
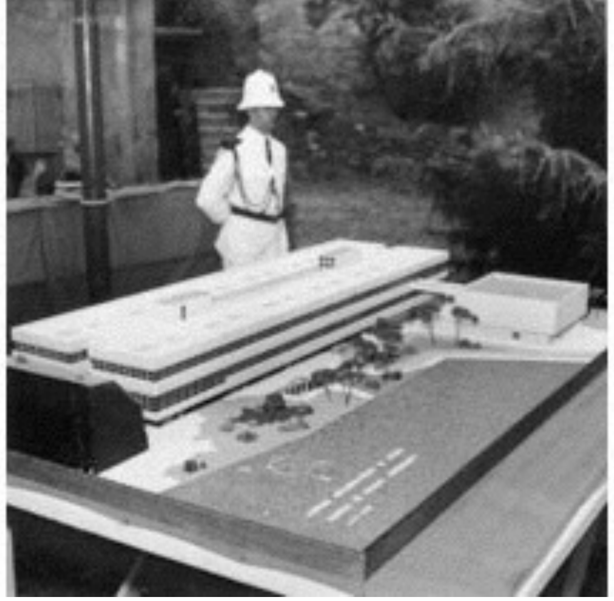

N=2 Born-Infeld Attractors

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It is a great honor to speak today in the occasion of the 50th Anniversary of the ICTP center.

I am old enough to have had the chance to meet and discuss in several occasions with its founder, Prof. ABDUS SALAM

Salam was one of the discoverers of many fundamental concepts in Physics (beyond the one for which he got the Nobel Prize)

Among them, he was one of the beginners of the field theories based on SUPERSYMMETRY (at the time called SUPERGAUGE invariance because of its inspiration from fermionic strings of Neveu-Schwarz, Ramond, Gervais-Sakita)

Two basic concepts he introduced (in 1974 with J. Strathdee) were

SUPERSPACE (x_μ, θ_α) $\theta_\alpha\theta_\beta = -\theta_\beta\theta_\alpha$

GOLDSTONE FERMIONS (Spontaneous breaking)

My talk will be devoted to subjects for which the above concepts are greatly used

At the same time, I would like to pay a tribute to

BRUNO ZUMINO

the founding-father of SUPERSYMMETRY

I had the privilege of having him as a friend and collaborator, and I witnessed and participated to some of the important discoveries attached to his name

Summary of the talk

- *Nilpotent Superfields in Superspace*
- *Applications to Rigid and Local Supersymmetry*
- *Partial Supersymmetry Breaking in Rigid N=2 Theories*
- *Emergence of Volkov-Akulov and Born-Infeld Actions*
- *Symplectic Structure and Black-Hole Attractors: analogies and differences*
- *New $U(1)^n$ Born-Infeld Actions and Theory of Invariant Polynomials*

Some of the material of this presentation originates from some recent work with Antoniadis, Dudas, Sagnotti; Kallosh, Linde and some work in progress with Porrati, Sagnotti

The latter introduces a generalization of the Born-Infeld Action for an arbitrary $U(1)^n$ N=2 supergravity with N=2 self-interacting vector multiplets

NILPOTENCY CONSTRAINTS IN SPONTANEOUSLY BROKEN N=1 RIGID SUPERSYMMETRY

(Casalbuoni, De Curtis, Dominici, Ferruglio, Gatto; Komargodski, Seiberg; Rocek; Lindstrom, Rocek)

X chiral superfield $(\bar{\mathscr{D}}_{\dot{\alpha}} X = 0)$ nilpotency: $X^2 = 0$

solution: $X = \frac{GG}{F_G} + i\sqrt{2}\theta\, G + \theta^2\, F_G$ (G Weyl fermion)

Lagrangian: $\mathrm{Re}\, X\, \bar{X}\Big|_D + f\, X\Big|_F$

Equivalent to Volkov-Akulov goldstino action

More general constraints (Komargodski,Seiberg)

$X^2 = 0\,, \quad XY = 0$ independent fields $\psi_X = G\,, \quad \psi_Y$

$XW_\alpha = 0$ independent fields $\psi_X = G\,,\ F_{\mu\nu}\,,\ \lambda = f(\psi_X, F_{\mu\nu})$

NILPOTENT CONSTRAINTS IN LOCAL SUPERSYMMETRY (SUPERHIGGS EFFECT IN SUPERGRAVITY)

Pure supergravity coupled to the Goldstino

$$W(X) = f\,X + m$$

theory of a massive gravitino coupled to gravity
and cosmological constant: $\Lambda = |f|^2 - 3m^2$

(Deser, Zumino; Rocek; Antoniadis, Dudas, Sagnotti, S.F.)

Volkov-Akulov-Starobinsky supergravity

(a linear (inflaton) multiplet T, and a non-linear Goldstino multiplet S, with $S^2=0$)

$$V = \frac{M^2}{12}\left(1 - e^{-\sqrt{\frac{2}{3}}\phi}\right)^2 + \frac{M^2}{18}\, e^{-2\sqrt{\frac{2}{3}}\phi}\, a^2$$

Starobinsky inflaton potential   axion field

Recently, nilpotent super fields have been used in more general theories of inflation

(Kallosh, Linde, S.F.; Kallosh, Linde)

For a class of models with SUPERPOTENTIAL

$$W = S\,f(T)$$

(Kallosh, Linde, Roest; Kallosh, Linde, Rube)

the resulting potential is a no-scale type, and has a universal form

$$V_{\text{eff}} = e^{K(T)}\,K^{S\bar{S}}\,|f(T)|^2 > 0$$ goldstino ψ_S

with $T_{\theta=0} = \varphi + ia\,,$ inflaton: φ or a

depending which one is lighter during inflation

GENERALITIES ON PARTIAL SUPERSYMMETRY BREAKING
GLOBAL SUPERSYMMETRY

(Witten; Hughes, Polchinski; Cecotti, Girardello, Porrati, Maiani, S.F.; Girardello, Porrati, S.F.; Antoniadis, Partouche, Taylor)

$a = 1 \dots N$ $\delta\chi^a$ fermion variations; $J^a_{\mu\alpha}(X)$ susy Noether current

Current algebra relation (Polchinski)

$$\int d^3y \, \{\bar{J}_{0\dot{\alpha}b}(y), J^a_{\mu\alpha}(x)\} = 2\sigma^\nu_{\alpha\dot{\alpha}} T_{\mu\nu}(x)\delta^a_b + \sigma_{\mu\alpha\dot{\alpha}} C^a_b$$

Relation to the scalar potential:

$$\delta\chi^a \, \delta\bar{\chi}_b = V \, \delta^a_b + C^a_b$$

In N=2 (APT, FGP)

$$C^a_b = \sigma^{xa}{}_b \, \epsilon_{xyz} Q^x \wedge Q^y = 2\,\sigma^{xa}{}_b (\vec{E} \wedge \vec{M})_x \, ; \quad Q^x = \begin{pmatrix} M^x \\ E^x \end{pmatrix}$$

In Supergravity (partial SuperHiggs) (CGP, FM, FGP)
there is an extra term in the potential which allows
supersymmetric anti-de-Sitter vacua

$$\delta\chi^a\,\delta\bar{\chi}_b = V\,\delta^a_b + 3\mathcal{M}^{ac}\bar{\mathcal{M}}_{bc}$$

$\mathcal{M}_{ab} = \mathcal{M}_{ba}$ *"gravitino mass" term*

$$\delta\psi^a_\mu = D_\mu\epsilon^a + \mathcal{M}^{ab}\gamma_\mu\bar{\epsilon}_b$$

N=2 RIGID (SPECIAL) GEOMETRY

$$R_{i\bar{j}k\bar{l}} = C_{ikp}\,\bar{C}_{\bar{j}\bar{l}\bar{p}}\,g^{p\bar{p}} \qquad\qquad V = \left(X^A\,,\,\frac{\partial \mathcal{U}}{\partial X^A}\right)$$

$$\partial_{\bar{\imath}} V = 0\,, \quad \mathscr{D}_j \partial_i V = C_{jik}\,g^{k\bar{k}}\,\partial_{\bar{k}}\,\bar{V}\,, \quad \partial_{\bar{\jmath}}\partial_i V = 0$$

$$C_{ikp} = \partial_i \partial_k \partial_p \mathcal{U} = \mathcal{U}_{ikp}$$

$$\mathcal{U} = X^2 + \frac{X^3}{M} + \ldots + \frac{X^{n+2}}{M^n}$$

for M large: $\mathcal{U} - X^2 \sim \dfrac{X^3}{M}$ $\qquad M\,\mathcal{U}_{ABC} = d_{ABC}$

N=2 rigid supersymmetric theory with Fayet-Iliopoulos terms (n vector multiplets, no hypermultiplets) in N=1 notations

Kahler potential: $K = i(X^a \bar{\mathcal{U}}_A - \bar{X}^A \mathcal{U}_A)\,, \quad \mathcal{U}_A = \frac{\partial \mathcal{U}_A}{\partial X^A}$

($\mathcal{U}$: N=2 prepotential)

Fayet-Iliopoulos terms:
triplet of (real) symplectic (Sp($2n$)) constant vectors

$$\vec{Q} = (\vec{M}^A, \vec{E}_A) = (Q_c, Q_3) = \begin{pmatrix} m_1^A + i\, m_2^A\,, & e_{1A} + i\, e_{2A} \\ m_3^A\,, & e_3^A \end{pmatrix}$$

$(A = 1, \ldots, n)$

Superpotential: $W = (\mathcal{U}_A\, m^A - X^A\, e_A)$ $Q_c = (m^A, e_A)$

D term N=1 F-I magnetic and electric charges: $Q_3 = (\epsilon^A, \xi_A)$

Due to the underlying symplectic structure of N=2 rigid special geometry, we can rewrite all expressions by using the symplectic metric

$$\Omega = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad \textit{and symplectic sections} \quad V = (X^A, \mathcal{U}_A)$$

$$W = (V, Q) = V\,\Omega\,Q\,, \qquad K = -i(V, \bar{V})$$

$$V_F = (\mathrm{Im}\,\mathcal{U}^{-1}_{AB})\,\frac{\partial W}{\partial X^A}\,\frac{\partial \bar{W}}{\partial \bar{X}^B} = \bar{Q}_c^T\,\mathcal{M}\,Q_c + i\,(\bar{Q}_c, Q_c)$$

$$V_D = Q_3^T\, \mathcal{M}\, Q_3 \quad \textit{so that}$$

$$V = V_F + V_D = \bar{Q}_c^T\, (\mathcal{M} + i\Omega)\, Q_c + Q_3^T\, \mathcal{M}\, Q_3$$

The matrix $\mathcal{M}$ is a real (positive definite) symmetric and symplectic $2n \times 2n$ matrix

$$\mathcal{M}^T = \mathcal{M} \qquad \mathcal{M}\,\Omega\mathcal{M} = \Omega \qquad \mathcal{M} > 0$$

It is related to the F^2 , $F\,\tilde{F}$ terms in the Lagrangian

$$\mathscr{L} = g_{AB}(X) F^A_{\mu\nu} F^{B\,\mu\nu} + \theta_{AB}(X)\, F^A_{\mu\nu} \tilde{F}^{B\,\mu\nu}$$

$$\mathcal{M} = \begin{pmatrix} g + \theta g^{-1}\theta & -\theta g^{-1} \\ -g^{-1}\theta & g^{-1} \end{pmatrix}$$

N=1 supersymmetric vacua require (Porrati, Sagnotti, S.F.)

$$\frac{\partial W}{\partial X^A} = 0\,, \qquad V_D = 0 \quad \text{since} \quad \mathcal{M} > 0$$

The $V_D = 0$ condition requires $Q_3 = 0$

The first equation implies

$$(\mathcal{M} + i\Omega)\, Q_c = 0 \quad \text{which requires} \quad i\bar{Q}_c\, \Omega\, Q_c < 0$$

Since $\bar{Q}_c\, \mathcal{M}\, Q_c$ is positive definite, and at the attractor point we have

$$\bar{Q}_c\, \mathcal{M}_{\text{crit}}\, Q_c = -i\bar{Q}_c\, \Omega Q_c$$

So it is crucial that Q_c is complex. To simplify, we will later consider m^A real and e_A complex, so that the previous condition is $m_1^A\, e_{2A} < 0$

The theory here considered is the generalization to n vector multiplets of the theory considered by Antoniadis, Partouche, Taylor (1995)

Later, this theory (n=1) was shown to reproduce in some limit (Rocek, Tseytlin, 1998) the supersymmetric Born-Infeld action (Cecotti, S.F., 1986). The latter was shown (Bagger, Galperin, 1996) to be the Goldstone action for N=2 partially broken to N=1 where the gauging $\lambda = W_\alpha \Big|_{\theta=0}$ is the Goldstone fermion of the second broken supersymmetry.

EXTREMAL BLACK HOLE ANALOGIES

In the case of asymptotically flat black holes, the so-called Black-Hole Potential for an extremal (single-center) black-hole solution is (Kallosh, S.F., 1996)

$$V_{\mathrm{BH}} = \frac{1}{2} Q^T \mathcal{M} Q \qquad\qquad Q = (m^A, e_A)$$

is the asymptotic black-hole charge

The theory of BH attractors (Kallosh, Strominger, S.F., 1995) tells that

$$V_{\mathrm{BH}}\Big|_{\mathrm{crit}} = \frac{1}{2} Q^c \mathcal{M}(X_{\mathrm{crit}})\, Q = \frac{1}{\pi} S(Q) = \frac{1}{4\pi} A_H$$

where $S(Q)$ is the BH entropy (Bekenstein-Hawking area formula)

In the N=2 case (Ceresole, D'Auria, S.F.; Gibbons, Kallosh, S.F.)

$$V_{\rm BH} = g^{i\bar{\jmath}}\,\mathscr{D}_i Z\,\mathscr{D}_{\bar{\jmath}}\bar{Z} + |Z|^2$$

so that at the BPS attractor point ($\mathscr{D}_i Z = 0$)

$$V_{\rm BH} = |Z_{\rm crit}|^2 = \frac{1}{\pi}\,S(Q)$$

In analogy to the N=2 partial breaking of the rigid case, the BPS black hole breaks N=2 down to N=1, and the central charge is the quantity replacing the super potential W.

The entropy and BI action are both expressed through W.

In our problem, the attractors occur at $V_{\text{crit}} = 0$,

(because of unbroken space time supersymmetry)

and this is only possible because the Fayet-Iliopoulos charge is an SU(2) triplet (charged in the F-term direction) and thus allows the attractor equation

$(\mathcal{M}_{\text{crit}} + i\Omega)\, Q_c = 0$ being satisfied.

We call these vacua Born-Infeld attractors,

for reasons will become soon evident

The superspace action of the theory in question is

$$\mathscr{L} = \mathrm{Im}\left(\mathcal{U}_{AB} W_\alpha^A W_\beta^B \epsilon^{\alpha\beta} + W(X) \right)\Big|_F + \left(X^A \bar{\mathcal{U}}_A - \bar{X}^A \mathcal{U}_A \right)\Big|_D$$

The Euler-Lagrange equations for X^A are

$$\mathcal{U}_{ABC} W^B W^C + \mathcal{U}_{AB} (m^B - \bar{D}^2 \bar{X}^B) - e_A + \bar{D}^2 \bar{\mathcal{U}}_A = 0$$

These are the complete X^A equations for the theory in question.
The first thing to note is that our attractor
gives a mass to the N=1 chiral multiplet X^A,
but not to the N=1 vector multiplets W_α.
So N=2 is broken.

Indeed, our action is invariant under a second supersymmetry η^α, which acts on the N=1 chiral super fields (X^A, W^A_α) (Bagger, Galperin n=1)

$$\delta X^A = \eta^\alpha W^A_\alpha ,$$
$$\delta W^A_\alpha = \eta_\alpha (m^A - \bar{D}^2 \bar{X}^A) - i\partial_{\alpha\dot{\alpha}} \bar{X}^A \bar{\eta}^{\dot{\alpha}}$$

and because of the m^A parameter, the second supersymmetry is spontaneously broken.

Note that the m^A, e_{2A} parameters are those which allow the equations

$$\frac{\partial W}{\partial Z^A} = \left(\mathcal{U}_{AB} m^B - e_A\right) = 0 \quad \text{to have solutions}$$

Expanding the fields around their "classical" value cancel the linear terms in the action and we obtain (Porrati, Sagnotti, S.F.)

$$\frac{\delta \mathscr{L}}{\delta X^A} = 0 \quad \Rightarrow \quad d_{ABC}\left[W^B W^C + X^B (m^C - \bar{D}^2\,\bar{X}^C)\right] + \bar{D}^2\,\bar{\mathcal{U}}_A = 0$$

The BI approximation corresponds to have $\bar{D}^2 \bar{\mathcal{U}}_A = 0$, (which we solve letting $\mathcal{U}_A = 0$ as operator condition)

The N=2 Born-Infeld generalized lagrangian turns out to be (Porrati, Sagnotti, S.F.)

$$\mathscr{L}_{\mathrm{BI}}^{N=2} = -\mathrm{Im}\, W(X) = \mathrm{Re}\, F^A e_{2A} + \mathrm{Im}\, F^A\, e_{1A}$$

with the chiral superfields X^A solutions of the above constraints

The goldstino is the fermion component of the superpotential superfield

$$\lambda_{g\alpha} = \psi_\alpha^A \, e_A$$

whose supersymmetry variation gives the BI Lagrangian

The super field BI constraint allows to write the n chiral fermions in terms of the n gauginos, so λ_g is a linear combination of the n (dressed) gauginos

In Black-Hole physics, the superpotential is the N=2 central charge, and

$$S_{\text{entropy}} = |Z|^2_{\text{crit}}$$

SOLUTION OF THE BORN-INFELD ATTRACTOR EQUATIONS

For n=1, the above equation is the BI constraint

$$X = \frac{-W^2}{m - \bar{D}^2 \bar{X}}$$

(electric magnetic self-dual BG inherited from the linear theory)

which also implies the nilpotency

(Komargodsky, Seiberg; Casalbuoni et al)

Type of constraints $X\,W_\alpha = 0\,, \qquad X^2 = 0$

This type of constraints have been recently used in inflationary supergravity dynamics to simplify and to provide a more general supergravity breaking sector

(Kallosh, Linde, S.F.; Kallosh, Linde; Antoniadis, Dudas, Sagnotti, S.F.)

The Born-Infeld action comes by solving the θ^2 component of the chiral superfield equation

$$\left.\frac{\partial \mathscr{L}}{\partial X^A}\right|_{\theta^2} \quad \Rightarrow \quad d_{ABC}\left[G_+^B G_+^C + F^B(m^C - \bar{F}^C)\right] = 0$$

$$G_\pm^A = F_{\mu\nu}^A \pm \frac{i}{2}\tilde{F}_{\mu\nu}^A \qquad \text{and we have set} \qquad D^A = 0$$

by taking the real and imaginary parts of this equation we have

$$d_{ABC}\left(H^B + \frac{m^B}{2}\right)\left(\frac{m^C}{2} - H^C\right) = d_{ABC}\left(-G^B G^C + \mathrm{Im}\,F^B\,\mathrm{Im}F^C\right)$$

$$d_{ABC}\,\mathrm{Im}\,F^B\,m^C = -d_{ABC}\,G^B\,\tilde{G}^C$$

and we have set $\mathrm{Re}\,F^A = \dfrac{m^A}{2} - H^A$

CLASSIFICATION OF d_{ABC}.

THEORY OF INVARIANT POLYNOMIALS

(Mumford, Gelfand, Dieudonné, et al)

$$\mathcal{U}(X) = \frac{1}{3!} d_{ABC} X^A X^B X^C$$

n=2 case: d_{ABC} (4 entries) spin 3/2 representation of SL(2)

It has a unique (quartic) invariant which is also the discriminant of the cubic (Cayley hyperdeterminant)

(Duff, q-bit entanglement in quantum information theory)

$$I_4 = -27\, d_{222}^2\, d_{111}^2 + d_{221}^2\, d_{112}^2 + 18 d_{222} d_{111} d_{211} d_{221} - 4 d_{111} d_{122}^3 - 4 d_{222} d_{211}^3$$

CLASSIFICATION OF d_{ABC}.

THEORY OF INVARIANT POLYNOMIALS

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$$I_4 = -27\, d_{222}^2\, d_{111}^2 + d_{221}^2\, d_{112}^2 + 18 d_{222} d_{111} d_{211} d_{221} - 4 d_{111} d_{122}^3 - 4 d_{222} d_{211}^3$$

Four orbits $I_4 > 0\,,\ I_4 < 0\,,\ I_4 = 0\,,\ \partial I_4 = 0$

Two of them ($I_4 < 0\,,\ \partial I_4 = 0$) give trivial models.

The other two ($I_4 \geq 0$) give non-trivial $U(1)^2$ BI theories

Extremal black-hole analogy:

the model in question corresponds to the T^3 model,

$\sqrt{|I_4|}$ is the BH Berenstein-Hawking entropy and the four orbits correspond to large BPS and non-BPS as well as small BH

An example: Explicit solution of the $I_4>0$ theory

$$\mathcal{U} = \frac{1}{3!}X^3 - \frac{1}{2}XY^2 \qquad (d_{111} = 1\,,\; d_{122} = -1)$$

$$\mathrm{Im}\,F_X = (m_X^2 + m_Y^2)^{-1}(m_X R_X + m_Y R_Y) \qquad R_X = -2G^X\tilde{G}^Y$$

$$\mathrm{Im}\,F_Y = (m_X^2 + m_Y^2)^{-1}(-m_Y R_X + m_X R_Y) \qquad R_Y = -G^X\tilde{G}^X + G^Y\tilde{G}^Y$$

$$H_X = \frac{1}{\sqrt{2}}\left(\sqrt{S_X^2 + S_Y^2} - S_X\right)^{1/2} \qquad S_X = T_X - \frac{m_X^2}{4} + \frac{m_Y^2}{4}$$

$$H_Y = \frac{1}{\sqrt{2}}\left(\sqrt{S_X^2 + S_Y^2} + S_X\right)^{1/2} \qquad S_Y = T_Y + \frac{m_X m_Y}{2}$$

$$-H_X^2 + H_Y^2 = S_X\,, \qquad 2H_X H_Y = S_Y \qquad (H^A \text{ equation})$$

$$T_X = -G^X G^X + \mathrm{Im}\,F^X\,\mathrm{Im}\,F^X + G^Y G^Y - \mathrm{Im}\,F^Y\,\mathrm{Im}\,F^Y$$

$$T_Y = 2(G^X G^Y + \mathrm{Im}\,F^X\,\mathrm{Im}\,F^Y)$$

Note: for $G_{\mu\nu}^{X,Y} = 0 \to F_X = F_Y = 0$