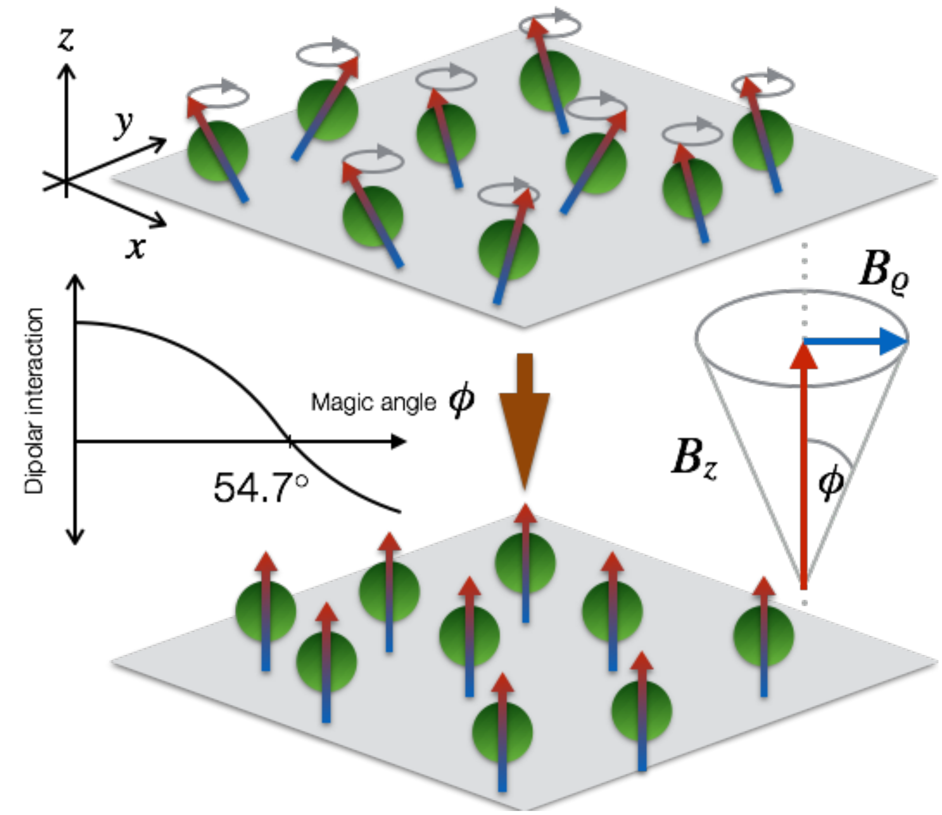
Quantum Quenches in Fermion Liquids

From the Generalized Gibbs Ensemble to Pre-thermalization



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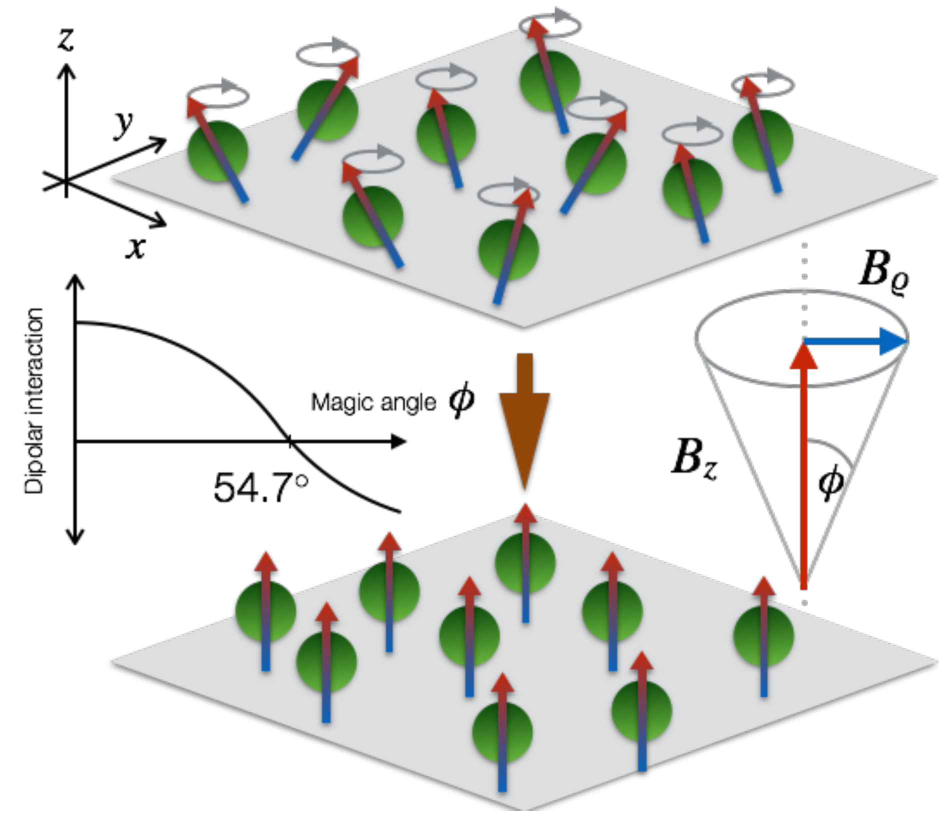
國立清華大學物理系

Department of Physics

National Tsing Hua University

ICTP (Trieste) *July 2, 2014*

What you can do with Quadratic Hamiltonians …



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Collaborators

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MAC, Phys. Rev. Lett. (2006)

A Iucci & MAC, Phys. Rev. A (2009)

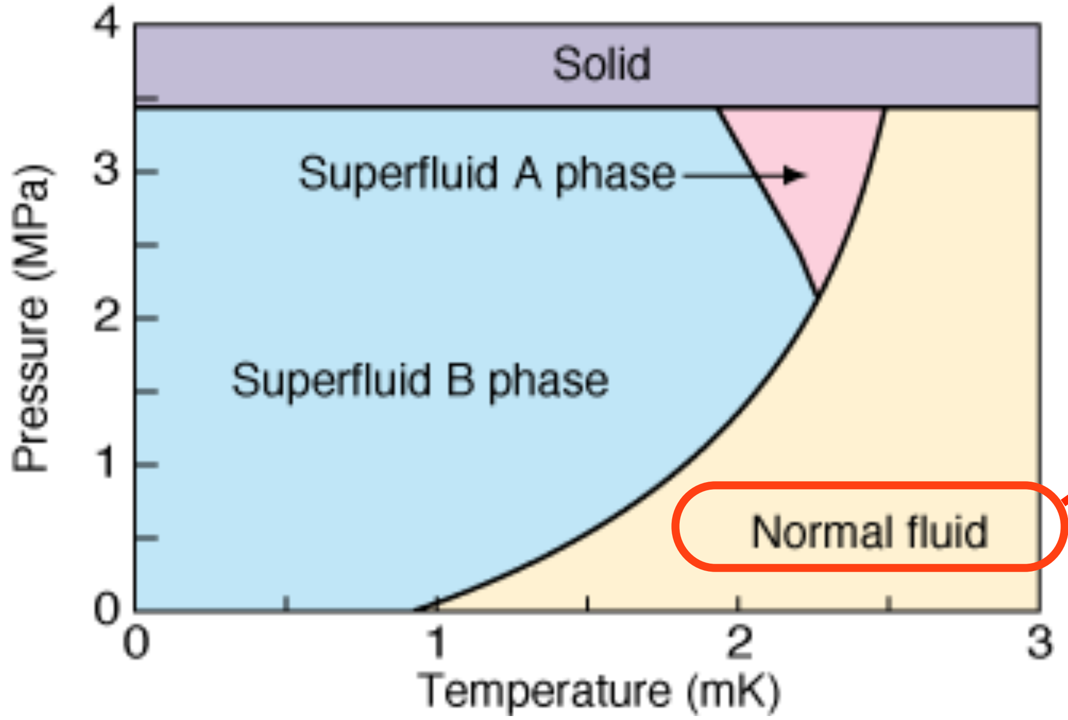
MAC, A Iucci & MC Chung Physical Review E (2012)

N Nessi, A Iucci & MAC, arXiv:1401.986

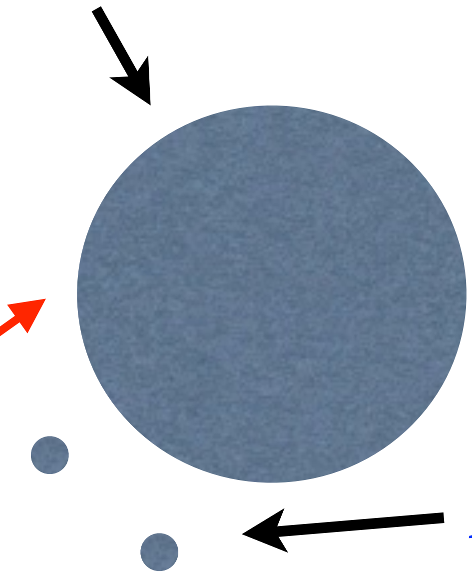
Fermion liquids in Equilibrium
(a crash course)

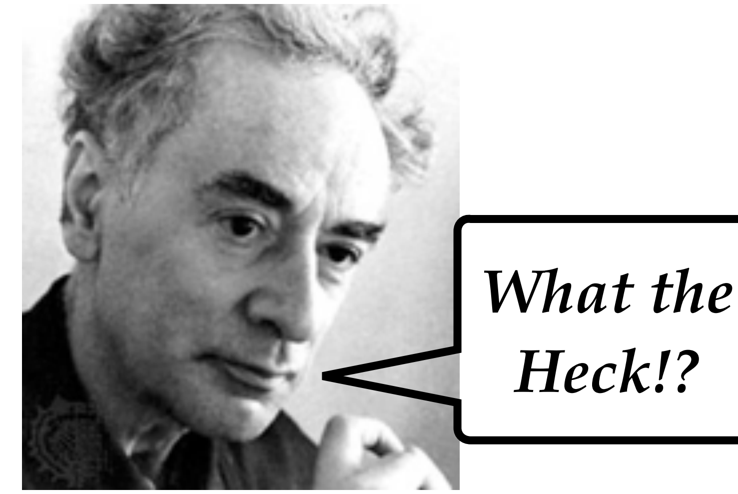
Fermi Liquid Theory

Helium 3 Phase diagram



Fermi Surface (FS)





Landau quasi-particles:
form a dilute gas

Quasi-particle occupation

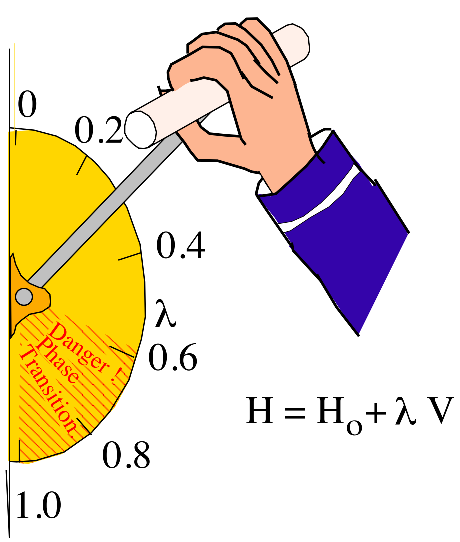
$$n_\sigma(\boldsymbol{p})$$

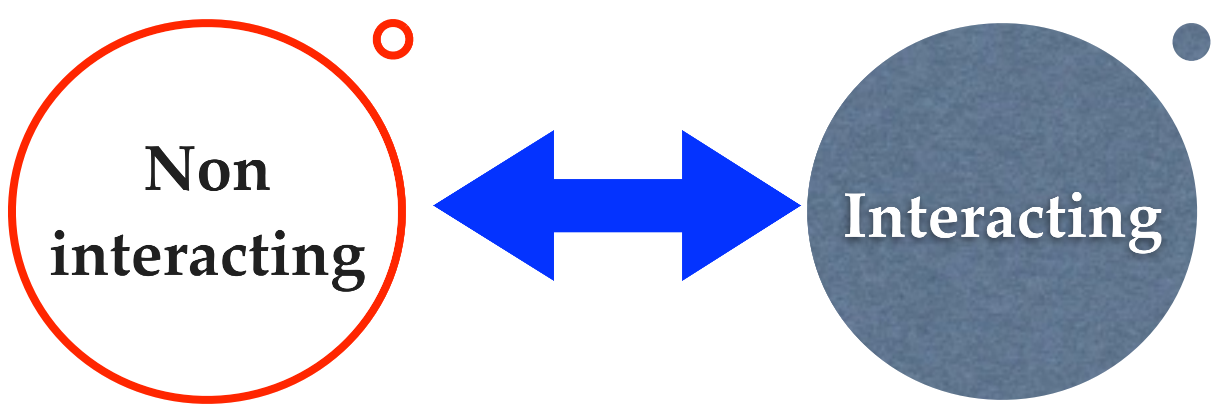
is a good quantum number

Quasi-particle scattering rate:

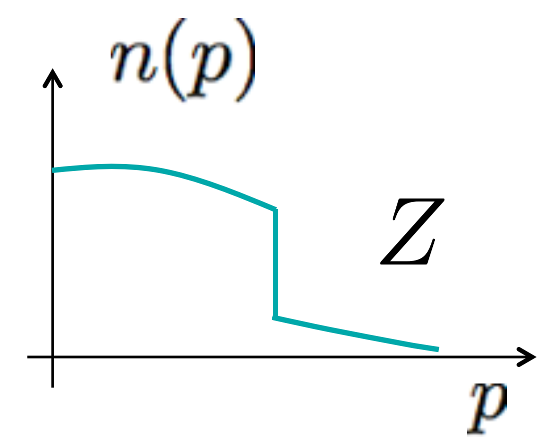
$$\Gamma_{\text{QP}} \sim \epsilon^2 + \pi^2 T^2$$

Adiabatic Continuity





Momentum distrib.

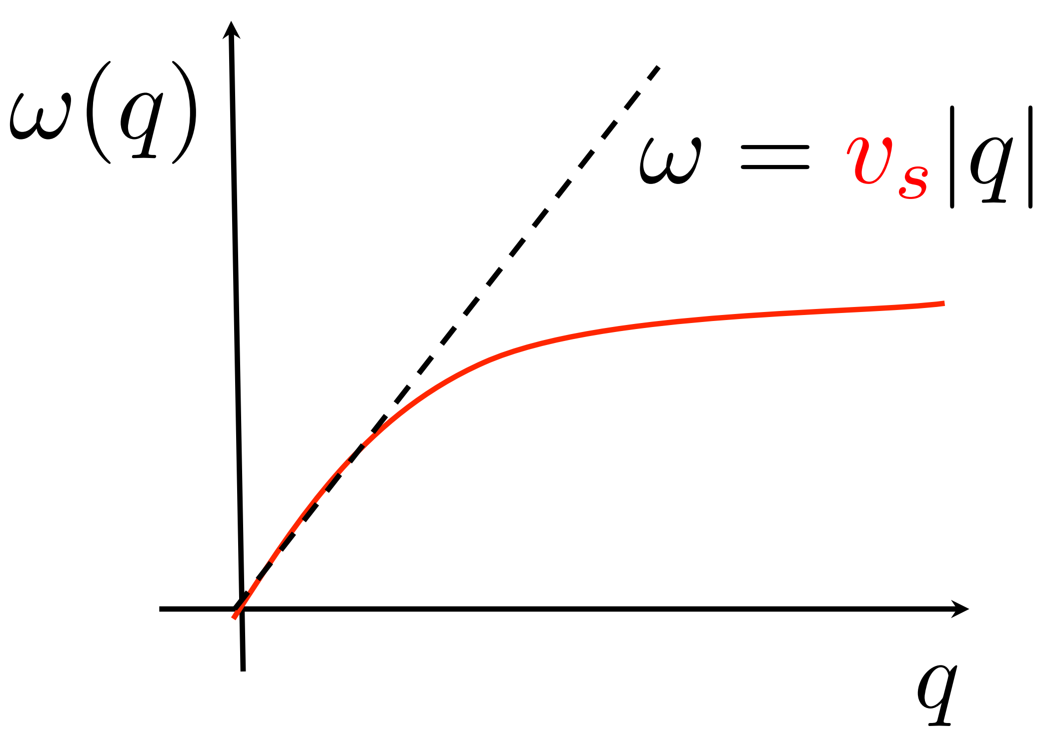


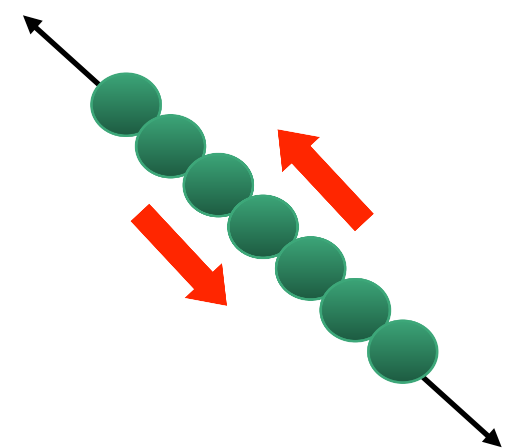
One Dimension: The Tomonaga-Luttinger Liquid

Luttinger *Mattis* *Lieb* *朝永* *Luther* *Emery* *Peschel* *Haldane*

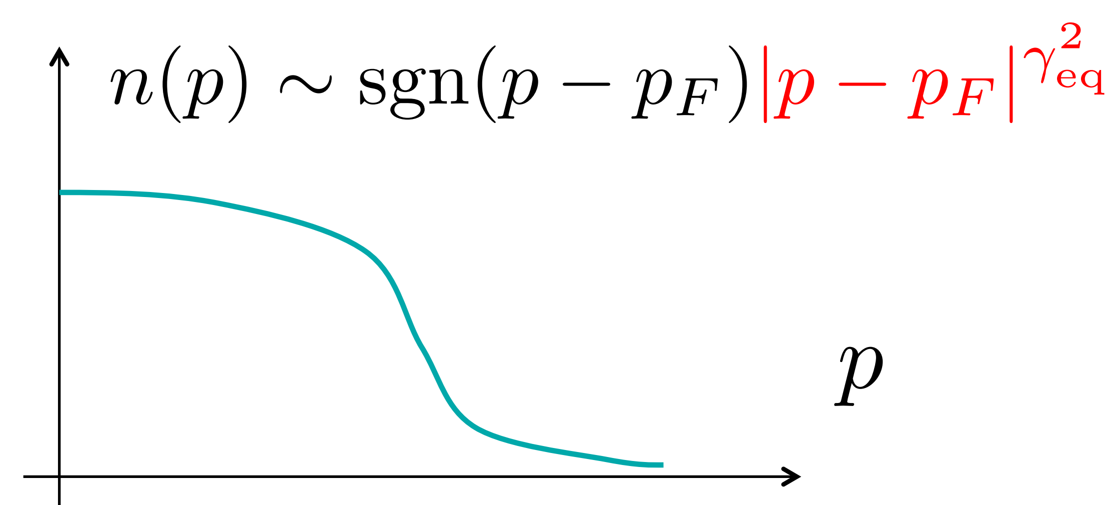
(There are more, but I simply couldn't fit in every one…)

Collective modes exhaust the low-energy spectrum

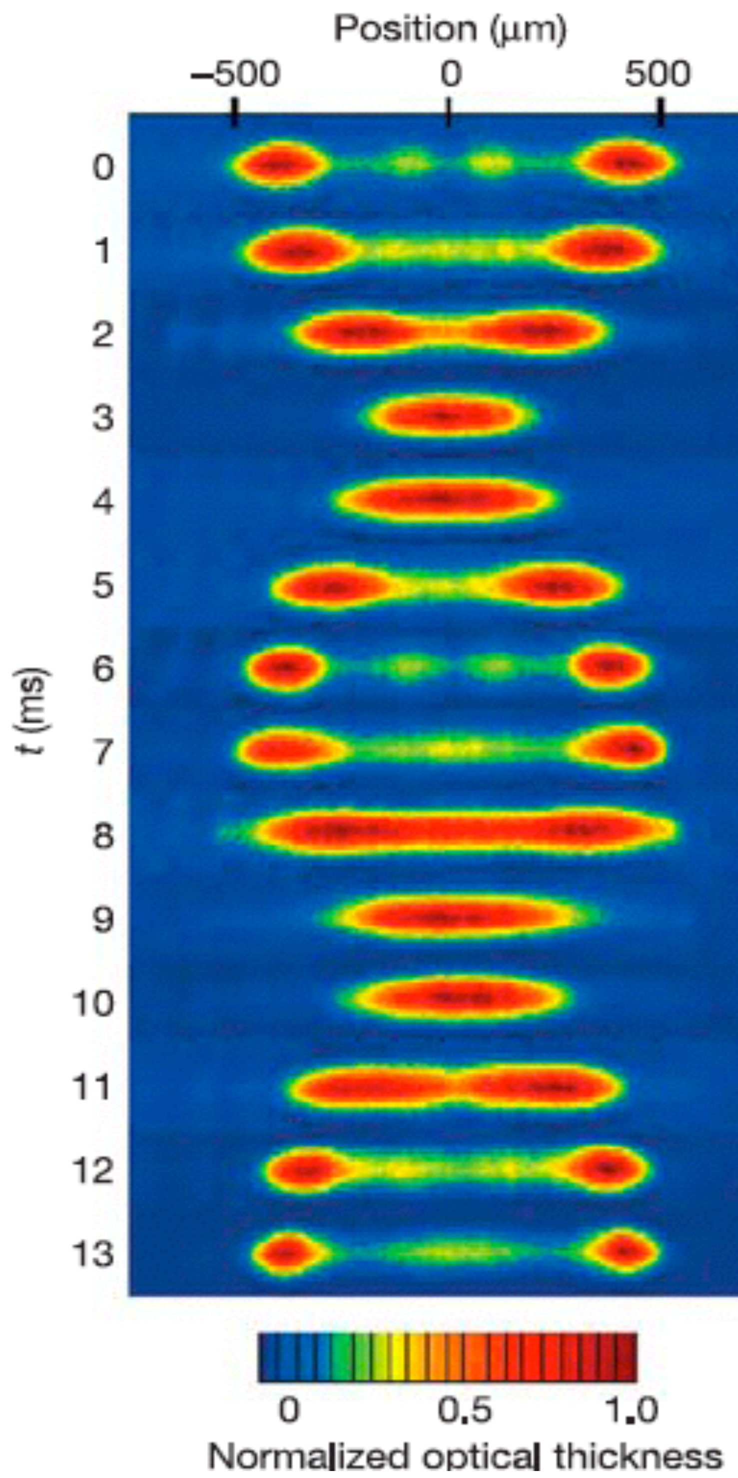




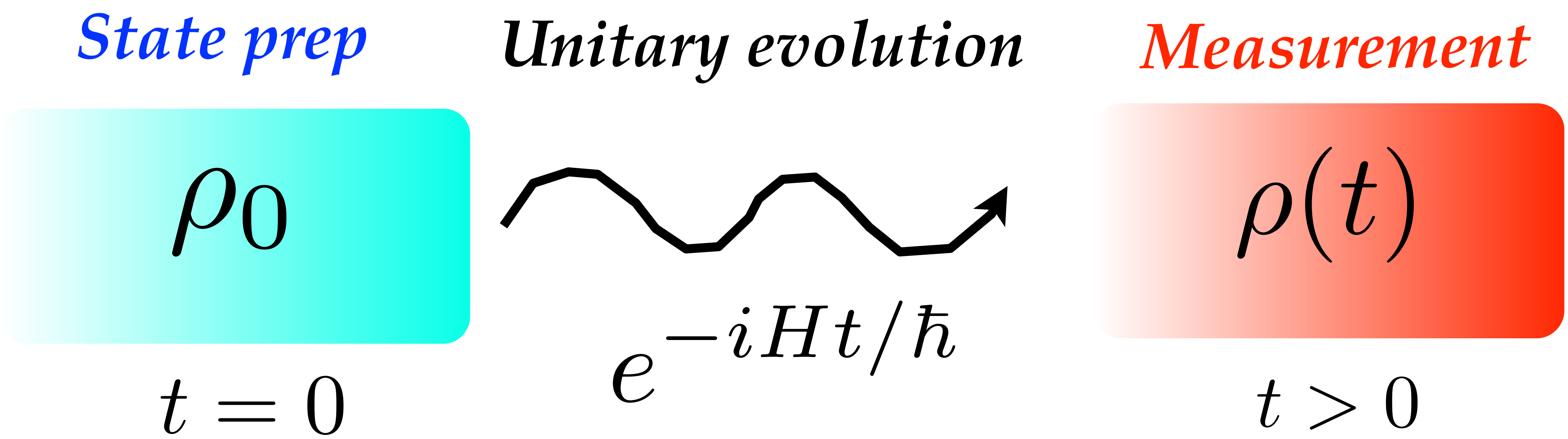
Power-law Momentum distribution



Out of Equilibrium Quantum (Fermi) Gases



Sudden Quantum Quenches



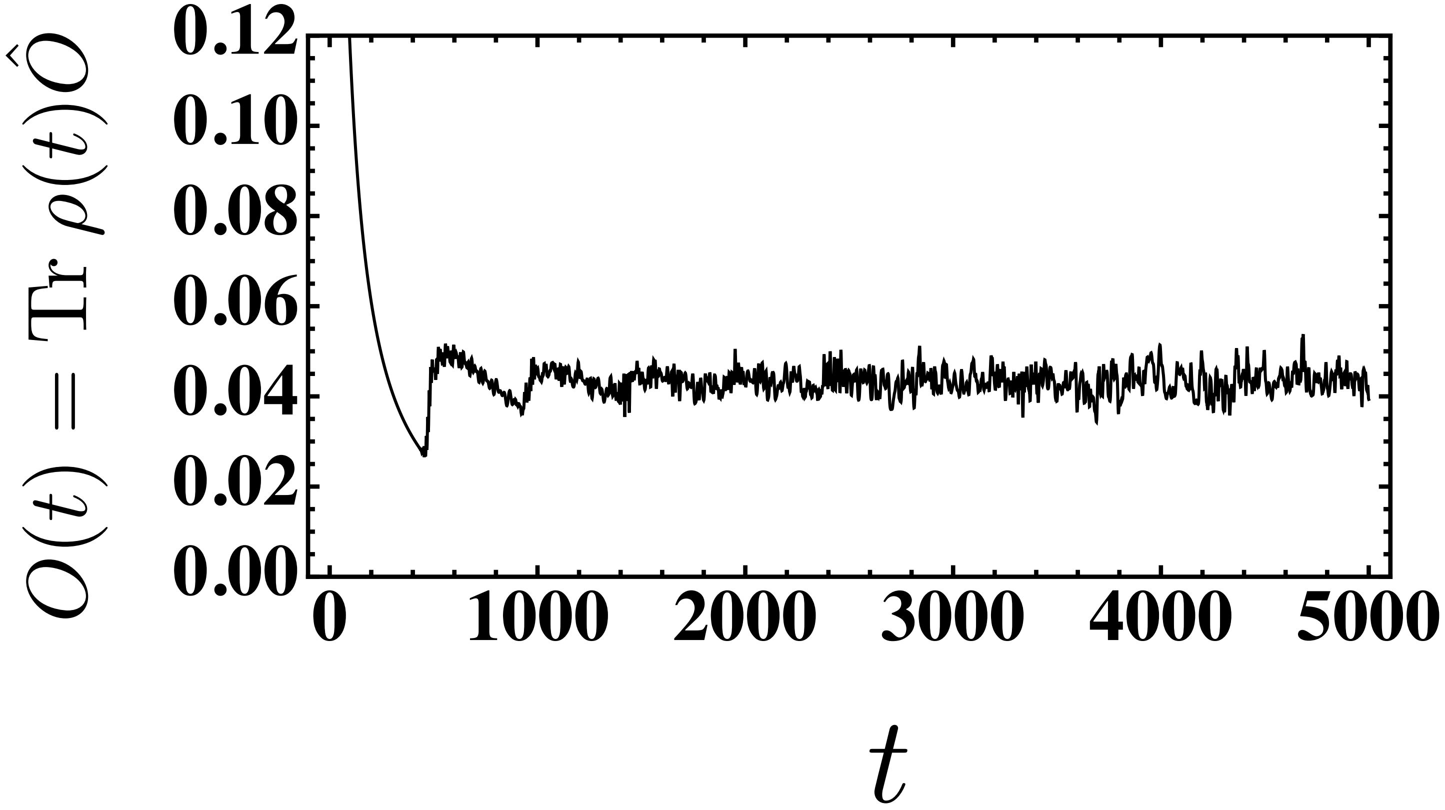
Some Important Questions

Does the system reach a steady state?

$$\overline{O} = \lim_{T \to +\infty} \frac{1}{T} \int_{t_0}^{t_0+T} dt\, O(t)$$

$$O(t) = \mathrm{Tr}\, \rho(t) \hat{O}$$

$$\hat{O} = n(\mathbf{r}), \Psi^\dagger(\mathbf{r})\Psi(\mathbf{0}), \ldots$$



If so, what are its properties? Does it thermalize?

$$\bar{O} = \mathrm{Tr}\, \rho_{\mathrm{steady}} \hat{O},$$

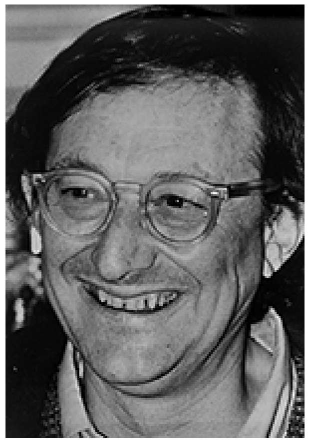
$$\rho_{\mathrm{steady}} \propto e^{-H/T_{\mathrm{eff}}} \;?$$

Fermions in 1D

Out of Equilibrium

The Luttinger (Thirring) Model

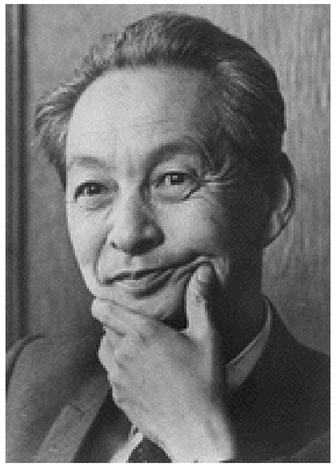
Luttinger



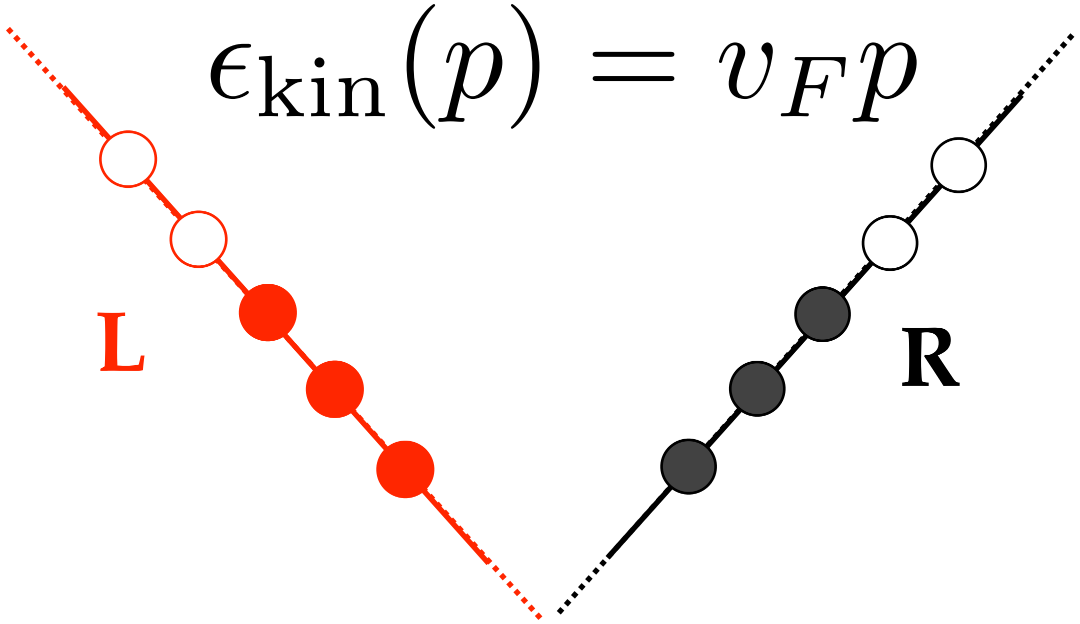
Mattis & Lieb

[J. Math. Phys. (1965)]

朝永



$$\epsilon_{\mathrm{kin}}(p) = v_F p$$



"Infinite story hotel" $|0\rangle_{\mathrm{Dirac}}$

'Anomalous' commutation relations

$$[\rho_R(q), \rho_R(-q')] = \frac{qL}{2\pi}\delta_{q,q'}$$

Quantum Quench in the LM

$$H_{\rm kin} = \sum_{q\neq 0} \hbar v_F |q| \, a^\dagger(q) a(q) \qquad H_{\rm LM} = \sum_{q\neq 0} \hbar v |q| \, b^\dagger(q) b(q)$$

Non-interacting fermions ($t \leq 0$) *Interacting fermions ($t > 0$)*

Equilibrium solution $b(q) = \cosh\varphi(q)\, a(q) + \sinh\varphi(q)\, a^\dagger(-q)$

Non-equilibrium (quench) solution:

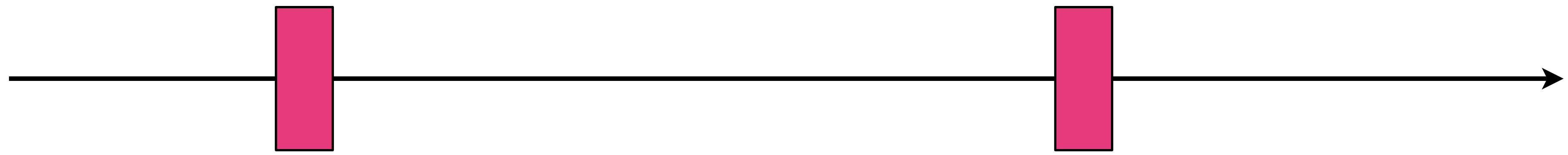
$$\begin{aligned} a(q,t) &= e^{iH_{LM}t/\hbar} a(q) e^{-iH_{LM}t/\hbar} = f(q,t) a(q) + g^*(q,t) a^\dagger(-q), \\ f(q,t) &= \cos v|q|t - i \sin v|q|t \, \cosh 2\varphi(q), \\ g(q,t) &= i \sin v|q|t \sinh 2\varphi(q) \end{aligned}$$

MAC, PRL 97 (2006)

One-particle density matrix

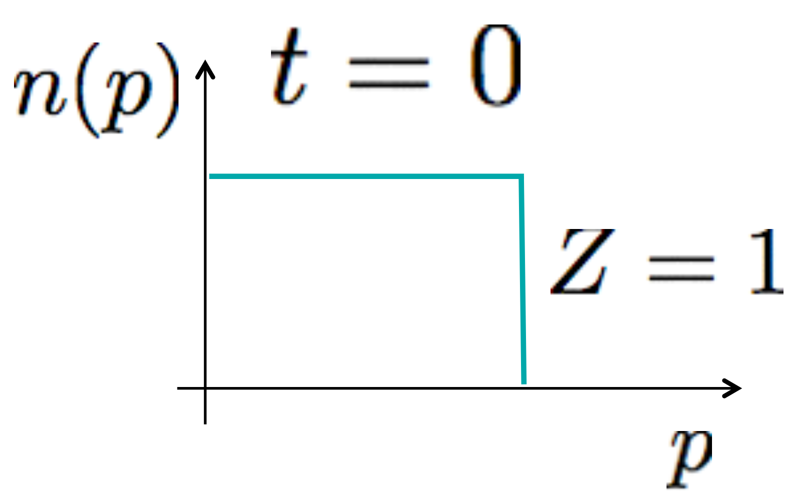
$$C_{\psi_r}(x, t>0) = \langle 0 | e^{iH_{LM}t/\hbar} \psi_r^\dagger(x) \psi_r(0) e^{-iH_{LM}t/\hbar} | 0 \rangle_{\rm Dirac}$$

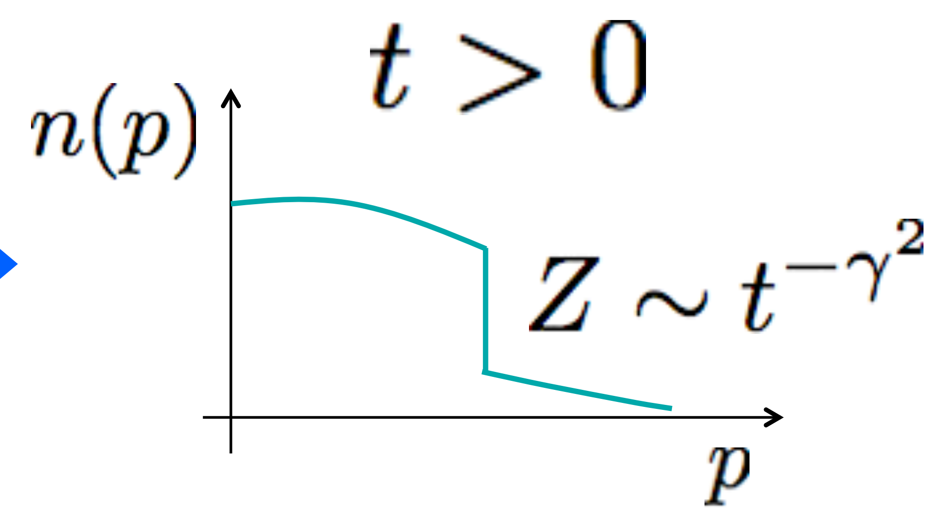
Interaction Quenches: Fermions in 1D



$$H_{\text{int}} = 0 \longrightarrow H_{\text{int}} \neq 0$$

Momentum distribution at time t :





MAC Phys Rev Lett (2006)

A Iucci & MAC Phys Rev A (2009)

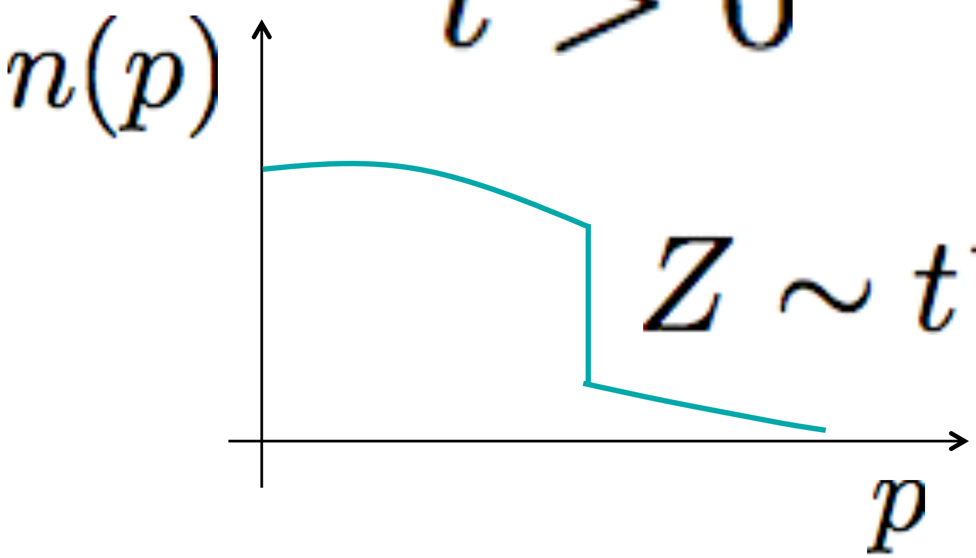
Does this work for Tomonaga-Luttinger liquids?

Model: XXZ + NN int

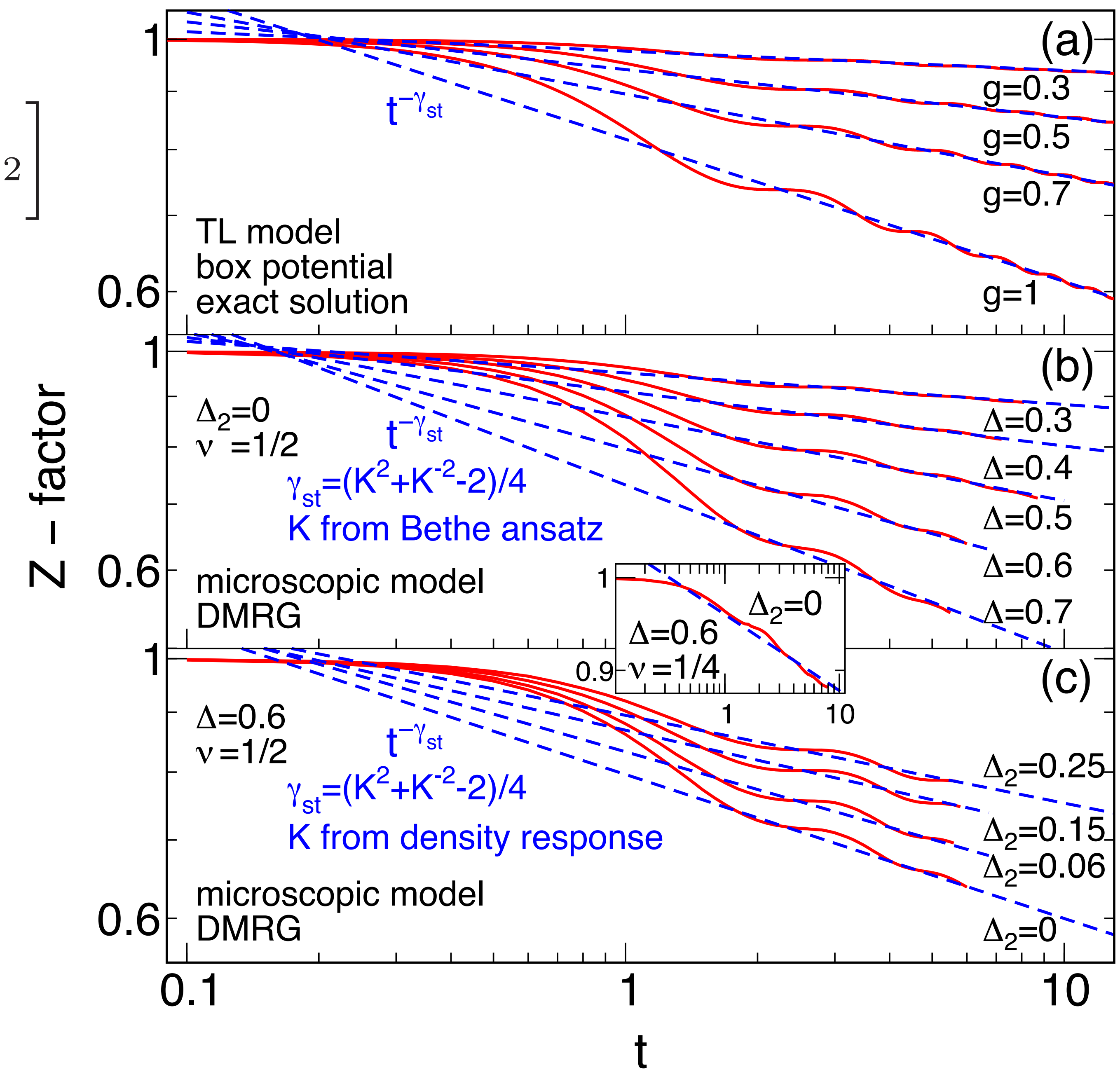
C Karrasch et al PRL (2012)

$$H = \sum_j \left[\frac{1}{2} c_j^\dagger c_{j+1} + \text{H.c.} + \Delta n_j n_{j+1} + \Delta_2 n_j n_{j+2} \right]$$

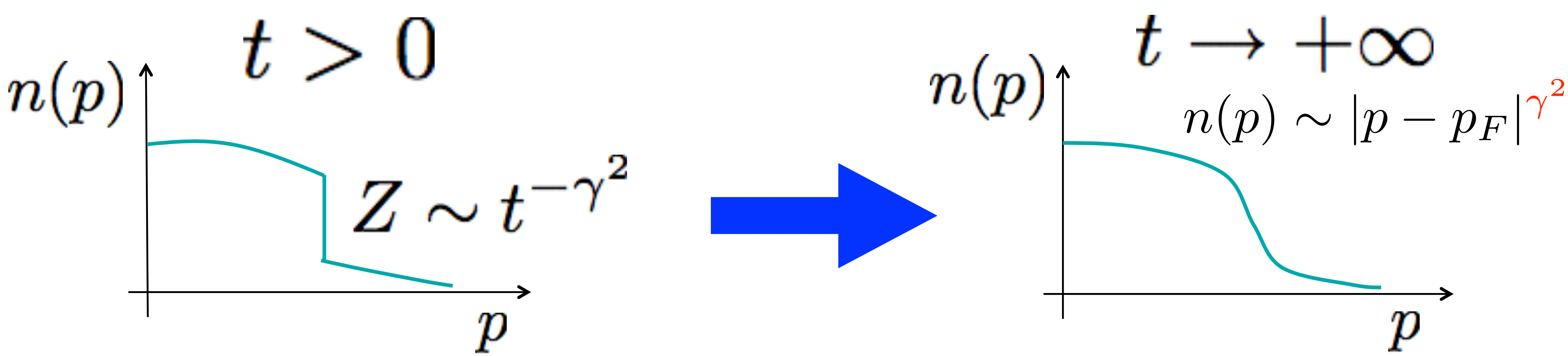
$$t > 0$$



$$Z \sim t^{-\gamma^2}$$



Where does the system go?



Non-equilibrium exponent : $\gamma > \gamma_{eq}$

The system does not thermalize! Why?

The GGE Conjecture

M Rigol, B Dunjko, V Yurovsky, and M Olshanii PRL (2007)

Apply the Maximum Entropy Principle

[E.T. Jaynes, PR (1957)]

$$\bar{O} = \lim_{t\to+\infty} \langle \Psi(t)|\hat{O}|\Psi(t)\rangle = \mathrm{Tr}\rho_{GGE}\,\hat{O},$$

$$\rho_{GGE} = \frac{e^{\sum_k \lambda_k I(k)}}{Z_{GGE}}, \quad \langle I(k)\rangle_{GGE} = \langle \Psi(t=0)|I(k)|\Psi(t=0)\rangle$$

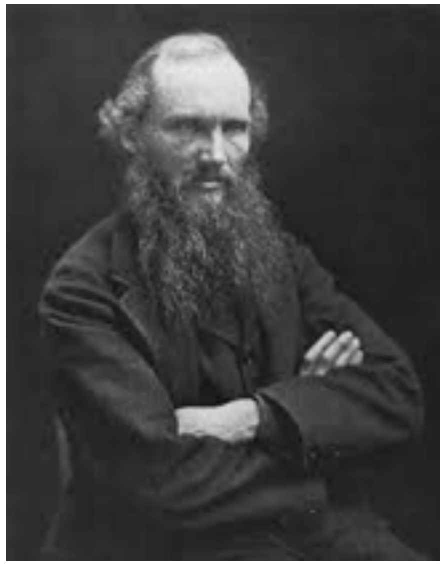
Need Integrals of Motion $[H, I(k)] = 0$

Luttinger Model Integrals of Motion $I(k) = b^\dagger(k)b(k)$

But only O(N) integrals are needed!

Why?

Other Evidence for the GGE



Falikov-Kimball Model M Eckstein and M Kollar PRL 2008

1/r Hubbard Model in 1D M Eckstein and M Kollar PRA 2008

Sine-Gordon model A Iucci & MAC PRA 2009, NJP 2010

Sine-Gordon model D Fioretto and G Mussardo, NJP 2010

Quantum Ising Model P Calabrese, Fagotti, FHM Essler, PRL 2011

+ Add your favorite paper here if not listed above …

"All Science is either Physics or stamp collecting"

Lord Kelvin

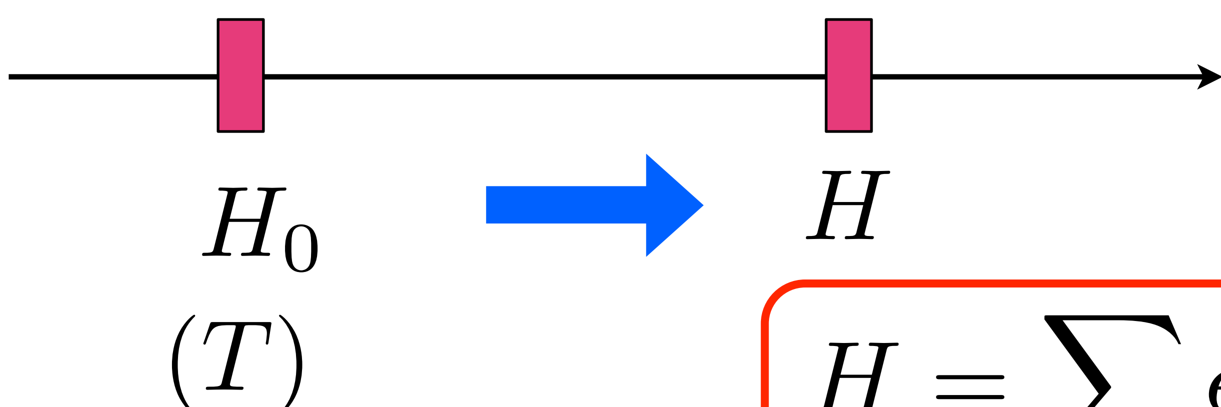
What principles behind the GGE?

Initial state

$$\boxed{\rho_0 = \frac{e^{-H_0/T}}{Z_0}}$$

$$H_0 = \sum_{k,k'} \left[\epsilon_0(k)\delta_{k,k'} + V_0(k,k')\right] f^\dagger(k) f(k') + \sum_{k,k'} \left[\Delta_0^*(k,k) f(k) f(k') + \Delta_0(k,k') f^\dagger(k') f^\dagger(k)\right]$$

Quantum Quench as a sudden change of Hamiltonian



$$\boxed{H = \sum_k \epsilon(k)\, f^\dagger(k) f(k)}$$

System eigenmodes
[O(L) bosons or fermions]

$$[H, f(k)] = \epsilon(k) f(k)$$

Uncorrelated Initial states

(Clustering) Wick's theorem

$$\langle f^\dagger(k_1) f^\dagger(k_2) f(k_3) f(k_4)\rangle = \langle f^\dagger(k_1) f^\dagger(k_2)\rangle\langle f(k_3) f(k_4)\rangle \pm \langle f^\dagger(k_1) f(k_3)\rangle\langle f^\dagger(k_2) f(k_4)\rangle + \cdots$$

Gaussian reduced density matrices

[M.C. Chung and I. Peschel PRB (2001)]
[S.-A. Cheong and C.~L. Henley, PRB (2004)]

$$\rho(k) = \mathrm{Tr}_{k' \neq k}\rho_0 = \frac{1}{Z(k)} e^{-\lambda(k) f^\dagger(k) f(k)}$$

$$\rho_{\mathrm{GGE}} = \bigotimes_k \rho(k)$$

Physically, the other modes act as a bath

Eigenmode dependent temperature

$$T(k) = \lambda(k)/\epsilon(k)$$

For correlated (i.e. Chaotic) initial states thermalization occurs

K He and M Rigol, PRA (2013)

Correlations of Local Operators

Local Operator $O(x) = \sum_k \varphi_k(x) f(k)$

Normalized orbitals

$\varphi_k \sim O(L^{-1/2})$

$$C_O^{(2)}(x_i, x_j, t) = \langle O^\dagger(x_i, t) O(x_j, t) \rangle$$
$$= \sum_{k,k'} \varphi_k^*(x_i) \varphi_{k'}(x_j) G_0(k, k') e^{-i[\epsilon(k) - \epsilon(k')]t/\hbar}$$

$G_0(k, k') = \langle f^\dagger(k) f(k') \rangle$ $O(L^2)$

$$\boxed{\lim_{\substack{t \to +\infty \\ L \to +\infty}} C_0(x_i, x_j, t) = \sum_k \varphi_k^*(x_i) \varphi_k(x_j) N_0(k)}$$

Dephasing ***Only*** **L** ***numbers***

$$\boxed{N_0(k) = \langle f^\dagger(k) f(k) \rangle =} \mathrm{Tr}\, \rho(k)\, f^\dagger(k) f(k) = \mathrm{Tr}\, \rho_{\mathrm{GGE}}\, f^\dagger(k) f(k)$$

$$\rho_{\mathrm{GGE}} = \bigotimes_k \rho(k)$$

MAC, A Iucci, MC Chung Phys. Rev. E (2012)

How about Nonlocal Operators?

Momentum distribution $S(k,t) = \frac{1}{L}\sum_{i,j}\langle\sigma_i^-(t)\sigma_j^+(t)\rangle\, e^{ik(x_i-x_j)}$

Non-local operator $\sigma_i^+ = \prod_{j<i}(1-2f_j^\dagger f_j)f_i,$

Toeplitz determinant

Using Wick's theorem

$$\lim_{t\to+\infty}\langle\sigma_i^-(t)\sigma_j^+(t)\rangle = \frac{1}{2}\begin{vmatrix} a_0 & a_1 & \cdots & a_{-n+1} \\ a_1 & a_0 & \cdots & a_{-n+2} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n-1} & a_{n-2} & \cdots & a_0 \end{vmatrix}$$

Tricky points with Wick's th.

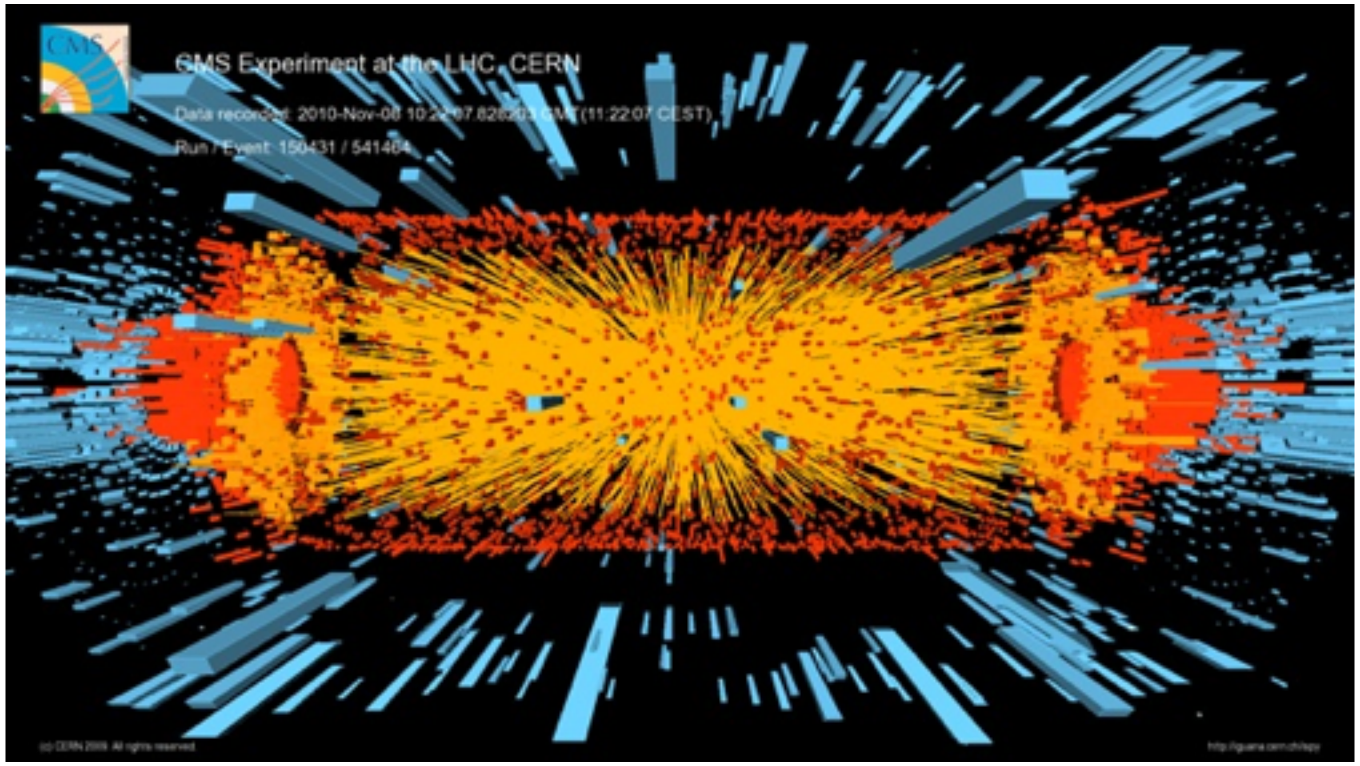
MAC, A Iucci, MC Chung PRE (2012)

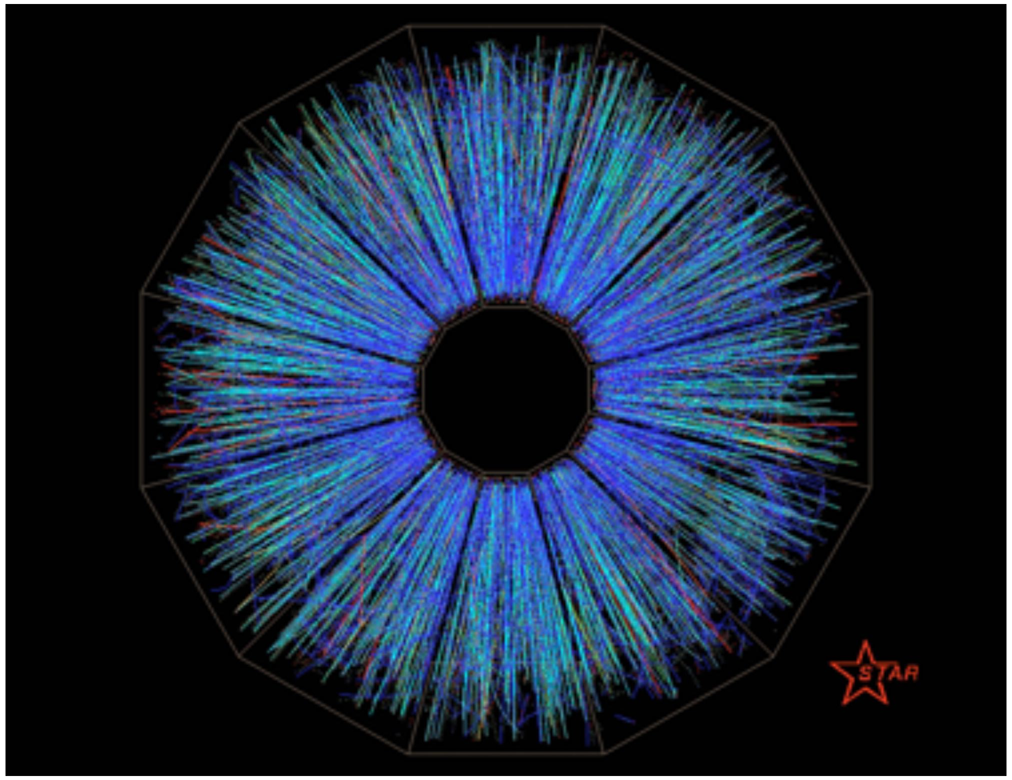
S Ziraldo and GE Santoro PRB 2013

$$\boxed{a_{i-j+1} = \delta_{ij} - 2\sum_k \varphi_k^*(x_i)\varphi_k(x_j)N_0(k)}$$

Depends only on diagonal correlations $N_0(k) = \langle f^\dagger(k)f(k)\rangle$

Pre-thermalization of Fermion fluids





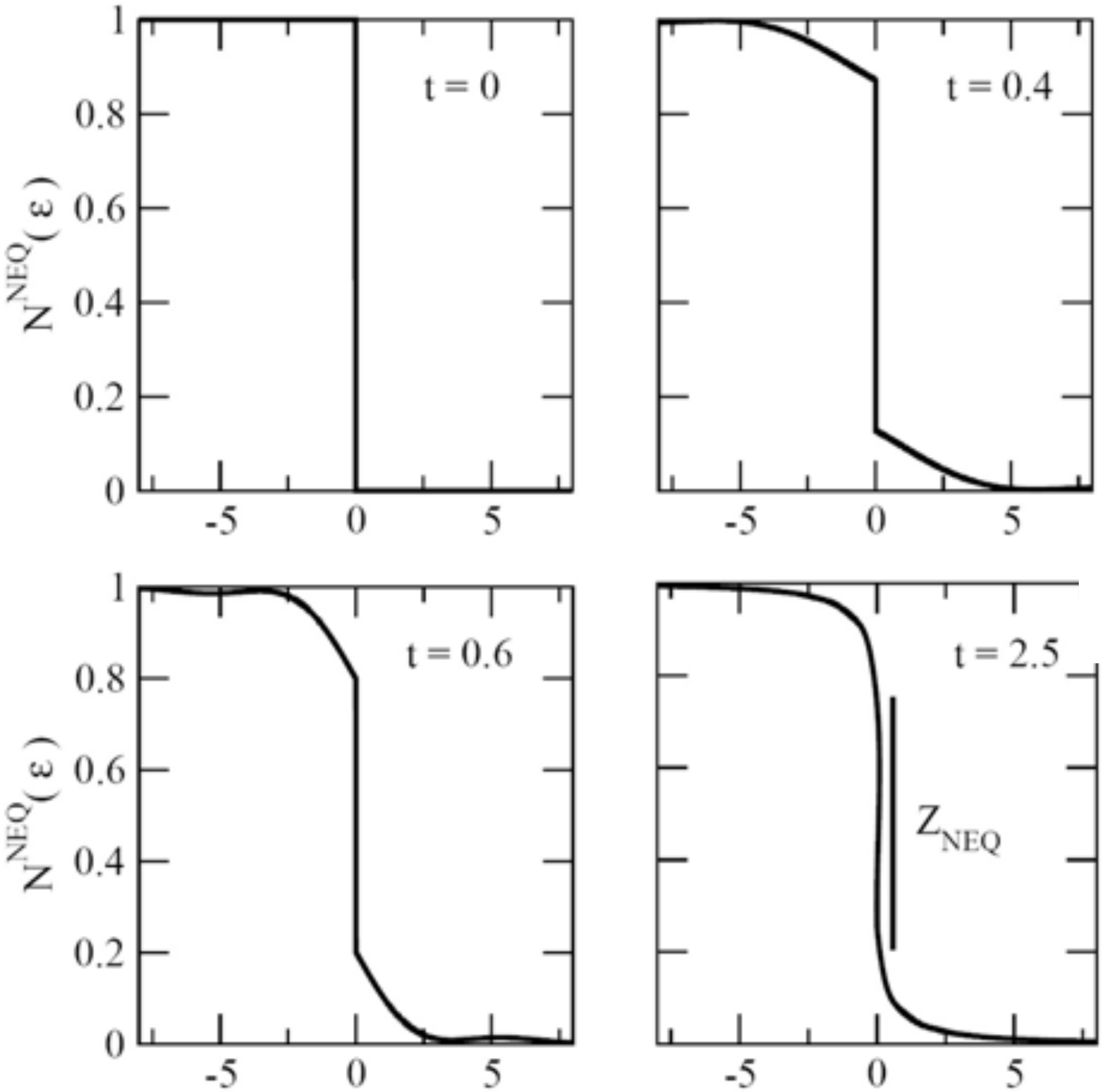
Prethermalization […] describes the very rapid establishment of [..] a kinetic temperature based on average kinetic energy […] the occupation numbers of individual momentum modes still show strong deviations from the late-time Bose-Einstein or Fermi-Dirac distribution.

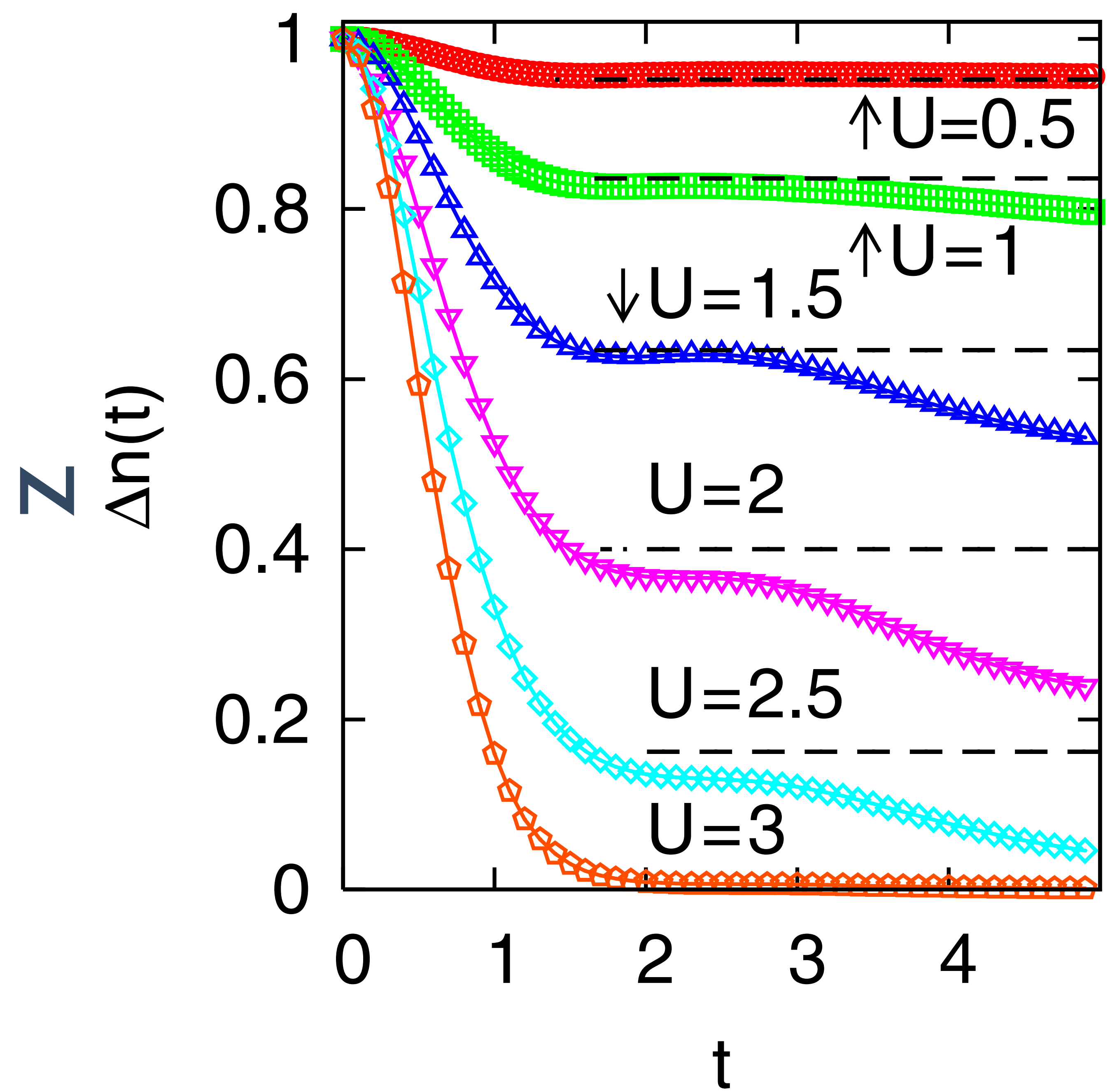
J Berges et al Phys Rev Lett 2004

Prethermalization in the Hubbard Model

$$H = \sum_{k,\sigma} \epsilon(k) c_k^\dagger c_k + \frac{U}{V}\theta(t) \sum_{kpq} c^\dagger_{k+q\uparrow} c^\dagger_{p-q\downarrow} c_{p\downarrow} c_{k\uparrow}$$

M Eckstein, M Kollar, &P Werner PRL (2009)





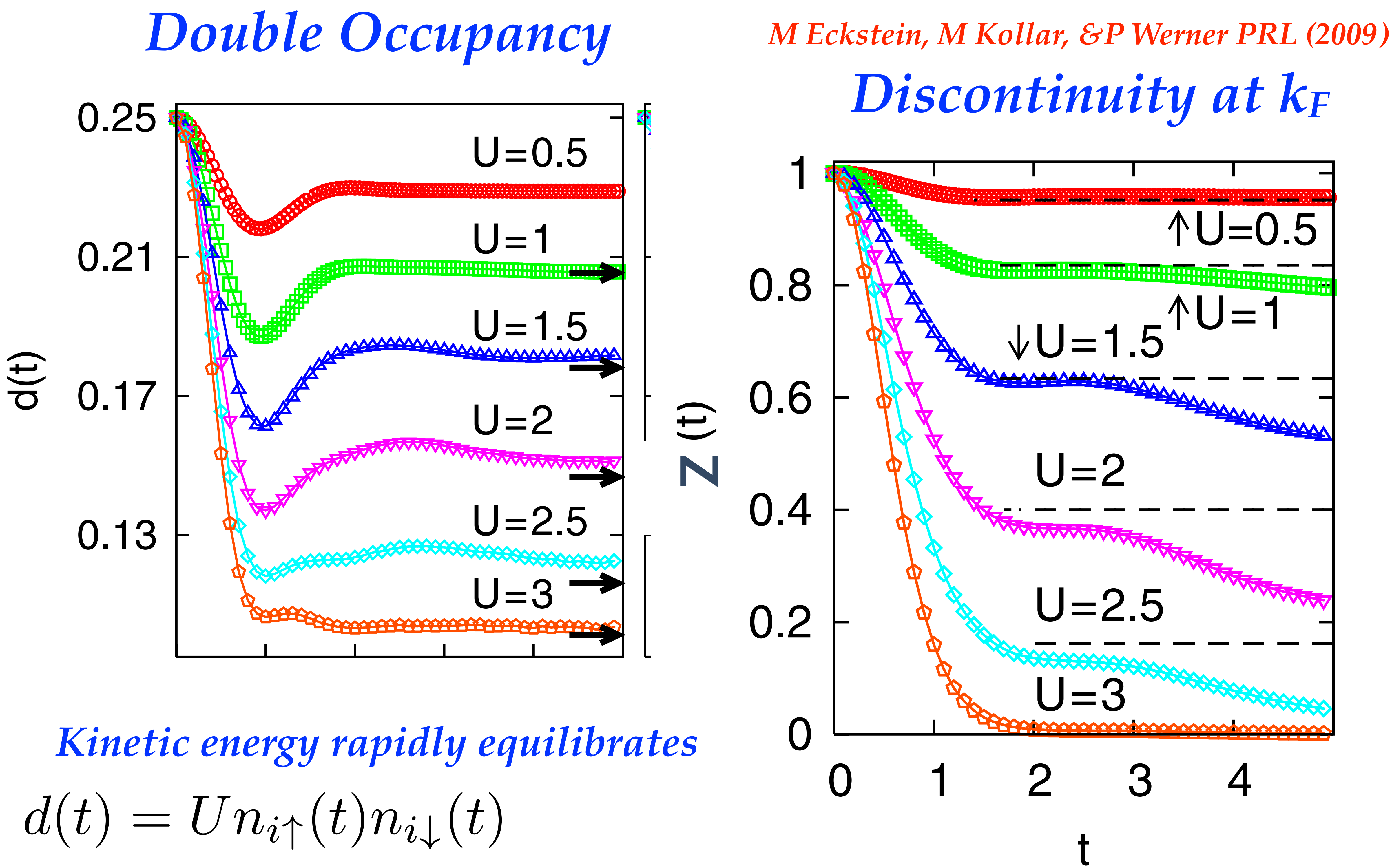
$$1 - Z_{\text{neq}} = 2(1 - Z_{\text{eq}})$$

M Moeckel & S Kehrein PRL (2006)

Prethermalization in the Hubbard Model

(in infinite dimensions)

M Eckstein, M Kollar, &P Werner PRL (2009)



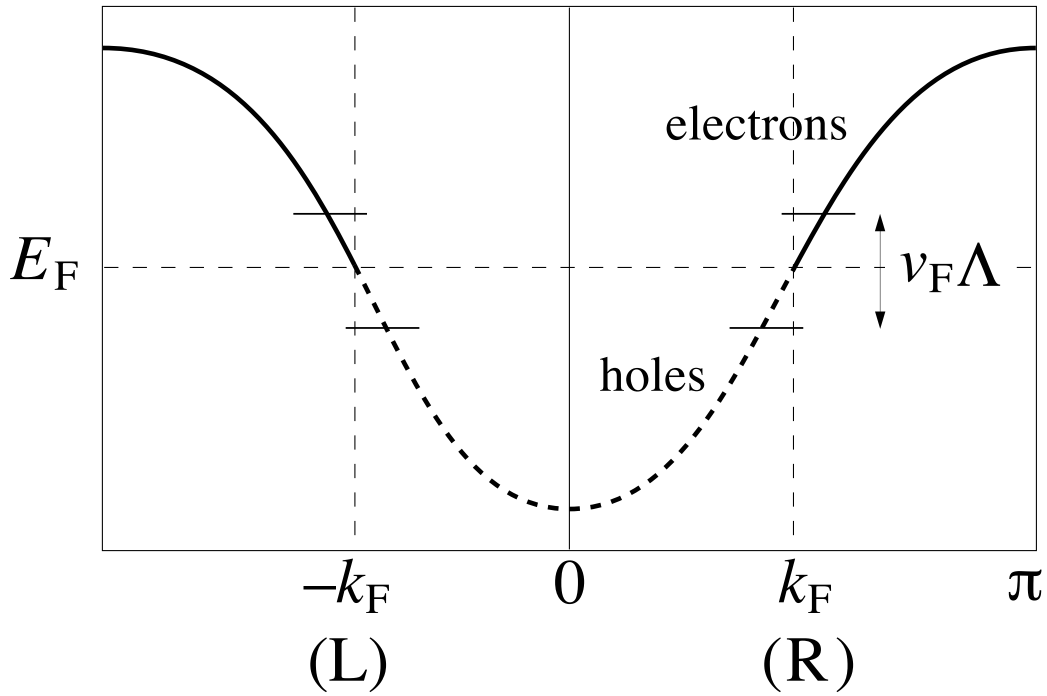
Kinetic energy rapidly equilibrates

$$d(t) = U n_{i\uparrow}(t) n_{i\downarrow}(t)$$

$$E = \langle K(t) \rangle + U N d(t) = \text{const.}$$

Two stage evolution to the Steady State

1D conductor



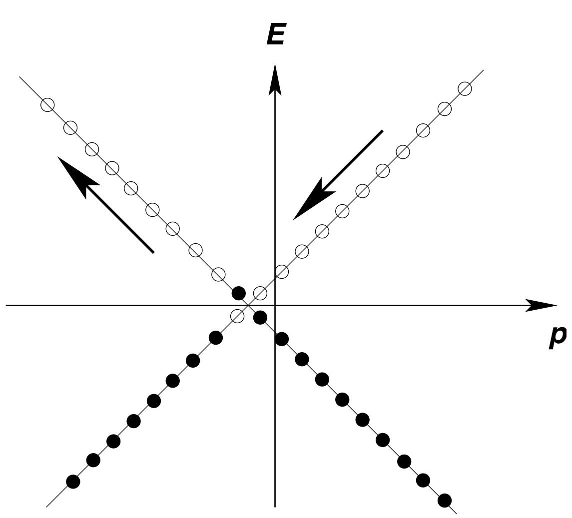
Quench of the electric field

$$\frac{dj(t)}{dt} + \frac{j(t)}{\tau} = \frac{e^2\rho_0}{m} E_0 \theta(t)$$

$$j(t \geq 0) = \frac{e^2\rho_0\tau}{m} E(1 - e^{-t/\tau})$$

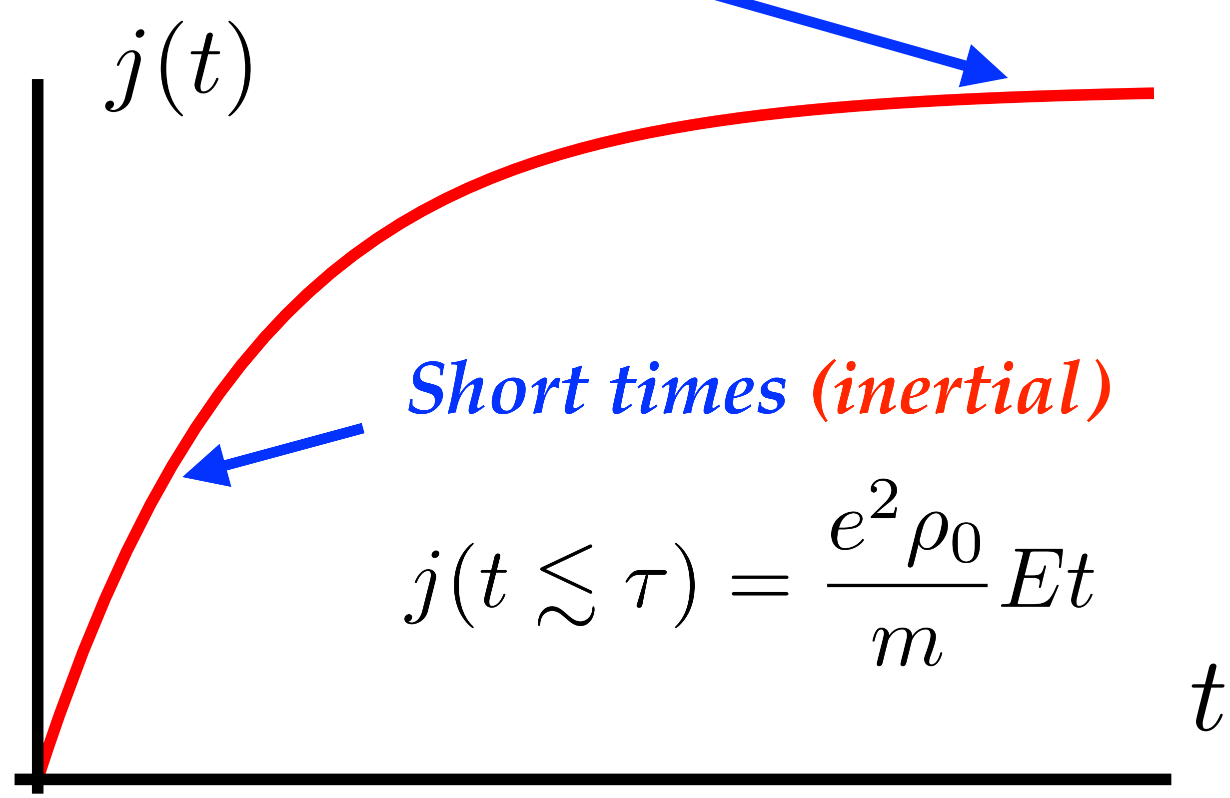
Chiral Anomaly

$$[\rho_R(q), \rho_R(-q')] = \frac{qL}{2\pi}\delta_{q,q'}$$



Long times (collision dominated)

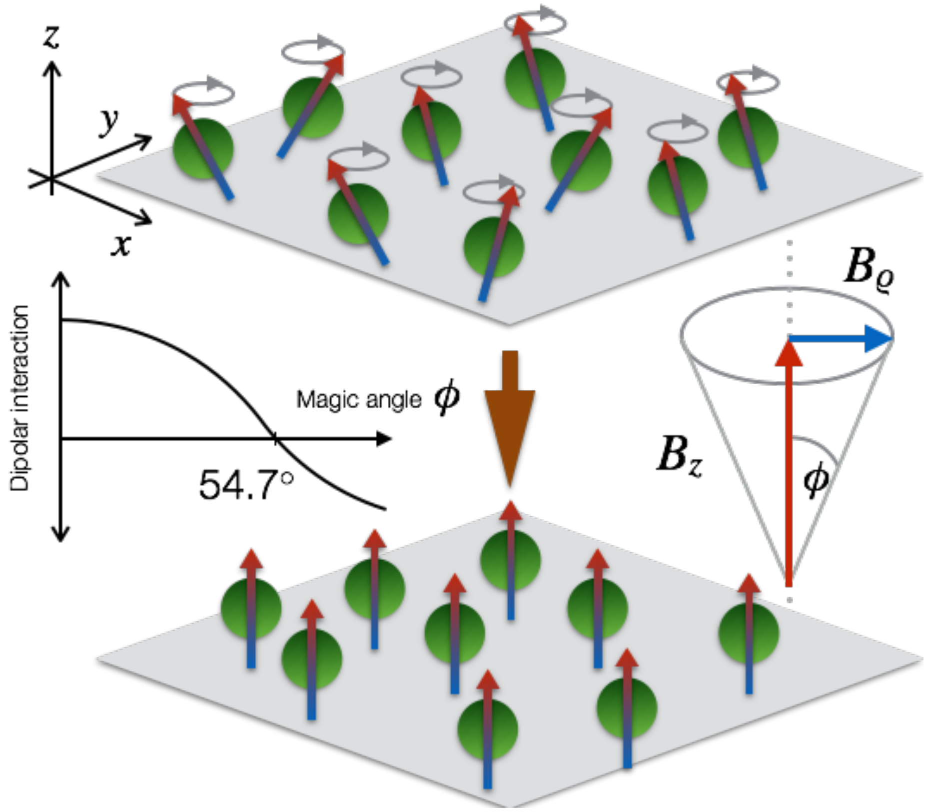
$$j(t \gtrsim \tau) = \frac{e^2\rho_0}{m} E\tau = \text{const.}$$



Short times (inertial)

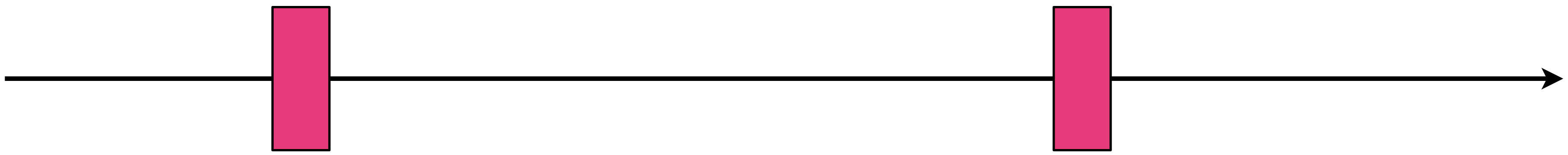
$$j(t \lesssim \tau) = \frac{e^2\rho_0}{m} Et$$

Pre-thermalization in a 2D Fermi gas with long range interactions



Quench in a 2D interacting Fermi Gas

N Nessi, A Iucci & MAC, arXiv:1401.1986



$$H_{\text{int}} = 0 \;\Rightarrow\; H_{\text{int}} \neq 0$$

Hamiltonian for t ≤ 0

$$H_0 = \sum_{\boldsymbol{k}} \epsilon(\boldsymbol{k}) c^\dagger_{\boldsymbol{k}} c_{\boldsymbol{k}}$$

Hamiltonian for t > 0

$$H = H_0 + H_{\text{int}} = \sum_{\boldsymbol{k}} \epsilon(\boldsymbol{k}) c^\dagger_{\boldsymbol{k}} c_{\boldsymbol{k}} + \frac{1}{V} \sum_{\boldsymbol{kpq}} f(q) c^\dagger_{\boldsymbol{k}+\boldsymbol{q}} c^\dagger_{\boldsymbol{p}-\boldsymbol{q}} c_{\boldsymbol{p}} c_{\boldsymbol{k}}$$

Long-range (non-singular) interaction $\quad q_c^{-1} \gg k_F^{-1}$

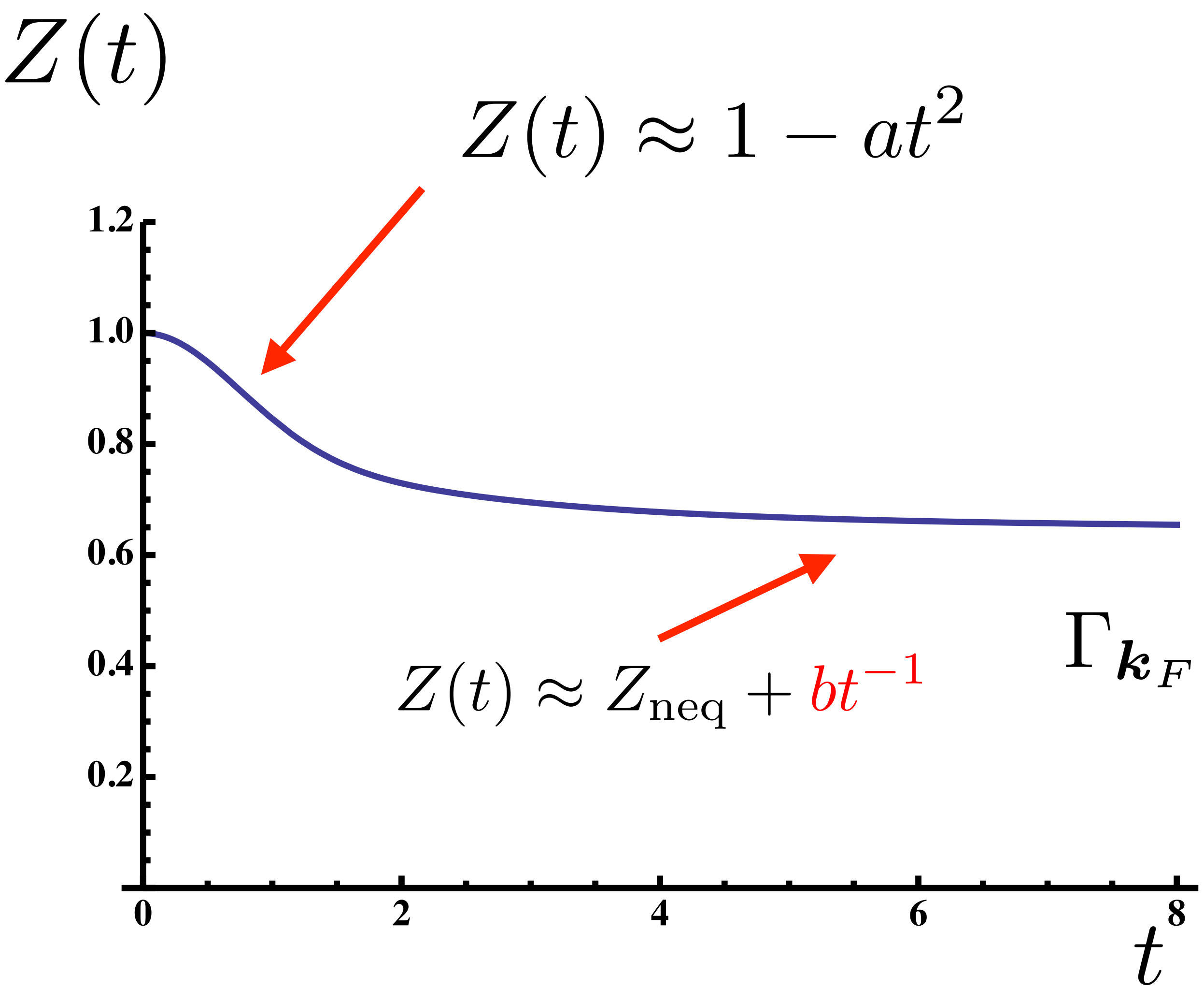
$$f(q) = f_0 F(q) \qquad F(q \gg q_c) \sim e^{-q/q_c} \qquad F(q=0) = \text{const.}$$

Pre-thermalization, perturbative? YES!

Perturbation theory valid for

$t \ll \tau_{coll} \sim [f_0^3 (N(0))^{-2}]^{-1}$

M Moeckel & S Kehrein PRL (2006)
M Eckstein, M Kollar &P Werner PRL (2009)
M Stark & M Kollar, arxiv:1308161

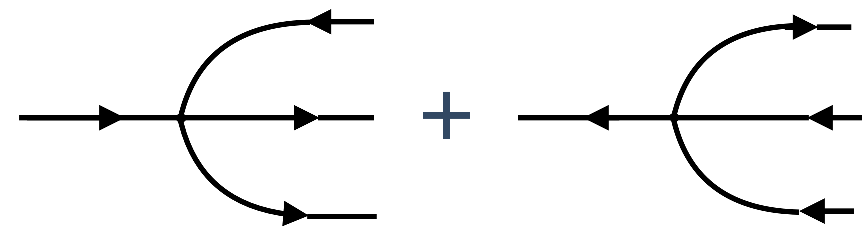


$$1 - Z_{\text{neq}} = 2(1 - Z_{\text{eq}})$$

Perturbation theory $[n(k), \rho_0] = 0 \Rightarrow O(f_0^2)$

$$Z(t) = 1 - 4 \int \frac{dE}{2\pi} \frac{\Gamma_{\boldsymbol{k}_F}(E)}{E^2} \sin^2\left(\frac{Et}{2}\right)$$

(Off-shell) scattering rate

$\Gamma_{\boldsymbol{k}_F}(E) =$  $= O(f_0^2)$

Fermi Liquid theory (Luttinger, PR 1963)

$$\Gamma_{\mathbf{k}}(E) \sim E^2$$

PT tell us there is a pre-thermalization plateau, but WHY?

Making an Interacting Gas Exactly Solvable

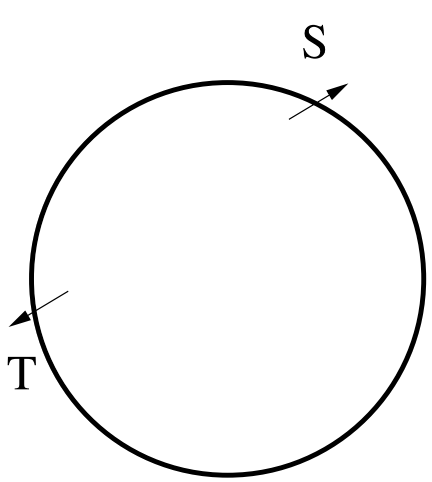
Hamiltonian for t > 0

Contains inelastic processes

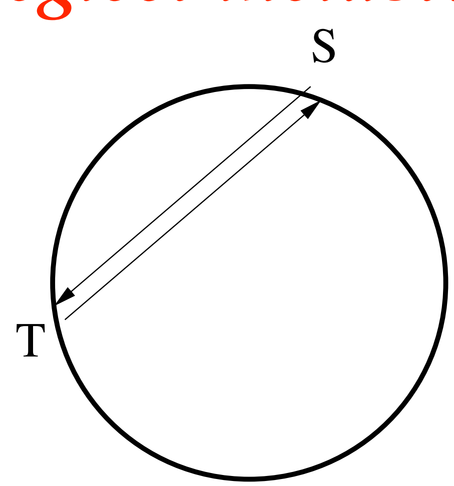
$$H = H_0 + H_{\text{int}} = \sum_{\boldsymbol{k}} \epsilon(\boldsymbol{k}) c^\dagger_{\boldsymbol{k}} c_{\boldsymbol{k}} + \frac{1}{V} \sum_{\boldsymbol{kpq}} f(q) c^\dagger_{\boldsymbol{k+q}} c^\dagger_{\boldsymbol{p-q}} c_{\boldsymbol{p}} c_{\boldsymbol{k}}$$

Fermi-liquid-like truncation of the bare Hamiltonian

(= Neglect inelastic processes at short times)



Forward



Exchange

FS Bosonization $J_{\boldsymbol{S}}(\boldsymbol{q}) \sim \sum_{\boldsymbol{k} \in \boldsymbol{S}} c^\dagger_{\boldsymbol{k+q}} c_{\boldsymbol{k}}$

$$H = \frac{1}{2} \sum_{\boldsymbol{S},\boldsymbol{T},\boldsymbol{q}} J_{\boldsymbol{S}}(\boldsymbol{q}) \left(\frac{v_F}{\Omega} \delta_{\boldsymbol{S},\boldsymbol{T}} + \frac{f(q)}{V} \right) J_{\boldsymbol{T}}(-\boldsymbol{q})$$

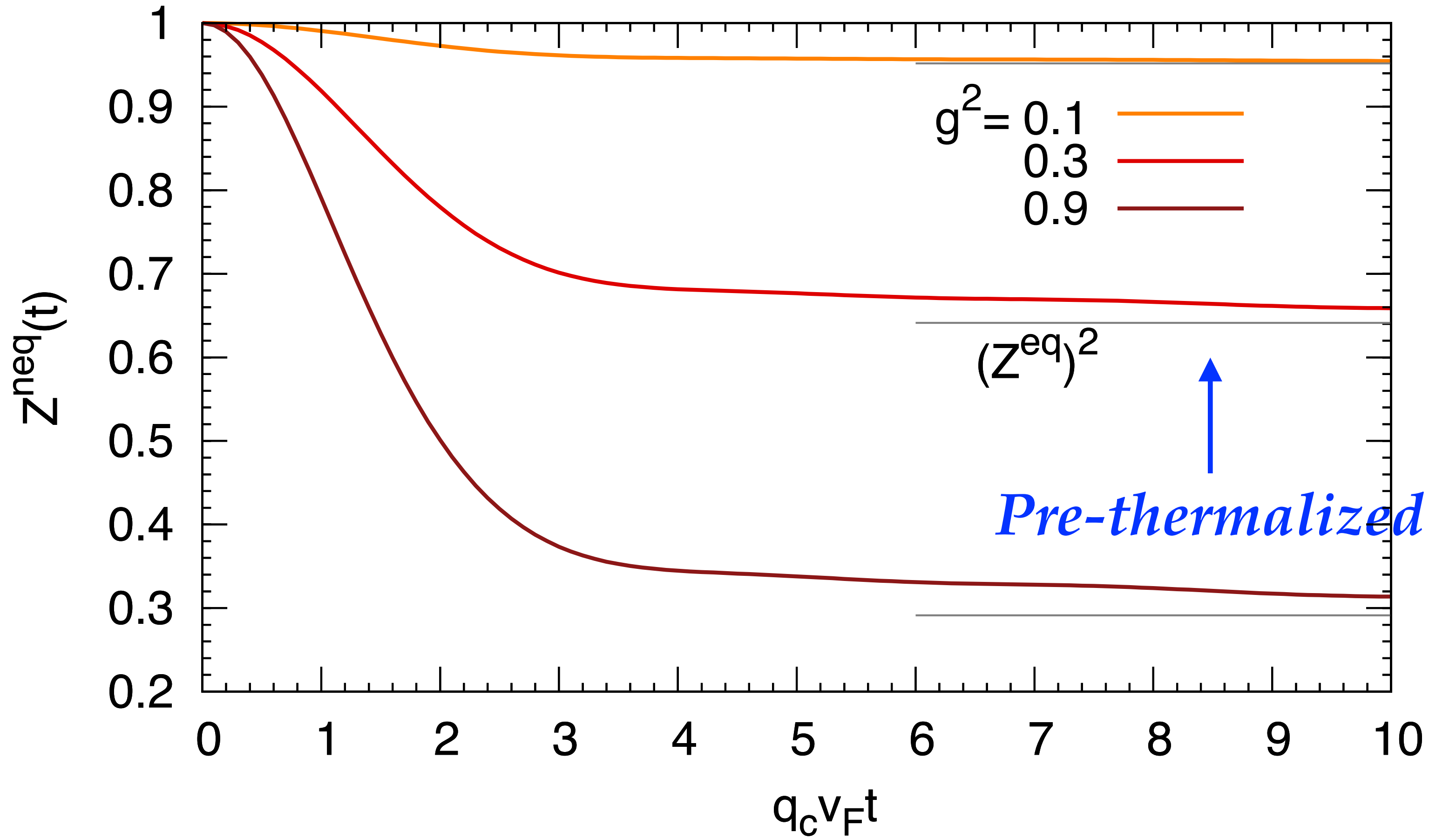
$$[J_{\boldsymbol{S}}(\boldsymbol{q}), J_{\boldsymbol{T}}(\boldsymbol{p})] = \delta_{\boldsymbol{S},\boldsymbol{T}} \delta_{\boldsymbol{q+p}}\, \Omega\, \hat{\boldsymbol{n}}_{\boldsymbol{S}} \cdot \boldsymbol{q}$$

Eigenmodes $H = \sum_{l,\boldsymbol{q}} \omega(\boldsymbol{q}) \alpha^\dagger(\boldsymbol{q}) \alpha_l(\boldsymbol{q})$

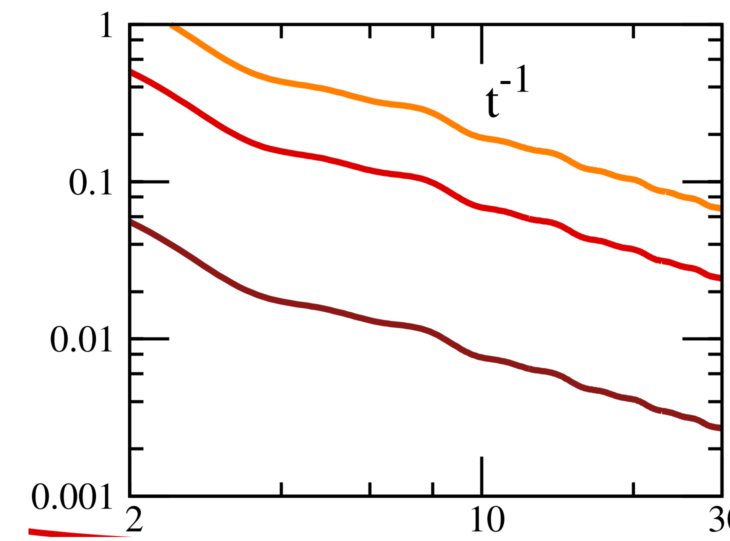
Houghton, Kwon & Marston Adv. in Phys. (2000)
+Haldane, Castro Neto & Fradkin,
Kim & Wen & Lee, …

Interaction quench in a 2D Fermi Gas

N Nessi, A Iucci, and MAC, arXiv:1401.1986



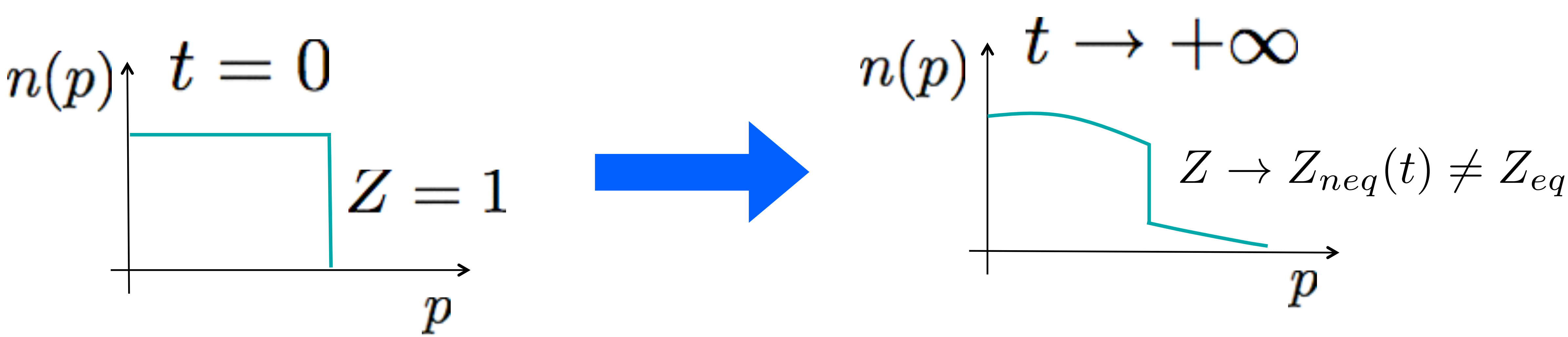
$Z^{\mathbf{eq}} = 1 + O(g^2)$



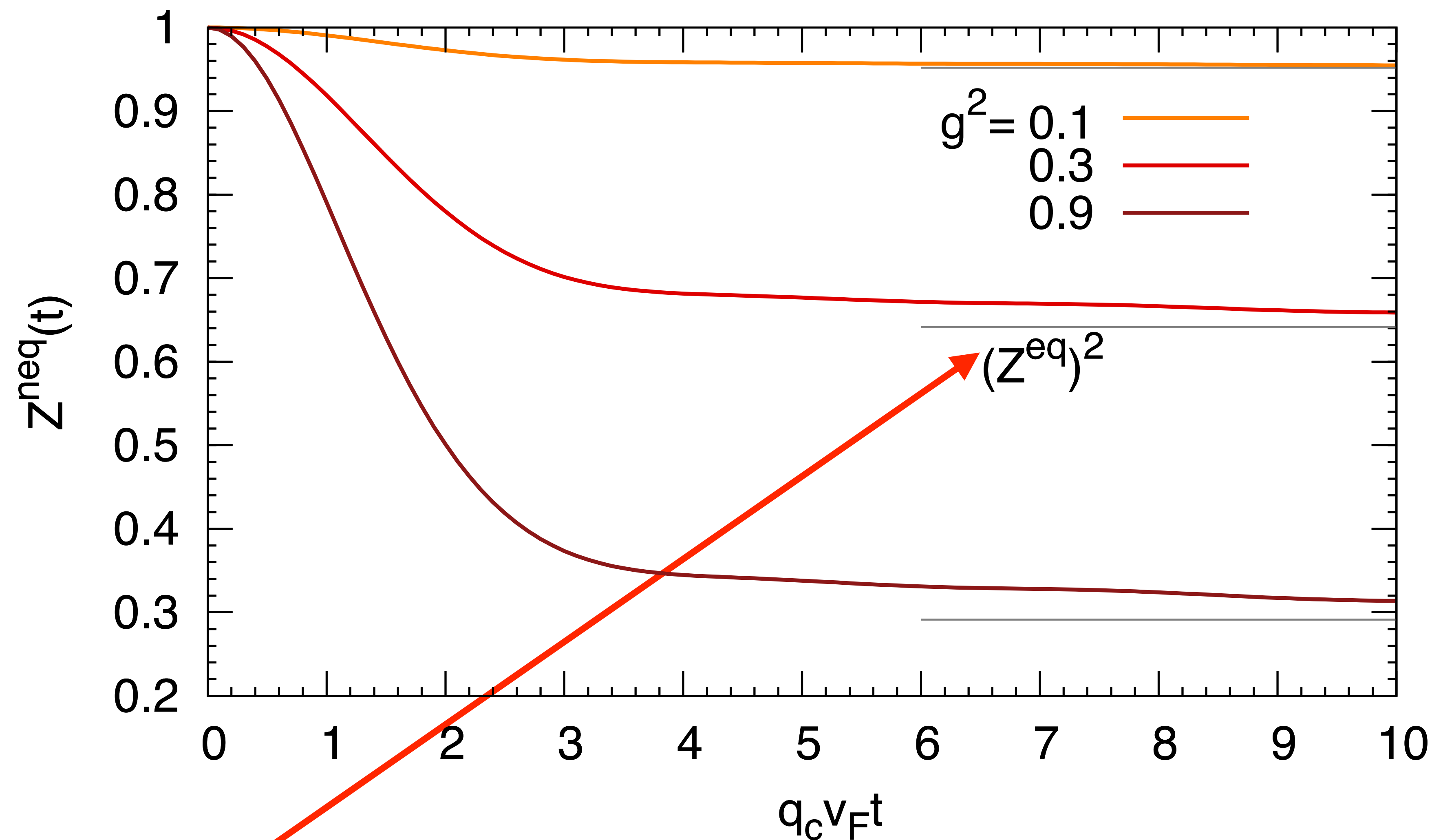
Kinetic Energy rapidly equilibrates

$$\langle K(t)\rangle \to \langle\Psi_0|(H - E_{\rm gs})|\Psi_0\rangle + O(g^3)$$

Momentum distribution



Prethermalized State = GGE



How do we describe the pre-thermalized state?

Generalized Gibbs Ensemble

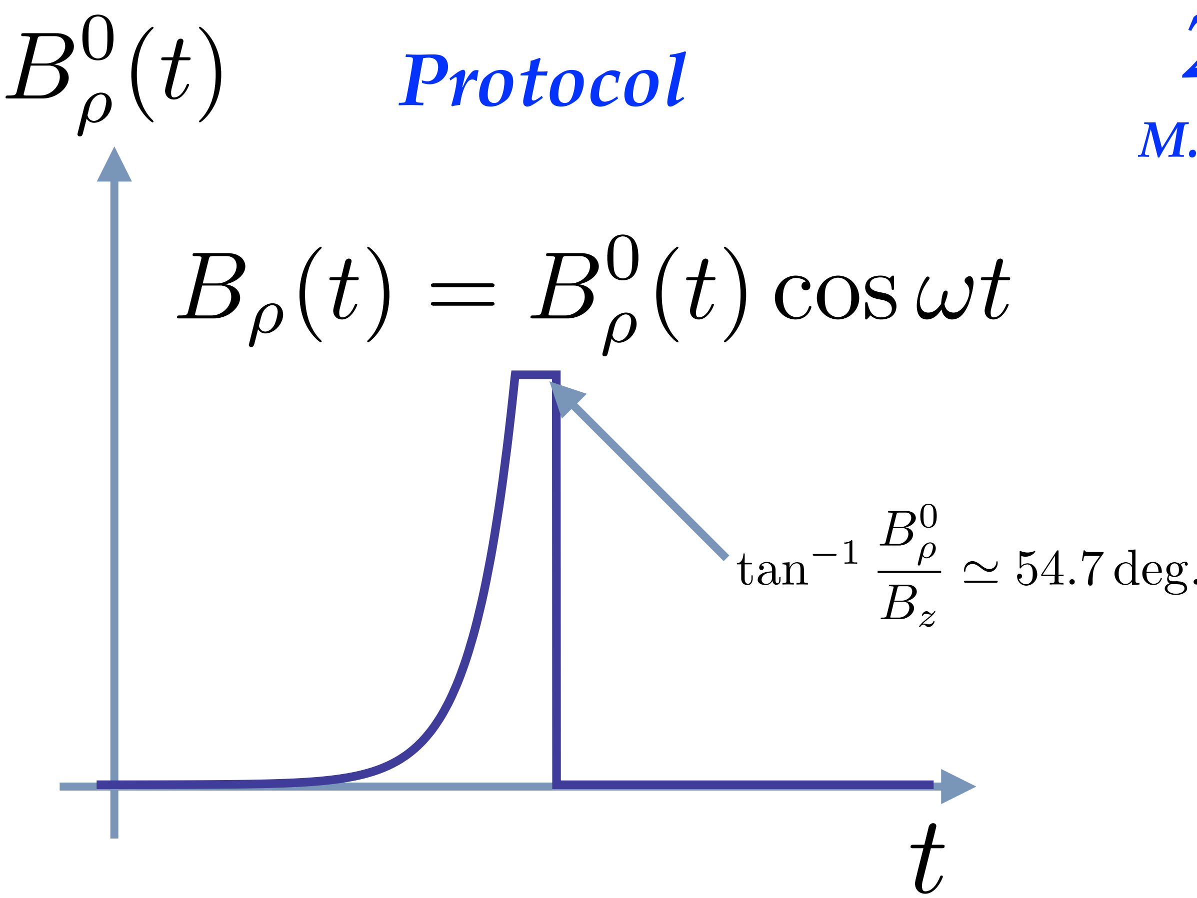
$$\rho_{\rm GGE} = \frac{1}{Z_{\rm GGE}} \exp\left[\sum_{l,\boldsymbol{q}} \lambda_l(\boldsymbol{q}) I_l(\boldsymbol{q})\right]$$

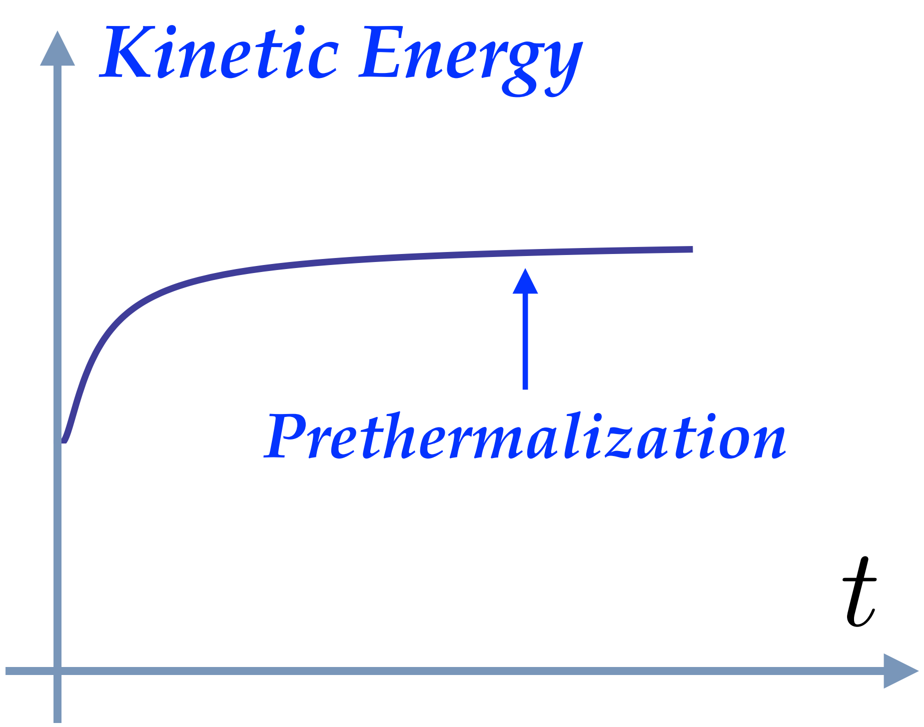
Eigenmodes $H = \sum_{l,\boldsymbol{q}} \omega(\boldsymbol{q}) \alpha^\dagger(\boldsymbol{q}) \alpha_l(\boldsymbol{q})$

$$I_l(\boldsymbol{q}) = \alpha_l^\dagger(\boldsymbol{q}) \alpha_l(\boldsymbol{q})$$

Fermi Surface eigenmodes

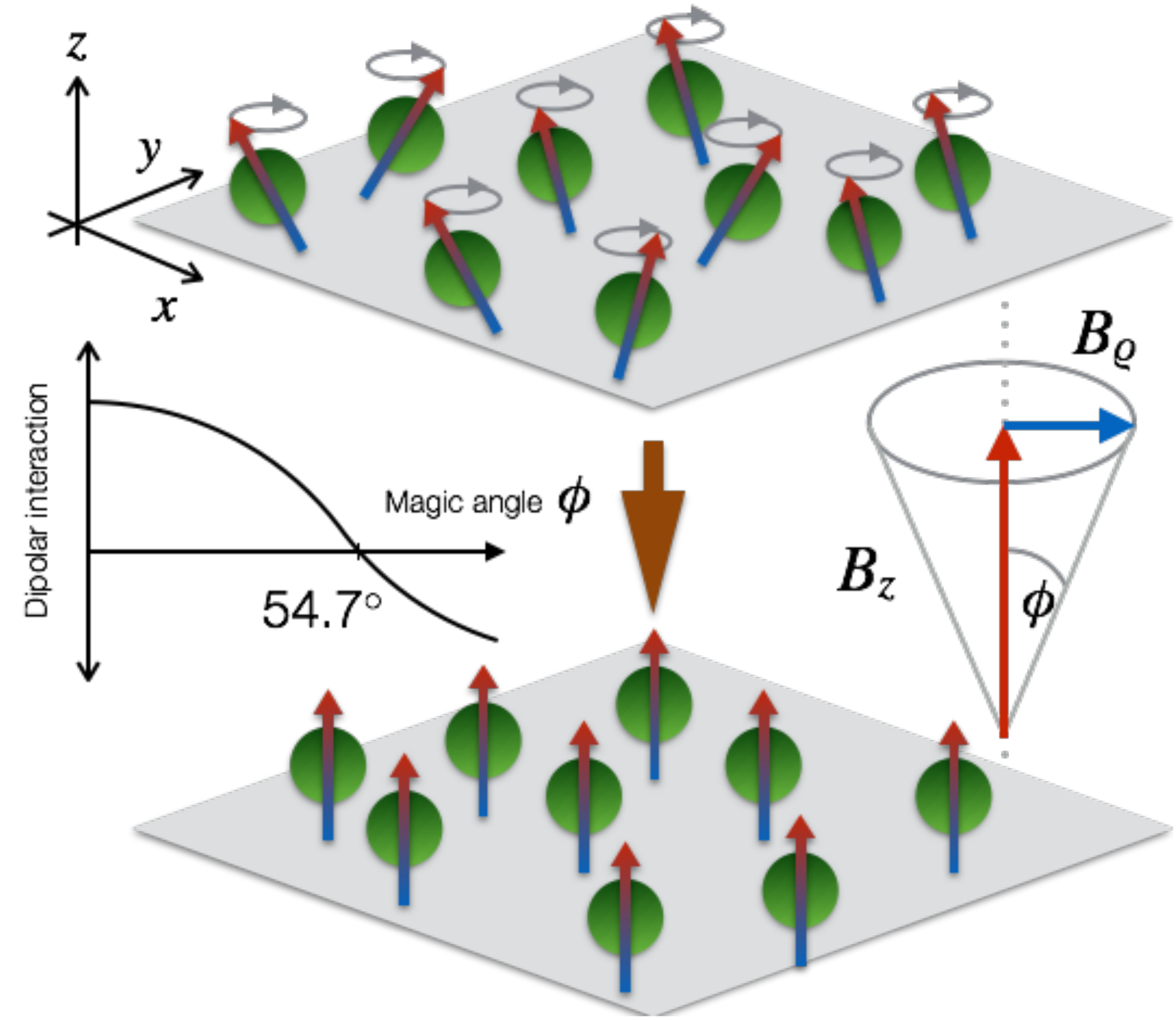
Interaction quench in a 2D Dipolar Gas





2D Dipolar Fermi Gas (^{167}Er?)

M. Ueda: Beware of inhomogeneity (Einstein-de Haas effect)



N Nessi, A Iucci, and MAC, arXiv:1401.1986

Conclusions

- *Fermions in 1D exhibit very slow relaxation dynamics following a quantum quench. At $T = 0$, the discontinuity at the Fermi energy vanishes as a power law.*
- *Generally speaking, systems that can be described in terms of quadratic Hamiltonians of Bosonic or Fermionic elementary excitations thermalize to a Generalized Gibbs Ensemble (GGE).*
- *Dephasing is the key mechanism and erases information about off diagonal normal mode correlations and leads to an asymptotic state described by the GGE.*
- *Even systems that eventually do thermalize can exhibit an intermediate regime known as pre-thermalization. The system dynamics may be describable for short times by a quadratic Hamiltonian, and therefore the pre-thermal state will be described by the GGE*
- *A dipolar Fermi liquid subject to a weak to moderate interaction quench should exhibit a pre-thermalized regime characterized by the kinetic energy rapidly reaching a constant value whilst the momentum distribution has not.*