DIFFERENTIABLE ERGODIC THEORY
PROBLEM SHEET 2

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1) Let $f : [0,1] \to [0,1]$ be a piecewise affine full branch map with a finite or countable set of branches. Show that Lebesgue measure is invariant and ergodic.

2) Consider the Gauss map

$$f(x) = \frac{1}{x} \mod 1$$

a) Show that the Gauss measure

$$\mu(A) = \frac{1}{\ln 2} \int_A \frac{1}{1+x} dx$$

is invariant.

b) Show that the Gauss map is uniformly hyperbolic and satisfies

$$\sup_{\omega \in \mathcal{P}} \sup_{x,y \in \omega} \frac{|f''(x)|}{|f'(y)|} < +\infty.$$

and deduce that the Gauss measure is ergodic.