Statistical Theory of Nuclear Reactions, Channel Widths and Level Densities

S. Hilaire - CEA,DAM,DIF
- Introduction

- General features about nuclear reactions
  - Time scales and associated models
  - Types of data needed
  - Data format = f (users)

- Nuclear Models
  - Basic structure properties
  - Optical model
  - Pre-equilibrium model
  - Compound Nucleus model
  - Miscellaneous: level densities, fission, capture

- From in depth analysis to large scale production with TALYS
  - General features about TALYS
  - Fine tuning and accuracy
  - Global systematic approaches

- What remains to be done?
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- What remains to be done?

TOMORROW
INTRODUCTION
RIPL – Reference Input Parameter Library for Calculation of Nuclear Reactions and Nuclear Data Evaluations


1 NAPC–Nuclear Data Section, International Atomic Energy Agency, A-1400 Vienna, Austria
2 National Nuclear Data Center, Brookhaven National Laboratory, Upton, NY 11973, USA
3 Los Alamos National Laboratory, Los Alamos, NM 87544, USA
4 Université Libre de Bruxelles, BE 1050 Brussels, Belgium
5 Institute of Isotope and Surface Chemistry, Chemical Research Center, H-1525 Budapest, Hungary
6 Institute of Physics and Power Engineering, 240033 Obninsk, Russia
7 Fuels Actinides and Isotopes NRG Nuclear Research and Consultancy Group, NL-1755 Petten, The Netherlands
8 CEA, DAM, DIF, F-91297 Arpajon, France
9 Taras Shevchenko National University, 03022 Kiev, Ukraine
10 National Institute of Physics and Nuclear Engineering “Horia Hulubei”, 077125 Bucharest-Magurele, Romania
11 Japan Atomic Energy Agency, Tokai-mura, Naka-gun, Ibaraki-ken, 319-1195 Japan
12 China Institute of Atomic Energy, Beijing 102413 China
13 Bhabha Atomic Research Center, Trombay, 400085 Mumbai, India
14 JUKO Research, NL-1817 Akkemar, The Netherlands
15 Joint Institute for Power and Nuclear Research – Soesia, BY-280109 Minsk, Belarus
16 Retired in 1998, Ente Nuove Tecnologie, Energetica e Ambiente (ENEA), 40129 Bologna, Italy
17 Nuclear Physics Department, Bucharest University, 077195 Bucharest-Magurele, Romania

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We describe the physics and data included in the Reference Input Parameter Library, which is devoted to input parameters needed in calculations of nuclear reactions and nuclear data evaluations. Advanced modelling codes require substantial numerical input, therefore the International Atomic Energy Agency (IAEA) has worked extensively since 1993 on a library of validated nuclear-model input parameters, referred to as the Reference Input Parameter Library.
Why do we need nuclear data and how much accurate?

Nuclear data needed for

Understanding basic reaction mechanism between particles and nuclei

Good accuracy if possible ⇒ good understanding or room for improvements
Why do we need nuclear data and how much accurate?

Nuclear data needed for

Understanding basic reaction mechanism between particles and nuclei
Astrophysical applications (Age of the Galaxy, element abundances …)

Good accuracy if possible ⇒ good understanding or room for improvements
Predictive power important ⇒ sound physics (first principles)
**Nuclear data needed for**

Understanding basic reaction mechanism between particles and nuclei

Astrophysical applications (Age of the Galaxy, element abundances …)

Existing or future nuclear reactor simulations

Good (Excellent) accuracy required ⇒ reproduction of data, safety

Predictive power less important ⇒ Reproductive power
Why do we need nuclear data and how much accurate?

Nuclear data needed for:

- Understanding basic reaction mechanism between particles and nuclei
- Astrophysical applications (Age of the Galaxy, element abundances …)
- Existing or future nuclear reactor simulations
- Medical applications, oil well logging, waste transmutation, fusion, …

Good accuracy required ⇒ reproduction of data
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Nuclear data needed for

Understanding basic reaction mechanism between particles and nuclei
Astrophysical applications (Age of the Galaxy, element abundances …)
Existing or future nuclear reactor simulations
Medical applications, oil well logging, waste transmutation, fusion, …

But

Finite number of experimental data (price, safety or counting rates)
Complete measurements restricted to low energies (< 1 MeV)
to scarce nuclei

Predictive & Robust Nuclear models (codes) are essential
GENERAL FEATURES ABOUT NUCLEAR REACTIONS
- Introduction

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- What remains to be done?
62 MeV$^{56}$Fe (p,xp)
Double differential cross sections

- Always evaporation peak
- Discrete peaks at forward angles
- Flat intermediate region
Low emission energy
Reaction time $\approx 10^{-18}$ s
Isotropic angular distribution
Reaction time

Emission energy

\[ \frac{d^2 \sigma}{d\Omega dE} \]

Compound Nucleus

Direct components

Reaction time

Emission energy

High emission energy
Reaction time \( \approx 10^{-22} \) s
Anisotropic angular distribution
- forward peaked
- oscillatory behavior
  \( \Rightarrow \) spin and parity of residual nucleus
Intermediate emission energy
Intermediate reaction time
Anisotropic angular distribution smoothly increasing to forward peaked shape with outgoing energy
Elastic

Inelastic
(n,n'), (n,α), (n,γ), etc...

Fission

Direct (shape) elastic

σ_{Reaction}

σ_{NC}

T_{ij}

Direct components

OPTICAL MODEL → PRE-EQUILIBRIUM → COMPOUND NUCLEUS
Reaction mechanisms

Compound nucleus

Pre-equilibrium: multiple emission

Elastic scattering

Fission

Emission spectrum

Elastic peak

Evaporation: low energy hump

High and medium energy structureless spectrum

High energy discrete peaks

62 MeV$^{56}$Fe (p,xp)
Double differential cross sections
TYPES OF DATA NEEDED

Cross sections:
- total, reaction, elastic (shape & compound), non-elastic, inelastic (discrete levels & total)
- total particle (residual) production
- all exclusive reactions (n,nd2a)
- all exclusive isomer production
- all exclusive discrete and continuum γ-ray production

Spectra:
- elastic and inelastic angular distribution or energy spectra
- all exclusive double-differential spectra
- total particle production spectra
- compound and pre-equilibrium spectra per reaction stage.

Fission observables:
- cross sections (total, per chance)
- fission fragment mass and isotopic yields
- fission neutrons (multiplicities, spectra)

Miscellaneous:
- recoil cross sections and ddx
- particle multiplicities
- astrophysical reaction rates
- covariances informations
DATA FORMAT

• Trivial for basic nuclear science: x,y,(z) file

• Complicated (even crazy) for data production issues: ENDF file
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**Values**
NUCLEAR MODELS
- Introduction

- General features about nuclear reactions
  - Time scales and associated models
  - Types of data needed
  - Data format $= f$ (users)

- Nuclear Models
  - Basic structure properties
  - Optical model
  - Pre-equilibrium model
  - Compound Nucleus model
  - Miscellaneous: level densities, fission, capture

- From in depth analysis to large scale production with TALYS
  - General features about TALYS
  - Fine tuning and accuracy
  - Global systematic approaches

- What remains to be done?
Nuclear Masses:
  ⇒ basic information to determine reaction threshold

Excited levels:
  ⇒ Angular distributions (depend on spin and parities)
  ⇒ Decay properties (branching ratios)
  ⇒ Excitation energies (reaction thresholds)

Target levels’ deformations:
  ⇒ Required to select appropriate optical model
  ⇒ Required to select appropriate coupling scheme

Many different theoretical approaches if experimental data is missing
Recommended databases (RIPL !)
Ground-state properties

- Audi-Wapstra mass compilation
- Mass formulas including deformation and matter densities
Discrete level schemes: $J$, $\pi$, $\gamma$-transitions, branching ratios

- $\approx 2500$ nuclei
- $> 110000$ levels
- $> 13000$ spins assigned
- $> 160000$ $\gamma$-transitions
### Macroscopic-Microscopic Approaches

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<th>Model</th>
<th>Reliability</th>
<th>Accuracy</th>
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<td>Liquid drop model (Myers &amp; Swiateki 1966)</td>
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<td>Droplet model (Hilf et al. 1976)</td>
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### Approximation to Microscopic models

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<td>ETFSI model (Aboussir et al. 1995)</td>
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### Mean Field Model

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Typical deviations for the best mass formulas:

\[ \text{rms}(M) = 600-700 \text{ keV on 2149 (Z \geq 8) experimental masses} \]
Comparison between several mass models adjusted with 2003 exp and tested with 2012 exp masses

**Current status**
- $\text{rms} < 1 \text{ MeV (masses } \approx \text{ GeV)}$
- micro ~ macro
- micro more predictive

**Microscopic models**
**Methodology**: \( E = E_{mf} + \delta E_{\infty} + \delta E_{bf} \)

- **Additional filters**
  - Collective properties (0+, 2+, BE2), RPA modes, backbending properties, pairing properties, fission properties, gamma strength functions, level densities

**BASIC STRUCTURE PROPERTIES (4/5)**

**HFB Mass models**

* Most advanced theoretical approach = multireference level*
Comparison with 2149 Exp. Masses

- $E_{\text{th}} = E_{\text{HFB}}$
  r.m.s. $\sim 4.4$ MeV

- $E_{\text{th}} = E_{\text{HFB}} - \Delta_{\infty}$
  r.m.s. $\sim 2.6$ MeV

- $E_{\text{th}} = E_{\text{HFB}} - \Delta_{\infty} - \Delta_{\text{quad}}$
  r.m.s. $\sim 2.9$ MeV
Comparison with 2149 Exp. Masses

D1S

D1N

D1M

\( \delta B \) (MeV)

r.m.s. \( \sim 2.5 \) MeV

r.m.s. \( \sim 0.95 \) MeV

\( \varepsilon = 0.126 \) MeV

r.m.s. = 0.798 MeV

\( N \)
Inelastic
$(n,n')$, $(n,\alpha)$, $(n,\gamma)$, etc…

Elastic

Fission

Direct components

Direct (shape) elastic

Elastic

The Optical Model

Optical Model

Pre-equilibrium

Compound Nucleus

$\sigma_{Reaction}$

$\sigma_{NC}$

$T_{ij}$

Direct components
Elastic Fission $(n,n')$, $(n,\alpha)$, etc…

Direct (shape) elastic

Optical Model

Pre-equilibrium Compound Nucleus

Direct components

$\sigma_{\text{Reactions}}$

$T_{ij}$

Inelastic $(n,n'), (n,\alpha), (n,\gamma)$, etc…

Elastic

Fission
Direct interaction of a projectile with a target nucleus considered as a whole
Quantum model → Schrödinger equation

\[
\left( -\frac{\hbar^2}{2\mu} \nabla^2 + U - E \right) \Psi = 0
\]

Complex potential:

\[
U = V + iW
\]

Refraction  Absorption

20 MeV $^{208}\text{Pb}(n,n)^{208}\text{Pb}$
THE OPTICAL MODEL

Direct interaction of a projectile with a target nucleus considered as a whole
Quantum model → Schrödinger equation

\[ \left( -\frac{\hbar^2}{2\mu} \nabla^2 + U - E \right) \Psi = 0 \]

Complex potential:

\[ U = V + iW \]

Refraction

Absorption
The optical model yields:

- Integrated cross sections
- Transmission coefficients
- Angular distributions

![Graphs showing integrated cross sections, transmission coefficients, and angular distributions for different reactions.]
TWO TYPES OF APPROACHES

Phenomenological
- Adjusted parameters
- Weak predictive power
- Very precise (≈ 1%)
- Important work

(Semi-)microscopic
- No adjustable parameters
- Usable without exp. data
- Less precise (≈ 5-10 %)
- Quasi-automated
- ≈ 20 adjusted parameters
- Very precise (1%)
- Weak predictive power
PHENOMENOLOGICAL OPTICAL MODEL

- OMP & its parameters
- Solution of the Schrödinger equation
- Calculated observables: $\sigma_{el-inl}(\theta), A_y(\theta), \sigma_{tot}, \sigma_{reac}, S_0, S_1$
- Experimental data: $\sigma_{el-inl}(\theta), A_y(\theta), \sigma_{tot}, \sigma_{reac}, S_0, S_1$
- $\sigma_{Reaction}, T_{ij}, \sigma_{direct}$
- No adjustable parameters
- Based on nuclear structure properties

⇒ usable for any nucleus

- Less precise than the phenomenological approach
Optical potential $= \frac{U(r',E)}{\rho(r')} \times \rho(r)$

Depends on the nucleus

Independent of the nucleus

Depends on the nucleus
Unique description of elastic scattering
Unique description of elastic scattering \((n,n)\)
Unique description of elastic scattering \((n,n), (p,p)\)
Unique description of elastic scattering \((n,n), (p,p)\) and \((p,n)\)
Enables to give predictions for very exotic nuclei for which there exist no experimental data.
Average neutron resonance parameters
• average s-wave spacing at $B_n$ ⇒ level densities
• neutron strength functions ⇒ optical model at low energy
• average radiative width ⇒ $\gamma$-ray strength function
OMP for more than 500 nuclei from neutron to $^4$He
- standard parameters (phenomenologic)
- deformation parameters (levels from levels’ segment)
- energy-mass dependent global models and codes (matter densities from mass segment)
THE PRE-EQUILIBRIUM MODEL

OPTICAL MODEL

PRE-EQUILIBRIUM

COMPOUND NUCLEUS

Shape elastic

Elastic

Fission

Inelastic

Direct components

\( \sigma_{\text{Reaction}} \)

\( T_{ij} \)

\( \sigma_{\text{NC}} \)

\( (n,n'), (n,\alpha), (n,\gamma), \text{etc...} \)
**The Pre-Equilibrium Model**

- **Shape elastic**
- **Elastic**
- **Inelastic**
  - \( (n,n') \), \( (n,\alpha) \), \( (n,\gamma) \), etc...
- **Direct components**

**Optical Model**
62 MeV$^{56}$Fe ($p, x_p$)
Double differential cross sections

- Always evaporation peak
- Discrete peaks at forward angles
- Flat intermediate region
Semi-classical approaches
- called « exciton model »
- « simple » to implement
- initially only able to describe angle integrated spectra (1966 & 1970)
- extended to ddx spectra in 1976
- link with Compound Nucleus established in 1987
- systematical underestimation of ddx spectra at backward angles
- complemented by Kalbach systematics (1988) to improve ddx description
- link with OMP imaginary performed in 2004

Quantum mechanical approaches
- distinction between MSC and MSD processes
  MSC = bound p-h excitations, symmetrical angular distributions
  MSD = unbound configuration, smooth forward peaked ang. dis.
- MSD dominates pre-equ xs above 20 MeV
- 3 approaches : FKK (1980)
  TUL (1982)
  NWY (1986)
- ddx spectra described as well as with Kalbach systematics
THE PRE-EQUILIBRIUM MODEL
(Exciton model principle)
THE PRE-EQUILIBRIUM MODEL
(Exciton model principle)

$E_F$

$0$

$E$

$1p$

$1n$
THE PRE-EQUILIBRIUM MODEL
(Exciton model principle)
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(Exciton model principle)
THE PRE-EQUILIBRIUM MODEL
(Exciton model principle)

Compounds:

- 1p 1n
- 2p-1h 3n
- 3p-2h 5n
- 4p-3h 7n

Energy levels:

- EF (Fermi energy)
- E (electronic energy)

Time progression:

- 1p 1n
- 2p-1h 3n
- 3p-2h 5n
- 4p-3h 7n

Compound Nucleus
THE PRE-EQUILIBRIUM MODEL
(Master equation exciton model)

\[ P(n, E, t) = \text{Probability to find for a given time } t \text{ the composite system with an energy } E \text{ and an exciton number } n. \]

\[ \lambda_{a, b} (E) = \text{Transition rate from an initial state } a \text{ towards a state } b \text{ for a given energy } E. \]
THE PRE-EQUILIBRIUM MODEL
(Master equation exciton model)

\[ P(n,E,t) = \text{Probability to find for a given time } t \text{ the composite system with an energy } E \text{ and an exciton number } n. \]

\[ \lambda_{a,b}(E) = \text{Transition rate from an initial state } a \text{ towards a state } b \text{ for a given energy } E. \]

Evolution equation

\[ \frac{dP(n,E,t)}{dt} = \text{Apparition} \]

- Disparition
THE PRE-EQUILIBRIUM MODEL
(Master equation exciton model)

\[ P(n, E, t) = \text{Probability to find for a given time } t \text{ the composite system with an energy } E \text{ and an exciton number } n. \]

\[ \lambda_{a, b} (E) = \text{Transition rate from an initial state } a \text{ towards a state } b \text{ for a given energy } E. \]

**Evolution equation**

\[
\frac{dP(n, E, t)}{dt} = P(n-2, E, t) \lambda_{n-2, n} (E) + P(n+2, E, t) \lambda_{n+2, n} (E)
\]

- **Disparition**
THE PRE-EQUILIBRIUM MODEL  
(Master equation exciton model)

\[ P(n,E,t) = \text{Probability to find for a given time } t \text{ the composite system with an energy } E \text{ and an exciton number } n. \]

\[ \lambda_{a,b}(E) = \text{Transition rate from an initial state } a \text{ towards a state } b \text{ for a given energy } E. \]

**Evolution equation**

\[
\frac{dP(n,E,t)}{dt} = P(n-2, E, t) \lambda_{n-2,n}(E) + P(n+2, E, t) \lambda_{n+2,n}(E) - P(n, E, t) \left[ \lambda_{n,n+2}(E) + \lambda_{n,n-2}(E) + \lambda_{n,\text{emiss}}(E) \right]
\]
THE PRE-EQUILIBRIUM MODEL
(Master equation exciton model)

\[ P(n, E, t) = \text{Probability to find for a given time } t \text{ the composite system with an energy } E \text{ and an exciton number } n. \]

\[ \lambda_{a, b}(E) = \text{Transition rate from an initial state } a \text{ towards a state } b \text{ for a given energy } E. \]

**Evolution equation**

\[
\frac{dP(n, E, t)}{dt} = P(n-2, E, t) \lambda_{n-2, n}(E) + P(n+2, E, t) \lambda_{n+2, n}(E) \\
- P(n, E, t) \left[ \lambda_{n, n+2}(E) + \lambda_{n, n-2}(E) + \lambda_{n, \text{emiss}}(E) \right]
\]

**Emission cross section in channel** \( c \)

\[
d\sigma_c(E, \varepsilon_c) = \sigma_R \int_0^\infty \sum_{n, \Delta n=2} P(n, E, t) \lambda_{n, c}(E) \, dt \, d\varepsilon_c
\]
THE PRE-EQUILIBRIUM MODEL
(Initialisation & transition rates)
THE PRE-EQUILIBRIUM MODEL
(Initialisation & transition rates)

**Initialisation**

\[ P(n,E,0) = \delta_{n,n_0} \text{ with } n_0=3 \text{ for nucleon induced reactions} \]

**Transition rates**

\[ \lambda_{n, n-2}(E) = \frac{2\pi}{\hbar} \langle M^2 \rangle \omega(p,h,E) \text{ with } p+h=n-2 \]

\[ \lambda_{n, n+2}(E) = \frac{2\pi}{\hbar} \langle M^2 \rangle \omega(p,h,E) \text{ with } p+h=n+2 \]

\[ \lambda_{n, c}(E) = \frac{2s_c+1}{\pi^2 \hbar^3} \mu_c \varepsilon_c \sigma_{c,inv}(\varepsilon_c) \frac{\omega(p-p_b,h,E-\varepsilon_c-B_c)}{\omega(p,h,E)} \]
Initialisation

\[ P(n,E,0) = \delta_{n,n_0} \text{ with } n_0=3 \text{ for nucleon induced reactions} \]

Transition rates

\[ \lambda_{n, n-2}(E) = \frac{2\pi}{\hbar} \langle M^2 \rangle \omega(p,h,E) \text{ with } p+h=n-2 \]

\[ \lambda_{n, n+2}(E) = \frac{2\pi}{\hbar} \langle M^2 \rangle \omega(p,h,E) \text{ with } p+h=n+2 \]

\[ \lambda_{n, c}(E) = \frac{2s_c+1}{\pi^2 \hbar^3} \mu_c \varepsilon_c \sigma_{c,\text{inv}}(\varepsilon_c) \frac{\omega(p-p_b,h,E-\varepsilon_c-B_c)}{\omega(p,h,E)} Q_c(n) \Phi_c \]

Corrections for proton-neutron distinguishability & complex particle emission
THE PRE-EQUILIBRIUM MODEL
(Initialisation & transition rates)

Initialisation

\[ P(n,E,0) = \delta_{n,n_0} \text{ with } n_0=3 \text{ for nucleon induced reactions} \]

Transition rates

\[ \lambda_{n, n-2}(E) = \frac{2\pi}{\hbar} \langle M^2 \rangle \omega(p,h,E) \text{ with } p+h=n-2 \]

\[ \lambda_{n, n+2}(E) = \frac{2\pi}{\hbar} \langle M^2 \rangle \omega(p,h,E) \text{ with } p+h=n+2 \]

\[ \lambda_{n, c}(E) = \frac{2s_c+1}{\pi^2 \hbar^3} \mu_c \varepsilon_c \sigma_{c,\text{inv}}(\varepsilon_c) \frac{\omega(p-p_b,h,E-\varepsilon_c-B_c)}{\omega(p,h,E)} Q_c(n) \Phi_c \]

State densities

\[ \omega(p,h,E) = \text{number of ways of distributing } p \text{ particles and } h \text{ holes on among accessible single particle levels with the available excitation energy } E \]
THE PRE-EQUILIBRIUM MODEL
(State densities)

State densities in ESM

- Ericson 1960: no Pauli principle
- Griffin 1966: no distinction between particles and holes
- Williams 1971: distinction between particles and holes as well as between neutrons and protons but infinite number of accessible states for both particle and holes

\[
\omega_{p_\pi h_\pi p_\nu h_\nu}(U) = g_{p_\pi}^{p_\pi+h_\pi} g_{p_\nu}^{p_\nu+h_\nu} \frac{(U-B)^{M-1}}{p_\pi! p_\nu! h_\pi! h_\nu!(M-1)!},
\]

where \( M \) is the total number of particles and holes of both kinds and

\[
B = \frac{1}{4} \left( \frac{p_\pi^2 + h_\pi^2 + p_\pi - h_\pi}{g_\pi} + \frac{p_\nu^2 + h_\nu^2 + p_\nu - h_\nu}{g_\nu} \right) - \frac{1}{2} \left( \frac{h_\pi}{g_\pi} + \frac{h_\nu}{g_\nu} \right)
\]
State densities in ESM

- Ericson 1960: no Pauli principle
- Griffin 1966: no distinction between particles and holes
- Williams 1971: distinction between particles and holes as well as between neutrons and protons but infinite number of accessible states for both particle and holes
- Běták and Doběs 1976: account for finite number of holes’ states
- Obložinský 1986: account for finite number of particles’ states (MSC)
- Anzaldo-Meneses 1995: first order corrections for increasing number of p-h
- Hilaire and Koning 1998: generalized expression in ESM
THE PRE-EQUILIBRIUM MODEL

Cross section

Outgoing energy

Total
Direct
Pre-equilibrium
Statistical

39%
45%
16%
12%
9%
79%

Cross section

Outgoing energy

25 MeV n + ^{56}\text{Fe}

\langle E_{\text{Tot}} \rangle = 12.1
\langle E_{\text{Dir}} \rangle = 24.3
\langle E_{\text{PE}} \rangle = 9.32
\langle E_{\text{Sta}} \rangle = 2.5
(MeV)
THE PRE-EQUILIBRIUM MODEL

14 MeV neutron + $^{93}$Nb

- **Pre-equilibrium**
- **Compound nucleus**

Graphs showing the relationship between incident and outgoin neutron energy, and their corresponding cross-sections.
Nuclear level densities (formulae, tables, codes)

- spin-, parity- dependent level densities fitted to $D_0$
- single particle level schemes
- $p$-$h$ level density tables
THE COMPOUND NUCLEUS MODEL

Shape elastic

Elastic

Fission

Inelastic
\((n,n'), (n,\alpha), (n,\gamma), \text{ etc...}\)

Direct components

\(\sigma_{\text{Reaction}}\)

\(\sigma_{\text{NC}}\)

\(T_{ij}\)
THE COMPOUND NUCLEUS MODEL

OPTICAL MODEL

Shape elastic

Direct components

Fission

Inelastic
(n,n'), (n,α), (n,γ), etc...

Elastic

COMPOUND NUCLEUS
After direct and pre-equilibrium emission

\[ \sigma_{\text{reaction}} = \sigma_{\text{dir}} + \sigma_{\text{pre-equ}} + \sigma_{\text{NC}} \]

- \( N_0 \) \( \rightarrow \) \( N_0 - dN_D \) \( \rightarrow \) \( N_0 - dN_D - dN_{PE} = E \)
- \( Z_0 \) \( \rightarrow \) \( Z_0 - dZ_D \) \( \rightarrow \) \( Z_0 - dZ_D - dZ_{PE} = Z \)
- \( E^*_0 \) \( \rightarrow \) \( E^*_0 - dE^*_D \) \( \rightarrow \) \( E^*_0 - dE^*_D - dE^*_{PE} = E^* \)
- \( J_0 \) \( \rightarrow \) \( J_0 - dJ_D \) \( \rightarrow \) \( J_0 - dJ_D - dJ_{PE} = J \)

\( N, Z, E^*, J \)
\( \rho(N, Z, E^*) \)
THE COMPOUND NUCLEUS MODEL
(initial population)

After direct and pre-equilibrium emission

\[ \sigma_{\text{reaction}} = \sigma_{\text{dir}} + \sigma_{\text{pre-equ}} + \sigma_{\text{NC}} \]

\[ N_0 \quad N_0 - dN_D \quad N_0 - dN_D - dN_{PE} = E \]
\[ Z_0 \quad Z_0 - dZ_D \quad Z_0 - dZ_D - dZ_{PE} = Z \]
\[ E^*_0 \quad E^*_0 - dE^*_D \quad E^*_0 - dE^*_D - dE^*_{PE} = E^* \]
\[ J_0 \quad J_0 - dJ_D \quad J_0 - dJ_D - dJ_{PE} = J \]

\[ N', Z', E'^*, J' \]

\[ \rho(N', Z', E'^*) \]
After direct and pre-equilibrium emission

\[ \sigma_{\text{reaction}} = \sigma_{\text{dir}} + \sigma_{\text{pre-equ}} + \sigma_{\text{NC}} \]

\[
\begin{align*}
N_0 & \rightarrow N_0 - dN_D \\
Z_0 & \rightarrow Z_0 - dZ_D \\
E^*_0 & \rightarrow E^*_0 - dE^*_D \\
J_0 & \rightarrow J_0 - dJ_D
\end{align*}
\]

\[
\begin{align*}
N_0 - dN_D - dN_{PE} &= E \\
Z_0 - dZ_D - dZ_{PE} &= Z \\
E^*_0 - dE^*_D - dE^*_{PE} &= E^* \\
J_0 - dJ_D - dJ_{PE} &= J
\end{align*}
\]

...
Compound nucleus hypothesis

- Continuum of excited levels
- Independence between incoming channel \( a \) and outgoing channel \( b \)

\[
\sigma_{ab} = \sigma_a^{(CN)} P_b
\]

\[
\sigma_a^{(CN)} = \frac{\pi}{k_a^2} T_a
\]

\[
P_b = \frac{T_b}{\sum_c T_c}
\]

⇒ Hauser- Feshbach formula

\[
\sigma_{ab} = \frac{\pi}{k_a^2} \frac{T_a T_b}{\sum_c T_c}
\]
THE COMPOUND NUCLEUS MODEL
(qualitative feature)

Compound angular distribution & direct angular distributions
THE COMPOUND NUCLEUS MODEL
(complete channel definition)

Channel Definition

\[ a + A \rightarrow (CN)^* \rightarrow b + B \]

Incident channel \( a = (l_a, j_a = l_a + s_a, J_A, \pi_A, E_A, E_a) \)

Conservation equations

• Total energy : \( E_a + E_A = E_{CN} = E_b + E_B \)

• Total momentum : \( p_a + p_A = p_{CN} = p_b + p_B \)

• Total angular momentum : \( \vec{l}_a + \vec{s}_a + \vec{J}_A = \vec{J}_{CN} = \vec{l}_b + \vec{s}_b + \vec{J}_B \)

• Total parity : \( \pi_A (-1)^l_a = \pi_{CN} = \pi_B (-1)^l_b \)
In realistic calculations, all possible quantum number combinations have to be considered.

\[
\sigma_{ab} = \frac{\pi}{k_a^2} \sum_{J=|I_A - S_a|}^{I_A + S_a + l_a^{\text{max}}} \sum_{\pi = \pm} (2J + 1) \frac{2I_A + 1 - (2S_a + 1)}{(2I_A + 1)(2S_a + 1)}
\]

Given by OMP.
In realistic calculations, all possible quantum number combinations have to be considered.

\[
\sigma_{ab} = \frac{\pi}{k_a^2} \sum_{J=|I_A - I_A|} (2J+1) \sum_{\pi = \pm} \frac{(2I_A+1)(2I_a+1)}{(2I_a+1)(2S_a+1)}
\]

Parity selection rules

\[
\delta_\pi(a) \delta_\pi(b) \sum_c T_{J\pi, l_c, j_c} \sum_{T_a, l_a, j_a} T_{J\pi, l_a, j_a} \sum_{T_b, l_b, j_b} T_{J\pi, l_b, j_b}
\]
In realistic calculations, all possible quantum number combinations have to be considered.

$$\sigma_{ab} = \frac{\pi}{k_a^2} \sum_{J = |I_A - I_a|}^{I_A + S_a + L_a^{\text{max}}} \sum_{\pi = \pm} (2J+1) \frac{(2I_a+1)(2s_a+1)}{(2I_b+1)(2s_b+1)}$$

Width fluctuation correction factor to account for deviations from independance hypothesis.

$$\delta_{\pi}(a) \delta_{\pi}(b) \frac{T_a, l_a, j_a \ T_b, l_b, j_b}{\sum_c T_c, l_c, j_c}$$

$$W^{J\pi}_{a, l_a, j_a, b, l_b, j_b}$$
Breit-Wigner resonance integrated and averaged over an energy width
Corresponding to the incident beam dispersion

\[ \langle \sigma_{ab} \rangle = \frac{\pi}{k_a^2} \frac{2\pi}{D} \left\langle \frac{\Gamma_a \Gamma_b}{\Gamma_{\text{tot}}} \right\rangle \]

Since

\[ T_\alpha \approx \frac{2\pi \left\langle \Gamma_\alpha \right\rangle}{D} \]

\[ \Rightarrow \left\{ \begin{array}{l}
\langle \sigma_{ab} \rangle = \frac{\pi}{k_a^2} \frac{T_a T_b}{\sum_c T_c} W_{ab} \\
\text{with} \quad W_{ab} = \frac{\left\langle \frac{\Gamma_a \Gamma_b}{\Gamma_{\text{tot}}} \right\rangle}{\left\langle \Gamma_a \right\rangle \left\langle \Gamma_b \right\rangle / \left\langle \Gamma_{\text{tot}} \right\rangle}
\end{array} \right. \]
THE COMPOUND NUCLEUS MODEL
(main methods to calculate WFCF)

• Tepel method

  Simplified iterative method

• Moldauer method

  Simple integral

• GOE triple integral

  « exact » result

Elastic enhancement with respect to the other channels
Inelastic enhancement sometimes in very particular situations?
\[
W_{a,l_a,j_a,b,l_b,j_b} = \int_0^{+\infty} d\lambda_1 \int_0^{+\infty} d\lambda_2 \int_0^1 d\lambda \frac{\lambda(1 - \lambda)|\lambda_1 - \lambda_2|}{\sqrt[4]{\lambda_1(1 + \lambda_1)\lambda_2(1 + \lambda_2)(\lambda + \lambda_1)^2(\lambda + \lambda_2)^2}}
\]

\[
\Pi_c \frac{(1 - \lambda T_{c,l_c,j_c}^J)}{\sqrt{(1 + \lambda_1 T_{c,l_c,j_c}^J)(1 + \lambda_2 T_{c,l_c,j_c}^J)}} \left\{ \delta_{ab}(1 - T_{a,l_a,j_a}^J) \right\}
\]

\[
\left[ \frac{\lambda_1}{1 + \lambda_1 T_{a,l_a,j_a}^J} + \frac{\lambda_2}{1 + \lambda_2 T_{a,l_a,j_a}^J} + \frac{2\lambda}{1 - \lambda T_{a,l_a,j_a}^J} \right]^2 + (1 + \delta_{ab})
\]

\[
\left[ \frac{\lambda_1(1 + \lambda_1)}{(1 + \lambda_1 T_{a,l_a,j_a}^J)(1 + \lambda_1 T_{b,l_b,j_b}^J)} + \frac{\lambda_2(1 + \lambda_2)}{(1 + \lambda_2 T_{a,l_a,j_a}^J)(1 + \lambda_2 T_{b,l_b,j_b}^J)} + \frac{2\lambda(1 - \lambda)}{(1 - \lambda T_{a,l_a,j_a}^J)(1 - \lambda T_{b,l_b,j_b}^J)} \right]
\]
THE COMPOUND NUCLEUS MODEL
(flux redistribution illustration)

\[ \Delta(\sigma_{\text{GOE}} - \sigma_{\text{Moldauer}})/\sigma \ (\%) \]
THE COMPOUND NUCLEUS MODEL
(multiple emission)

+ Loop over CN spins and parities
\[ n + ^{238}\text{U} \]

**Optical model** +

**Statistical model** +

**Pre-equilibrium model**

\[ \sigma_R = \sigma_d + \sigma_{\text{PE}} + \sigma_{\text{CN}} \]

= \sigma_{nn'} + \sigma_{nf} + \sigma_{n\gamma} + \ldots
THE COMPOUND NUCLEUS MODEL
(compact expression)

\[ \sigma_{NC} = \sum_b \sigma_{ab} \quad \text{où } b = \gamma, \text{ n, p, d, t, ..., fission} \]

\[ \sigma_{ab} = \frac{\pi}{k_a} \sum_{J,\pi} \sum_{\alpha,\beta} \frac{(2J+1)}{(2s+1)(2l+1)} \frac{J_\pi}{T_{lj}(\alpha)} \frac{\langle T_b(\beta) \rangle}{\sum_\delta \langle T_d(\delta) \rangle} W_{\alpha\beta} \]

with \( J = l_\alpha + s_\alpha + I_A = j_\alpha + I_A \) and \( \pi = (-1)^{l_\alpha} \pi_A \)

and \( \langle T_b(\beta) \rangle = \) transmission coefficient for outgoing channel \( \beta \)

associated with the outgoing particle \( b \)
Possible decays

• Emission to a discrete level with energy $E_d$

$$
\langle T_b(\beta) \rangle = T_{lj}^{J\pi}(\beta) \quad \text{given by the O.M.P.}
$$

• Emission in the level continuum

$$
\langle T_b(\beta) \rangle = \int_{E}^{E+\Delta E} T_{lj}^{J\pi}(\beta) \rho(E,J,\pi) \, dE
$$

$\rho(E,J,\pi)$ density of residual nucleus’ levels $(J,\pi)$ with excitation energy $E$

• Emission of photons, fission

Specific treatment
Two types of strength functions:

- the «upward» related to photoabsorption

\[
\bar{\int} \, XL(\epsilon) = \frac{\epsilon^{-2L+1}}{(\pi \hbar c)^2} \frac{\langle \sigma_{XL}(\epsilon) \rangle}{2L+1}.
\]

- the «downward» related to $\gamma$-decay

\[
\int \, XL(\epsilon) = \epsilon^{-2L+1} \frac{\langle \Gamma_{XL}(\epsilon) \rangle}{D_l}
\]

Spacing of states from which the decay occurs

**Standard Lorentzian (SLO)**

[D.Brink, PhD Thesis(1955); P. Axel. PR 126(1962)]

\[
\bar{f} = \frac{E_\gamma \Gamma_r^2}{(E_\gamma^2 - E_r^2)^2 + E_\gamma \Gamma_r^2} \quad \Rightarrow \quad O \quad E_\gamma \to 0
\]
Two types of strength functions:
- the « upward » related to photoabsorption

\[
\overrightarrow{f}_{XL}(\epsilon_\gamma) = \frac{\epsilon_\gamma^{2L+1}}{2(\pi \hbar c)^2 \frac{2L + 1}{2L + 1}} \langle \sigma_{XL}(\epsilon_\gamma) \rangle.
\]

- the « downward » related to \( \gamma \)-decay

\[
\overleftarrow{f}_{XL}(\epsilon_\gamma) = \epsilon_\gamma^{-(2L+1)} \frac{\langle \Gamma_{XL}(\epsilon_\gamma) \rangle}{D_l}.
\]

### Standard Lorentzian (SLO)


\[
\overrightarrow{f} = \overrightarrow{f} \sim \frac{E_\gamma \Gamma_r^2}{(E_\gamma^2 - E_r^2)^2 + E_\gamma \Gamma_r^2} \quad \Rightarrow \quad E_\gamma \to 0
\]
MISCELLANEOUS : THE PHOTON EMISSION
(strength function and selection rules)

\[
T^{k\lambda}(E, \varepsilon_\gamma) = 2\pi \int_{E}^{E+\Delta E} \Gamma^{k\lambda}(\varepsilon_\gamma) \rho(E) \, dE
\]

\[
= 2\pi f(k, \lambda, \varepsilon_\gamma) \varepsilon_\gamma^{2\lambda+1}
\]

\[f(k, \lambda, \varepsilon_\gamma) : \text{gamma strength function (several models)}\]

Decay selection rules from a level \(J_i\) to a level \(J_f\):

Pour \(E\lambda\): \(\pi_i=(-1)^\lambda \pi_i\), \(|J_i-\lambda| \leq J_i \leq J_i+\lambda\)

Pour \(M\lambda\): \(\pi_i=(-1)^{\lambda+1} \pi_i\)

Renormalisation method for thermal neutrons

\[
<T_\gamma> = \sum_{J_i, \pi_i} \sum_{k\lambda} \sum_{J_f, \pi_f} \int_{0}^{B_n} T^{k\lambda}(\varepsilon) \rho(B_n-\varepsilon, J_f, \pi_f) \, S(\lambda, J_i, \pi_i, J_i, \pi_f) \, d\varepsilon = 2\pi <\Gamma_\gamma> \rho(B_n)
\]

\(k\): transition type EM (E ou M)

\(\lambda\): transition multipolarity

\(\varepsilon_\gamma\): outgoing gamma energy
MISCELLANEOUS : THE PHOTON EMISSION  
(strength function and selection rules)

\[ T^{k\lambda}(E,\varepsilon_\gamma) = 2\pi \int_{E}^{E+\Delta E} \Gamma^{k\lambda}(\varepsilon_\gamma) \rho(E) \, dE \]

\[ = 2\pi \, f(k,\lambda,\varepsilon_\gamma) \, \varepsilon_\gamma^{2\lambda+1} \]

\[ f(k,\lambda,\varepsilon_\gamma) : \text{gamma strength function} \quad \text{(several models)} \]

Decay selection rules from a level \( J_i^{\pi_i} \) to a level \( J_f^{\pi_f} \):

Pour \( E\lambda \): \( \pi_i = (-1)^{\lambda} \pi_i \)

Pour \( M\lambda \): \( \pi_i = (-1)^{\lambda+1} \pi_i \) \( |J_i-\lambda| \leq J_i \leq J_i+\lambda \)

Renormalisation method for thermal neutrons

\[ \langle T_\gamma \rangle = C \sum_{J_i,\pi_i} \sum_{k\lambda} \sum_{J_f,\pi_f} \int_0^{B_n} T^{k\lambda}(\varepsilon) \rho(B_n-\varepsilon,J_f,\pi_f) \, S(\lambda,J_i,\pi_i,J_i,\pi_f) \, d\varepsilon = 2\pi \, \langle \Gamma_\gamma \rangle \frac{1}{D_0} \]

\( k \): transition type EM (E ou M)  
\( \lambda \): transition multipolarity  
\( \varepsilon_\gamma \): outgoing gamma energy
Improved analytical expressions:
- 2 Lorentzians for deformed nuclei
- Account for low energy deviations from standard Lorentzians for E1
  - Kadmenskij-Markushef-Furman model (1983)
  - Enhanced Generalized Lorentzian model of Kopecky-Uhl (1990)
- Reconciliation with electromagnetic nuclear response theory
  - Modified Lorentzian model of Plujko et al. (2002)
  - Simplified Modified Lorentzian model of Plujko et al. (2008)
FIG. 42: $E1$ $\gamma$-decay strength function plotted against energy $\epsilon_\gamma$ for $^{90}$Zr; experimental data are taken from Ref. [327].
MISCELLANEOUS : THE PHOTON EMISSION
(strength function and selection rules)

Improved analytical expressions:
- 2 Lorentzians for deformed nuclei
- Account for low energy deviations from standard Lorentzians for E1
    ⇒ Enhanced Generalized Lorentzian model of Kopecky-Uhl (1990)
    ⇒ Hybrid model of Goriely (1998)
- Reconciliation with electromagnetic nuclear response theory
  ⇒ Modified Lorentzian model of Plujko et al. (2002)
  ⇒ Simplified Modified Lorentzian model of Plujko et al. (2008)

Microscopic approaches: RPA, QRPA
« Those who know what is (Q)RPA don’t care about details,
  those who don’t know don’t care either », private communication
  ⇒ Systematic QRPA with Gogny force under work (300 Mh!!!)
MISCELLANEOUS : THE PHOTON EMISSION
(phenomenology vs microscopic)

Capture cross section @ $E_n=10$ MeV for Sn isotopes

⇒ Weak impact close to stability but large for exotic nuclei
Surface $^{238}$U
MISCELLANEOUS : THE FISSION PROCESS
(fissile or fertile ?)

Fission barrier with height \( V \)

\[ B_n < V \]

Fertile target (\(^{238}\)U)
MISCELLANEOUS: THE FISSION PROCESS
(fissile or fertile?)

Fission barrier with height $V$

$B_n < V$
Fertile target ($^{238}$U)

$B_n > V$
Fissile target ($^{235}$U)
**MISCELLANEOUS : THE FISSION PROCESS**

(fissile or fertile ?)

---

**Incident neutron data / ENDF/B-VI.8 / U235 / MT=19 : (n,f) / Cross section**

---

**Incident neutron data / ENDF/B-VI.8 / U238 / MT=19 : (n,f) / Cross section**

**Fission barrier**
MISCELLANEOUS : THE FISSION PROCESS
(multiple chances)

Energy

Nucleus (Z,A)

Nucleus (Z,A-1)

B_n

1st chance

2nd chance

B_n

σ fission (barn)

Incident neutron energy (MeV)

elongation
MISCELLANEOUS: THE FISSION PROCESS
(multiple chances)

Energy

Nucleus (Z,A)

1\textsuperscript{st} chance

B_n

Nucleus (Z,A-1)

2\textsuperscript{nd} chance

B_n

\textbf{ elongation }

\textbf{ B_n }
MISCELLANEOUS : THE FISSION PROCESS
(multiple chances)

1st chance

2nd chance

3rd chance

Incident neutron energy (MeV)

σ fission (barn)
MISCELLANEOUS : THE FISSION PROCESS
(multiple chances)

Energy

Nucleus (Z,A)

1\textsuperscript{st} chance

B_n

Nucleus (Z,A-1)

2\textsuperscript{nd} chance

B_n

Nucleus (Z,A-2)

3\textsuperscript{rd} chance

\begin{align*}
\text{Nucleus (Z,A)} & \quad 1\textsuperscript{st} \text{ chance} \\
\text{B}_n & \\
\text{Nucleus (Z,A-1)} & \quad 2\textsuperscript{nd} \text{ chance} \\
\text{B}_n & \\
\text{Nucleus (Z,A-2)} & \quad 3\textsuperscript{rd} \text{ chance}\end{align*}

\begin{align*}
\text{Elongation} & \\
\sigma \text{ fission (barn)} & \\
\text{Incident neutron energy (MeV)} &
\end{align*}
MISCELLANEOUS : THE FISSION PROCESS
(multiple chances)

Energy

Nucleus (Z,A)

V

1st chance

B_n

Nucleus (Z,A-1)

V

2nd chance

B_n

Nucleus (Z,A-2)

3rd chance

B_n

V

1st chance

Nucleus (Z,A)

V

2nd chance

B_n

3rd chance

B_n

σ fission (barn)

Incident neutron energy (MeV)

0 5 10 15 20 25 30 35 40

0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8 2.0

+ 238U
+ 235U
+ 233U
+ 237Pa
+ 239U
+ 235Th
+ (n,4nd)
+ (n,3n)
+ (n,αf)
+ (n,αn)
+ (n,3f)
+ (n,2f)
+ (n,4f)
+ (n,5f)
+ (n,6f)
+ (n,7f)
+ (n,8f)
+ (n,9f)
MISCELLANEOUS: THE FISSION PROCESS (multiple chances)

1st chance

Nucleus (Z,A)

2nd chance

Nucleus (Z,A-1)

3rd chance

Nucleus (Z,A-2)

Energy

Bn

Bn

Bn

1st chance

2nd chance

3rd chance

Nucleus (Z,A-2)

σ fission (barn)

Incident neutron energy (MeV)
MISCELLANEOUS : THE FISSION PROCESS
(Fission penetrability: Hill-Wheeler)

\[ T_{hw}(E) = \frac{1}{1 + \exp\left(\frac{2\pi(V-E)}{\hbar\omega}\right)} \]

For one barrier!

Energy

E

Bn

Elongation

+ transition state on top of the barrier!

Bohr hypothesis
\[ T_f(E, J, \pi) = \sum_{\text{discreets } J, \pi} T_{hw}(E - \varepsilon_d) + \int_{E_s}^{E+B_n} \rho(\varepsilon, J, \pi) T_{hw}(E - \varepsilon) \, d\varepsilon \]

\[ T_{hw}(E) = \frac{1}{1 + \exp(2\pi(V - E)/\hbar\omega)} \]

Discrete transition states with energy \( \varepsilon_d \)

Hill-Wheeler

\[ E + B_n \]

Energy

\[ V \]

Elongation
MISCELLANEOUS : THE FISSION PROCESS
(multiple humped barriers)

Fission barrier
\[ (V, \hbar\omega) \]

Energy

\[ B_n \]

elongation
MISCELLANEOUS : THE FISSION PROCESS
(multiple humped barriers)

Fission barrier
\[ (V, \hbar \omega) \]

\[ B_n \]

+ transition states on top of the barrier!
MISCELLANEOUS : THE FISSION PROCESS
(multiple humped barriers)

Barrier A  ( $V_A$, $\hbar \omega_A$ )  Barrier B  ( $V_B$, $\hbar \omega_B$ )

+ transition states on top of the barrier!
MISCELLANEOUS : THE FISSION PROCESS
(multiple humped barriers)

+ transition states on top of each barrier!
MISCELLANEOUS : THE FISSION PROCESS
(multiple humped barriers)

+ transition states on top of each barrier !
+ class II states in the intermediate well !

Barrier A
$\left( V_A, \hbar \omega_A \right)$

Barrier B
$\left( V_B, \hbar \omega_B \right)$
+ transition states on top of each barrier!
+ class II states in the intermediate well!
MISCELLANEOUS: THE FISSION PROCESS
(multiple humped barriers)

Barrier A: $(V_A, \hbar \omega_A)$
Barrier B: $(V_B, \hbar \omega_B)$

+ transition states on top of each barrier!
+ class II states in the intermediate well!
Two barriers A et B

\[ T_f = \frac{T_A T_B}{T_A + T_B} \]

Three barriers A, B and C

\[ T_f = \frac{T_A T_B}{T_A + T_B} + T_C \]

Resonant transmission

\[ T_f = \frac{T_A T_B}{T_A + T_B} \]

More exact expressions in Sin et al., PRC 74 (2006) 014608
MISCELLANEOUS : THE FISSION PROCESS
(multiple humped barriers with maximum complexity)

See in Sin et al., PRC 74 (2006) 014608
Bjornholm and Lynn, Rev. Mod. Phys. 52 (1980) 725.
**MISCELLANEOUS : THE FISSION PROCESS**
(Impact of class II states)

\[ ^{239}\text{Pu} \,(n,f) \]

![Graph showing cross section (barn) vs. neutron energy (MeV) for \( ^{239}\text{Pu} \,(n,f) \) with and without class II states. The graph indicates two chances:

- **1st chance**: A decrease in cross section as neutron energy increases.
- **2nd chance**: A rise in cross section at higher energies.

The graph highlights the impact of class II states on the fission process, showing a deviation from the baseline at certain neutron energies.}
Case of a fertile nucleus

Partially damped class II states. No class III states
MISCELLANEOUS : THE FISSION PROCESS
(impact of class II and class III states)

Case of a fertile nucleus

Class II + III states. Partial damping.
For exotic nuclei: strong deviations from Hill-Wheeler.
MISCELLANEOUS : THE LEVEL DENSITIES
(Principle)
Exponential increase of the cumulated number of discrete levels $N(E)$ with energy

\[ U(E) = \sum \text{odd-even effects} \]

Mean spacings of s-wave neutron resonances at $B_n$ of the order of few eV

\[ \rho(E) = \frac{dN(E)}{dE} \quad \text{increases exponentially} \]

\[ \rho(B_n) \text{ of the order of } 10^4 \text{ – } 10^6 \text{ levels / MeV} \]
\[
\frac{1}{D_0} = \rho \left( B_n, 1/2, \pi_t \right) \text{ for an even-even target} \\
= \rho \left( B_n, I_t + 1/2, \pi_t \right) + \rho \left( B_n, I_t - 1/2, \pi_t \right) \text{ otherwise}
\]
\[ \rho (U, J, \pi) = \frac{1}{2} \frac{\sqrt{\pi}}{12} \frac{\exp \left( 2 \sqrt{aU} \right)}{a^{1/4} U^{5/4}} \frac{2J+1}{2 \sqrt{2\pi} \sigma^3} \exp - \left[ \frac{(J+\frac{1}{2})^2}{2\sigma^2} \right] \]

\[ + \quad \sigma^2 = I_{\text{rig}} \sqrt{\frac{U}{a}} \]
\[ \rho (U, J, \pi) = \frac{1}{2} \frac{\sqrt{\pi}}{12} \exp \left( 2\sqrt{aU} \right) \frac{2J+1}{2 \sqrt{2\pi} \sigma^3} \exp \left[ -\frac{(J+\frac{1}{2})^2}{2\sigma^2} \right] \]
MISCELLANEOUS : THE LEVEL DENSITIES
(Quantitative analysis 1/2)

\[
\rho (U, J, \pi) = \frac{1}{2} \frac{\sqrt{\pi}}{12} \frac{\exp \left( 2 \sqrt{aU'} \right)}{a^{1/4}U^{5/4}} \frac{2J+1}{2\sqrt{2\pi}\sigma^3} \exp \left[ -\frac{(J+1/2)^2}{2\sigma^2} \right]
\]

Odd-even effects accounted for

\[ U \rightarrow U^* = U - \Delta \]

\[ \Delta = \begin{cases} 
0 & \text{odd-odd} \\
12/\sqrt{A} & \text{odd-even} \\
24/\sqrt{A} & \text{even-even} 
\end{cases} \]

Shell effects
\[ a\ (N, Z, U^*) = \tilde{a}\ (A) \left[ 1 + \delta W(N,Z) \frac{1 - \exp(-\gamma U^*)}{U^*} \right] \]
Discrete levels (spectroscopy)

Temperature law

\[ N(E) = \exp \left( \frac{E - E_0}{T} \right) \]

Fermi gaz (adjusted at \( B_n \))

\[ \rho(E) = \alpha \frac{\exp \left( 2\sqrt{aU^*} \right)}{a^{1/4} U^{5/4}} \]
MISCELLANEOUS : THE LEVEL DENSITIES
(More sophisticated approaches)

- **Superfluid model & Generalized superfluid model**
  *Ignatyuk et al., PRC 47 (1993) 1504 & RIPL3 paper (IAEA)*

  ⇒ More correct treatment of pairing for low energies
  ⇒ Fermi Gas + Ignatyuk beyond critical energy
  ⇒ Explicit treatment of collective effects

\[
\rho(U) = K_{vib}(U) \ast K_{rot}(U) \ast \rho_{int}(U)
\]

\[
a_{eff} \approx A/8\quad \text{Several analytical or numerical options}\quad \quad \quad a \approx A/13\]

⇒ Collective enhancement only if \(\rho_{int}(U) \neq 0\) not correct for vibrational states
**Superfluid model & Generalized superfluid model**  
Ignatyuk et al., PRC 47 (1993) 1504 & RIPL2 Tecdoc (IAEA)

⇒ More correct treatment of pairing for low energies  
⇒ Fermi Gas + Ignatyuk beyond critical energy  
⇒ Explicit treatment of collective effects

**Shell Model Monte Carlo approach**  
Agrawal et al., PRC 59 (1999) 3109

⇒ Realistic Hamiltonians but not global  
⇒ Coherent and incoherent excitations treated on the same footing  
⇒ Time consuming and thus not yet systematically applied

**Combinatorial approach**  

⇒ Direct level counting  
⇒ Total (compound nucleus) and partial (pre-equilibrium) level densities  
⇒ Non statistical effects  
⇒ Global (tables)
THE LEVEL DENSITIES
(The combinatorial method 1/3)

See PRC 78 (2008) 064307 for details

- HFB + effective nucleon-nucleon interaction $\Rightarrow$ single particle level schemes

- Combinatorial calculation $\Rightarrow$ intrinsic p-h and total state densities $\omega(U, K, \pi)$
Level density estimate is a counting problem: \[ \rho(U) = \frac{dN(U)}{dU} \]

\( N(U) \) is the number of ways to distribute the nucleons among the available levels for a fixed excitation energy \( U \)
- HFB + effective nucleon-nucleon interaction  \( \Rightarrow \) single particle level schemes

- Combinatorial calculation  \( \Rightarrow \) intrinsic p-h and total state densities \( \omega(U, K, \pi) \)

- Collective effects  \( \Rightarrow \) from state to level densities \( \rho(U, J, \pi) \)

1) folding of intrinsic and vibrational state densities
2) construction of rotational bands for deformed nuclei

\[
\rho(U, J, \pi) = \sum_{K} \omega(U-E_{rot}^{JK}, K, \pi)
\]

2) spherical nuclei

\[
\rho(U, J, \pi) = \omega(U, K=J, \pi) - \omega(U, K=J+1, \pi)
\]

- Phenomenological transition for deformed/spherical nucleus
THE LEVEL DENSITIES
(The combinatorial method 2/3)

Structures typical of non-statistical feature
THE LEVEL DENSITIES
(The combinatorial method 3/3)

\[ f_{\text{rms}} = \exp \left[ \frac{1}{N_e} \sum_{i=1}^{N_e} \ln^2 \frac{D_{\text{th}}^i}{D_{\text{exp}}^i} \right]^{1/2} \]
THE LEVEL DENSITIES
(The combinatorial method 3/3)

D₀ values (s-waves & p-waves)

Description similar to that obtained with other global approaches
**CONCLUSIONS & PROSPECTS**

- Nuclear reaction modeling complex and no yet fully satisfactory
  - ⇒ pre-equilibrium phenomenon must be improved
  - ⇒ fission related phenomena (fission, FF yields & decay) must be improved

- Formal and technical link between structure and reactions has to be pushed further
  - ⇒ pre-equilibrium and OMP efforts already engaged
  - ⇒ computing time is still an issue

- Fundamental $\nu$-$\nu$ interaction knowledge (and treatment) has to be improved
  - ⇒ Ab-initio not universal (low mass or restricted mass regions)
  - ⇒ Relativistic aspects not included systematically
  - ⇒ Human & computing time is still an issue