Report on Uncertainty Estimation Practical Exercises
External Beam Therapy

Tony Aalbers

Introduction

During the practical session on external beam therapy only exercise 2 was addressed. However also the results and associated uncertainty estimates of exercise 1 are presented here and may serve as additional training material for the participants. All students are encouraged to contact me, if you have questions, remarks or other comments concerning the lectures, exercises, etc.

Exercise 1: ^{60}Co absorbed dose to water calibration

1 Objective of the exercise

Determine the *calibration coefficient* of a user's measurement assembly (ionization chamber + electrometer) in terms of absorbed dose to water in a reference beam of ^{60}Co gamma radiation.

Estimate the *expanded uncertainty* in the absorbed dose calibration coefficient $N_{D,w}^{\text{user}}$.

2 Available data/information/documentation

Measurement procedure in SSDL

- As a *first step* the absorbed dose to water rate has been determined with a traceable calibrated reference dosimeter of the SSDL. You may assume that this part of the calibration procedure has been carried out.

  The absorbed doserate $\dot{D}_w$ at the reference point in water is: 12.942 mGy/s. Assume a standard uncertainty of 0.117 mGy/s (k=1) in the absorbed dose rate.

- During the calibration procedure the collimator setting of the ^{60}Co source is fixed and between measurements the source moves to its fully shielded position.

- The user chamber is set up in a water phantom with a surface to surface distance (SSD) of 95 cm at a depth of 5 cm in water, to within 0.2 mm.

- The chamber orientation is checked visually, the mark on the stem is aligned towards the source. Any misalignment and resulting uncertainty contribution may be neglected.

- Reference conditions: Beam size at the reference position: 10x10 cm^2, Source detector distance (SDD) = 100 cm, reference depth in water: 5 cm, temperature: 20 ^oC, air pressure: 1013.25 hPa, relative humidity (RH): 50%

- The following series of measurement values is obtained:

measurement	number	M	t
		nC	s
leakage #1		0.000	88.48
	1	23.899	88.46
	2	23.911	88.48
	3	23.920	88.48
	4	23.927	88.48
	5	23.915	88.47
	6	23.901	88.47
	7	23.910	88.47
	8	23.912	88.47
leakage #2		0.000	88.47

- During the measurements was the average temperature in air 19.5 ^oC, the average temperature in the water phantom 19.8 ^oC and the average air pressure 1013.10 hPa.

- The expanded uncertainty of the calibrated temperature and pressure devices are 0.5 ^oC and 0.1 kPa. The resolution of the temperature device is 0.1 ^oC.

- Ambient humidity lies within 20% to 70% RH during the calibration procedure, over which the humidity correction is constant to within 0.1%.

- No corrections or uncertainty contributions for change in source position, time, polarity, recombination and resolution of the pressure device are applied.

- The gradient of the normalized depth dose curve at the reference point in water is 0.5% per mm.

3 Results

- **Formulate the *measurement equation*.**

$$N_{D,w}^{user} = \dot{D}_w / ((M_{uncorr}/t) \bullet k_{T,P})$$

Note that in de measurement equation no additional correction factors are included for humidity and the chamber position at the reference depth, although we have assumed uncertainty contributions (see note on humidity and on the dosimeter position at the reference depth below)!

- **Determine the *value* for the calibration coefficient $N_{D,w}^{\text{user}}$ in *mGy/nC* and the *expanded uncertainty* .**

Absorbed doserate at reference depth in water

The absorbed doserate $\dot{D}_w$ at the reference depth in water is: 12.942 ± 0.117 mGy/s (k=1). This value has been determined by measurement for the same measurement set-up and reference conditions as stated in section 2. The *relative standard uncertainty* in $\dot{D}_w$ is: 0.90% (Type B).

Dosimeter readings

We calculate the average value of the ratio's ($M_{uncorr.}/t$) from the data presented in the table above. Note that we neglect the leakage radiation signal. Uncertainty in time measurement is very small and may be neglected. In general in the calibration laboratory the same polarizing voltage and polarity are adopted as during routine use of the dosimeter assembly. Moreover the calibration is carried out in a reference beam of ^{60}Co gamma rays and the effect of recombination is usually negligible. Therefore no corrections are applied for polarity and recombination.

measurement	number	M	t	M/t
		nC	s	nA
leakage #1		0	88.48	
	1	23.899	88.46	0.2702
	2	23.911	88.48	0.2702
	3	23.92	88.48	0.2703
	4	23.927	88.48	0.2704
	5	23.915	88.47	0.2703
	6	23.901	88.47	0.2702
	7	23.91	88.47	0.2703
	8	23.912	88.47	0.2703
leakage #2		0	88.47	

Mean	0.2703	nA
Experimental standard deviation	0.0001	nA
sample size	8	
Experimental standard deviation of the mean	3.12439E-05	nA
Relative experimental standard deviation of the mean	0.012	%

The contribution to the *relative combined standard uncertainty* of the calibration coefficient due to the dosimeter readings is: 0.01% (Type A contribution)

Correction factor $k_{T,P}$

$k_{T,P} = (TxP_0)/(T_0xP)$ corrects for temperature and pressure. Using the "available data":

average water temperature	19.8 °C
average pressure	1013.10 hPa
reference temperature	20 °C
reference pressure	1013.25 hPa
$k_{T,P}$	0.9995

uncertainty table $k_{T,P}$

quantity	*value quantity*	*uncertainty (expanded)*	*uncertainty type*	*probablity distribution*	*divisor k-factor*	*rel. standard uncertainty (%)*	*$U_i(y)$ %*
average water temperature	293.0 K						
1. thermometer calibration		0.5 K	B	normal	2	0.09	0.09
2. thermometer resolution		0.1 K	B	rectangular	1.73	0.02	0.02
average pressure	1013.10 hPa						
3. barometer calibration		1 hPa	B	normal	2	0.05	0.05

1. To evaluate the uncertainty contribution in the temperature we must state all temperature values in kelvin. Average water temperature: 19.8 ^{0}C -> 293.0 K Note that the <u>absolute uncertainty</u> in the calibration of the temperature device is numerically equal in terms of ^{0}C and K. That is the expanded uncertainty of 0.5 ^{0}C = 0.5 K.
   $U_i(y)$ denotes the contribution to the uncertainty in the calibration coefficient

Humidity

No additional correction is applied for the relative humidity, but for the interval 20 to 70% the humidity is assumed constant within 0.1%. This is a type B uncertainty with rectangular probability distribution. The *relative standard uncertainty* is: 0.1/1.73 = 0.06 %.

Positioning of user dosimeter assembly (depth)

The deviation in the reference depth is estimated to be 0.2 mm. This is a type B uncertainty assuming a Gaussian probability distribution (k=2). Note that the gradient of the normalized depth dose curve at the reference depth in water is 0.5% per mm. This results in a *relative standard uncertainty*: (0.2 mm/2)x 0.5 (%/mm) = 0.05%

The value of the calibration coefficient $N_{D,w}^{\text{user}}$ is:

12.942×10^{-3} (Gy/s) / [0.2703×10^{-9} (C/s) x 0.9995] =4.79×10^{7} (Gy/C)

The ***combined relative standard uncertainty*** in $N_{D,w}^{\text{user}}$ is derived from the table below summarizing all estimated uncertainty contributions.

quantity	*value quantity*	*uncertainty (expanded)*	*uncertainty type*	*probablity distribution*	*divisor k-factor*	*rel. standard uncertainty*	*$U_i(y)$ %*
doserate	12.942 mGy/s						
uncertainty in doserate		0.117 mGy/s	B	normal	1	0.90	0.90
average reading	0.2703 nA						
uncertainty of mean reading		3.124×10^{-5} nA	A	normal	1	0.01	0.01
average water temperature	293.0 K						
thermometer calibration		0.5 K	B	normal	2	0.09	0.09
thermometer resolution		0.1 K	B	rectangular	1.73	0.02	0.02
average pressure	1013.10 hPa						
barometer calibration		1 hPa	B	normal	2	0.05	0.05
humidity							
uncertainty humidity correction		0.10%	B	rectangular	1.73	0.06	0.06
user chamber positioning							
deviation in depth in phantom		0.2 mm	B	normal	2	0.05	0.05

Combined relative standard uncertainty in $N_{D,w}^{\text{user}}$ = sqrt $[U_i(y)]^2$ = 0.91 % (k=1)

The *expanded uncertainty* in $N_{D,w}^{\text{user}}$ is: 1.8 % (k=2)

<u>Exercise 2:</u> Determination absorbed dose to water in a high-energy photon beam

1. Objective of the exercise

Determine the *absorbed dose to water* in a high-energy accelerator photon beam in a radiotherapy environment following the recommendations of the IAEA Code of Practice TRS398 and estimate the *expanded uncertainty* in the determined absorbed dose to water.

2. Available data/information/documentation

<u>Measurement procedure according to TRS398</u>

- The absorbed dose to water $D_{w,Q}$ at the reference depth (10 cm) in water for a photon beam of quality Q and in absence of the chamber, is given by

$$D_{w,Q} = M_{corr} N_{D,w,Q_0} k_{Q,Q_0} ,$$

where M_{corr} is the reading of the dosimeter with the reference point of the chamber positioned at the reference depth and corrected for the influence quantities for temperature and pressure, polarity effect and recombination.

N_{D,w,Q_0} is the calibration coefficient in terms of absorbed dose to water for the dosimeter at the reference quality Q_o, and k_{Q,Q_0} is a chamber specific factor which corrects for the difference between the reference beam quality Q_o and the actual quality Q.

Note that Q_o is the reference beam quality of ^{60}Co used in the standards laboratory (e.g. SSDL). For the sake of simplicity we omit symbol Q_o.

- The following *reference conditions* apply: Beam size at the reference position: 10x10 cm^2, source detector distance (SDD) = 100 cm, reference depth in water: 10 cm, temperature: 20 ^{o}C, air pressure: 1013.25 hPa, relative humidity (RH): 50%.

- The dosimeter system used in the hospital for absorbed dose to water measurements consists of a Farmer type cylindrical ionization chamber PTW 30012 and a PTW Unidos electrometer. The dosimeter system is calibrated at a standards laboratory in a ^{60}Co reference beam. The absorbed do to water calibration coefficient $N_{D,w}$ is: 51.81 ± 0.57 (k=1) mGy/nC.

- The ionization chamber is set up in a 10 MV accelerator photon beam (*beam quality specifier* $TPR_{20,10}$ = 0.73) in a water phantom with a surface to surface distance (SSD) of 90 cm at a depth of 10 cm (reference depth) in water, to within 0.1 cm.

- During the measurements the ionization chamber was fitted in a PMMA sleeve (0.5 mm). Note that the chamber has been calibrated including the sleeve.

- The chamber orientation is checked visually, the mark on the stem is aligned towards the source. Any misalignment and resulting uncertainty contribution may be neglected.

- The following series of measurement values is obtained:

measurement	number	M
		nC
leakage #1		0.00013
	1	28.03
	2	28.07
	3	28.07
	4	28.08
	5	28.08
leakage #2		0.00015

- During the measurements was the average temperature in air 22.8 ^{o}C, the average temperature in the water phantom 22.5 ^{o}C and the average air pressure 101.4 kPa.

- The expanded uncertainty of the calibrated temperature and pressure devices are 0.5 ^{o}C and 0.1 kPa. The resolution of the temperature and the pressure device are respectively 0.1 ^{o}C. and 0.05 kPa.

- Ambient humidity lies within 20% to 70% RH during the calibration procedure, over which the humidity correction is constant to within 0.1%.

- A correction factor for recombination was applied k_s: 1.002 with a relative standard uncertainty of 0.1%

- No corrections for the influence of the sleeve, time and polarity were applied, resulting uncertainty contributions may be neglected.

- The gradient of the normalized depth dose curve at the reference point in water is 0.6% per mm.

- Part of table 14 from TRS398 is reproduced below to allow the selection of k_Q. The combined relative standard uncertainty in k_Q is estimated as 1%. Note that the influence of the sleeve is incorporated in the calculations of k_Q.

TABLE 14. CALCULATED VALUES OF k_Q FOR HIGH ENERGY PHOTON BEAMS FOR VARIOUS CYLINDRICAL IONIZATION CHAMBERS AS A FUNCTION OF BEAM QUALITY $TPR_{20,10}$ *(adapted from Andreo [20])*

Ionization chamber type[a]	Beam quality $TPR_{20,10}$														
	0.50	0.53	0.56	0.59	0.62	0.65	0.68	0.70	0.72	0.74	0.76	0.78	0.80	0.82	0.84
Capintec PR-05P mini	1.004	1.003	1.002	1.001	1.000	0.998	0.996	0.994	0.991	0.987	0.983	0.975	0.968	0.960	0.949
Capintec PR-05 mini	1.004	1.003	1.002	1.001	1.000	0.998	0.996	0.994	0.991	0.987	0.983	0.975	0.968	0.960	0.949
Capintec PR-06C/G Farmer	1.001	1.001	1.000	0.998	0.998	0.995	0.992	0.990	0.988	0.984	0.980	0.972	0.965	0.956	0.944
Exradin A2 Spokas	1.001	1.001	1.001	1.000	0.999	0.997	0.996	0.994	0.992	0.989	0.986	0.979	0.971	0.962	0.949
Exradin T2 Spokas	1.002	1.001	0.999	0.996	0.993	0.988	0.984	0.980	0.977	0.973	0.969	0.962	0.954	0.946	0.934
Exradin A1 mini Shonka	1.002	1.002	1.001	1.000	1.000	0.998	0.996	0.994	0.991	0.986	0.982	0.974	0.966	0.957	0.945
Exradin T1 mini Shonka	1.003	1.001	0.999	0.996	0.993	0.988	0.984	0.980	0.975	0.970	0.965	0.957	0.949	0.942	0.930
Exradin A12 Farmer	1.001	1.001	1.000	1.000	0.999	0.997	0.994	0.992	0.990	0.986	0.981	0.974	0.966	0.957	0.944
NE 2515	1.001	1.001	1.000	0.999	0.997	0.994	0.991	0.988	0.984	0.980	0.975	0.967	0.959	0.950	0.937
NE 2515/3	1.005	1.004	1.002	1.000	0.998	0.995	0.993	0.991	0.989	0.986	0.982	0.975	0.969	0.961	0.949
NE 2577	1.005	1.004	1.002	1.000	0.998	0.995	0.993	0.991	0.989	0.986	0.982	0.975	0.965	0.961	0.949
NE 2505 Farmer	1.001	1.001	1.000	0.999	0.997	0.994	0.991	0.988	0.984	0.980	0.975	0.967	0.955	0.950	0.937
NE 2505/A Farmer	1.005	1.003	1.001	0.997	0.995	0.990	0.985	0.982	0.978	0.974	0.969	0.962	0.955	0.947	0.936
NE 2505/3, 3A Farmer	1.005	1.004	1.002	1.000	0.998	0.995	0.993	0.991	0.989	0.986	0.982	0.975	0.965	0.961	0.949
NE 2505/3, 3B Farmer	1.006	1.004	1.001	0.999	0.996	0.991	0.987	0.984	0.980	0.976	0.971	0.964	0.957	0.950	0.938
NE 2571 Farmer	1.005	1.004	1.002	1.000	0.998	0.995	0.993	0.991	0.989	0.986	0.982	0.975	0.969	0.961	0.949
NE 2581 Farmer	1.005	1.003	1.001	0.998	0.995	0.991	0.986	0.983	0.980	0.975	0.970	0.963	0.956	0.949	0.937
NE 2561/2611 Sec. Std	1.006	1.004	1.001	0.999	0.998	0.994	0.992	0.990	0.988	0.985	0.982	0.975	0.969	0.961	0.949
PTW 23323 micro	1.003	1.003	1.000	0.999	0.997	0.993	0.990	0.987	0.984	0.980	0.975	0.967	0.960	0.953	0.941
PTW 23331 rigid	1.004	1.003	1.000	0.999	0.997	0.993	0.990	0.988	0.985	0.982	0.978	0.971	0.964	0.956	0.945
PTW 23332 rigid	1.004	1.003	1.001	0.999	0.997	0.994	0.990	0.988	0.984	0.980	0.976	0.968	0.961	0.954	0.943
PTW 23333	1.004	1.003	1.001	0.999	0.997	0.994	0.990	0.988	0.985	0.981	0.976	0.969	0.963	0.955	0.943
PTW 30001/30010 Farmer	1.004	1.003	1.001	0.999	0.997	0.994	0.990	0.988	0.985	0.981	0.976	0.969	0.962	0.955	0.943
PTW 30002/30011 Farmer	1.006	1.004	1.001	0.999	0.997	0.994	0.992	0.990	0.987	0.984	0.980	0.973	0.967	0.959	0.948
PTW 30004/30012 Farmer	1.006	1.005	1.002	1.000	0.999	0.996	0.994	0.992	0.989	0.986	0.982	0.976	0.969	0.962	0.950
PTW 30006/30013 Farmer	1.002	1.002	1.000	0.999	0.997	0.994	0.990	0.988	0.984	0.980	0.975	0.968	0.960	0.952	0.940
PTW 31002 flexible	1.003	1.002	1.000	0.999	0.997	0.994	0.990	0.988	0.984	0.980	0.975	0.968	0.960	0.952	0.940
PTW 31003 flexible	1.003	1.002	1.000	0.999	0.997	0.994	0.990	0.988	0.984	0.980	0.975	0.968	0.960	0.952	0.940
SNC 100730 Farmer	1.004	1.003	1.001	0.999	0.997	0.993	0.990	0.988	0.985	0.981	0.977	0.970	0.963	0.956	0.944
SNC 100740 Farmer	1.006	1.005	1.002	1.000	0.999	0.996	0.994	0.992	0.990	0.987	0.983	0.977	0.971	0.963	0.951
Victoreen Radocon III 550	1.005	1.004	1.001	0.998	0.996	0.993	0.989	0.986	0.983	0.979	0.975	0.968	0.961	0.954	0.943

3. Results

- ***Measurement equation***

$$D_{w,Q} = N_{D,w} k_Q M_{uncorr} k_{T,P} k_s$$

Note that the symbol of the reference quality *Qo* has been omitted in the equation and that no correction factors have been applied for leakage radiation, influence of the sleeve, deviation of the reference depth, polarity and humidity. However in the estimation of the uncertainty in $D_{w,Q}$ contributions for some of these influence quantities will be included.

- **Determine $D_{w,Q}$ at the refence depth**:

We calculate the average value for the dosimeter readings *M* including the *relative standard deviation* of the mean value. Note that the leakage radiation is neglected.

measurement	number	M	
		nC	
leakage #1		0.00013	
	1	28.03	28.03
	2	28.07	28.07
	3	28.07	28.07
	4	28.08	28.08
	5	28.08	28.08
leakage #2		0.00015	
average leakage		0.00014 nC	neglected
mean		28.066	nC
exp. st.dev.		0.021	nC
sample size		5	
exp.st.dev. mean		0.009	nC
rel. st.dev. mean		0.033	%

Calculation correction factor $k_{T,P}$:

average water temperature		22.5	°C
average pressure		1014.00	hPa
reference temperature		20	°C
reference pressure		1013.25	hPa
$k_{T,P}$		1.008	

Calculation beam quality correction factor k_Q:

For the ionization chamber type: *PTW 30012 Farmer* we have to interpolate between the k_Q values for the $TPR_{20,10}$ values 0.72 and 0.74 to find a value for beam quality $TPR_{20,10} = 0.73$:

$k_Q = (0.989 + 0.986)/2 = 0.988$

The value of $D_{w,Q}$ is:

$D_{w,Q}$ = 51.81(mGy/nC) x 28,066 (nC) x 1.008 x 1.002 x 0.988 = 1450.73 mGy

≈ 1.45 Gy

- **Estimation of the expanded uncertainty in $D_{w,Q}$:**

quantity	*value quantity*	*uncertainty (expanded)*	*uncertainty type*	*probablity distribution*	*divisor k-factor*	*rel. standard uncertainty %*	*U_i (y) %*
1. calibration coefficient $N_{D,w}$	51.81 mGy/nC						
uncertainty of calibration		0.57 mGy/nC	B	normal	1	1.10	1.1
2. reading of the instrument	28.07 nC						
uncertainty of mean reading		0.01 nC	A	normal	1	0.03	0.03
3. average water temperature	295.7 K						
thermometer calibration		0.5 K	B	normal	2	0.08	0.08
thermometer resolution		0.1 K	B	rectangular	1.73	0.02	0.02
4. average pressure	101.4 kPa						
barometer calibration		0.1 kPa	B	normal	2	0.05	0.05
barometer resolution		0.05 kPa	B	rectangular	1.73	0.03	0.03
5. relative humidity							
uncertainty contribution		0.10%	B	rectangular	1.73	0.06	0.06
6. recombination correction	1.002						
uncertainty contribution		0.10%	B	normal	1	0.10	0.1
7. chamber positioning ref. depth							
deviation chamber depth in phantom		0.1 cm	B	normal	2	0.30	0.3
8. beam quality correction factor							
combined relative standard uncertainty		1.0%	B	normal	1	1.0	1.0

Combined relative standard uncertainty in $D_{w,Q}$ = sqrt $[U_i(y)]^2$ = 1.52 % (k=1)

The *expanded uncertainty* in $D_{w,Q}$ **is:** 3.0 % (k=2)

Notes:

1. See section 2 “available data”
2. See calculation of mean reading *M*, no correction made for leakage and the resulting uncertainty is negligible.
   The resolution uncertainty of the electrometer reading is neglected here. Usually it is included in the uncertainty budget. For further reading consult IAEA-TECDOC-1585 “Measurement Uncertainty” (2008)
3. Data taken from calibration certificate with expanded uncertainty (k=2). All temperature data must be stated in K (see results section exercise 1)
4. Data taken from calibration certificate with expanded uncertainty (k=2)
5. No additional correction factor for humidity, but constant uncertainty of 0.1% within 20 % to 70 % relative humidity
6. Relative standard uncertainty 0.1% (see “available data”)
7. Deviation of chamber depth in phantom of 0.1cm=1mm. Assumed uncertainty contribution stated as expanded uncertainty (k=2) with Gaussian (normal) probability distribution. The relative standard uncertainty is: (1 mm / 2) x 0.6 (%/mm) = 0.3 %
8. The combined standard uncertainty (k=1) for k_Q is taken from TRS 398 (2000) (see page 183, table 39)