

GISM

Global Ionospheric Scintillation Model

<http://www.ieea.fr/en/gism-web-interface.html>

Y. Béniguel, IEAA

- Béniguel Y., P. Hamel, "A Global Ionosphere Scintillation Propagation Model for Equatorial Regions", Journal of Space Weather Space Climate, 1, (2011), doi: 10.1051/swsc/2011004

Contents

- Bias
- Scintillations
- Modelling Results vs Measurements
- Scattering Function Calculation (SAR observations)
- Conclusion
- Positioning errors

Measurement Campaigns

PRIS

Béniguel Y., J-P Adam, N. Jakowski, T. Noack, V. Wilken, J-J Valette, M. Cueto, A. Bourdillon, P. Lassudrie-Duchesne, B. Arbesser-Rastburg, Analysis of scintillation recorded during the PRIS measurement campaign, Radio Sci., Vol 44, (2009), doi:10.1029/2008RS004090

<http://telecom.esa.int/telecom/www/object/index.cfm?fobjectid=29210>

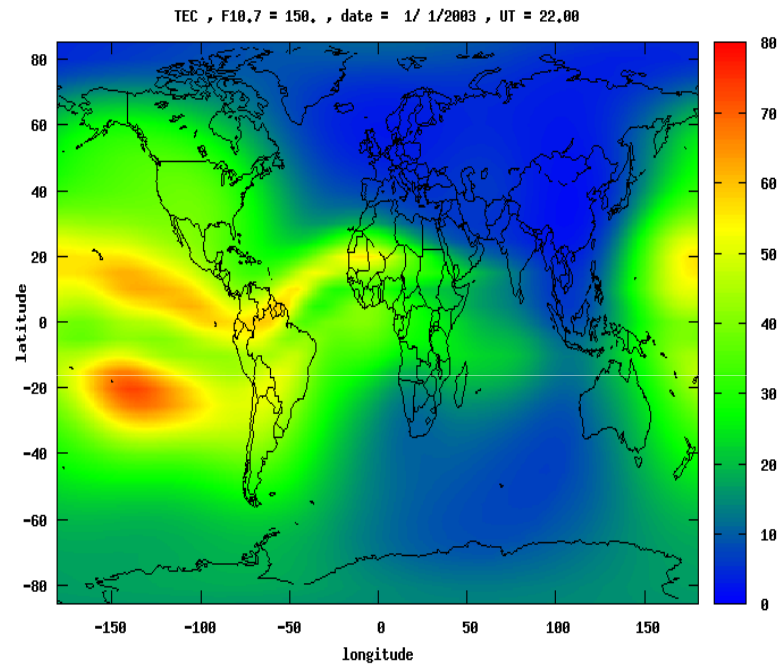
MONITOR

Prieto Cerdeira R., Y. Béniguel, "The MONITOR project: architecture, data and products", Ionospheric Effects Symposium, Alexandria VA, May 2011

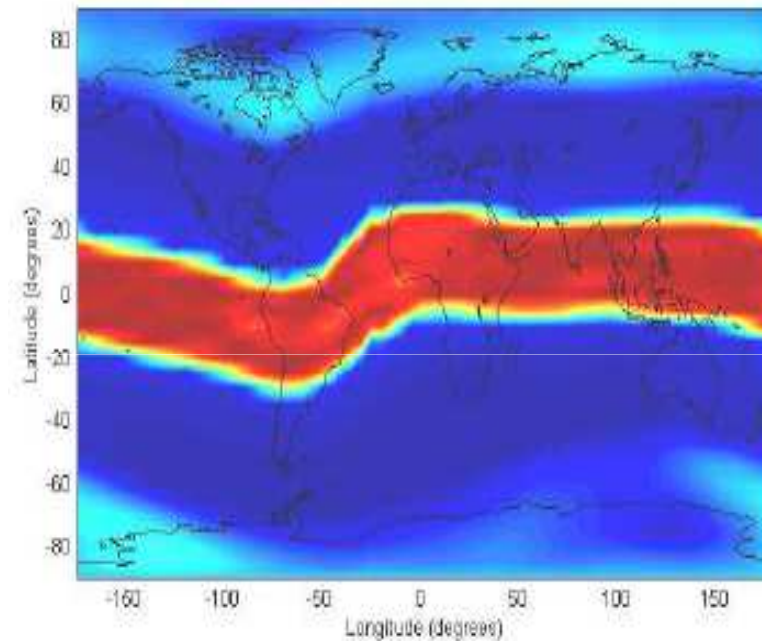
MONITOR Extension

On going; start : 30 june 2014

Ionosphere Variability



TEC Map



S4 map cumulated over 24 hours

Mean Errors (ray technique calculation)

$$n^2 = 1 - \frac{X}{1 - \frac{Y^2 \sin^2 \vartheta}{2(1-X)} \pm \left[\left(\frac{Y^2 \sin^2 \vartheta}{2(1-X)} \right)^2 + Y^2 \cos^2 \vartheta \right]^{1/2}}$$

$$X = \frac{\omega_P^2}{\omega^2} \qquad Y = \frac{\omega_b}{\omega}$$

Haselgrove equations (Simplified)

$$\frac{d x_i}{d t} = \frac{c^2 k_i}{\omega} \qquad \frac{d k_i}{d t} = - \frac{\omega_P}{\omega} \frac{\partial \omega_P}{\partial x_i}$$

Bias

Range error

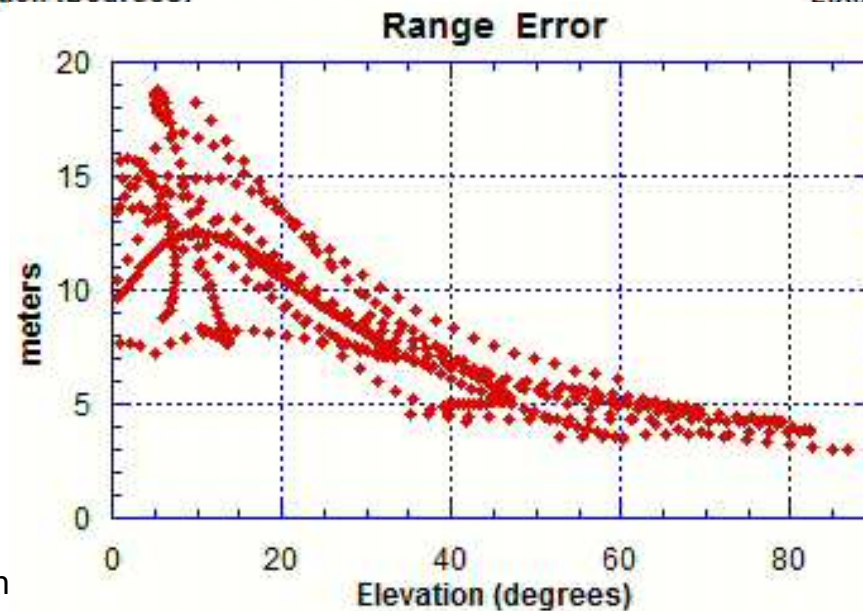
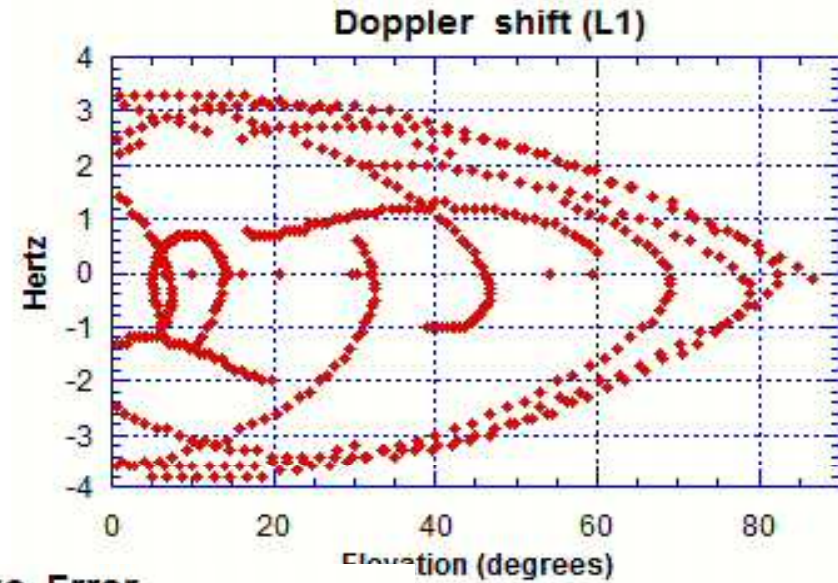
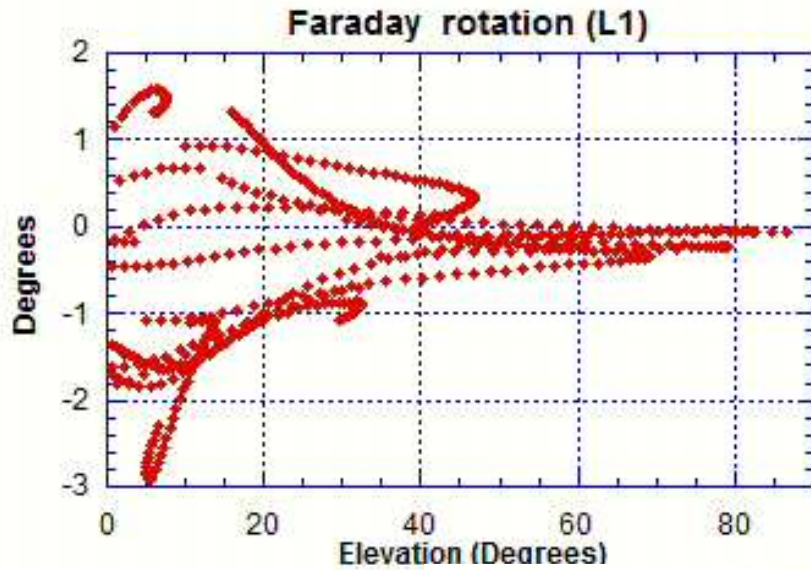
$$\Delta L = \frac{\lambda^2 r_e}{2\pi} N_T \quad N_T = \int_0^z N_e ds \quad \Delta L = \frac{40.3 N_T}{f^2}$$

Faraday rotation

$$\Psi = \frac{e^3}{2\epsilon_0 c m^2 \omega^2} \int_0^z N_e B \cos\vartheta ds$$

Inputs : N_e ; B at any point inside ionosphere

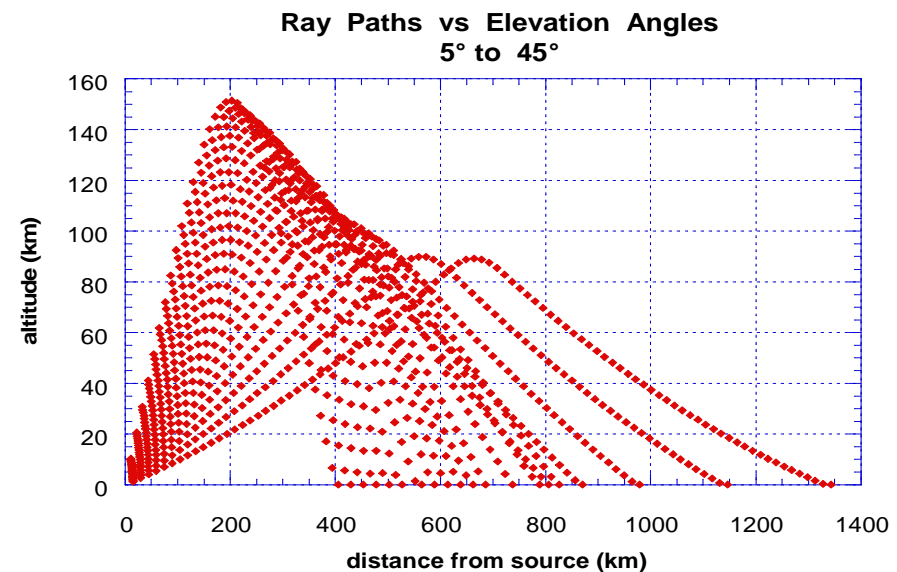
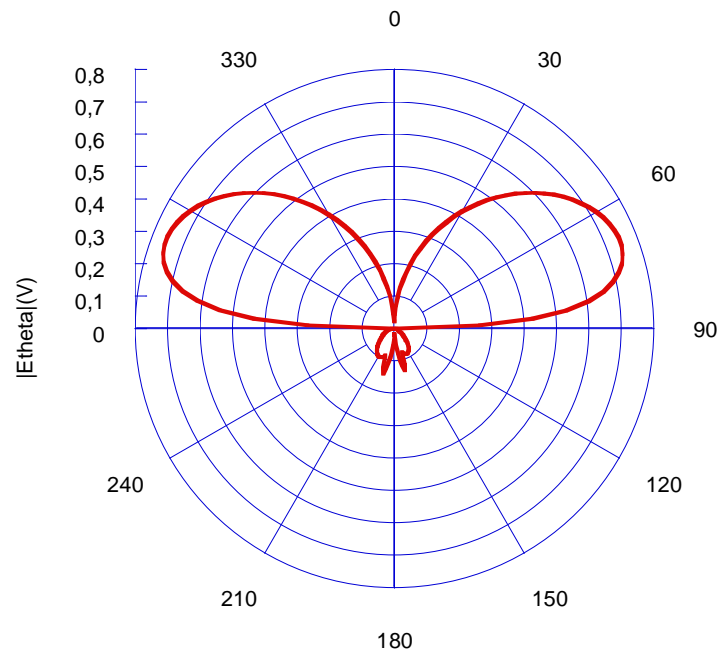
Example of results / Solar Flux 150



HF Ground to Ground Propagation (Sky Wave)

$$\frac{d x_i}{d t} = \frac{c^2 k_i}{\omega}$$

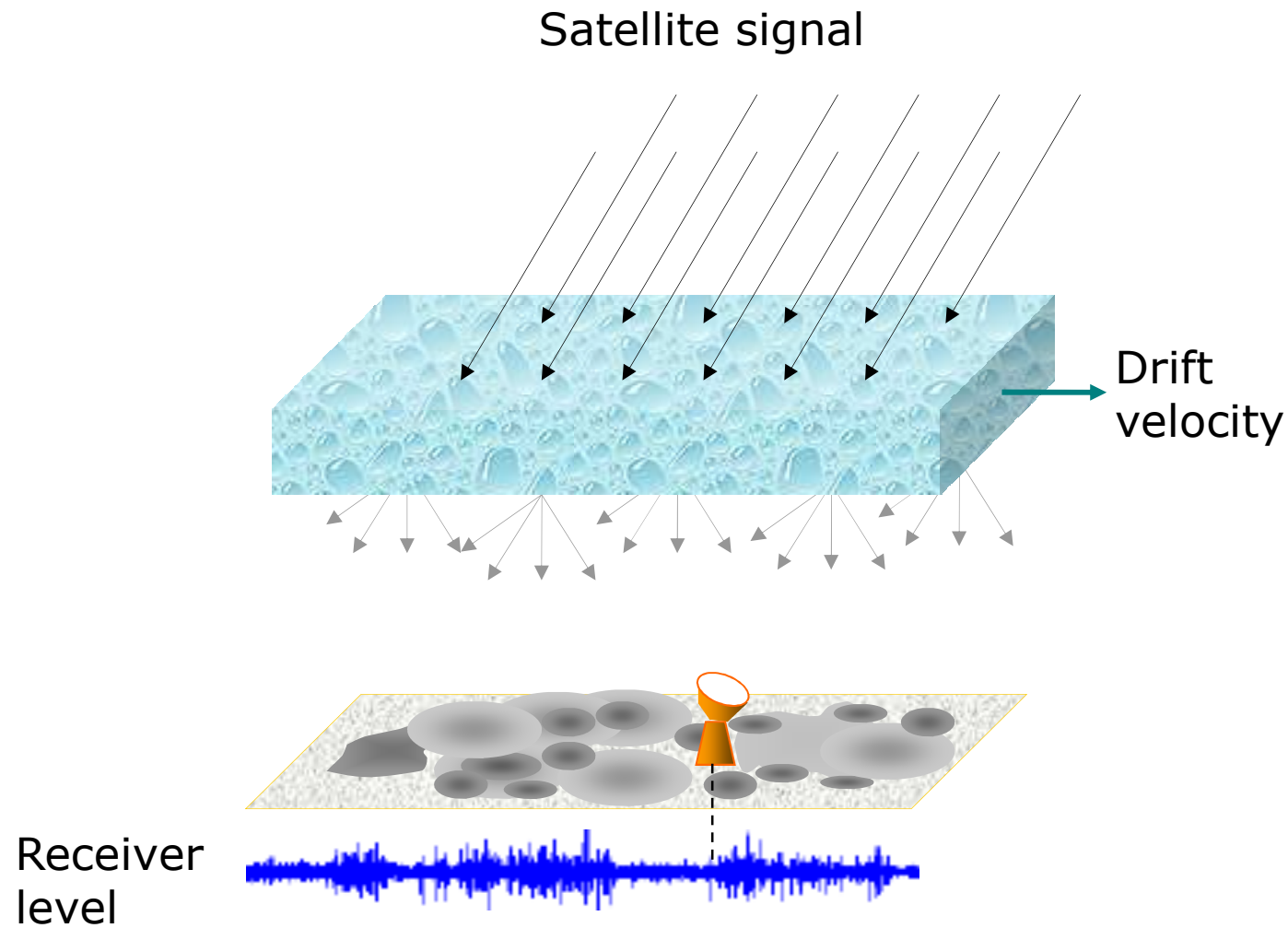
$$\frac{d k_i}{d t} = - \frac{\omega_p}{\omega} \frac{\partial \omega_p}{\partial x_i}$$



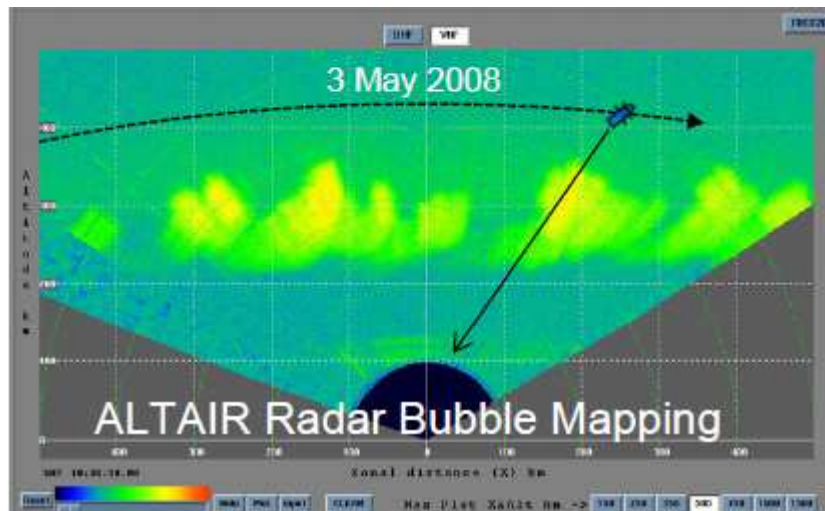
HF Antenna Pattern as an input

Turbulent Ionosphere (scintillation)

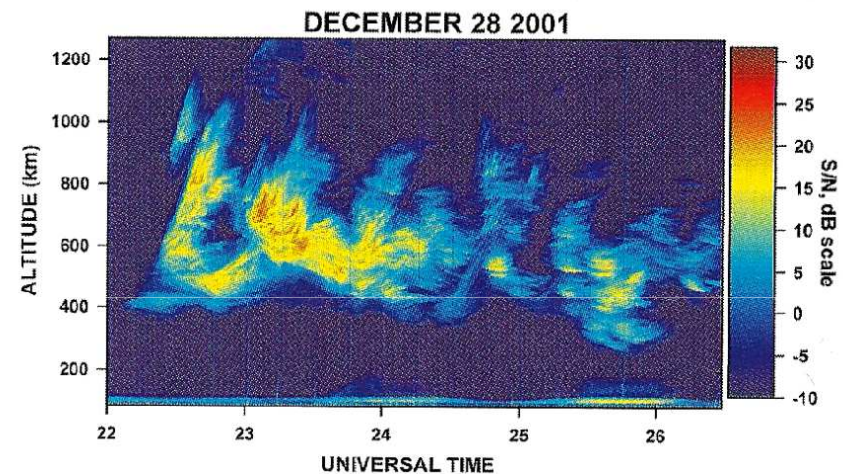
Physical Mechanism



Medium Radar Observations



Observations at Kwajalen Islands
Courtesy K. Groves, AFRL

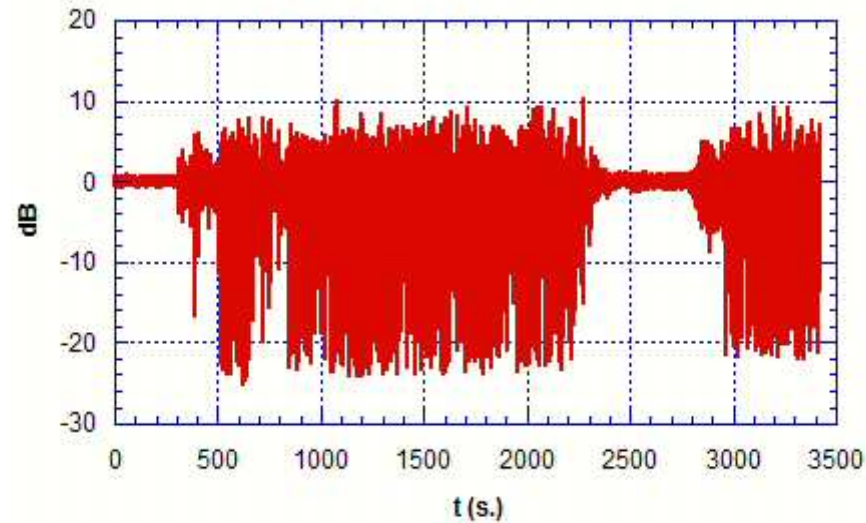


Observations in Brazil
Courtesy E. de Paula, INPE

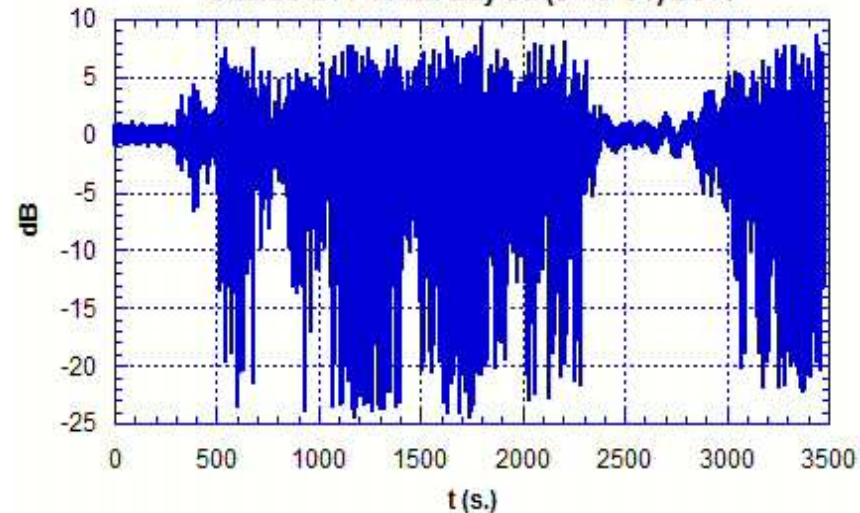
The vertical extent may reach hundreds of kilometers

Scintillation on Galileo Satellites L1 vs E5a

Galileo E5a / Tahiti day 85 (8 - 9 UT) 2013



Galileo L1 / Tahiti day 85 (8 - 9 UT) 2013



Field Propagation Equation

$$E(\rho, z, \omega, t) = U(\rho, z, \omega) \exp\left\{j\left(\omega t - \int \langle k(z') \rangle dz'\right)\right\}$$

The field amplitude value U is a solution of the the parabolic equation

$$2jk \frac{\partial U(\rho)}{\partial z} + \nabla_t^2 U(\rho) + k^2 \epsilon_1(\rho) U(\rho) = 0$$

Method of solution : phase screen technique

Field Propagation Equation

Solution of the parabolic equation

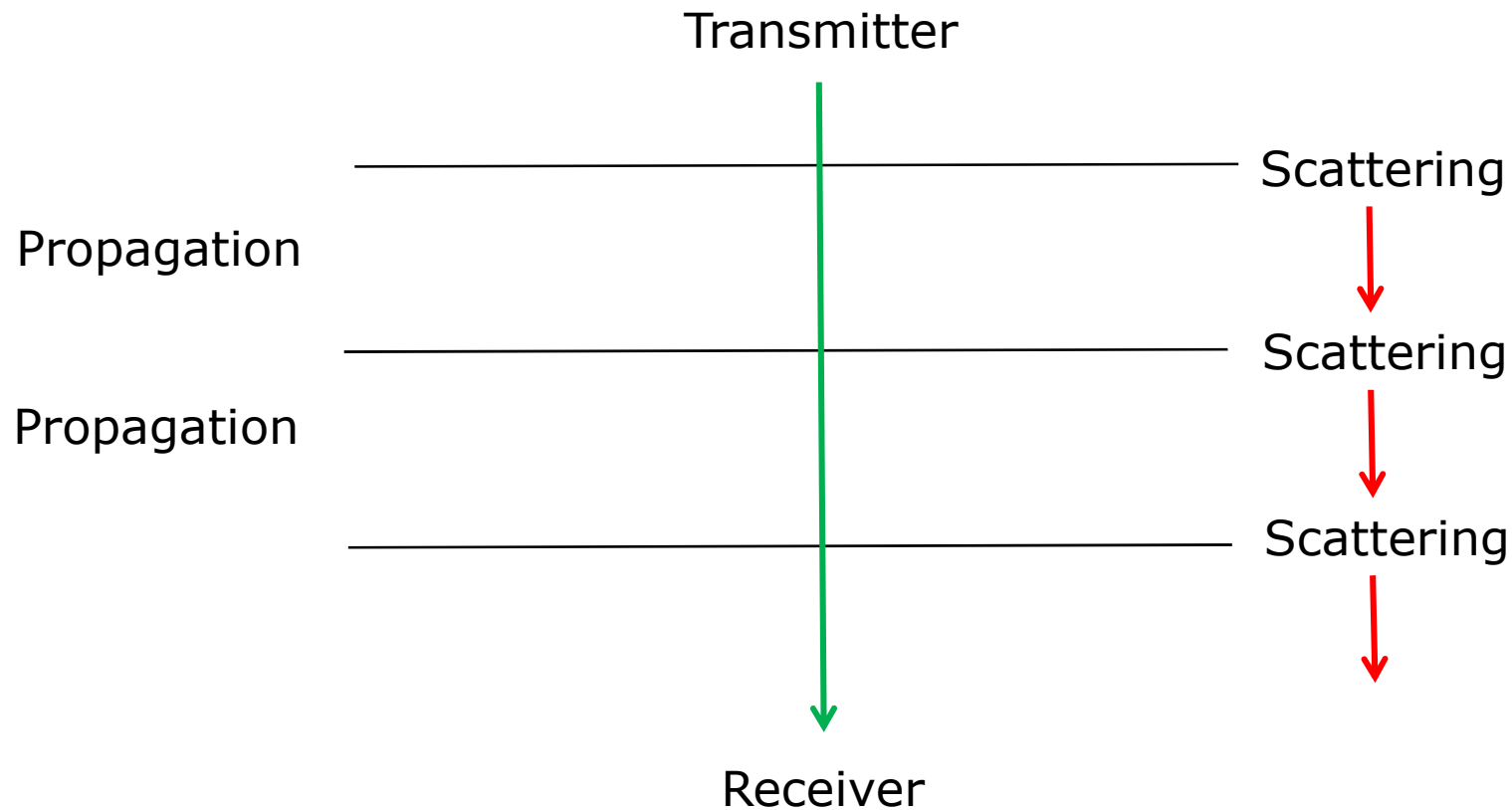
$$2jk \frac{\partial}{\partial z} \langle U(\mathbf{r}) \rangle + \nabla_t^2 \langle U(\mathbf{r}) \rangle + k^2 \langle \epsilon(\mathbf{r}) U(\mathbf{r}) \rangle = 0$$

$$2jk \frac{\partial}{\partial z} \langle U(\mathbf{r}) \rangle + \nabla_t^2 \langle U(\mathbf{r}) \rangle + j \frac{k^3}{4} A(0) \langle U(\mathbf{r}) \rangle = 0$$

Using the phase index autocorrelation function

$$B(z, \rho) = \langle \epsilon(\rho_1) \epsilon(\rho_2) \rangle \quad A(\rho) = \int B(z, \rho) dz$$

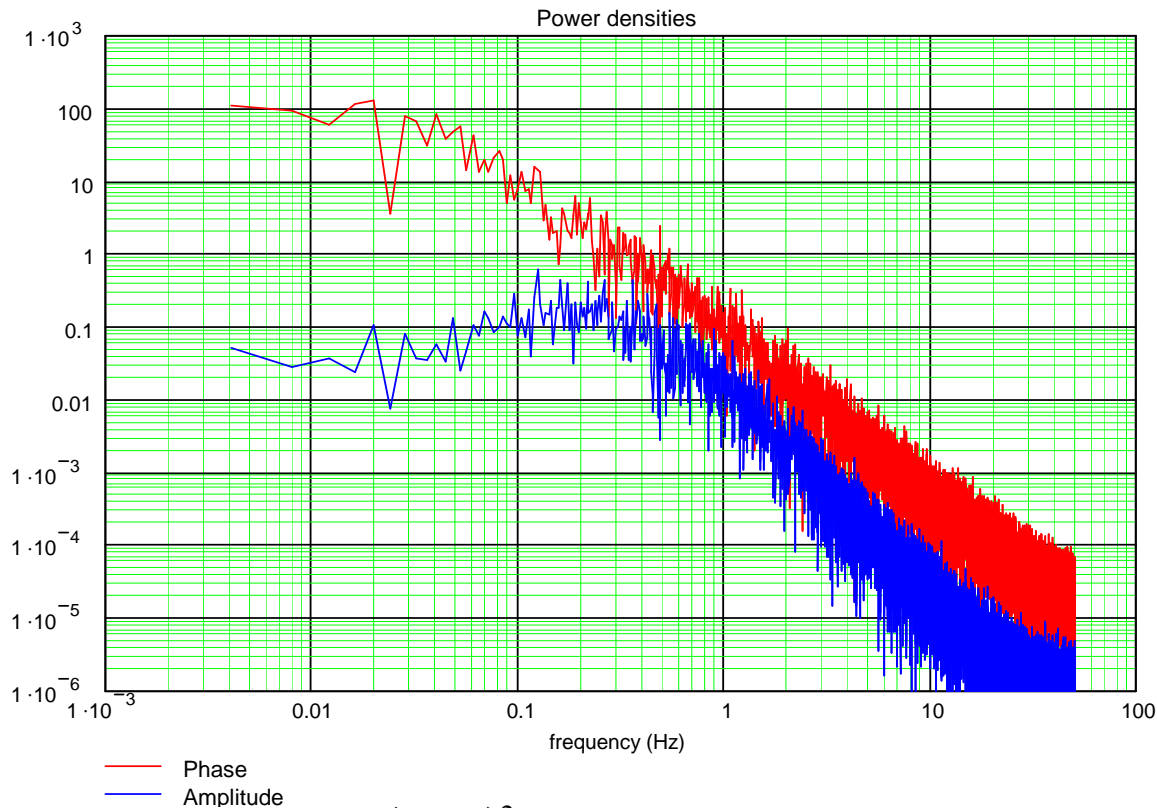
Phase Screen Technique



Propagation : 1st & 3rd terms ; scattering : 2nd & 3rd terms

Medium Characterization Index Spectral Density

Medium's Phase Spectrum

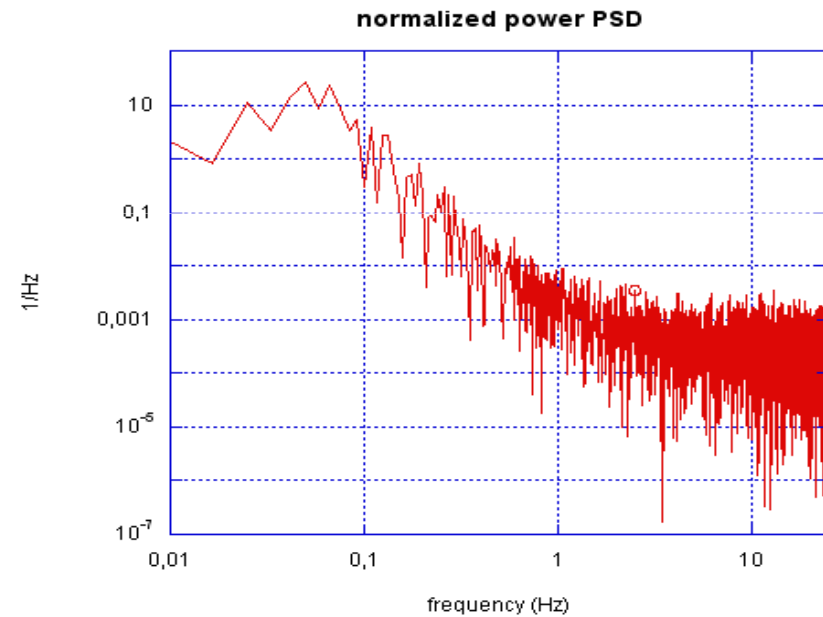
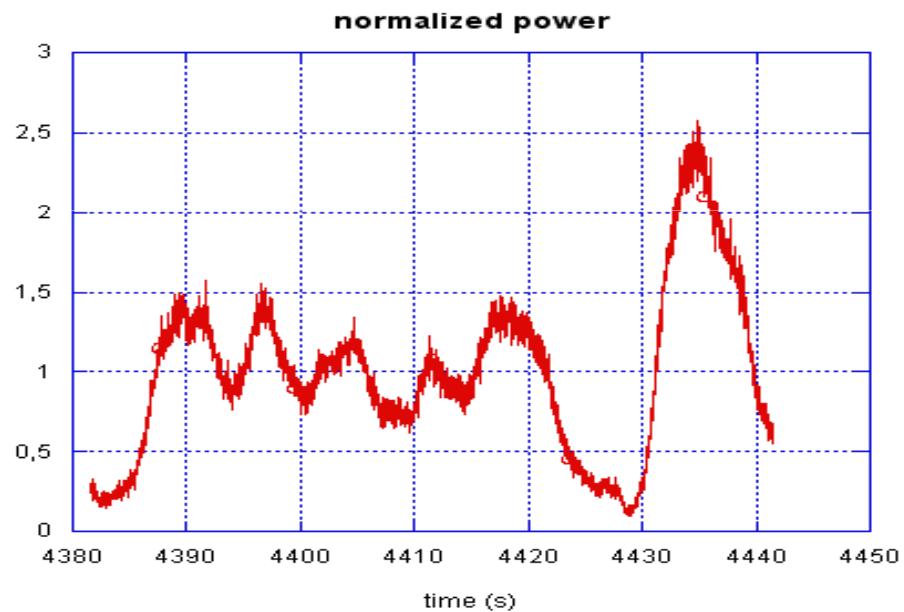


$$\gamma_{\Phi} (K) = \frac{(\lambda r_e)^2 L C_S \sigma_{Ne}^2}{(K^2 + q_0^2)^{p/2}} = \frac{C_P}{(K^2 + q_0^2)^{p/2}}$$

3 parameters : σ_{Ne} ; q_0 ; p

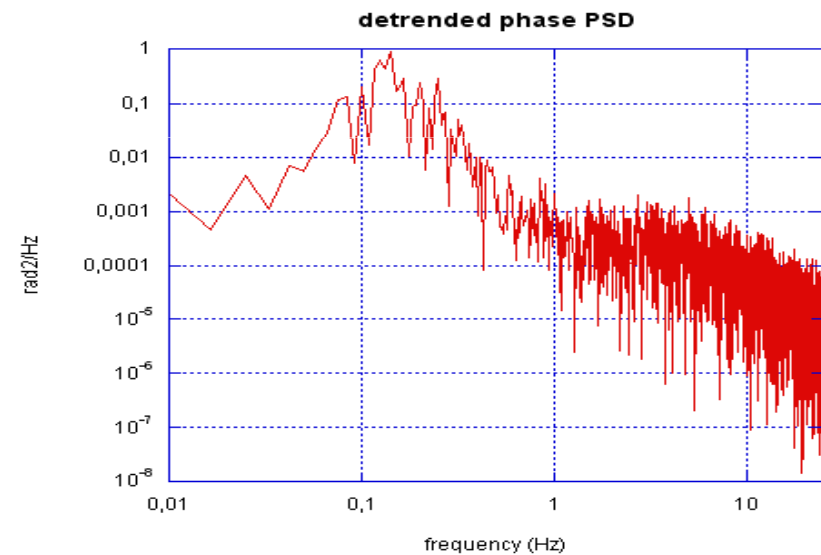
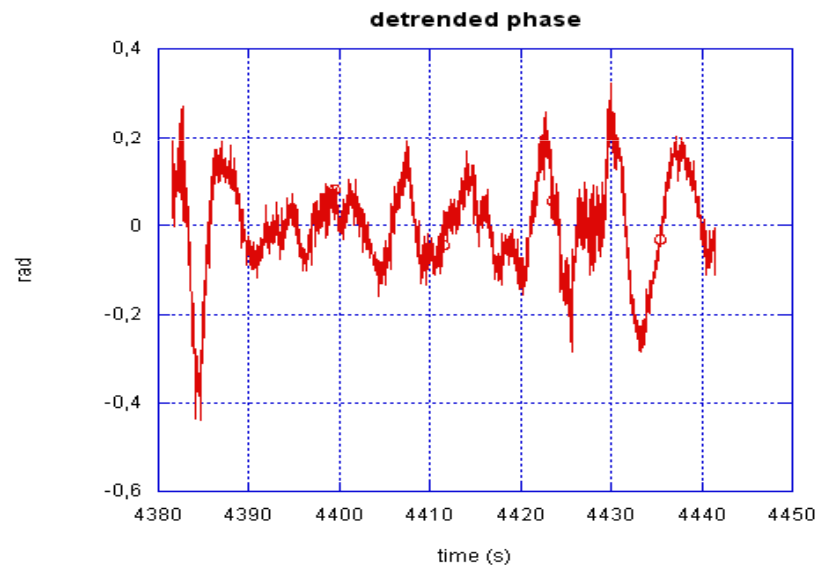
One Sample : Intensity

Sample characteristics : $S4 = 0.51$, $\sigma\phi = 0.11$



One Sample : Phase

Sample characteristics : $S4 = 0.51$, $\sigma \phi = 0.11$

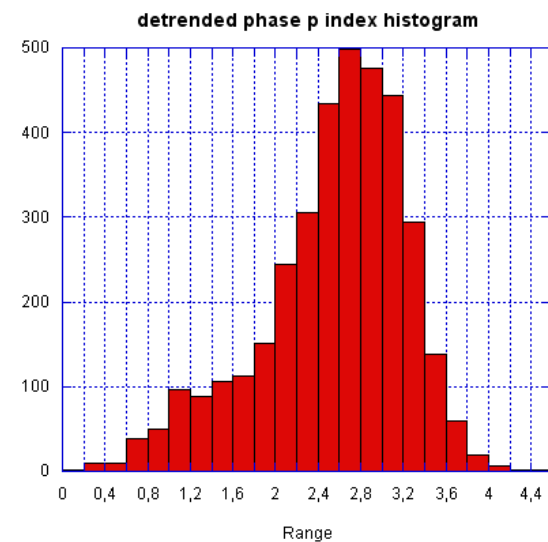
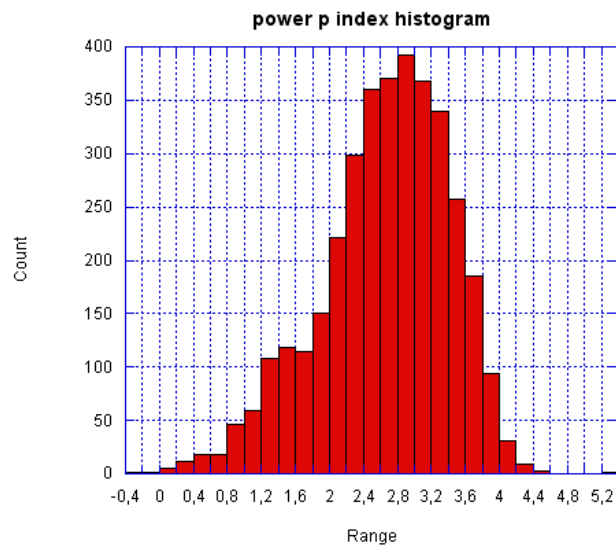


Spectrum Parameters

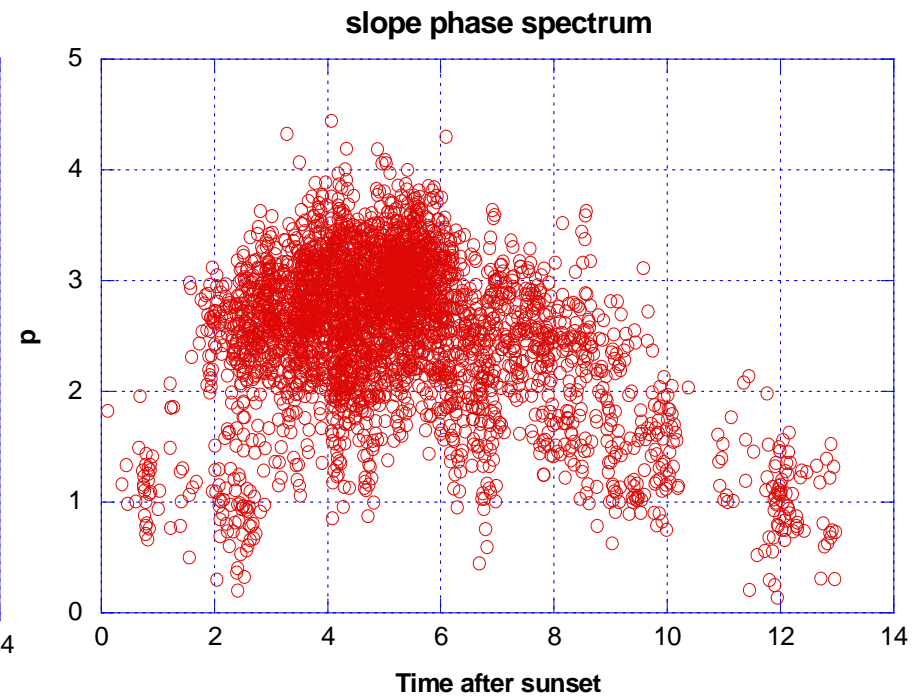
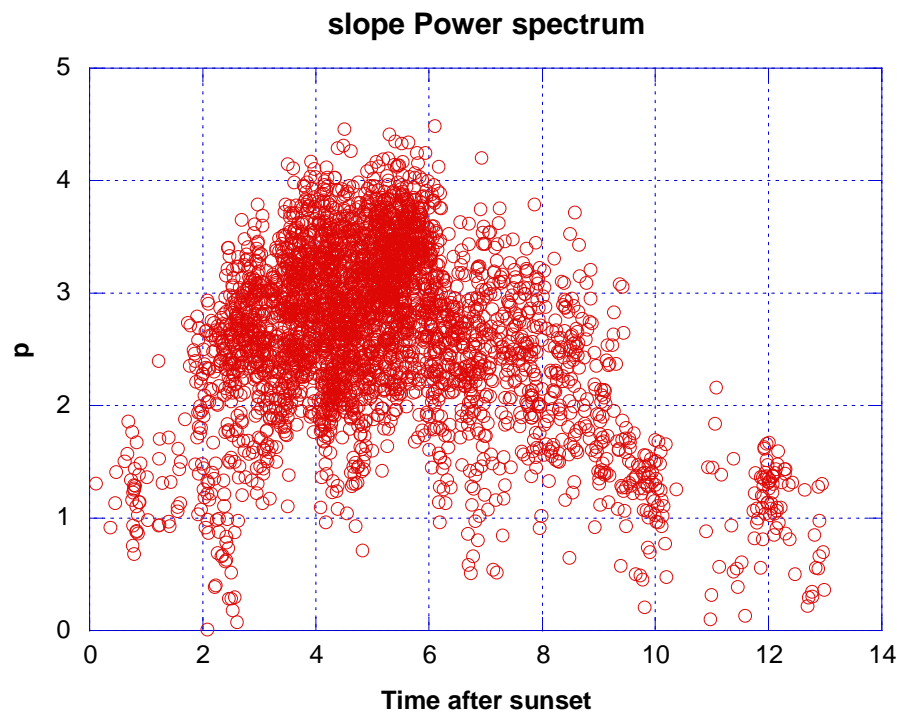
5 days RINEX files considered in the analysis

$S_4 > 0.2$ & $\sigma_{\phi} < 2$ (filter convergence)

2 parameters to define the spectrum : T (1 Hz value) & p



Slope spectrum vs time after sunset

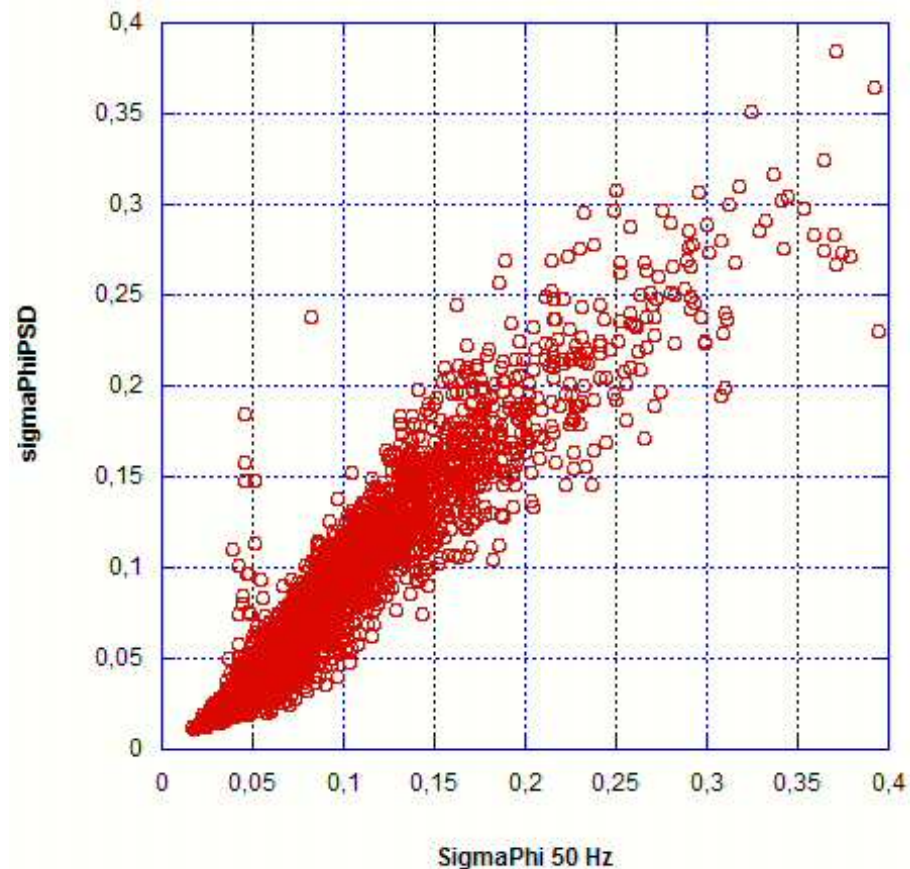


Phase variance

Time domain vs frequency domain

$$\sigma_{\phi}^2 = 2 \int_{f_c}^{\infty} PSD(f) df = 2 \int_{f_c}^{\infty} T f^{-p} df = 2T \left[\frac{f^{-p+1}}{-p+1} \right]_{f_c}^{\infty} = \frac{2T}{(p-1)f_c^{p-1}} \quad (\text{if } p > 1)$$

Slope set to 2.8



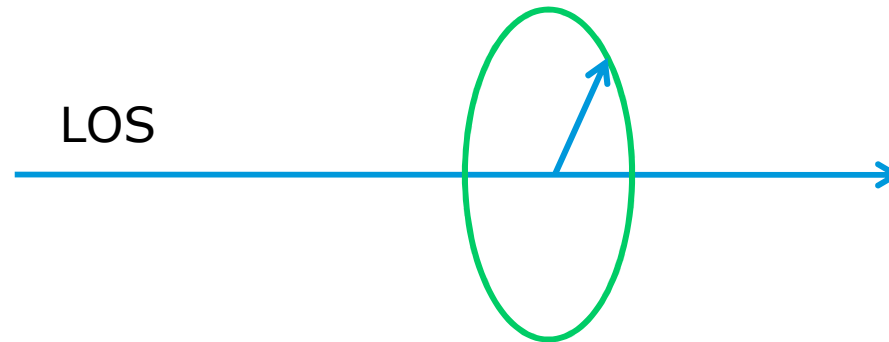
Medium Characterization (Correlation Function)

Isotropic

1D

Anisotropic

2D Analysis : Isotropic Medium



$$B_{\Phi}(\rho) = \frac{C_P}{(2\pi)^2} \iint \gamma_{\Phi}(\mathbf{K}) \exp(-j\bar{\mathbf{K}} \cdot \bar{\rho}) d\mathbf{K}$$

$$\rightarrow [B_{\Phi}(\rho)]_{\text{iso}} = \frac{\sigma_{\Phi}^2}{2^{(p-4)/2} \Gamma((p-2)/2)} (\rho q_0)^{((p-2)/2)} K_{((p-2)/2)}(\rho q_0)$$

$$\sigma_{\Phi}^2 = B_{\Phi}(0) = (\lambda r_e)^2 L L_0 \sigma_{\text{Ne}}^2$$

1D Analysis Isotropic Medium



$$B_{\Phi}(\rho) = \frac{C_P}{2\pi} \int \gamma_{\Phi}(k) \exp(-jk\rho) dk$$

$$[B_{\Phi}(\rho)]_{1D} = \frac{C_P}{2\pi} \frac{\sqrt{\pi}}{2^{(p-3)/2} \Gamma(p/2)} q_0^{1-p} (\rho q_0)^{(p-1)/2} K_{(p-1)/2}(\rho q_0)$$

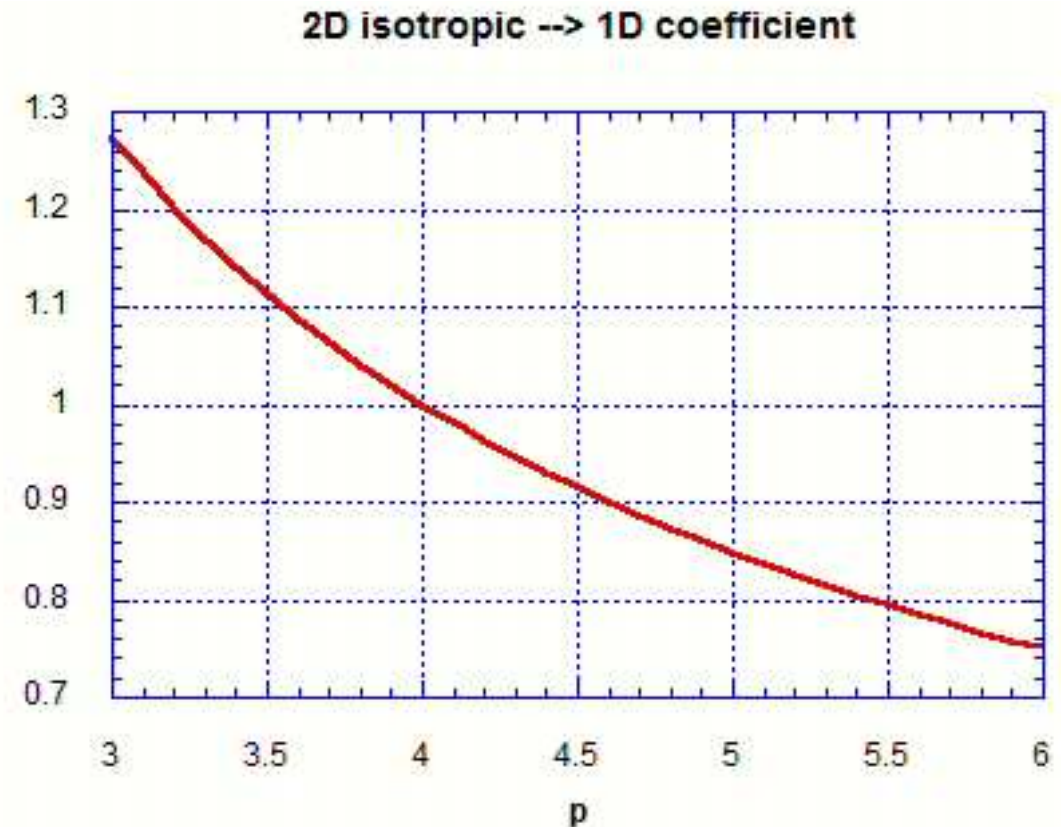
1D vs Isotropic

Slope

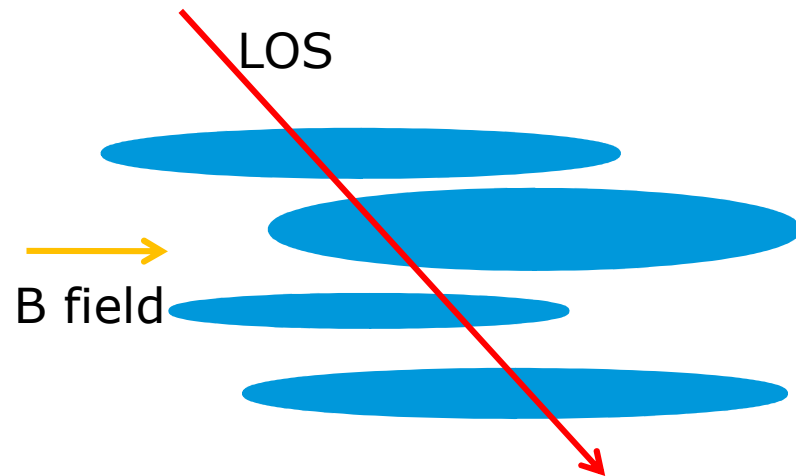
$$p \rightarrow p - 1$$

Multiplicative factor

$$\frac{2 \Gamma((p - 1)/2)}{\sqrt{\pi} \Gamma(p/2)}$$



Anisotropic vs Isotropic



$$\gamma_{\Phi}(\mathbf{K}) = \frac{(\lambda r_e)^2 L C_S \sigma_{Ne}^2 ab}{(q^2 + q_0^2)^{p/2}}$$

$$\gamma_{\Phi}(\mathbf{K}) = \frac{ab C_P}{\left((AK_{x\perp}^2 + BK_{x\perp} q_{y\perp} + CK_{y\perp}^2)^2 + q_0^2 \right)^{p/2}}$$

Additional geometric factor with respect to the 2D case $G = \frac{ab}{(AC - B^2/4)^{1/2}}$

a, b ellipses axes

A, B, C trigonometric terms resulting from rotations related to variable changes

Phase Synthesis (1D)

$$\Phi(\rho) = \text{FFT}^{-1}(\text{FFT}(u) * \gamma_{\Phi}(k))$$

u random number with a uniform spectral density

Done at each successive layer

Numerical Constraints

Frequency band	Medium parameters ⁽¹⁾	Phase variance ⁽²⁾ $\sigma_{\Phi}^2 = (\lambda r_c)^2 \Delta z L_0 \sigma_{N_e}^2$	Aliasing ⁽³⁾ $L > \frac{z \lambda \sigma_{\Phi}}{L_0 \sqrt{2}}$	Propagation $\Delta z < \frac{2L \Delta x}{\lambda}$
P 450 MHz	$L_0 = 500\text{m.}$ $N_e = 10^{12} \text{el/m}^3$ RMS = 20 %	$\sigma_{\Phi}^2 = 1.41$ ($\sigma_{\Phi} = 1.19$)	$L > 282 \text{ m.}$ $z=3.10^6; \sigma_{\Phi}=1$	$\Delta z < 25\text{km}$ $L = 2500\text{m.}$ $\Delta x = 3.3\text{m.}$
L 1.5 GHz	$L_0 = 500\text{m.}$ $N_e = 10^{12} \text{el/m}^3$ RMS = 20 %	$\sigma_{\Phi}^2 = 0.12$ ($\sigma_{\Phi} = 0.36$)	$L > 85 \text{ m.}$ $z=3.10^6; \sigma_{\Phi}=1$	$\Delta z < 122\text{km}$ $L = 2500\text{m.}$ $\Delta x = 4.88 \text{ m.}$ (FFT :1024 pts)
S 2.5 GHz	$L_0 = 500\text{m.}$ $N_e = 10^{12} \text{el/m}^3$ RMS = 20 %	$\sigma_{\Phi}^2 = 0.05$ ($\sigma_{\Phi} = 0.22$)	$L > 51 \text{ m.}$ $z=3.10^6; \sigma_{\Phi}=1$	$\Delta z < 203\text{km}$ $L = 2500\text{m.}$ $\Delta x = 4.88 \text{ m.}$ (FFT :1024 pts)

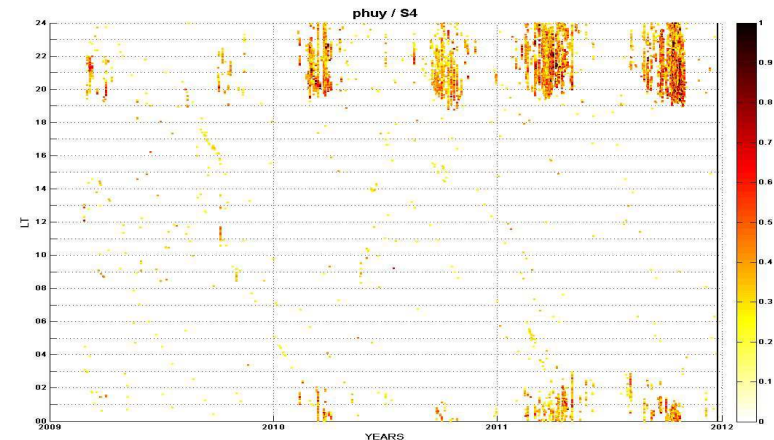
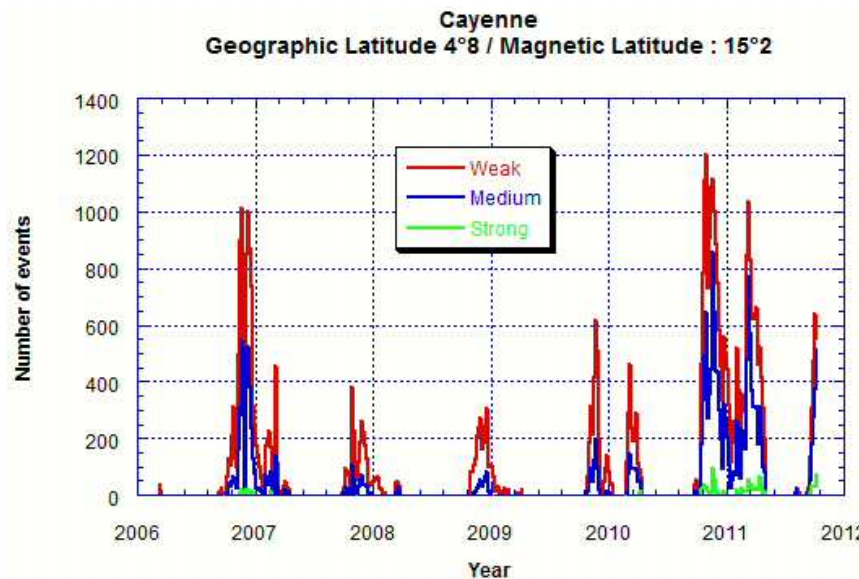
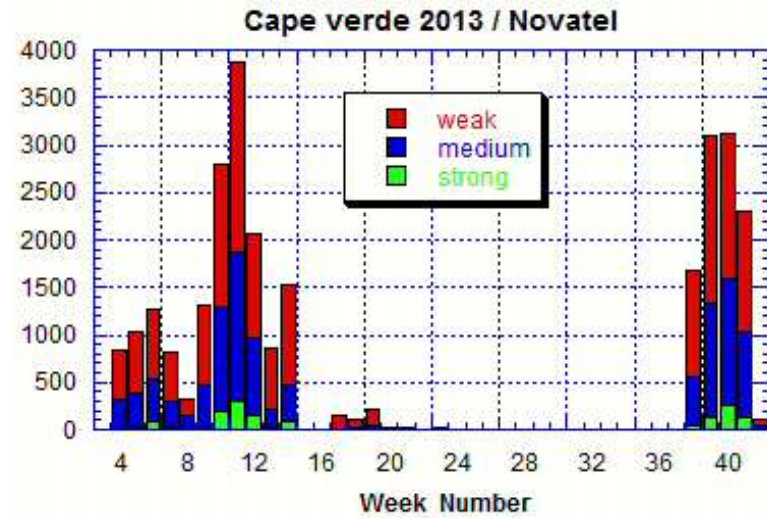
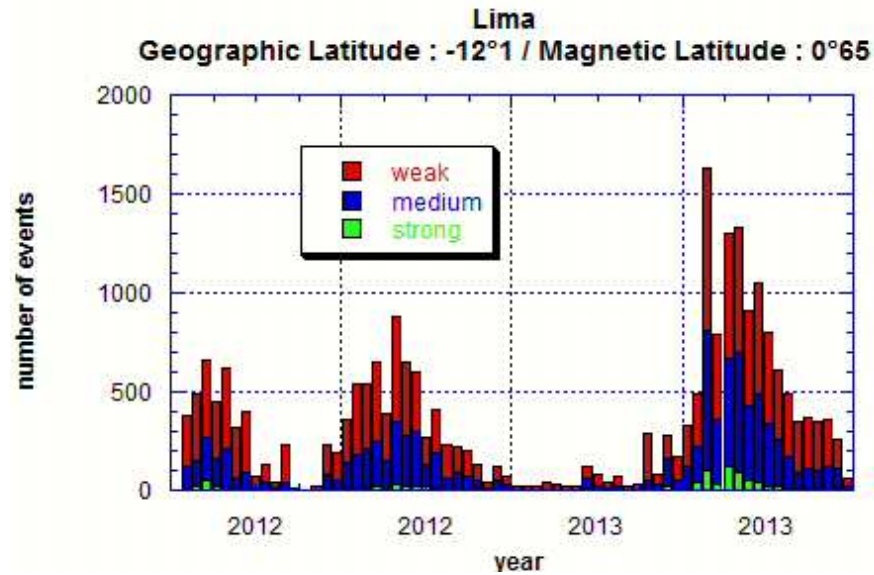
Sub Models

(1 / 2)

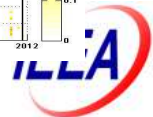
Seasonal Dependency

(Low Latitude Scintillations)

Scintillation Events Histograms

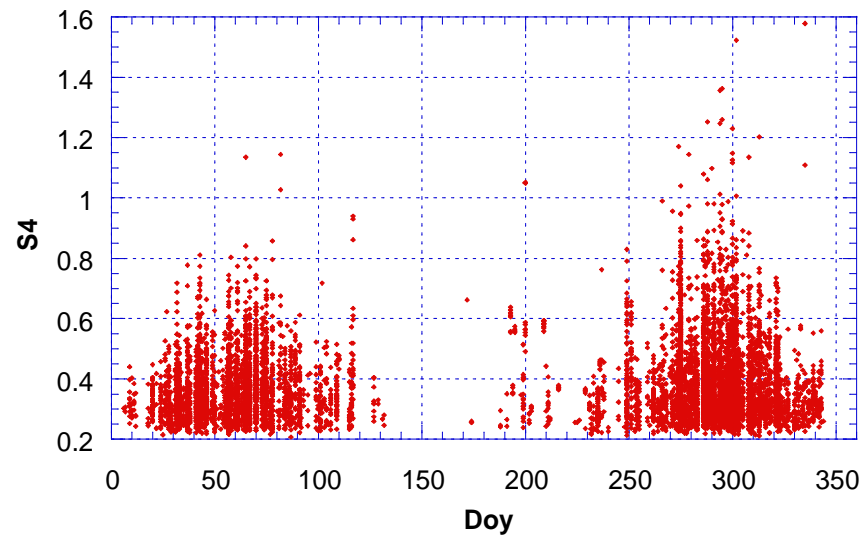


Rwanda - 30 june 2014 - 11 july 2014

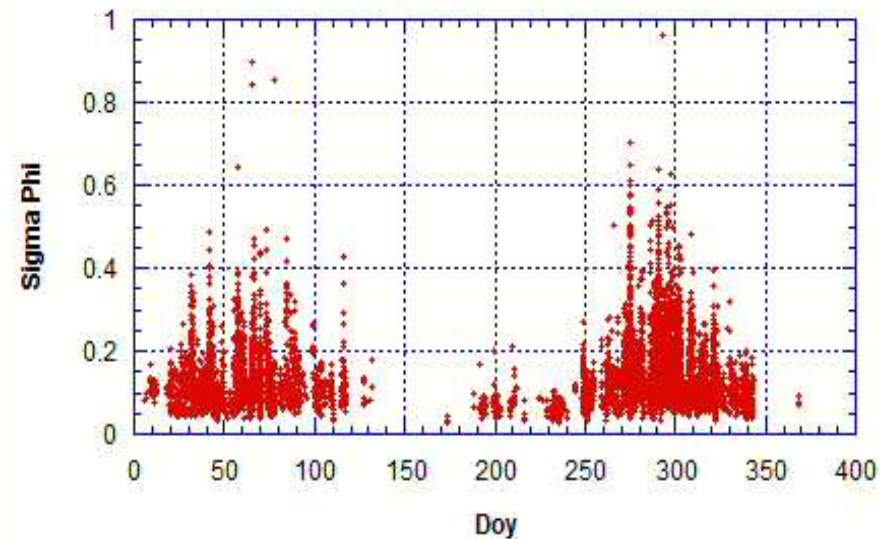


Scintillation Events / Lima 2012

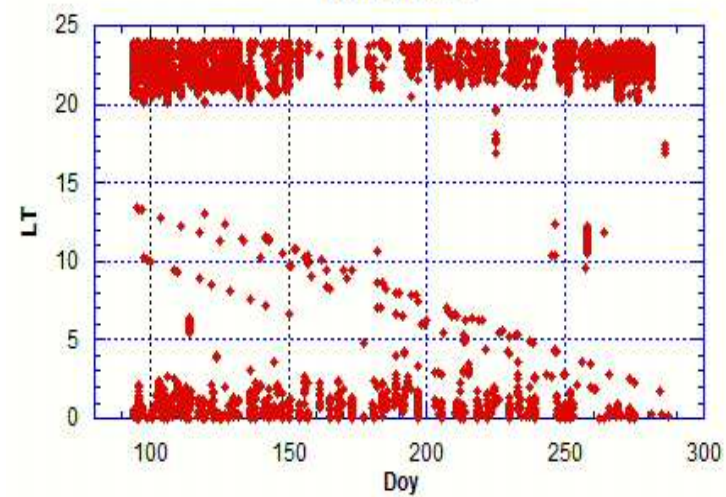
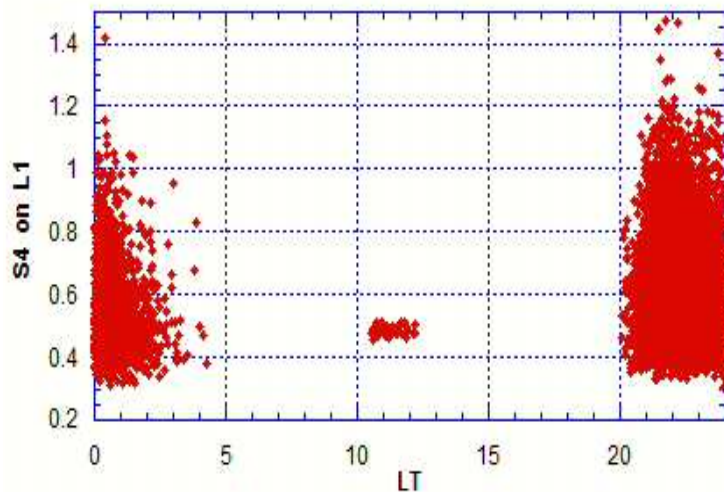
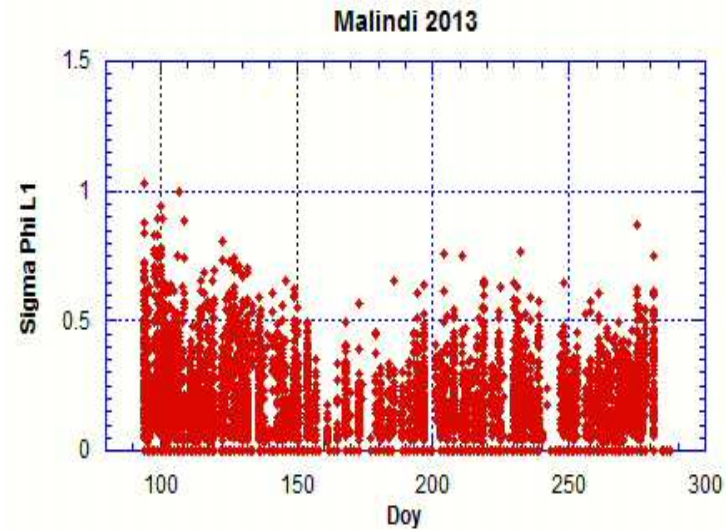
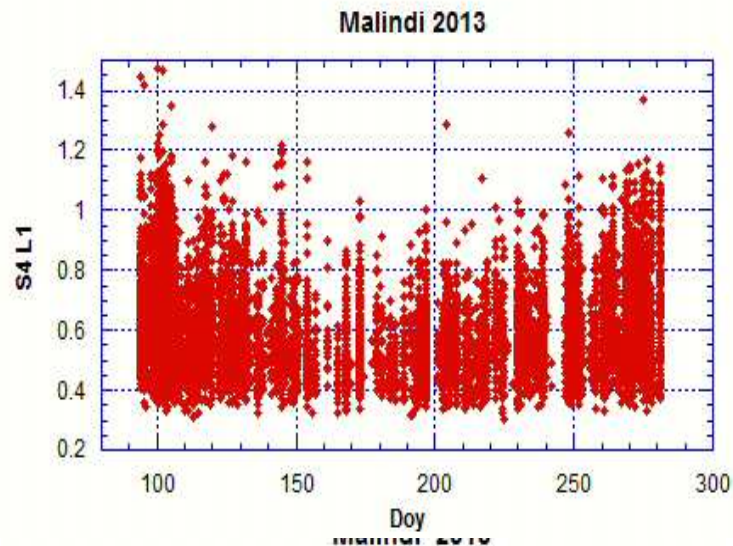
S4 vs Day of year 2012 Lima



Lima 2012 / Novatel

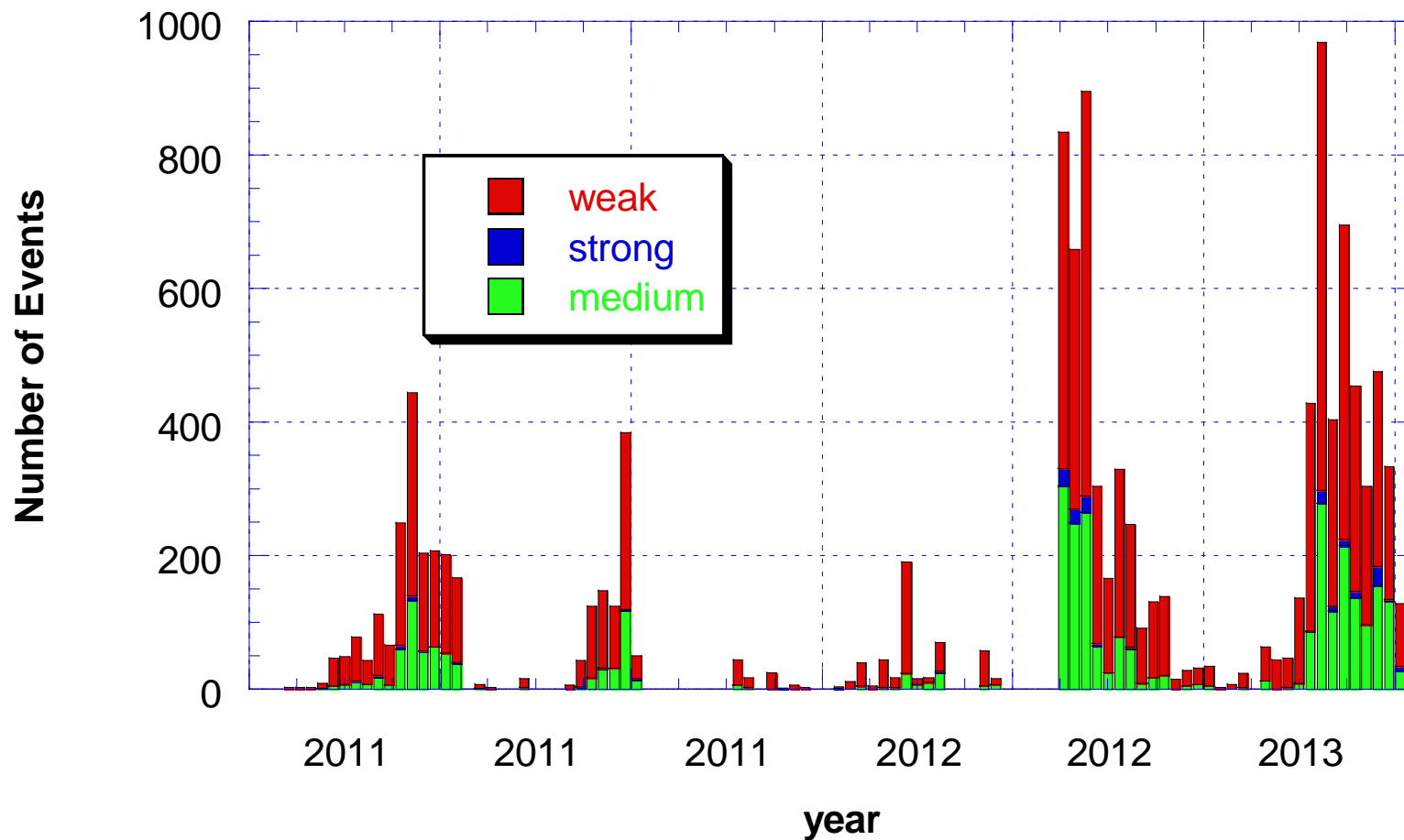


Measurements in Malindi, Kenya



Measurements in Burkina Fasso

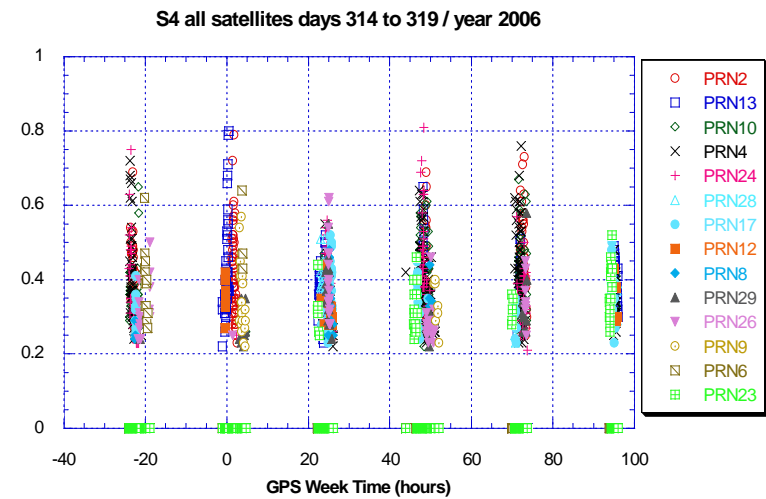
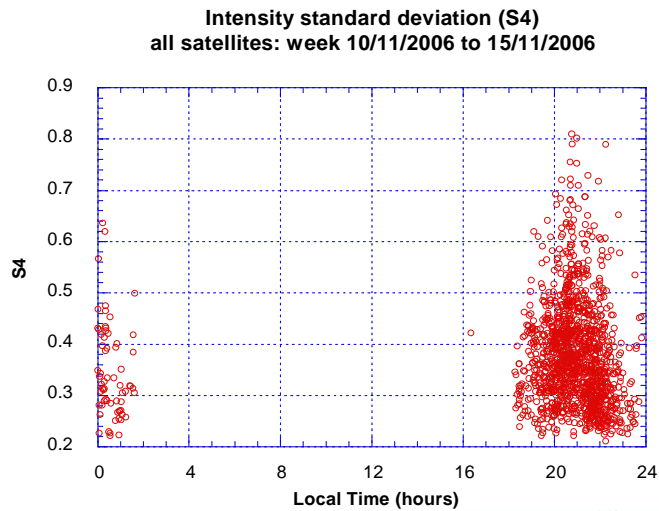
Koudougou (Burkina Fasso)
Geographic Latitude : $12^{\circ}15$ / Magnetic Latitude : $-1^{\circ}14$



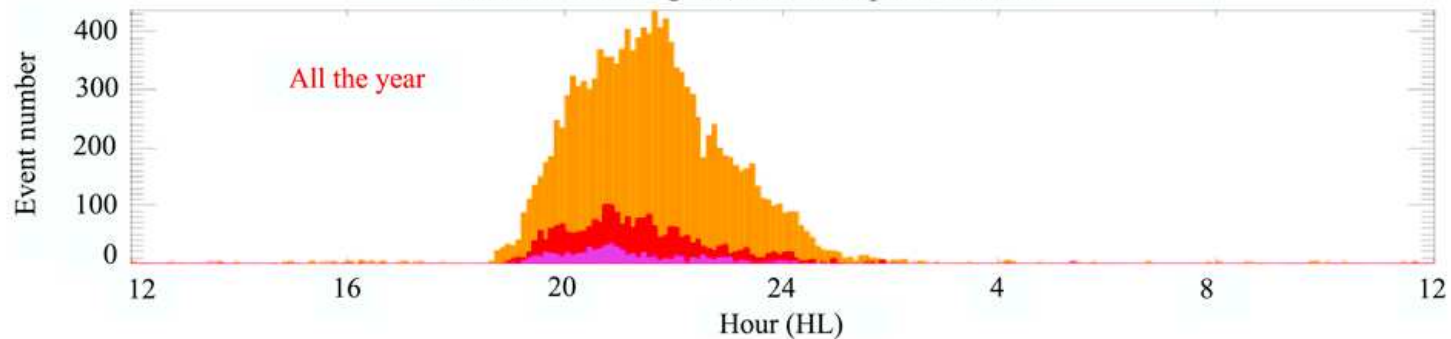
Sub Models (2 / 2)

Local Time Dependency (Low Latitude Scintillations)

One week of measurements in Guiana



Cayenne : Data from June 06 to July 07
S4 level : (0.25 - 0.4), (0.4, 0.55), (> 0.55)
Elevation > 20 degrees, After multipath correction



Local time : post sunset hours (CLS measurements, PRIS Campaign)

Checking Results

- Indices
- Inter frequency correlation
- Probability of intensity
- Fades distribution
- Loss of Lock

Medium Characterisation

Mean Effects (Sub Models)

NeQuick, Terrestrial Magnetic Field (NOAA)

Geophysical Parameters

SSN, Medium Drift Velocity

LT & Seasonal dependency

Scintillations (Fluctuating medium)

Spectrum slope (p), BubblesRMS, OuterScale (L_0)

Anisotropy ratio

Numerical Implementation

The model includes an orbit generator (GPS, Glonass, Galileo, ...)

Inputs

Medium Characterisation

Geophysical Parameters

Scenario

Intermediate calculation : LOS, Ionisation along the LOS

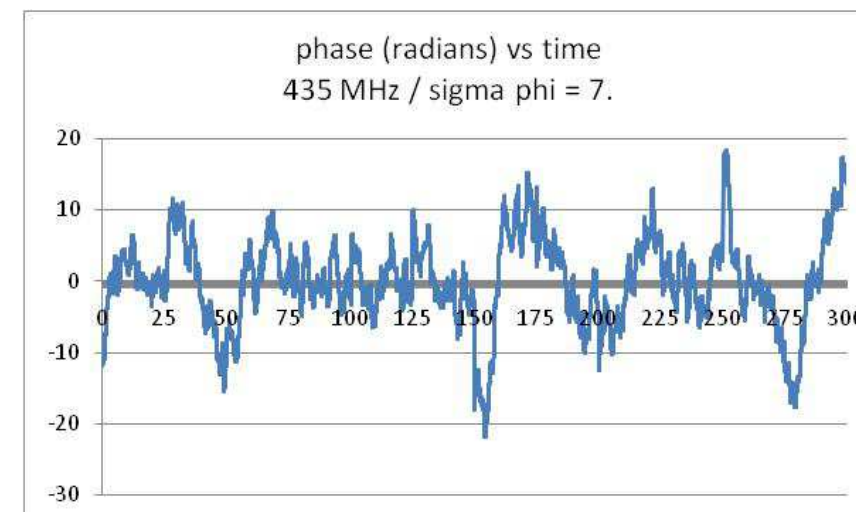
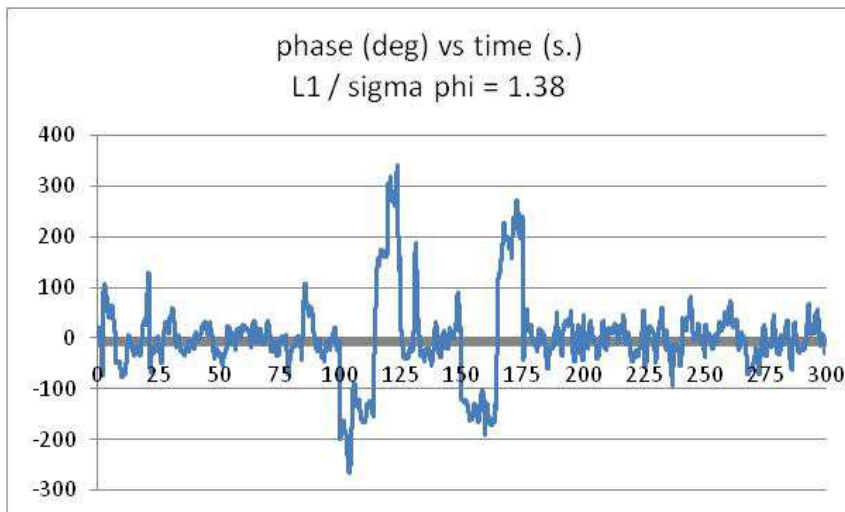
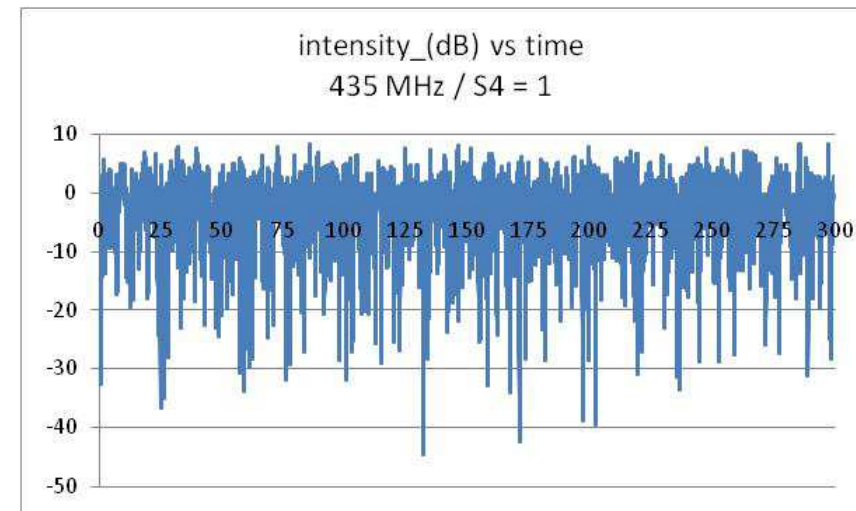
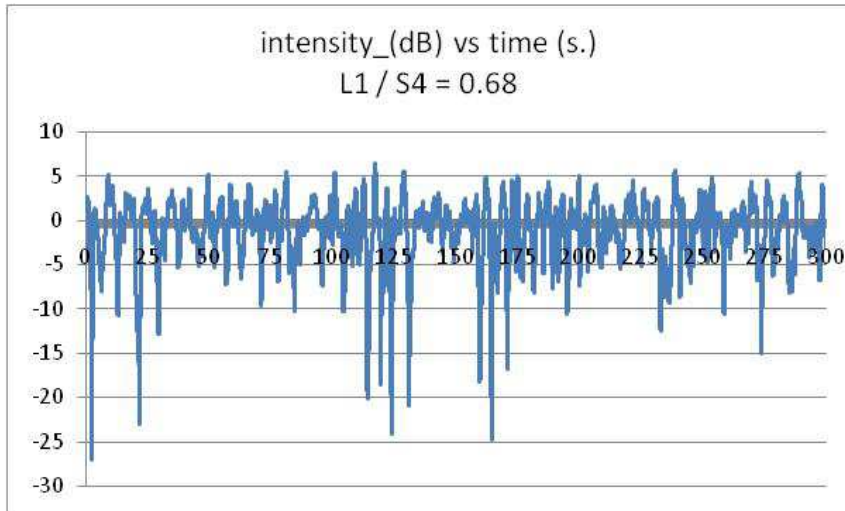
Outputs

Scintillation indices

Correlation Distances (Time & Space)

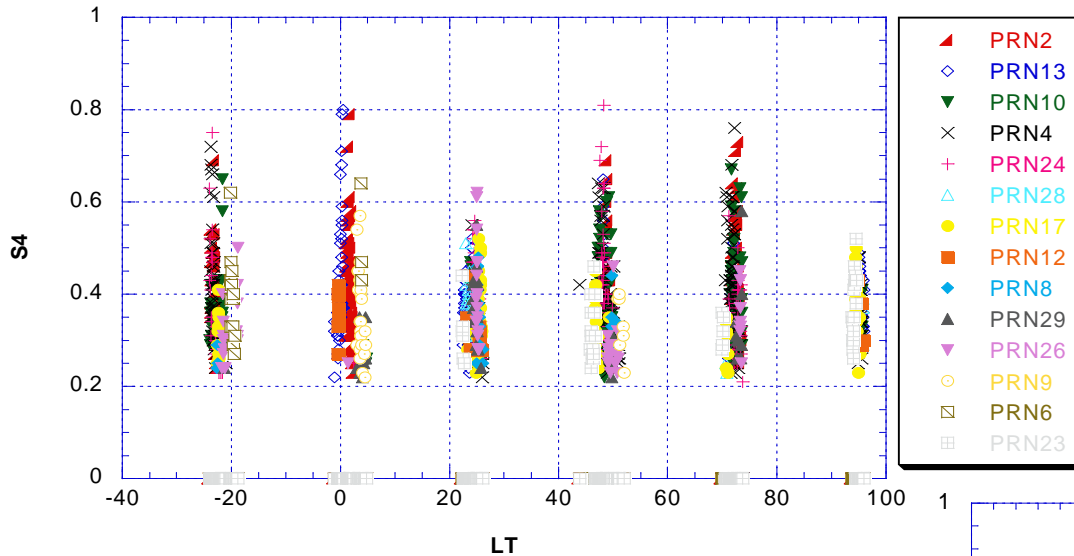
Scattering function

Signal at receiver level



Modelling vs Measurements (Intensity)

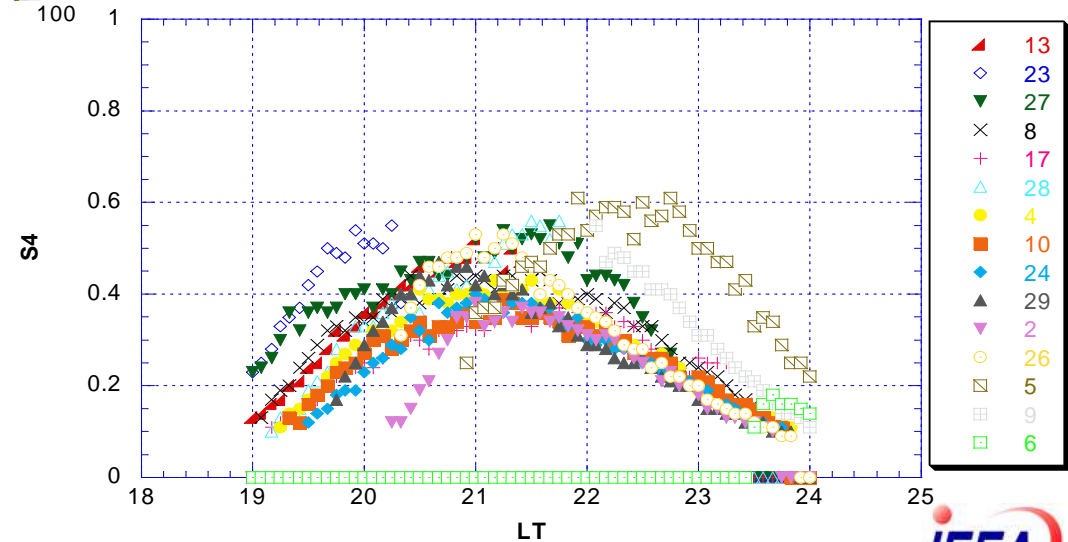
S4 all satellites
Cayenne days 314 to 319 : year 2006



← Measurements

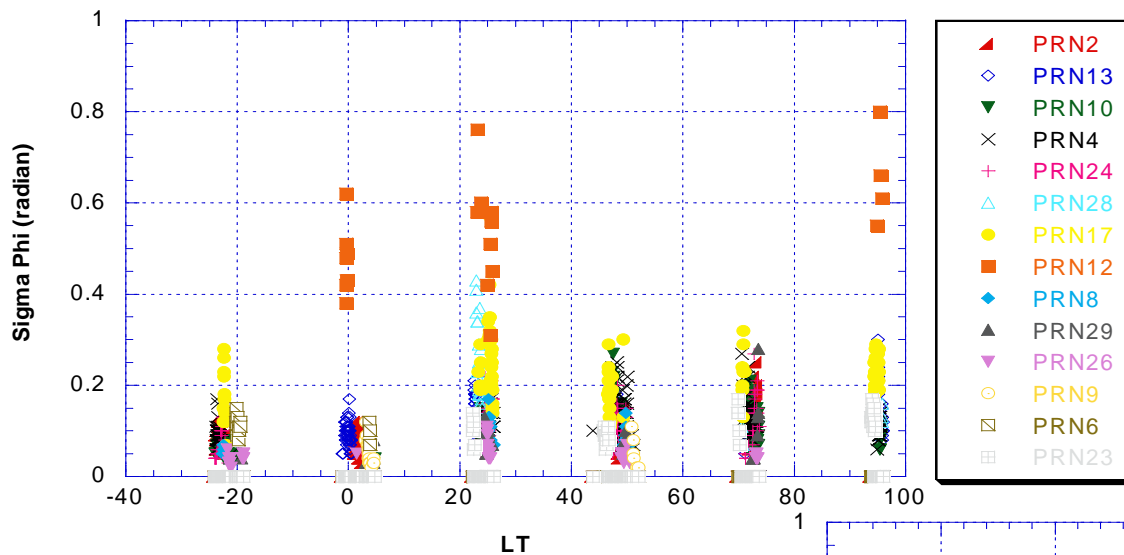
Modelling →

Cayenne day 314 / 2006
GISM



Modelling vs Measurements (Phase)

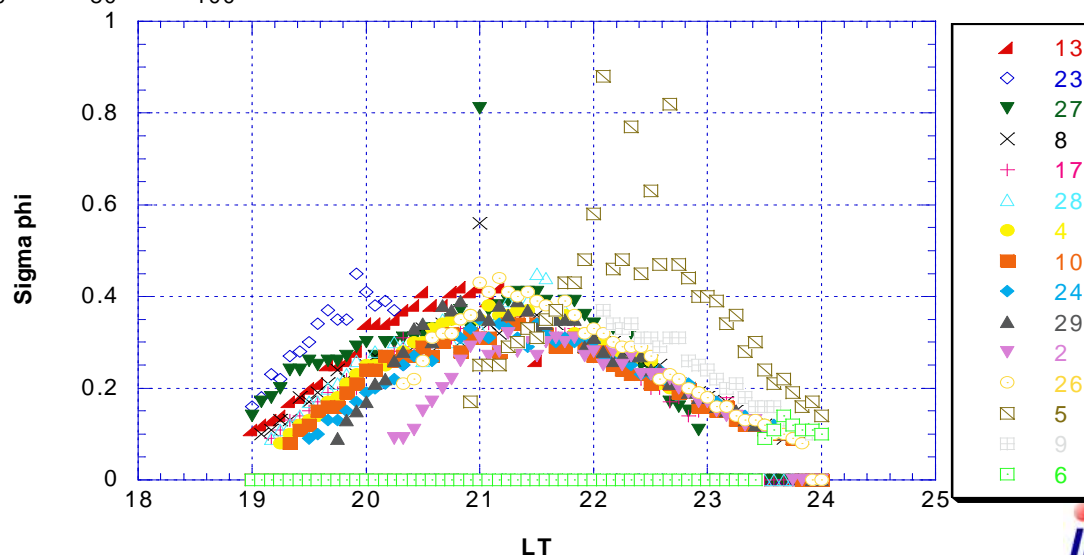
Sigma Phi all satellites
Cayenne, days 314 to 319 / year 2006



The phase RMS value is slightly lower than the S4 value

← **Measurements**

one day 314 / 2006
GISM

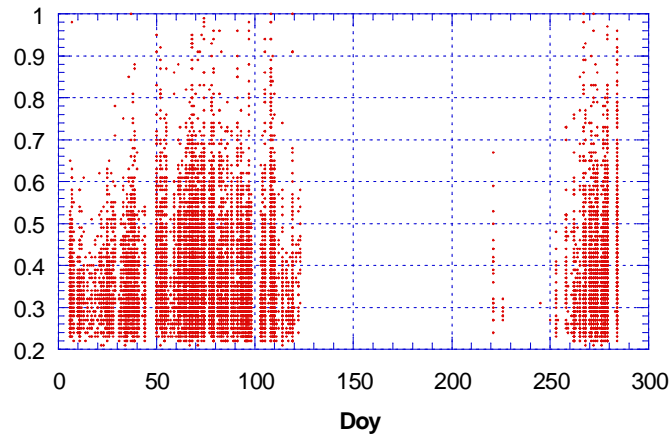


Modelling →

Some samples exhibit high values (both measurements and modelling) due to the phase jumps

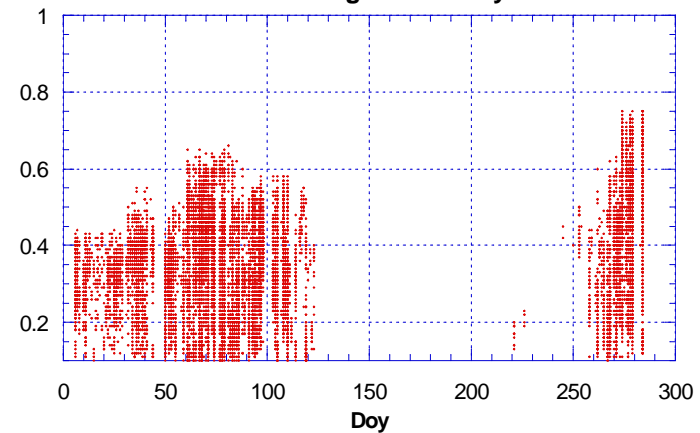
Scintillation Modelling vs Measurements

S4 Measurement in Cayenne
Latitude 4°8 Longitude 37°6 year 2011



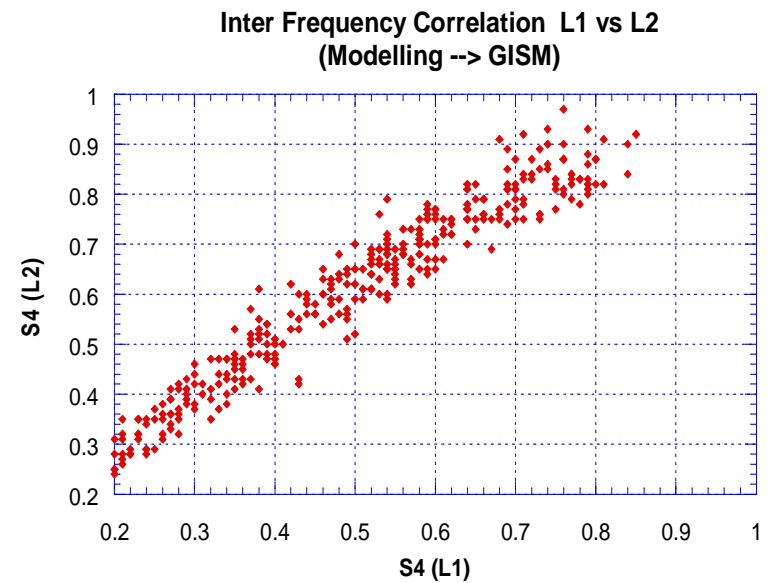
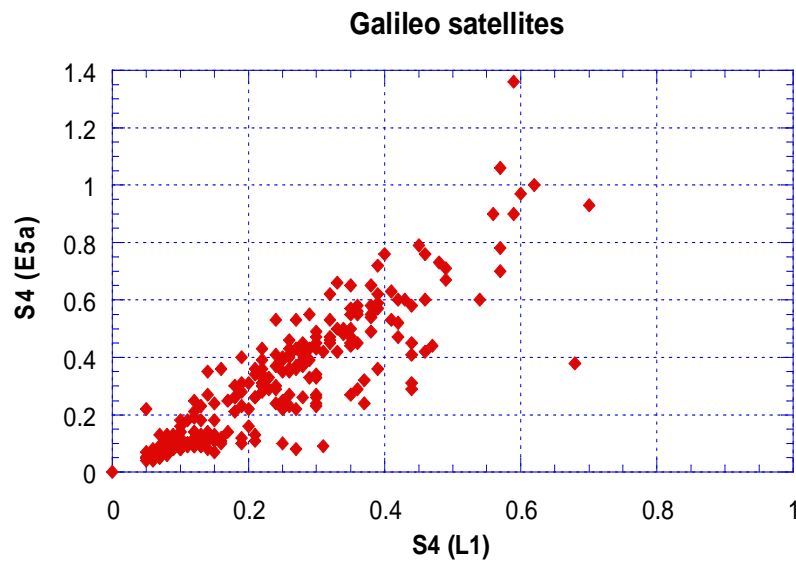
Measurements

S4 Calculated (GISM) in Cayenne
Latitude 4°8 Longitude 37°6 year 2011



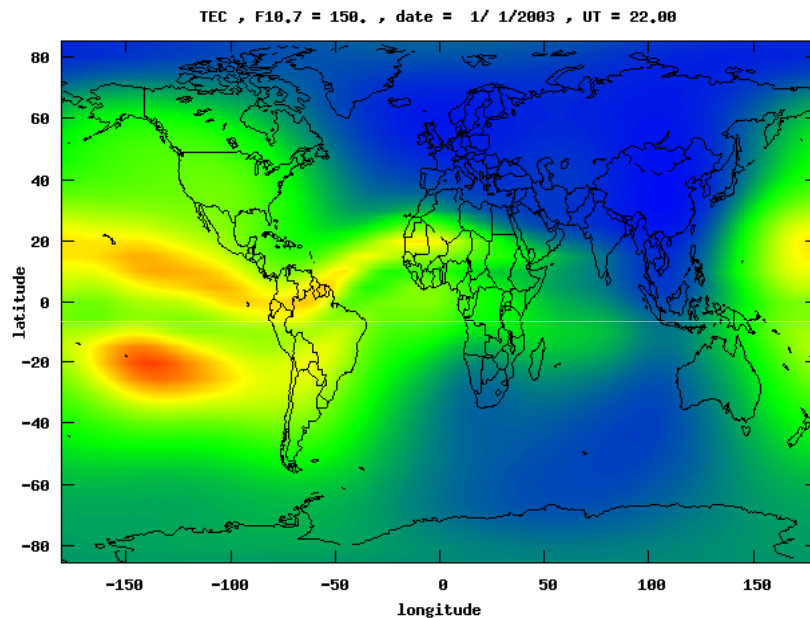
Modelling

Scintillation Index Dependency on Frequency

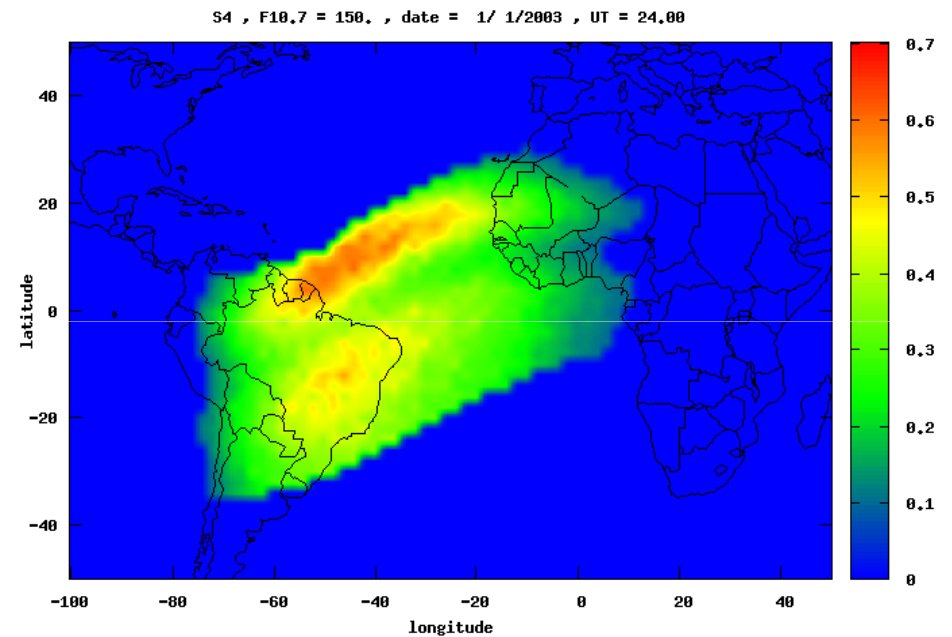


using Yuma files

Global Maps



TEC Map Modelling

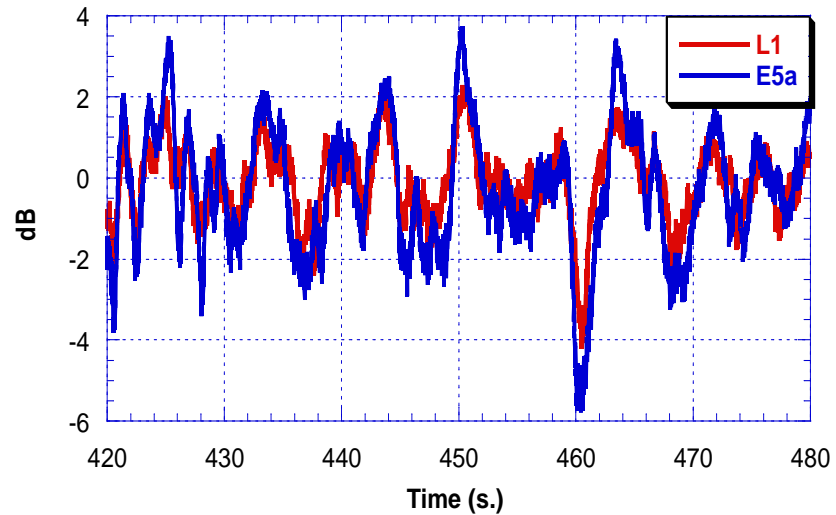


Scintillation Map Modelling

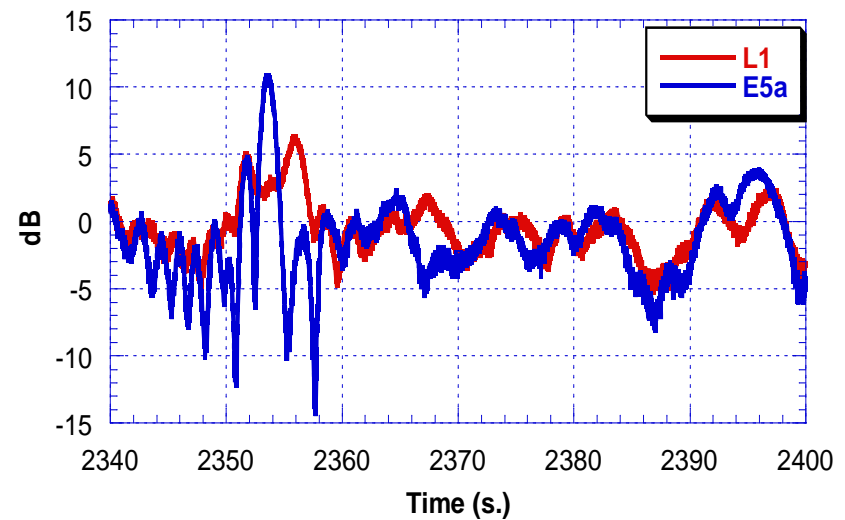
Inter Frequency Correlation

Weak scintillations vs strong scintillations

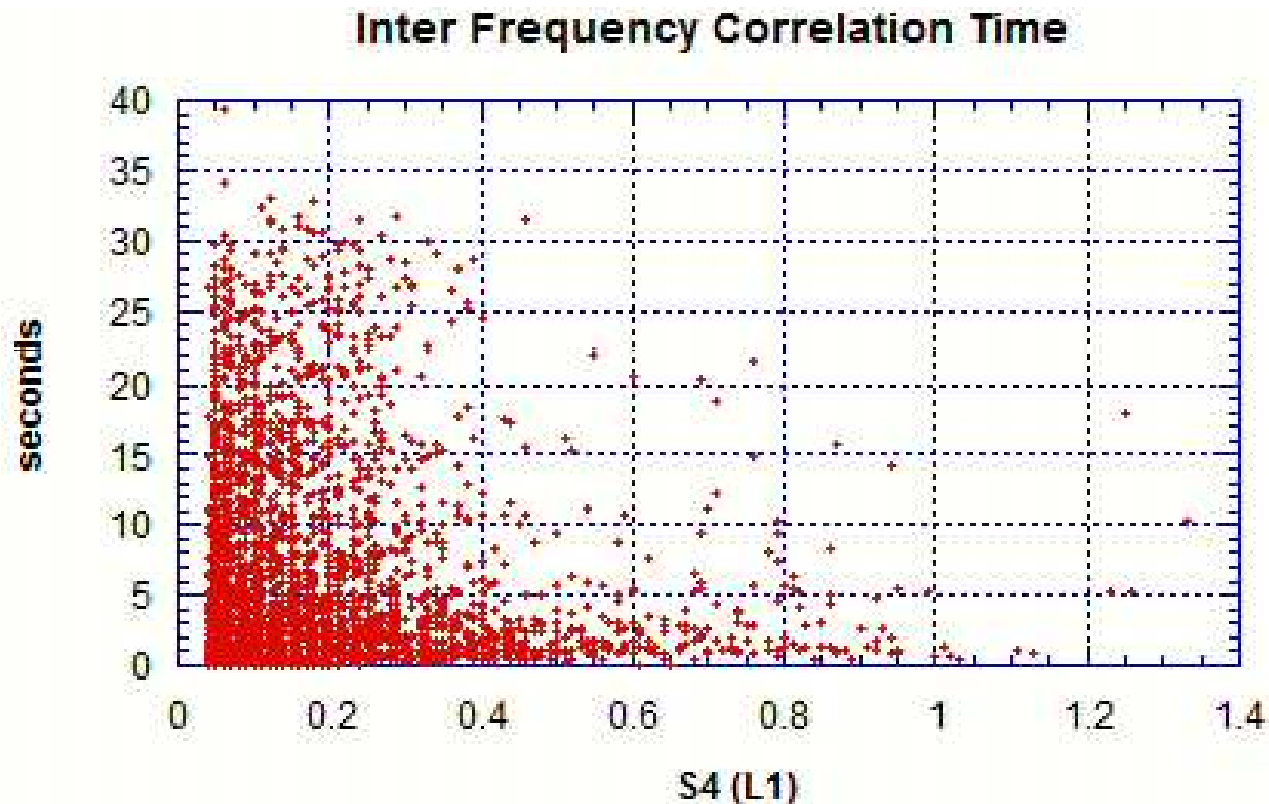
Tahiti Galileo N° 12 doy 85 / 2013
S4 (L1) = 0.21 ; S4 (E5a) = 0.36



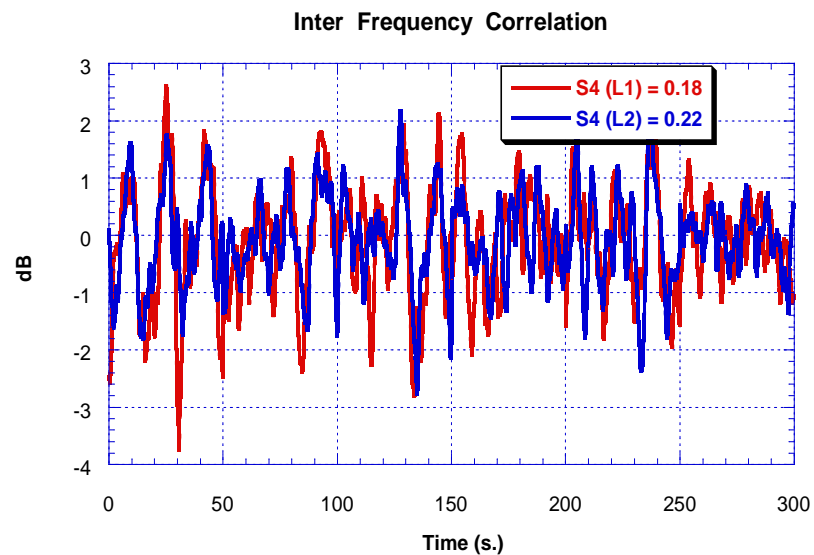
Tahiti Galileo N° 12 doy 85 / 2013
S4(L1) = 0.59 ; S4(E5a) = 1.36



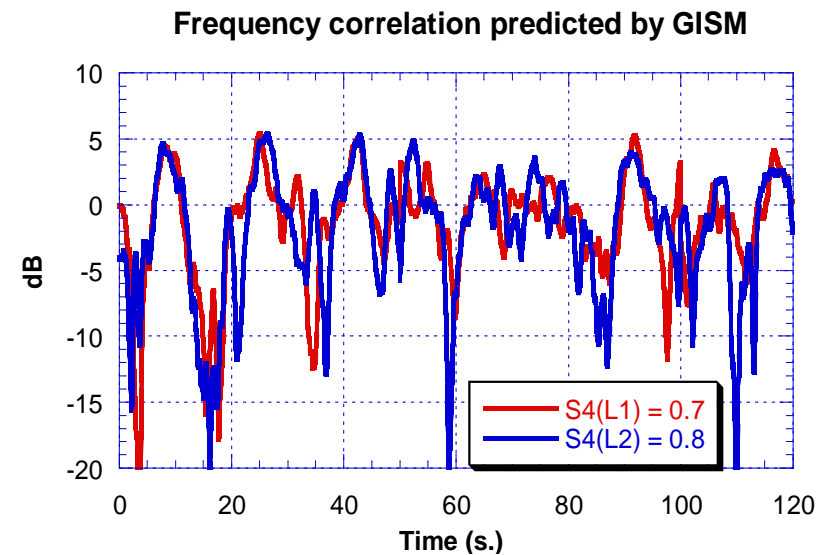
Inter Frequency Correlation Time Using 1 week of measurements in Tahiti



Frequency Correlation (Modelling)



Weak scintillations

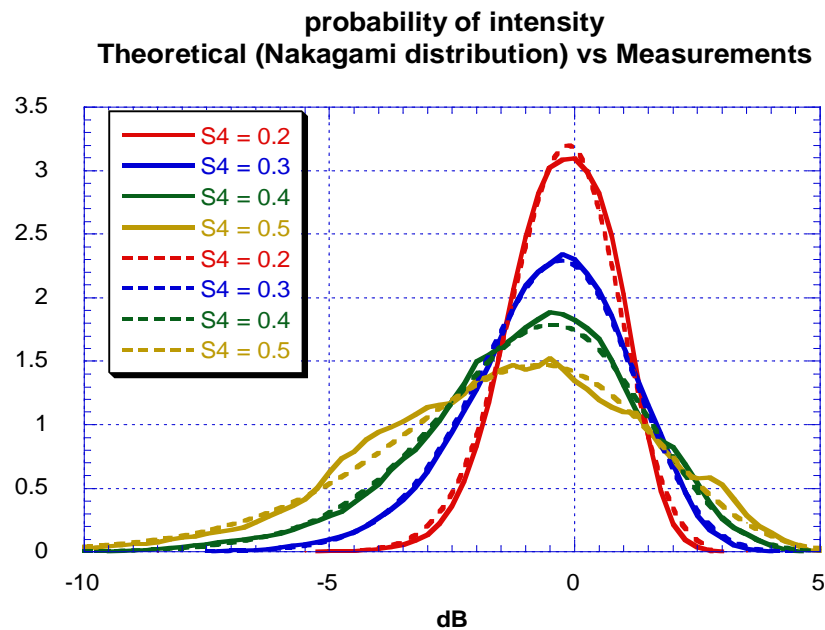


Strong scintillations

Loss of Lock

Probability of Loss of Lock

Loss of Lock when $\sigma_\phi >$ threshold value



The phase noise is related to the Intensity of the received signal

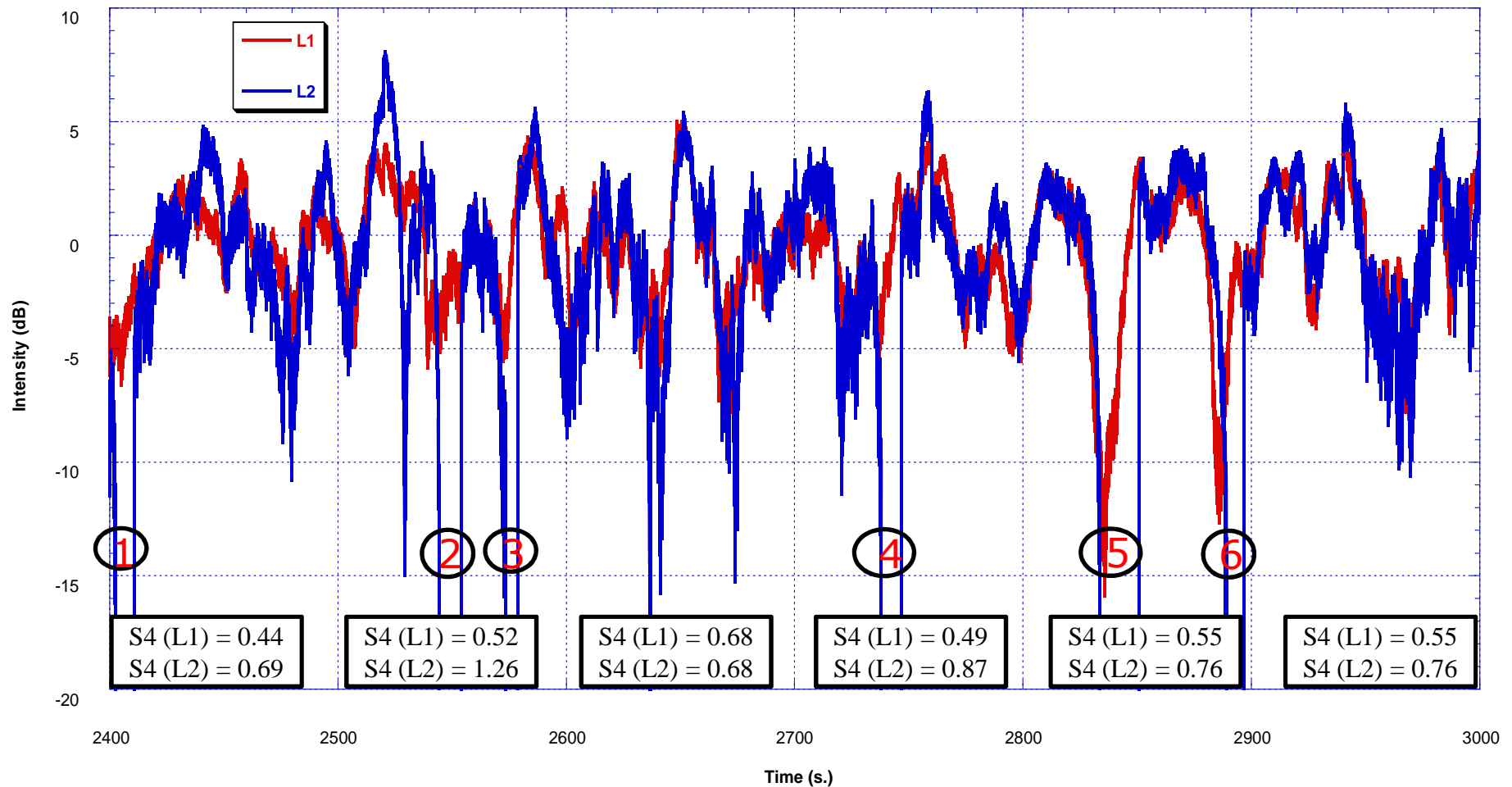
$$\sigma_{\Phi T}^2 = \frac{B_n}{(c/n_0) I} \left[1 + \frac{1}{2\eta (c/n_0) I} \right]$$

$p(I)$ Nakagami distributed

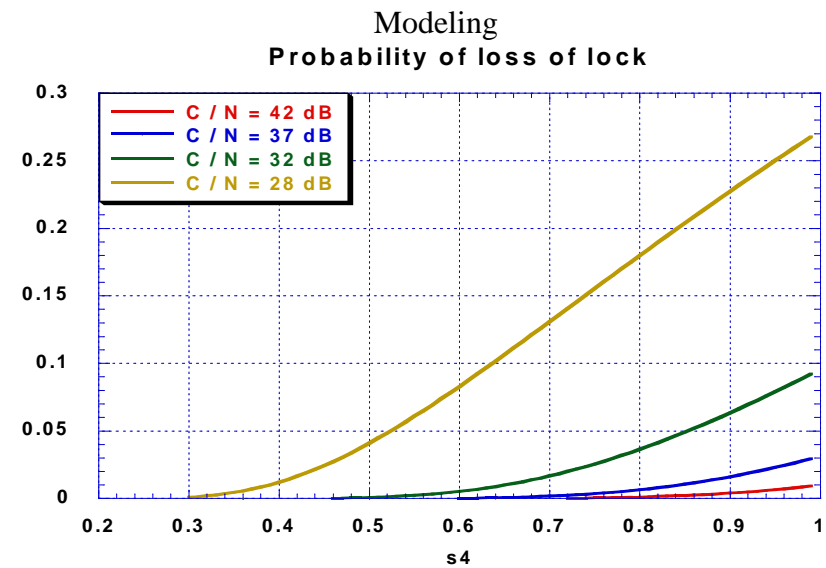
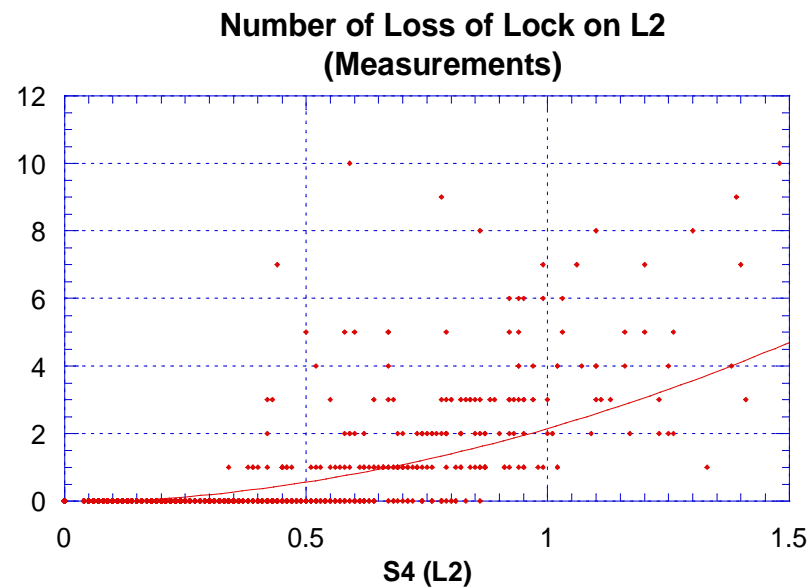
Probability of Loss of Lock is $p(\sigma_\phi) >$ threshold value

Loss of Lock (Measurements in Tahiti)

Loss of Locks L2 / strong scintillations



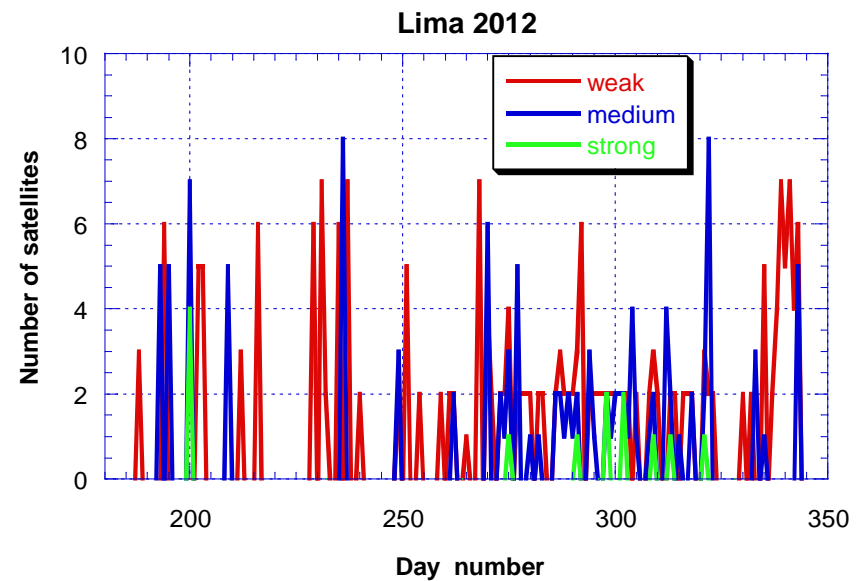
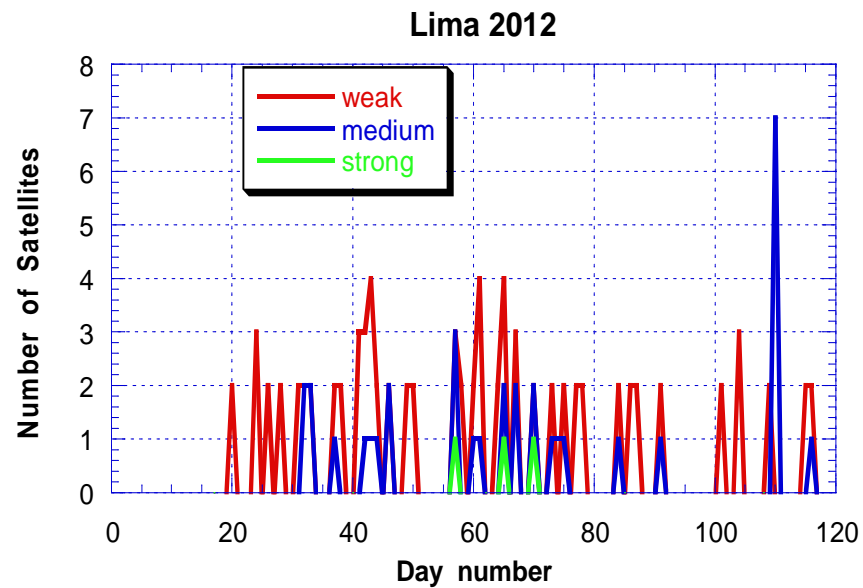
Loss of Lock Measurements vs Modelling



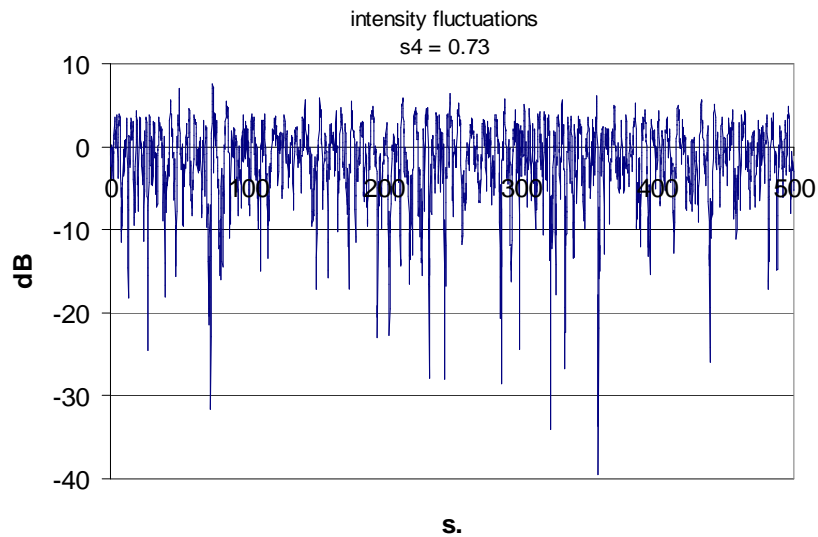
3 days of analysis in Tahiti

Geographical Extent Simultaneous Scintillation

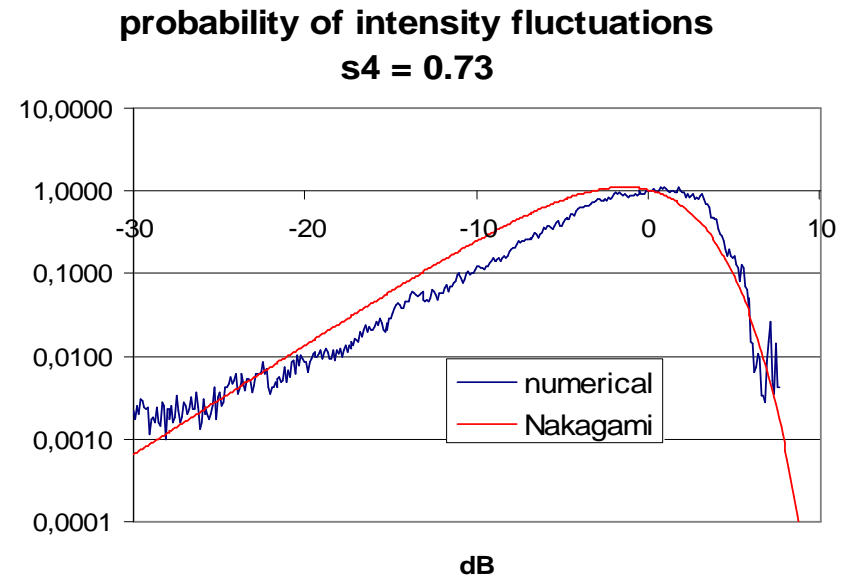
Number of Satellites Simultaneously Corrupted by Scintillation



Probability of intensity / Modelling



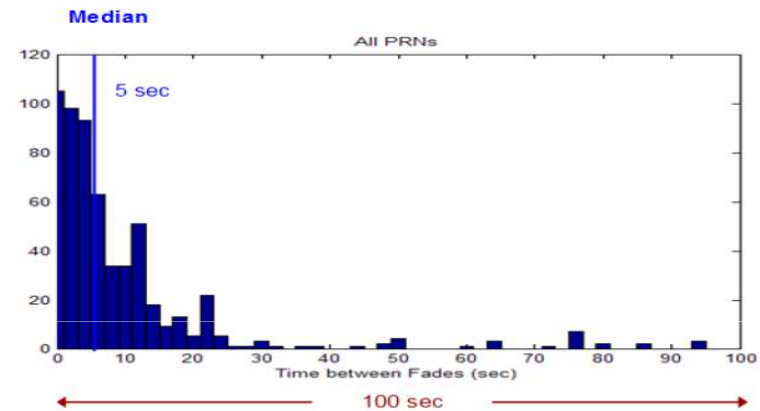
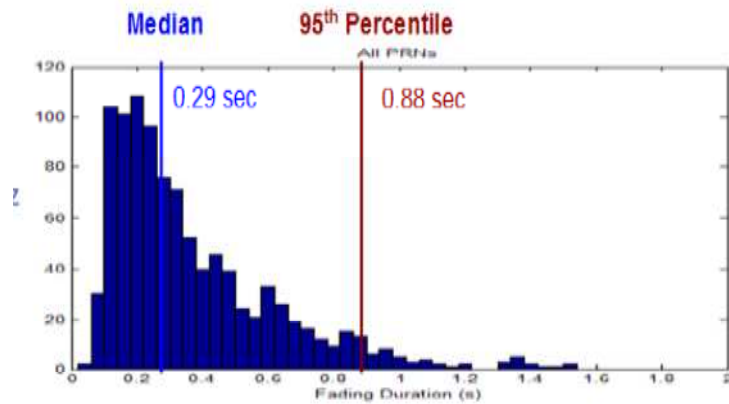
GISM output



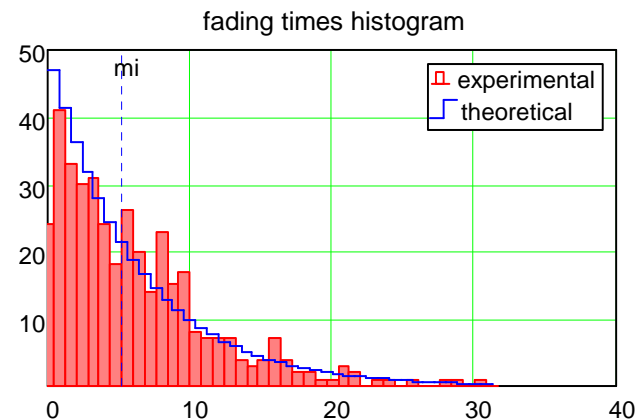
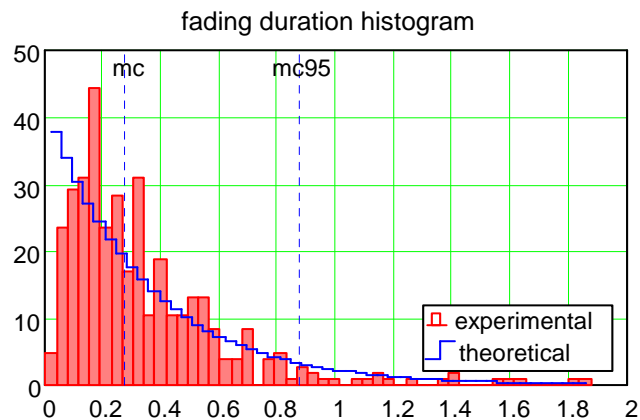
Nakagami law
$$p(A) = \frac{2 m^m A^{2m-1}}{\Gamma(m)} \exp(-mA^2) \quad \text{with} \quad m = 1 / S_4^2$$

Fades Statistics

Example of equatorial scintillation in Ascension Island, in solarmax conditions (2001)



1. Real data



2. GISM simulation

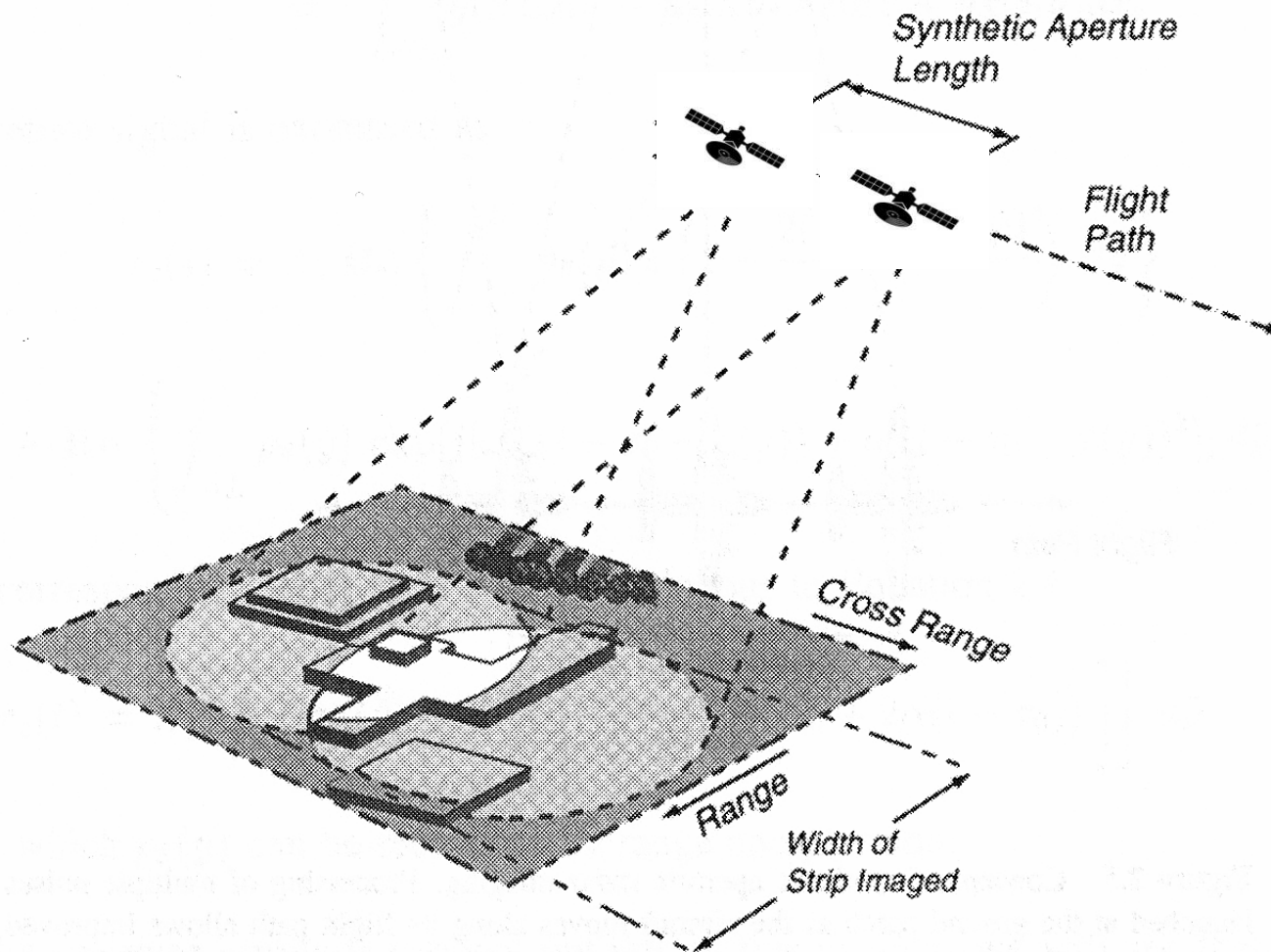
Probability of intensity / Modelling

Nakagami vs measurements

Radar Observations

Mutual Coherence Function

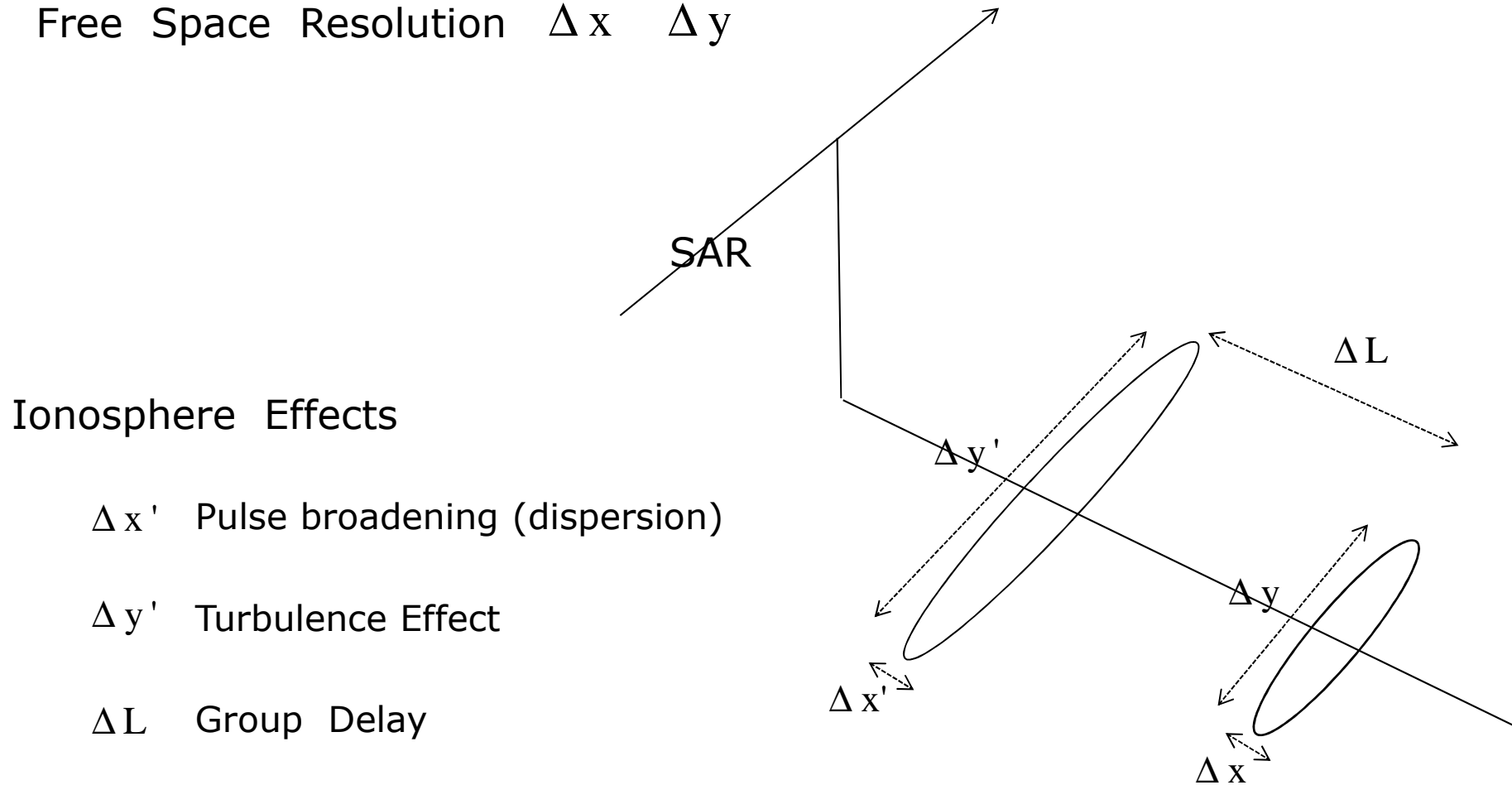
Correlation distance vs LSAR



L_e Synthetic Aperture Length \rightarrow 10 km

Ionosphere Effects

Free Space Resolution Δx Δy



Ionosphere Effects

$\Delta x'$ Pulse broadening (dispersion)

$\Delta y'$ Turbulence Effect

ΔL Group Delay

Two Points - Two Frequencies Coherence Function

$$\Gamma(z, k_1, k_2, \rho_1, \rho_2) = \langle U_1(z, k_1, \rho_1) U_2^*(z, k_2, \rho_2) \rangle$$

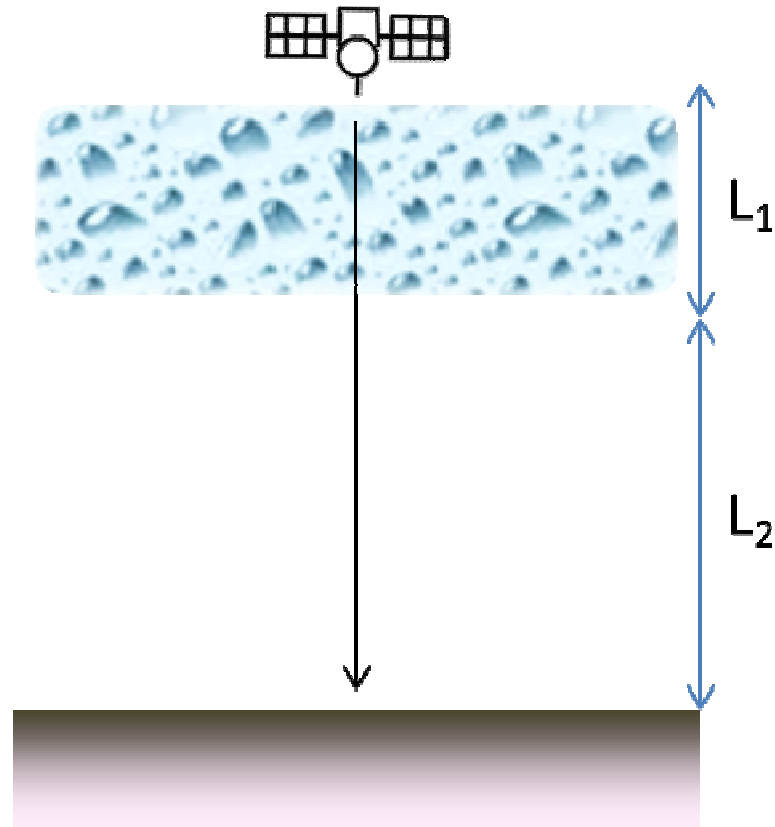
Using the parabolic equation

$$\left[\frac{\partial}{\partial z} - \frac{j}{2} \frac{k_d}{k_0^2} \nabla_d^2 + \frac{k_p^4}{8 k_0^2} \left[\frac{k_d^2}{k_0^2} A_\xi(0) + D_\xi(\rho) \right] \right] \Gamma(k_d, z, \rho) = 0$$

The structure function $D_\Phi(z, \rho) = 2[B_\Phi(0) - B_\Phi(\rho)]$ is quadratic with respect to the distance

Same process than previously : propagation 1st & 3rd terms ;
Diffraction : 2nd & 3rd terms

Two Points - Two Frequencies Coherence Function



➔ $\Gamma(z, k_1, k_2, \rho_1, \rho_2) = \langle U_1(z, k_1, \rho_1) U_2^*(z, k_2, \rho_2) \rangle$

Solution (one single screen)

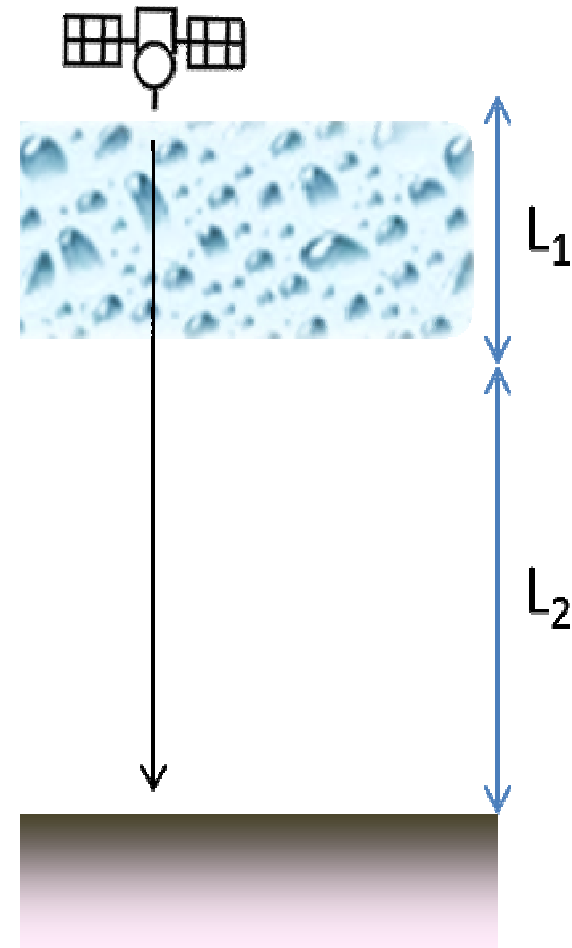
Scattering → 2 constants

$$B = \frac{\sigma_{\Phi}^2}{2\omega^2}$$

$$S = \sigma_{\Phi}^2 (L_1)^2 \frac{\text{Log}(L_0 / \ell_i)}{6L_0^2}$$

Propagation → 1 constant

$$P = \frac{1}{2ck^2} \left(\frac{1}{L_1 + L_2} - \frac{1}{L_1} \right)$$



* Nickisch, RS 92, Knepp & Nickisch, RS, 2010

African School on Space Science – Kigali, Rwanda – 30 june 2014 – 11 july 2014

One Single Screen

Analytical solution

$$\Gamma(\tau, K_x, z) = \frac{z}{2\sqrt{BS}} \exp\left(-\frac{z^2 K_x^2}{4S}\right) \exp\left(-\frac{(\tau + z^2 K_x^2 P)^2}{4B}\right)$$

The equation is annotated with two boxes and arrows. A red box surrounds the term $\exp\left(-\frac{z^2 K_x^2}{4S}\right)$, with a red arrow pointing upwards to the label K_x . A yellow box surrounds the term $\exp\left(-\frac{(\tau + z^2 K_x^2 P)^2}{4B}\right)$, with a yellow arrow pointing upwards to the label τ and K_x .

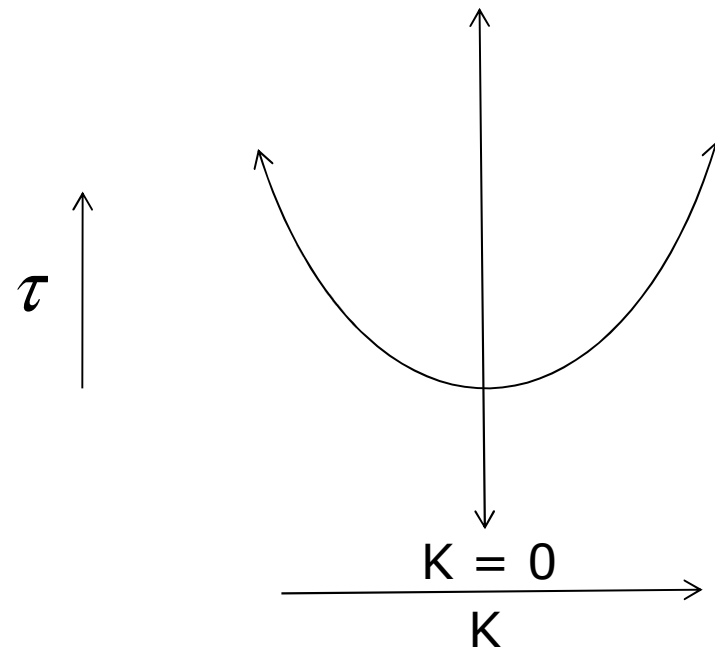
Spreading Extent

$$K = 0 \rightarrow \tau_1 = 2A\sqrt{B}$$

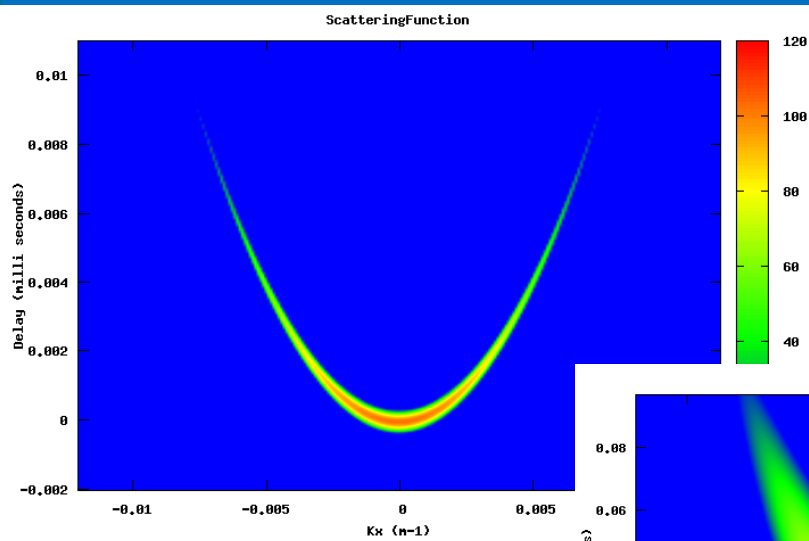
$$K = K_{\text{Max}} \rightarrow \tau_2 = 4SP A$$

Spread factor

$$Q = \frac{\tau_2}{\tau_1} = \frac{2\sqrt{A} S P}{\sqrt{B}}$$

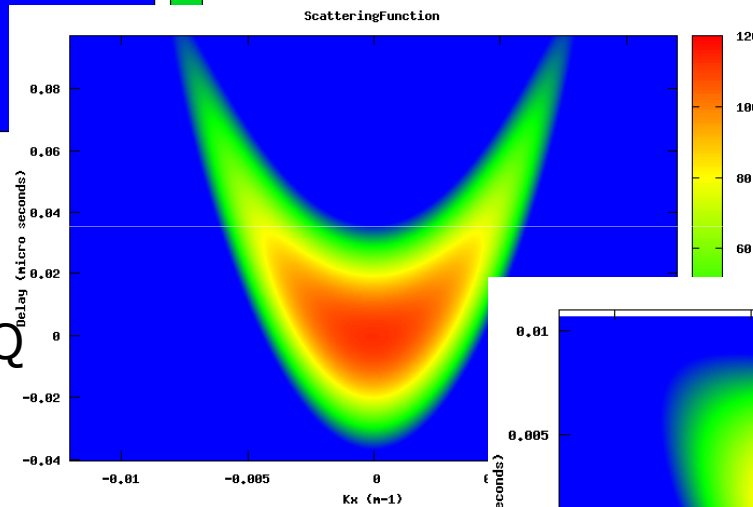


Spreading Extent

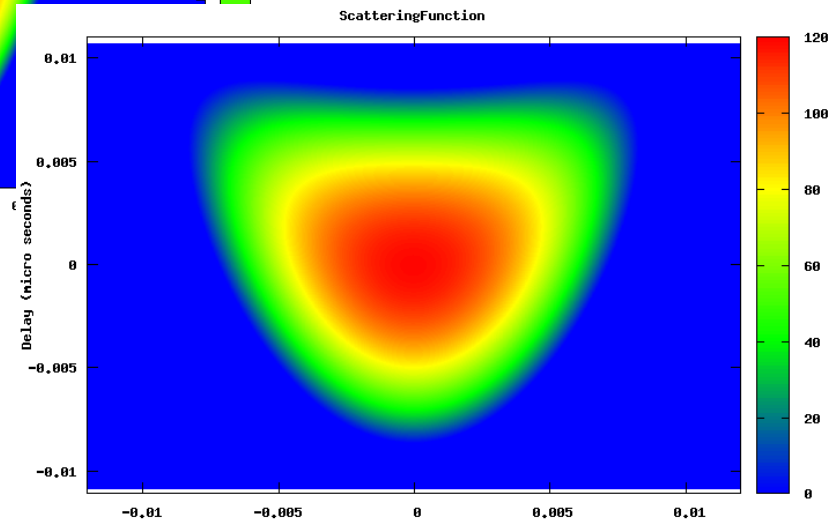


10 MHz
Very large value of Q

100 MHz
Large value of Q

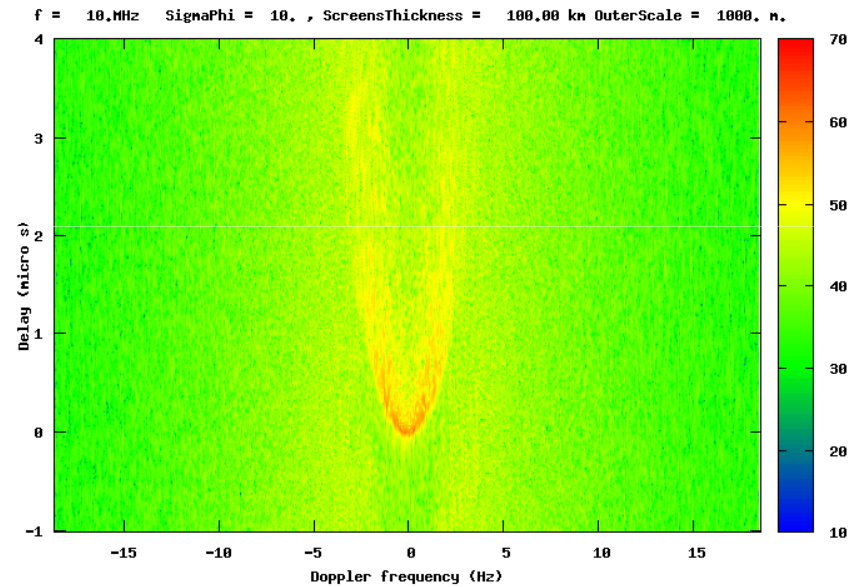
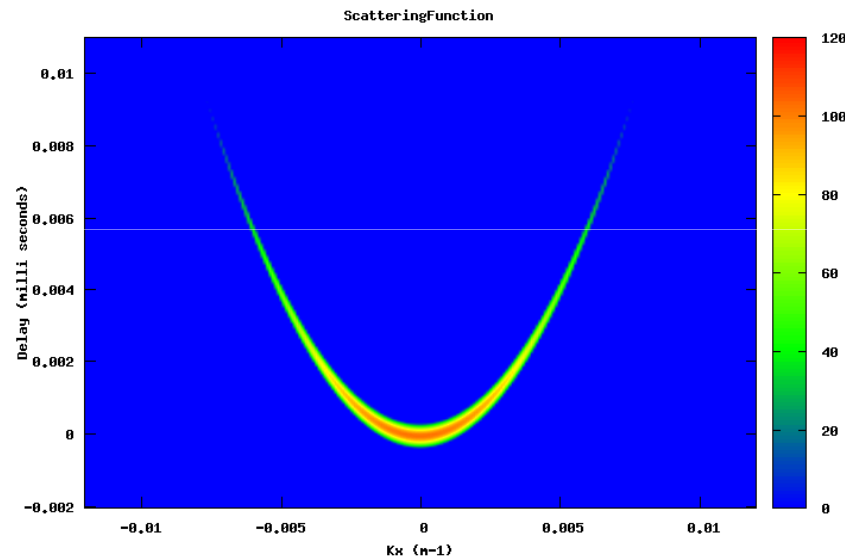


435 MHz
Q around 1



Rogers, N., P. Cannon, and K. Groves, Measurements and simulations of ionospheric scattering on VHF and UHF radar signals: Channel scattering function, Radio Sci., 44, RS0A33, DOI: 10.1029/2008RS004033, 2009.

Analytic vs Numerical



Several Screens Refined Analysis

- The algorithm can easily be generalized
- Different statistical properties may be assigned to the different layers
- Numerical FFT (1D) shall be performed to get the coherence function

Ambiguity Function

$$\chi(r, r_0) = \sum_n \int g_n(t, r_n) f_n^*(t, r_{0n}) dt$$

g_n is the received signal and f_n is the matched filter

$$\chi_n(r_n, r_{0n}) = \frac{1}{(4\pi r_n)^2} \exp(j\Phi_0) \int \exp(-j(\omega - \omega_0)\Phi_1 - (\omega - \omega_0)^2 \Phi_2) d\omega$$

Value of Coherent field received

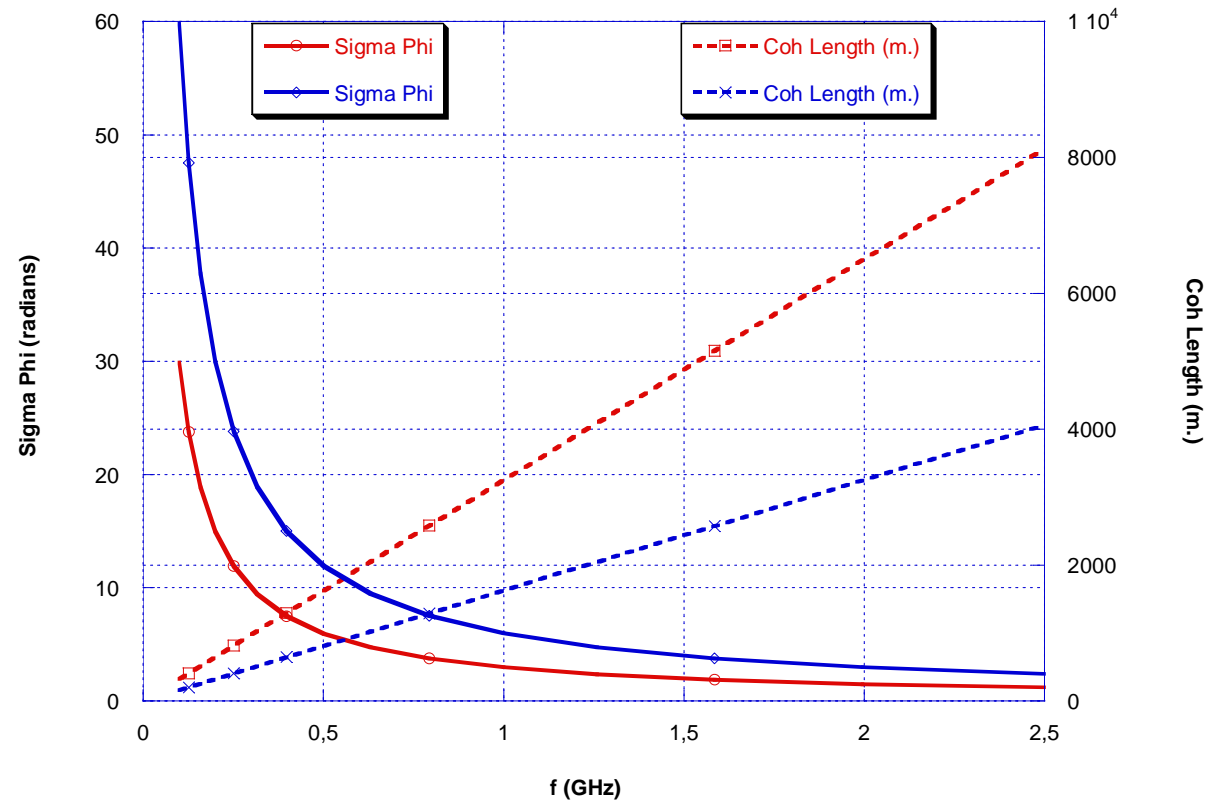
$$\langle \chi(r, r_0) \rangle = \frac{\exp(-\sigma_\Phi^2)}{(4\pi r_0)^2} \int_{-L_e/2}^{L_e/2} \exp\left(2jk_0\left(r_0 + \frac{\rho^2}{2r_0}\right)\right) d\rho = \frac{\exp(-\sigma_\Phi^2)}{(4\pi r_0)^2} \text{sinc}(k_0 \rho L_e / r_0)$$

An attenuation factor on the coherent component is included

Coherent Length

It is given by function $\Gamma(\omega_d, \rho, r) = \sqrt{\frac{D}{S}} \exp\left(-B \omega_d^2 - D \left(\rho / r_0\right)^2\right)$

Phase Standard Deviation and related Coherent Length
in the case of strong fluctuations (red lines) & very strong fluctuations (blue lines)



Positioning Errors

Modelling

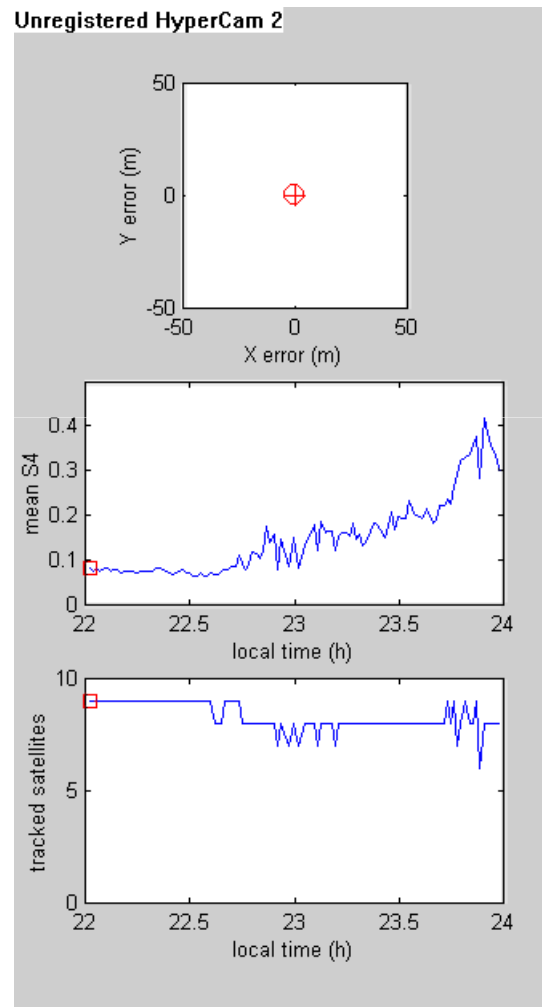
S4 measured for each tracked satellite

σ_{τ} is calculated taking the thermal noise as the main contribution

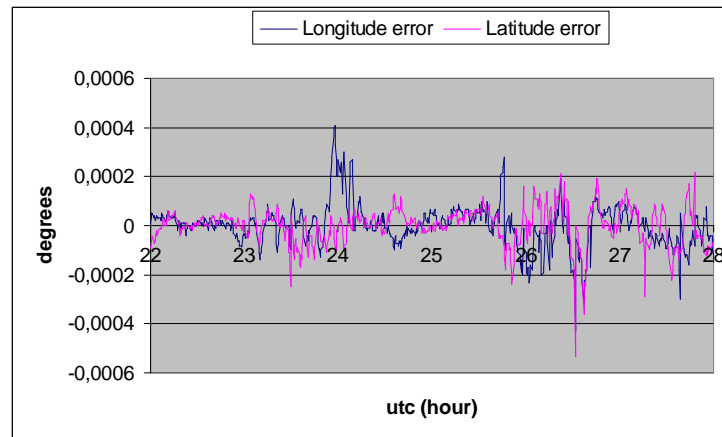
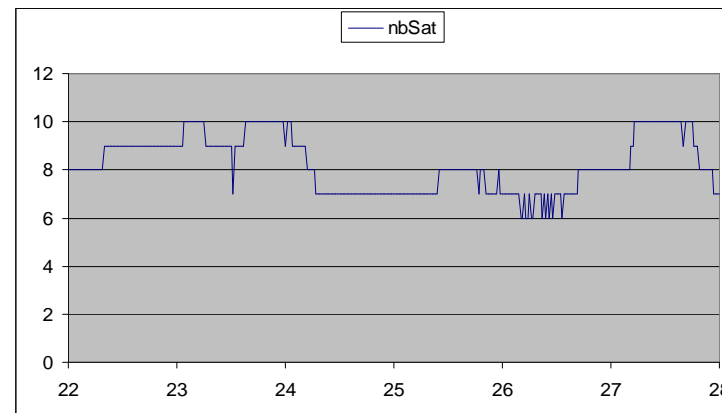
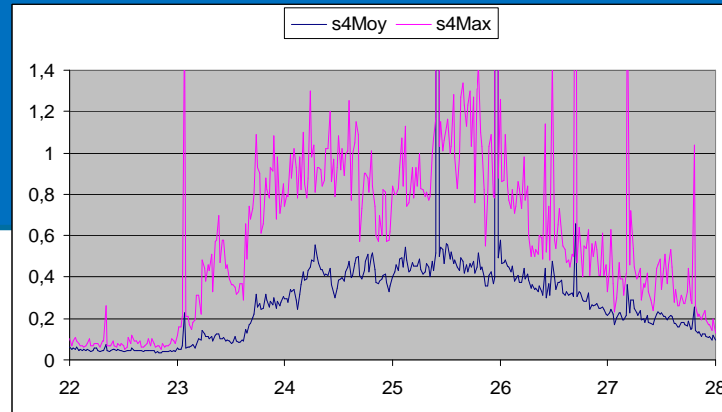
Range error calculated assuming a gaussian distribution

GPS constellation simulated with a yuma file

GPS Positioning Errors



Positioning errors from Measurements in Brazil in 2001



Conclusion

- Reasonable agreement between modelling (GISM) and measurements
- Positioning errors due to scintillations may reach values up to 50 meters
- The azimuthal resolution of a SAR may be significantly decreased
- All results will be updated taking measurements campaign data into account