Quantum Quenches in a model with tuneable integrability breaking interactions

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in collaboration with

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Outline

A. Quantum quenches and integrability.

B. Interacting Peierls Insulator.

C. Integrable quenches.

D. Effects of integrability breaking interactions: "prethermalization plateaux"

E. Characterization of the plateaux.

F. Termalization?

Quantum Quenches

A. Consider an **isolated** quantum system in the **thermodynamic limit**; Hamiltonian H(h) (short-ranged), h e.g. bulk magnetic field

B. Prepare the system in the ground state $|\psi\rangle$ of $H(h_0)$

C. At time t=0 change the Hamiltonian to H(h)

D. (Unitary) time evolution $|\psi(t)\rangle = \exp(-iH(h)t)\,|\psi\rangle$

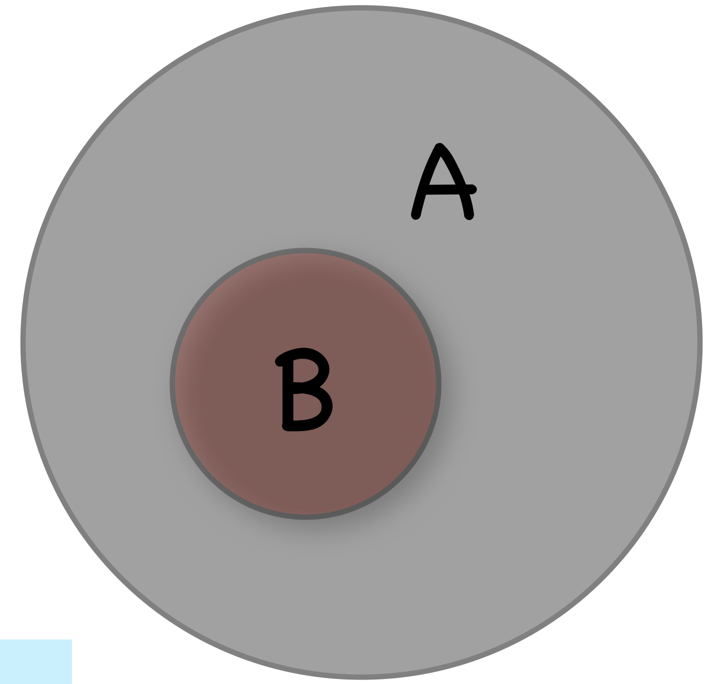
E. Goal: study time evolution of local (in space) observables

$\langle\psi(t)|\mathcal{O}(x)|\psi(t)\rangle$, $\langle\psi(t)|\mathcal{O}_1(x)\mathcal{O}_2(y)|\psi(t)\rangle$ etc

Time evolution after a quantum quench

Density matrix of entire system: $\rho(t)$

Reduced density matrix: $\rho_B(t) = \mathrm{tr}_A\, \rho(t)$



ρ_B contains **all** local correlation functions in B:

$$\rho_B(t) = \frac{1}{2^\ell} \sum_{\alpha_1,\ldots,\alpha_\ell} \mathrm{Tr}\left[\rho(t)\ \sigma_1^{\alpha_1} \ldots \sigma_\ell^{\alpha_\ell}\right]\ \sigma_1^{\alpha_1} \ldots \sigma_\ell^{\alpha_\ell}$$

$\alpha_j = 0, x, y, z$

for $B = [1, \ldots, \ell]$ in a spin-1/2 quantum spin chain

Late time behaviour in generic systems: Thermalization

Deutsch '91, Srednicki '94,...

- Define **Gibbs ensemble** for the entire system A∪B :

$$\rho_G = \frac{1}{Z} e^{-\beta H(h)}$$

- β fixed by:

$$\lim_{L\to\infty} \frac{\mathrm{Tr}\left(\rho_G \; H(h)\right)}{L} = \lim_{L\to\infty} \frac{\mathrm{Tr}\left(\rho(0) \; H(h)\right)}{L}$$

- Reduced density matrix for B:

$$\rho_{G,B} = \mathrm{Tr}_A \left(\rho_G\right)$$

The system **thermalizes** if for any finite subsystem B

$$\rho_B(\infty) = \rho_{G,B}$$

$t \to \infty$ behaviour in integrable systems: GGE

Rigol et al '07

Let I_m be integrals of motion $[I_m, I_n]=[I_m, H(h)]=0$

Define GGE density matrix by: $\rho_{gG}=\exp(-\sum \lambda_m I_m)/Z_{gG}$

λ_m **fixed** by $$\lim_{L\to\infty}\frac{1}{L}\mathrm{Tr}\left[\rho_{gG}\ I_m\right] = \lim_{L\to\infty}\frac{1}{L}\mathrm{Tr}\left[\rho(0)\ I_m\right]$$

Reduced density matrix of B: $\rho_{gG,B}=\mathrm{tr}_A[\rho_{gG}]$

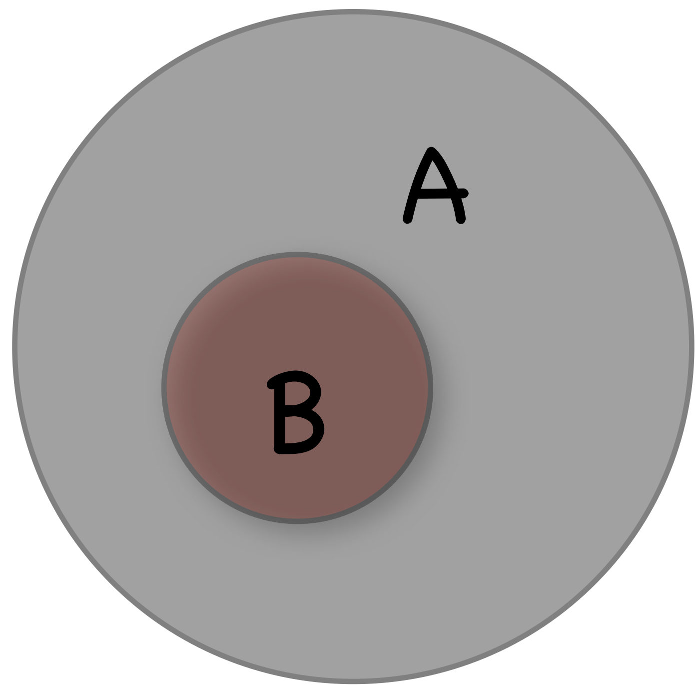
The system is described by a GGE if for any finite subsystem B

$$\rho_B(\infty)= \rho_{gG,B}$$

Barthel & Schollwöck '08

Cramer, Eisert et al '08

Physical Picture

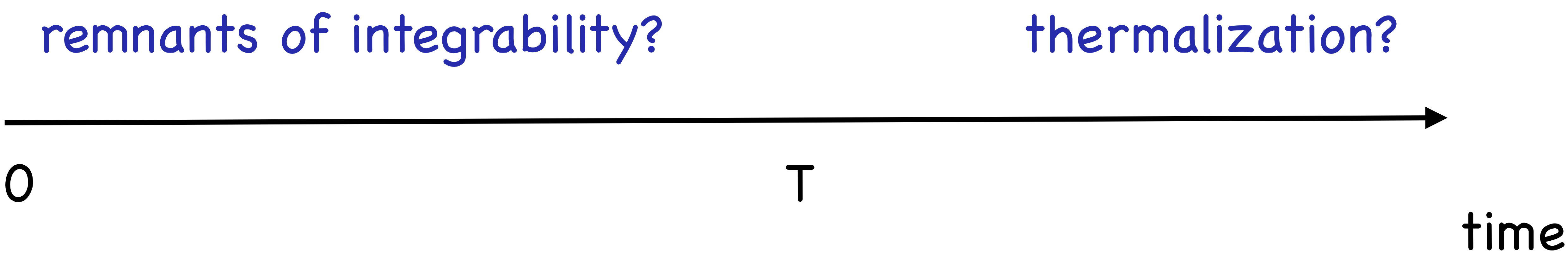


Thermalization: A acts as a heat bath with T_{eff}

Integrable models: A is **not** a standard heat bath: ∞ information about the initial state is retained.

Question:

What happens if we "weakly" break integrability?



Question:

What happens if we "weakly" break integrability?

remnants of integrability? thermalization?

0 T time

Moeckel&Kehrein '08
Kollar et al '11
Marcuzzi et al '13
Brandino et al '13
Nessi et al '14

Manmana et al '07
Kollath et al '07
Rigol& Santos '09, '10

Interacting Peierls Insulator

- Want a D=1 model so that we can do tDMRG;
- Preserve U(1) to aid tDMRG;
- 2 tuneable parameters: one for quench in integrable model, one to break integrability;
- **integrable model = free theory for simplicity**

$$H(\delta, U) = -J\sum_{l=1}^{L}\left[1+\delta(-1)^l\right]\left(c_l^\dagger c_{l+1} + \text{h.c.}\right) + U\sum_{l=1}^{L} n_l n_{l+1}$$

$$H(\delta, 0) = \sum_{0<k<\pi}\sum_{\alpha=\pm}\epsilon(k,\delta)a_\alpha^\dagger(k)a_\alpha(k)$$

2 bands of free fermions

$$c_l = \frac{1}{\sqrt{L}}\sum_{k>0}\sum_{\alpha=\pm}\gamma_\alpha(l,k|\delta)a_\alpha(k)\ .$$

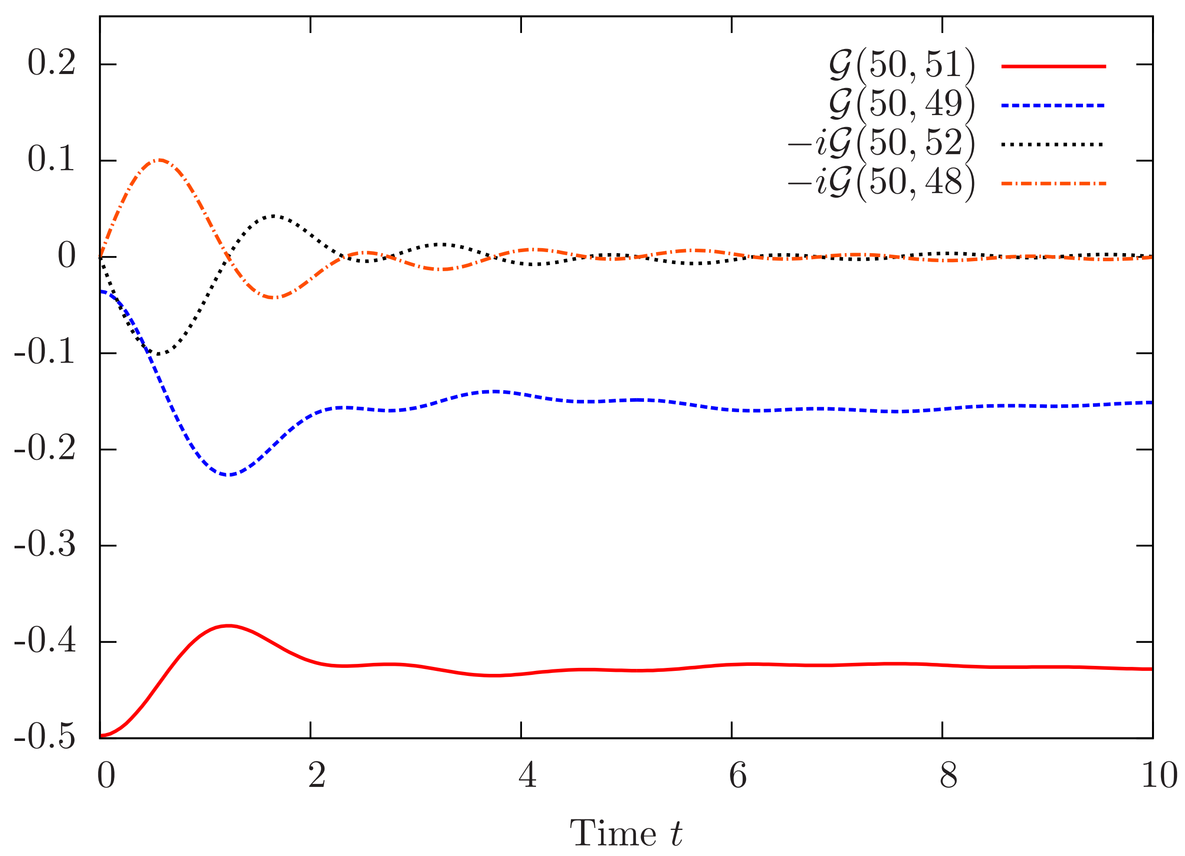
Quenches in the free theory

Prepare the system in the ground state $|\Psi_0\rangle$ of $H(\delta_i, 0)$

At t=0 quench $\delta_i \longrightarrow \delta_f$

Single particle Green's function $G(j,\ell,t) = \langle\Psi_0(t)|c_j^\dagger c_\ell|\Psi_0(t)\rangle$

$\delta_i = 0.75$
$\delta_f = 0.25$

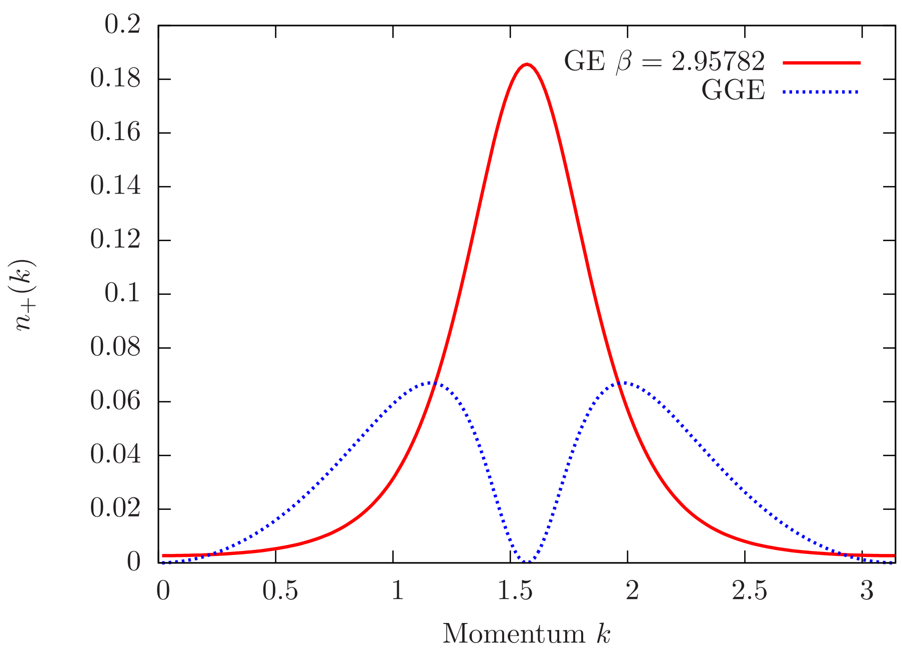


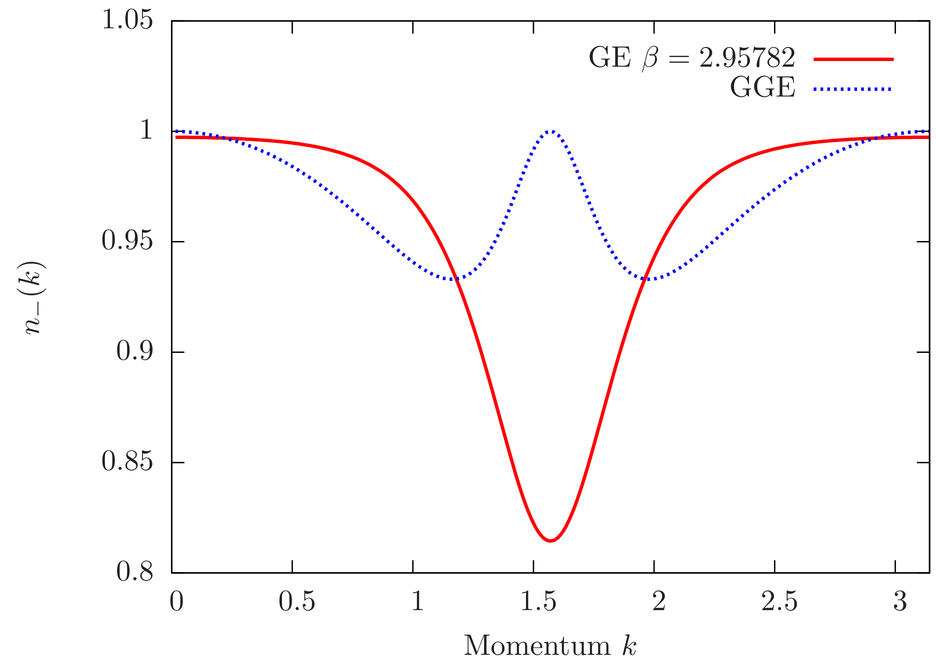
$$\lim_{t\to\infty} G_0(j,\ell,t) \sim g_1(j,\ell) + g_2(j,\ell)t^{-3/2} + \ldots$$

given by a GGE

Momentum occupation numbers for the two bands:

$\delta_i = 0.75$
$\delta_f = 0.25$





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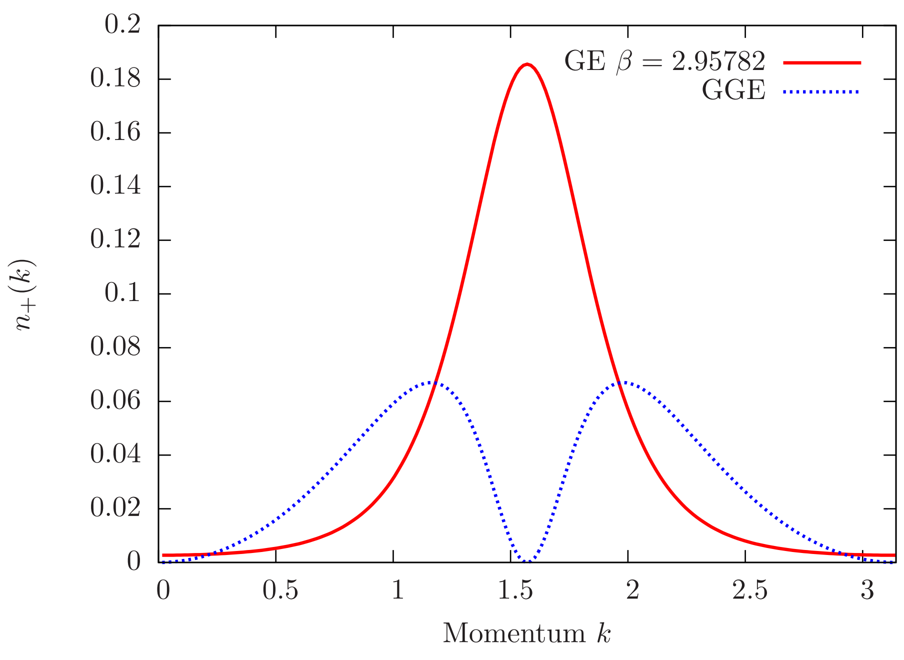
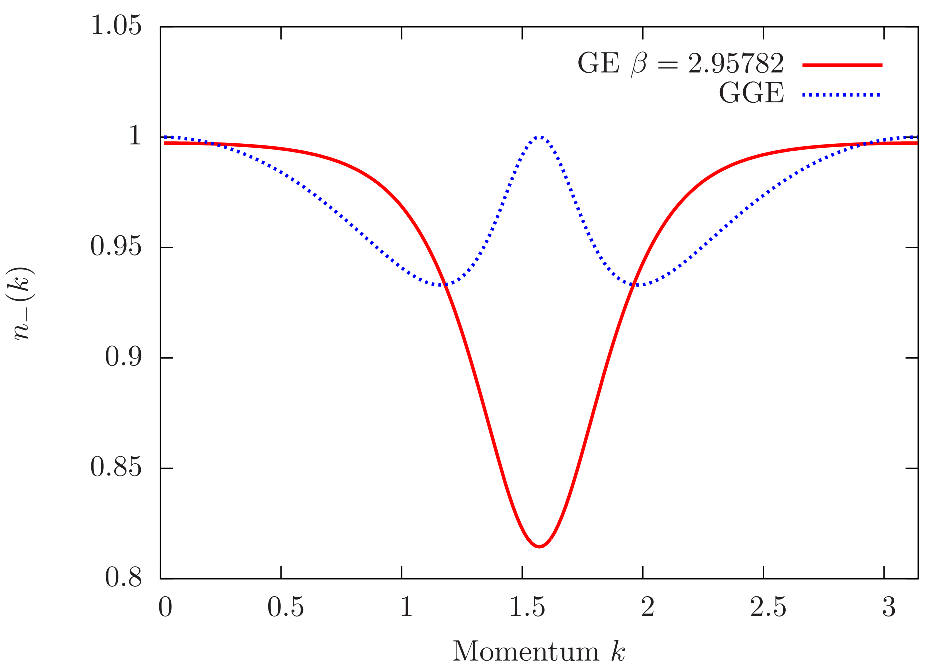
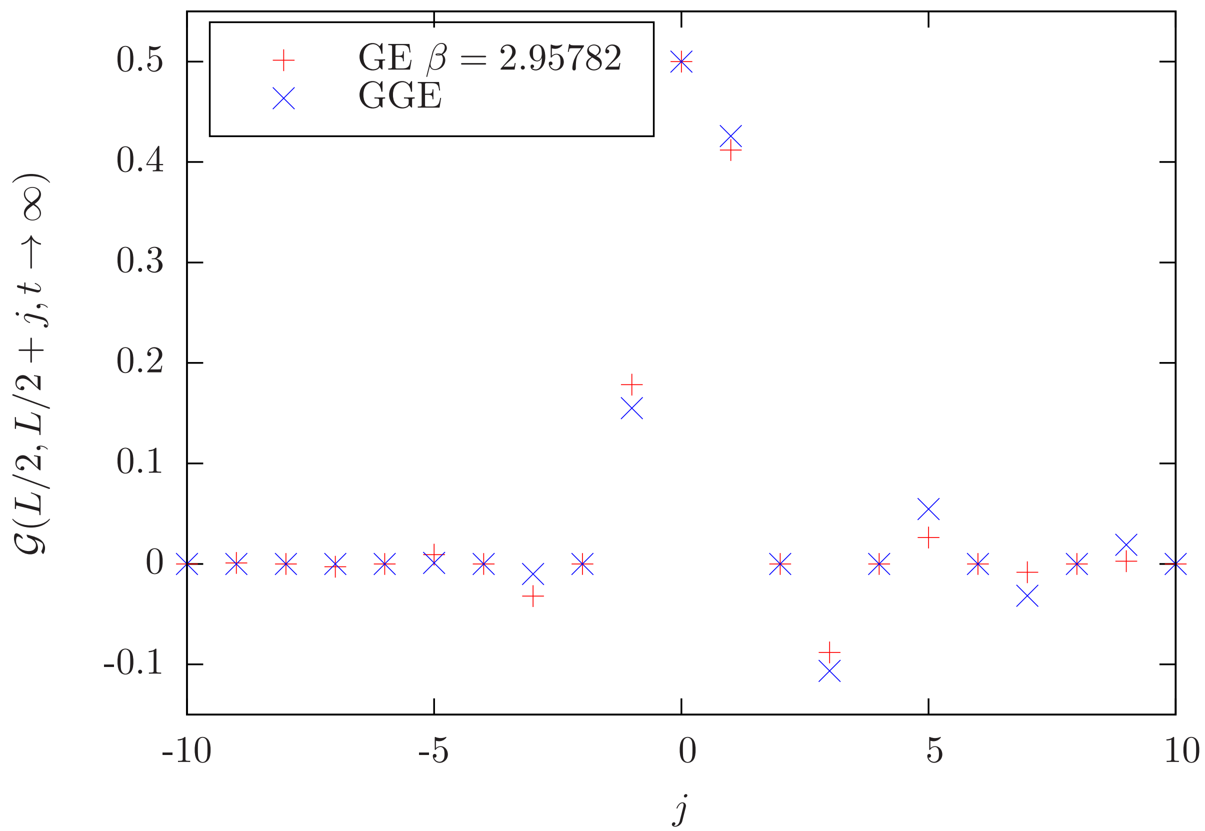




Very non-thermal!

Green's function at t=∞: $G\left(\frac{L}{2}, \frac{L}{2}+j, t \to \infty\right)$

$\delta_i = 0.75$
$\delta_f = 0.25$



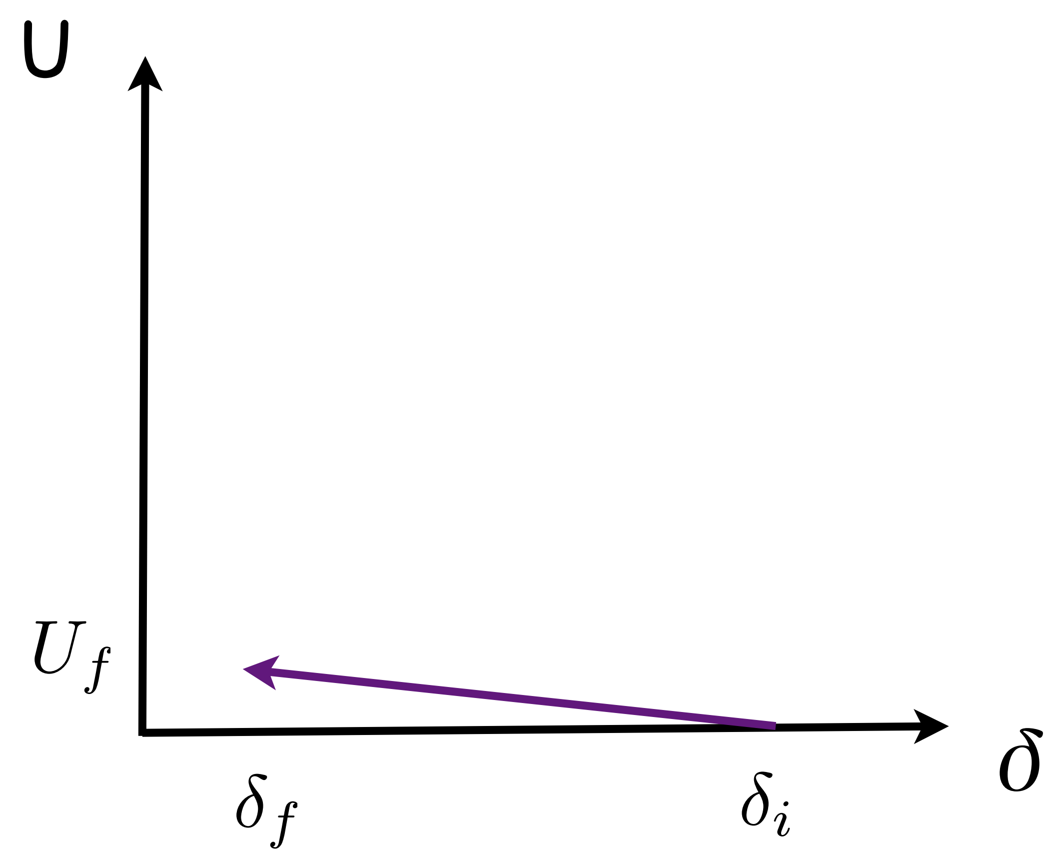
smaller differences.

Break integrability through Interactions

Prepare the system in the ground state $|\Psi_0\rangle$ of $H(\delta_i, 0)$

At t=0 quench $\delta_i \longrightarrow \delta_f \quad U_i = 0 \longrightarrow U_f > 0$

Single particle Green's function $G(j, \ell, t) = \langle \Psi_0(t) | c_j^\dagger c_\ell | \Psi_0(t) \rangle$



Analytic treatment through Continuous Unitary Transformation method

Glazek&Wilson '93; Wegner '93
Moeckel&Kehrein '08

- Express H in the form $H = \underbrace{H_0}_{\text{quadratic}} + \underbrace{H_{\rm int}}_{\propto U}$
- Construct unitarily equivalent family of Hamiltonians

$$H(B) = e^{\eta(B)} H e^{-\eta(B)}$$

- "Canonical" choice of generator $\eta(B) = [H_0(B), H_{\rm int}(B)]$

→ H(∞) is energy diagonal

- In practice expand H(B), η(B) in power series in U
- Local operators transform as $\mathcal{O}(B) = e^{\eta(B)} \mathcal{O} e^{-\eta(B)}$

In our case:

$$H_0(B) = \sum_{\alpha=\pm} \sum_{k>0} \epsilon_\alpha(k|B) a_\alpha^\dagger(k) a_\alpha(k)$$

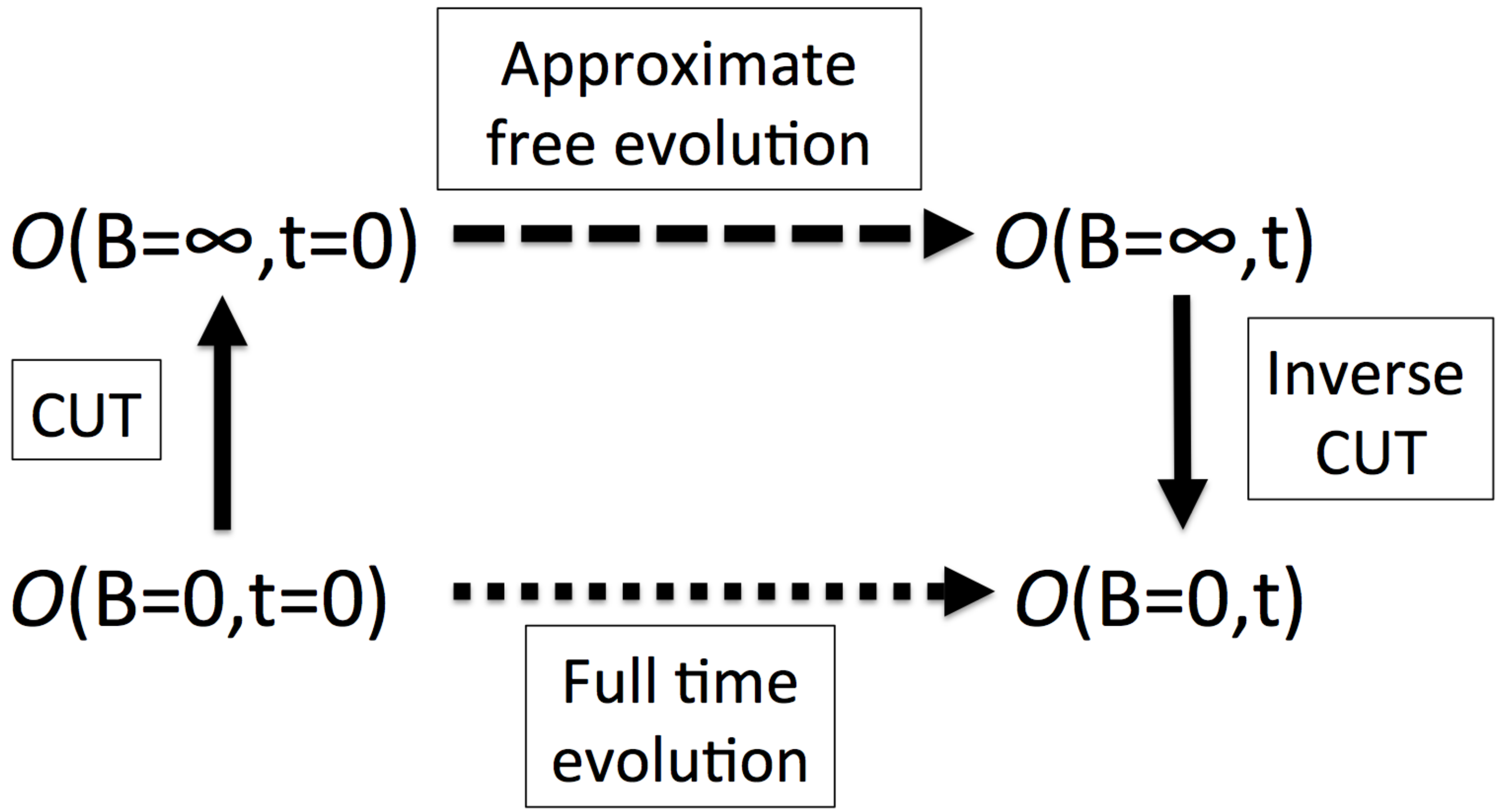
$$H_{\text{int}}(B) = \sum_{k_j>0} V_\alpha(\mathbf{k}|B) a_{\alpha_1}^\dagger(k_1) a_{\alpha_2}(k_2) a_{\alpha_3}^\dagger(k_3) a_{\alpha_4}(k_4) + \ldots$$

$$\epsilon_\alpha(k|B) = \epsilon_\alpha(k|B=0)$$

$$V_\alpha(\mathbf{k}|B) = V_\alpha(\mathbf{k}|B=0) e^{-B\left(\epsilon_{\alpha_1}(k_1|B) - \epsilon_{\alpha_2}(k_2|B) + \epsilon_{\alpha_3}(k_3|B) - \epsilon_{\alpha_4}(k_4|B)\right)^2}$$

energy diagonal at B=∞!

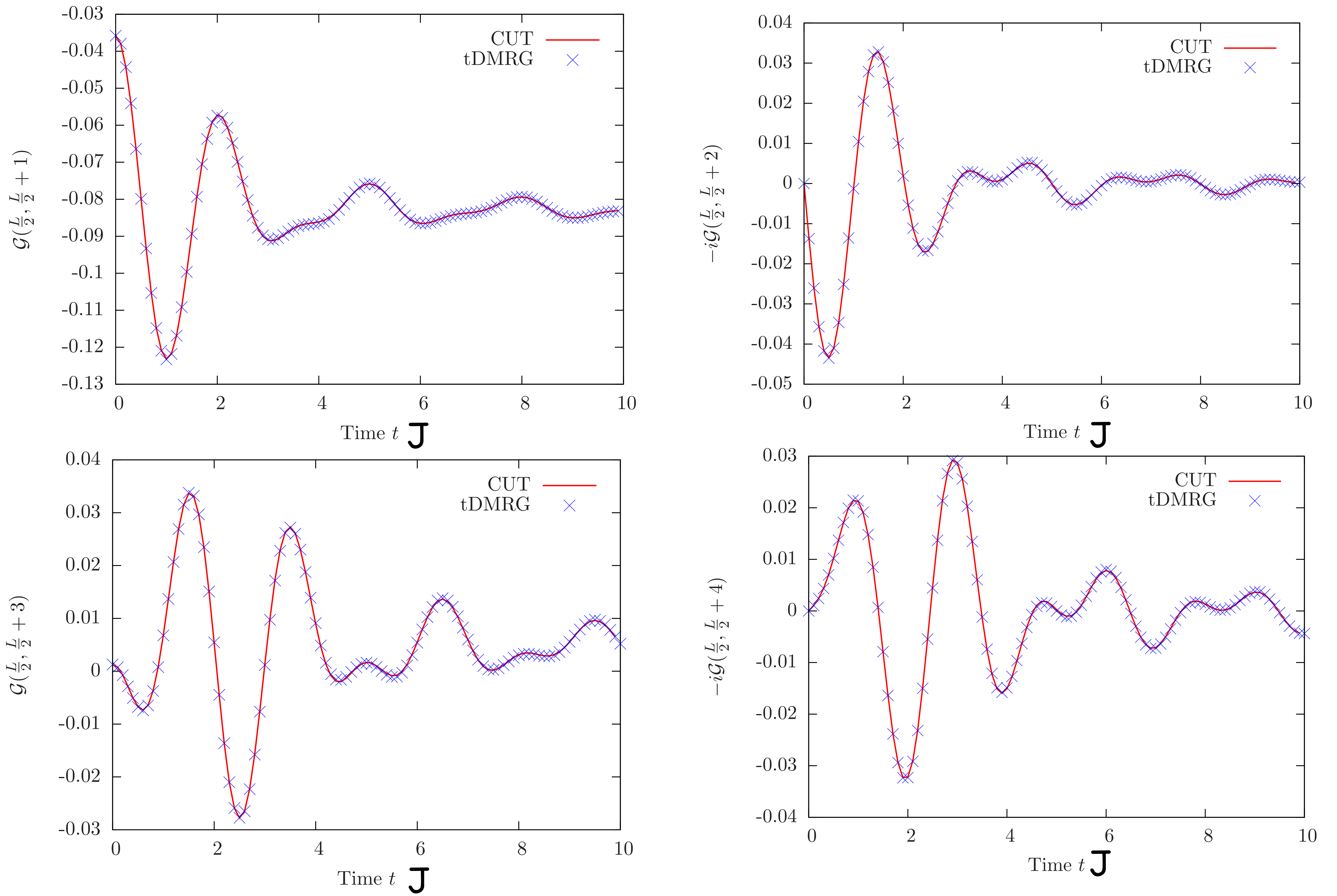
Time evolution:



$$H(B=\infty) = H_0(B=\infty) + H_{\rm int}(B=\infty)$$
$$= H' + :H_{\rm int}(B=\infty): \ ,$$
$$\mathcal{U}(t) \approx \exp(-iH't) \ ,$$

Quenches in the interacting theory

$$\delta_i = 0.75, \quad \delta_f = 0.5, \quad U = 0.15J$$

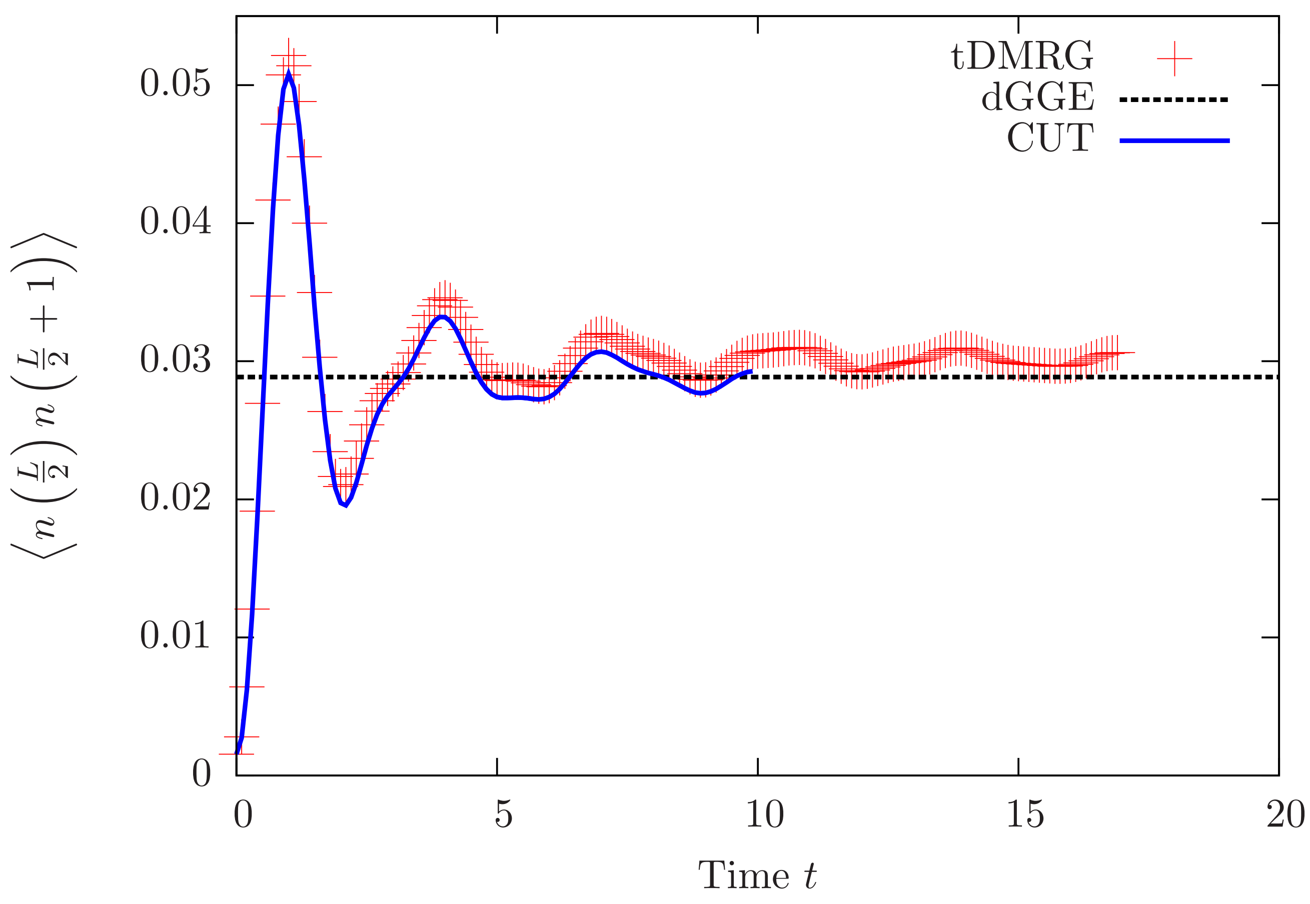


- Good agreement between CUT and tDMRG up to U~J/2 on time scales accessible by tDMRG (Jt<30)

  N.B. system size always large enough to avoid revivals

- Good agreement also for density-density correlator

$$\delta_i = 0.8, \quad \delta_f = 0.4, \quad U = 0.4J$$



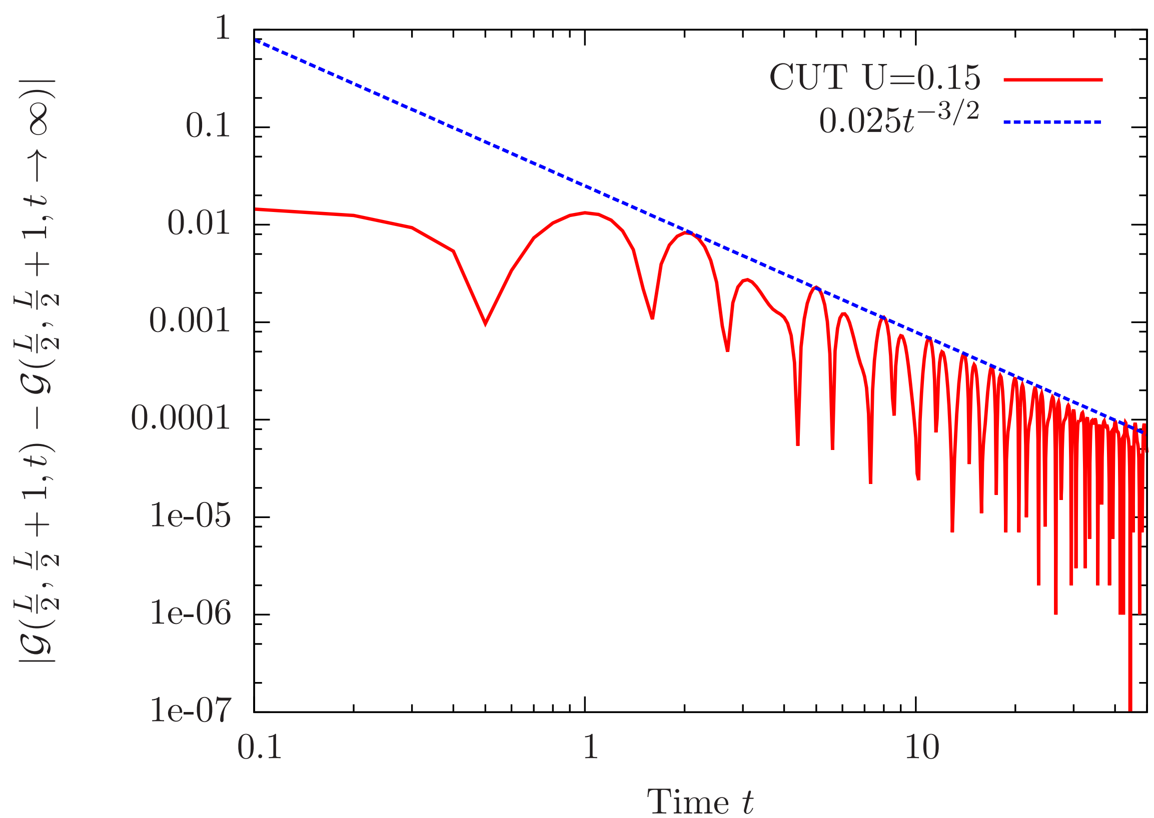
compatible with differences ~O(U^2)

- Good agreement between CUT and tDMRG up to U~J/2 on time scales accessible by tDMRG (Jt<30)

  N.B. system size always large enough to avoid revivals

- Good agreement also for density-density correlator

- $t^{3/2}$ power-law decay to constant values Thermalization?



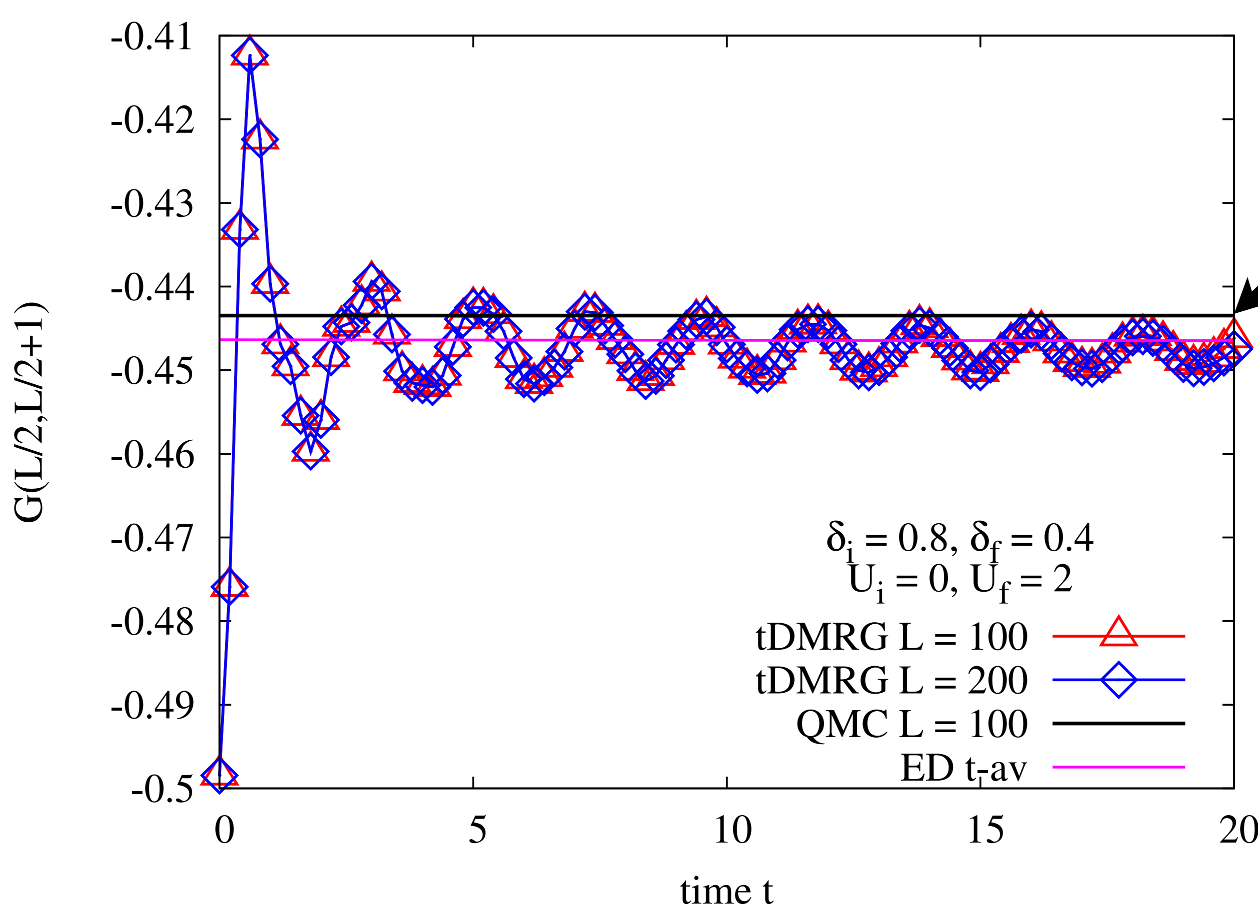
- CUT approach shows that they are **neither** thermal **nor** GGE

→ "Prethermalization Plateaux"

Moeckel&Kehrein '08
Kollar et al '11 — D>1

Marcuzzi et al '13 — D=1

- tDMRG suggests PPs exist in an extended U range



thermal value

U=2

Description of PPs: "Deformed GGE"

generator of time evolution in CUT: $H' = \sum_{\alpha=\pm} \sum_{k>0} \tilde{\epsilon}_\alpha(k) a^\dagger_\alpha(k) a_\alpha(k).$

→ mode occupation numbers conserved $n_\alpha(k) = a^\dagger_\alpha(k) a_\alpha(k)$

pre-image under CUT

$$\mathcal{Q}_\alpha(k) = a^\dagger_\alpha(k) a_\alpha(k) - U \sum_{q_j>0} N^\gamma_{\alpha\alpha}(\mathbf{q}|k,k,B=\infty)\ a^\dagger_{\gamma_1}(q_1) a_{\gamma_2}(q_2) a^\dagger_{\gamma_3}(q_3) a_{\gamma_4}(q_4)$$

$$+\mathcal{O}(U^2)$$

Physical interpretation as **quasiparticle** occupation numbers

Commutation relations:

$$[\mathcal{Q}_\alpha(k), \mathcal{Q}_\beta(p)] = \mathcal{O}(U^2). \qquad [\mathcal{Q}_\alpha(k), H(\delta_f, U)] = \mathcal{O}(U),$$

→ charges not (perturbatively) conserved at the operator level, but only **weakly**

$$\mathrm{Tr}\,(\rho(t)\mathcal{Q}_\alpha(k)) - \langle\Psi_0|\mathcal{Q}_\alpha(k)|\Psi_0\rangle = \mathcal{O}(U^2)$$

Define a density matrix ("deformed GGE") by

$$\varrho_{\mathrm{PT}} = \frac{1}{Z_{\mathrm{PT}}} \exp\left(\sum_{k,\alpha} \lambda_k^{(\alpha)} \mathcal{Q}_\alpha(k)\right).$$

fix Lagrange multipliers by $\quad \mathrm{tr}\left[\varrho_{\mathrm{PT}}\ \mathcal{Q}_\alpha(k)\right] = \langle\Psi_0|\mathcal{Q}_\alpha(k)|\Psi_0\rangle.$

ρ_{PT} reproduces the prethermalization plateaux values to o(U) for both two-point and 4-point functions.

At this stage the picture is as follows:

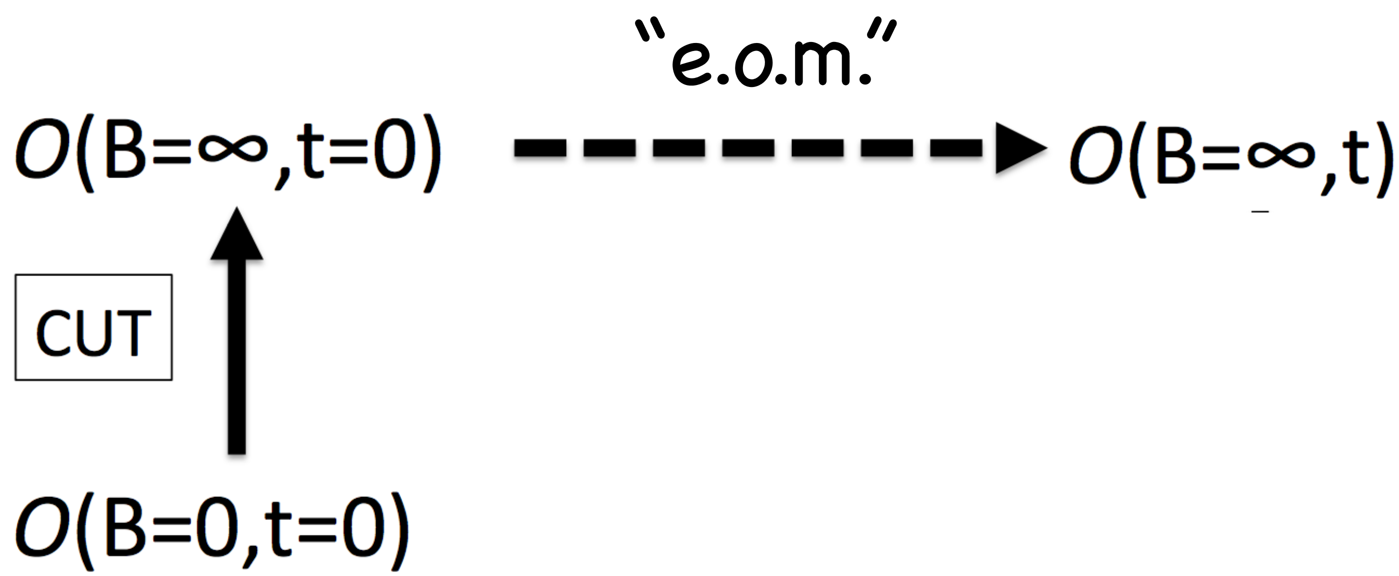
- Free theory: relaxation to a GGE
- Weakly interacting (non-integrable) theory: relaxation to a prethermalization plateau
- PP can be described by a "deformed GGE"

Late time behaviour ?

Bertini, Essler & Robinson

Both CUT and tDMRG break down at late times.

→ try "equations of motion" approach in B=∞ basis



Consider O(B=∞)=quasiparticle mode occupation numbers: good diagnostic for leaving the prethermalization plateau

Proceed as in derivation of quantum kinetic equation

$$\rho_{\alpha\beta}(k,t) = \langle a^\dagger_\alpha(q,t) a_\beta(q,t)\rangle e^{-it\left(\tilde{\epsilon}_\alpha(k) - \tilde{\epsilon}_\beta(k)\right)}$$

$$\tilde{\epsilon}_\alpha(q) = \epsilon_\alpha(q) + 4U\rho_{\gamma\gamma}(q,0)V_{\alpha\gamma\gamma\alpha}(q,k,k,q)$$

$$\dot{\rho}_{\alpha\alpha}(q) = -U^2 \sum_{k_1,k_2} \int_0^t dt' \; K^{\vec{\nu}}_\alpha(k_1,k_2,t'|q) \; \rho_{\nu_1\nu_2}(k_1)\rho_{\nu_3\nu_4}(k_2)$$

$$- U^2 \sum_{k_1,k_2,k_3} \int_0^t dt' \; L^{\vec{\mu}}_\alpha(k_1,k_2,k_3,t'|q) \; \rho_{\mu_1\mu_2}(k_1)\rho_{\mu_3\mu_4}(k_2)\rho_{\mu_5\mu_6}(k_3),$$

$$\dot{\rho}_{+-}(q) = 4iU \sum_{k,\gamma} M^\gamma_{+-}(q,k,k,q)\left[\rho_{\gamma\gamma}(q,t) - \rho_{\gamma\gamma}(q,0)\right]\rho_{+-}(q)$$

Dropping the O(U^2) terms precisely recovers the CUT PP results.

$$\dot{\rho}_{\alpha\alpha}(q) = -U^2 \sum_{k_1,k_2} \int_0^t dt' \; K_\alpha^{\vec{\nu}}(k_1,k_2,t'|q) \; \rho_{\nu_1\nu_2}(k_1)\rho_{\nu_3\nu_4}(k_2)$$

$$- U^2 \sum_{k_1,k_2,k_3} \int_0^t dt' \; L_\alpha^{\vec{\mu}}(k_1,k_2,k_3,t'|q) \; \rho_{\mu_1\mu_2}(k_1)\rho_{\mu_3\mu_4}(k_2)\rho_{\mu_5\mu_6}(k_3),$$

$$\dot{\rho}_{+-}(q) = 4iU \sum_{k,\gamma} M_{+-}^{\gamma}(q,k,k,q) \left[\rho_{\gamma\gamma}(q,t) - \rho_{\gamma\gamma}(q,0)\right] \rho_{+-}(q)$$

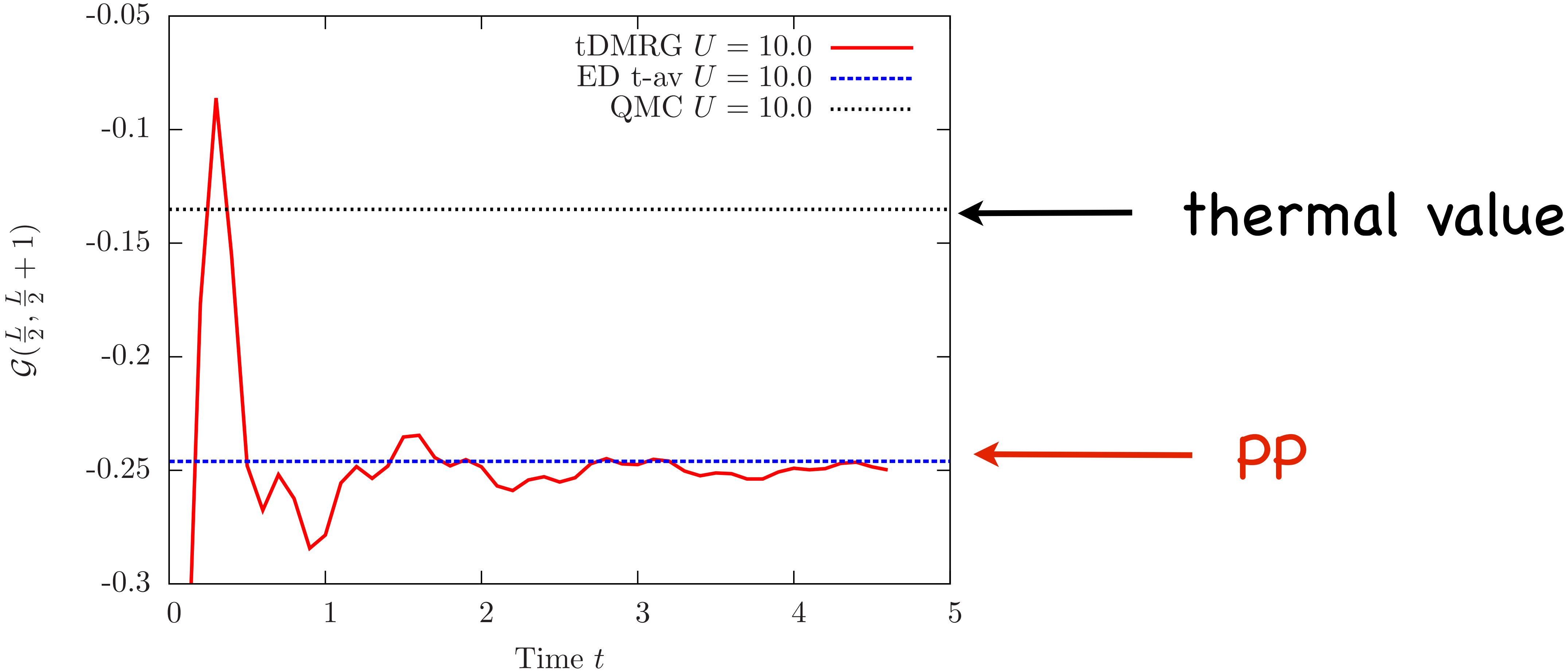
Structure is **different** from quantum kinetic equation:

- cannot remove integrals (E-diagonal interactions)
- mixture of first-/second order equations

work in progress on numerical integration   Thermalization?

Quenches to strongly interacting systems

$$\delta_i = 0.8, \quad \delta_f = 0.4, \quad U = 10J$$



???

$$H(\delta, U) = -J \sum_{l=1}^{L} \left[1 + \delta(-1)^l\right] \left(c_l^\dagger c_{l+1} + \text{h.c.}\right) + U \sum_{l=1}^{L} n_l n_{l+1}$$

H(0,U) is **integrable** (spin-1/2 XXZ chain)

→ quench to XXZ chain with weak integrability-breaking term

$$-J\delta \sum_{l=1}^{L} (-1)^l \left(c_l^\dagger c_{l+1} + \text{h.c.}\right)$$

No analytic understanding of this limit.

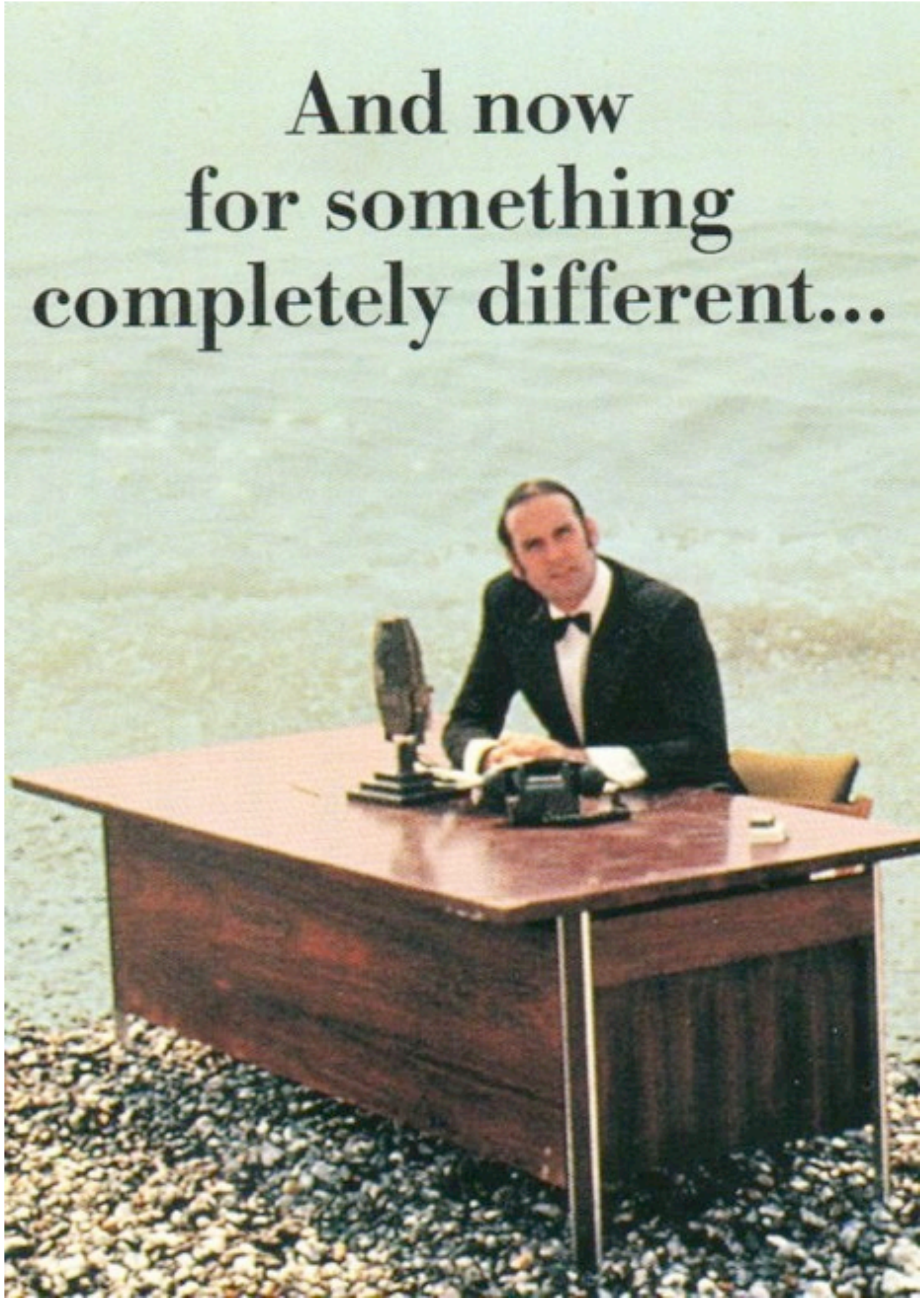
Summary and Outlook

1. Nice prethermalization plateaux in quenches to **weakly interacting** (weakly non-integrable) models in D=1

2. Statistical description of these plateaux through **"deformed GGE"** (charges do **not** commute with H)

3. Derived differential equations that hopefully describe both the plateaux and the eventual thermalization.

4. How general is all this? PPs in **interacting** integrable theories with weak integrability breaking terms?

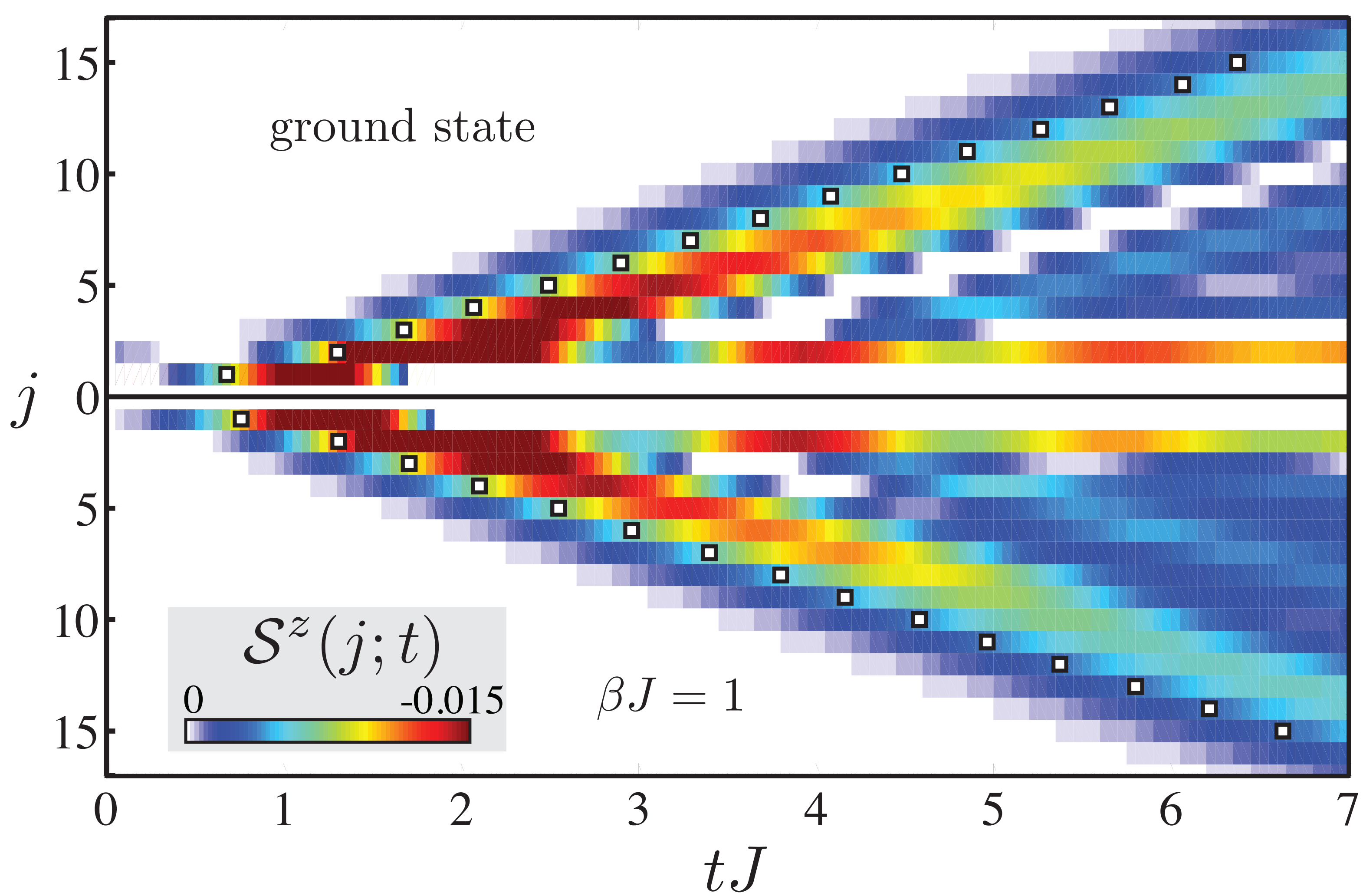

And now
for something
completely different...

Yesterday: I. Bloch, U. Schollwoeck

"Light-cone" effects after quantum quenches

L. Bonnes, FE& A. Laeuchli arXiv:1404.4062

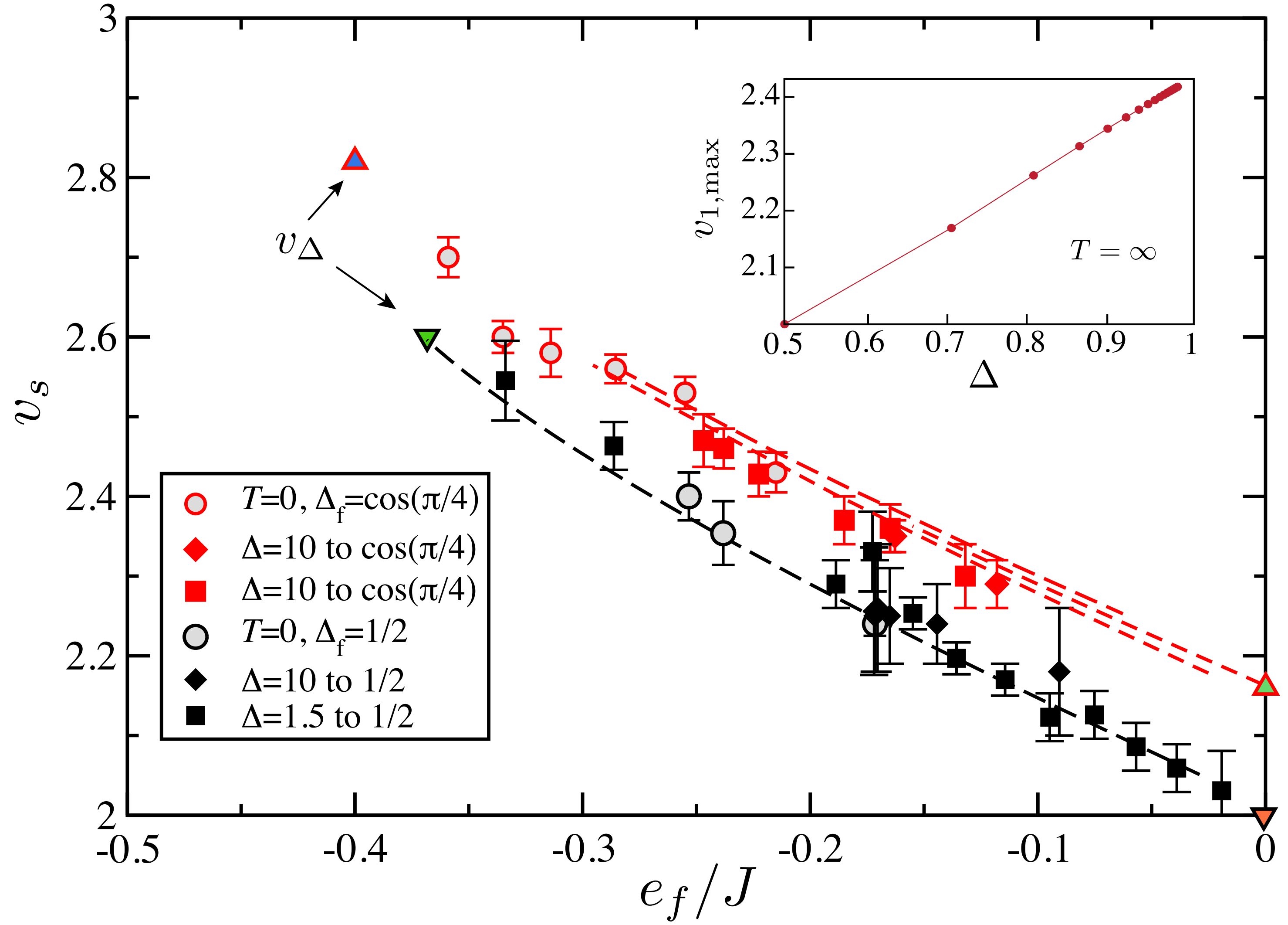
Results for quenches $\Delta_i = 4 \longrightarrow \Delta = \cos(\pi/4)$



$$\rho(t=0) = |\mathrm{GS}_{\Delta_i}\rangle\langle\mathrm{GS}_{\Delta_i}|$$

$$\rho(t=0) = \frac{1}{Z} e^{-\beta H(\Delta_i)}$$

Observation: Light-cone velocity **depends on initial state** (and not just final Hamiltonian):



Have a **theory** for this effect in terms of "excitations at finite energy density"