures may possess some unexpected properties (in particular, symplectic leaves may be chaotically embedded in the phase space). It is shown how various geometric methods can be used for a qualitative analysis of such systems.
Alexey Borisov & Ivan Mamaev	**Superintegrable systems on a plane and sphere** Here we consider superintegrable systems which are an immediate generalization of the Kepler and Hook problems, both in two-dimensional spaces — the plane R^2 and the sphere S^2 — and in three-dimensional spaces R^3 and S^3. Using the central projection and the reduction procedure proposed in [1], we show an interrelation between the superintegrable systems found previously and show new ones. In all cases the superintegrals are presented in explicit form. [1] A. V. Borisov, A. A. Kilin, I. S. Mamaev. Superintegrable system on a sphere with the integral of higher degree. Regul. Chaotic Dyn., 2009, vol. 14, no. 6, p. 615–620.

Monique Chyba	**The Submerged Rigid Body: Application to Guidance and Navigation for Autonomous Underwater Vehicles** Recent advances in robotics, control theory, portable energy sources and automation increase our ability to create more intelligent robots, and allows us to conduct more explorations by use of autonomous vehicles. This facilitates access to higher risk areas, longer time underwater, and more efficient exploration as compared to human occupied vehicles. The use of underwater vehicles is expanding in every area of ocean science. Such vehicles are used by oceanographers, archaeologists, geologists, ocean engineers, and many others. These vehicles are designed to be agile, versatile and robust, and thus, their usage has gone from novelty to necessity for any ocean expedition. Formally, Autonomous Underwater Vehicles (AUV) are characterized by a Lagrangian of the form kinetic energy minus potential energy. This is commonly referred to as the class of simple mechanical control systems. Theoretically, an AUV is represented by a complex, non-linear, dynamic system of equations to model and control. A practical concern for AUV implementation is under-actuation. Some vehicles are designed to operate in an under-actuated condition, while others fully-actuated vessels need to be prepared to deal with actuator failure(s) resulting from any number of mechanical issues. One approach to control strategy design for under-actuated vehicles is by use of kinematic reductions. Therefore, the mathematical framework of geometric control theory is well-suited to take into account the possibility of loosing some degrees of freedom due to actuator failures. In this lecture, control strategies for under-actuated AUVs and results of their implementation onto a test-bed vehicle will be presented.
Yassir Dinar	**Algebraic Frobenius manifolds, classical W-algebras and semiuniversal deformation of simple singularties** We obtain algebraic Frobenius manifolds from classical W-algebras associated to subregular nilpotent orbits in simple Lie algebras of type D_r (where r is even) and E_r. The resulting Frobenius manifolds can be realized as certain subvarieties in the total spaces of semiuniversal deformation of simple hypersurface singularities of the same types. These examples support conjecture due to Dubrovin which relates algebraic Frobenius manifolds to primitive conjugacy classes in Coxeter groups.
Vladimir Dragovic, Borislav Gajic	**Applications of the Kowalevski-Painleve method to the motion of a rigid body in an ideal fluid** We will present the Kowalevski-Painleve analysis in the diagonal case of the Kirchhoff equations of motion of a rigid body in an ideal incompressible fluid. In the nonsymmetric and not positive-definite case, a class of systems with all integer Kowalevski exponents is found. In the symmetric case, search for the systems with the general meromorphic solutions can be reduced to a finite number of families of systems. This is work in progress.

Holger Dullin	**Two Integrable Systems related to the Euler fluid equations on a rotating sphere** The Euler fluid equations on a rotating sphere can be projected to spherical harmonics with a Lie-Poisson structure of SU(N) using a construction due to Zeitlin. We will show that for N=3 and N=4 this gives integrable systems on the sphere of dimension N^2-1. The systems are super-integrable, can be integrated using representation theory, and have torus actions with a moment map whose image is a convex polytope.
Yuri Fedorov	**Generic hyperelliptic Prym varieties and separation of variables in some algebraic integrable systems** Many algebraic integrable systems possess matrix Lax representations whose spectral curves admit involutions. The Jacobians of the curves contain Abelian (Prym) subvarieties whose open subsets are identified with the complex invariant manifolds of the systems. It is known that if the spectral curve S is a 2-fold covering of a hyperelliptic curve ramified at two points, then the Jacobain of S contains a hyperelliptic Prym variety. Recently this variety was given an explicit algebraic description. We apply it to find separation of variables in a class of systems which include the integrable Henon--Heiles systems with non-polynomial potentials and the integrable Somos 6 map. The talk will be presented in a self-contained form, all the necessary notions will be explained.
Tatiana Ivanova	**Dynamics and Control of a Spherical Robot with an Axisymmetric Pendulum Actuator** This research is a continuation of analysis of the dynamics of a spherical shell rolling without slipping on a horizontal plane with Lagrange’s top fixed at the center of the shell [1,2]. In [2] the free motion of the ball with Lagrange’s top was considered, the stability of periodic solutions was analyzed and the trajectories of the contact point were constructed. This paper investigates the controlled motion of a ball with a pendulum. Special attention is paid to the control of the ball using gaits — the simplest motions (such as acceleration to a certain velocity and deceleration in a straight line and rotation through a given angle) and their combination for one oscillation of the pendulum and specific examples of such motions are shown. The control torque can be generated by the engine which is installed at the point of attachment of the pendulum to the ball and sets the pendulum and, accordingly, the ball in motion. This approach (using gaits) implies that each motion must be performed for one oscillation of the pendulum, which is the necessary condition for a new motion to start. By combining such motions we can get any complex trajectory (which is useful, for example, for bypassing an obstacle). In the second part of the research we consider the controlled motion of the ball along a straight line with fixed parameters according to a predetermined law of motion, indicate shortcomings of this approach and consider an algorithm for computing the control torques by example. [1] Borisov, A. V. and Mamaev, I. S., Two Non-Holonomic Integrable Problems Tracing Back to Chaplygin, Regul. Chaotic Dyn., 2012, vol. 17, no. 2, pp. 191–198. [2] Pivovarova, E. N. and Ivanova, T. B., Stability Analysis of Periodic Solutions in the Problem of the Rolling of a Ball with a Pendulum, Nonlin. Dyn. Mob. Robot., 2014, 2(1), pp. 21–32.

Bozidar Jovanovic	**Virtual billiards and Heisenberg model in pseudo-Euclidean spaces** We construct L-A representations of geodesic flows on quadrics and of billiard problems within ellipsoids in the pseudo-Euclidean spaces. A geometric interpretation of the integrability analogous to the classical Chasles theorem for symmetric ellipsoids is given. We also consider a generalization of the billiard within arbitrary quadric allowing virtual billiard reflections (joint work with Vladimir Jovanovic, University of Banja Luka). In the second part of the talk, analogues of the classical Heisenberg spin chain model (or the discrete Neumann system) on pseudo-spheres and light--like cones in the pseudo-Euclidean spaces will be presented. It is known that the light-like billiard flow provides a natural example of a discrete contact completely integrable system. We prove that the Heisenberg model on a light-like cone leads to a integrable discrete contact system as well. References V. Dragovic and M. Radnovic, Ellipsoidal billiards in pseudo-euclidean spaces and relativistic quadrics, Adv. Math. 231 (2012), 1173-1201. B. Jovanovic, The Jacobi-Rosochatius problem on a ellipsoid: the Lax representations and billiards, Arch. Rational Mech. Anal. 210 (2013), 101--131. B. Jovanovi c, Heisenberg model in pseudo-Euclidean spaces, Reg. Ch. Dyn. 19 (2014), 245-250. B. Khesin and S. Tabachnikov, Pseudo-Riemannian geodesics and billiards, Adv. Math. 221 (2009), 1364-1396. J. Moser, and A. Veselov, Discrete versions of some classical integrable systems and factorization of matrix polynomials Comm. Math. Phys. 139 (1991) 217-243.

Alexey Kazakov	**Regular and Chaotic Phenomena in the Nonholonomic Model of an Unbalanced Ball Moving on a Plane** We consider a model of unbalanced ball moving on a rough plane. By an unbalanced ball we call the dynamically asymmetric ball with a displaced center of gravity. The roughness of the plane means that a body moves without slipping. It is well known [1] that such a motion is governed by the system of 6 differential equations in variables ω (angular velocities) and γ (projection of the vertical unit vector to the body frame). These equations admit 2 integrals: energy and geometric one which reduce the problem dimension from 6 to 4. To visualize the dynamics of this system we construct three-dimensional Poincare map on some cross-section using the Andoyer-Deprit variables [1]. If the gravity center of the ball is displaced along all 3 axis of the body frame, the dynamics of the ball looks as very complex. In this case we found few types of strange attractors such as torus-chaos and figure-eight attractor. The existence of the figure-eight attractor in three dimensional maps was predicted in [2]. As we know, the system under consideration is the first system from applications where an attractor of such the type was found. In this talk we present results related to scenarios of the appearance of figure-eight attractor and its properties. In more details results will appear in [3]. The work was supported by Ministry of Education and Science of the Russian Federation within the framework of its basic part, and RFBR grants No.13-01-00589 and 13-01-97028-Povolzhie. [1] Borisov A.V. and Mamaev I.S. Rigid Body Dynamics: Hamiltonian Methods, Integrability, Chaos. Moscow–Izhevsk: R&C Dynamics, Institute of Computer Science, 2005 (Russian). [2] Gonchenko A.S., Gonchenko S.V., Shilnikov L.P. Towards scenarios of chaos appearance in three-dimensional maps. Rus. J. Nonlin. Dyn., 2012, V.8, No.1, p. 3-28 [3] Borisov A.V., Kazakov A.O., Sataev I.R. Regular and chaotic phenomena in the nonholonomic model of the unbalanced ball rolling on a plane Regular and Chaotic Dynamics, 2014, Vol.18, No.4, to appear.
Boris Khesin	**Integrability and non-integrability of pentagram maps** We describe recent results on integrable cases for higher-dimensional generalizations of the 2D the pentagram map. We also discuss a numerical evidence that certain generalizations of the integrable 2D pentagram map are non-integrable and present a conjecture for a necessary condition of their discrete integrability. This is a joint work with Fedor Soloviev (Univ. of Toronto).

Giorgi Khimshiashvili	**Poncelet Porism for Biquadratic Curves** We will discuss two seemingly unrelated topics having in fact a common feature that they naturally lead to consideration of certain involutive transformations of biquadratic curves. The first topic is concerned with the so-called Darboux transformation on the moduli space of planar quadrilateral linkage. We will explain how this transformation can be related to involutions of an appropriate biquadratic curve and discuss various aspects of Poncelet porism in this setting. The second topic is concerned with the uniqueness of solution to the Dirichlet problem for string equation in bounded domain. If the boundary is a convex biquadratic curve we show that an analog of Poncelet porism for the so-called John's mapping can be established in the same way as for Darboux transformation. We will also discuss connections with related results of other authors and formulate several open problems arising in our setting.
Alexander Kilin	**The controllability and self-propulsion of bodies with an invariable shell in a fluid** In this work we consider the problem of motion of a rigid body in an ideal fluid with two material points moving along circular trajectories. The controllability of this system on the zero level set of first integrals is shown. Elementary "gaits" are presented which allow the realization of the body's motion from one point to another. The existence of obstacles to a controlled motion of the body along an arbitrary trajectory is pointed out. An example of a solution to the problem of optimal control of the body's motion is given using genetic algorithms.
Valery Kozlov	**Conservation Laws of Generalized Billiards That Are Polynomial in Momenta** We deal with dynamics particles moving on a Euclidean n-imensional torus or in an n-dimensional parallelepiped box in a force field whose potential is proportional to the characteristic function of the region D with a regular boundary. After reaching this region, the trajectory of the particle is refracted according to the law which resembles the Snell – Descartes law from geometrical optics. When the energies are small, the particle does not reach the region D and elastically bounces off its boundary. In this case, we obtain a dynamical system of billiard type (which was intensely studied with respect to strictly convex regions). In addition, we discuss the problem of the existence of nontrivial first integrals that are polynomials in momenta with summable coefficients and are functionally independent with the energy integral. Conditions for the geometry of the boundary of the region D under which the problem does not admit nontrivial polynomial first integrals are found. Examples of nonconvex regions are given; for these regions the corresponding dynamical system is obviously nonergodic for fixed energy values (including small ones), however, it does not admit polynomial conservation laws independent of the energy integral.

Valery Kozlov *Special Public Lecture*	**On explicitly solvable ODE's** We discuss the problem of integration of ODE systems by quadratures. The main classical results on exact integration of general nonlinear systems are the Euler-Jacobi theorem on the integrating factor and the Lie theorem on the solvable algebra of symmetries. We present a general theory uniting these two approaches. We generalize Darboux's classical results on the integrability of linear non-autonomous systems with an incomplete set of particular solutions. Special attention is paid to linear Hamiltonian systems. We also discuss the general problem of integrability of autonomous ODE systems in an n-dimensional space which admit an algebra of symmetry fields of dimension greater than or equal to n.
Stefano Luzzatto	**Physical measures for partially hyperbolic diffeomorphisms** We consider partially hyperbolic C^{1+} diffeomorphisms of compact Riemannian manifolds of arbitrary dimension which admit a partially hyperbolic tangent bundle decomposition $E^s \oplus E^{cu}$. Assuming the existence of a set of positive Lebesgue measure on which f satisfies a weak nonuniform expansivity assumption in the centre-unstable direction, we prove that there exists at most a finite number of transitive attractors each of which supports an SRB measure. As part of our argument, we prove that each attractor admits a Gibbs-Markov-Young geometric structure with integrable return times. We also characterize in this setting SRB measures which are liftable to Gibbs-Markov-Young structures. This is joint work with Alves, Dias, Pinheiro.
Andrey Mironov	**Integrable geodesic flows on 2-torus: formal solutions and variational principle** We study quasi-linear system of differential equations which describes the existence of the polynomial in momenta first integral of the integrable geodesic flow on 2-torus. This system is a semi-Hamiltonian system. We show that the metric associated with the system is a metric of Egorov type. We use this fact in order to prove that in the case of integrals of degree three and four the system is in fact equivalent to a single remarkable equation of order 3 and 4 respectively. Remarkably the equation for the case of degree four has variational meaning: it is Euler-Lagrange equation of a variational principle. We prove that this equation for n=4 has formal double periodic solutions as a series in a small.

Oktay Pashaev	**Classical Images and Quantum Symmetries in Annular Domain** The annular domain has been subject of several studies as a canonical region for the classical and quantum motion, including classical billiard, transition from regular to chaotic motion and quantum chaotic effects. In present talk I will show that geometry of the region is related naturally with “quantum symmetries” associated with q-elementary functions. I will formulate the two circle theorem as descriptive of arbitrary hydrodynamic flow in this region. The construction is based on q-periodic extension of complex potential as an analytic function, leading to fixed scale-invariant complex velocity, where parameter q-the relative size of the region, is determined by geometry of the region. It leads naturally to self-similar fractal structure of the flow with q-periodic modulation as a solution of q-difference equation. The theorem allows describe the problem of point vortices, dipoles, etc. in terms of images located at geometric progression. We show that for point vortices an infinite set of images is determined completely by pole singularities of q-logarithm and zeroes of q-exponential functions. The Hamiltonian, the Kirchhoff–Routh, and the Schottky–Klein prime functions as well as the uniformly rotating exact N-vortex polygon solutions with the rotation frequency expressed in terms of q-logarithms are found. In particular, we show that a single vortex orbits the cylinders with constant angular velocity, given as the q-harmonic series. If time allows I will discuss also some implications of the method of images for constructing quantum states. Considering image points in generalized circles in complex plane as a phase space for qubit coherent states we obtain Fibonacci and Lucas type entangled n-qubit states. Related to this construction complex Fibonacci curve shows a reach set of geometric patterns. 1. O.K. Pashaev, Two-circle theorem, q-periodic functions and entangled qubit states, Journal of Physics: Conference Series 482 (2014) 012033 2. O.K. Pashaev Vortex images, q-calculus and entangled coherent states, Journal of Physics: Conference Series 343 (2012) 012093 3. O.K. Pashaev and O. Yilmaz, Hamiltonian dynamics of N vortices in concentric annular region, J. Phys. A: Math. Theor. 44 (2011) 185501 4. O.K. Pashaev and O. Yilmaz, Vortex images and q-elementary functions, J. Phys. A: Math. Theor. 41 (2008) 135207
Milena Radnovic	**Integrable line congruences and double reflection nets** The billiard systems within quadrics, playing the role of discrete analogues of geodesics on ellipsoids, are incorporated into the theory of integrable quad-graphs. A notion of the double-reflection nets as a subclass of dual Darboux nets associated with pencils of quadrics is introduced. Basic properies and several examples are presented.

Vasilisa Shramchenko	**Poncelet theorem, Painlevé-VI and Schlesinger equations** In 1995 Hitchin constructed explicit algebraic solutions to the Painlevé VI (1/8,-1/8,1/8,3/8) equation starting with any closed Poncelet trajectory inscribed in a conic and circumscribed about another conic. In this talk I will show that Hitchin's construction is nothing but the Okamoto transformation between Picard's solution and the general solution of the Painlevé VI (1/8,-1/8,1/8,3/8) equation. Moreover, this Okamoto transformation can be written in terms of an Abelian differential of the third kind on the associated elliptic curve, which allows to write down solutions to the corresponding Schlesinger system in terms of this differential as well. This solution of the Schlesinger system admits a natural generalization to hiperelliptic curves. This is a joint work with V. Dragovic.
Fedor Soloviev	**Algebraic-geometric integrability of pentagram maps** We discuss higher dimensional pentagram maps, their Lax representations, and how various integrability properties follow from the latter.
Peter Topalov	**Spectral properties of Liouville billiard tables** I will discuss a new class of isospectral invariants for the Laplace-Beltrami operator on Riemannian manifolds with boundary. The construction of these invariants is based on the dynamical properties of the underlying billiard system.
Dmitry Treschev	**A family of locally integrable billiards** We study symmetric billiard tables for which the billiard map is locally (near an elliptic periodic orbit of period 2) conjugated to a rigid rotation. We obtain an equation (below the conjugacy equation) for such tables, prove that if α, the rotation angle, is rationally incommensurable with π, the conjugacy equation has a solution in the category of formal series. We present numerical evidence that for "good" rotation angles the series have positive radii of convergence. We discuss symmetries of the conjugacy equation, dependence of the convergence radius on α, and other aspects.
Andrey Tsiganov	**Simultaneous separation for the Neumann and Chaplygin systems** The Neumann and Chaplygin systems on the sphere share a simple separation of variables originated from the standard 2x2 Lax matrix for the Neumann system. A family of similar pairs of simultaneously separable systems can be constructed using standard curvilinear coordinate systems and special Backlund transformations, associated with 2x2 Lax matrices. This family includes two systems with quartic potentials, two Henon-Heiles system and the Kowalevski top on the sphere.
Jean-Claude Zambrini	**A Theorem of Jacobi on complete integrals and its stochastic deformation**