Quench Dynamics in Interacting and Disordered Field Theories in 1D

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References:
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Marco Tavora and AM, Phys. Rev. B, 88, 115144 (2013)
Marco Schiro and AM, PRL 2014
Marco Tavora, Achim Rosch, and AM arXiv:1404.0885 (PRL in print)

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Supported by NSF-DMR

A quantum quench

Start initially in a state $\left|\Phi_{H_i}\right\rangle$ which is the ground state of some Hamiltonian Hi

Drive the system out of equilibrium by a sudden change in parameters of the Hamiltonian Hi → Hf

Explore the time-evolution and the long-time behavior.

a). Is the system thermal at long times?

b). What does it mean to be thermal for an isolated quantum system in a pure state? $\rho = \left|\Phi_{H_i}\right\rangle\left\langle\Phi_{H_i}\right|$ $Tr\left[\rho^2\right] = 1; Tr\left[\rho_{th}^2\right] < 1$

c). Prethermalization: "Glassy behavior" with intermediate long-lived metastable states.

d). Quenches across and in the vicinity of critical points: new kinds of nonequilibrium phase transitions? Universality out of equilibrium?

Model

$$H_i = \frac{u}{2\pi} \int dx \left[K \left(\pi \Pi(x)\right)^2 + \frac{1}{K} \left(\partial_x \phi(x)\right)^2 \right]$$

Quench involves switching on a leading irrelevant or marginal perturbation V

$$H_f = H_i + V$$

Quench 1: Sudden switching on of a commensurate periodic potential: dynamics in the vicinity of superfluid-Mott quantum critical point

$$V = -\left(\frac{gu}{\alpha^2}\right) \int dx \cos[2\varphi(x)]$$

Quench 2: Sudden switching on of a disordered potential: Dynamics in the vicinity of the superfluid-Bose glass quantum critical point

$$H_f = H_i + V_{\text{dis}}$$

$$V_{\text{dis}} = \int dx \left[-\frac{1}{\pi} \eta(x) \partial_x \phi + \left(\xi^* e^{2i\phi} + \xi e^{-2i\phi} \right) \right]$$

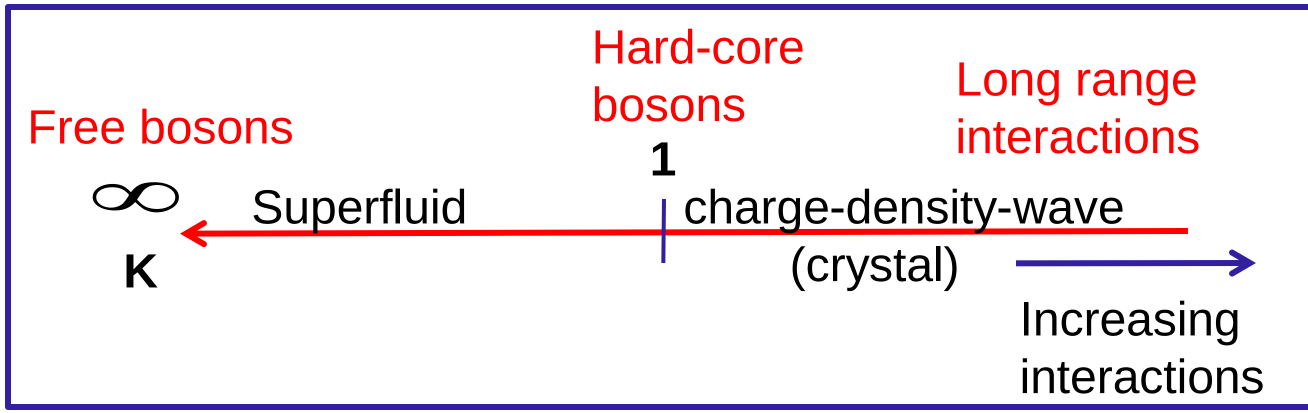
η, ξ :Random variables

Quench 3: Sudden switching on of a local impurity potential (Kane-Fisher problem).

Equilibrium and low-energy (T=0) properties of 1D Interacting Bose gas

1D interacting Bose gas
characterized by an interaction parameter K:
K = 1/(Interaction Strength)

$$H_i = \frac{u}{2\pi}\int dx \left[K\left(\pi\Pi(x)\right)^2 + \frac{1}{K}\left(\partial_x\phi(x)\right)^2 \right]$$



ψ Boson creation operator

$\rho(r)$ Boson density at r=(x,t)

Due to quantum-fluctuations, only quasi-long range order

Density-density correlator:

$$\langle \rho(x)\rho(0)\rangle \approx \frac{1}{x^{2K}}\cos(2\pi\rho_0 x)$$

Dual fields

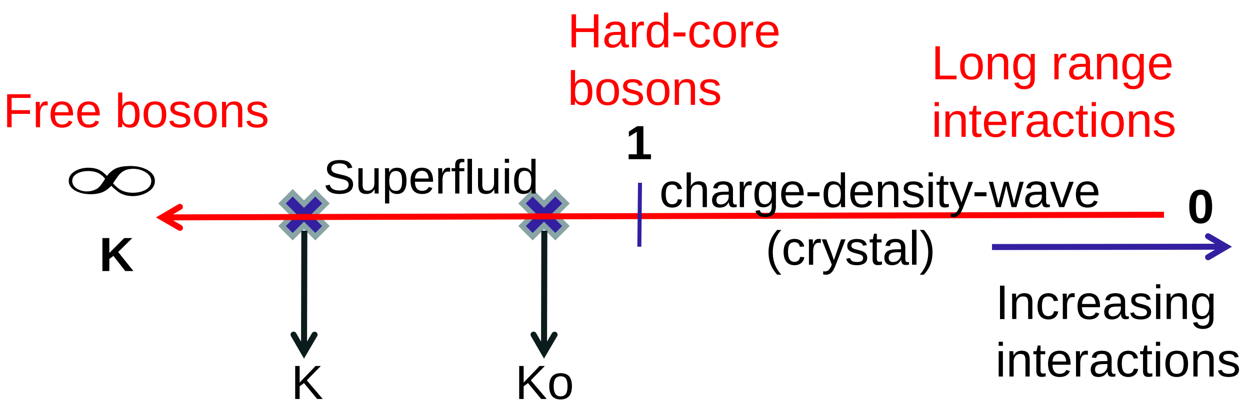
Boson propagator (superfluid order parameter):

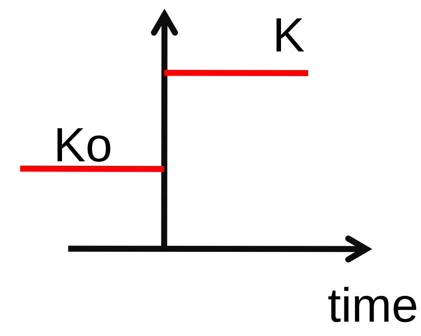
$$\langle \psi(r)\psi^+(0)\rangle \approx \frac{1}{r^{1/2K}}$$

Non-zero temperature T, exponential decay: exp[-T x]

Review of results for quenches within the harmonic LL theory

Ko=1, Cazalilla 2006
Iucci and Cazalilla, 2009





Hi : Bosons with interaction Ko

Hf : Bosons with interaction K

$\left| \Phi_{H_i, H_f} \right\rangle$ **Ground-state of Hi, Hf**

$$\left\langle \Phi_{H_i} \left| e^{iH_f t} A e^{-iH_f t} \right| \Phi_{H_i} \right\rangle \xrightarrow{t \to \infty}$$

What connection does this have with

$$\left\langle \Phi_{H_f} \left| A \right| \Phi_{H_f} \right\rangle$$

Equal time correlations at long times after the quench from Ko→K

Ko=1, Cazalilla 2006,
Iucci and Cazalilla, 2009

Density-density correlator:

$$C_{\varphi\varphi} = \left\langle e^{i2\varphi(r,t)} e^{-i2\varphi(0,t)} \right\rangle \xrightarrow{t\to\infty} \frac{1}{r^{2K_{neq}}}$$

Dual fields

Boson propagator:

$$C_{\theta\theta} = \left\langle e^{i\theta(r,t)} e^{-i\theta(0,t)} \right\rangle \xrightarrow{t\to\infty} \frac{1}{r^{2K_{neq}^{\theta}}}$$

$$K_{neq} = \frac{K_0}{2}\left(1+\frac{K^2}{K_0^2}\right) \longrightarrow K_{eq} = K$$

Compare with equilibrium (K=Ko)

$$K_{neq} > K_{eq}$$

$$K_{neq}^{\theta} > K_{eq}^{\theta}$$

$$K_{neq}^{\theta} = \frac{1}{8K_0}\left(1+\frac{K_0^2}{K^2}\right) \longrightarrow K_{eq}^{\theta} = \frac{1}{4K}$$

All correlations always decay faster after the quench as compared to the decay in the ground state of Hf.
In some sense like an effective-temperature,
yet decay is still a power-law

REASON BEHIND NEW EXPONENTS: Infinite number of conserved quantities

Initial state is ground state of → $H_i = \sum_p \epsilon_p^a a_p^\dagger a_p$ → **Density modes of the Bose gas with interaction Ko**

Time-evolution is due to → $H_f = \sum_p \epsilon_p^b b_p^\dagger b_p$ → **Density modes of the Bose gas with interaction K**

$$b_p = \cosh\Theta_p a_p + \sinh\Theta_p a_{-p}^\dagger$$

Initial state a ground state of Hi $\langle a_p^+ a_p \rangle = 0$

Hence the initial distribution function which is also conserved during the dynamic is $\langle b_p^\dagger b_p \rangle = \sinh^2\Theta_p$

Generalized Gibbs Ensemble can recover new exponents

M. Rigol, V. Dunjko, V. Yurovsky, and M. Olshanii, *Phys. Rev. Lett.* 98, 050405 (2007).

$$\rho_{GGE} = \frac{1}{Z_{GGE}} e^{-\sum_p \beta_p \varepsilon_p b_p^+ b_p}$$

where

$$\langle \Phi_i | b_p^+ b_p | \Phi_i \rangle = Tr\left[\rho_{GGE} b_p^+ b_p\right]$$

Time-evolution of correlation functions for K0 to K quench: HORIZON EFFECT

Calabrese and Cardy, 2007
Ko=1, Iucci and Cazalilla, 2009

$$R = C_{\varphi\varphi} = \left\langle e^{i2\varphi(r,T_m)} e^{-i2\varphi(0,T_m)} \right\rangle$$

$T_m = \frac{r}{2}$ Is the first instant at which excitations emitted from the same point in space reach the two operators at the same time (sound velocity=1)

Inside the light-cone r<<2Tm

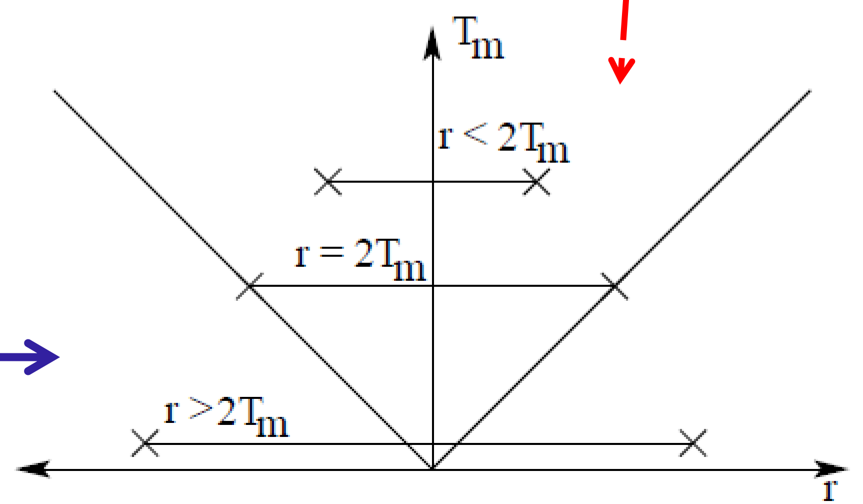
$$R(r << 2T_m) \approx \frac{1}{r^{2K_{neq}}}$$

Outside the light-cone r>>2Tm

$$R(r >> 2T_m) \approx \frac{1}{r^{2K_0}} [T_m]^{f(K-K_0)}$$

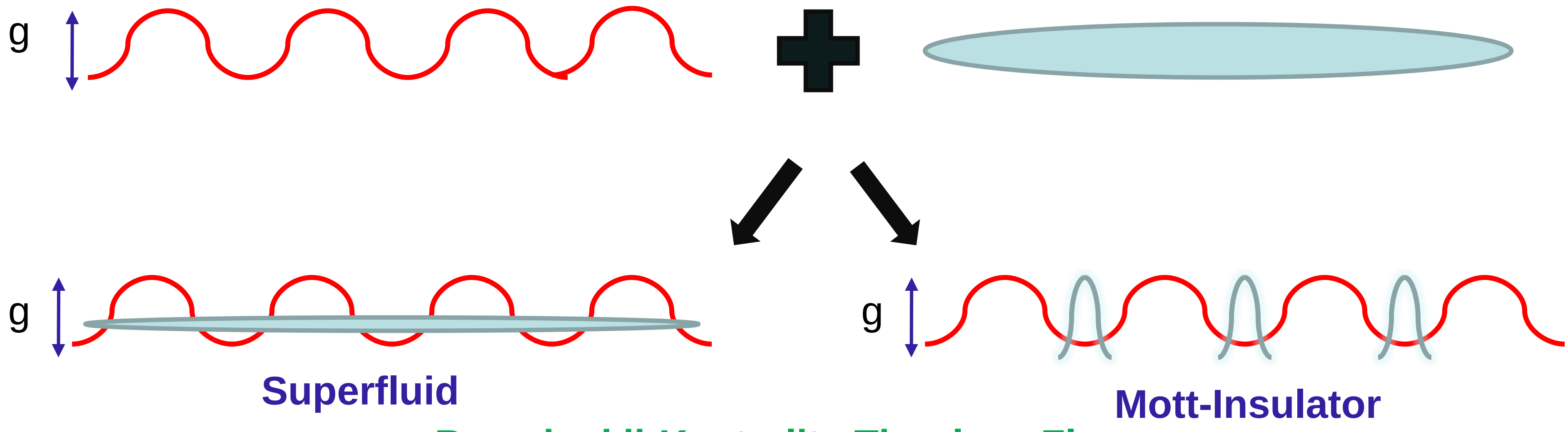
Wavefunction renormalization or formation of the new quasiparticles

HORIZON EFFECT

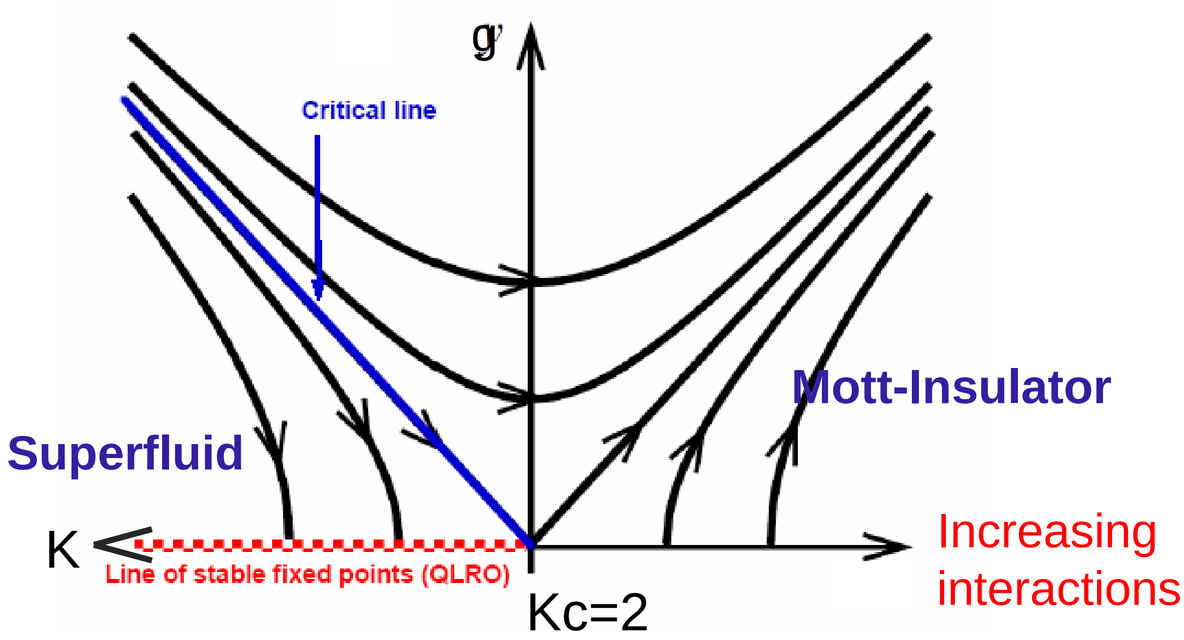


NEXT: What happens in the presence of non-linearities that take the system away from exact solvability?

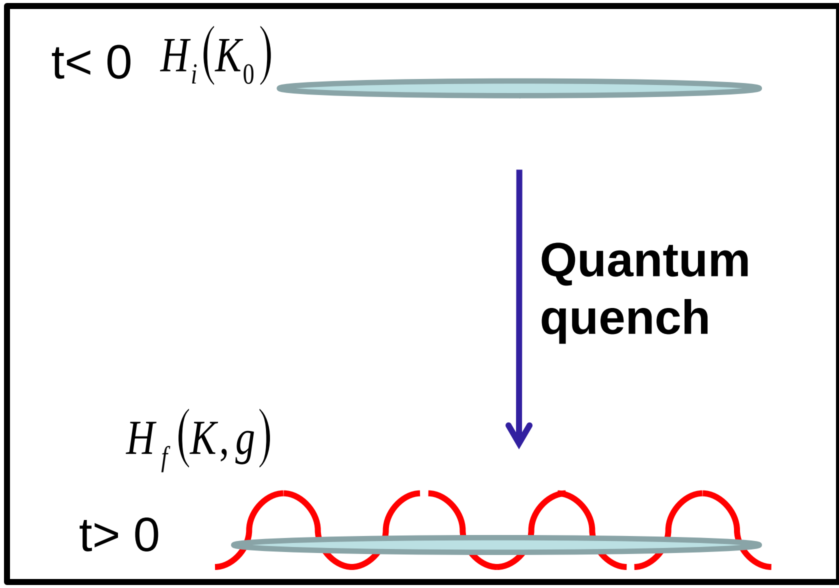
Ground state properties: Interacting bosons in a periodic potential



Berezinskii-Kosterlitz-Thouless Flows



$$H_i = \frac{u_0}{2\pi}\int dx\left\{K_0\left[\pi\Pi(x)\right]^2 + \frac{1}{K_0}\left[\partial_x\phi(x)\right]^2\right\}$$



$$H_f = \frac{u}{2\pi}\int dx\left\{K\left[\pi\Pi(x)\right]^2 + \frac{1}{K}\left[\partial_x\phi(x)\right]^2\right\} - \frac{gu}{\alpha^2}\int dx\cos\left(\gamma\phi(x)\right)$$

When g=0 (no periodic potential), exactly solvable problem.

FORMALISM: Schwinger-Keldysh "Action"

$$Z_K = Tr\left[e^{-iH_f t} \left| \Phi_i \right\rangle \left\langle \Phi_i \right| e^{iH_f t} \right]$$

$$Z_K = \int \mathcal{D}\left[\phi_{cl}, \phi_q\right] e^{i(S_0 + S_{sg})}$$

Quadratic part that describes the nonequilibrium Luttinger liquid

$$S_0 = \frac{1}{2} \int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} dx_2 \int_0^t dt_1 \int_0^t dt_2 \left(\phi_{cl}(1) \;\; \phi_q(1)\right) \begin{pmatrix} 0 & G_A^{-1}(1,2) \\ G_R^{-1}(1,2) & -\left[G_R^{-1} G_K G_A^{-1}\right](1,2) \end{pmatrix} \begin{pmatrix} \phi_{cl}(2) \\ \phi_q(2) \end{pmatrix}$$

$$\left[G^{R,A}(1,2)\right]^{-1} = -\delta(x_1 - x_2)\delta(t_1 - t_2)\frac{1}{\pi K u}\left[\partial^2_{t_1 \pm i\delta} - u^2 \partial^2_{x_1}\right]$$

Action for the sine-Gordon potential

$$S_{sg} = \frac{gu}{\alpha^2} \int_{-\infty}^{\infty} dx_1 \int_0^t dt_1 \left[\cos\{\gamma\phi_-(1)\} - \cos\{\gamma\phi_+(1)\}\right]$$

$$\gamma = 2$$

Two-point correlation function: Derivation of a Callan-Symanzik equation

$$R(x_1 t, x_2 t) = 4\langle \cos\left(\frac{\gamma\phi(x_1 t)}{2}\right)\cos\left(\frac{\gamma\phi(x_2 t)}{2}\right)\rangle$$

Related to staggered antiferromagnetic order in a spin-chain,

$$R(j) = (-1)^j \langle S_0^z S_j^z \rangle$$

$$\gamma = 2$$

$$R(x_1 t, x_2 t) = Tr\left[\rho(t) O(x_1 0) O(x_2 0)\right]$$
$$= Tr\left[e^{-iH_f t}|\phi_i\rangle\langle\phi_i| e^{iH_f t}\hat{O}(x_1)\hat{O}(x_2)\right]$$
$$= \int \mathcal{D}\left[\phi_{cl}, \phi_q\right] e^{i(S_0 + S_{sg})}\hat{O}_I(x_1 t)\hat{O}_I(x_2 t)$$

Renormalization group procedure where fast modes are gradually integrated out

Split fields into slow and fast modes in momentum space. Fast modes oscillate rapidly both in position and in time.

$$\phi_{\pm} = \phi_{\pm}^{<} + \phi_{\pm}^{>}$$

$$G_{0,\Lambda} = G^{<}_{0,\Lambda-d\Lambda} + G^{>}_{\Lambda-d\Lambda,\Lambda}$$

$$G^{>}_{\Lambda-d\Lambda,\Lambda} = d\Lambda \frac{dG_{0,\Lambda}}{d\Lambda}$$

Go=Correlators for the slow and fast fields

Integrate fast modes perturbatively in g, then rescale action in position and time. This leads to corrections to the quadratic action and to the correlation function. The corrections will depend on the time after the quench.

This procedure leads to the derivation of a Callan-Symanzik like differential equation for the correlation function.

$$R_{0,\Lambda} = R_{0,\Lambda-d\Lambda}\left[1-\frac{d\Lambda}{\Lambda}(\cdots)\right]$$

Derivation of a Callan-Symanzik equation

Lower the cut-off $\Lambda \longrightarrow \Lambda - d\Lambda$ reabsorb changes to coupling parameters such that R remains invariant.

$\frac{r\Lambda_0}{l} \gg 1, \frac{T_{m0}\Lambda_0}{l} \gg 1$

$$\left[\frac{\partial}{\partial \ln l} + \beta(g_i)\frac{\partial}{\partial g_i} \underbrace{- \gamma_{an,0} + 2\pi g I_C}\right] \times R\left[\frac{r\Lambda_0}{l}, \frac{\Lambda_0 T_{m0}}{l}, g_i(l)\right] = 0$$

Anomalous dimension of R

$$R\left(\Lambda_0 r, \Lambda_0 T_{m0}, g_0\right) = e^{-\int_{g_0}^{g(l)} dg' \frac{\gamma_{an}(g')}{\beta(g')}} \times R\left(\frac{r\Lambda_0}{l}, \frac{T_{m0}\Lambda_0}{l}, g(l)\right)$$

Correlation function at large distances may be related to correlation function at short distances but with renormalized couplings, where the latter may be evaluated within perturbation theory.

Integrate upto $l^* = \Lambda_0 \min(r, T_{m0})$

Qualitatively different behavior depending upon whether the distance r is large, small or the same order as the time after the quench.

Lets discuss the β -function

Corrections to the action: Generation of new terms out of equilibrium representing dissipation and noise

Aditi Mitra, PRL 2012
Aditi Mitra, PRB 2013
Mitra and Giamachi, PRL 2011

$$S = S_0^< + \delta S^<$$

$$S_0^< = \int_{-\infty}^{\infty} dR \int_0^{ut/\sqrt{2}} d(uT_m) \frac{1}{2\pi K} \Big[\phi_q^< \left(\partial_R^2 - \partial_{uT_m}^2\right) \phi_{cl}^< + \phi_{cl}^< \left(\partial_R^2 - \partial_{uT_m}^2\right) \phi_q^< + \frac{\delta u}{u} \phi_q^< \left(\partial_R^2 + \partial_{uT_m}^2\right) \phi_{cl}^<$$

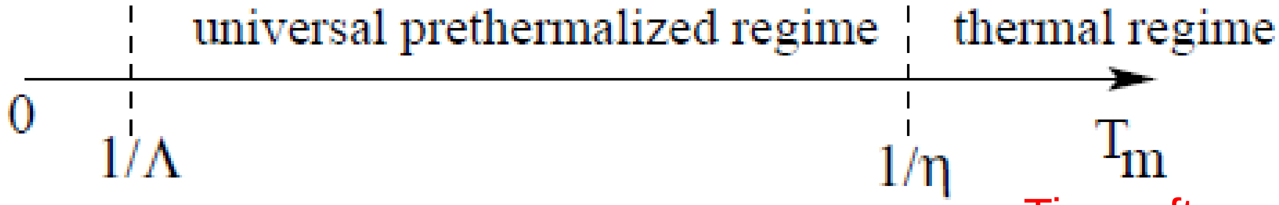
$$+ \frac{\delta u}{u} \phi_{cl}^< \left(\partial_R^2 + \partial_{uT_m}^2\right) \phi_q^< - 2\frac{\eta}{u} \phi_q^< \partial_{uT_m} \phi_{cl}^< + i\frac{4\eta T_{eff}}{u^2} \left(\phi_q^<\right)^2 \Big]$$

Dissipation of long wavelength modes

Noise on long wavelength modes

The magnitude of these corrections $\eta(T_m), \eta(T_m) T_{eff}(T_m)$ depend on time.

The inelastic scattering rate implies a new energy scale in the problem



Time after quench

$$\eta \sim g^2 (K_0 - K)^4 \ll 1$$

$$\eta \propto g^2 [Energydensity]^2$$

Next:

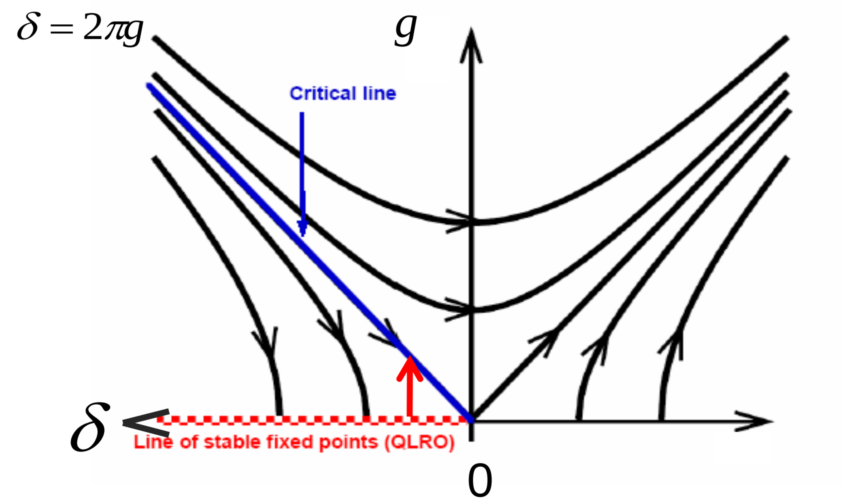
1. Results for the correlation function in the prethermalized regime where inelastic effects are weak.

2. A kinetic equation approach to understand dynamics in the thermal regime where dissipative effects are strong.

Correlation functions after a quantum quench in the prethermalized regime

$$R = \langle O(r, T_m) O(0, T_m) \rangle$$

$$1 < T_m < \frac{1}{\eta}$$



Outside the light-cone r>>2Tm

$$R(r >> 2T_m) \approx \frac{\sqrt{\ln(T_m)}}{r}$$

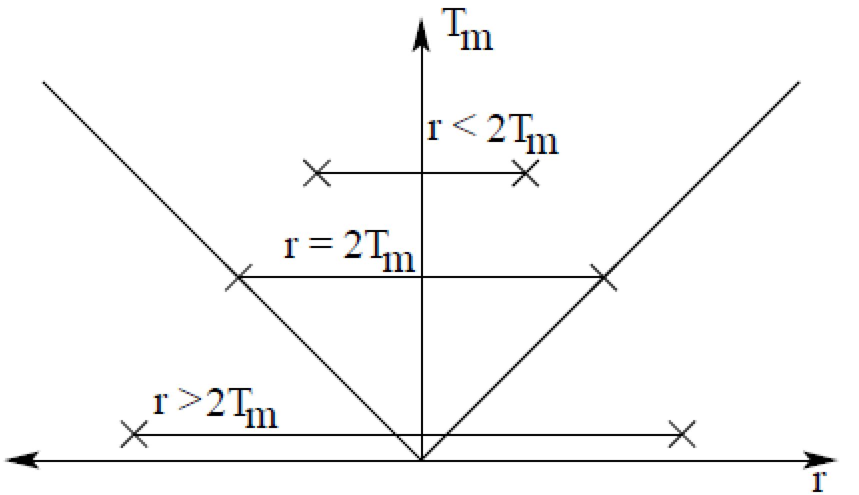
Inside the light-cone r<<2Tm

$$R(r << 2T_m) \approx \frac{\sqrt{\ln(r)}}{r}$$

On the light-cone r=2Tm

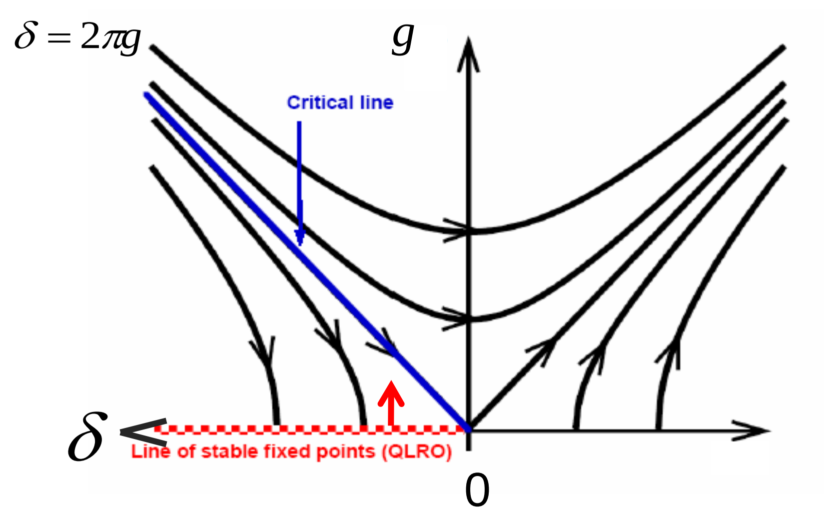
$$R \approx \frac{\ln(r)}{r}$$

HORIZON EFFECT



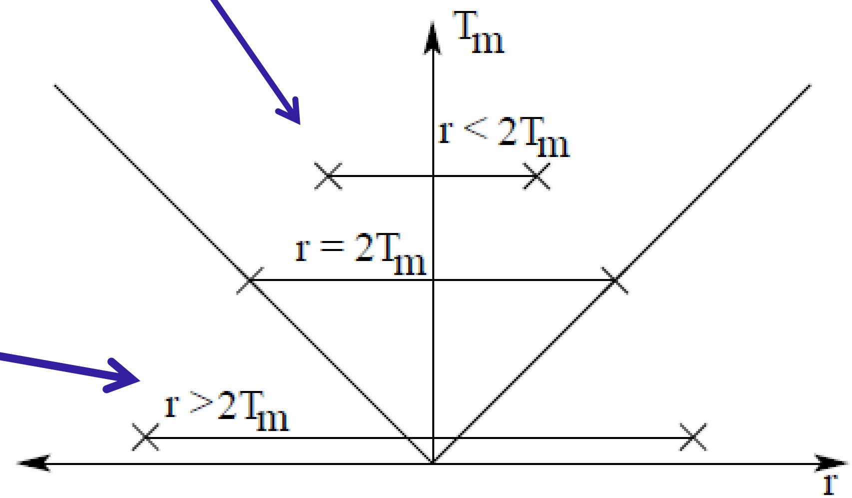
Aditi Mitra, PRB 2013

Correlation functions after a quench slightly away from the critical point

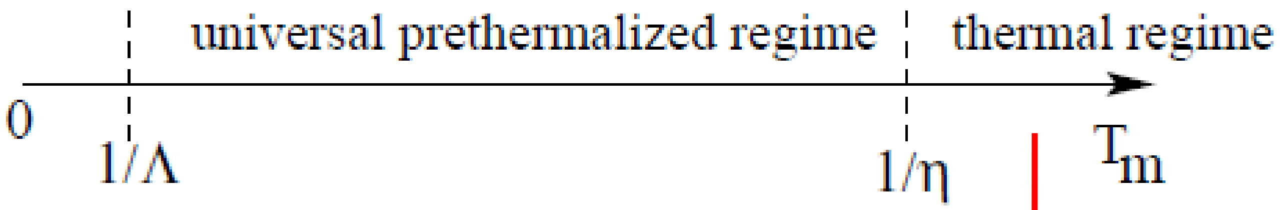


$$R\left(\Lambda_0 r, \Lambda_0 T_{m0}, g_0, r \ll 2T_{m0}\right) \sim \frac{1}{r^{1+\frac{1}{2}\sqrt{\delta^2-(2\pi g)^2}}}$$

HORIZON EFFECT



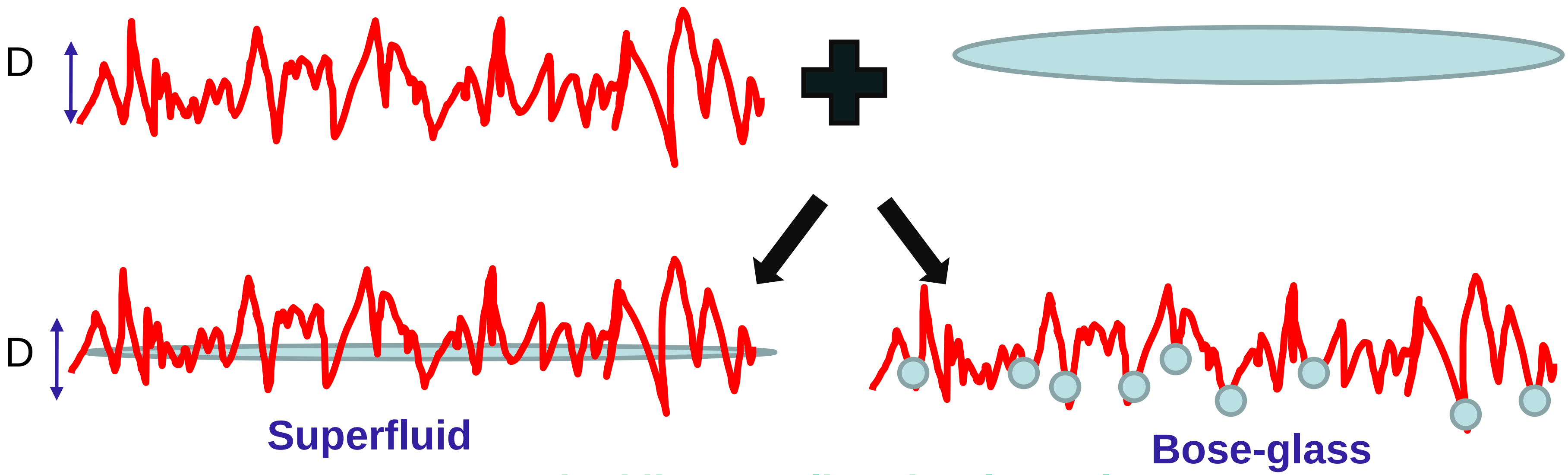
$$R\left(\Lambda_0 r, \Lambda_0 T_{m0}, g_0, r \gg 2T_{m0}\right) \sim \left(\frac{1}{r}\right)^{1+\delta/2} \left(T_{m0}\right)^{\frac{\delta}{2}-\frac{\sqrt{\delta^2-(2\pi g)^2}}{2}}$$



NEXT: WHAT HAPPENS WHEN INELASTIC EFFECTS BECOME IMPORTANT ?

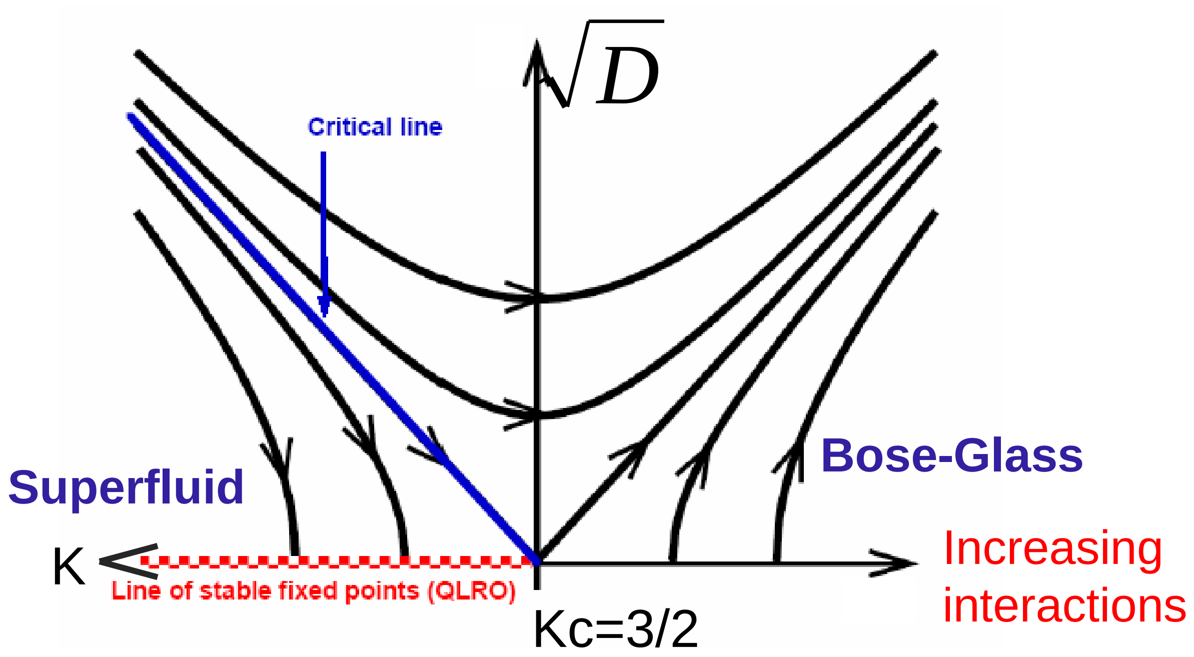
BUT FIRST: Disorder quench in the intermediate time pre-thermal regime

Ground state properties: Interacting bosons in a periodic potential



Berezinskii-Kosterlitz-Thouless Flows

Giamarchi&Schulz, 87



$$H_i = \frac{u}{2\pi}\int dx \left[K\left(\pi\Pi(x)\right)^2 + \frac{1}{K}\left(\partial_x\phi(x)\right)^2\right]$$

Quench 2: Sudden switching on of a disordered potential: Dynamics in the vicinity of the superfluid-Bose glass quantum critical point

$$H_f = H_i + V_{\text{dis}}$$

$$V_{\text{dis}} = \int dx \left[-\frac{1}{\pi}\eta(x)\partial_x\phi + \left(\xi^* e^{2i\phi} + \xi e^{-2i\phi}\right)\right]$$

η, ξ :Random variables

$$\overline{\eta(x)\eta(x')} = D_f\delta(x - x')$$

$$\overline{\xi(x)\xi^*(x')} = D_b\delta(x - x')$$

Time evolution of correlation functions:

Density-correlator: $R_{\phi\phi}(r,t) = \langle\psi_i|e^{iH_f t}e^{2i\phi(r)}e^{-2i\phi(0)}e^{-iH_f t}|\psi_i\rangle$

Boson propagator: $R_{\theta\theta}(r,t) = \langle\psi_i|e^{iH_f t}e^{i\theta(r)}e^{-i\theta(0)}e^{-iH_f t}|\psi_i\rangle$

Correlators may be written as a Keldysh path-integral

$$\langle\psi_i|R_{aa}(t)|\psi_i\rangle = Tr\left[e^{-iH_f t}|\psi_i\rangle\langle\psi_i|e^{iH_f t}R_{aa}\right]$$
$$= \int \mathcal{D}\left[\phi_{cl},\phi_q\right] e^{i(S_0+S_{\rm dis})} R_{aa}\left[\phi_{cl/q}(t),\theta_{cl/q}(t)\right]$$

Disorder-average within Keldysh formalism: (no replicas required)

$$\overline{\langle\psi_i|R_{aa}(t)|\psi_i\rangle} = \int \mathcal{D}\left[\eta,\xi,\xi^*\right] e^{-\frac{\eta^2(x)}{2D_f}} e^{-\frac{\xi(x)\xi^*(x)}{D_b}}$$
$$\times\langle\psi_i|R_{aa}(t)|\psi_i\rangle$$

Results when only forward scattering disorder is present

$$H_f = \frac{u}{2\pi}\int dx \left[K\left(\pi\Pi(x)\right)^2 + \frac{1}{K}\left(\partial_x\phi(x)\right)^2\right] + \int dx \left[-\frac{1}{\pi}\eta(x)\partial_x\phi\right]$$

$$\tilde{\phi}(x) = \phi(x) - \frac{K}{u}\int^x dy\eta(y)$$

$$H_f = \frac{u}{2\pi}\int dx \left[K\left(\pi\Pi(x)\right)^2 + \frac{1}{K}\left(\partial_x\tilde{\phi}(x)\right)^2\right] = \sum_p u|p|\Gamma_p^\dagger\Gamma_p$$

Initial state is the ground state of the unshifted fields, and corresponds to a nonequilibrium distribution of the quasiparticles of Hf

$$R^{(0)}_{\phi\phi}(r,t) = \langle\psi_i|e^{2i\phi(r,t)}e^{-i2\phi(0,t)}|\psi_i\rangle_{D_f=0}$$
$$\times e^{-\frac{iK}{u}\sum_{\epsilon=\pm}\left[\int_r^{r+\epsilon ut} dy\eta(y) - \int_0^{\epsilon ut} dy\eta(y)\right]}$$

$$R^{(0)}_{\theta\theta}(r,t) = \langle\psi_i|e^{i\theta(r,t)}e^{-i\theta(0,t)}|\psi_i\rangle_{D_f=0}$$
$$\times e^{-\frac{i}{2u}\left[\int_{r-ut}^{r+ut} dy\eta(y) - \int_{-ut}^{ut} dy\eta(y)\right]}$$

Random phases due scattering off the random forward scattering potential

Disorder averaging leads to correlators that decay exponentially in min(t,r/2)

$$\overline{R}^{(0)}_{\phi\phi}(r,t) = \left[\frac{1}{\sqrt{1+r^2\Lambda^2}}\right]^{2K} \exp\left\{-\frac{K^2 D_f}{u}\left[2t\Theta(|r|/u - 2t) + (4t - |r|/u)\,\Theta(2t - |r|/u)\Theta(|r|/u - t) +3|r|\Theta(t - |r|/u)\right]\right\}$$

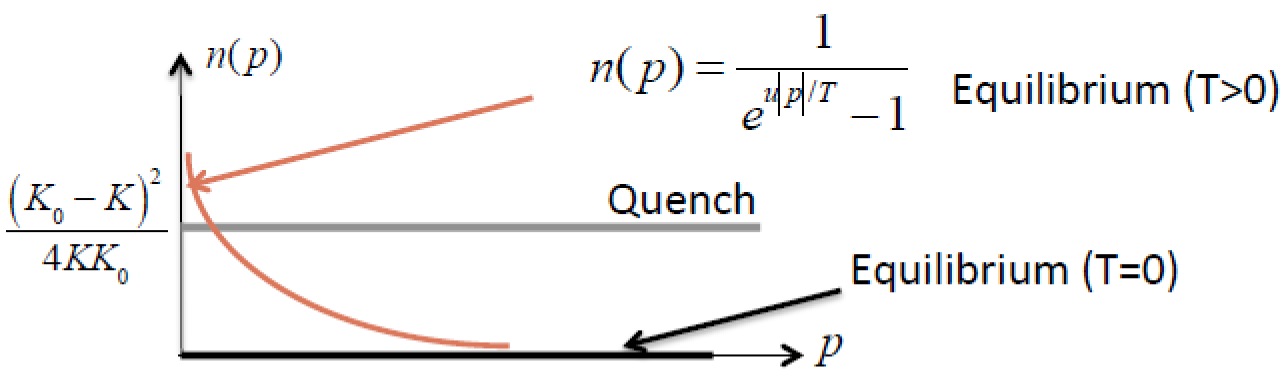
$$\overline{R}^{(0)}_{\theta\theta}(r,t) = \left[\frac{1}{\sqrt{1+r^2\Lambda^2}}\right]^{1/(2K)} \exp\left\{-\frac{D_f}{4u}\left[2t -(2t - |r|/u)\Theta(2t - |r|/u)\right]\right\} \quad (8)$$

Forward scattering disorder causes dephasing that destroys superfluidity even at short times .

For times $T_m \geq \frac{1}{\eta}$ when the clean or disordered backscattering potential causes inelastic scattering: A quantum kinetic equation approach.

Tavora and Mitra (2013)
Tavora, Rosch, Mitra (2014)

In the absence of the uniform and/or disordered backscattering potential, the boson distribution n(p) is out of equilibrium and conserved.



For disordered bosons, nonequilibrium distribution:

$$n_p(t \simeq 0) = \langle \psi_i | \Gamma_p^\dagger \Gamma_p | \psi_i \rangle = \frac{T_{\text{eff}}^f + T_{\text{eff}}^b}{u|p|} = \frac{T_{\text{eff},0}}{u|p|}$$

$$T_{\text{eff}}^f = \frac{K\mathcal{D}_f \Lambda}{2\pi}, T_{\text{eff}}^b = \Lambda 8\pi K \mathcal{D}_b \left[\frac{\Gamma(2K-2)}{\Gamma(2K)}\right]$$

Quantum Sine-Gordon ($g \neq 0$) Derivation similar for clean and disordered case

- The exact Green's function $G(xt, yt')$ obeys the Dyson equation:

$$G = g \circ (1 + \Sigma \circ G) \quad \begin{cases} -\dfrac{1}{\pi K u}\left(\partial_t^2 + u^2 q^2\right) G_R = \delta(t-t') + (\Sigma^R \circ G_R)(t,t') \\ G_K = G_R \circ F - F \circ G_A \\ -\dfrac{1}{\pi K u}\left(\partial_t^2 - \partial_{t'}^2\right) F(t,t') = \Sigma^K(t,t') - (\Sigma^R \circ F)(t,t') + (F \circ \Sigma^A)(t,t') \end{cases}$$

$$(F(q,T) = 1 + 2n(q,T))$$

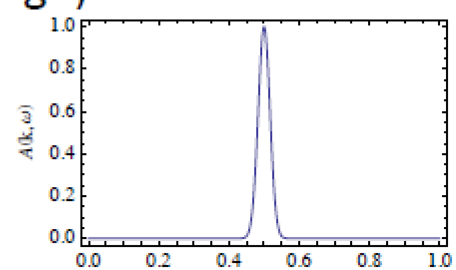
Dyson equation

Quantum Kinetic Equation

- We simplify the Dyson equations by performing a gradient expansion to lowest order, which is equivalent to a quasi-particle approximation (energy levels are not modified but only the occupation numbers change)

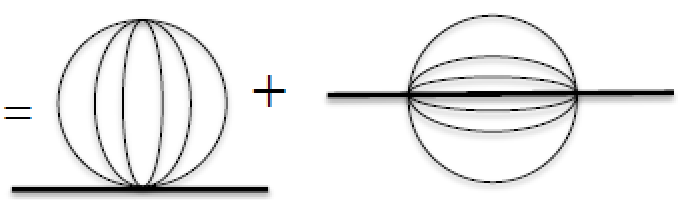
$$\frac{\partial F}{\partial T}(q,T) = \left(\frac{i\pi K}{2|q|}\right)\left[\Sigma^K(q,T) - (\Sigma^R(q,T) - \Sigma^A(q,T))F(q,T)\right]$$

$$\frac{\partial (G_R - G_A)}{\partial T}(q,T) = 0$$



$$A(k,\omega) \sim \delta(\omega - |q|)$$

$\Sigma[G] =$



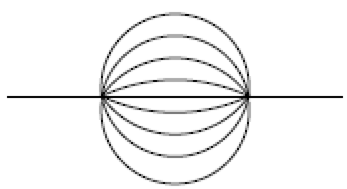
This potential describes multi-particle scattering and even to leading order in the gradient expansion the system has dynamics and has the capacity to thermalize.

$$(F(q,T)=1+2n(q,T))$$

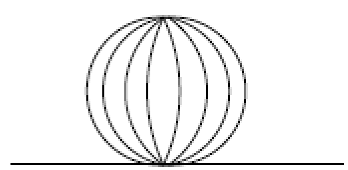
$$\frac{\partial F}{\partial T}(q,T)=I[F(q,T)]\begin{cases} I[n(k)]\sim\int_{p,q}\left[(1+n(k))(1+n(p))n(q)n(r)-n(k)n(p)(1+n(q))(1+n(r))\right] & \Phi^4\text{- theory} \\ I[n(q)]\sim\exp\left[-\int_q n(q)\right] & \text{Sine-Gordon}\end{cases}$$

$$(F(q,T)=1+2n(q,T))$$

$$\Sigma\ \sim\ \langle\cos(\gamma\phi)\cos(\gamma\phi)\rangle\ \sim\ e^{-\gamma^2\langle\phi\phi\rangle}$$

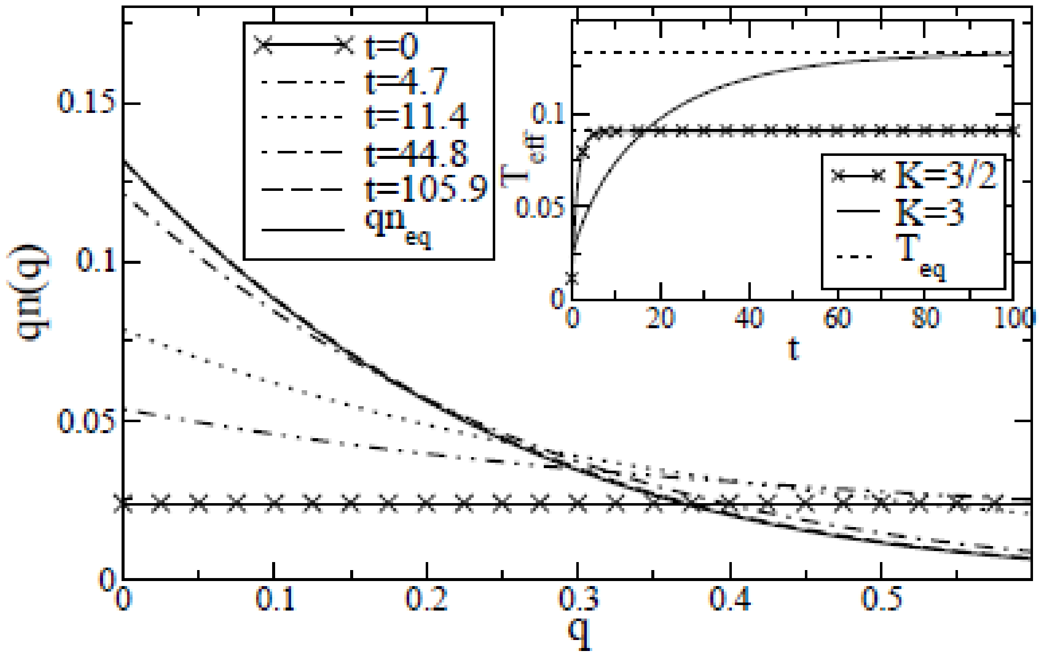


a



b

$$\Psi(q,T)=qF(q,T)\xrightarrow{\text{Equilibrium}}q\coth\left(\frac{q}{2T_{\text{eq}}}\right)$$



Thermalization time shortens on approaching the critical point at K=3/2: Critical speeding up. Relaxation rate from the long time tail of the time-evolution is found to be

$$\gamma_{\rm th} \sim \mathcal{D}_b\, [T_{\rm eff,0}]^{K-1}$$

Faster relaxation near critical points also observed in numerics and experiments

J. P. Ronzheimer, M. Schreiber, S. Braun, S. S. Hodgman, S. Langer, I. P. McCulloch, F. Heidrich-Meisner, I. Bloch, and U. Schneider, Phys. Rev. Lett. **110**, 205301 (2013).

C. P. Grams, M. Valldor, M. Garst, and J. Hemberger, ArXiv e-prints (2013), arXiv:1307.8287 [cond-mat.str-el].

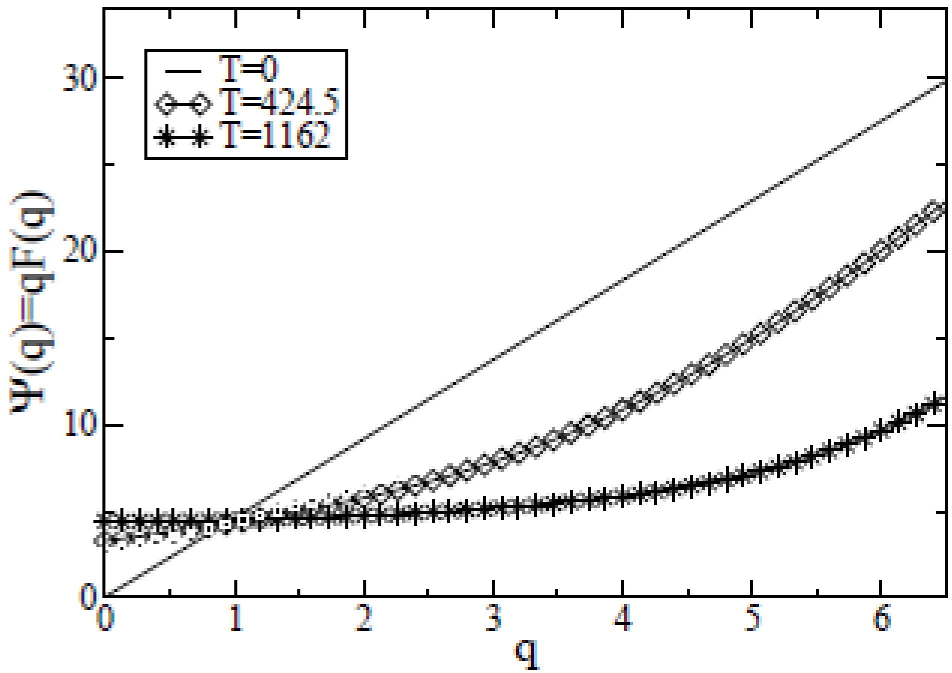
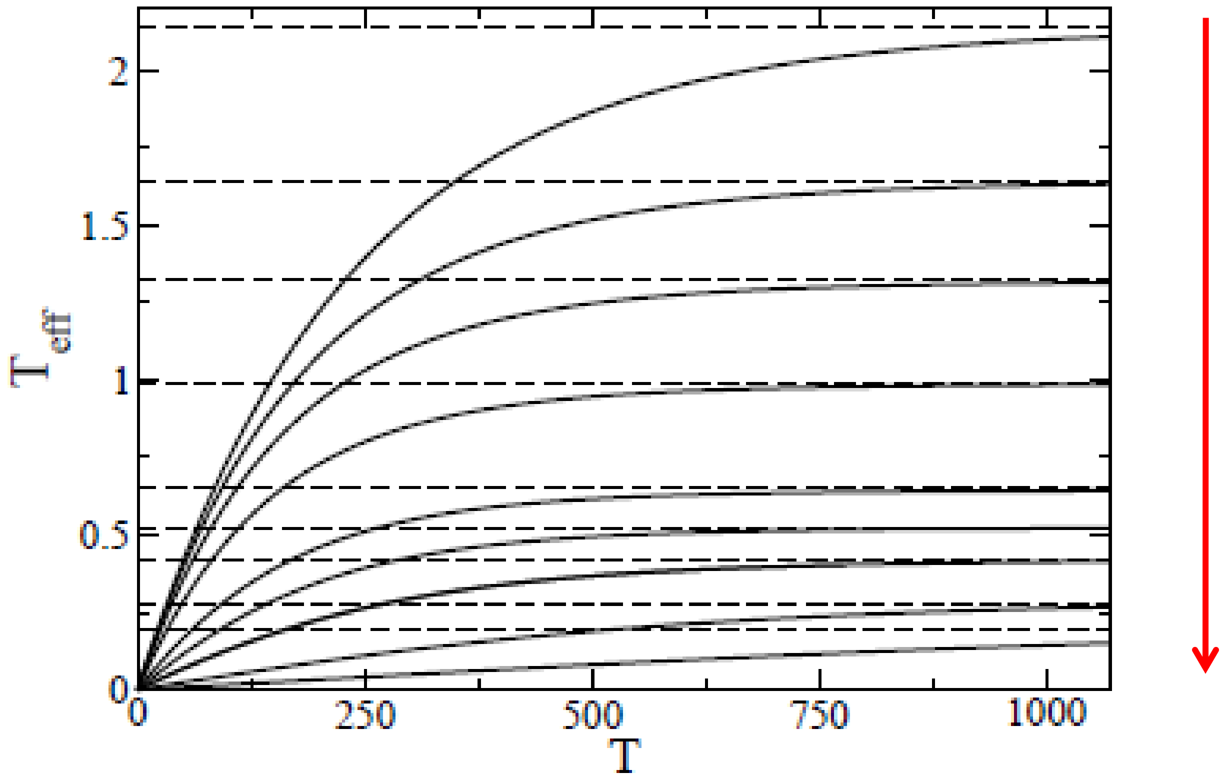


FIG. 3. The distribution function $\Psi(q) = qF(q)$ plotted for increasing times for a quench where $K_{\rm eq} = 3, K_{\rm neq} = 13.8$. At the initial time, the quench generates the distribution given by the straight line $\Psi(q) = q\frac{K_{\rm neq}}{K_{\rm eq}}$. The distribution converges to the equilibrium distribution given by $\Psi(q) = q\coth\left(\frac{uq}{2T_{\rm eq}}\right)$.

$$T_{\mathrm{eff}} = \frac{1}{2}\Psi(q=0)$$

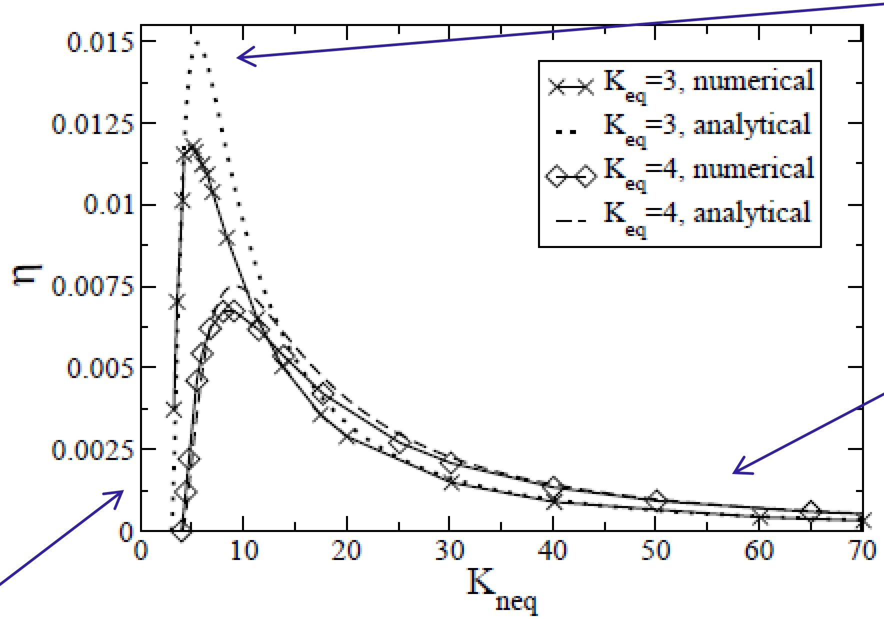


Decreasing (interaction K0 to K) quench amplitude

FIG. 6. Time-evolution of the effective temperature for (bottom to top) $K_{\mathrm{neq}} = 4.40, 4.72, 5.38, 6.0, 6.72, 9.08, 11.48, 13.91$ and 17.72 with $K_{\mathrm{eq}} = 4$. For all quenches the system thermalizes to the equilibrium temperate T_{eq} (dashed line).

Non-monotonic dependence of the thermalization time on the quench amplitude

Scattering rate increases as one approaches the critical point



$$1/K_{neq}^2$$

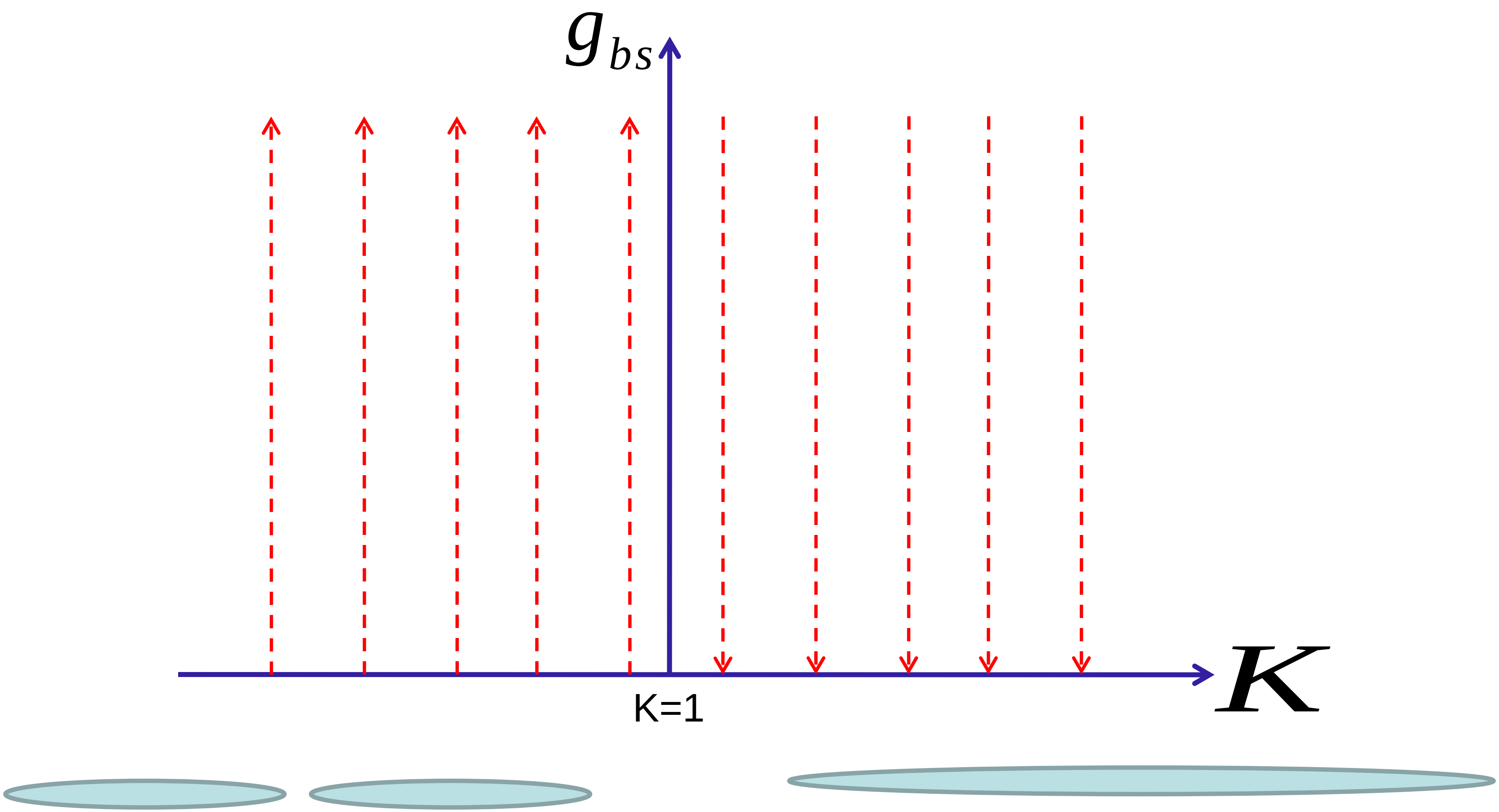
$$\Sigma \sim \langle \cos(\gamma\phi)\cos(\gamma\phi)\rangle \sim e^{-\gamma^2 \langle \phi \breve{\phi} \rangle}$$

$$(K_{neq} - K_{eq})^2$$

Reason for longer thermalization times for large quenches: Orthogonality catastrophe arising due to a poorer overlap between initial wavefunction and low energy eigenstates of Hf

Thermalization time shortens on approaching the critical point

Recall Equilibrium (T=0) Phase Diagram of Kane-Fisher Problem



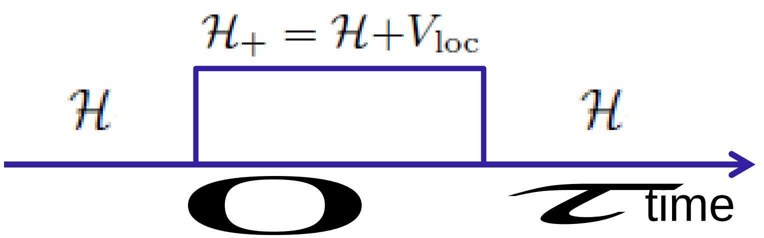
Impurity cuts the chain

Impurity has no effect

Loschmidt-Echo and the orthogonality catastrophe

$$\mathcal{H} = \frac{u}{2\pi}\int dx \left[K\left(\partial_x\theta(x)\right)^2 + \frac{1}{K}\left(\partial_x\phi(x)\right)^2\right]$$

$$V_{\rm loc} \equiv V_{\rm fs} + V_{\rm bs} = g_{\rm fs}\,\partial_x\,\phi(x)|_{x=0} + g_{\rm bs}\,\cos 2\phi(x=0)\,.$$



Suppose initially the system is in the ground state $|\Psi_0\rangle$ of $\mathcal{H}$
We then switch on the local potential for a certain period of time, and switch it off again and study the overlap between the resulting state and the initial state.

$$D_{\rm eq}(\tau) = \langle\Psi_0|e^{i\,\mathcal{H}\tau}\,e^{-i\,\mathcal{H}_+\tau}|\Psi_0\rangle$$

$$\sim \left(\frac{1}{\Lambda\tau}\right)^{\delta_{\rm oc}^{\rm eq}}$$

K>1

$$\delta_{\rm oc}^{\rm eq} = K g_{\rm fs}^2/2u^2$$

$$D_{eq}(\tau\to\infty)=0$$

Overlap between ground state wavefunction with and without the local potential vanishes: “orthogonality catastrophe”.

What about overlap between excited states??
We will study Loschmidt echo in excited states generated by a quench

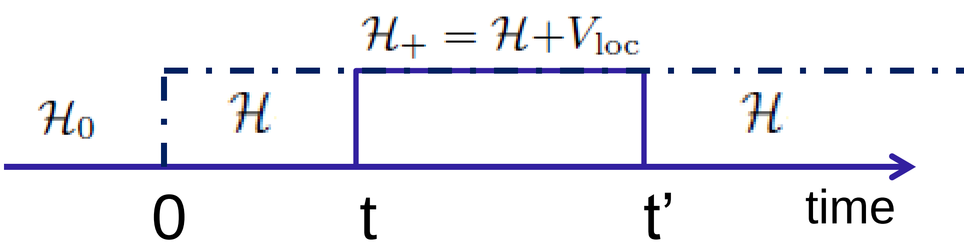
Orthogonality Catastrophe or Loschmidt Echo (LE) when the bulk is out of equilibrium: Quench induced decoherence

$$|\Psi(t)\rangle = e^{-i\mathcal{H}t}|\Psi_0\rangle \text{ while } \mathcal{H}_+ = \mathcal{H} + V_{\rm loc}.$$

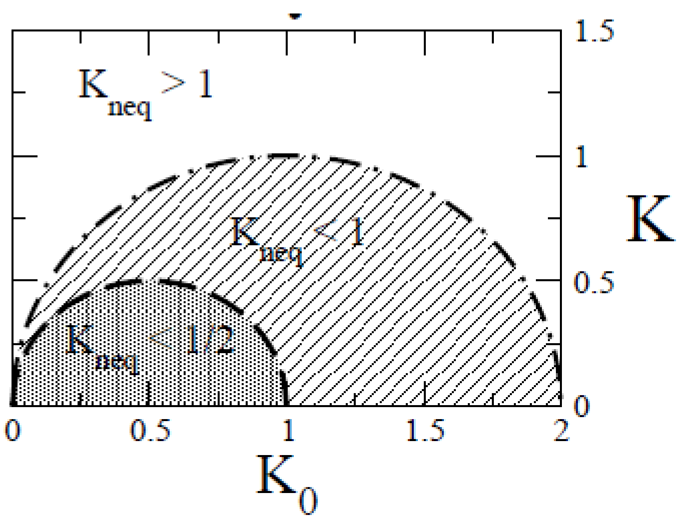
$$\mathcal{D}(t',t) \equiv \langle\Psi(t)|\, e^{i\mathcal{H}(t'-t)}\, e^{-i\mathcal{H}_+(t'-t)}|\Psi(t)\rangle$$

$$\mathcal{D}(t',t) = \mathcal{D}_{\rm fs}(t',t)\,\mathcal{D}_{\rm bs}(t',t).$$

$$\mathcal{D}_{\rm bs}(\tau) \sim \exp\left(-\gamma_* \tau\right)$$



$$\gamma_* = g_{\rm bs}^2 \left(\frac{2\pi}{2^{K_{\rm neq}}\left[2K_{\rm neq}-1\right]}\right) \frac{\Gamma\left(2K_{\rm neq}\right)}{\Gamma\left(K_{\rm neq}+K\right)\Gamma\left(K_{\rm neq}-K\right)}$$



$$\frac{dg_{\rm bs}}{d\ln l} = g_{\rm bs}\left[1 - \left(K_{\rm neq} + \frac{K_{\rm tr}}{1+4T_m^2}\right)\right]$$

$$\frac{d\eta}{d\ln l} = 2\, g_{\rm bs}^2\, I_\eta(T_m)$$

$$\frac{d(\eta T_{\rm eff})}{d\ln l} = \eta T_{\rm eff} + g_{\rm bs}^2\, I_{T_{\rm eff}}(T_m);\ \frac{dT_m}{d\ln l} = -T_m$$

Exponential decay of LE also seen in XXZ chain with impurity: Santos et al (2014)

Conclusions

1. Results for quenches in a system with disorder and interactions in one dimension.
2. An RG approach is used to show that for quenches near a critical point there is a separation of time-scales. There is a short-time perturbative regime, an intermediate time “prethermal” regime, and a long time thermal regime. Different theoretical methods can be used to study these three different cases.
3. In the prethermalized regime, universal behavior emerges in the clean system with new nonequilibrium scaling regimes related to the “horizon effect”. In the disordered system, no universal behavior in the prethermal regime, rather forward scattering disorder causes dephasing, causing correlators to decay exponentially fast.
4. A new quantum kinetic equation which accounts for multiple scattering between bosons captures the long time regime.
5. Thermalization becomes more efficient on approaching the critical point due to non-linearities becoming more important (relevant).
6. Loschmidt echoes that study excited state overlaps as a new way of probing the state generated by a quantum quench. These show thermal behavior caracterized by an exponential decay of the echo with time.