ICTP 50

Jogesh Pati

Preliminary Remarks

On this occasion of the 50th Anniversary of ICTP, first let me say that we are indeed very fortunate to have an institution like the ICTP in the world, because it serves some UNIQUE PURPOSES of not only creating good science, but most importantly spreading good science in the developing part of the world, thanks to the visionary efforts and dedication of Abdus Salam, who is the founder of this institution.

We are also fortunate that the Salam spirit is continuing to thrive in the hands of the new directors, thanks to the generous support of governments & Int'l organizations.

I won't go into Salam's many contributions, and won't describe how my collaboration with Salam started spontaneously from a tea-conversation, here at the Centre in the summer of 1972, & grew to take a shape in about 2 weeks, leading to a creative idea → Lepton Number as the 4th color → & how the collaboration grew to encompass other ideas → Including Unification of Forces & Proton-Decay.

The starting phases of our collaboration, and Salam's many contributions including his dreams of creating a World University & 20 institutions like the ICTP in different parts of the world are described in the two articles I wrote:

C

1. J. Pati, "Twenty Years Later, Why I still Believe in SU(4)- Color", SALAMFESTSCHRIFT, Ed. by A. Ali, J. Ellis, S. Randjbar-Daemi, Publ. by World Scientific, pages 368-391 (1993)

2. J. Pati, "Recollections of Abdus Salam, Scientist and Humanitarian" (1997), Extracts has appeared in Physics Today (1997) and Elsewhere.

↓

In ICTP Library.

GRAND UNIFICATION & PROTON DECAY: PROSPECTS FOR MAJOR DISCOVERIES AT NEXT-GENERATION DETECTORS

JOGESH PATI
SLAC, STANFORD UNIVERSITY

ICTP 50, Oct. 2014

REFERENCES

Babu, Pati, Tavartkiladze / hep-ph/1003-2625
JHEP 2010

Babu, Badziak, Pati, Tavartkiladze –
– 2014 (TO APPEAR)

Babu, Pati, Wilczek – hep-ph/9812538 –
Nucl. Phys. (2000)

A SUMMARY

Talk Based on Recent Works in Grand Unification

⇓

ROOTS IN THE TWO IDEAS (1972-73).

SU(4) - COLOR & **PROTON DECAY**

BOTH IDEAS VERY MUCH ALIVE TODAY

(1) ROUTE TO HIGHER UNIF BASED ON $SU(4)^C$ Supported by Several Experiments

↓

(i) DISCOVERY OF ATM. ν oscillation (1998)

↓

(a) $SU(4)^C \Rightarrow$ ν_R // $m(\nu^{\tau}_{Dirac}) \approx m_{top}(M_{GUT}) \approx 120$ GeV

(b) Seesaw $\Rightarrow$ superheavy Majorana ν_R

(c) Coupling Unif. Scale $\Rightarrow$ Scale of M_{ν_R}

⇓

$\Delta m^2_{23}(\text{Atm}) \sim (1/10\ \text{eV})^2$ ✓

(ii) Majorana ν_R → Leptogenesis → Baryogenesis

(iii) $m_b(\text{GUT}) \approx m_\tau$

(2) PROTON DECAY YET TO BE SEEN

Will present recent studies by

1. Babu, Pati, Tavartkiladze – JHEP 2010
2. Babu, Badziak, Pati, Tavartkiladze – – (2014, To Appear)

We incorporate GUT-Scale Threshold Corr

↓

A BONUS: AN INVERSE RELATIONSHIP BETWEEN $d=6$ ($p \to e^+ \pi^0$) & $d=5$ ($p \to \bar{\nu} K^+$) Amplitudes

↓

⇒ UPPER LIMITS ON LIFETIMES OF BOTH MODES

IN CONTRAST TO COMMONLY OBTAINED ORDERS OF MAGNITUDE ESTIMATES FOR p-lifetimes, UNCERTAIN BY SEVERAL POWERS OF TEN.

INCLUDING LHC RESULTS ON SUSY SEARCHS, FIND BOTH MODES FULLY ACCESSIBLE TO NEXT-GENERATION DETECTORS!

↓

STRONG MOTIVATION TO BUILD SUCH DETECTORS: p-decay, ν-OSC, Supernova ν's

I. INTRODUCTION

A) Improved studies of

① Proton Decay — $qqql/M^2$

② ν oscillations — $LL\langle\phi_H\rangle\langle\phi_H\rangle/M$

2 indispensable tools to probe nature at truly high energies ($\sim 10^{16}$ GeV) → short dist. ($\sim 10^{-30}$ cm) → Not possible any other way.

- Both are predictions of a Well-Motivated class of GRAND UNIFICATION MODELS
- With coupling unification & ν-oscillations discovered, one can argue, within such models, p-decay should be discovered in next Gen. detectors, exceeding SuperK sensitivity by a factor $\approx$ 10.

→ Would serve multiple purpose:

- Improve sensitivity to ν-physics ($\theta_{13}, \delta, \delta m^2_{23}$, Hier.)
- p-decay search → Hallmark of Grand Unif.
- Supernova physics
- $0\nu\,2\beta$-decay → $|\Delta L| = 2$
- n-$\bar{n}$ oscillation → $|\Delta B| = 2$
- Dark Matter

B) Alt. Routes To Grand Unification

$$G(213) = SU(2)_L \times U(1)_Y \times SU(3)^C$$

$$\begin{pmatrix} u_r & u_y & u_b \\ d_r & d_y & d_b \end{pmatrix}_L^{1/3}, \; (u_{r,y,b})_R^{4/3}, \; (d_{r,y,b})_R^{-2/3}, \; \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L^{-1}, \; (e_R^-)^{-2}$$

5 disconn. multiplets // Arb. Y_W, SU(2), $SU(3)^C$ Q.NOS.

↓

1972 $G(224) = SU(2)_L \times SU(2)_R \times SU(4)^C_{L+R} \times (L \leftrightarrow R)$

$$F^e_{L,R} = \begin{bmatrix} u_r & u_y & u_b & \nu_e \\ d_r & d_y & d_b & e^- \end{bmatrix}_{L,R} \quad F^e_L = (2,1,4) \quad F^e_R = (1,2,4)$$

$$Q_{em} = I_{3L} + I_{3R} + \frac{B-L}{2}$$

Advantages of G(224)

- All 16 in one L-R Conj. multiplet // L-R symm
- Explain Y_W & All Q.NOS. // Quantize Q_{em} //
- $Q_{e^-} = -Q_p$ // ν_R // B-L ← Need For Seesaw & Leptogenesis

↓

All Advantages of G(224) retained by SO(10), not SU(5)

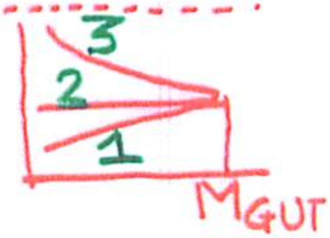
SO(10): 16 (one coupling)

SU(5): $\bar{5} + 10$
NO ν_R, NO B-L } → Problem with ν Masses & Leptogenesis.

D) EVIDENCE FOR GRAND UNIFICATION

1) Family Structure - Quantum Nos. Predicted as observed
2) Charge Quantization
3) $Q_{e^-} = -Q_p$

4) Meeting of The 3 gauge Couplings → SUSY Grand Unification → $M_{GUT} \simeq 2 \times 10^{16}$ GeV



5) $\sqrt{\Delta m^2(\nu)_{23}} \approx 1/20$ eV ↔ $SU(4)^c \to \nu_R$ // B-L; $m(\nu^\tau)_{Dirac}$ // $M_{GUT} \to$ Maj ν_R; Seesaw

6) $m_b(GUT) = m_\tau$
7) $m(\nu_\tau)_{Dirac} = m_{top}(GUT)$
→ $SU(4)^c$

8) $\Theta(\nu_\mu - \nu_\tau) \approx \pi/4 \leftrightarrow V_{cb} \simeq 0.04$

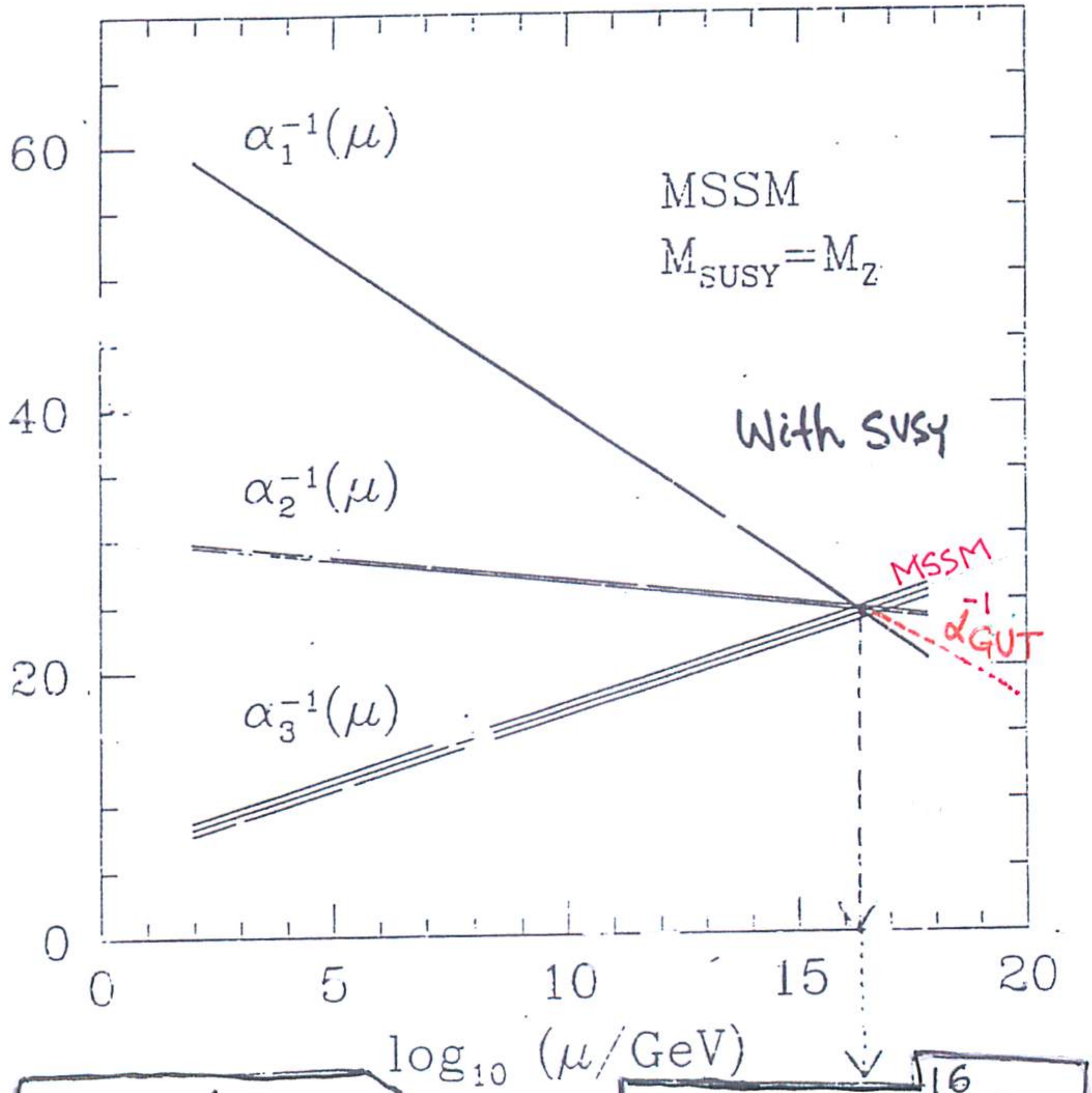
9) LEPTOGENESIS → $Y_B \sim 10^{-10}$ → ν_R/B-L

SO(10) // G(224) (items 6–9)

SUCCESS OF ALL 9 FEATURES NON-TRIVIAL!

TOGETHER MAKE A STRONG CASE FOR SUSY GRAND UNIF // SIMULT. FOR AN EFF. SYMMETRY LIKE SO(10) OR MINIMALLY A STRING-DERIVED G(224) SYMMETRY

Gauge Coupling Unification With SUSY



Supports SUSY Unification

$M_{GUT} \approx 2 \times 10^{16}$ Gev

$\sin^2\theta_W(m_Z)_{th} = 0.2315 \pm .003$

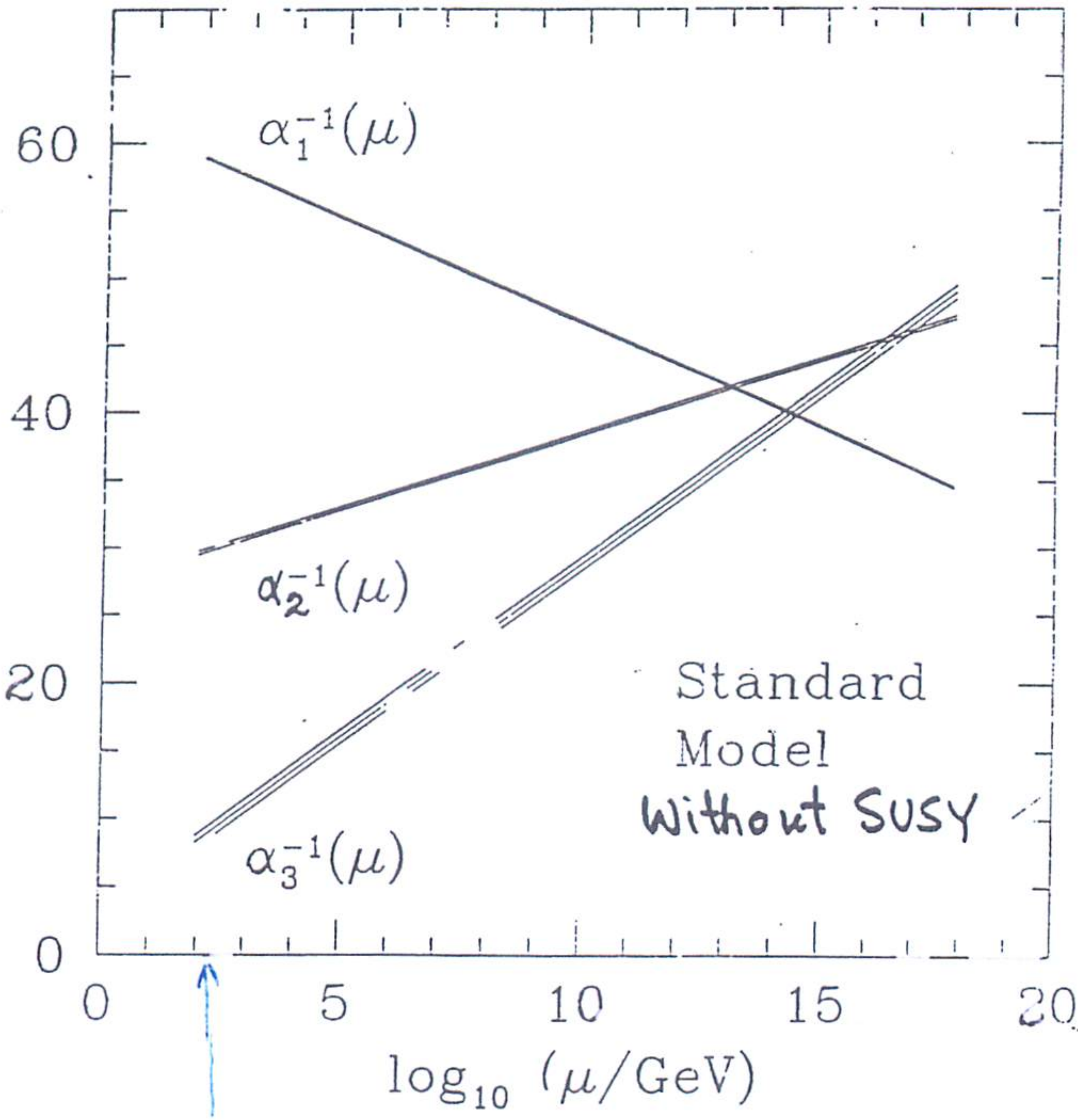
$\sin^2\theta_W(m_Z)_{EXPT} = 0.23124 \pm .00017$

$\alpha_3(m_Z)^0_{theory} \simeq 0.125 - 0.13$

$\alpha_3(m_Z)_{obs} = 0.118 \pm .0007$

THRESHOLD CORRECTIONS

Coupling Unification Without SUSY



α_i's Measured at LEP
at ~ 100 GeV

E) FOUR STRIKING / OBSERVABLE CONSEQUENCES

① Meeting of the 3 gauge couplings ✓✓

② $m_\nu \neq 0$ & Small as observed $\to \Delta m^2_{23}$ ✓✓

③ With ν_R, B-L, Seesaw $\to$ Baryogenesis via Leptogenesis : $Y_B \sim 10^{-10}$ (Quite Natural) ✓

④ PROTON DECAY : A GENERIC FEATURE

(d=6)

$p \to e^+ \pi^0$

With SUSY (d=5)

$p \to \bar{\nu} K^+$

$\to \bar{\nu} \pi^+$

$\Gamma^{-1}(p \to e^+ \pi^0)_{Th}$	$\sim 10^{35 \pm 2}$ Yrs (Estimate)	$\xrightarrow{\text{In a class}}$	$< 10^{35}$ Yrs
$\Gamma^{-1}(p \to \bar{\nu} K^+)_{Th}$	SUSY SO(10) or SUSY G(224)		$\approx 10^{30} \leftrightarrow$ Few $\times 10^{34}$ Yrs

PROTON DECAY } $\to$ **THE MISSING PIECE**

NEED LARGE UNDERGROUND DETECTOR EXCEEDING SUPERK SENSITIVITY BY A FACTOR 10 TO PROBE INTO THESE PREDICTIONS.

II SO(10) - BREAKING

LOW DIM. HIGGSES

$45_H, 16_H, \overline{16}_H, 10_H$

HIGH DIM. HIGGSES

$126, \overline{126}, 120, 54, 210$

Generically Large GUT SCALE THRESHOLD CORR.

$$SO(10)$$

$$\langle 45_H \rangle \sim \ \ \langle 16_H \rangle = \langle \overline{16}_H \rangle \sim M_{GUT}$$

$\propto B-L$

$$SU(2)_L \times U(1)_Y \times SU(3)^C$$

$$\langle 10_H \rangle \sim \langle 16_H^d \rangle \sim M_{EW}$$

$$U(1)_{EM} \times SU(3)^C$$

Babu, Pati, Wilczek

$16_i\, 16_j\, 10_H$

$16_i\, 16_j\, 10_H . 45_H / M$

$16_i\, 16_j\, 16_H\, 16_H^d / M$

ASSUME HIER. YUKAWA $\rightarrow U(1)_F$

$16_i\, 16_j\, \overline{16}_H \overline{16}_H / M$ $\rightarrow$ MAJORANA MASS $\nu_R^i \nu_R^j$

$\rightarrow$ A PREDICTIVE SUCCESSFUL FRAMEWORK FOR FERMION MASSES / MIXINGS / NEUTRINO OSC. / CP // WITH PREDICTIONS FOR $\mu \rightarrow e\gamma$, $(edm)_{n,e}$...

III PROTON DECAY: HALLMARK OF GRAND UNIFICATION

① d = 6 Gauge Med. SUSY SU(5)/SO(10)

$M_X \approx M_Y \sim 10^{16}$ GeV

$\Rightarrow$ Amp ($p \to e^+ \pi^0$) $\propto g^2/M_X^2$

Chiral Lag Param (D+F) $\simeq$ 1.25 // $A_R \simeq 3.4$

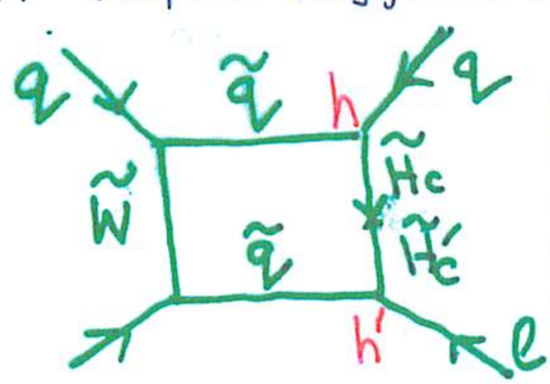
$\alpha_H = 0.012\ \text{GeV}^3$ $\Gamma^{-1}(p \to e^+ \pi^0) \simeq 10^{35} (M_X/10^{16}\text{GeV})^4$

LARGE UNCERTAINTY $\to \sim 10^{33} - 10^{37}$ yrs ? ESTIMATE

② SUSY SU(5)/SO(10) Standard d=5

Color Triplet Higgsino Med — Sakai, Yanagida/Weinberg

Need D-T Splitting



$\tilde{H}_c \subset (10_H)_{SO(10)} = (2,2,1) + (1,1,6)$

Amp $\propto (hh'/M)(m_{\tilde{W}}/m^2_{\tilde{q}})\alpha_2$

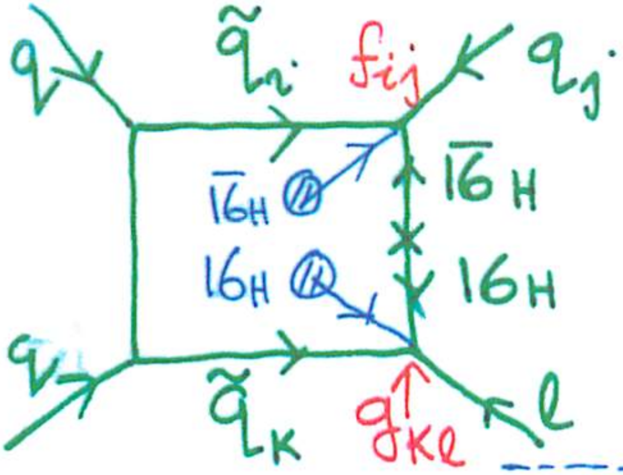
$M = M_{H_c}$ (SU(5))
$M = M_{eff}$ (SO(10))

$p \to \bar{\nu} K^+$ (dominant)
$\to \mu^+ K^0$ (suppressed) BR $\sim 10^{-5}$

$\Gamma^{-1}(p \to \bar{\nu} K^+) \sim 10^{30} - 10^{34}$ yrs.

③ New $d=5$ ν-Mass Related operators

Babu, Pati, Wilczek



$f_{ij}\, 16_i\, 16_j\, \overline{16}_H \overline{16}_H / M$

⇓

Maj Masses of ν_R's

Generically $16_i\, \overline{16}_H$ in 45 & 1 of SO(10)

⇢ $g_{k\ell}\, 16_k\, 16_\ell\, 16_H\, 16_H / M$

↳ $V_{CKM} \neq 1\!\!1$

Indep of $\tan\beta$

$\bar{\Gamma}^{-1}(p \to \bar{\nu} K^+)_{\text{New } d=5} \approx 10^{33} - 10^{34}$ yrs

SUSY SO(10) or G(224)

$BR(p \to \mu^+ K^0)_{\text{New } d=5} \approx (10 - 30)\%$

Note these contributions from new $d=5$ would generically be present, even if "standard" $d=5$ (with color triplets $\subset 10_H$) absent, as in SUSY G(224) !

The $\mu^+ K^0$ mode a signature of this mechanism.

$d=5$ $p \to \bar{\nu} K^+$ in Non-GUT string Solutions

For example String/M Theory ($d=10/11$)

↓

3 Families
Hierarchical Yukawas

$G(2,2,4) \times U(1)$'s $\times$ (Hidden)

D-T splitting done naturally by string (*) compactification

↓

But Color-triplets $\sim M_{string}$ Exist with coupling to fermions.

⇓

$d=5$ $p \to \bar{\nu} K^+$ will occur

They have not calculated eff. Couplings $\neq 0$

THUS p-decay a Generic Feature in GUT & String Th with rates ~ observable range

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(*) Recently Christodoulides, Faraggi, Rizos
→ Exophobic (224) string solutions
→ hep-ph/1104.2264

Assel, Christodoulides, Faraggi, Kounnas, Rizos
- 2010 // 2011

Earlier, S. Raby & Collaborators -
& others

IV SUSY SO(10) WITH STABILIZED D-T SPLITTING, INCLUDING GUT-THRESHOLD EFF.

Babu, Pati, Tavartkiladze (JHEP 2010)

Stabilized DW D-T splitting with no Fine Tuning

Low Dim. Higgses $\supset 45_H, 16_H, \overline{16}_H, 10_H, 10'_H, \ldots$

(no Higher Dim. Higgses: $126, \overline{126}, 120, 210 \ldots$)

$$\alpha_U^{-1}(\Lambda) = \alpha_i^{-1}(M_Z) - \frac{b_i}{2\pi} \ln \frac{\Lambda}{M_Z} + \Delta_{i,W}^{(2)} + \Delta_i^{GUT} \quad (1)$$

$b_i \rightarrow$ 1-loop MSSM β-FUNCN COEFFICIENTS

$\Delta_{i,W}^{(2)} \rightarrow$ 2-loop running (incl. gauge & Yukawa Int.) from $m_Z \rightarrow \Lambda$, and Weak scale SUSY Threshold EFF.

$$\Delta_i^{GUT} = \text{1-loop GUT Scale Th. Eff} = -\frac{1}{2\pi} \sum_a b_i^{(a)} \ln \frac{\Lambda}{M_a}$$

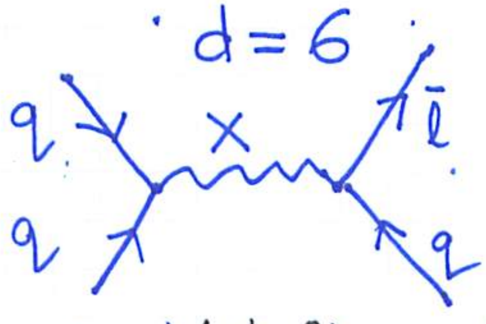
↓

Fewer Parameters due to Low Dim. Higgses

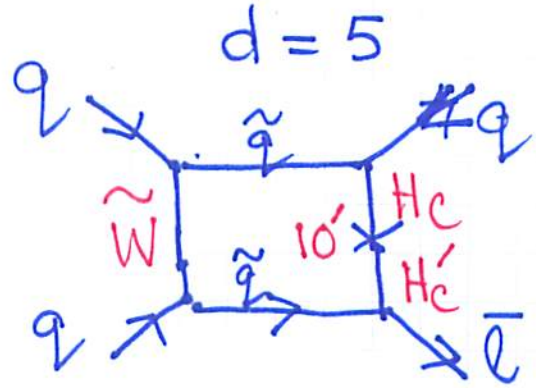
↓

Can Eliminate & Get a NOVEL INVERSE CORRELATION BETWEEN $d=6$ & $d=5$ AMPLITUDES

Recall



$A_{d=6}(e^+\pi^0) \propto \frac{1}{M_X^2}$



$A_{d=5}(\bar{\nu}K^+) \propto 1/M_{eff}$ ☒

UNIFICATION OF 3 COUPLINGS WITH GUT-THRESHOLD

$$\Rightarrow \quad \boxed{M_{eff} \propto \frac{K_{SUSY}}{M_X^3} \text{ (A)}} \quad \text{INVERSE CORRELATION}$$

$\Gamma^{-1}(e^+\pi^0) \propto M_X^4$; $\Gamma^{-1}(\bar{\nu}K^+) \propto M_{eff}^2$

$\therefore \Gamma^{-1}_{expt}(\bar{\nu}K^+) > 6\times10^{33}$ yrs $\Rightarrow M_{eff} > M_{eff}^{Min.}$ (depending on SUSY SPECT)

By (A) $\Rightarrow$ $\boxed{M_X < M_X^{Max}}$ $\Rightarrow$ UPPER LIMIT ON $\Gamma^{-1}(e^+\pi^0)$!

Similarly $\Gamma^{-1}_{expt}(e^+\pi^0) > 1.3\times10^{34}$ yrs $\Rightarrow M_X > M_X^{Min}$

By (A) $\Rightarrow$ $\boxed{M_{eff} < M_{eff}^{Max}}$ $\Rightarrow$ UPPER LIMIT ON $\Gamma^{-1}(\bar{\nu}K^+)$!

☒ $M_{eff} \sim M^2_{Color\ Triplet}/M_{10'} \sim M^2_{GUT}/M_{10'}$
$\gg M_{GUT}$

Recent Update

Babu, Badziak, Pati, Tavartkiladze (To appear, 2014)

Using LHC results on SUSY searches & $m_h \simeq 125$ GeV, Vary SUSY spectrum & parameters with GUT-Scale Boundary Condns & Reasonable Naturalness ($\Delta_a \equiv \left|\frac{\partial \ln m_h}{\partial \ln a}\right| \sim (150-300)$), in accord with phenomenology of DM, $b \to s\gamma$, etc.

- primarily with light stops ($\tilde{t}_1 \approx 500-700$ GeV, $\tilde{t}_2 \approx 2$ TeV), Heavy 1st Two Families ($\sim 15-20$ TeV), gluino ~ 3 TeV, Wino $\sim 550-800$ GeV, $A_0 = 0 \to -2$ TeV, $\mu \sim 800$ GeV $- 1$ TeV.

To be specific – e.g. Badziak, Dudas, Olechowski, Pokorski (ArXiv: 1205.1675), & other cases with Intermediate 1st Two Families $\sim 3-4$ TeV.

$\hookrightarrow$ $m_0(1,2) \gg m_0(3) \gg m_{1/2}$, $|A_0| = 0 \to 2$ TeV

INVERTED HIERARCHY SUSY SPECTRUM OF BDOP TYPE * MODIFIED BY GUT THRESHOLD CORRECTIONS

GUT SCALE INPUTS:

$m_{1/2} = 2031\ GeV,\ \ m_0(3) = 3.4$ TeV,
$m_0(1{,}2) = 21.1$ TeV

$$\tan\beta = 10, \qquad A_0 = 0$$

EW SCALE SPECTRUM:

$$m_h \simeq 125 \text{ GeV}$$

$$M_{\tilde{B}} = 652.7 \text{ GeV} \quad M_{\tilde{W}} = 1295.3 \text{ GeV} \quad M_{\tilde{g}} = 3561 \text{ GeV}$$

$$m_{\tilde{q}_{1,2}} = 21122\,\text{GeV} \qquad m_{\tilde{l}_3} = 3115\,\text{GeV}$$

$$m_{\tilde{t}_1} = 735.02 \text{ GeV}, \quad m_{\tilde{t}_2} = 1503.2 \text{ GeV}$$

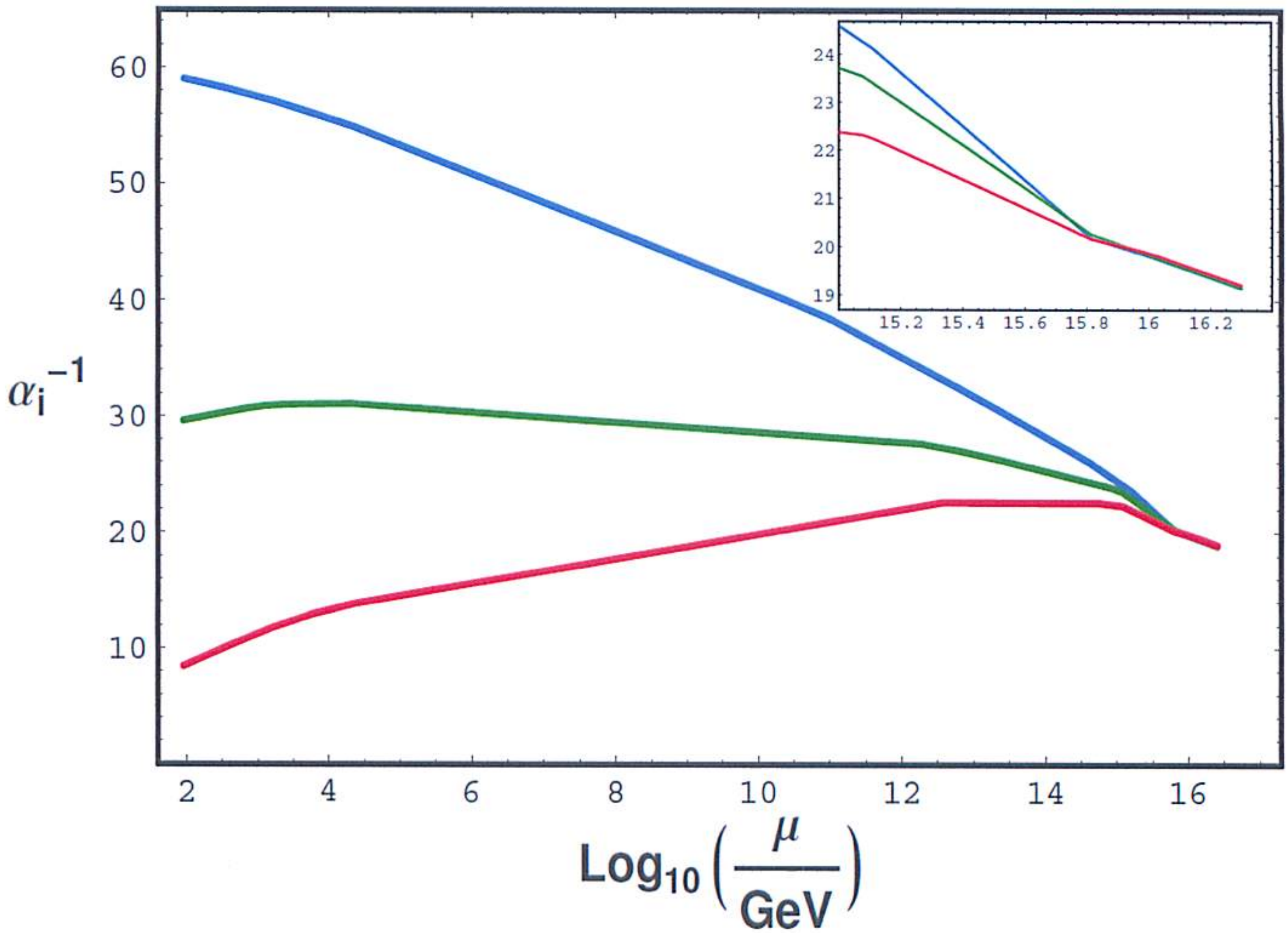
$$m_{\tilde{\tau}^c} = 3227.6\,\text{GeV} \qquad \mu = 759.6 \text{ GeV}$$

$$\text{Neutralinos}: \qquad m(\tilde{\chi}_i^0) \simeq (653 - 1295) \text{ GeV}$$

$$\text{Charginos}: \qquad m(\tilde{\chi}_i^\pm) \simeq (759 - 1295) \text{ GeV}$$

* Badziak, Dudas, Olechowski, Pokorski, Arxiv: 1205.1675

UNIFICATION FOR SUSY SO(10) MODEL INCLUDING GUT THRESHOLD EFFECTS



$\alpha_3(M_Z)=0.1184,\ (M_{\rm eff}, M_X)=(7\cdot 10^{19}, 6.52\cdot 10^{15})$ GeV

$(r, p, \hat{p}/p)=\left(\frac{1}{1752},\ 4,\ 6.431\cdot 10^{-5}\right)$, $\tan\beta = 10$

Varying SUSY Spectrum With Reasonable naturalness & Using the Inverse Correlation (A) → Get Theoretical Upper Limits:

⇓

$$\Gamma^{-1}(p \to e^+ \pi^0)^{Th} \lesssim (2-10) \times 10^{34}\ \text{yrs}$$

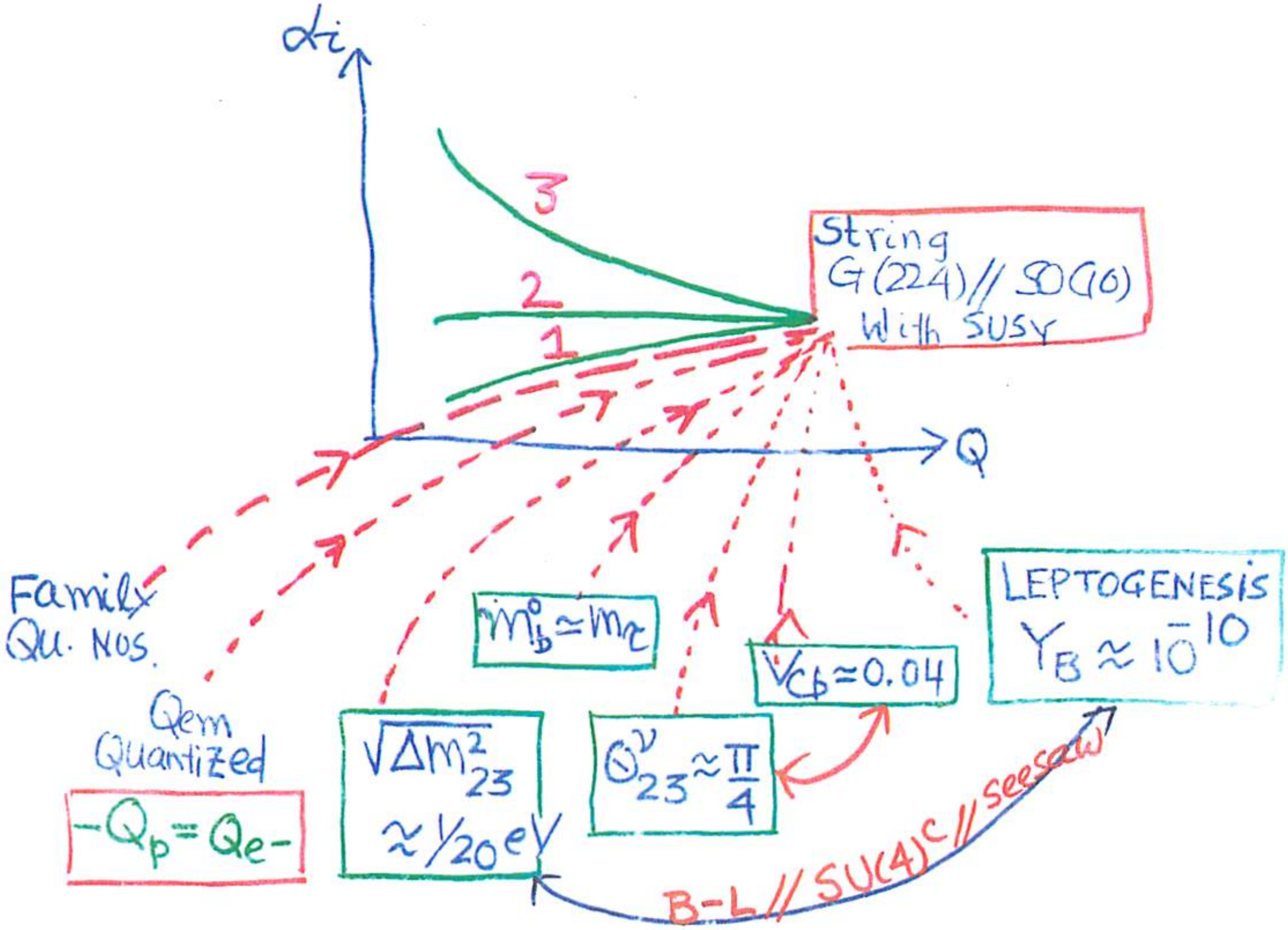
$$\Gamma^{-1}(p \to \bar{\nu} K^+)^{Th} \lesssim (1-8) \times 10^{34}\ \text{yrs}$$

Within Factors of 5 to 10 above SuperK limits → Within striking distance.

Thus Prospects of Discovery of p-decay, within these well-studied & Well-motivated Class of SUSY SO(10) Models, in the next-Generation Detectors (Both Water Cherenkov ~500 kt & Liquid Argon ~ 30-50 kt) High.

Will Nature oblige wrt Both

SUSY &
PROTON Decay?



ALL THESE FEATURES & MORE HANG TOGETHER NEATLY IN A SINGLE UNIFIED FRAMEWORK → HARD TO IMAGINE THIS CAN BE MERE COINCIDENCE

⇓

TWO MISSING PIECES

- PROTON DECAY → Need NEXT GEN. DETECTOR
- SUPERSYMMETRY → LHC

PROTON DECAY THE HALLMARK OF GRAND UNIF.

Provides a Unique Window To View Physics at truly short dist $\lesssim 10^{-30}$ cm

$p \to \bar{\nu} K^+$ If Seen $\Rightarrow$ { q-ℓ, q-$\bar{q}$, q-$\bar{\ell}$ unif //, SUSY UNIF //, $M_X \sim 10^{16}$ GeV }

$p \to \mu^+ K^0$ If Seen $\Rightarrow$ { ν MASS Rel op IMPORTANT }

If $e^+\pi^0$ not seen Say upto 10^{36} yrs, but $\bar{\nu}K^+$ seen $\Rightarrow$ { EFF. G(224) & ν MASS Rel, $d=5$ IMPORTANT }

THUS PROTON DECAY IF SEEN WILL BRING A WEALTH OF KNOWLEDGE THAT CAN NOT BE GAINED BY ANY OTER MEANS

$\Downarrow$

DISCOVERY POTENTIAL HIGH

$\Downarrow$

MEGATON SIZE DETECTOR (or EQUIVALENT)

SENSITIVE TO
- PROTON DECAY,
- ν oscillations (LONG BASELINE)

AND
- SUPERNOVA ν's.

Backup Slides

③. $\sqrt{\Delta m^2(\nu_\mu \nu_\tau)_{SuperK}} \approx \frac{1}{20}$ eV

See Saw (IGNORE MIXING For a Moment

$$\begin{array}{c|cc} & \nu_L^\tau & \overline{\nu_R^\tau} \\ \nu_L^\tau & 0 & m(\nu_D^\tau) \\ \overline{\nu_R^\tau} & m(\nu_D^\tau) & M_R \end{array} \longrightarrow \begin{bmatrix} -\frac{m(\nu_D^\tau)^2}{M_R} & 0 \\ 0 & M_R \end{bmatrix}$$

$m(\nu_D^\tau)$: Dirac Mass

M_R: Majorana Mass, Superheavy

$$\Rightarrow \boxed{m(\nu_L^\tau) \approx m(\nu_D^\tau)^2/M_R}$$

(i) Get M_R From SUSY Unif Scale $M_X \sim 2\times10^{16}$ GeV

$$f_{33}\, 16_3 16_3 \frac{\langle\overline{16}_H\rangle\langle\overline{16}_H\rangle}{M_{Pl}} \Rightarrow M_R \approx \frac{(2\times10^{16}\text{ GeV})^2}{2\times10^{18}\text{ GeV}} \approx \boxed{2\times10^{14}\text{ GeV}}$$

f_{33} (≈ 1), M_{Pl} (2×10^{18} GeV)

(ii) Get $m(\nu_D^\tau)$ Using SU(4)-Color

$m_b^0 = m_\tau^0$

$$m(\nu_D^\tau) = m_t(m_X) \approx \boxed{100\text{ GeV}}$$

$$\Rightarrow \boxed{m(\nu_L^\tau) \approx \frac{(100\text{ GeV})^2}{2\times10^{14}\text{ GeV}} \sim (1/20\text{ eV})(1/3 - 3)}$$

Also get $m(\nu_L^\mu) \sim m(\nu_L^\tau)/10 \Rightarrow \sqrt{\Delta m^2(\nu_\mu \nu_\tau)_{Th}} \sim (\frac{1}{20})\text{eV}(\frac{1}{3}-3)$

Thus the SuperK result brings to light the existence of $\boxed{\nu_R}$ // Reinforces the ideas of a) SeeSaw, b) SU(4)-Color & c) SUSY-unification...

(12)

Summary on Fermion Masses & Mixings (Babu, Pati, Wilczek)

Predictions	Observations
$m_b(m_b) \approx (4.7-4.9)\,\mathrm{GeV}$	$\approx 4.2\,\mathrm{GeV}$
$\sqrt{\Delta m^2_{23}(\nu)} \sim (1/24\,\mathrm{eV})(\frac{1}{2}-2)$	$\approx (1/15 - 1/25)\,\mathrm{eV}$ (*)
$V_{cb} \approx 0.043$	≈ 0.04
$\sin^2 2\Theta^{osc}_{\nu_\mu \nu_\tau} \approx \boxed{0.92} \leftrightarrow \boxed{0.995}$ $m_{\nu_2}/m_{\nu_3} \approx 1/10 - 1/5$	$\approx 0.92 \leftrightarrow 1$
$V_{us} \approx 0.23$	≈ 0.22
$\lvert V_{ub} \rvert \approx 0.0032$	$\approx 0.003 - 0.004$
$m_d(1\,\mathrm{GeV}) \simeq 8\,\mathrm{MeV}$	$\approx 6-8\,\mathrm{MeV}$

$m(\nu_2) \approx (2-10)\times 10^{-3}\,\mathrm{eV} \longleftrightarrow$ { SMA $\sim 3\times 10^{-3}\,\mathrm{eV}$; $\boxed{\text{LMA} \simeq 7\times 10^{-3}\,\mathrm{eV}}$ }

$m(\nu_1) \sim (1 \text{ to few})\times 10^{-3}\,\mathrm{eV}$

Consistent with the framework

$M(\text{"}\nu_R^3\text{"}, \text{"}\nu_R^2\text{"}, \boxed{\nu_R^1}) \approx (10^{15}, 2\times 10^{12}, (1/3 - 3)\times 10^{10})$ GeV

Just right for Lepto/Baryo genesis

NOTE: ν_L Masses Necessarily Hierarchical.

SMA or LMA ?

$\rightarrow f_{ij}\, 16_i 16_j \overline{16}_H \overline{16}_H / M$

Just with standard See-saw ν_L-masses, SMA rather generic

$m(\nu_L^1) \sim 2\times10^{-5} - 2\times10^{-6}\,\text{eV}$ // $m(\nu_L^2) \sim 3\times10^{-3}\,\text{eV}$

$\Theta^{osc}_{\nu_e\nu_\mu} = \Theta^{l}_{e\mu} - \Theta^{\nu}_{e\mu} \approx 0.05$

Situation alters once allow for direct Maj masses of ν_L's — Most likely to arise through Higher Dim. op. involving GUT & EW VEV's - Through tiny $\sim 10^{-3}$ eV entries → IMPORTANT For $(\nu_e - \nu_\mu)$

$$W \supset g_{12}\, 16_1\, 16_2\, 16_H\, 16_H\, 10_H\, 10_H / M^3_{GUT}$$

$$g_{12}\,(\nu_L^e \nu_L^\mu)\left(\langle \tilde{\bar{\nu}}_{RH}\rangle / M_{GUT}\right)^2 \; (\upsilon_u^2)$$

$(\sim 175\ \text{GeV})^2$

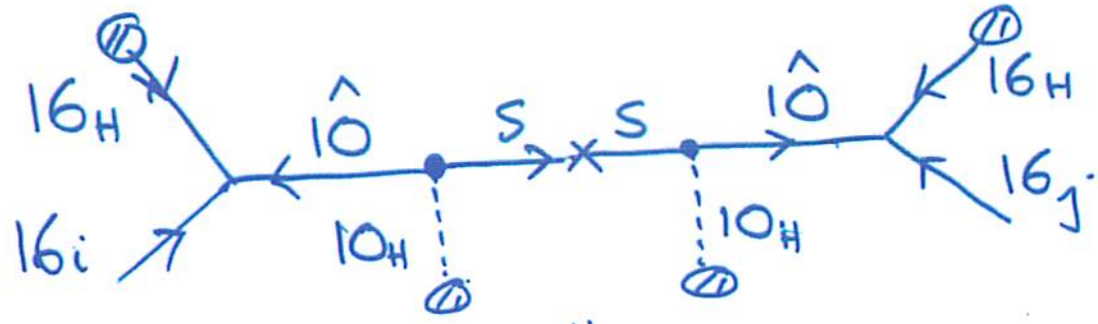
$$\boxed{\sim g_{12}(\nu_L^e \nu_L^\mu)\,(1.5-6)\times10^{-3}\,\text{eV}} \quad (\langle 16_H\rangle \approx (1\text{-}2) M_{GUT})$$

$$\begin{array}{cc} \nu_L^e & \nu_L^\mu \end{array}$$

$$\begin{bmatrix} \approx 0 & (3-4) \\ (3-4) & 7 \end{bmatrix} \times 10^{-3}\,\text{eV} \Longrightarrow \begin{bmatrix} \Theta^{\nu}_{\nu_e\nu_\mu} \approx 1/2 \\ \sin^2 2\Theta^{osc}_{\nu_e\nu_\mu} \sim 0.7 \end{bmatrix} \text{Quite Plausib[le]}$$

Thus LMA not strictly a prediction, but perfectly plausible within the framework.

GUT-SCALE EFFECTIVE OPERATORS



$$\Downarrow$$

$$g_{ij}\; 16_i\, 16_j\, 16_H\, 16_H\, 10_H\, 10_H / (M^2_{\hat{10}} M_S)$$

Θ_{13}

$$m(\nu_L^e \nu_L^\tau)_{\text{Non-seesaw}} \sim (2-6) \times 10^{-3}\,\text{eV}$$

$$\Theta_{13} \sim \frac{(2-6) \times 10^{-3}\,\text{eV}}{5 \times 10^{-2}\,\text{eV}}$$

$$\sim .03 - .1$$

ν-Less 2β decay : $\Delta L = \pm 2$

$$m_{ee} = \left| \sum_i m_i U_{ei}^2 \right|$$

$m_1 \sim$ few $\times 10^{-3}$ eV, $m_2 \approx (6-8) \times 10^{-3}$ eV

$m_3 \approx 5 \times 10^{-2}$ eV

$\Theta_{12} \approx 1/2$, $\Theta_{13} \sim 0.03 - 0.1$

$$\Downarrow$$

$$m_{ee} \sim (1 \text{ to } 6) \times 10^{-3}\,\text{eV}$$

CORRELATION EQN

$$M_{eff} \simeq (10^{19}\,\text{GeV})\left(\frac{10^{16}\,\text{GeV}}{M_X}\right)^3\left(\frac{3}{\tan\beta}\right)\left(\frac{1/100}{r}\right)$$
$$\times\left\{\frac{\exp\left[2\pi\left(\Delta_W^{(2)}-\Delta_W^{(3)}-\delta\alpha_3^{-1}\right)\right]}{2.07\times 10^{-2}}\right\}$$

→ FAIRLY INSENSITIVE TO SUSY SPECTRUM

$r \equiv m_\Sigma / M_X$ $\quad (\Sigma = \underset{\sim}{8}^c \subset 45_H)$

$\delta\alpha_3^{-1}$ = DeV. of α_3^{-1} FROM CENTRAL VALUE

$= (\cdot 147, 0, -\cdot 1422) \rightarrow (\alpha_3(m_Z) = \cdot 1156, \cdot 1176, \cdot 1196)$

$\Rightarrow$ $\quad M_{eff} \propto 1/M_X^3$

$A(p \to e^+\pi^0) \propto g^2/M_X^2$; $A(p \to \bar{\nu} K^+) \propto \frac{1}{M_{eff}}$

$\Rightarrow$ If RAISE M_X TOO MUCH TO SUPPRESS $d=6$ $(p \to e^+\pi^0)$, M_{eff} WILL BE TOO LOW → EXCLUDED BY EMPIRICAL LOWER LIMIT ON $\Gamma^{-1}(p \to \bar{\nu} K^+)$.