Universal dynamics in Many-body localized states and the many-body localization transition

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Minerva foundation

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European Research Council

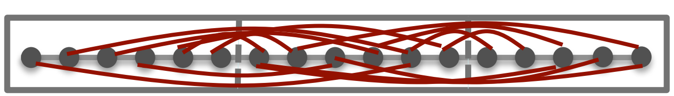
Dynamics of closed quantum systems

Thermalization

Quantum information stored in local objects is rapidly lost

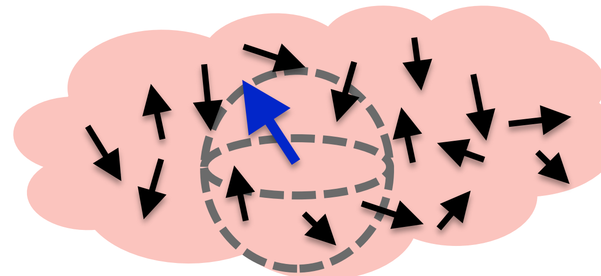
Classical hydro description of remaining slow modes (e.g. diffusion)

Thermal eigenstates (highly entangled):



$S_A \sim L^d$

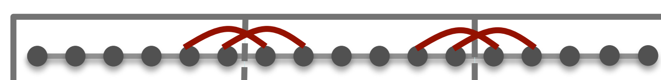
Many-body localization



Local quantum information persists indefinitely

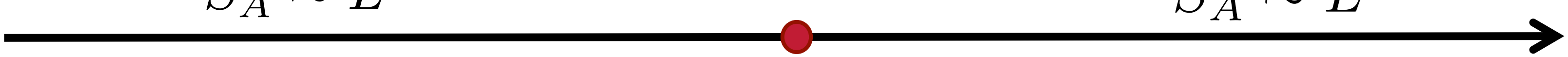
Need quantum description of long time dynamics.

Ground-state-like high energy eigenstates (low entanglement):



$S_A \sim L^{d-1}$

?



Outline

- Thermalization in closed quantum systems
  Eigenstate thermalization hypothesis and its breaking

- What we understand about MBL dynamics
  RG, distinct phases, dynamical critical points.

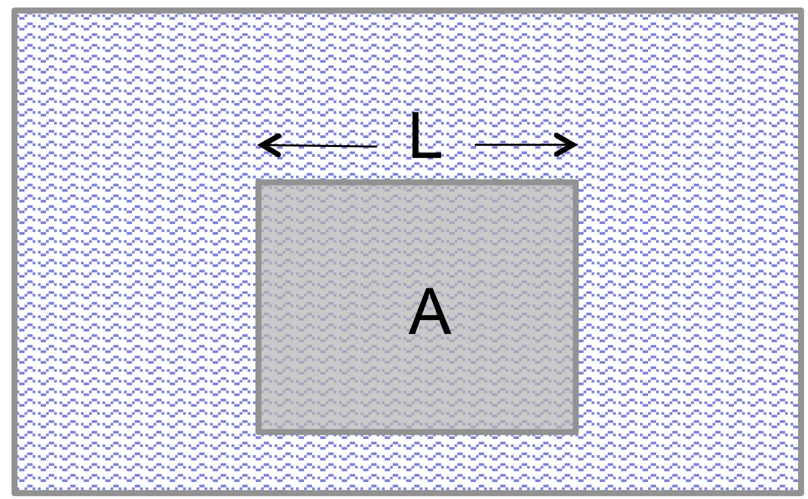
- The many-body localization phase transition
  RG approach: transport, entanglement scaling and a surprise!

Eigenstate thermalization hypothesis (ETH)

Deutsch 91, Srednicki 94

In a high energy eigenstate:

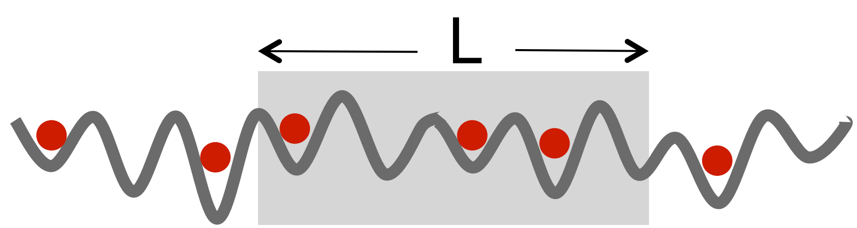
$$\rho_A = \frac{1}{Z_A} e^{-\beta H_A}$$



Extensive Von-Neuman entropy:

$$S_A \propto L^d$$

Example where ETH fails:
Anderson localization



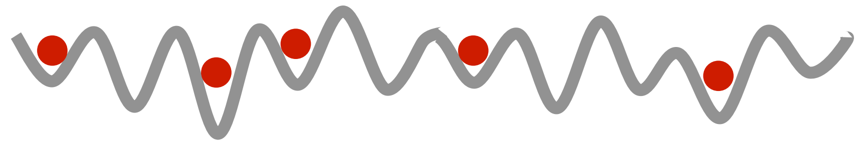
“Area law” entropy as in ground state also holds in high energy eigenstates

$$S_A \propto L^{d-1}$$

Is this situation stable to adding interaction between the particles?

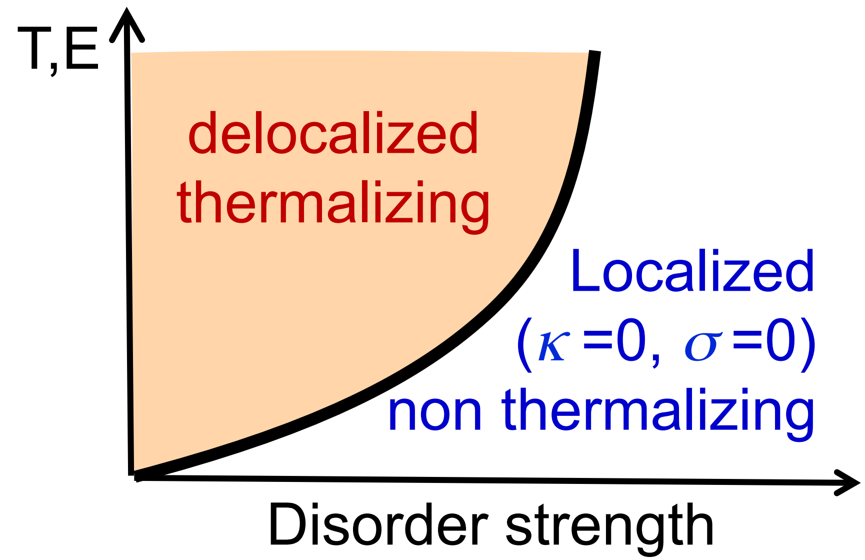
Generic exception to ETH: Many body localization

Anderson localization of
non interacting particles:



Perturbative stability to interactions
(Basko, Aleiner, Altshuler 2005)

Delocalization transition at a critical
energy density, disorder or interaction
strength.



Stability of MBL supported by other approaches:

Numerics – Oganesyan & Huse 2010, Pal & Huse, Bardarson et. al 2012 …

RG – Vosk an EA 2012, Vosk and EA 2013, Pekker et. al. 2013 .

Mathematical proof – Imbrie 2014

➡ A lot of insight into the nature of the MBL phase

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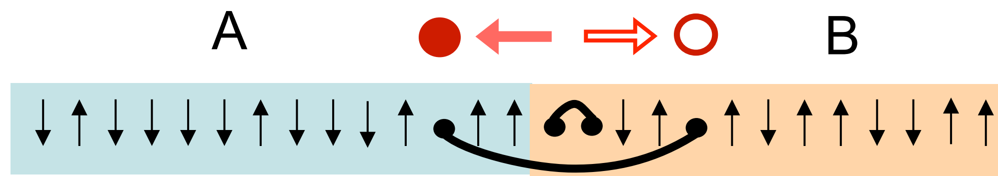
Ultra slow growth of the entanglement entropy

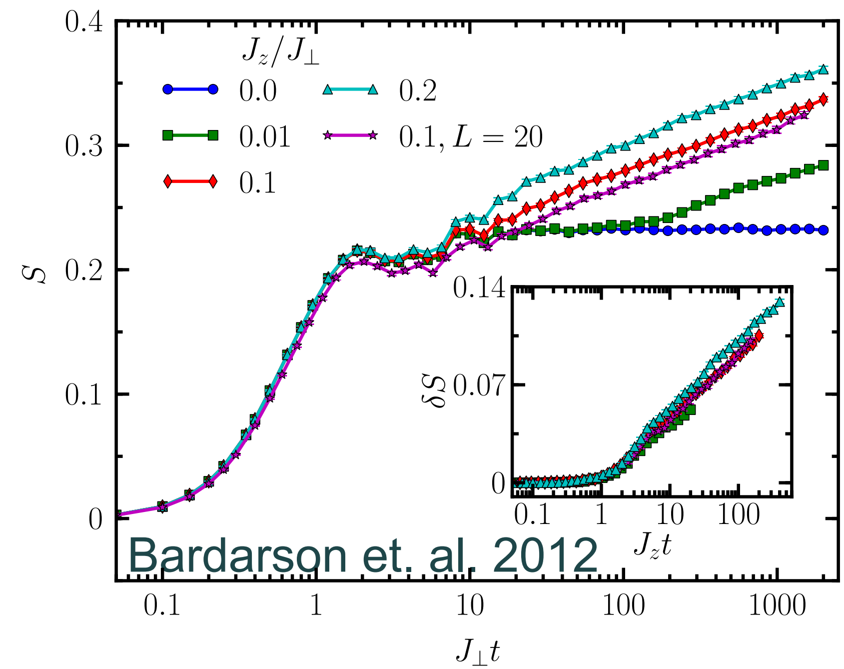
Zindaric et. al. 2008; Bardarson, Pollmann & Moore. 2012

$$H_0 = J_\perp \sum_i (S_i^x S_{i+1}^x + S_i^y S_{i+1}^y) + \sum_i h_i S_i^z \qquad H_{\text{int}} = J_z \sum_i S_i^z S_{i+1}^z$$

$$e^{-iHt} \,|\, \Psi_0 \,\rangle$$



$$S_A(t) = -Tr\left[\rho_A(t)\ln\rho_A(t)\right]$$



Result:

Non-interacting AL: Saturation

Interacting MBL: $S_A(t) \sim \log t$

Finite subsystem: $S_A(\infty) = s_\infty L$

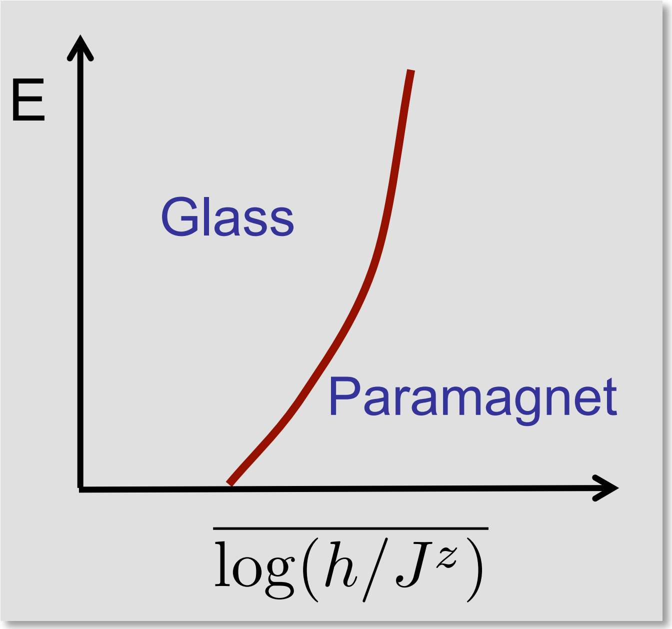
saturate to subthermal volume law

There can be distinct localized phases

Huse et. al. (2013)

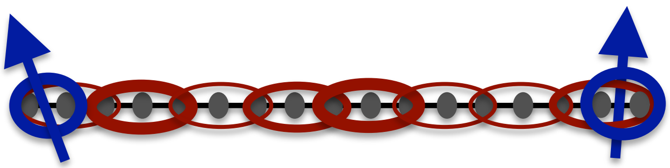
Ground state like entanglement properties allow quantum “phases” in high energy eigenstates.

$$H = \sum_i \left[J_i^z \sigma_i^z \sigma_{i+1}^z + h_i \sigma_i^x + J_i^x \sigma_i^x \sigma_{i+1}^x + \ldots \right]$$



Can even support topological eigenstates.
Edge modes protected at high energy with dramatic effects on the dynamics!

Bahri, Vosk, EA and Vishawanth (2013)



Universal dynamical signatures of the phases and transitions between them?

RG Solution of time evolution

R. Vosk and EA, PRL (2013); R. Vosk and EA, arXiv:1307.3256

$$H = \sum_i \left[J_i^z \sigma_i^z \sigma_{i+1}^z + h_i \sigma_i^x + J_i^x \sigma_i^x \sigma_{i+1}^x + \ldots \right] \qquad e^{-iHt} \,|\, \Psi_0 \,\rangle$$

$$|\Psi_0\rangle = \downarrow\uparrow\downarrow\downarrow\downarrow\downarrow\uparrow\downarrow\downarrow\downarrow\uparrow\downarrow\uparrow\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow\uparrow\downarrow\uparrow\uparrow\downarrow\downarrow\uparrow\uparrow\downarrow\downarrow\uparrow\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\downarrow\uparrow\downarrow\uparrow$$

H_{fast}

Pick out largest couplings $\quad \Omega = \max\left(J_i^z, h_i\right)$

<u>Short times</u> ($t \approx 1/\Omega$): System evolves according to H_{fast}
Other spins essentially frozen on this timescale.

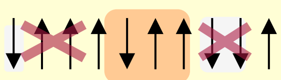
<u>Longer times</u> ($t \gg 1/\Omega$): Eliminate fast modes (order Ω) perturbatively to obtain effective evolution for longer timescales.

Related RSRG-X: Pekker, Refael, EA, Demler & Oganesyan arXiv:1307.3253

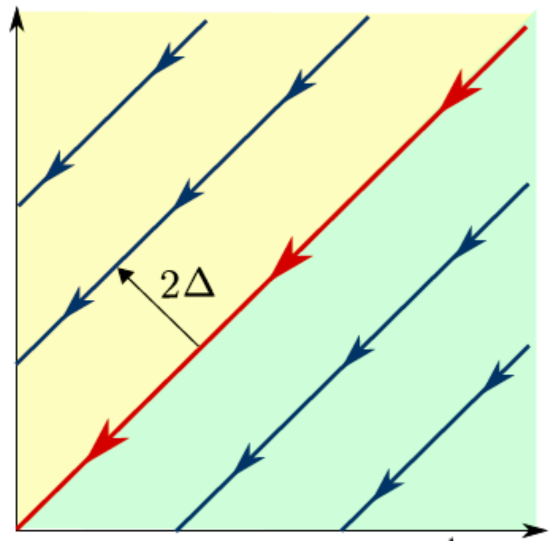
Result from the RG flow

"Paramagnet"

$$\langle \sigma_i^z(t) \rangle \xrightarrow[t\to\infty]{} \frac{\ln t}{t^{\alpha}}$$



$\log h_{\text{typ}}$



$\log J^z_{\text{typ}}$

Critical point:

$$\langle \sigma_i^z(t) \rangle \sim \frac{1}{\ln^{2-\Phi} t}$$

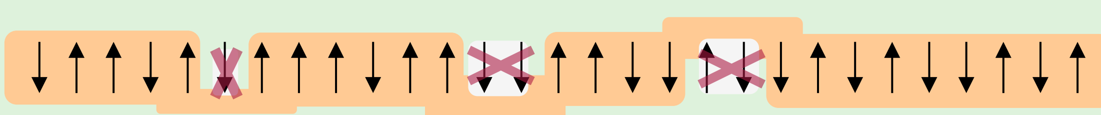
$$\phi = (1+\sqrt{5})/2 \approx 1.618$$

(golden ratio)

$$S_A(t) \sim \log^{2/\phi} t$$

$$S_A(t) \sim \log t$$

"glass": $\langle \sigma_i^z(t) \rangle \xrightarrow[t\to\infty]{} const \sim \Delta^{2-\Phi}$



σ_i^z is an emergent integral of motion in the glass

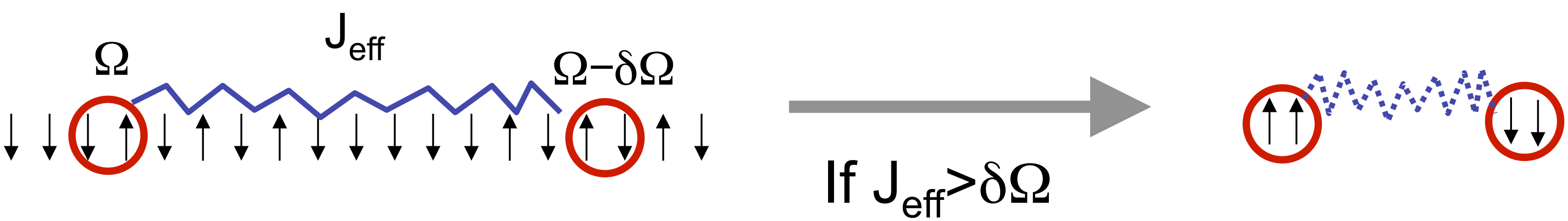
Glass order parameter!

<u>Important:</u> this is a quantum dynamical transition at high energies!

Takes place in the localized phase.

Limitation of the RG scheme: resonances

Resonances between decimated sites can generate a slow mode that is not accounted for by the RG



Resonances do not proliferate in MBL phase! (Irrelevant in RG sense).

(Vosk and EA 2013)

This RG scheme is limited to the MBL phase!

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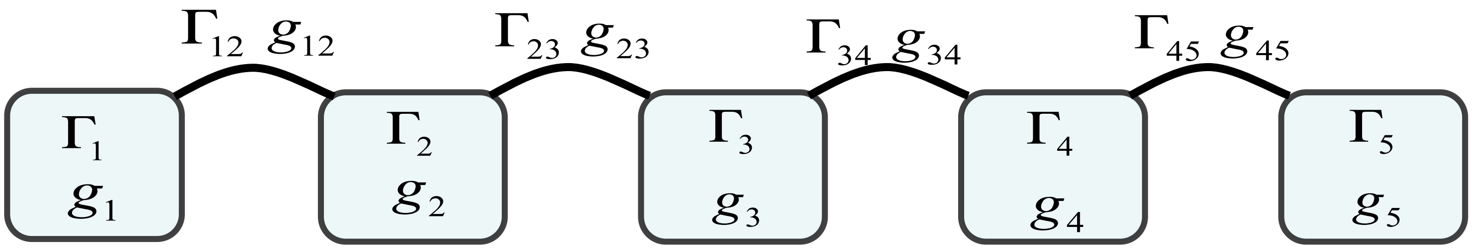
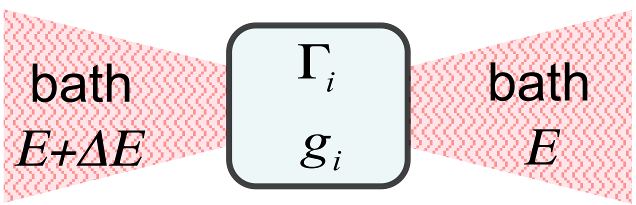
Coarse Grained Model of coupled blocks



The block parameters:



Γ_i Energy transport rate through block

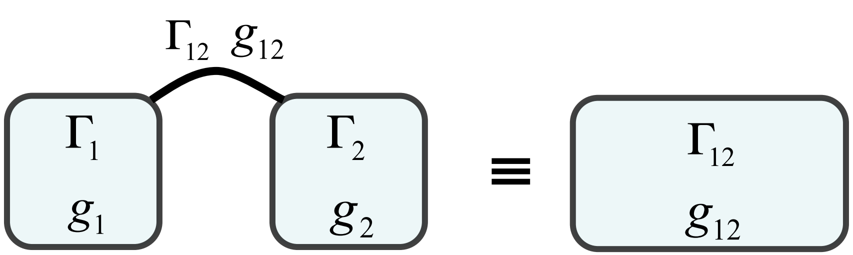
Δ_i Single block level spacing

$g_i = \Gamma_i / \Delta_i$ Number of coupled levels

$g_i \ll 1$ "insulating block"

$g_i \gg 1$ "thermalizing block"

Link parameters: Parameters of new block if blocks 1 and 2 were joined

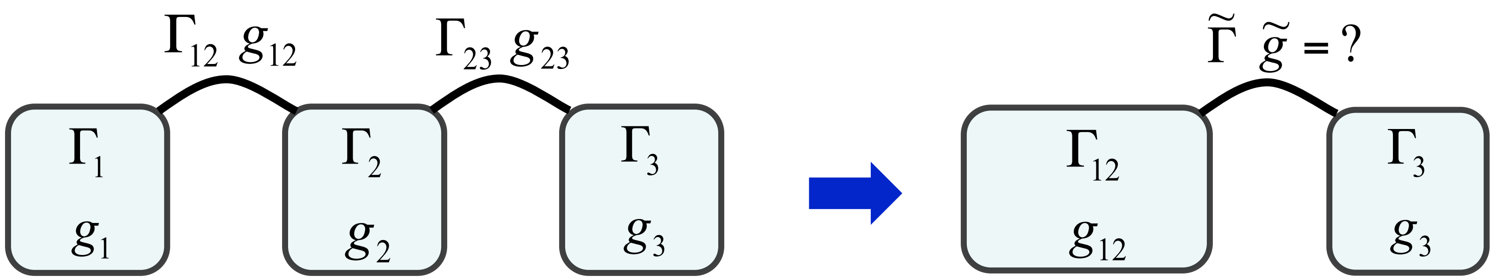


$g_{12} = \frac{\Gamma_{12}}{\Delta_{12}}$ ~ Effectiveness of coupling

Requirement: $\{\Gamma_{ii+1}\} < \{\Gamma_i\}$

RG scheme

1. Join blocks coupled by the fastest rate Γ_{12}



2. Renormalize couplings to left and right blocks

Two cases:

(i) If $g_{12} \ll 1$ or $g_{23} \ll 1$ then we show

$$\tilde{\Gamma} = \frac{\Gamma_{12}\Gamma_{23}}{\Gamma_2} \qquad \tilde{g} = \frac{g_{12}g_{23}}{g_2}$$

(ii) If $g_{12}, g_{23} \gg 1$

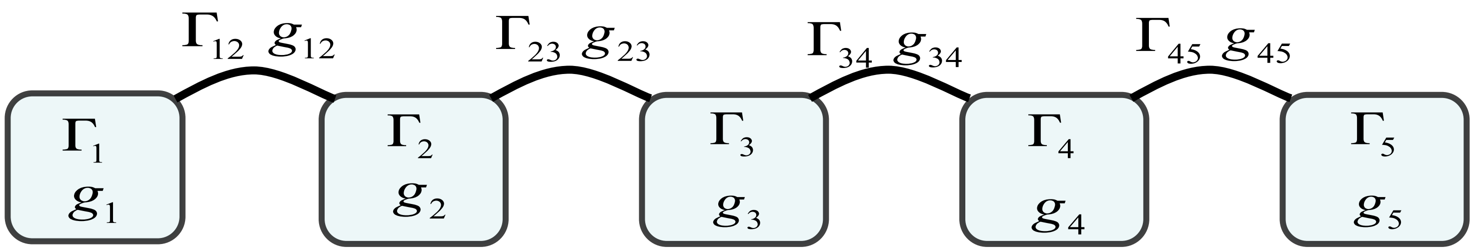
then assume ohmic transport

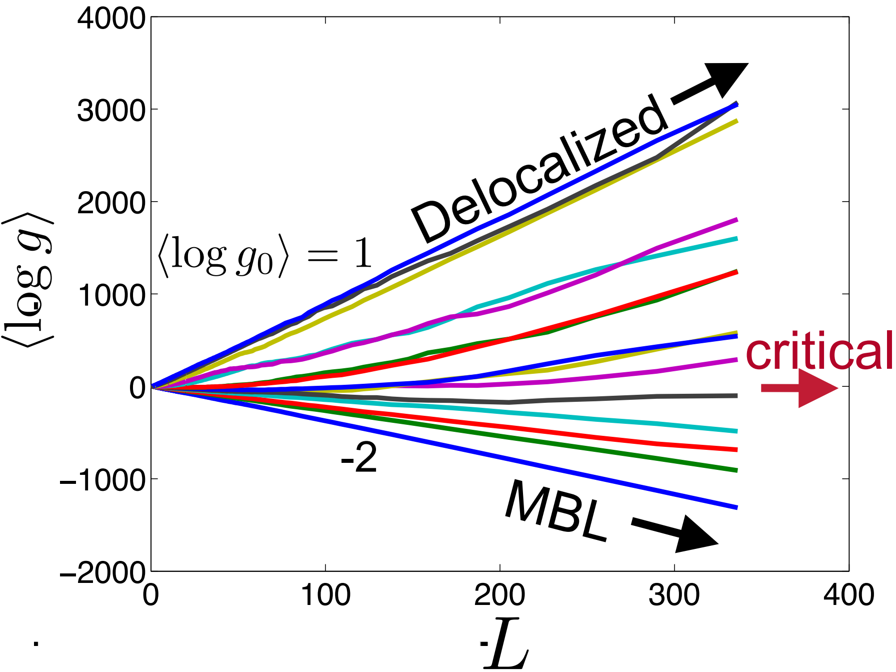
$$\frac{1}{(l_1 + l_2 + l_3)\tilde{\Gamma}} = \frac{1}{(l_1 + l_2)\Gamma_{12}} + \frac{1}{(l_2 + l_3)\Gamma_{23}}$$

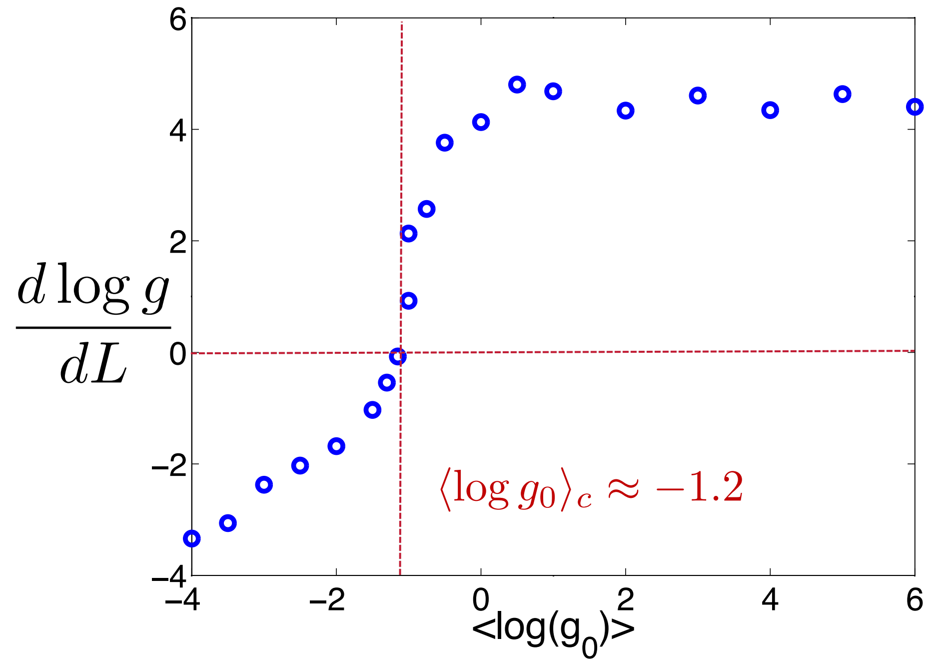
$$\left(lengh \sim \sqrt{time}\right)$$

Note: the scheme is controlled if the distribution of g_{ij} is wide

Outcome of the RG flow



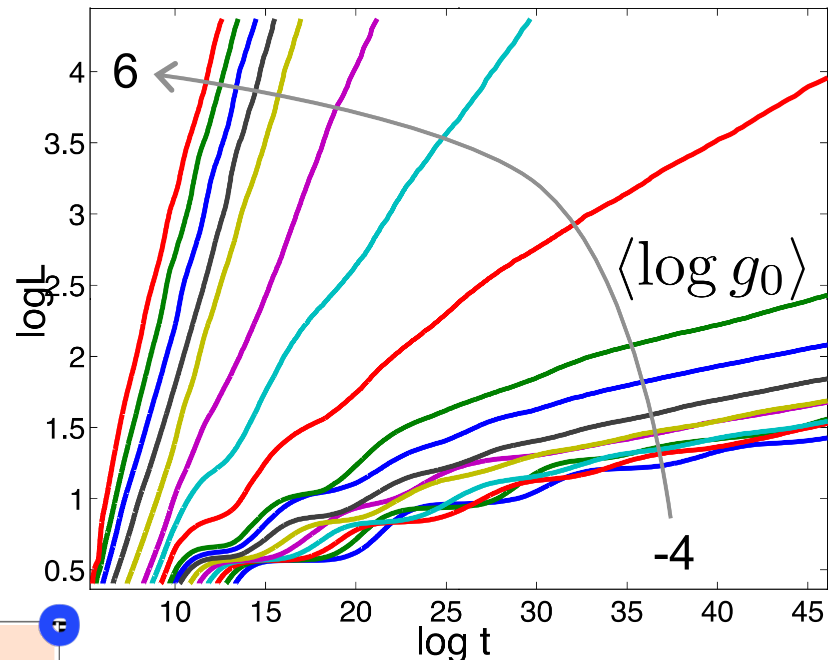


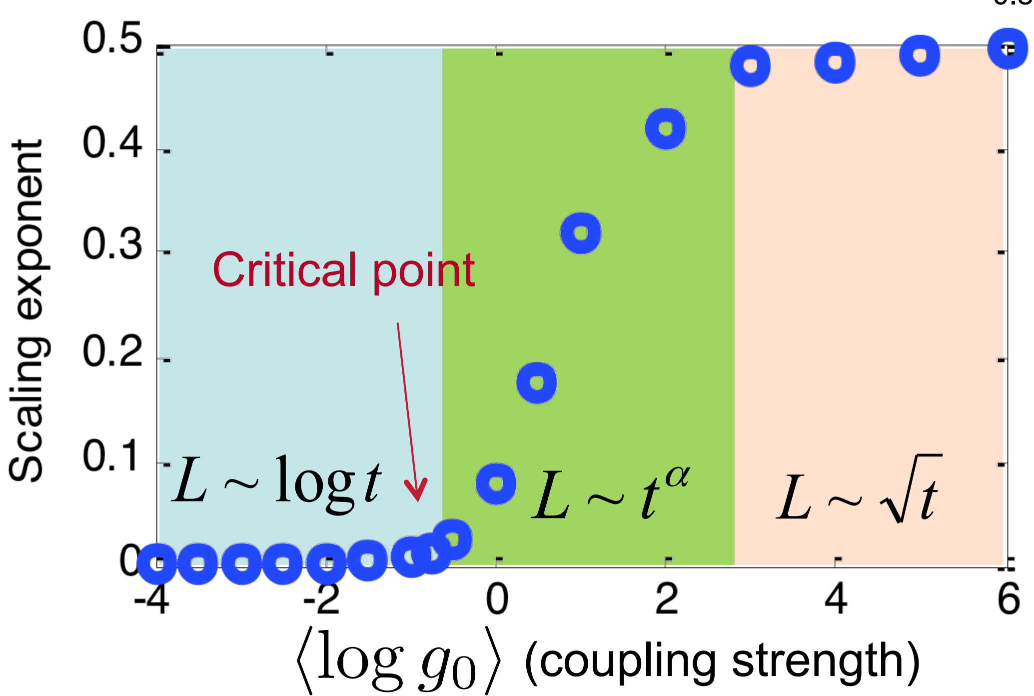


How does diffusion disappear ?

RG results – dynamical scaling exponent

Relation between time scale ($t = \Gamma^{-1}$) and length L of blocks

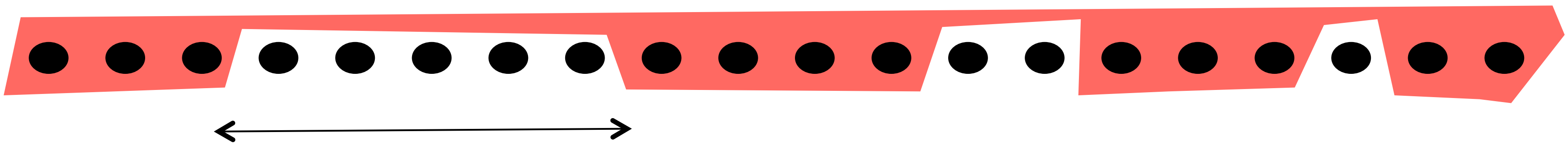




Surprise!

Anomalous diffusion phase between the MBL and thermal metal phases.

Anomalous diffusion = Griffith phase



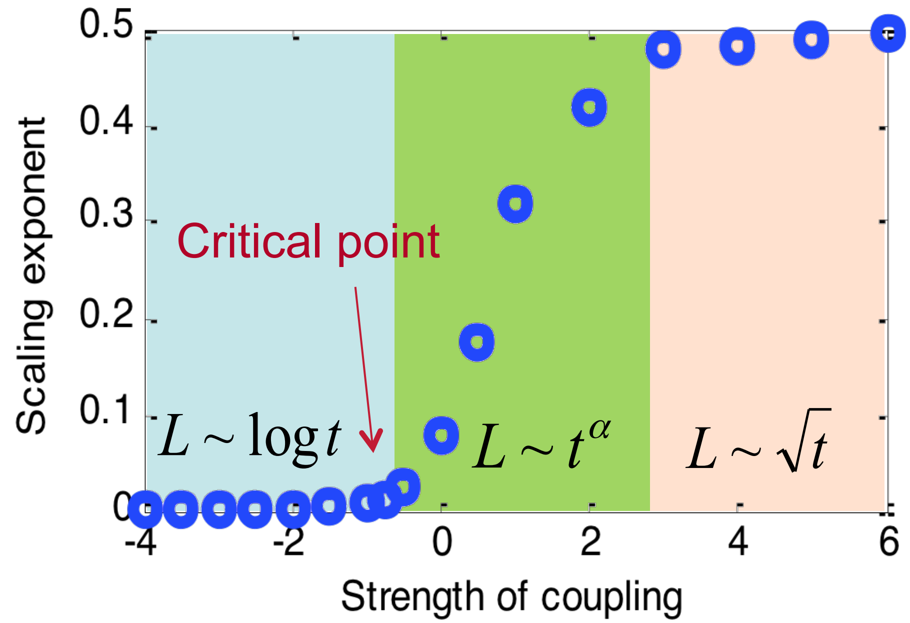
$$l \gg \xi \geq l_o$$

Exponentially rare insulating puddles in the metal

$$P(l) \sim l_0^{-1} e^{-l/\xi}$$

$$\tau(l) = \tau_0 e^{l/l_0}$$

Exponentially long delay



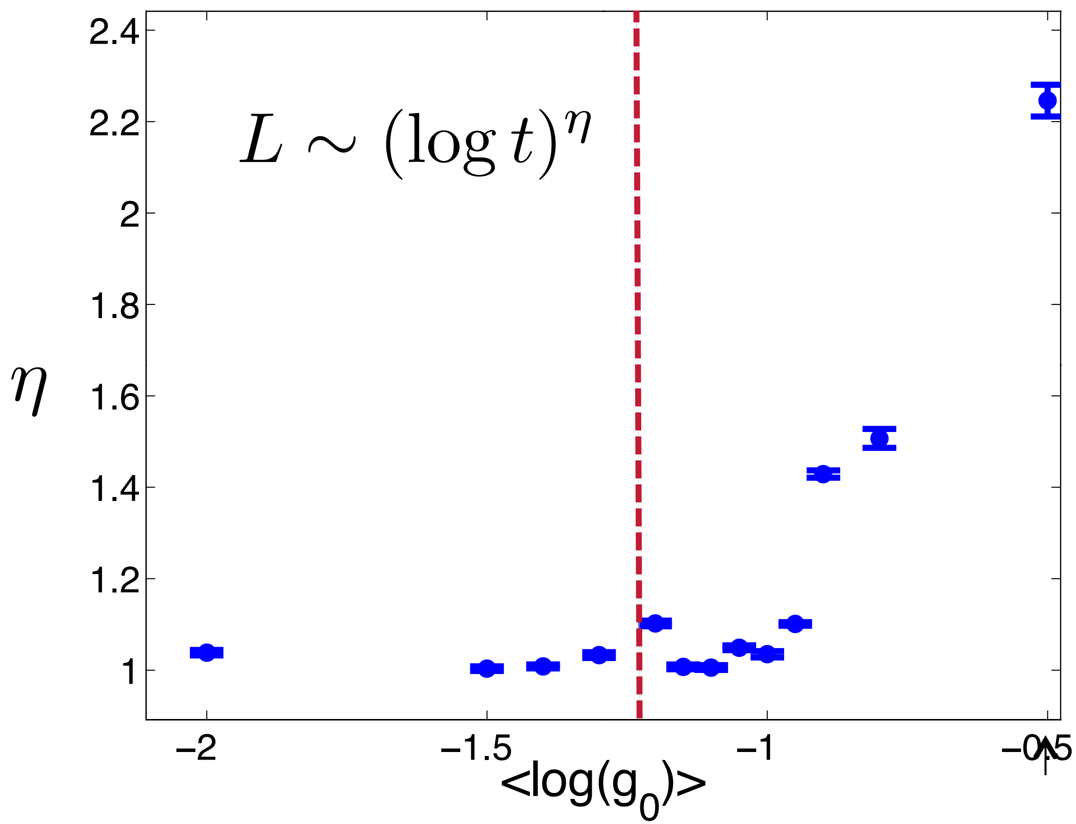
Broad distribution of times:

$$P(\tau) = \tau_0^{-1} \left(\frac{\tau_0}{\tau} \right)^{1+\frac{\xi}{l_0}}$$

All "insulating" puddles ultimately thermalize but at broadly distributed times!

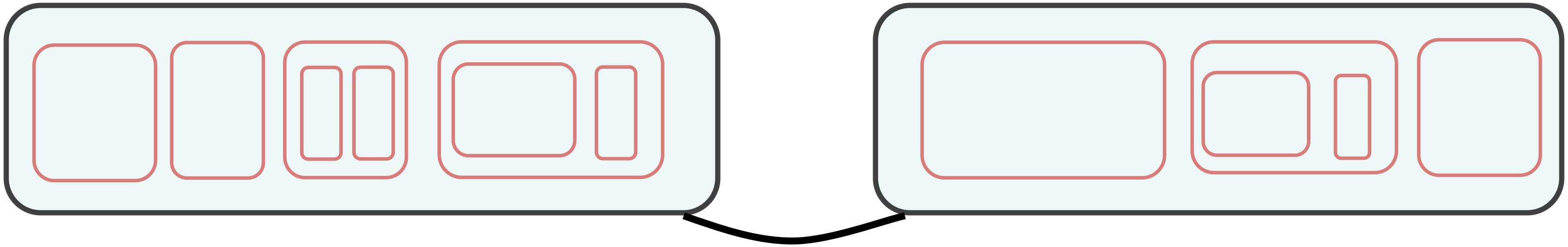
Infinite randomness but thermal critical point at $\xi \to \infty$

Scaling in the localized phase



Suggests also: $S_A \sim \log t$

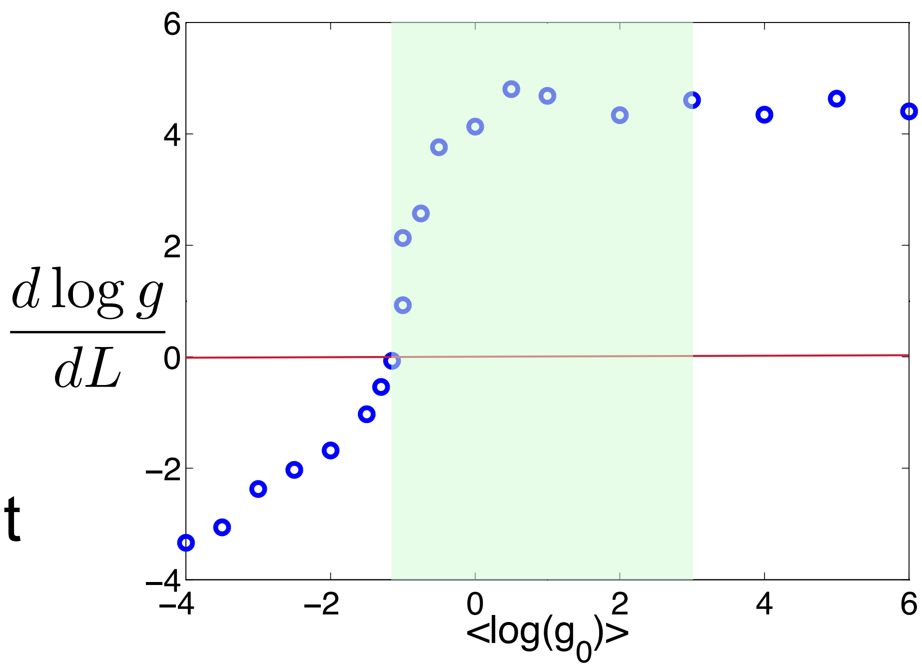
Entanglement scaling in eigenstates



g_{12} ~ # of 2-block product states in an eigenstate of the coupled system

$$\log_2\left[g(L)+1\right] \sim S_E(L/2)$$

(Gives only the volume law contribution. To obtain full entropy need to integrate over the flow)



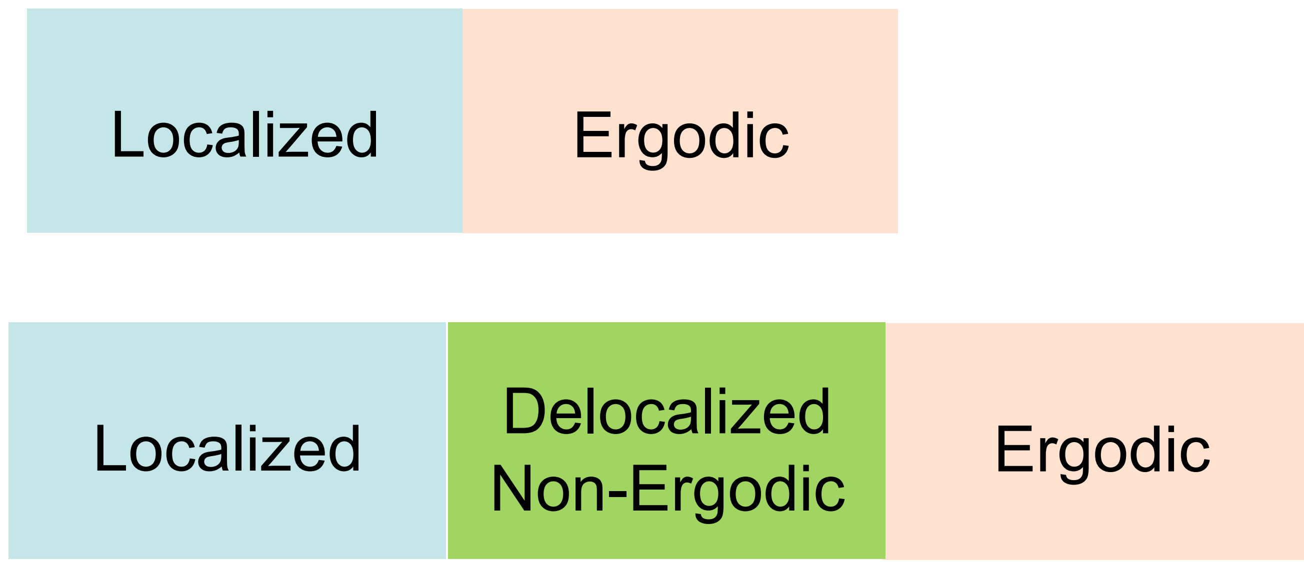
RG result indicates thermal entropy in eigenstates throughout the Griffiths phase

The Many-Body Localization Transition

Two possible transitions are consistent with entanglement entropy strong subadditivity (T. Grover arXiv:1405.1471)

1)

2)



It appears that our scheme gives case1.
The Griffith phase is ergodic!

$S_A(l)$ at the critical point is thermal for a subsystem of an infinite system.
But finite size effects are highly annomalous.

Transport versus entanglement dynamical scaling: Heuristic argument

(David Huse)

$$\Gamma_{12} = \tau_E^{-1}$$

$$E_1 = L(\epsilon + \delta\epsilon) \qquad E_2 = L\epsilon$$

L

To equilibrate the two blocks need L transitions with $\delta E = O(1)$

But one such transition is enough to fully entangle two blocks in the delocalized phase! $\tau_E = L\tau_S$

Time τ_S entanglement-entropy propagates a distance L:

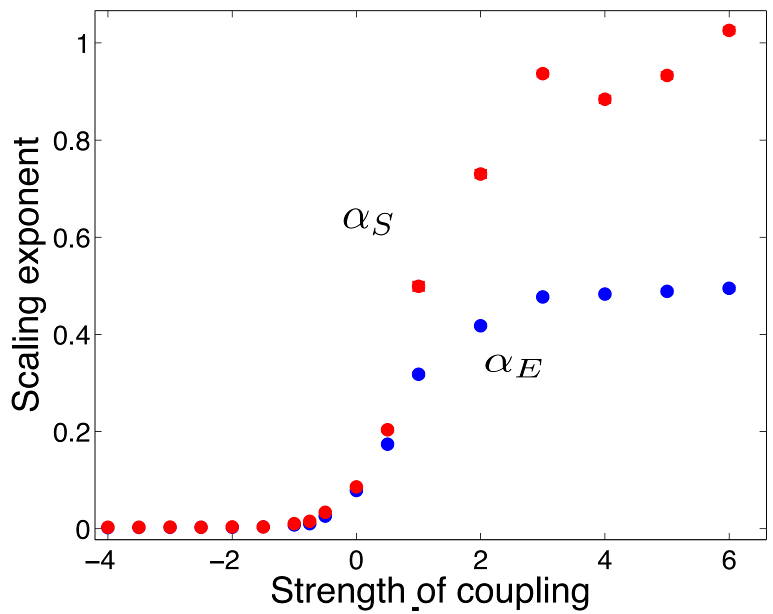
$$\tau_E \sim L^{1/\alpha} \Rightarrow \tau_S \sim L^{-1+\frac{1}{\alpha}} \Rightarrow S_A(t) \sim t^{\frac{\alpha}{1-\alpha}}$$

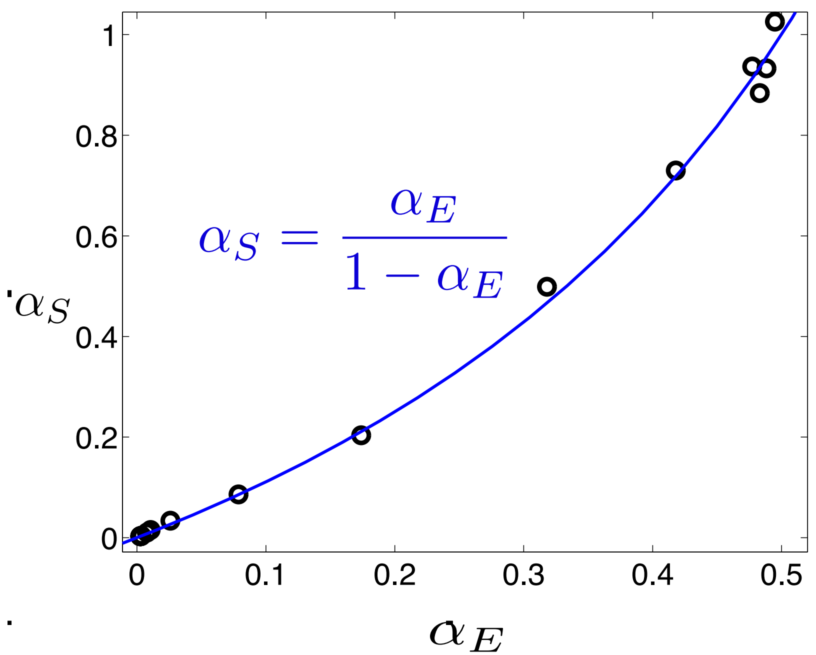
Diffusive energy implies ballistic entanglement propagation.

Test of energy versus entanglement scaling

Compare previous RG result with RG scheme done for coupled blocks with no energy conservation (Floquet)

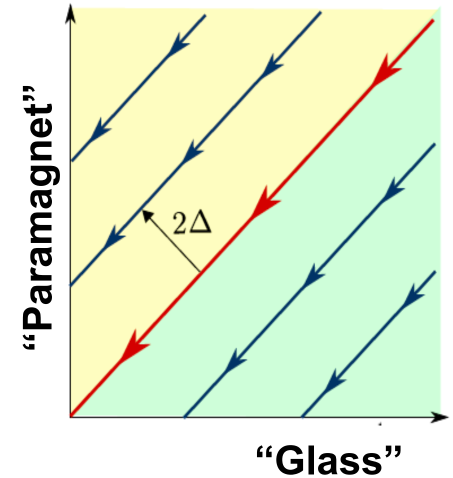
Entanglement is the only propagating quantity





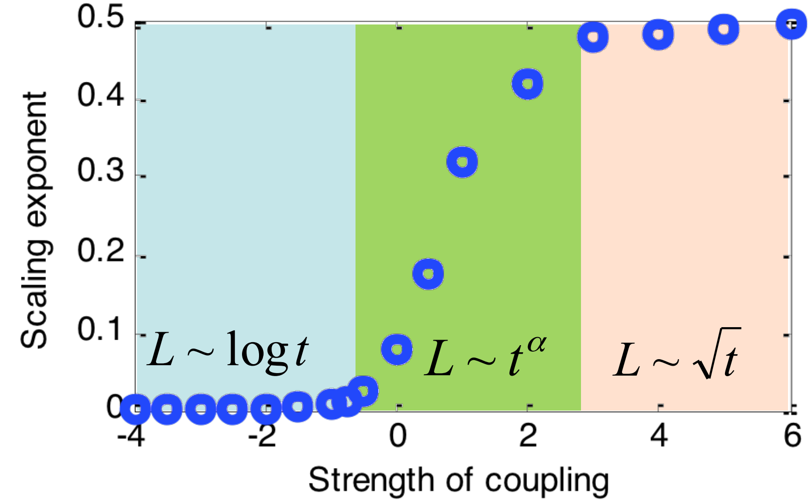
Summary

1. RG approach in the MBL state:
   - Dynamical phases and phase transitions.
   - Emergent integrals of motion.



2. RG theory of the MBL transition.

   Found intermediate phase!
   Thermal but anomalous diffusion.
   “Griffiths phase”



Many open questions

1. Generalization to 2d and 3d ? Does the Griffiths phase survive?
2. How to see MBL physics in experiments? Cold atoms?