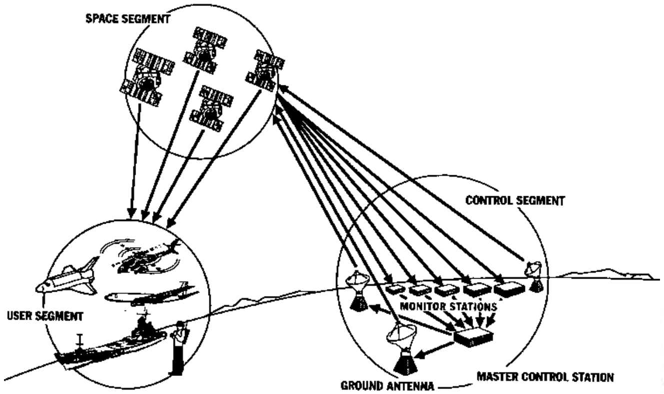
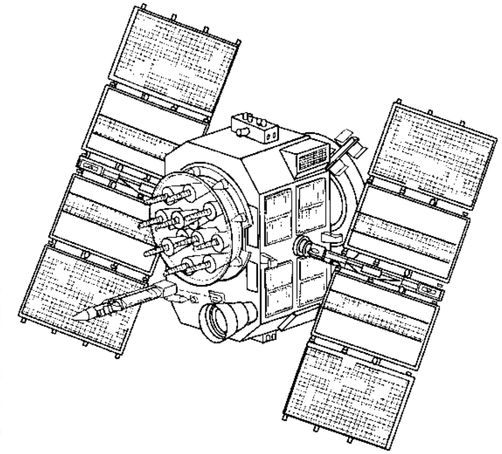
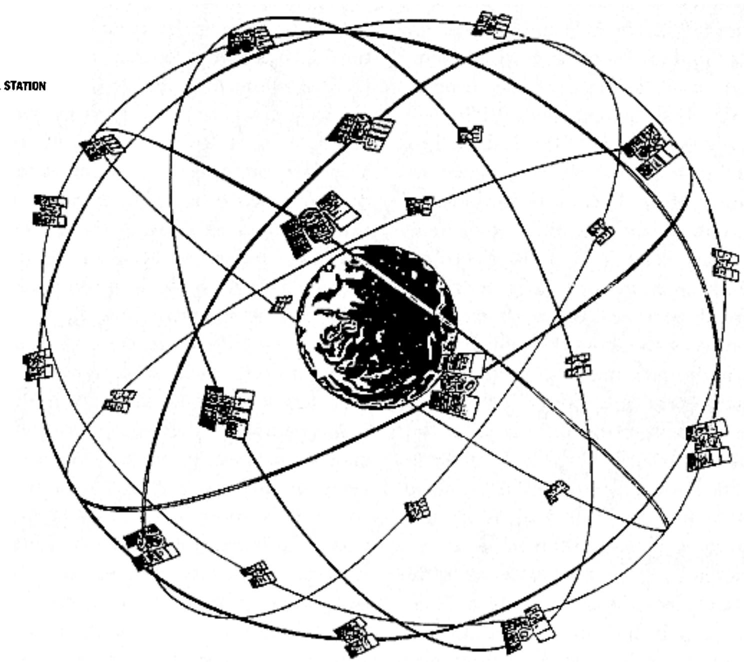






GNSS Fundamentals

Dr. John Raquet

Kigali, Rwanda

July 7, 2014

GNSS Fundamentals: Outline

Section 1: GPS History

Section 2: Background—
Time of Arrival (TOA) Positioning

Section 3: GPS System Overview

Section 4: Receiver measurements

Section 5: Signal Structure

Section 6: Measurement Errors

Section 7: Differential GPS

U.S. Air Force photo by Carleton Bailie,
http://www.af.mil/weekinphotos/040625-04.html

SECTION 1:
GPS HISTORY

GPS History

- **Navigation technology has always been a military (and commercial) asset**
  - Example: Seagoing clock[1]
- **Ability to determine position from satellites known since Sputnik**
- **Predecessors to GPS**
  - Navy TRANSIT system
    - Measured Doppler shift of satellites in polar orbits
    - Required stationary (or slowly moving) vehicles
  - NRL TIMATION satellites
    - First to orbit precise clocks
    - Provided precise time transfer between points on the Earth
    - Provided side-tone ranging capability
  - Air Force Project 621B
    - Demonstrated ranging based on pseudorandom noise (PRN)
    - Allowed all satellites to transmit at same frequency
- **GPS Joint Program Office (JPO) formed in 1973**
  - Combining TIMATION with Project 621B created the Navigation Technology Satellite (NTS-1 & NTS-2)
    (Note: NTS-2 was designated the first GPS Phase I SV)

TRANSIT Satellite[2]

TIMATION Satellite[3]

Navigation Technology Sat (NTS)
(result of TIMATION
and 621B)[3]

[1]Info can be found at http://www.oldnewspublishing.com/harrison.htm
[2]Image from https://www.patrick.af.mil/heritage/6555th/6555ch4/images/wcgtsz.jpg
[3]Images from http://code8200.nrl.navy.mil/nts.html

SECTION 2: TIME-OF-ARRIVAL POSITIONING (TRILATERATION)

- **How can a receiver figure out where it is?**

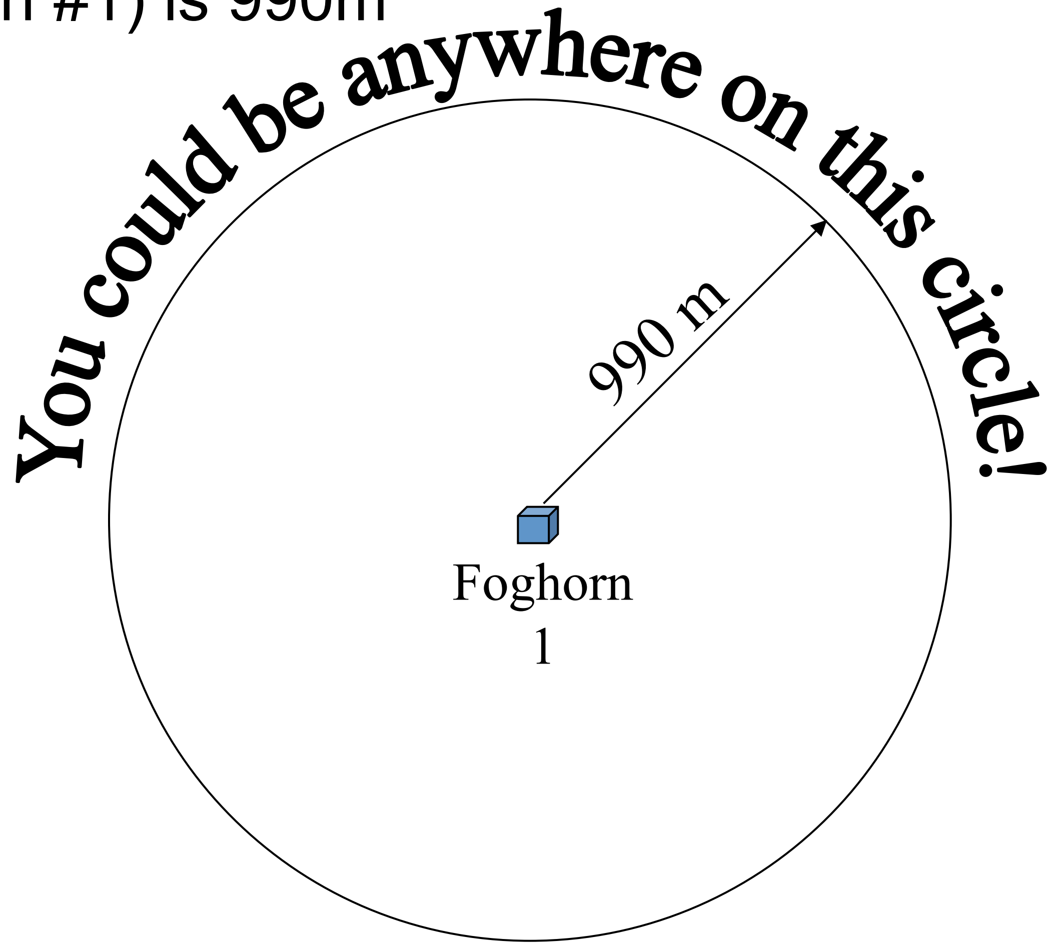
Ranging Using Time-Of-Arrival

- **Time-of-arrival (TOA) is one method that can be used to perform positioning**
- **Basic concept**
  - You must know
    - When a signal was transmitted
    - How fast the signal travels
    - Time that the signal was received
  - Then you can determine how far away you are from the signal emitter
- **Foghorn example:**

> Assume there is a foghorn that goes off at exactly 12:00:00 noon every day.
> You know that the velocity of sound around the foghorn is 330 m/sec.
> You have a device that measures the time when the foghorn
> blast is received, and it says it heard a foghorn blast at 12:00:03.
> What is the distance between the foghorn and the foghorn "receiver"?
> Now that you know how far you are from the foghorn, the question is-
> "Where are you?"

Two-Dimensional Positioning Using Single Range Measurement

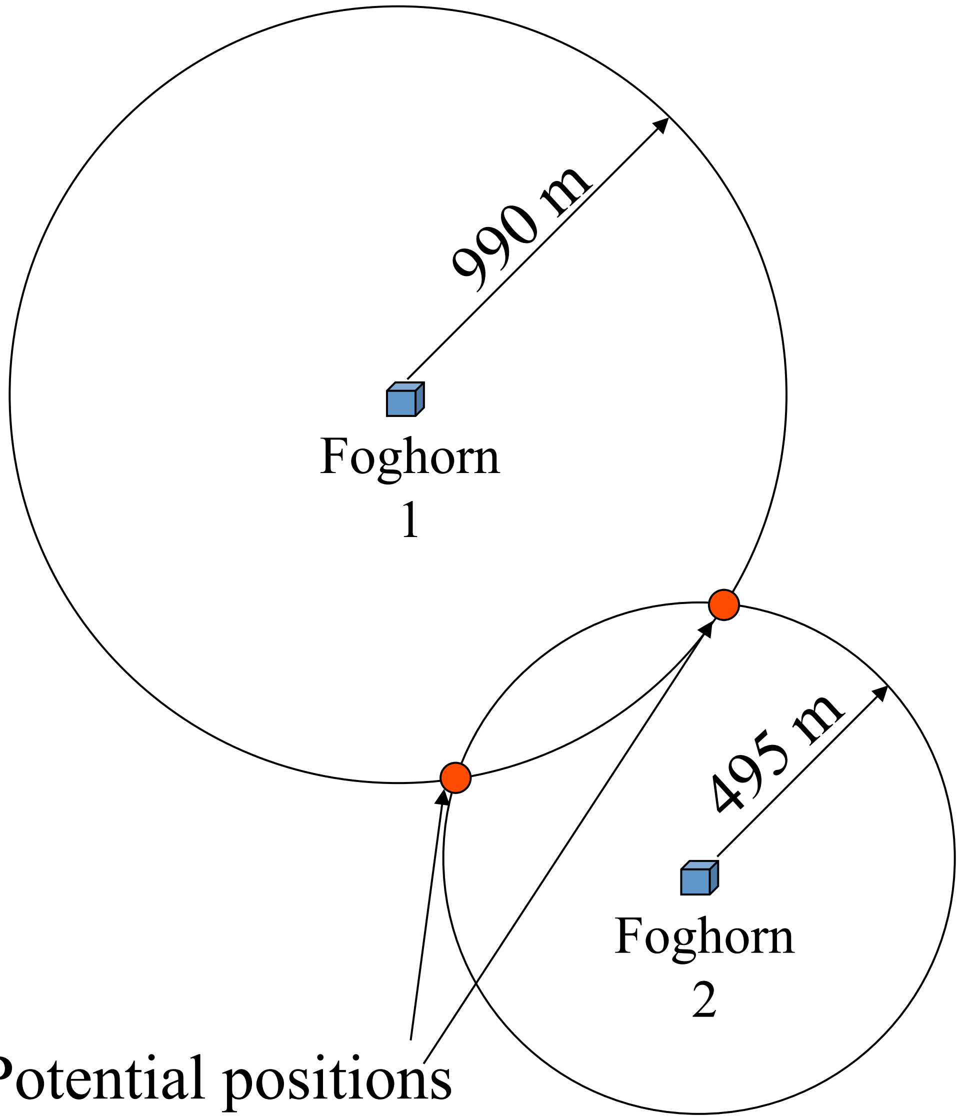
- Range between you and the foghorn (we'll call it foghorn #1) is 990m



- Unable to determine exact position in this case

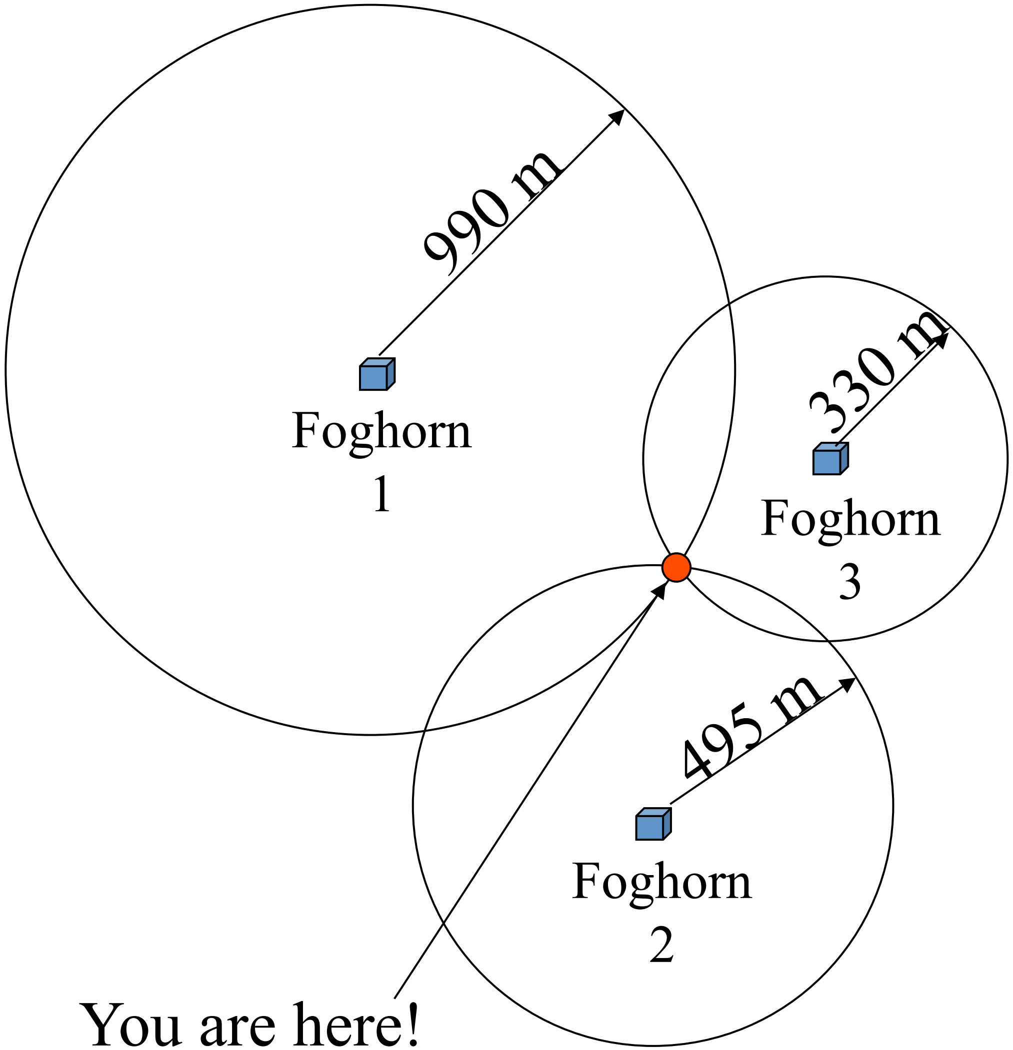
Two-Dimensional Ranging Using Two Measurements

- Now, you take a measurement from foghorn #2 at 12:00:01.5 (for a range of 495 m)
- Yields two potential solutions
  - How would you determine the correct solution?



Resolving Position Ambiguity Using Three Measurements

- You get a third measurement from foghorn #3 at 12:00:01 (Range = 330 m)
  - Now there's a unique solution



Receiver Clock Errors
(one way time transfer)

- The foghorn example assumed that the foghorn “receiver” had a perfectly synchronized clock, so the measurements were perfect
- What happens if there is an unknown receiver clock error?
- Effect on range measurement
  - Without clock error

$$R = v_{sound} \Delta t$$

R = range

v_{sound} = velocity of sound

Δt = transmit/receive time difference

  - With clock error δt

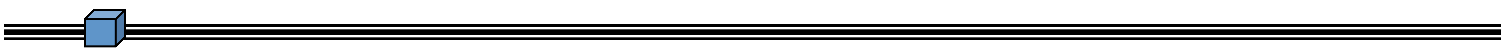
$$R' = v_{sound} \left(\Delta t + \delta t \right)$$

where

R' = range with error (pseudo-range)

Receiver Clock Errors
One-Dimensional Example (1/3)

- Now, we'll look at the foghorn example, except in only one dimension
  - The foghorn(s) and receiver are constrained to be along a line
  - We want to determine the position of the receiver on that line

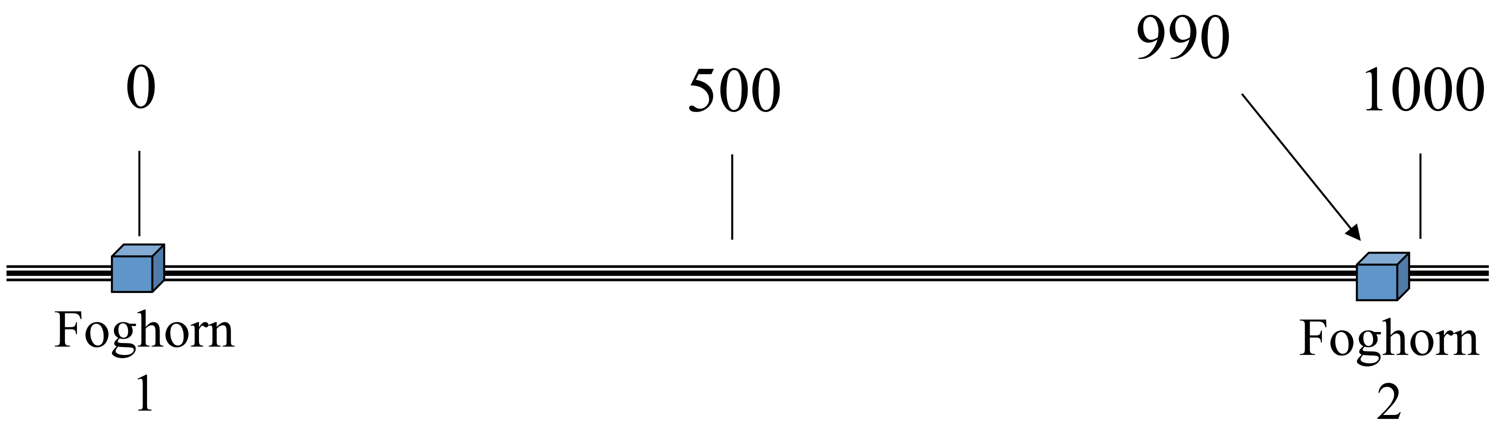


Foghorn 1

- If the receiver measured a signal at 12:00:10, where is it on the line?
- Now, assume an unknown clock bias δt in the clock used by the foghorn receiver
- Your foghorn receiver measures a foghorn blast at 12:00:10
- What can you say about where you are?

Receiver Clock Errors
One-Dimensional Example (2/3)

- Clearly, more information is needed
- Assume that there is a second foghorn located 990 m away from the first



- You receive a signal from the second foghorn at 12:00:09
- What can you tell about where you are at this point?

Receiver Clock Errors One-Dimensional Example (3/3)

- Here are the measurements we have:

$$\text{Pseudorange 1} = 330 \times 10 = 3300 = R_1'$$

$$\text{Pseudorange 2} = 330 \times 9 = 2970 = R_2'$$

- From the pseudorange equation:

$$R_1' = v_{sound}\left(\Delta t_1 + \delta t\right) = \quad x \quad + v_{sound}\delta t = 3300$$

$$R_2' = v_{sound}\left(\Delta t_2 + \delta t\right) = 990 - x + v_{sound}\delta t = 2970$$

- Rearranging terms we get

$$x + v_{sound}\delta t = 3300$$

$$x - v_{sound}\delta t = -1980$$

- We can then solve for the two unknowns

$$\delta t = 8 \text{ seconds}$$

$$x = 660 \text{ m}$$

Does this work?

Receiver Clock Errors
Extending to Three Dimensions

- **In the single-dimensional case**
  - We needed two measurements to solve for the two unknowns, x and δt.
  - The quantities x and (990 - x) were the “distances” between the position of the receiver and the two foghorns.
- **In three-dimensional case**
  - We need four measurements to solve for the four unknowns, x, y, z, and δt.
  - The distances between receiver and satellite are not linear equations (as was case in single-dimensional case).
  - The four equations to be solved simultaneously, for pseudorange measurements R_1’… R_4’ and transmitter positions $(x_1,y_1,z_1)\ldots(x_4,y_4,z_4)$:

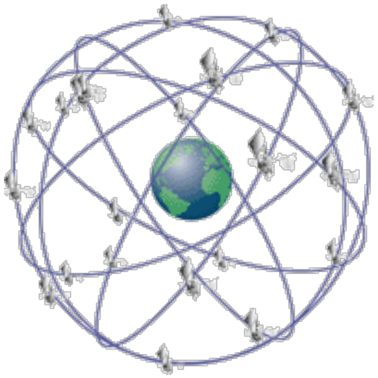
$$R_1' = \sqrt{(x-x_1)^2+(y-y_1)^2+(z-z_1)^2} + c\delta t$$

$$R_2' = \sqrt{(x-x_2)^2+(y-y_2)^2+(z-z_2)^2} + c\delta t$$

$$R_3' = \sqrt{(x-x_3)^2+(y-y_3)^2+(z-z_3)^2} + c\delta t$$

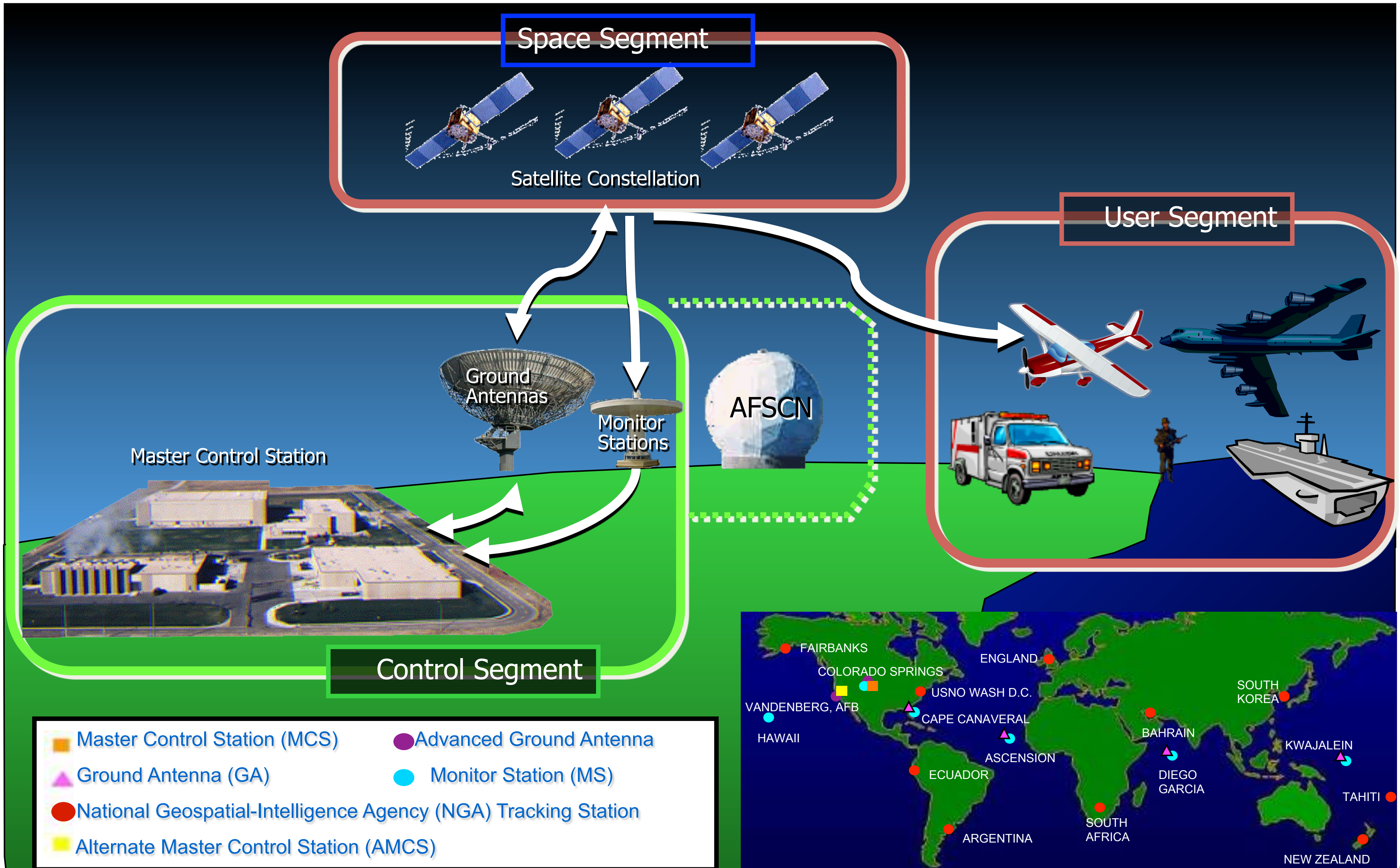
$$R_4' = \sqrt{(x-x_4)^2+(y-y_4)^2+(z-z_4)^2} + c\delta t$$

SECTION 3: GPS SYSTEM OVERVIEW



- Three segments of GPS system
- Differential GPS
- GPS performance

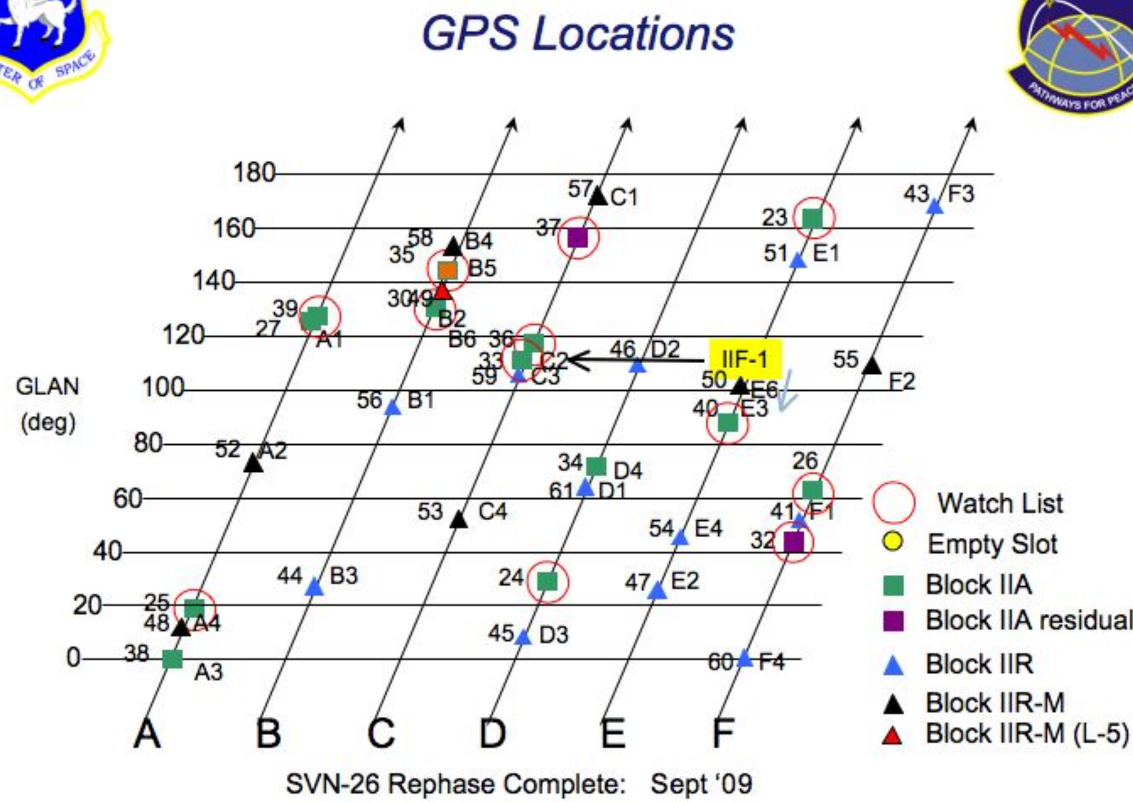
GPS Overview: Three Interactive Segments



GPS - Space Segment

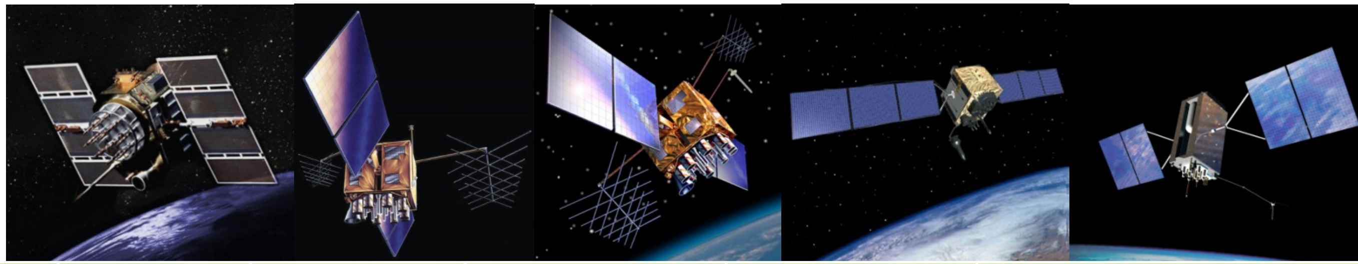
- **Nominally, there are 24 active satellites**
  - Originally “21 operational and 3 active spares” (but distinction not really made any more)
  - Current Constellation described as the “24+3”
  - Have been 30+ satellites recently

- **Orbit characteristics**
  - Six orbital planes
  - Four SVs per plane nominally
  - 55° inclination angle



*figures from *The Global Positioning System: A Shared National Asset*, National Research Council, Washington, D.C., May 1995

Space Segment – Satellite Characteristics

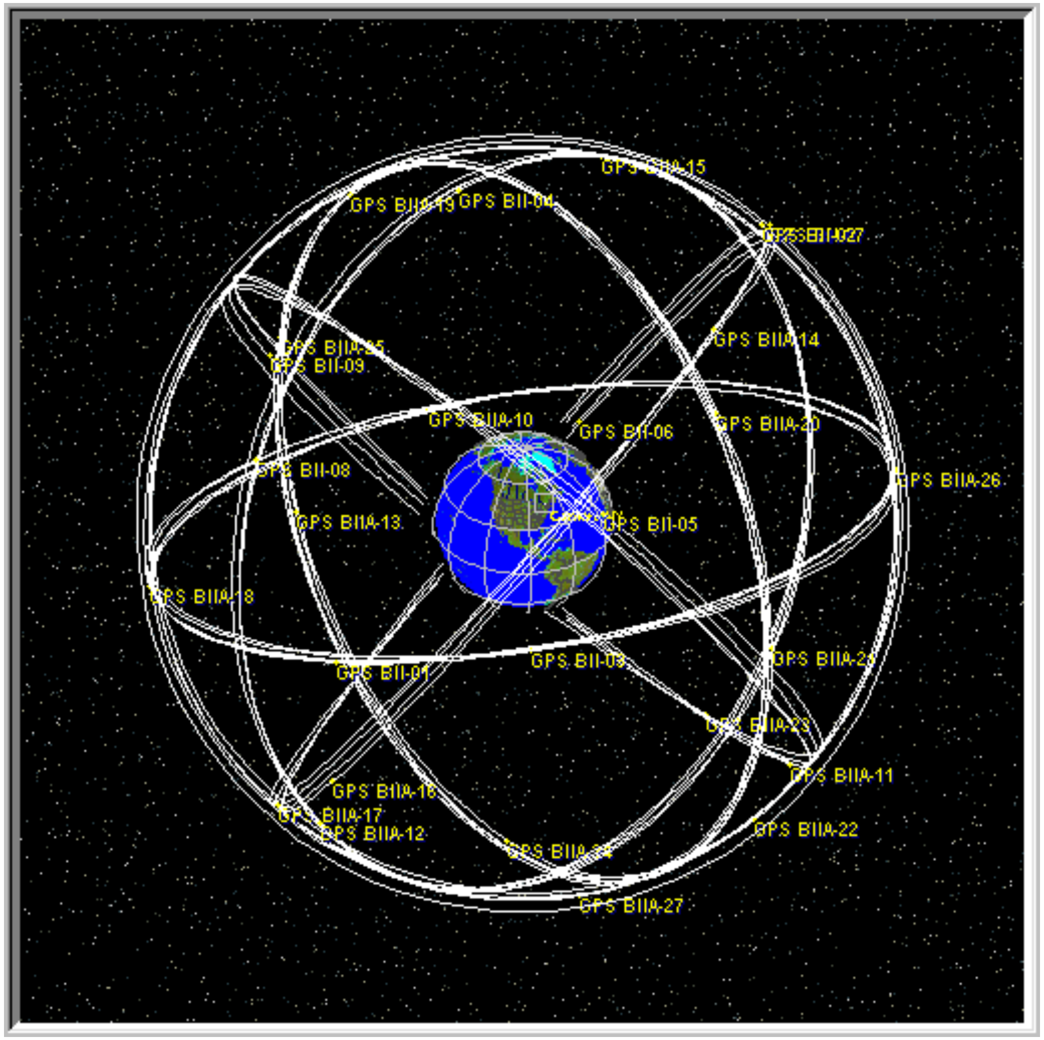


	II/IIA	IIR	IIR-M	IIF	III (A,B,C)
Number SV's	28	13	8	12	32
First Launch	1989	1997	2005	2010	2016+
Satellite Weight (Kg)	900	1100	1100	844	Tbd
Power (W)	1100	1700	1700	Not Avail	Tbd
Design Life (Years)	7.5	7.5	7.5	12	15
In Use (as of Jun 2014)	6	12	7	6	0
L1 Signals	C/A, P(Y)	C/A, P(Y)	C/A, P(Y), M	C/A, P(Y), M	C/A, P(Y), M, L1C
L2 Signals	P(Y)	P(Y)	P(Y), L2C, M	P(Y), L2C, M	P(Y), L2C, M
L5 Signals	-	-	-	L5	L5

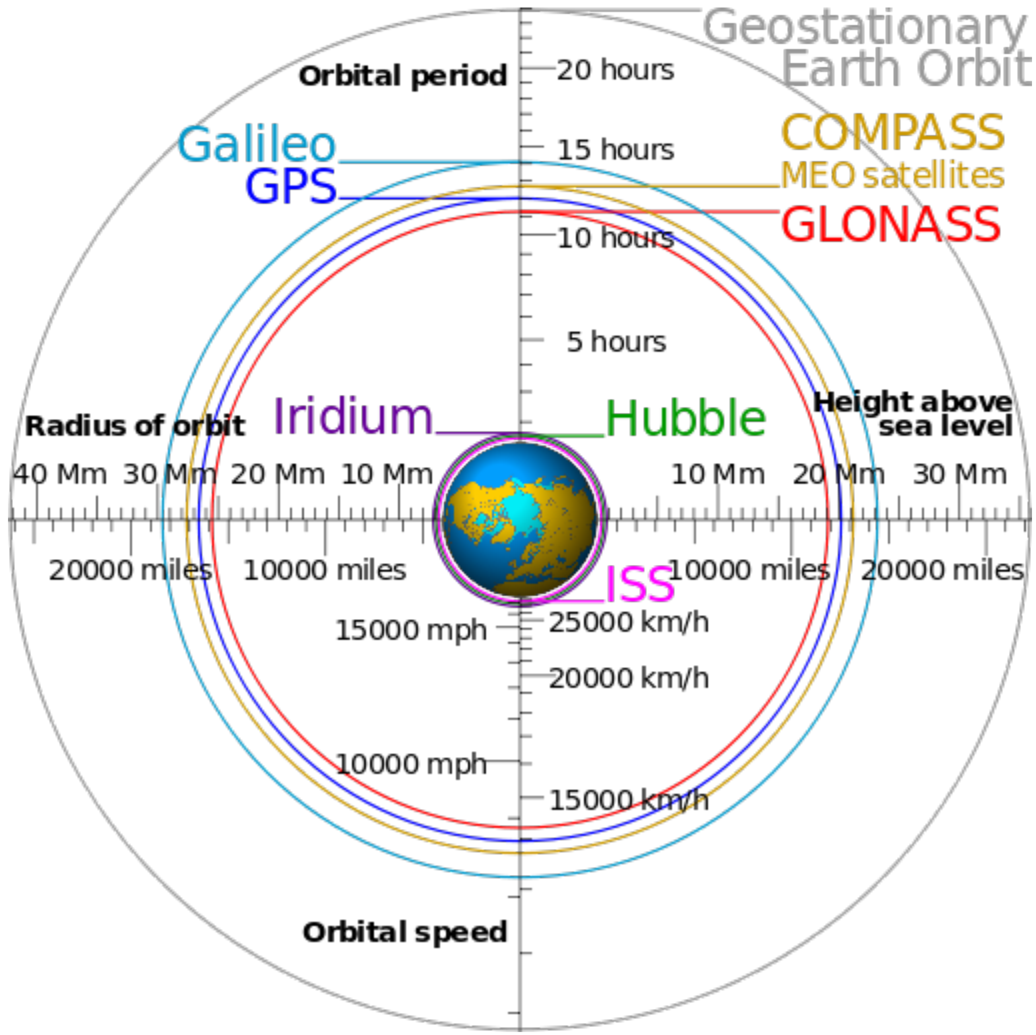
*Estimates

Sources: ftp://tycho.usno.navy.mil/pub/gps/gpsb2.txt
Misra and Enge, *Global Positioning System: Signals, Measurements, and Performance, 2001*
http://www.deagel.com/C3ISTAR-Satellites/GPS-Block-IIR_a000238003.aspx
http://www.deagel.com/C3ISTAR-Satellites/GPS-Block-IIF_a000238004.aspx
https://www.gps.gov/systems/gps/space

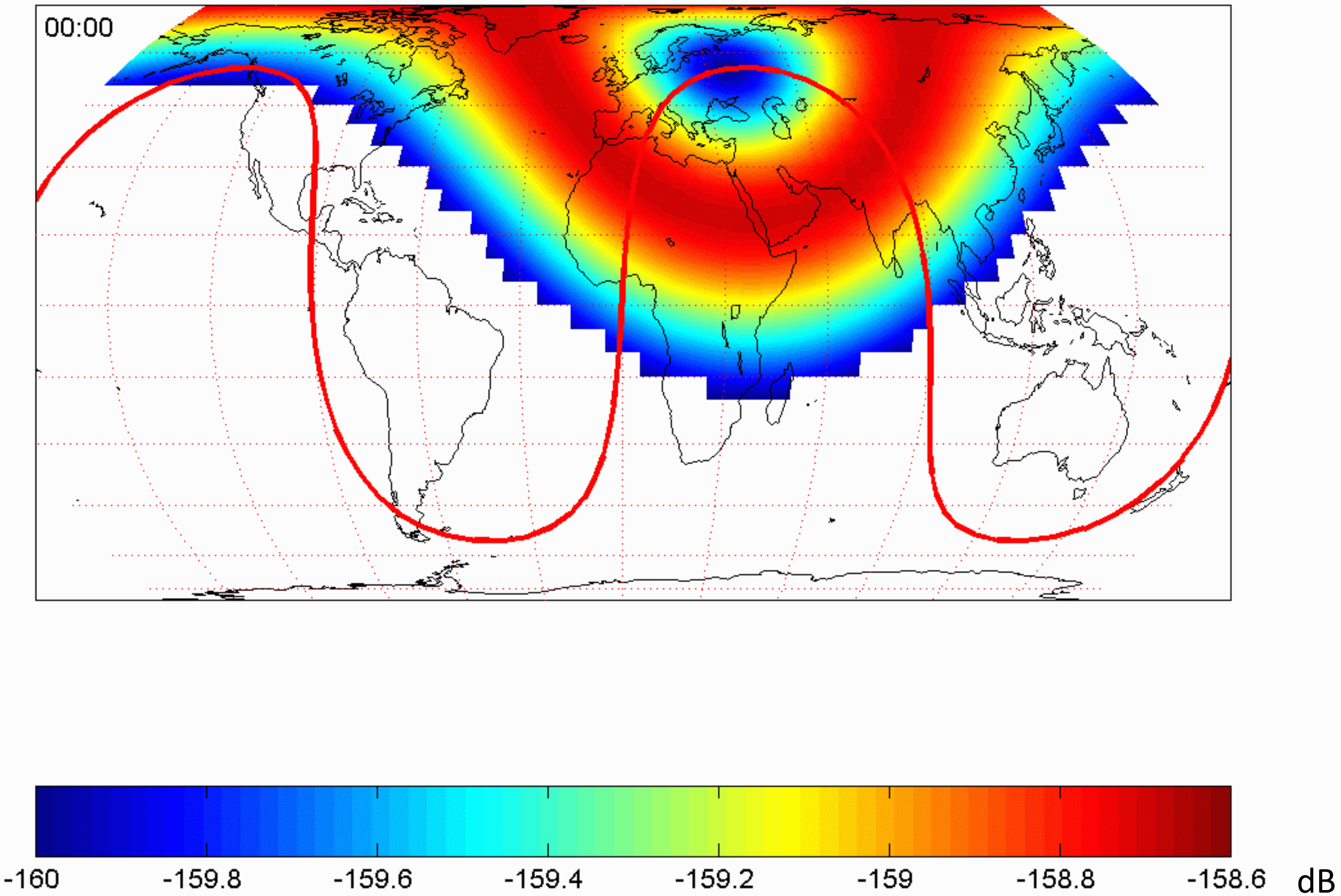
Space Segment –
GPS Constellation as Viewed from Space



Comparison of GPS to Other Satellite Orbits

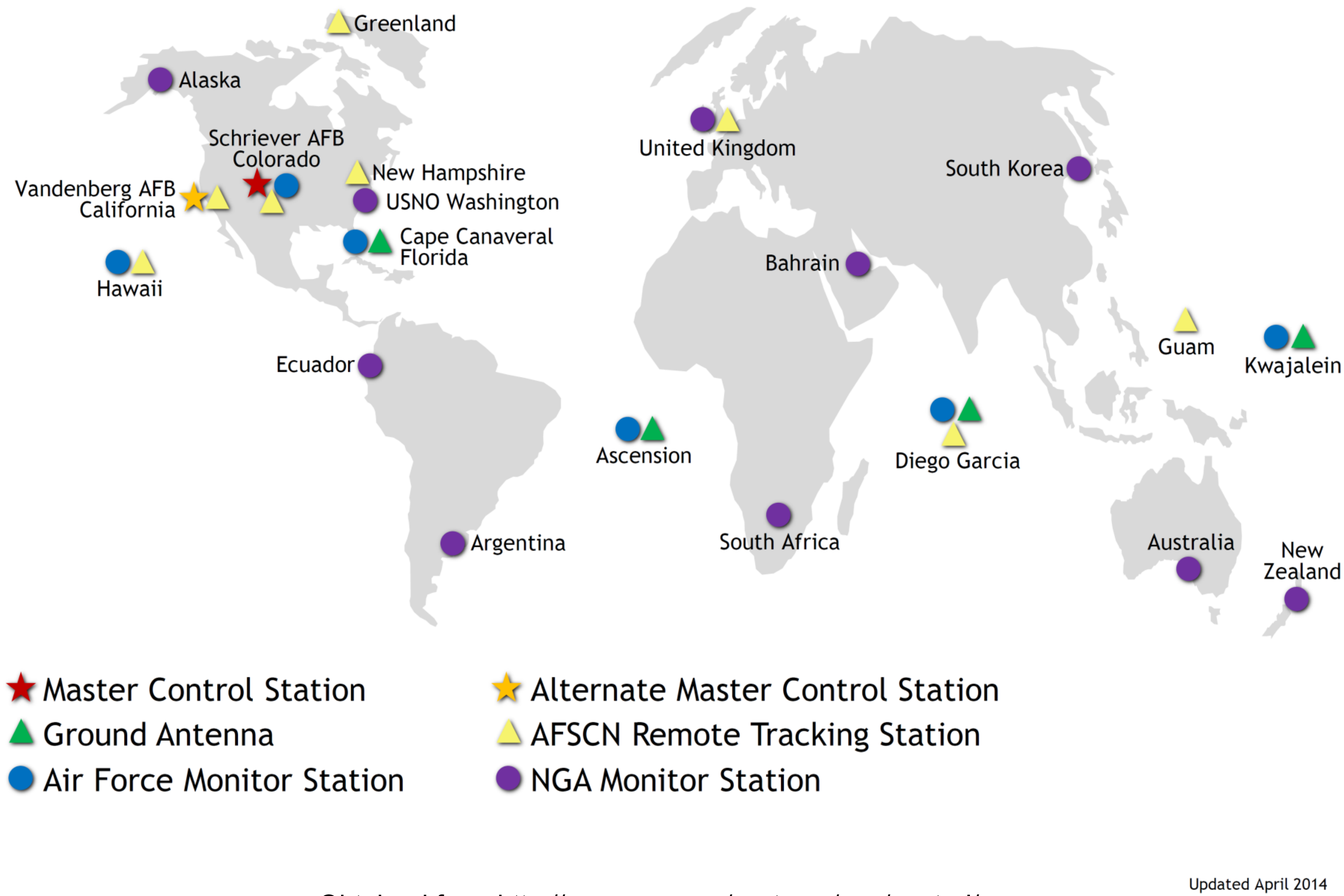


Space Segment - Representative GPS Ground Track



D. Goldstein, GPS Joint Program Office

Control Segment

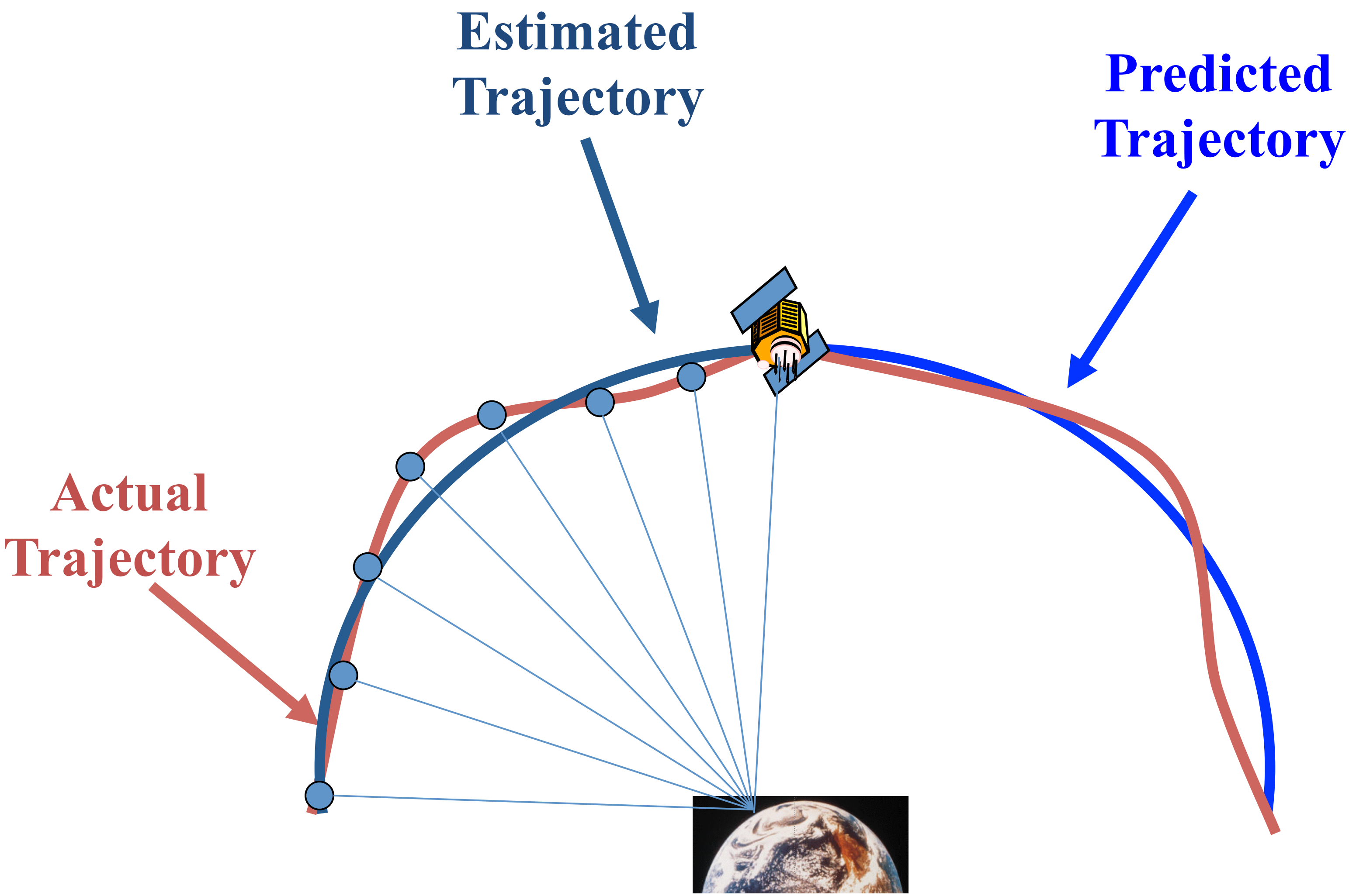


Obtained from http://www.gps.gov/systems/gps/control/

GPS - Control Segment

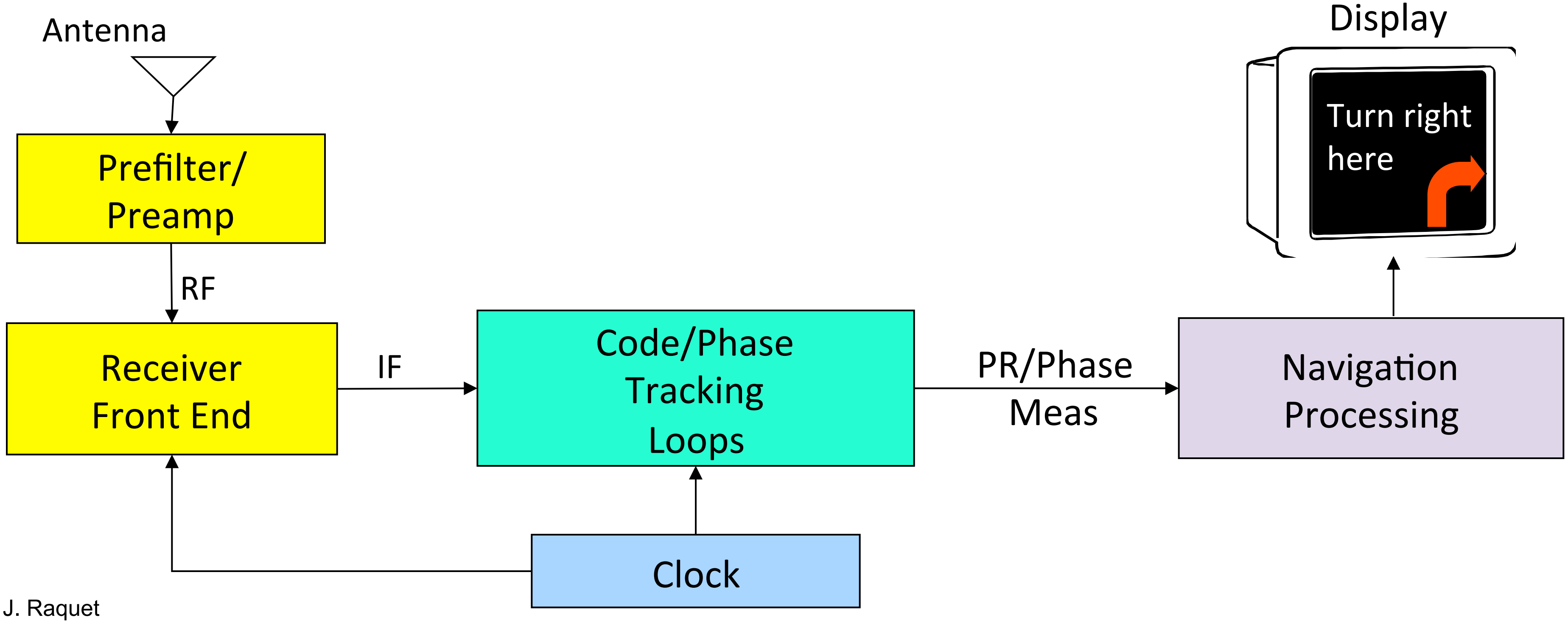
- **GPS Master Control Station (MCS) located at Schriever AFB, CO (2nd Space Operations Squadron, or 2SOPS)**
  - Manages constellation (flies satellites)
  - Monitors GPS system performance
  - Calculates data sent over the 50 bps navigation message
    - Orbit ephemeris data
    - Satellite clock error correction coefficients
    - Ionospheric model parameters
    - System status
    - GPS time information
- **Communications with satellite using S-band data link**
  - Types of communication
    - Satellite control
    - Navigation message upload
  - S-band communications are intermittent

Control Segment – Trajectory Estimation/Prediction



GPS - User Segment

- **User segment consists of all GPS receivers**
  - Space
  - Air
  - Ground
  - Marine
- **Typical GPS receiver components**



SECTION 4:
GPS RECEIVER MEASUREMENTS

- **What does the receiver measure?**

GPS Measurements (Overview)

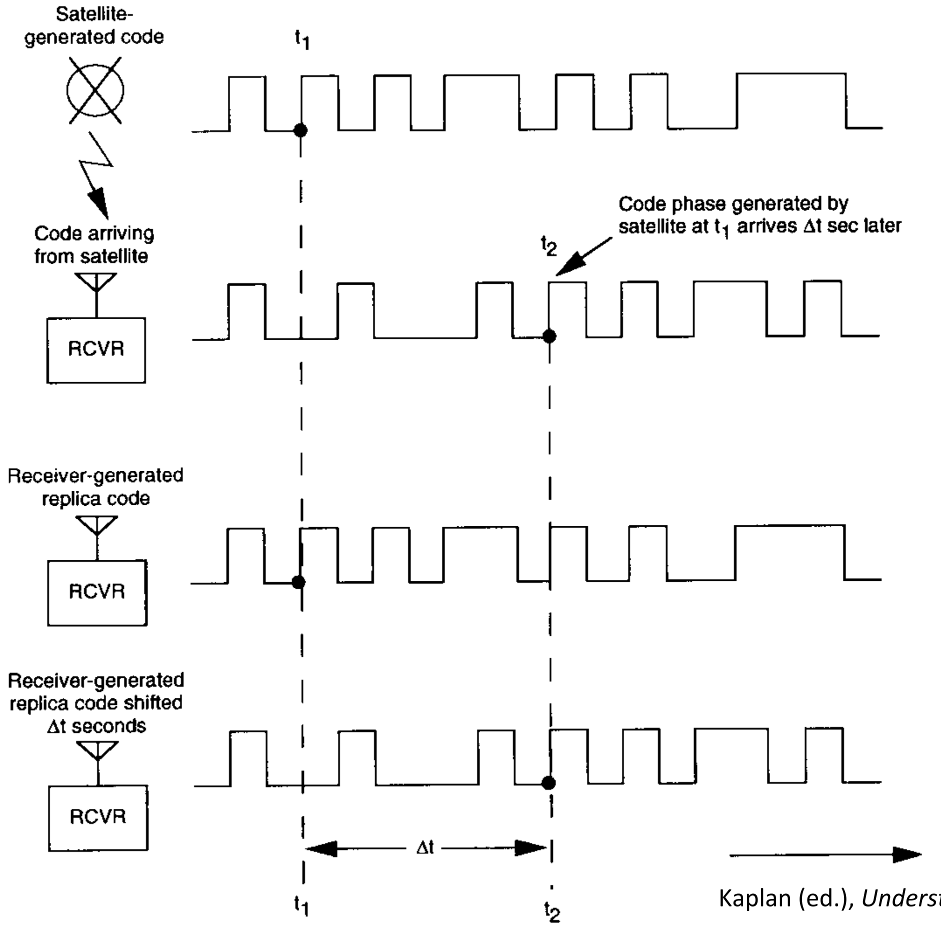
- **Each separate tracking loop typically can give 4 different measurement outputs**
  - Pseudorange measurement
  - Carrier-phase measurement (sometimes called integrated Doppler)
  - Doppler measurement
  - Carrier-to-noise density C/N_0
- **Actual output varies depending upon receiver**
  - NovAtel, Trimble, Leica, etc. give them all
  - RCVR-3A gives just C/N_0
- **Note: We're talking here about *raw measurements***
  - Almost all receivers generate navigation processor outputs (position, velocity, heading, etc.)

Measurement Rates and Timing

- **Most receivers take measurements on all channels/ tracking loops simultaneously**
  - Measurements time-tagged with the receiver clock (receiver time)
  - The time at which a set of measurements is called a data epoch.
- **The data rate varies depending upon receiver/ application. Typical data rates:**
  - Static surveying: One measurement every 30 seconds (120 measurements per hour)
  - Typical air, land, and marine navigation: 0.5-2 measurement per second (most common)
  - Specialized high-dynamic applications: Up to 50 measurements per second (recent development)

GPS Pseudorange Measurement

- **Pseudorange is a measure of the difference in time between signal transmission and reception**



Kaplan (ed.), *Understanding GPS: Principles and Applications*, Artech House, 1996

Doppler Shift
(The "Original" Satellite Navigation)

- **For electromagnetic waves (which travel at the speed of light), the received frequency f_R is approximated using the standard Doppler equation**

$$f_R = f_T\left(1 - \frac{(\boldsymbol{v}_r \cdot \boldsymbol{a})}{c}\right)$$

$$\begin{aligned}
f_R &= \text{received frequency (Hz)} \\
f_T &= \text{transmitted frequency (Hz)} \\
\boldsymbol{v}_r &= \text{satellite-to-user relative velocity vector (m/s)} \\
\boldsymbol{a} &= \text{unit vector pointing along} \\
&\quad \text{line-of-sight from user to SV} \\
c &= \text{speed of light (m/s)}
\end{aligned}$$

  – Note that $\boldsymbol{v}_r$ is the (vector) velocity difference

$$\boldsymbol{v}_r = \boldsymbol{v} - \dot{\boldsymbol{u}}$$

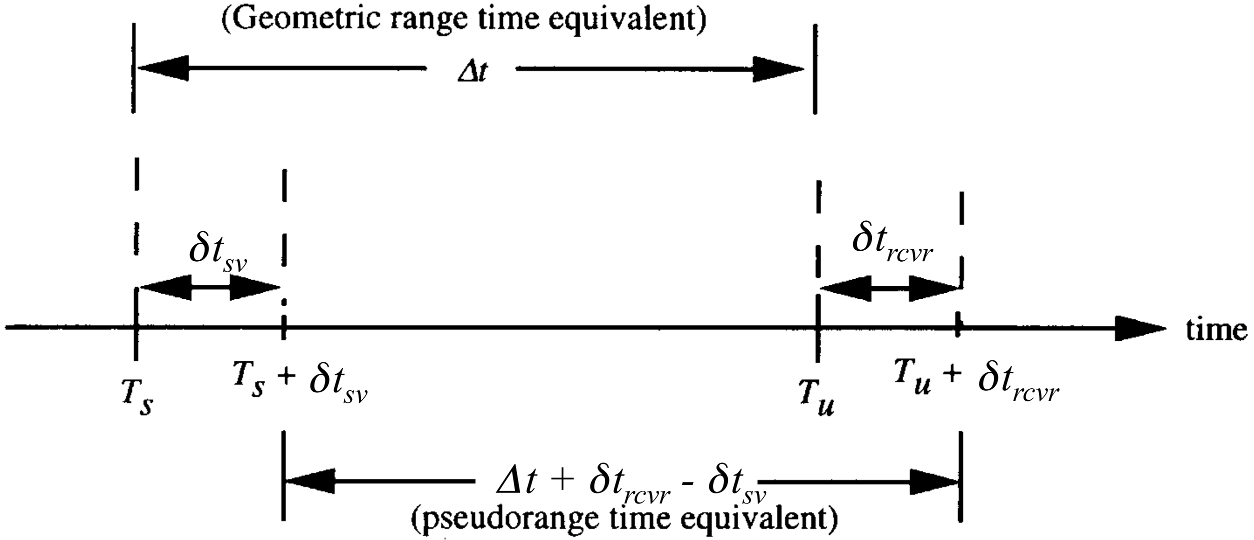
$$\begin{aligned}
\boldsymbol{v} &= \text{velocity vector for satellite (m/s)} \\
\dot{\boldsymbol{u}} &= \text{velocity vector for user (m/s)}
\end{aligned}$$

- **The Doppler shift Δf is then**

$$\Delta f = f_R - f_T \quad \text{(Hz)}$$

Effect of Clock Errors on Pseudorange

- **Since pseudorange is based on time difference, any clock errors will fold directly into pseudorange**



- **Small clock errors can result in large pseudorange errors (since clock errors are multiplied by speed of light)**
- **Satellite clock errors (δt_{sv}) are very small**
  - Satellites have atomic time standards
  - Satellite clock corrections transmitted in navigation message
- **Receiver clock (δt_{rcvr}) is dominant error**

Kaplan (ed.), *Understanding GPS: Principles and Applications*, Artech House, 1996

Doppler Measurement

- **The GPS receiver locks onto the carrier of the GPS signal and measures the received signal frequency**
  - Relationship between true and measured received signal frequency:

$$f_{R_{meas}}$$

$$f_R = f_{R_{meas}}(1 + \delta\dot{t}_{rcvr})$$

$$f_R = \text{true received signal frequency (Hz)}$$

$$f_{R_{meas}} = \text{measured received signal frequency (Hz)}$$

$$\delta\dot{t}_{rcvr} = \text{receiver clock drift rate (sec/sec)}$$

  - Doppler measurement formed by differencing the measured received frequency and the transmit frequency:

$$\Delta f_{meas} = f_{R_{meas}} - f_T$$

  - Note: transmit frequency is calculated using information about SV clock drift rate given in navigation message

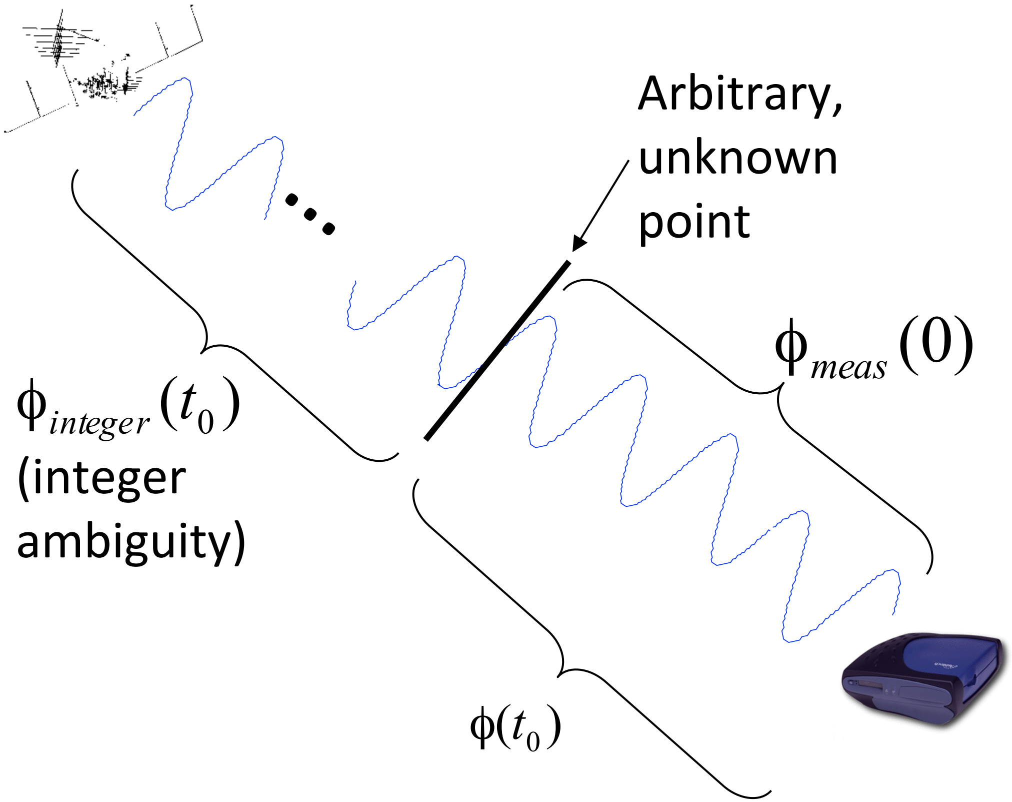
Carrier-Phase (Integrated Doppler) Measurement

- The carrier-phase measurement $\phi_{meas}(t)$ is calculated by integrating the Doppler measurements

$$\text{range}(t) = \underbrace{\int_{t_o}^{t} \Delta f_{meas}(t)dt + \phi(t_0)}_{\substack{\phi_{meas}(t) \\ \text{(can be measured by receiver)}}} + \phi_{integer}(t_0) + \text{clock error} + \text{other errors}$$

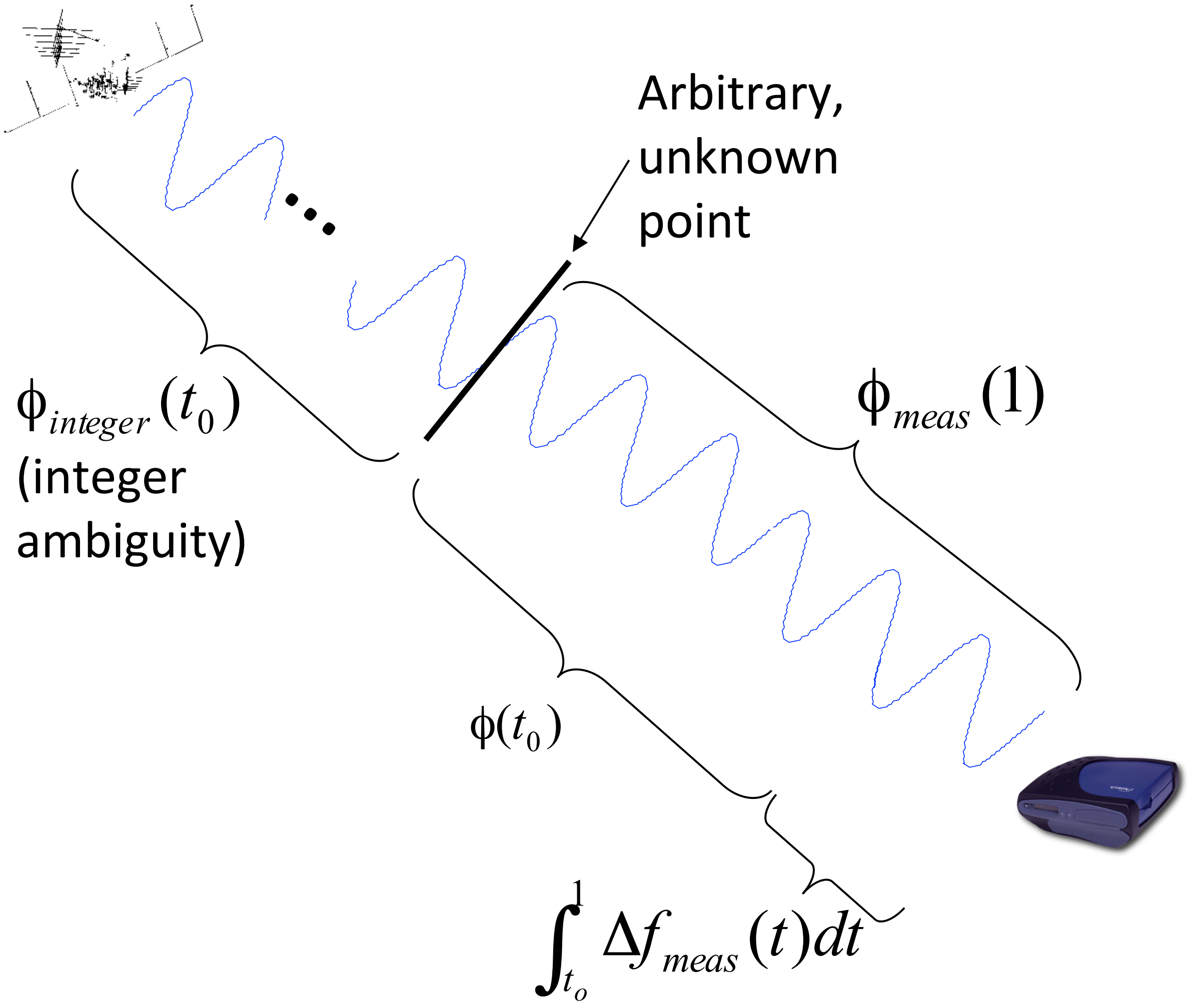
- The integer portion of the initial carrier-phase at the start of the integration ($\phi_{integer}(t_0)$) is known as the “carrier-phase integer ambiguity”
  – Because of this ambiguity, the carrier-phase measurement is not an absolute measurement of position
  – Advanced processing techniques can be used to resolve these carrier-phase ambiguites (carrier-phase ambiguity resolution)
- Alternative way of thinking: carrier-phase measurement is the “beat frequency” between the incoming carrier signal and receiver generated carrier.

Phase Tracking Example
At Start of Phase Lock (Time = 0 seconds)



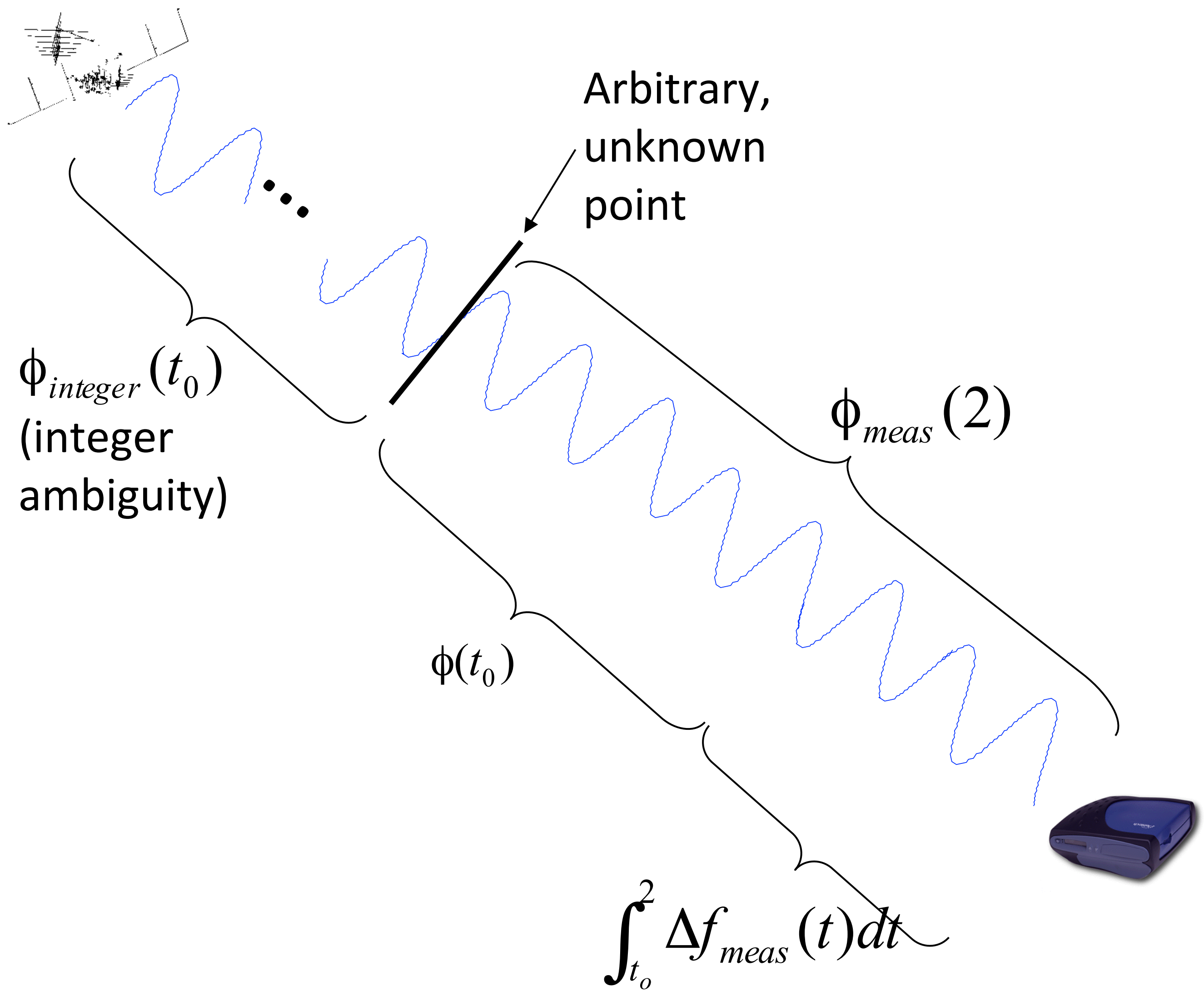
Ignoring clock and other errors

Phase Tracking Example
After Movement (for 1 Second)



Ignoring clock and other errors

Phase Tracking Example
After Movement (for 2 Seconds)



Ignoring clock and other errors

Comparison Between Pseudorange and Carrier-Phase Measurements

	Pseudorange	Carrier-Phase
Type of measurement	Range (absolute)	Range (ambiguous)
Measurement precision	~1 m	~0.01 m
Robustness	More robust	Less robust (cycle slips possible)

Necessary for high precision GPS

Carrier-to-Noise Density (C/N_0)

- **The carrier-to-noise density is a measure of signal strength**
  - The higher the C/N_0, the stronger the signal (and the better the measurements)
  - Units are dB-Hz
  - General rules-of-thumb:
    - $C/N_0 > 40$: Very strong signal
    - $32 < C/N_0 < 40$: Marginal signal
    - $C/N_0 < 32$: Probably losing lock (unless using high sensitivity receiver)
- **C/N_0 tends to be receiver-dependent**
  - Can be calculated many different ways
  - Absolute comparisons between receivers not very meaningful
  - Relative comparisons between measurements in a single receiver are very meaningful

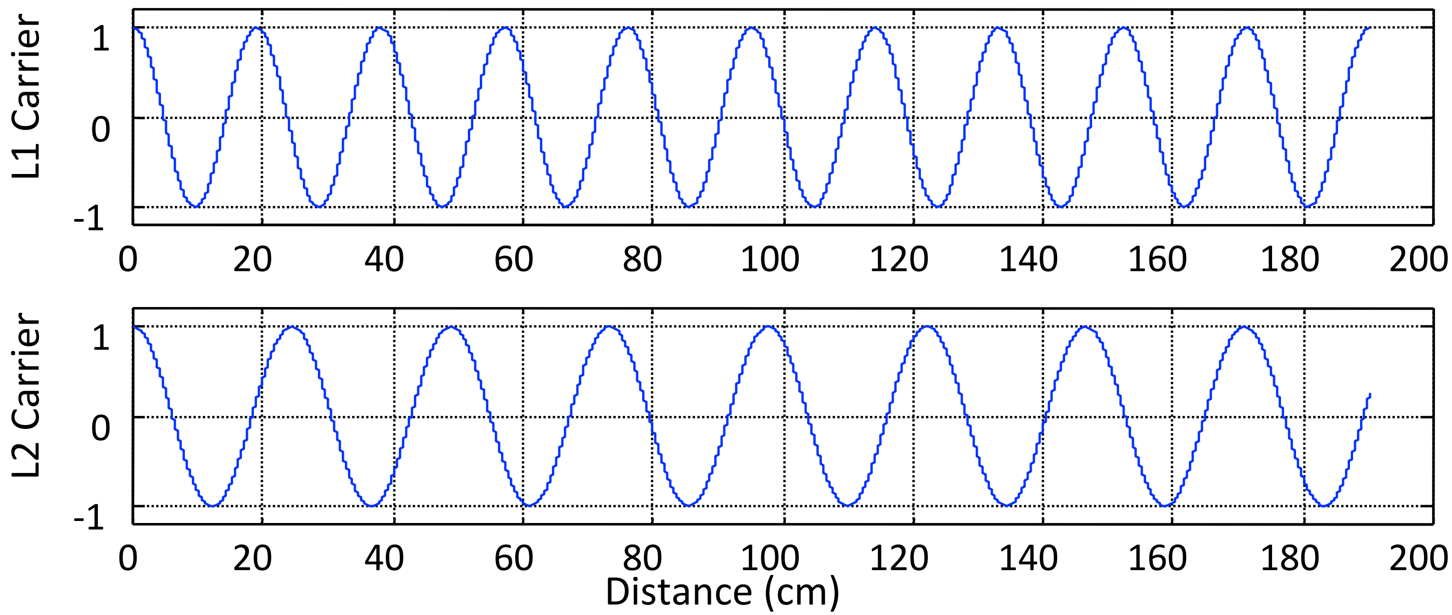
SECTION 5: GPS SIGNAL STRUCTURE

- **So what do those satellites transmit anyway?**

GPS Carrier Frequencies

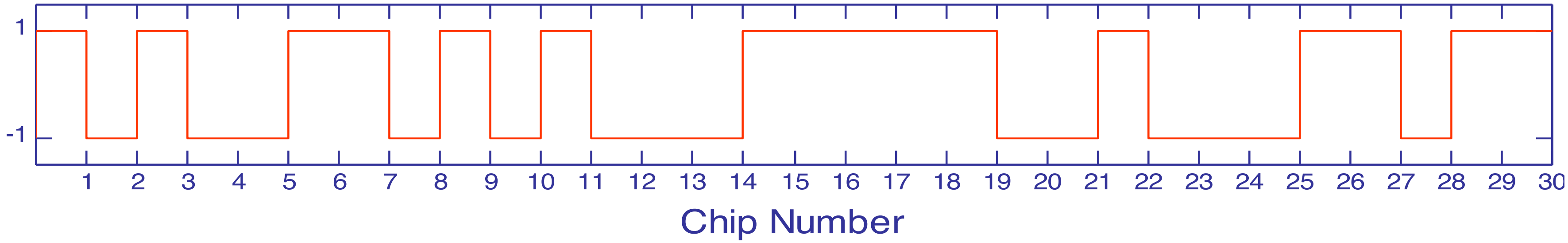
- **Fundamental frequency f_0 = 10.23 MHz**
- **GPS carrier (or center) frequencies**
  - f_{L1} = 1575.42 MHz = 154 f_0
  - f_{L2} = 1227.6 MHz = 120 f_0
  - f_{L5} = 1176.45 MHz = 115 f_0
- **Wavelengths of carriers**

$\lambda_{L1} = c/f_{L1} \approx$ 19.03 cm $\quad \lambda_{L2} = c/f_{L2} \approx$ 24.42 cm $\quad \lambda_{L2} = c/f_{L2} \approx$ 25.48 cm



GPS Pseudo-Random Noise (PRN) Codes

- **A PRN code is a binary sequence that appears to be random. Example:**



  - Not called a data bit, because it is not data being transmitted
  - The number of chips per second is called the "chipping rate"
- **PRN code sequence generated in hardware using a tapped feedback shift register**
  - Sequence of bits where the new bit is generated by an exclusive-or of two previous bits in the sequence
  - Very easy to implement in hardware

C/A and P-Codes

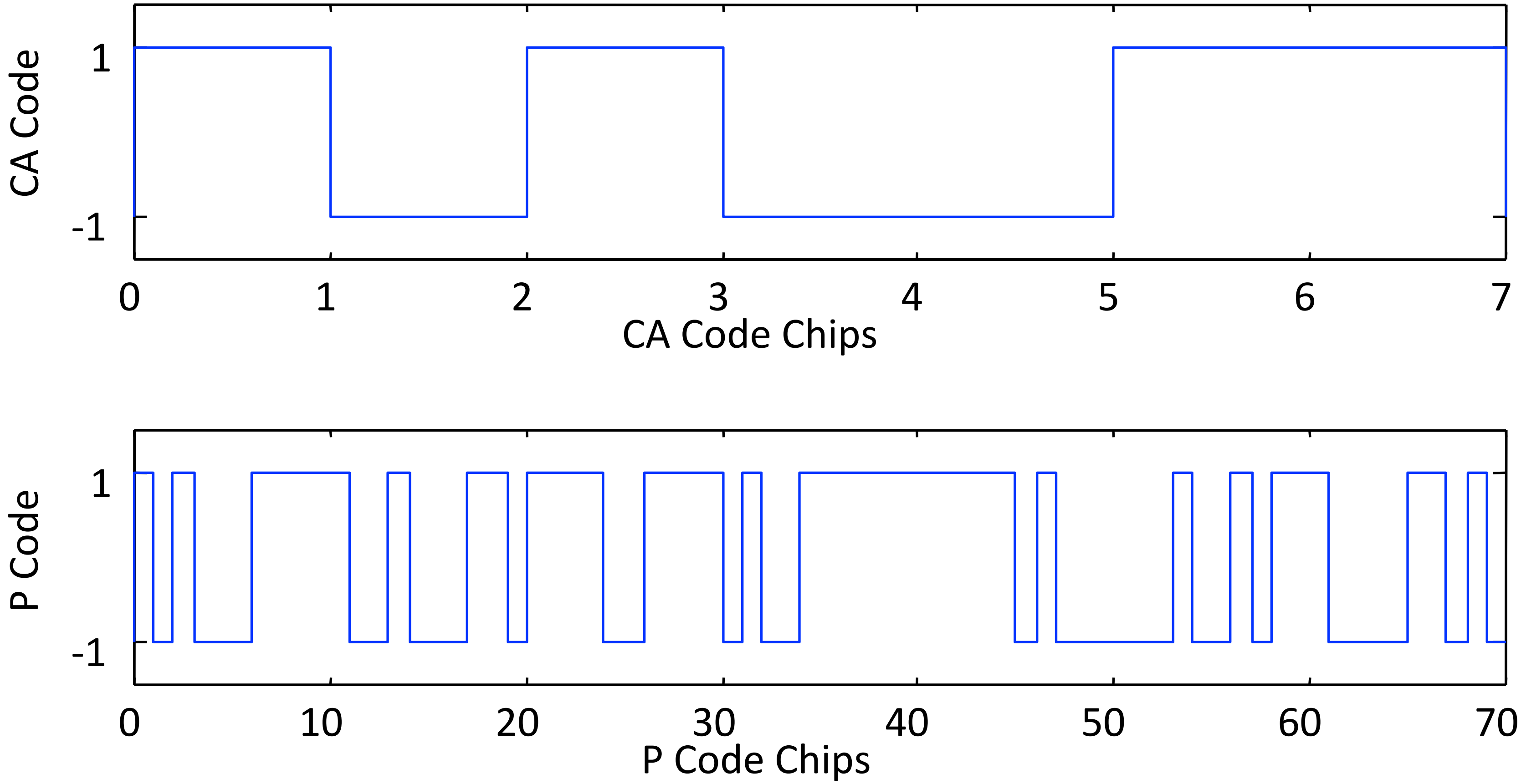
- **GPS uses two classes of codes**
  - Coarse-Acquisition (C/A) code
    - Intended for initial acquisition of the GPS signal
  - Precise (P) code
    - Higher chipping rate, so provides better performance
  - Comparison between C/A and P codes:

Parameter	C/A-Code	P-Code
Chipping Rate (chips/sec)	1.023×10^6	10.23×10^6
Chipping Period (nsec)	977.5 nsec	97.75 nsec
Range of One Chip	293.0 m	29.30 m
Code Repeat Interval	1 msec	1 week

- **It's more difficult to lock onto the P-code (due to length of code)**
  - Requires accurate knowledge of time
  - Normally, C/A-code locked onto first
    - Easier, since there's only 1ms to search over
    - Once locked onto C/A-code, receiver has accurate time information for locking onto P-code
  - Using accurate timing information to lock onto P-code without initial C/A-lock called "direct P(Y)-code acquisition"

Example C/A and P-Codes

- **Simulated C/A and P-Codes are given below.**
  - Note that the P-code chipping rate is 10 times higher than the C/A-code

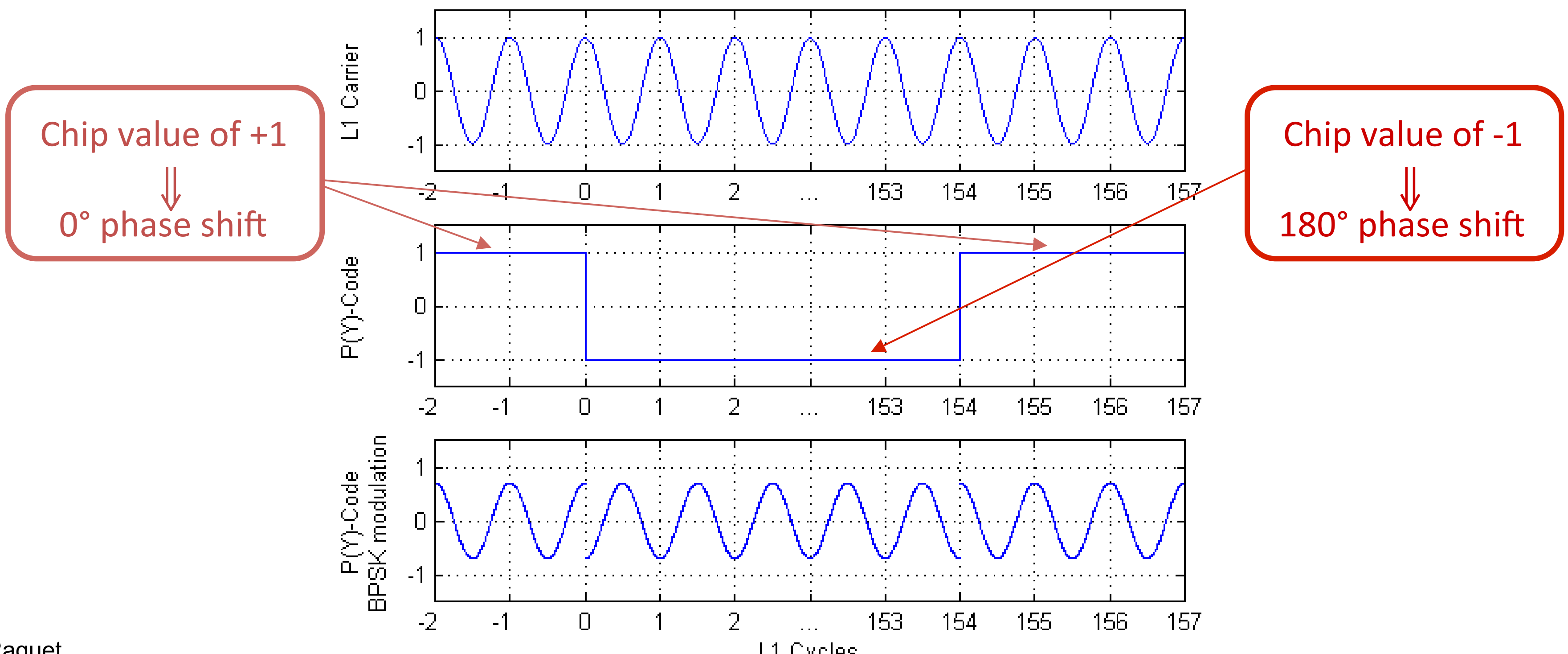


P-Code Encryption for Anti-Spoofing

- **P-code is unclassified and defined in ICD-GPS-200.**
- **Satellites don't normally transmit P-code, however.**
  - P-code is encrypted by an encryption code
  - The encrypted P-code is called Y-code
    - Often referred to as P(Y)-code
    - Y-code is classified, so unauthorized users cannot
      - Directly lock onto the Y-code
      - Spoof the Y-code (i.e., make a fake signal that appears to be coming from a GPS SV)
    - Correlation techniques exist that allow advanced civilian receivers to lock onto P(Y)-code.
      - Degraded capability vs. direct Y-code tracking
      - Requires C/A-code lock

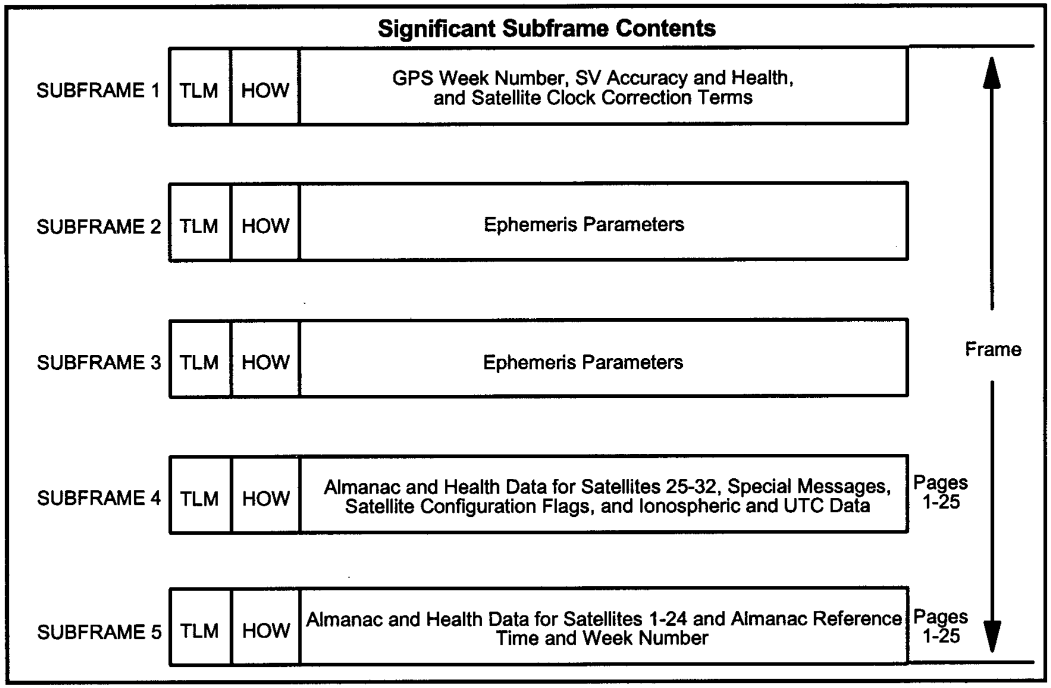
Code Modulation of Carrier

- **So far, we've covered**
  - GPS L1 and L2 carrier frequencies
  - C/A-code and P-code
- **These need to be combined through modulation**
  - GPS uses biphase shift key (BPSK) modulation



GPS Navigation Message

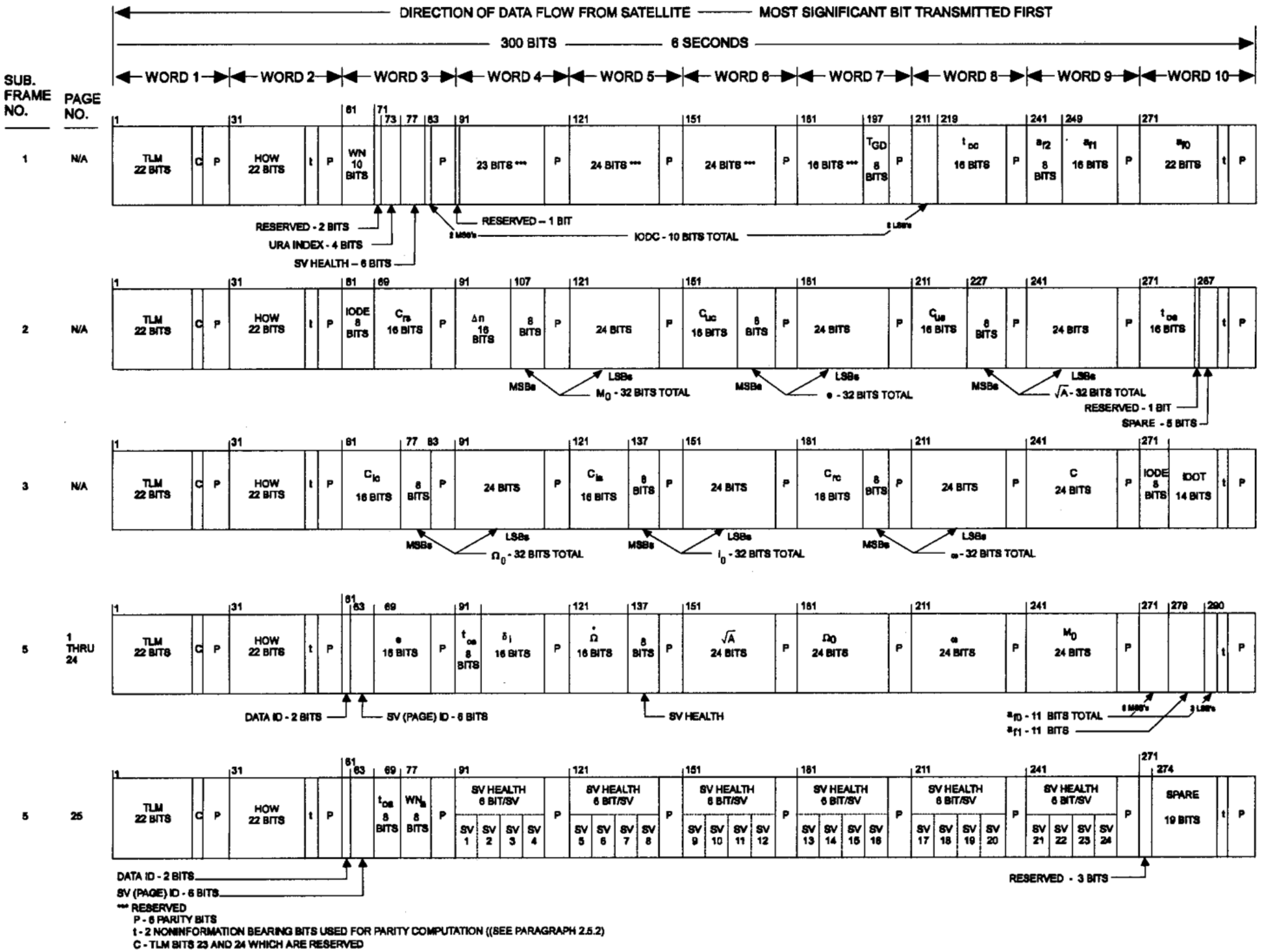
- **In addition to the C/A or P(Y)-codes, the signal is also modulated with the 50 bit/sec navigation message**
  - One “frame” is 1500 bits (30 seconds), and is broken into 5 300-bit “sub-frames” (6 seconds each):



Subframes 4 and 5 change each frame
(Total of 12.5 minutes to cycle through all messages)

- **Navigation message is combined with code**
  - For 1/0 representation: exclusive-or
  - for 1/-1 representation: multiplication

Data Format of Subframes 1, 2, 3, and 5



Obtained from SPS Signal Spec (http://www.spacecom.af.mil/usspace/gps_support/documents/SPSMAIN.pdf)

L1 and L2 Signal Breakdown

- **Note: 50 bps navigation message modulated on all of the codes**
- **L1 signal**
  - P-code
  - C/A-code modulated on carrier that is 90° out of phase from P-code carrier

$$s_{L1}(t) = \underbrace{A_{P_{L1}} Y(t) N(t) \cos(\omega_1 t)}_{\text{P-Code}} + \underbrace{A_{C/A} CA(t) N(t) \sin(\omega_1 t)}_{\text{C/A-Code}}$$

$$N(t) = \text{50 bps navigation message}$$

$$A_{P_{L1}} = \text{Amplitude of L1 P-code signal} \approx -163 \text{ dBW}$$

$$A_{C/A} = \text{Amplitude of C/A-code signal} \approx -160 \text{ dBW}$$

$$\omega_1 = 2\pi f_{L1}$$

- **L2 signal**
  - P-code only

$$s_{L1}(t) = \underbrace{A_{P_{L2}} Y(t) N(t) \cos(\omega_2 t)}_{\text{P-Code}}$$

$$A_{P_{L2}} = \text{Amplitude of L2 P-code signal} \approx -166 \text{ dBW}$$

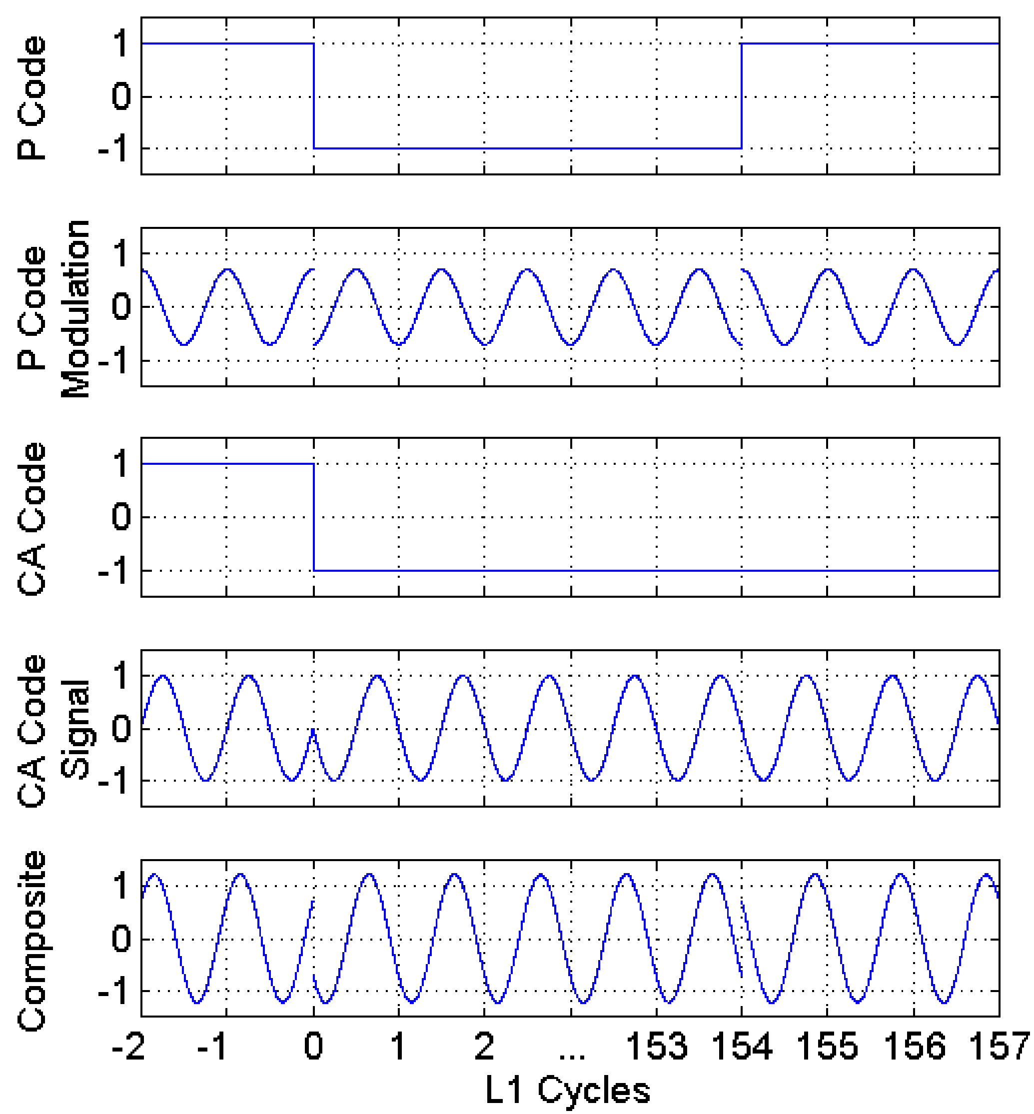
$$\omega_2 = 2\pi f_{L2}$$

Sample of How L1 Signal is Generated

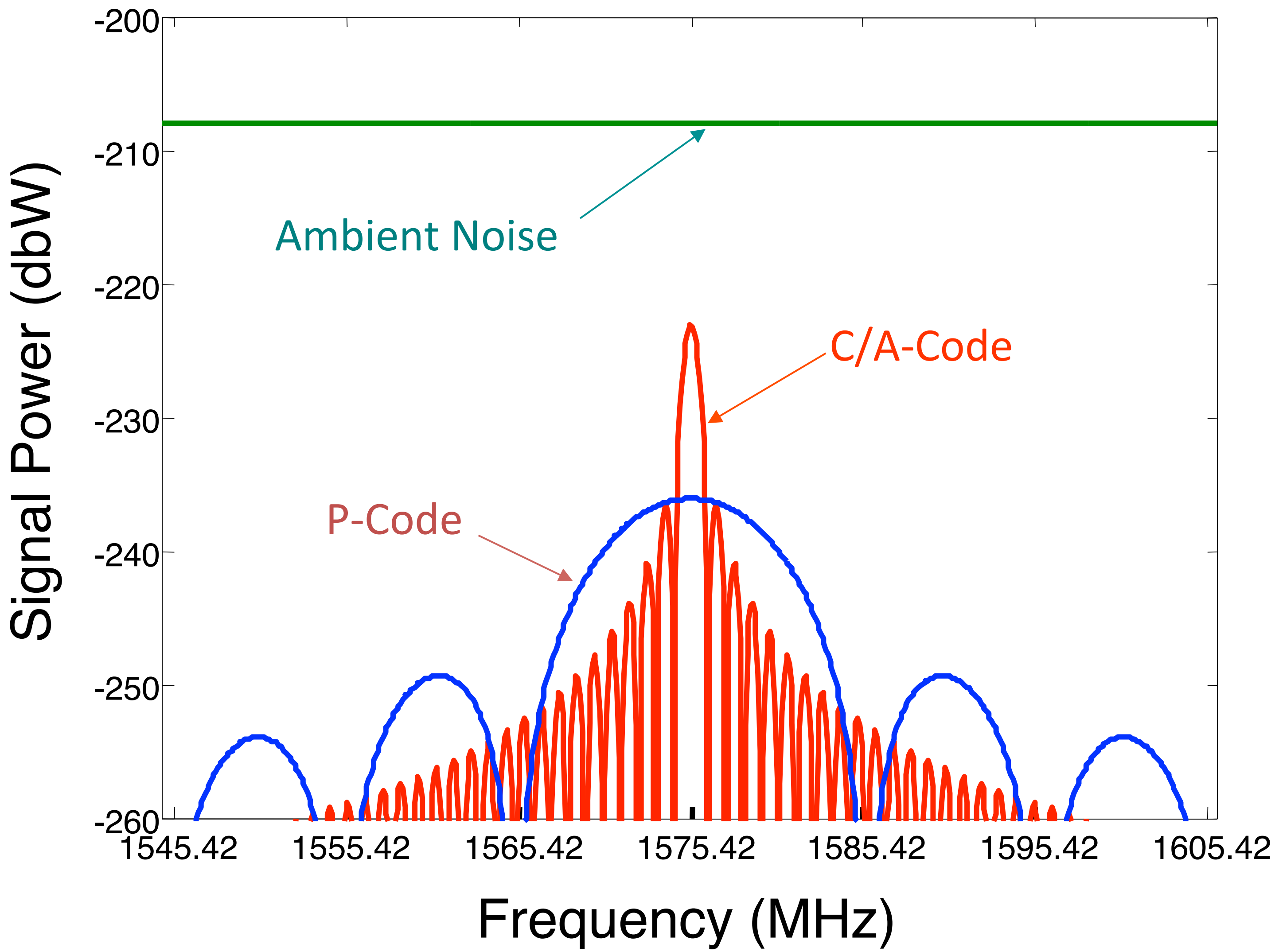
P-Code Portion

C/A-Code Portion

Total Signal



Comparison of GPS C/A-Code and P-Code Power Spectral Densities with Noise

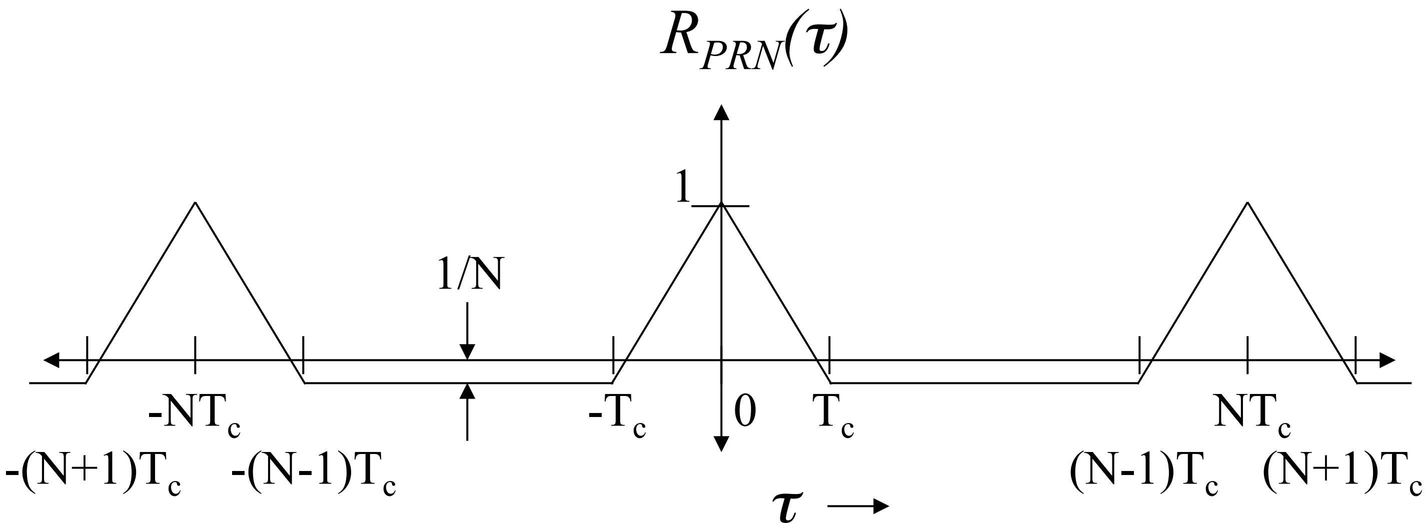


GPS Signal Autocorrelation

- **Definition of autocorrelation for function *g(t)*:**

$$R(\tau) = \int_{-\infty}^{\infty} g(t)g(t+\tau)dt$$

- **Autocorrelation function for maximum length PRN sequence (code amplitude of +/- 1)**

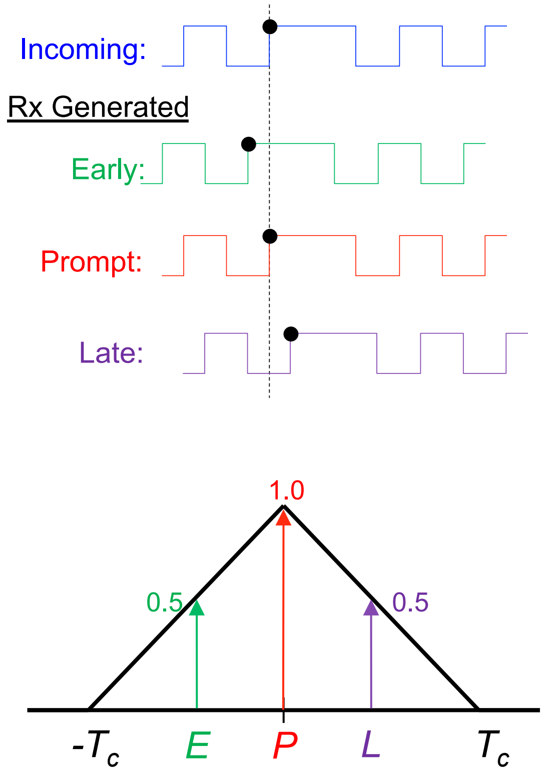


T_c = chipping period (1/chipping rate)
NT_c = code repeat period (i.e., repeats after N chips)
(N = 1023 for C/A-code)

How the PRN Code is Tracked

Aligned

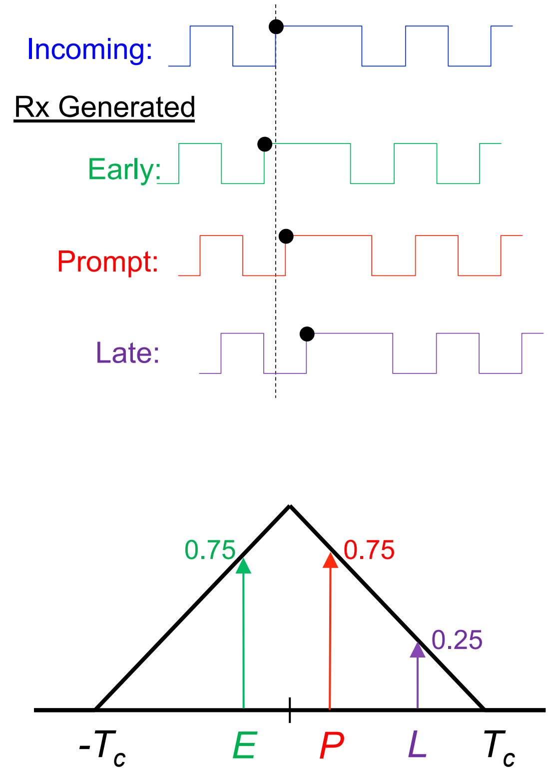
$\tau = 0$



E – L = 0

0.25 Chip Early

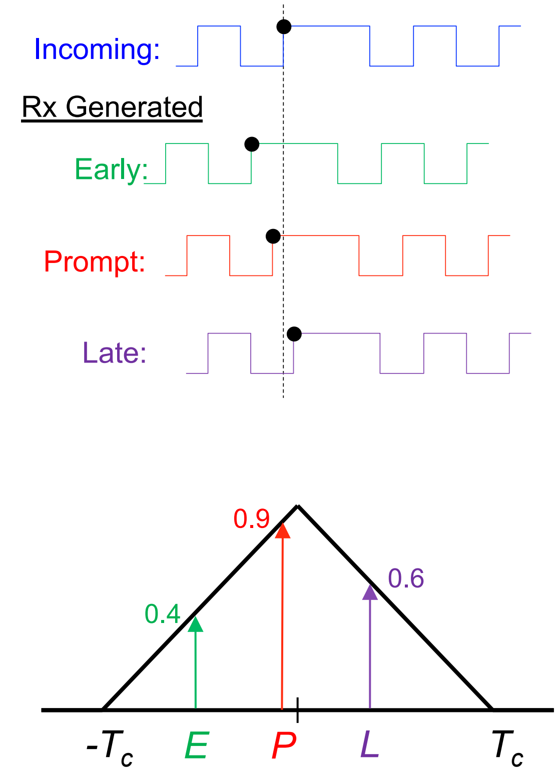
$\tau = 0.25$



E – L = 0.5

0.1 Chip Late

$\tau = -0.1$

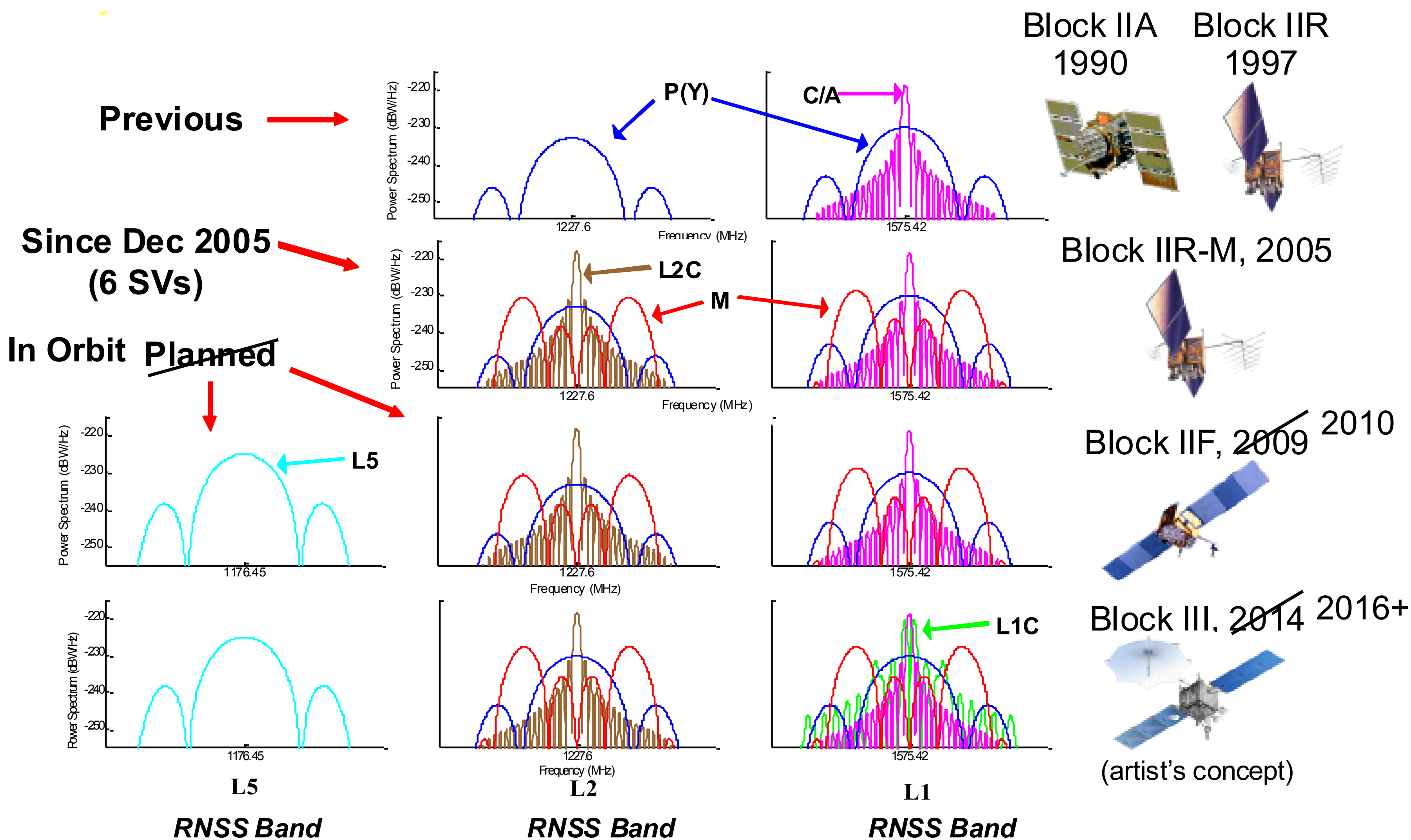


E – L = -0.2

Modernized GPS Signals

- **L2C – Block IIR-M SV's and later**
  - Contains CM and CL Codes (Civilian Moderate and Long)
    - CM has CNAV Data Modulation
    - CL has NO Data Modulation
  - CNAV is half rate of 'standard NAV' and has several important improvements including Forward Error Correction and information to link GPS to other GNSS systems
- **M – Block IIR-M SV's and later**
  - Centered on L1 and L2 frequencies
  - Binary Offset Carrier (BOC) 5.2 w/ bandwidth of 24 MHz
- **L5 – Block IIF (and tested on late IIR-M's)**
  - Two ranging codes transmited- I5 and Q5 (in-phase and quad)
  - I5 and Q5 10,230 bit sequences transmitted at 10.23 MHz

GPS Signal Modernization



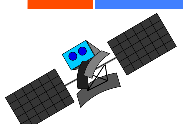
 Extracted from presentation by D. Goldstein, presented at ENC GNSS 09 (http://www.gps.gov/multimedia/presentations/2009/05/ENC/), with updates

SECTION 6:
GPS MEASUREMENT ERRORS

Pseudorange Errors

- **The pseudorange measurement includes**
  - True (geometric) range between receiver and satellite
  - Satellite clock error (or residual error after SV clock correction Δt_{sv} is applied)
  - Receiver clock error
  - Other errors due to atmosphere, selective availability, receiver hardware, etc (δt_D)
    - Lumped together in δt_D term

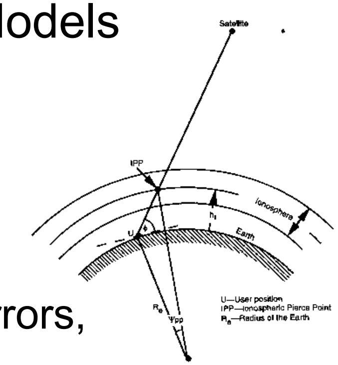
SV Clock Error | True Range | Receiver Clock Error | Other Errors (δt_D)



- For single point positioning
  - Receiver clock error is estimated
  - Other errors are ignored (and cause errors in position and time solution)
  - It’s important to understand these errors

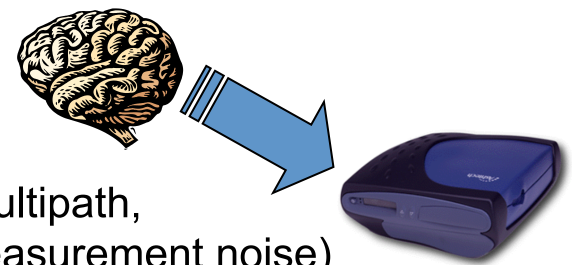
Ways of Correcting Additional Errors

Apply Error Models



(atmospheric errors, multipath(?))

Improved Receiver Processing Algorithms



(multipath, measurement noise)

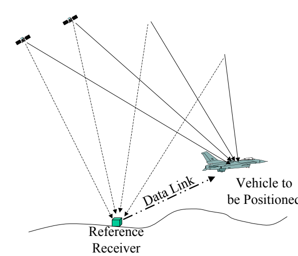
Use Externally Generated Data

SP3

Ionospheric Grid

(SV position, ionosphere)

Differential GPS



(SV position, atmospheric errors, clock errors)

Pseudorange Error Equation

- **Pseudorange ρ is**

$$\rho = r + c(\delta t_u - \delta t_{sv} + \delta t_D)$$

$r = c(T_u - T_s) = c\Delta t =$ geometric range

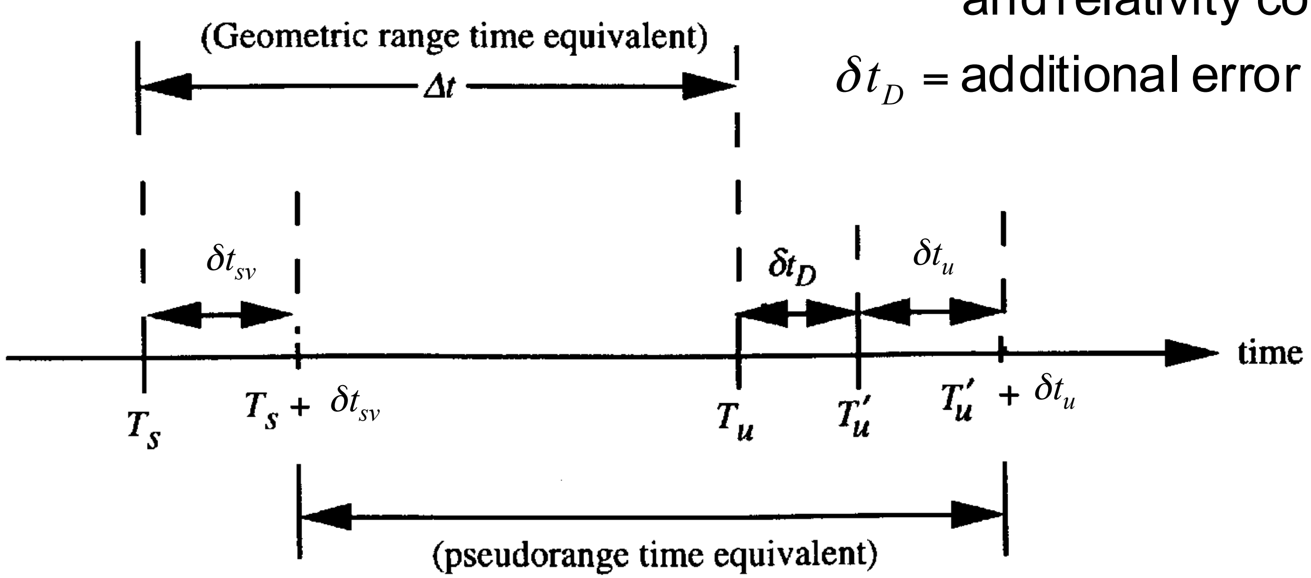
$T_u =$ time signal would have been received if there were no errors (theoretical)

$T_s =$ true signal transmit time

$\delta t_u =$ receiver clock error

$\delta t_{sv} =$ satellite clock error (after polynomial and relativity corrections applied)

$\delta t_D =$ additional error effects



Kaplan, *Understanding GPS: Principles and Applications*, Artech House, 1996

Pseudorange Errors

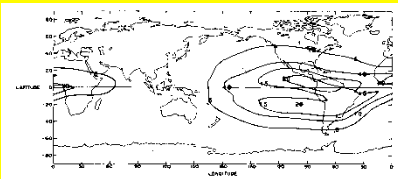
- **All non-clock errors are combined in the δt_D error term**

$$\delta t_D = \delta t_{iono} + \delta t_{trop} + \delta t_{noise\&res} + \delta t_{mp} + \delta t_{hw}$$

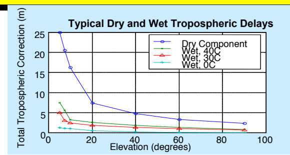
$$\begin{aligned}
\delta t_{iono} &= \text{delay due to ionosphere} \\
\delta t_{trop} &= \text{delay due to troposphere} \\
\delta t_{noise\&res} &= \text{receiver noise and resolution error} \\
\delta t_{mp} &= \text{multipath error} \\
\delta t_{hw} &= \text{hardware errors (inter-channel or} \\
&\quad \text{inter-frequency biases)}
\end{aligned}$$

- **Additionally, satellite position error will contribute to an error in the solution**
  – Not actually a measurement error, but “seems” like a measurement error
- **Each error will be covered separately**
  – Emphasis on pseudorange errors
  – Carrier-phase errors often related

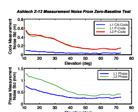
GPS Measurement Errors



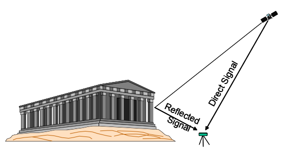
Ionosphere



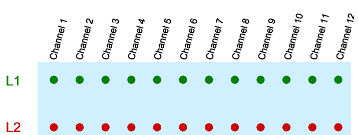
Troposphere



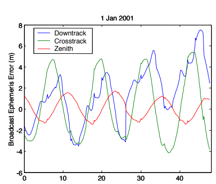
Receiver noise



Multipath



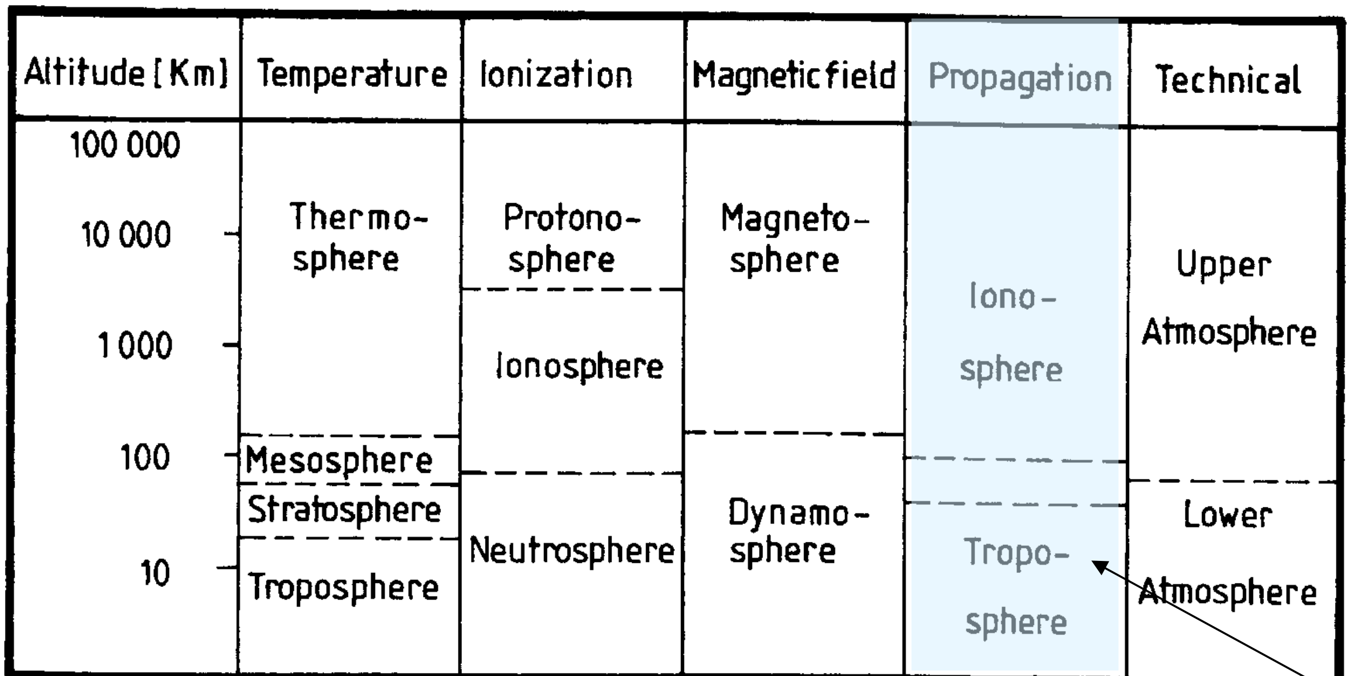
Hardware errors



Broadcast ephemeris error

Atmospheric Errors with GPS

- **There are many different ways to describe the atmosphere**



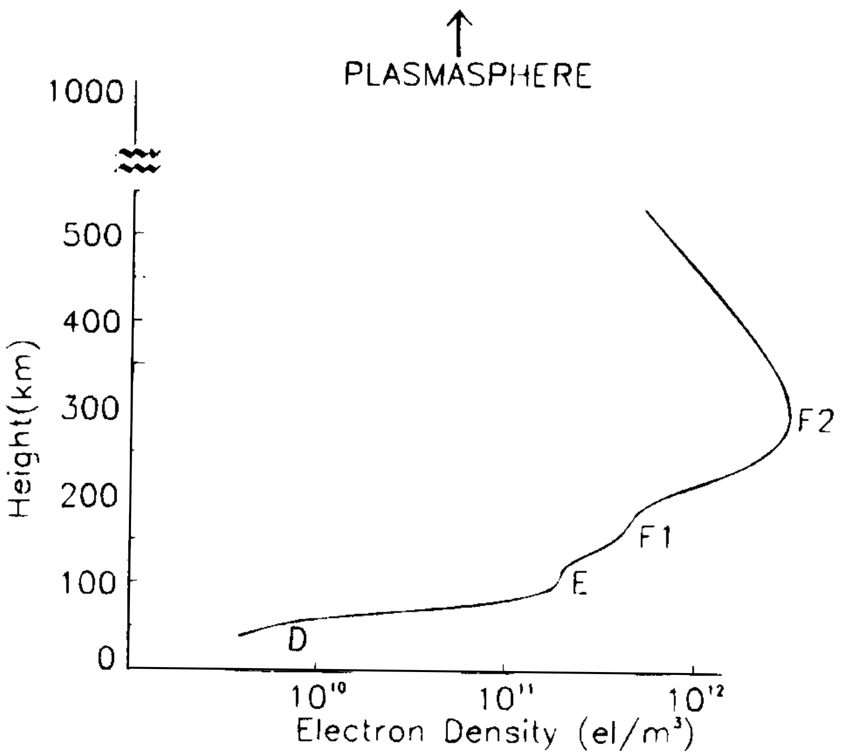
Seeber, *Satellite Geodesy*, Walter de Gruyter & Co., 1993.

Our Definition

- **For GPS, atmosphere usually divided up into troposphere and ionosphere**
  - Troposphere: neutral (uncharged) atmosphere
  - Ionosphere: free ions

Ionosphere

- Weakly ionized plasma that can affect radio waves
  - Caused by ultraviolet radiation from the sun
  - Maximum density between 300-400 km
  - Effects
    - Code delay (phase advance)
    - Doppler shift
    - Refraction (bending) of radio wave
    - Amplitude or phase scintillations

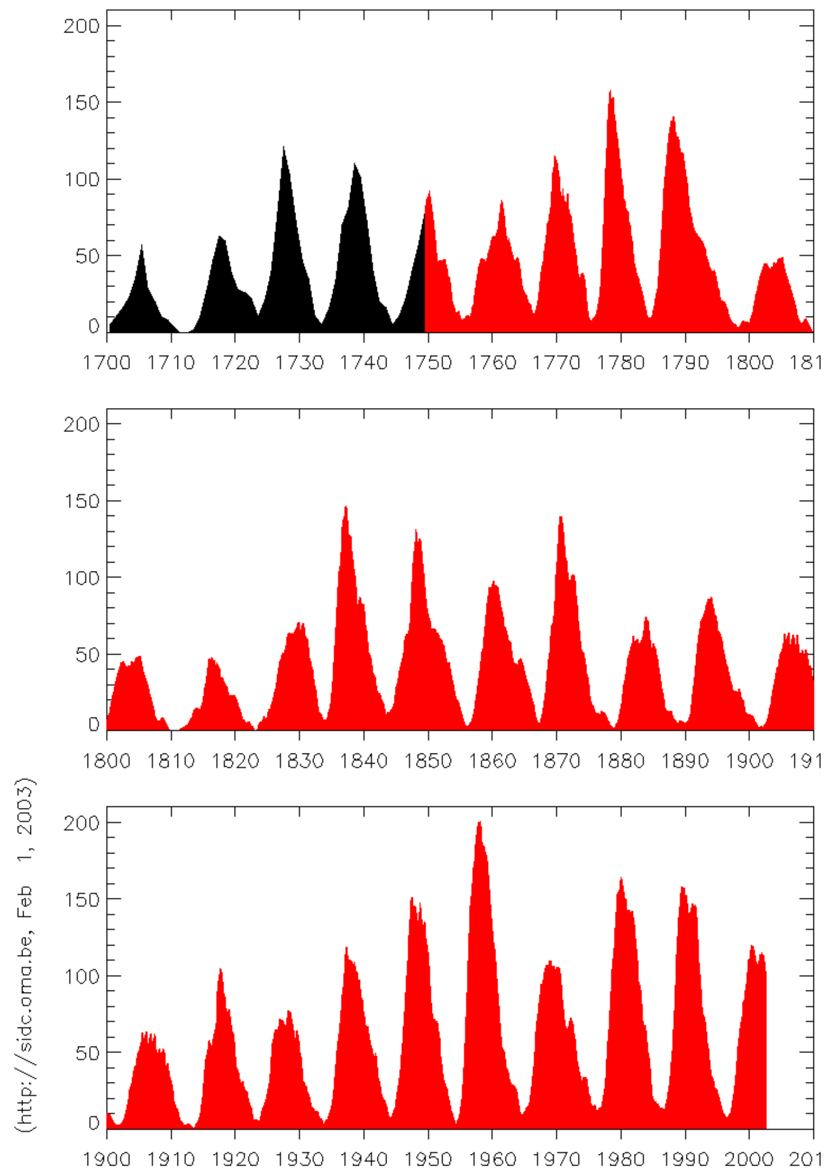


 Klobuchar, *Ionospheric Effects on GPS*, Chapter 12 of Parkinson et al (ed.), *Global Positioning System: Theory and Aplications*, AIAA, 1996.

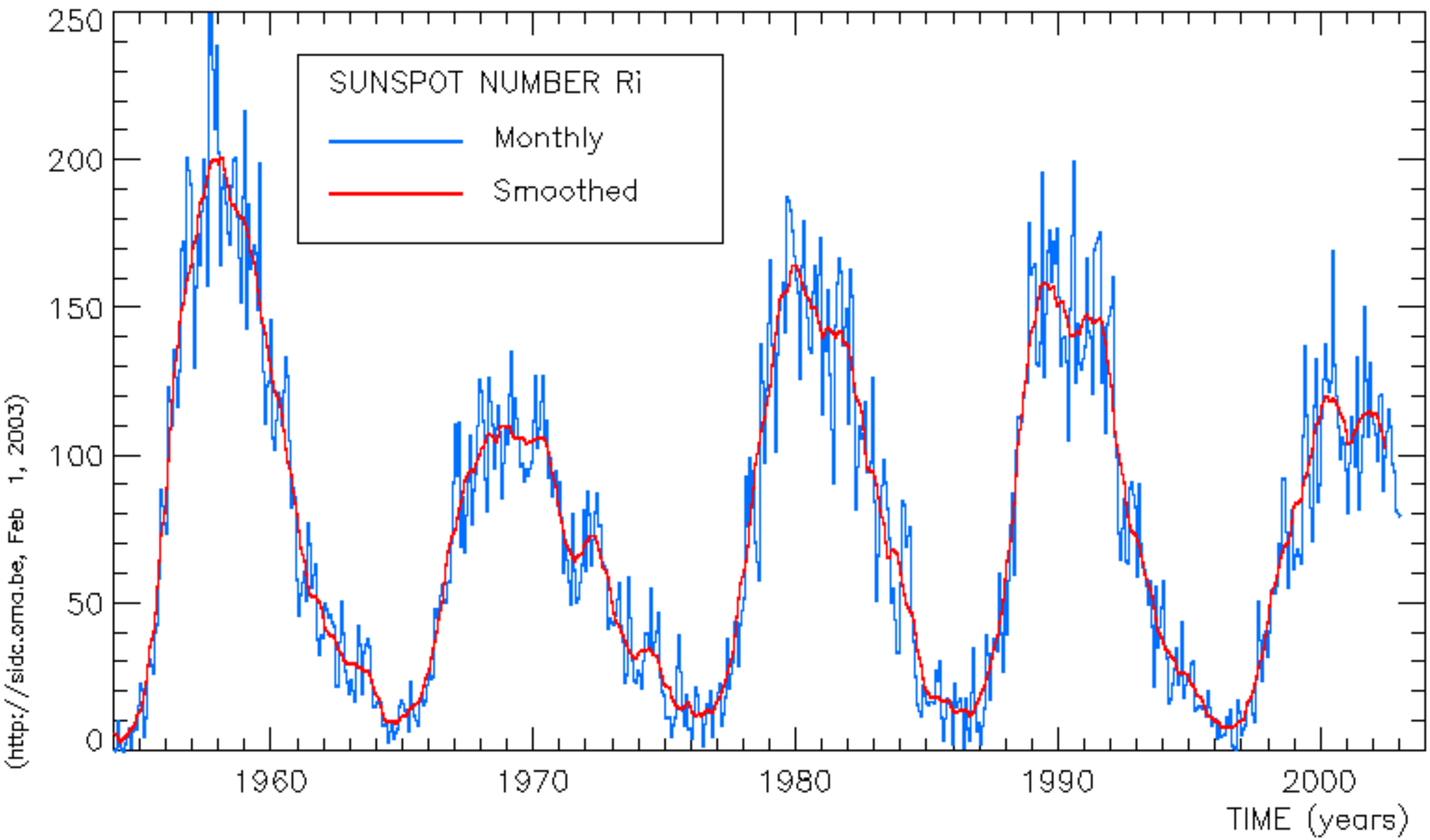
Solar Cycle

- **Ionospheric activity is directly related to the solar cycle**
  - Measured by sunspot number
  - Varies on ~11 year cycle
  - In peak around 2001-2002

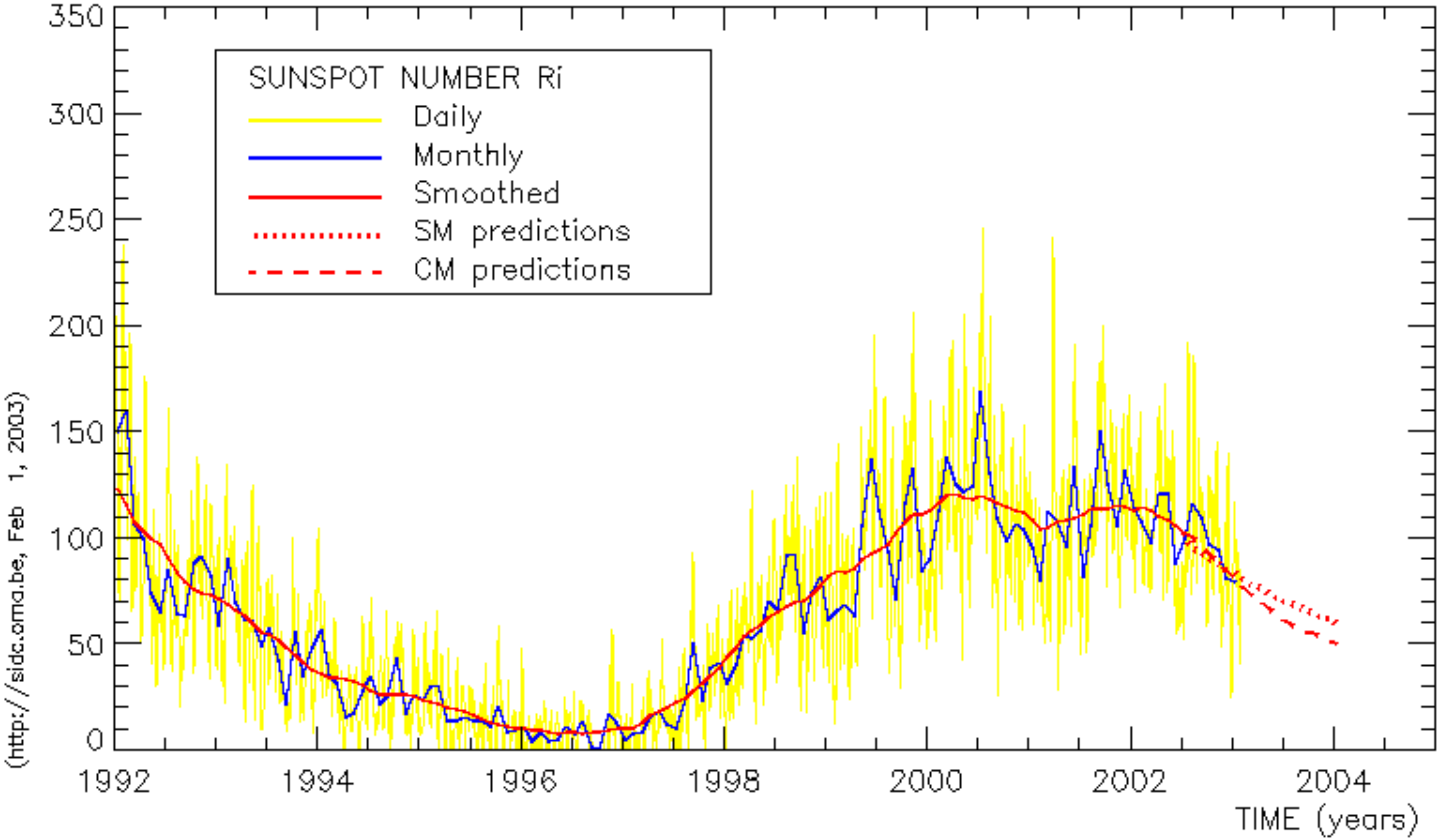
A long history of sunspot numbers



Recent Solar Cycles

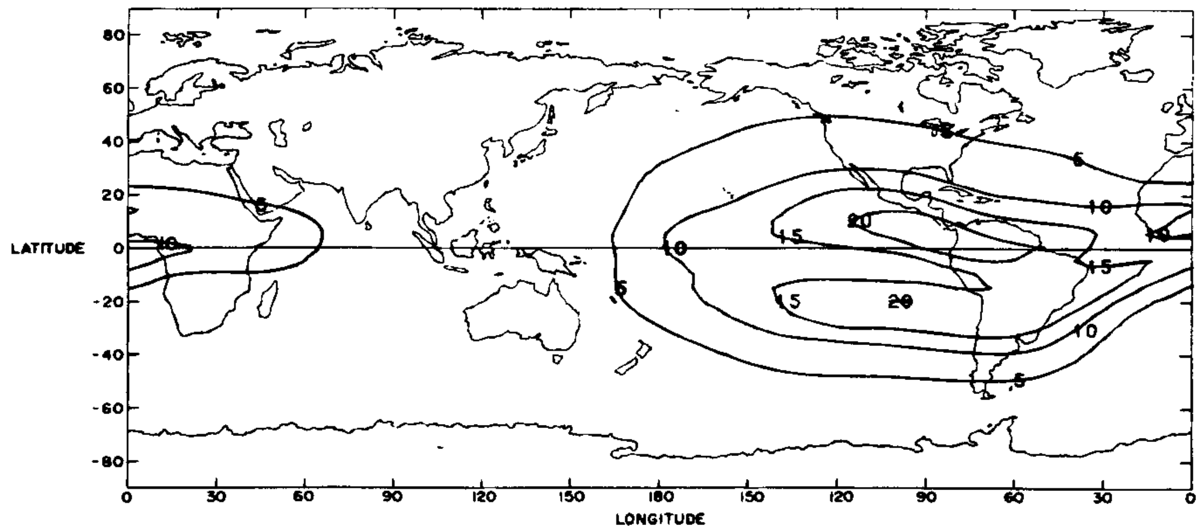


Sunspot Numbers – More Detailed View, Including One Year Predictions

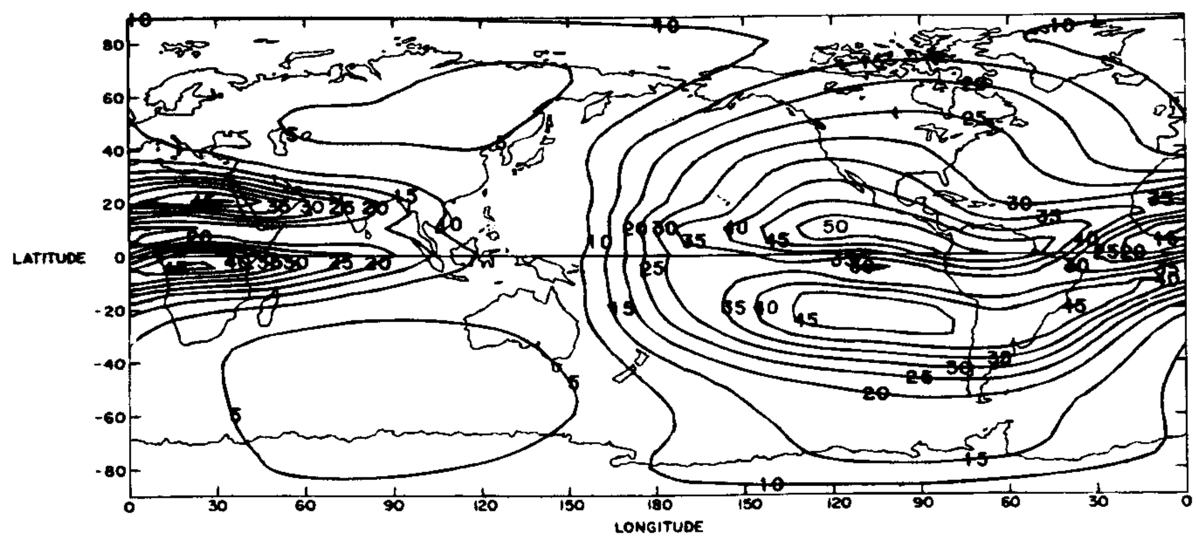


Spatial and Temporal Variation of L1 Ionospheric Delay

Zenith L1 Ionospheric Time Delay (ns) - Solar Minimum (sunspot number = 10)

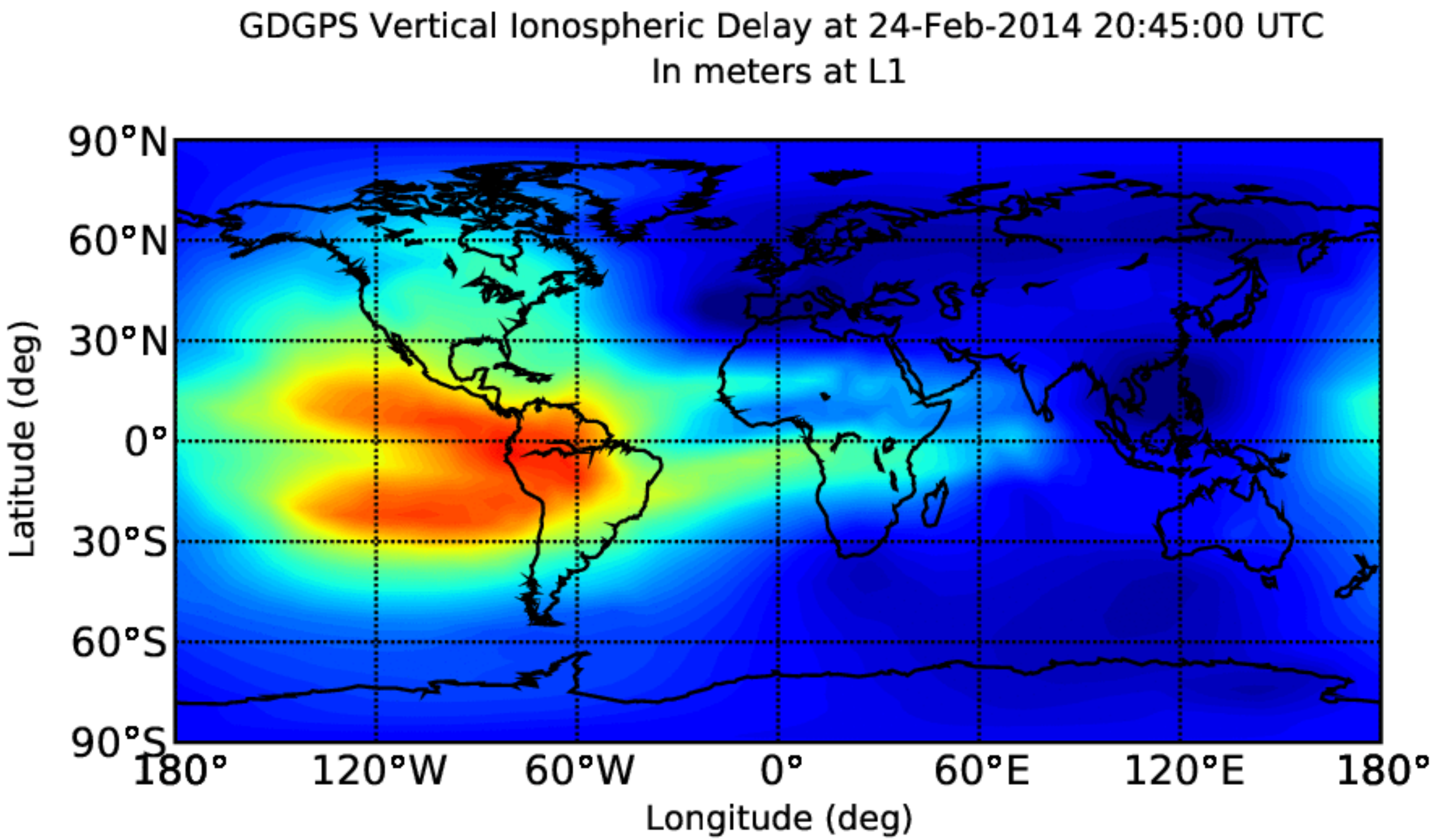


Zenith L1 Ionospheric Time Delay (ns) - Solar Maximum (sunspot number = 153)



Klobuchar, *Ionospheric Effects on GPS*, Chapter 12 of Parkinson et al (ed.), *Global Positioning System: Theory and Aplications*, AIAA, 1996.

Sample Ionosphere Plot



http://www.gdgps.net/products/tec-maps.html

Ionospheric Effects on GPS Pseudorange Measurement

- **Total Electron Content (TEC)**
  - Number of electrons in a 1m x 1m rectangular column from the receiver up through the ionosphere (units of electrons/m^2)

$$TEC = \int_{SV}^{User} n_e \, dl$$

n_e = electron density

Units of TECU (=1x10^{16} TEC)

(1 TECU = 0.16 m pseudorange error on L1)

- **Group (pseudorange) delay**

$$\Delta S_{iono,g} = \frac{40.3 \ \mathrm{TEC}}{f^2}$$

$\Delta S_{iono,g}$ = group (pseudorange) delay (m)

f = carrier frequency (L1or L2 for GPS)

- **Carrier-phase delay**

$$\Delta S_{iono,p} = -\frac{40.3 \ \mathrm{TEC}}{f^2}$$

  - Carrier-phase is actually *advanced* by ionosphere by same magnitude as group delay

Use of Dual Frequency Measurements to Calculate Ionospheric Delay

- **L1 ionospheric delay calculated by**

$$\Delta S_{iono,corr_{L1}} = \left(\frac{f_2^2}{f_2^2 - f_1^2}\right)(\rho_{L1} - \rho_{L2})$$

$$\Delta S_{iono,corr_{L1}} = \text{L1 ionospheric delay (m)}$$

$$f_1, f_2 = \text{L1 and L2 carrier frequencies}$$

$$\rho_{L1}, \rho_{L2} = \text{L1 and L2 pseudorange measurements}$$

- **L2 ionospheric delay can be calculated by**

$$\Delta S_{iono,corr_{L2}} = \left(\frac{f_1}{f_2}\right)^2 \Delta S_{iono,corr_{L1}}$$

- **Ionospheric-free pseudorange:**

$$\rho_{IF} = \frac{\rho_{L2} - \gamma\rho_{L1}}{1-\gamma}, \qquad \gamma = \left(\frac{f_{L1}}{f_{L2}}\right)^2 = \left(\frac{77}{60}\right)^2$$

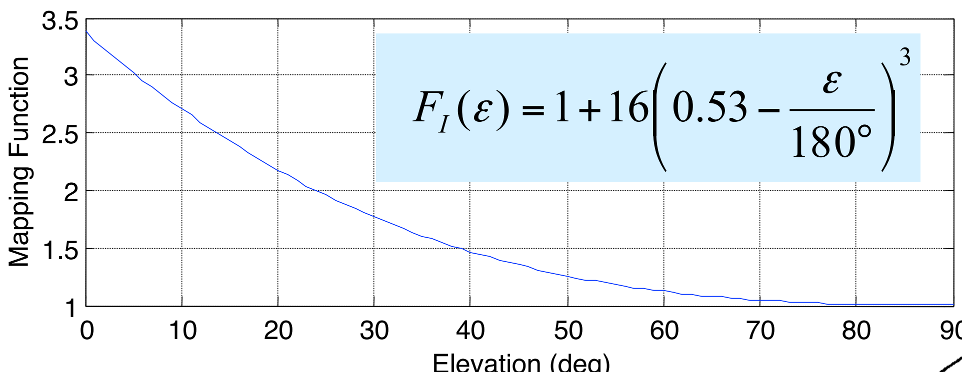
- **Multipath and measurement noise will corrupt this measurement of ionosphere**

Ionospheric Mapping Functions

- **Mapping function (or obliquity factor) is used to relate ionospheric error at elevation ε with ionospheric error at zenith:**

$$\delta t_{iono} = F_I(\varepsilon)\,\delta t_{iono,zenith}$$

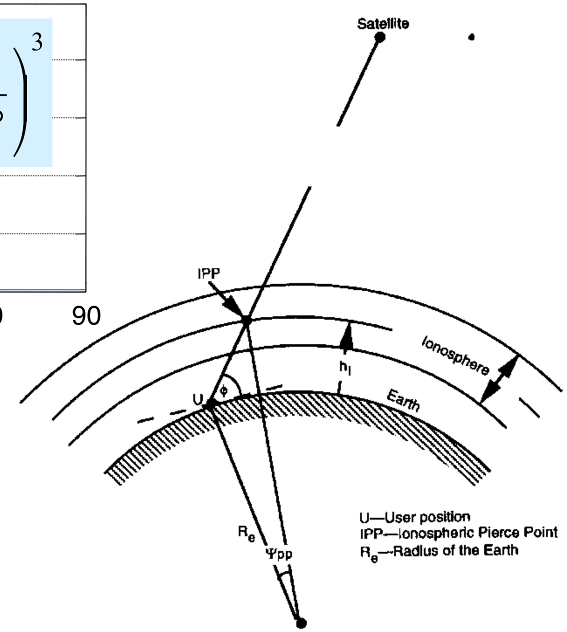
  – Mapping function used in Klobuchar model (from ICD-GPS-200) (elevation ε in degrees)



$$F_I(\varepsilon) = 1 + 16\left(0.53 - \frac{\varepsilon}{180^\circ}\right)^3$$

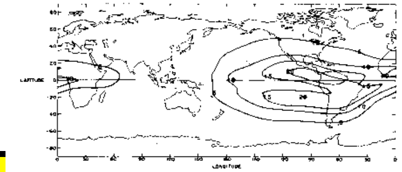
  – Mapping function used by WAAS

$$F_I(\varepsilon) = \left[1 - \left(\frac{R_e \cos\phi}{R_e + h_I}\right)^2\right]^{-\frac{1}{2}}$$

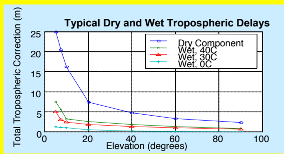


Kaplan, *Understanding GPS: Principles and Applications*, Artech House, 1996.

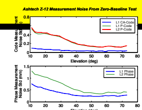
GPS Measurement Errors



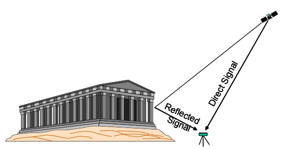
Ionosphere



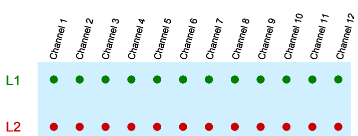
Troposphere



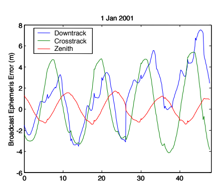
Receiver noise



Multipath



Hardware errors



Broadcast ephemeris error

Tropospheric Delay (δt_{trop})

- **Troposphere is defined as the neutral atmosphere**
  - Actual troposphere (~0-10 km)
    - Contributes about 75% of "tropospheric" error
  - Tropopause (~10-16 km)
  - Stratosphere (~16-50 km)

  $\left.\begin{array}{l}\text{Tropopause} \\ \text{Stratosphere}\end{array}\right\}$ Contribute about 25% of error

- **Tropospheric delay expressed as**

$$\delta t_{trop} = \int_{path} (n-1)\,ds + \Delta_g$$

n = refractive index $= \dfrac{c_v}{c_m}$

c_v = velocity in vacuum

c_m = velocity in medium (air in this case)

Δ_g = error due to path curvature

- **Sometimes refractivity (N) is used**

$$N = (n-1)\times 10^6$$

  - Average N on Earth's surface: 320

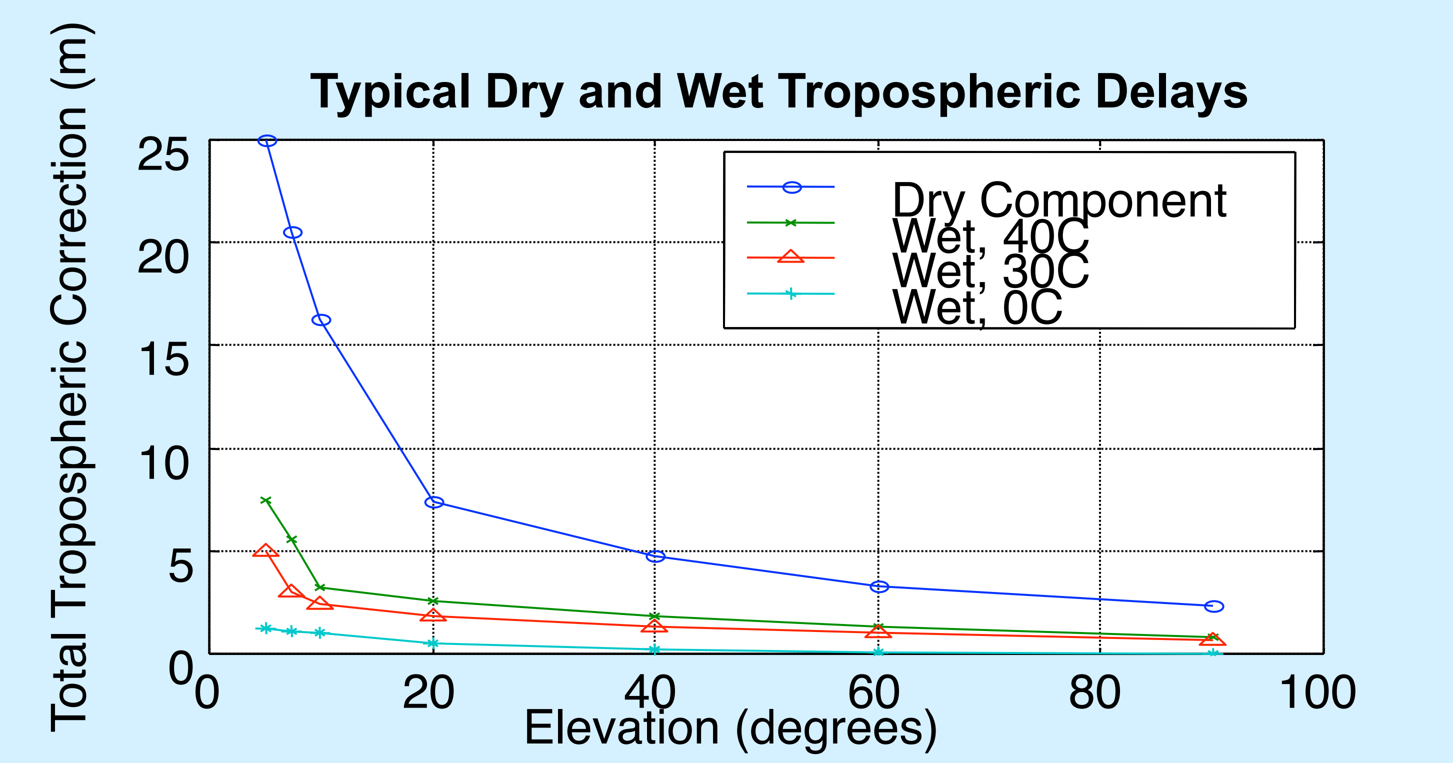
Characteristics of Tropospheric Error

- **Two different terms can be calculated**

Dry — 80-90% of total error
Predictable to 1% @ zenith

Wet — 10-20% of total error
Predictable to 10-20% @ zenith

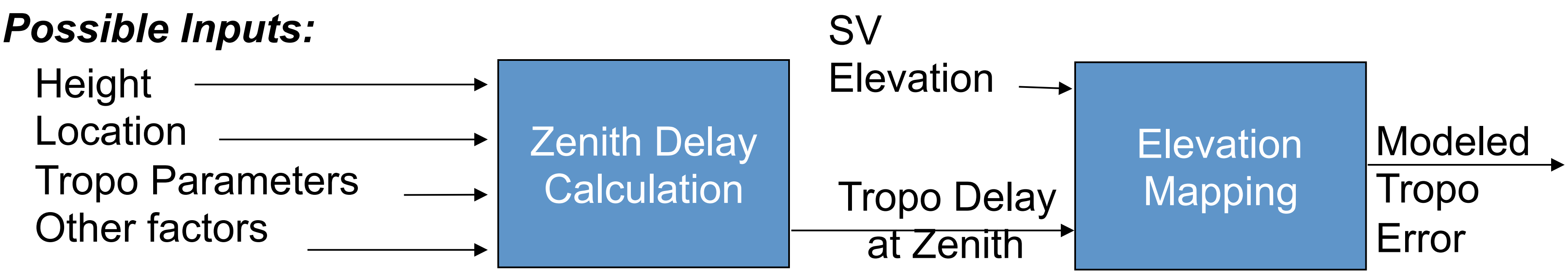
Total = Wet + Dry



Data from Black and Eisner, “Correcting Satellite Doppler Data for Tropospheric Effects”, *Proc. Of International Symposium on the Use of Artificial Satellites for Geodesy and Geodynamics*, National Technical Univ. of Athens, 1984.

Tropospheric Models

- **Many models exist for modeling troposphere**



  - Most are accurate at elevation angles > 15°
  - Can be significant differences for low elevations
  - More common models:
    - Saastamoinen (Saastamoinen, "Contributions to the Theory of Atmospheric Refraction." In three parts. *Bulletin Geodesique*, No. 105, pp. 279-298; No. 106, pp. 383-397; No. 107, pp. 13-34.)
    - Black & Eisner (Black and Eisner, "Correcting Satellite Doppler Data for Tropospheric Effects." *Journal of Geophysical Research*, Vol. 89, No. D2, pp. 2616-2626.)
    - Marini & Murray (Marini and Murray, "Correction of Laser Range Tracking Data for Atmospheric Refraction at Elevations Above 10 Degrees." Goddard Space Flight Center Report X-591-73-351, NASA GSFC, Greenbelt, MD, 1973.)

Tropospheric Mapping Functions

- **Mapping function is used to relate tropospheric error at elevation ε with tropospheric error at zenith:**

$$\delta t_{trop} = F_T(\varepsilon)\,\delta t_{trop,zenith}$$

- Chao mapping function[1]
  - Wet component:

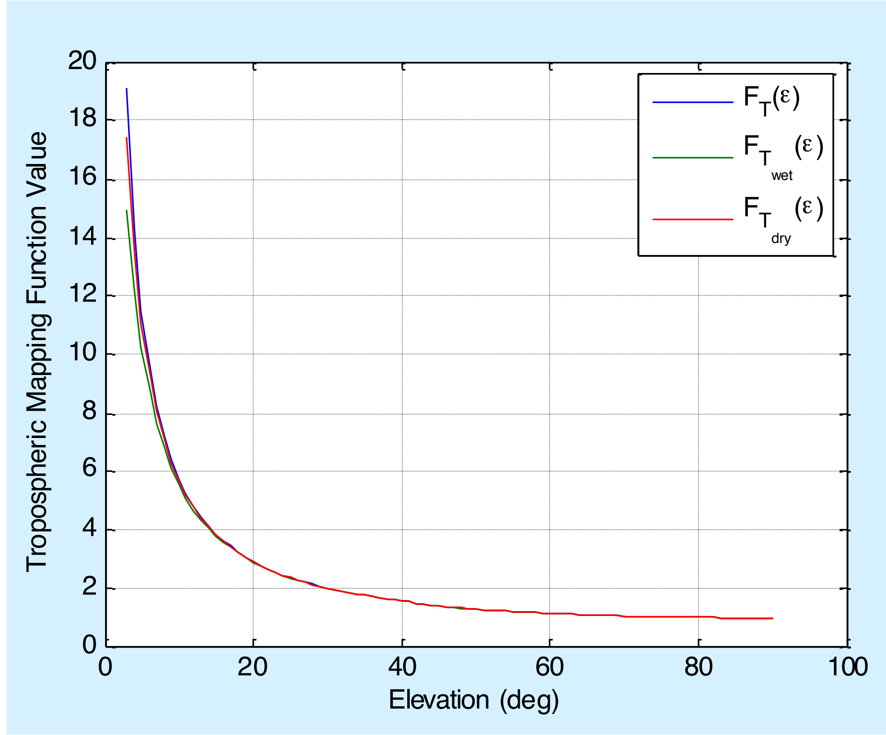
$$F_{T_{wet}}(\varepsilon) = \frac{1}{\sin\varepsilon + \dfrac{0.00143}{\tan\varepsilon + 0.0445}}$$

  - Dry component

$$F_{T_{dry}}(\varepsilon) = \frac{1}{\sin\varepsilon + \dfrac{0.00035}{\tan\varepsilon + 0.017}}$$

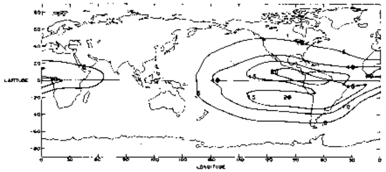
- Simplified troposphere mapping function (useful for many applications)

$$F_T(\varepsilon) = \frac{1}{\sin\varepsilon}$$

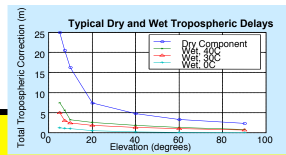


[1] C. Chao, “The Tropospheric Calibration Model for Mariner Mars, 1971,” JPL TR 32-1587, Jet Propulsion Laboratory, 1974.

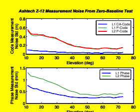
GPS Measurement Errors



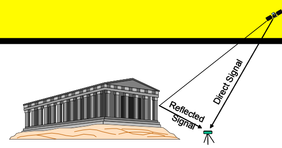
Ionosphere



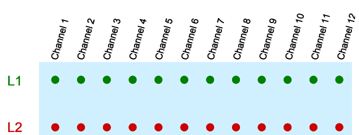
Troposphere



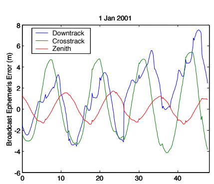
Receiver noise



Multipath



Hardware errors



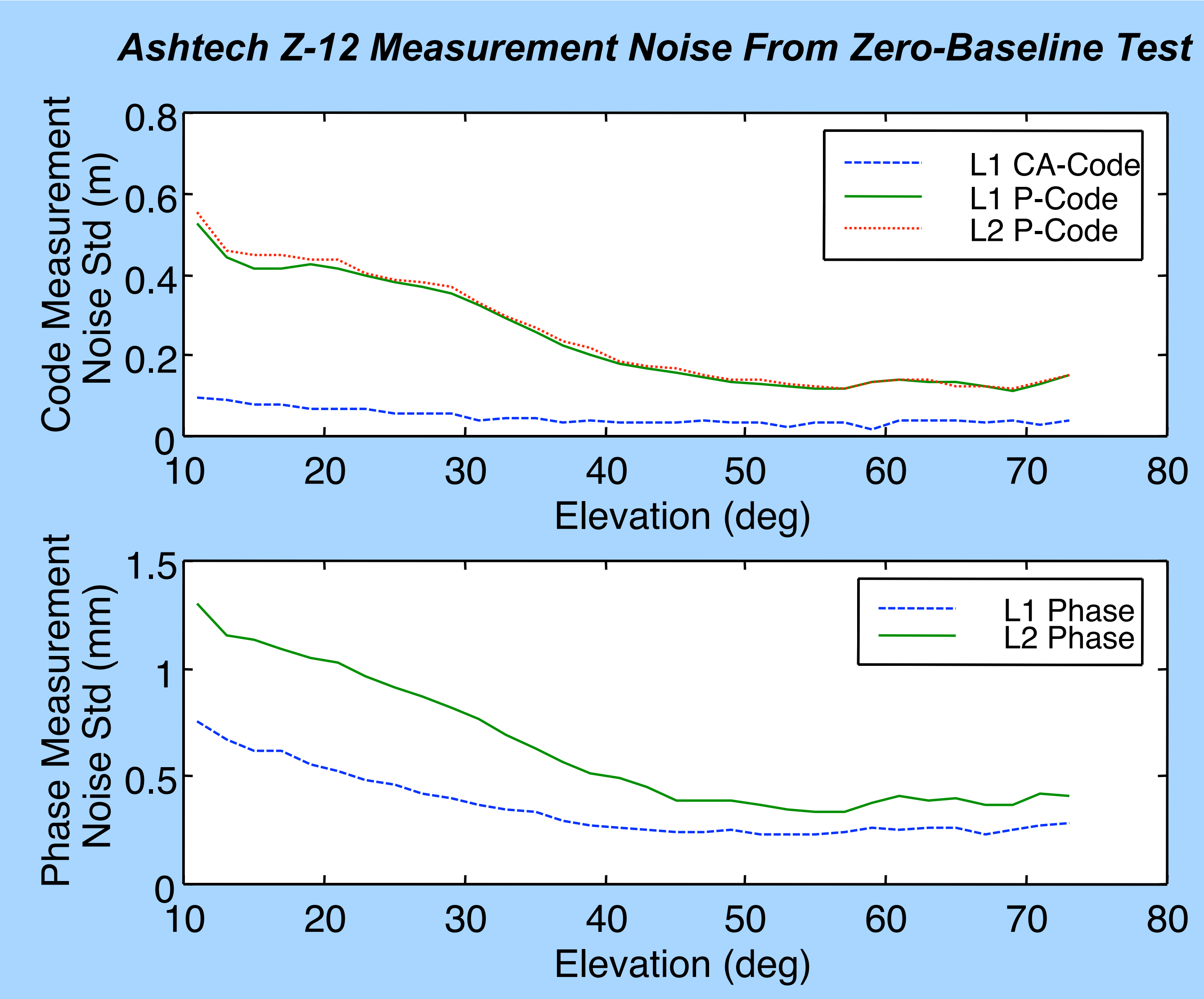
Broadcast ephemeris error

Measurement Noise

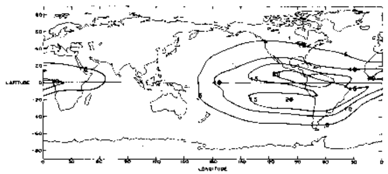
- **As with any measuring device, measurements from a GPS receiver exhibit measurement noise**
  - Uncorrelated in time at typical sampling intervals (i.e., white noise)
  - Gaussian probability density function
  - Zero-mean
  - Correlated with C/N_0 (lower $C/N_0 \rightarrow$ more noise)
- **Quantization error also lumped with noise**
  - Error due to LSB roundoff
  - Depends upon receiver implementation and the way data is reported from the receiver
- **Normally, measurement noise is not a significant problem**
  - Easy to remove by filtering
  - No bias

Sample of Measurement Noise vs. Satellite Elevation

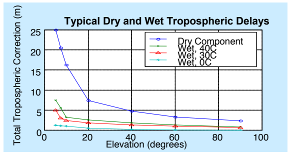
- **Measurement noise vs. elevation**
  - Based upon "zero-baseline" test (single antenna, two receivers)
  - Low elevation satellites typically have lower C/N_0 values



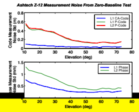
GPS Measurement Errors



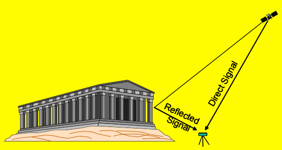
Ionosphere



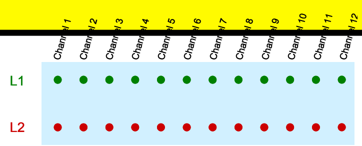
Troposphere



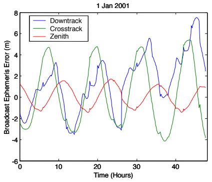
Receiver noise



Multipath



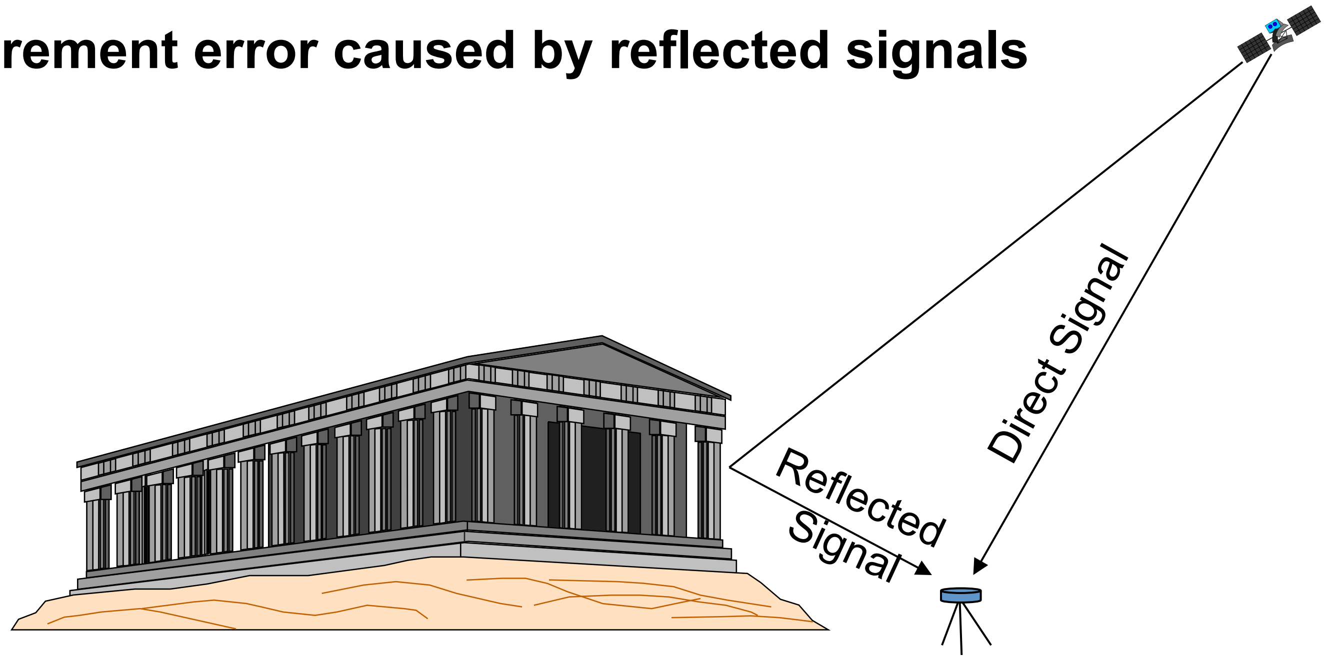
Hardware errors



Broadcast ephemeris error

Multipath

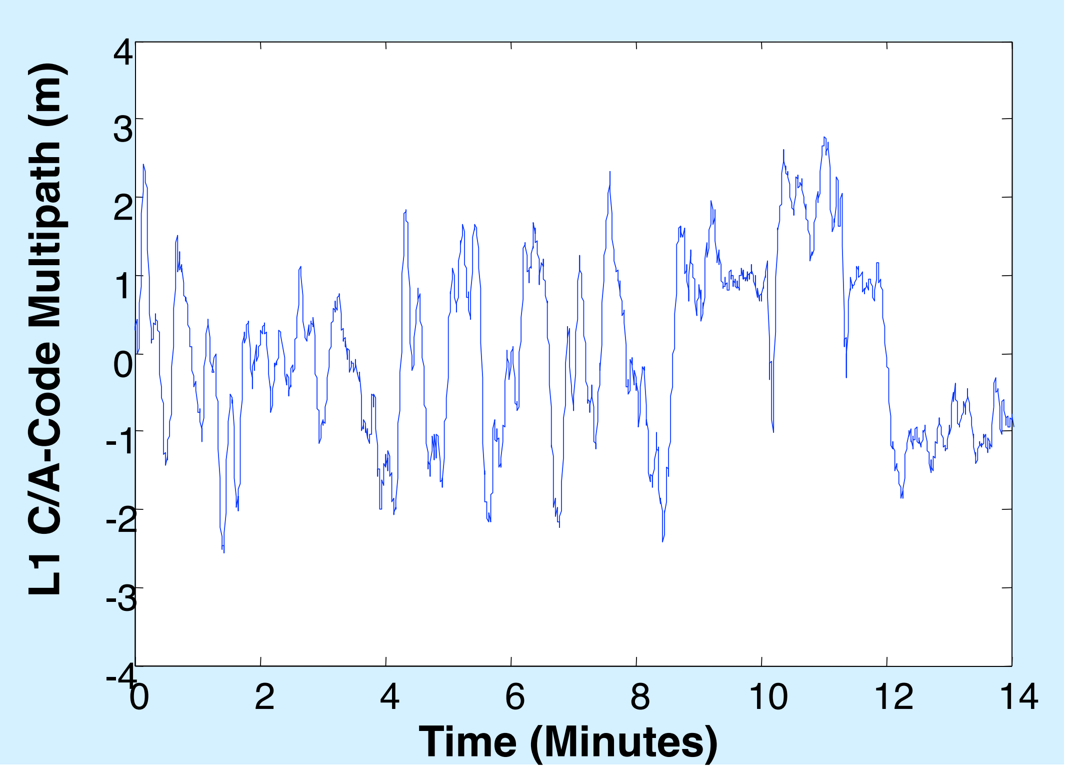
- **Measurement error caused by reflected signals**



  – Can affect both code and carrier-phase measurements
  – Potential multipath sources
    - Ground
    - Water
    - Buildings
    - Heating ducts

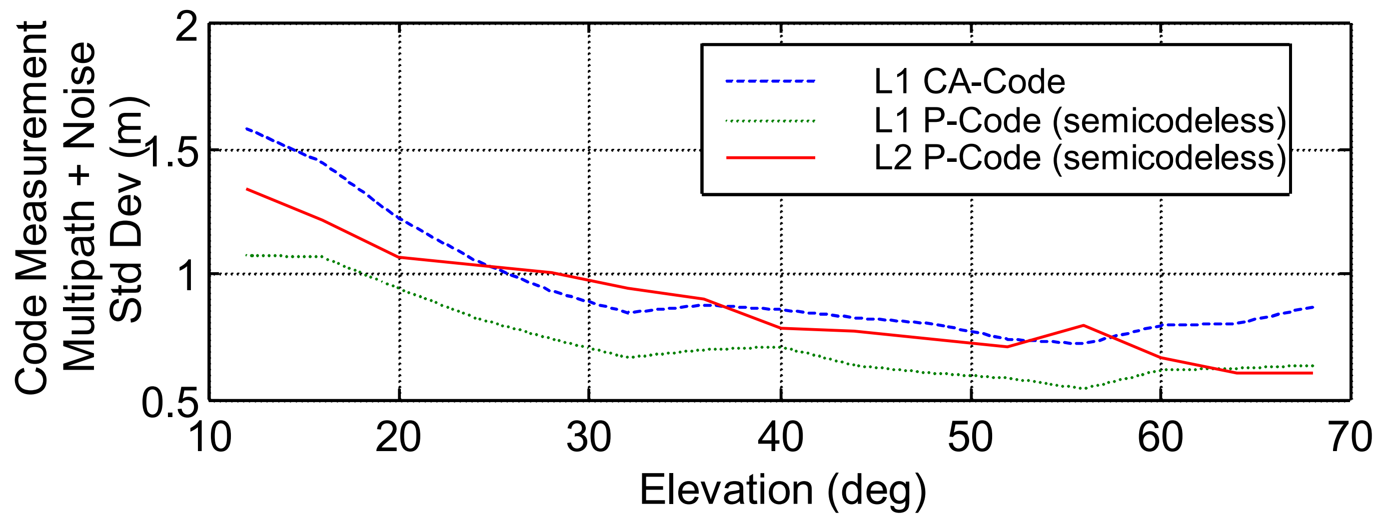
Pseudorange Multipath Example--Aircraft Sitting on Tarmac

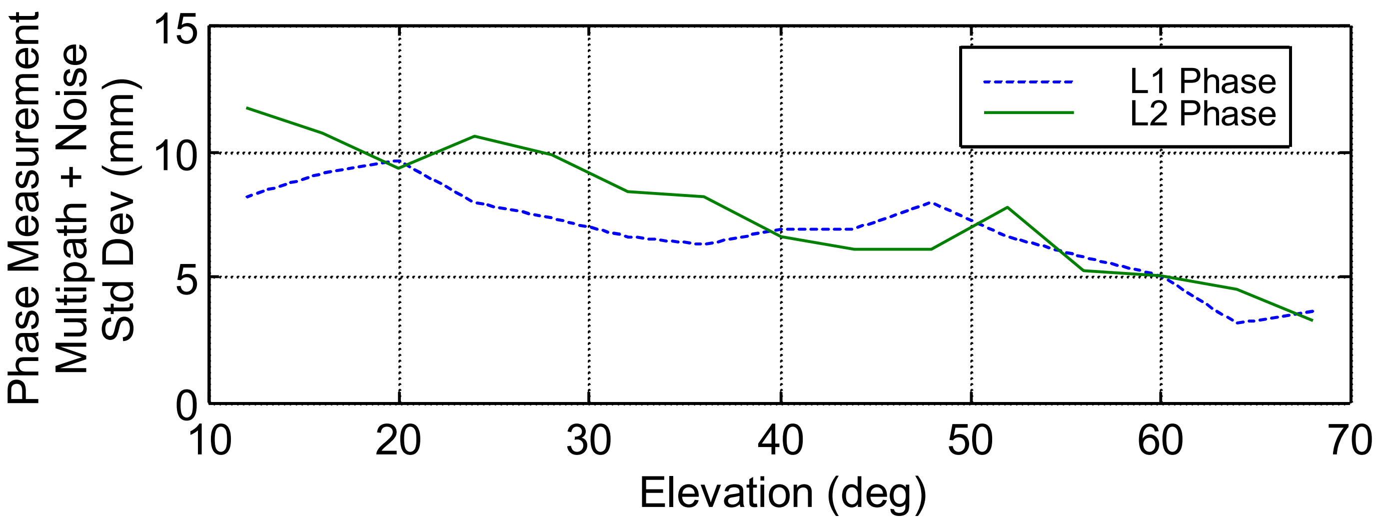
- **Plot of multipath plus measurement noise**
  - Ashtech Z-Surveyor in T-38 on tarmac
  - Generated from code-minus-carrier observable
    - Absolute bias unknown
  - Code-carrier ionospheric divergence removed by fitting to 2nd order polynomial
  - Note periodic nature



Examples of Multipath (plus Noise) vs. SV Elevation Ashtech Z-12

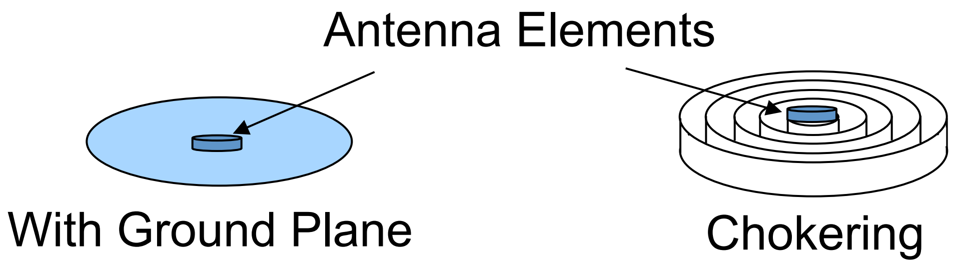
Ashtech Z-12 Receiver, Dorne-Margolin Groundplane Antenna





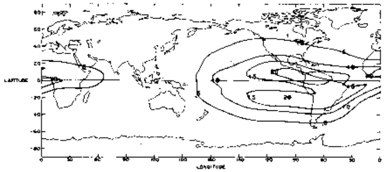
Multipath Mitigation Techniques

- **Antenna-based approaches**
  - Place antenna in low-multipath environment
  - Use groundplane or chokering antenna

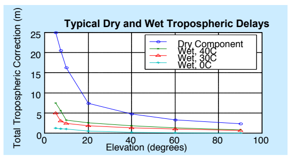


- **Measurement-based approaches**
  - Carrier-phase smoothing of the code
  - Use of C/N_0 and antenna gain pattern to estimate multipath
- **Receiver processing-based approaches**
  - Narrow correlator spacing
  - Advanced signal processing techniques
- **Other approaches**
  - Use of multiple receivers
  - Modeling of environment around antenna

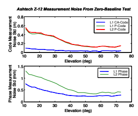
GPS Measurement Errors



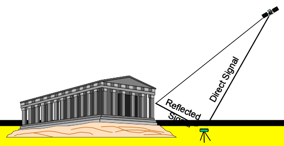
Ionosphere



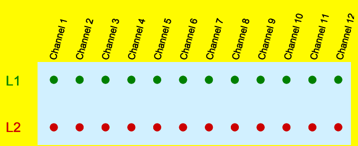
Troposphere



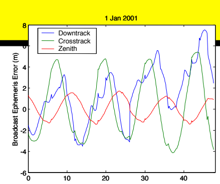
Receiver noise



Multipath

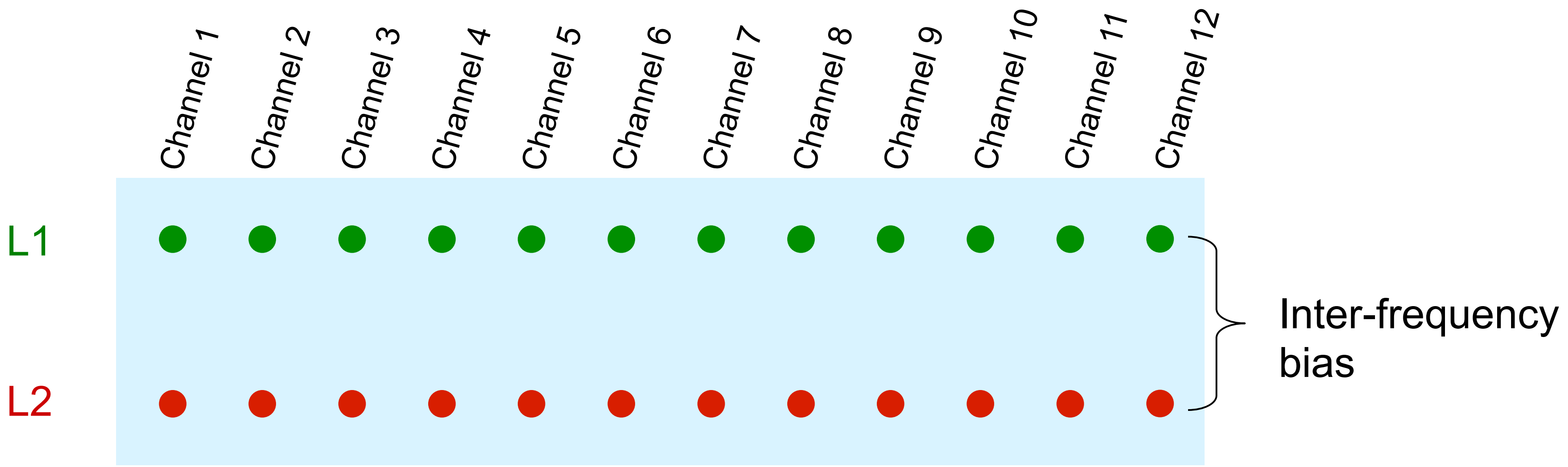


Hardware errors



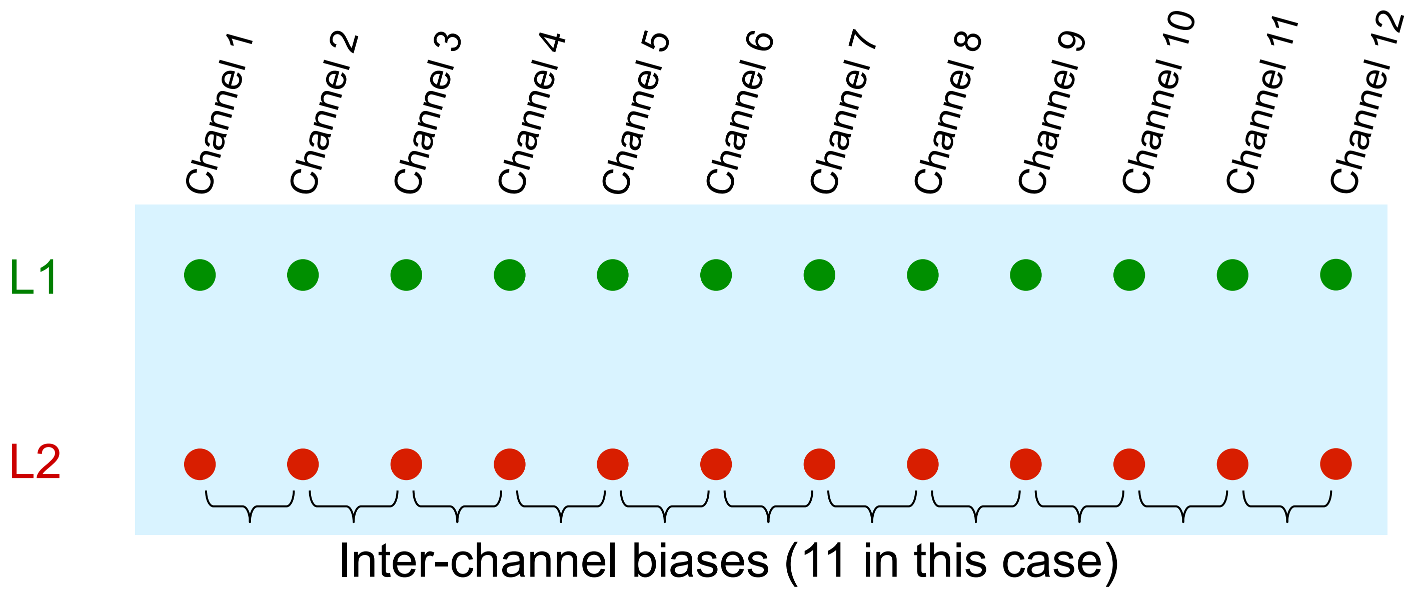
Broadcast ephemeris error

Hardware Errors – Inter-frequency Bias



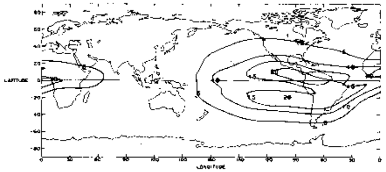
- **Measurement bias between L1 and L2 measurements**
  – Consistent across all receiver channels
- **Adverse effects**
  – Does not affect position solution (Why?)
  – Can affect ionospheric study
  – Can affect clock estimate

Hardware Errors – Inter-channel Bias

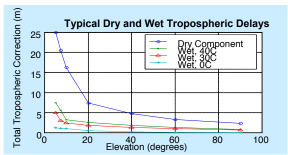


- **Measurement bias between channels in a receiver**
  - Typically very small (can be calibrated out in the factory)
  - Can be determined/removed by tracking a common satellite (like RCVR-3A)
- **Adverse effects**
  - Does not affect ionospheric study
  - Does affect position solution (although typically too little to worry about)
  - Does affect timing solution (although typically too little to worry about)

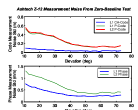
GPS Measurement Errors



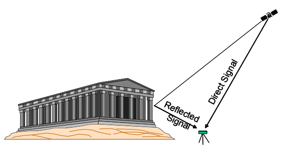
Ionosphere



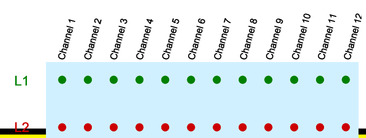
Troposphere



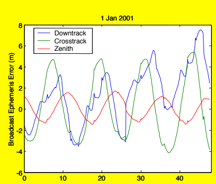
Receiver noise



Multipath



Hardware errors



Broadcast ephemeris error

Satellite Position Error

- **Perturbing forces and measurement error (by OCS) prevent perfect prediction of satellite positions**
  - Most can be modeled accurately
  - Even small errors can lead to significant position errors (due to integration)
    - Example: Venus gravity can result in up to 80 cm of error after a day, if not accounted for

Force	Acceleration (m/s^2)
Earth gravity modeled as point mass	6.1×10^{-1}
Earth gravity oblateness modeled by the J2 coefficient	1.0×10^{-4}
Lunar gravity	3.9×10^{-6}
Solar gravity	1.0×10^{-6}
Summed effect of Earth gravity field, coefficients 2,1 to 4,4	2.2×10^{-7}
Solar radiation pressure	7.2×10^{-8}
Summed effect of Earth gravity field, coefficients 5,0 to 8,8	5.9×10^{-9}
Albedo (or Earthshine)[1]	1.5×10^{-9}
Thermal re-radiation[2]	1.4×10^{-9}
Solid Earth tide, raised by the Moon	1.3×10^{-9}
Solid Earth tide, raised by the Sun	4.5×10^{-10}
Venus gravity	1.1×10^{-10}

[1]Force due to electromagnetic radiation reflected by the Earth and thermal radiation emitted by the Earth
[2]Caused by anisotropic radiation of heat from the spacecraft

Ziebart, Cross, and Adhya, Modeling Photon Pressure: The Key to High-Precision GPS Satellite Orbits, *GPS World*, Jan 2002

Satellite Position (Ephemeris) Error

- **The broadcast ephemeris are generated real-time by the control segment using data from ground GPS receivers**
  - Typically has ~1 m accuracy[1]
- **Errors in ephemeris result in satellite position errors**
  - Not truly *measurement* errors
  - Do cause errors when the measurements are used, however
    - Cause error in expected measurement value
  - In terms of error analysis, can be treated as measurement errors
- **Precise orbits (~3 cm accuracy) can be obtained from National Geodetic Survey (and other organizations)**
  - Calculated using days of data from hundreds of reference stations
  - Can be obtained over internet (http://www.ngs.noaa.gov/GPS/GPS.html)
  - Serves as useful truth reference for broadcast ephemeris errors

[1]http://www.gps.gov/systems/gps/performance/accuracy/URE.pdf

Improvement in Satellite Position/ Clock Accuracy over Time

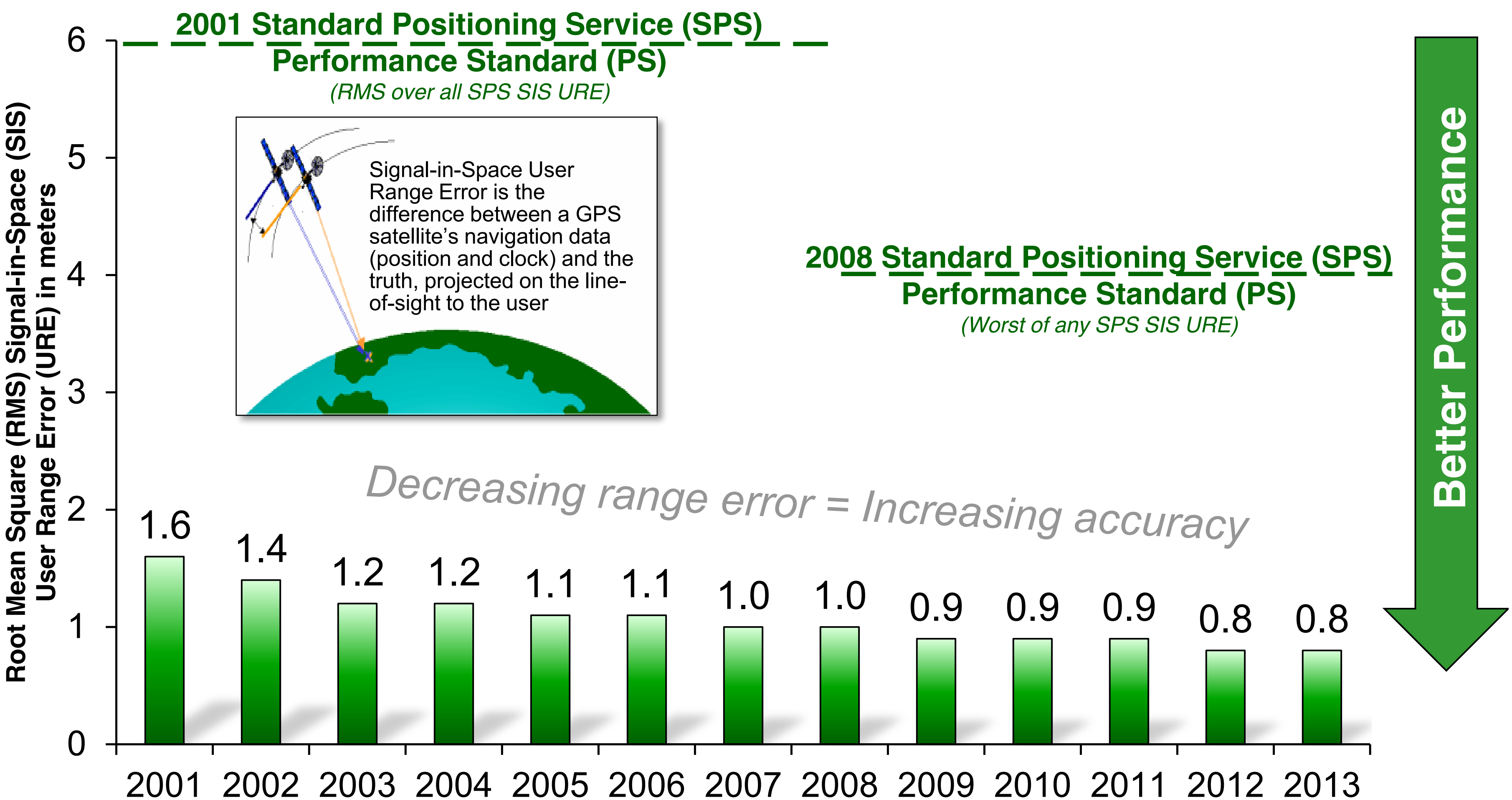
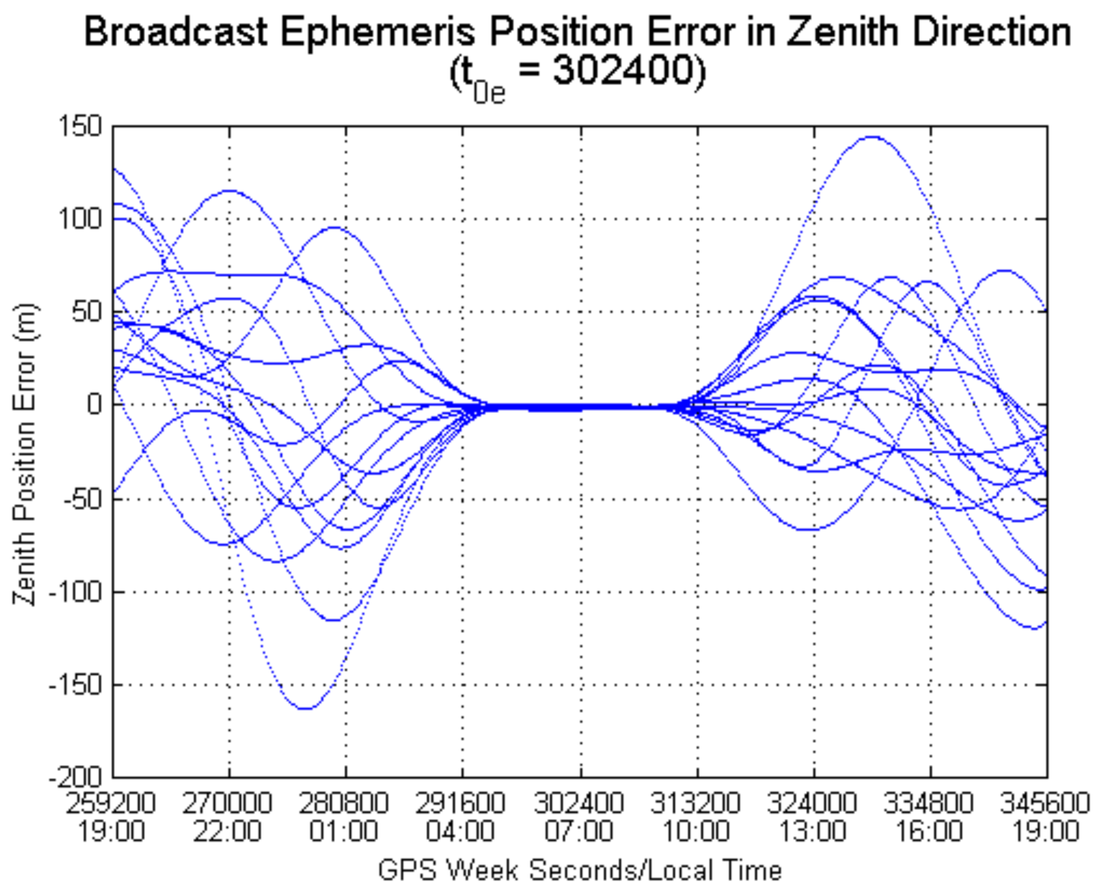


Figure taken from http://www.gps.gov/systems/gps/performance/accuracy/URE.pdf

Example of Satellite Position Errors

- Errors in broadcast ephemeris (compared to NGS precise ephemeris)
  - Traces shown for PRNs 1, 3, 5, 8, 9, 15, 17, 21, 23, 25, 26, 29, 30, and 31
  - Ephemeris for all these PRNs had t_{0e} within 100 seconds of 302400 GPS week seconds ($302300 < t_{0e} < 302500$) (20 Jan 99)
  - Demonstrates 4-hour validity window



Observed GPS Pseudorange Positioning Errors with Typical SPS and PPS Receivers

Error Source	Typical Range Error Magnitude (meters, 1σ)		
	SPS (w/ SA)	SPS (w/o SA)	PPS
Selective Availability	~~24.0~~	0.0	0.0
Ionosphere[a]	~~7.0~~	7.0	0.01
Troposphere[b]	~~0.7~~	0.7	0.7
SV Clock & Ephemeris	~~3.6~~	~~3.6~~ 0.8	~~3.6~~ 0.8
Receiver Noise	~~1.5~~	1.5	0.6
Multipath[c]	~~1.2~~	1.2	1.8
Total User Equivalent Range Error (UERE)	~~25.3~~	~~8.1~~ 7.3	~~4.1~~ 2.5

[a]For SPS: 7.0 is typical value of ionosphere after applying ionospheric model. Actual values can range between approximately 1-30 m.
[b]Residual error after using tropospheric model
[c]For PPS: includes increase in multipath that results from using L1 and L2 code measurements to remove ionospheric error.

 Original table from National Research Council, *The Global Positioning System: A Shared National Asset*, National Academy Press, 1995.

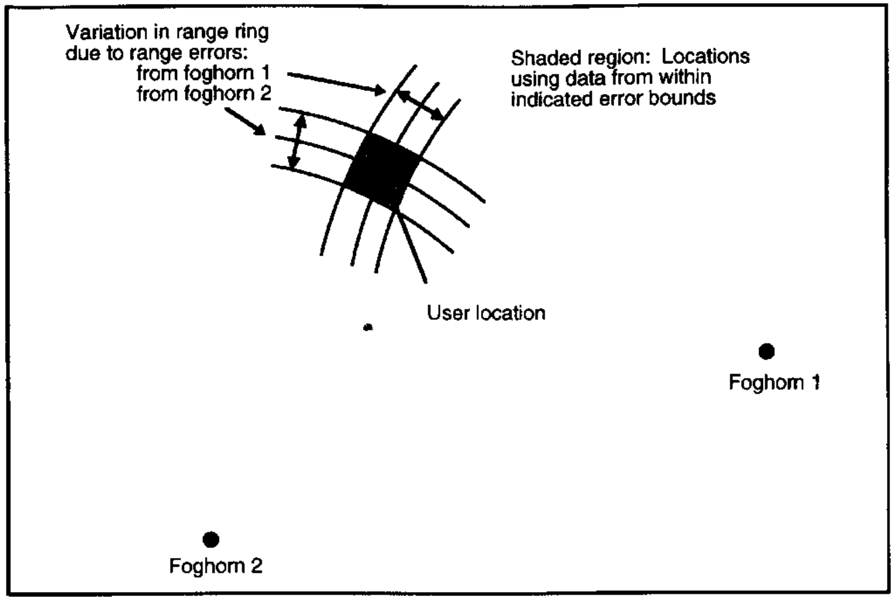
Measurement Domain vs. Position Domain

- **Pseudorange errors are errors in "measurement domain"**
  - Errors in the measurements themselves
  - UERE is one example
- **Ultimately, we'd like to know errors in "position domain"**
  - The position errors that result when using the measurements
  - Errors in position domain are different than measurement errors!
    - Can be larger
    - Can be smaller
  - Dependent on measurement geometry
- **Mathematical representation**
  - We have covariance matrix of measurements ($\mathbf{C}_\rho$).
  - We want covariance matrix of calculated position and clock error ($\mathbf{C}_\mathbf{x}$)
- **In GPS applications, this problem is approached using concept called Dilution of Precision (DOP)**

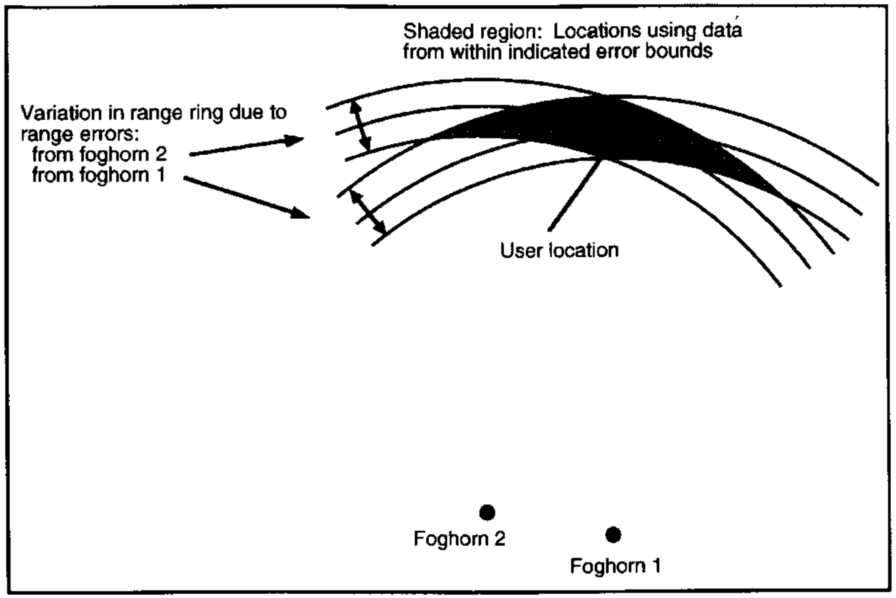
Effect of Geometry on Positioning Accuracy (Foghorn Example)

Consider the foghorn example, except allow for a measurement error

Good Geometry Example



Poor Geometry Example



Types of DOPs

- **The "Big Three"**
  - GDOP (Geometric DOP)

$$GDOP = \sqrt{D_{11} + D_{22} + D_{33} + D_{44}}$$

$$\sqrt{\sigma_e^2 + \sigma_n^2 + \sigma_u^2 + \sigma_{\delta t_u}^2} = GDOP \times \sigma_{UERE}$$

  - PDOP (Position DOP)

$$PDOP = \sqrt{D_{11} + D_{22} + D_{33}}$$

$$\sqrt{\sigma_e^2 + \sigma_n^2 + \sigma_u^2} = PDOP \times \sigma_{UERE}$$

  - HDOP (Horizontal DOP)

$$HDOP = \sqrt{D_{11} + D_{22}}$$

$$\sqrt{\sigma_e^2 + \sigma_n^2} = HDOP \times \sigma_{UERE}$$

- **Less common (for navigators, at least!)**
  - VDOP (Vertical DOP)

$$VDOP = \sqrt{D_{33}}$$

$$\sqrt{\sigma_u^2} = VDOP \times \sigma_{UERE}$$

  - TDOP (Time DOP)

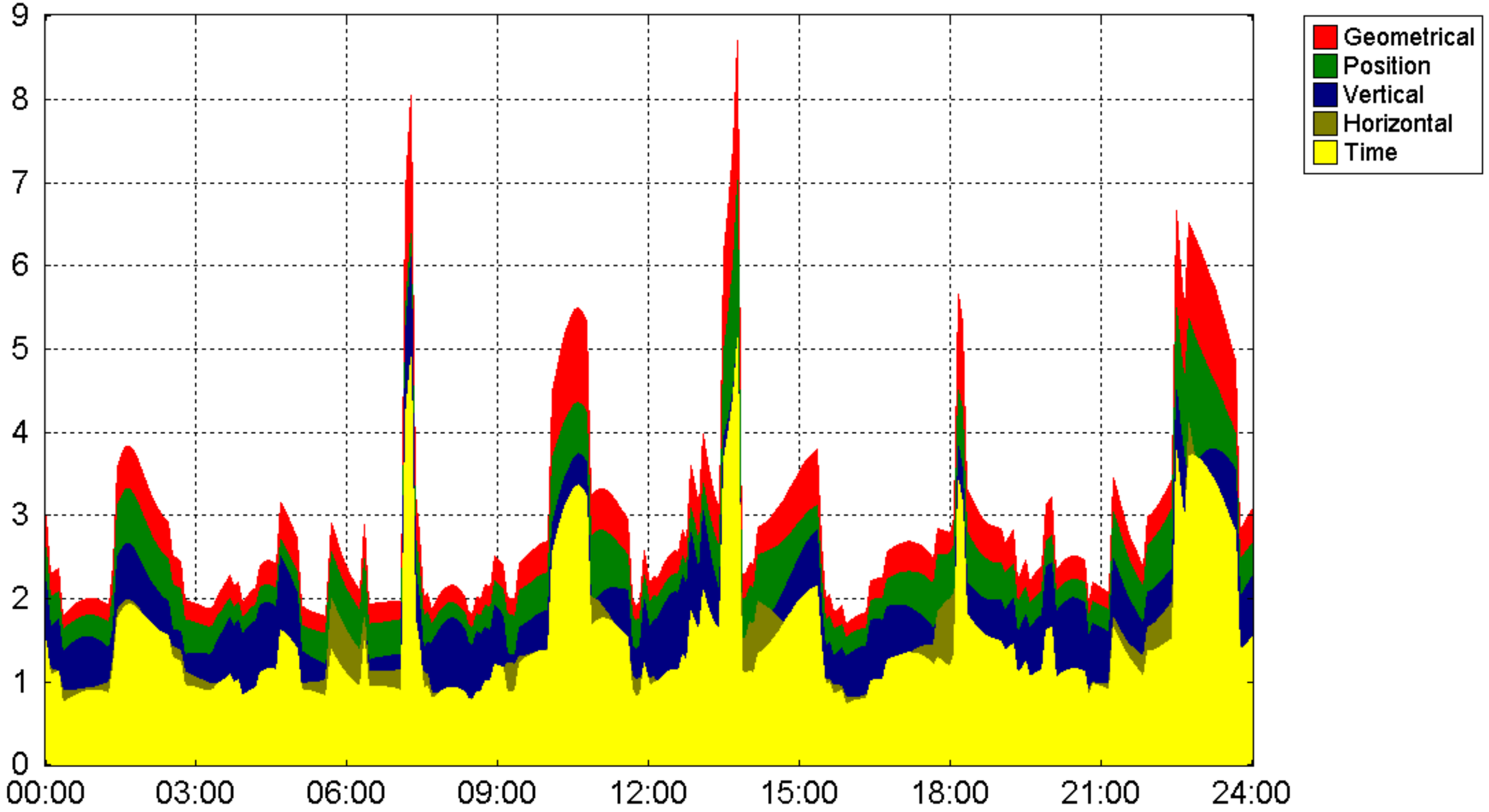
$$TDOP = \sqrt{D_{44}}$$

$$\sqrt{\sigma_{\delta t_u}^2} = TDOP \times \sigma_{UERE}$$

    - Note: time is in units of meters

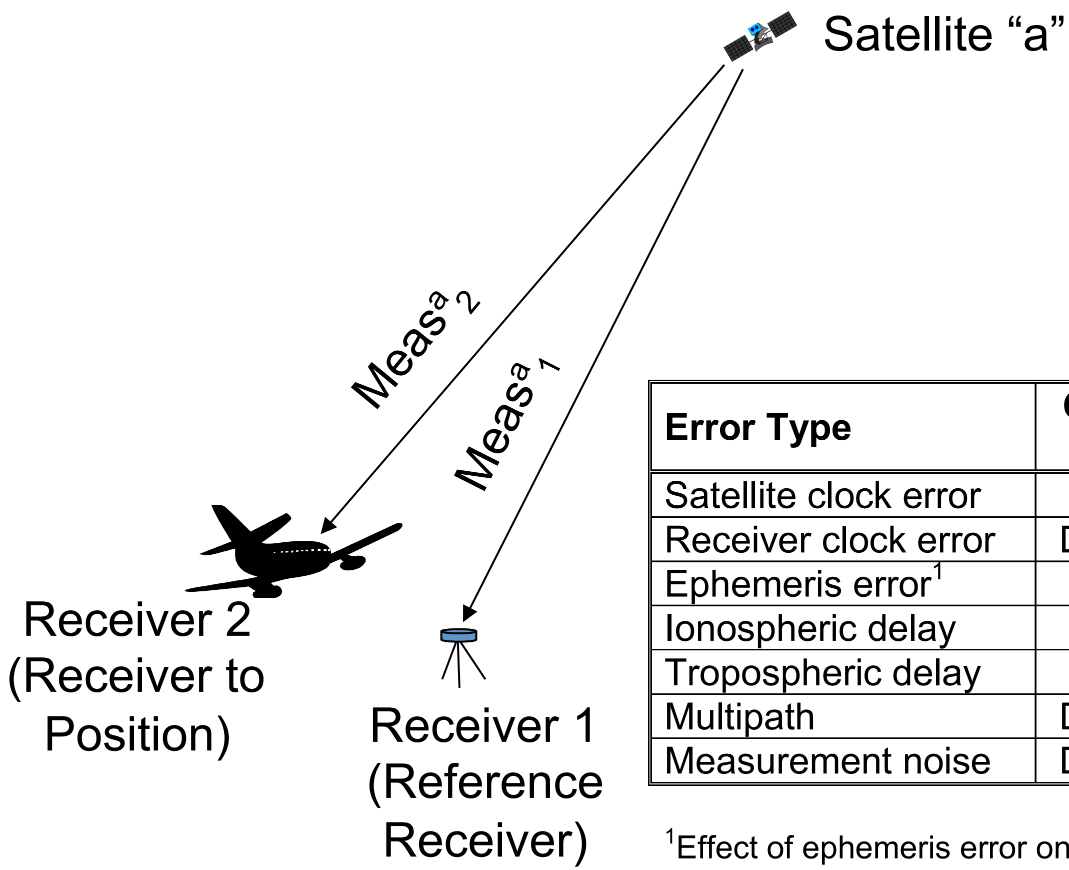
Typical DOP Plot

Dayton Ohio – 24 Apr 2003 – All Visible SVs (above 10° elevation)



SECTION 7: DIFFERENTIAL GPS

Differential GPS Concept



Error Type	Comparison between $Meas_1^a$ and $Meas_2^a$	DGPS Effect on Error
Satellite clock error	Same	Removed
Receiver clock error	Different (uncorrelated)	Added
Ephemeris error[1]	Very similar[2]	Reduced[2]
Ionospheric delay	Very similar[2]	Reduced[2]
Tropospheric delay	Very similar[2]	Reduced[2]
Multipath	Different (uncorrelated)	Added (and amplified)
Measurement noise	Different (uncorrelated)	Added (and amplified)

[1]Effect of ephemeris error on positioning (actually only affects the calculated range, not the actual measurement)
[2]Errors grow as the separation distance between receivers 1 and 2 increases. (The errors are the same and are removed for very short baseline distances).

DGPS Variations

- **DGPS is a broad term, and there are many different ways DGPS can be applied.**
  - Measurements used
    - Code only
    - Carrier-smoothed code
    - Carrier-phase
  - Application type
    - Positioning
    - Attitude
  - Position domain vs. measurement domain
  - Post-processing vs. real-time
  - Type of correction
  - Number of reference receivers
  - Area of coverage
    - LADGPS
    - RADGPS
    - WADGPS
  - Differencing method used
    - Single-differencing

576 possible combinations!

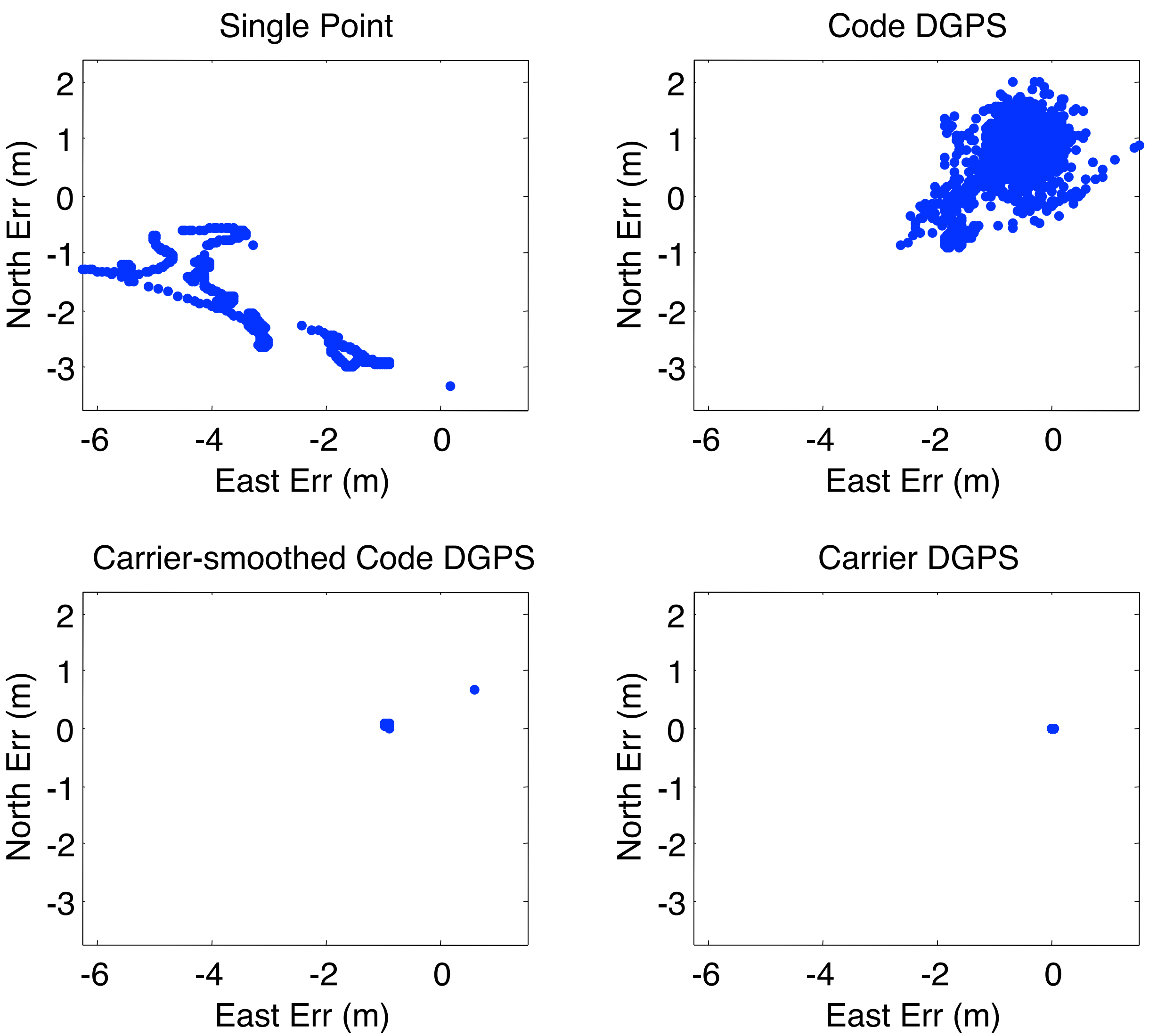
DGPS - Measurements Used (1 of 4)

- **The type of measurements is one of the primary distinguishing factors between different DGPS implementations**
  - Code only
    - Simplest to implement
    - Based purely on pseudorange measurements
    - In best case (short baseline), errors include code multipath and noise
    - Typical accuracy: 2-4 m
  - Carrier-smoothed code
    - Carrier-phase measurement is very precise (~1 cm), but it is not an absolute measurement (due to unknown integer ambiguity).
    - Code (pseudorange) measurement is absolute, but it is much less precise (~1-2 m).
    - A filter can be used to combine the carrier-phase and the code measurements to take advantage of their respective strengths.
      - Filter time constant limited by code-carrier ionospheric divergence (due to different signs on ionospheric error term)
    - Carrier-phase smoothing of the code essentially removes most of the code multipath and noise
    - Typical carrier-smoothed code DGPS accuracy: 0.1-0.5 m
    - Relatively easy to implement

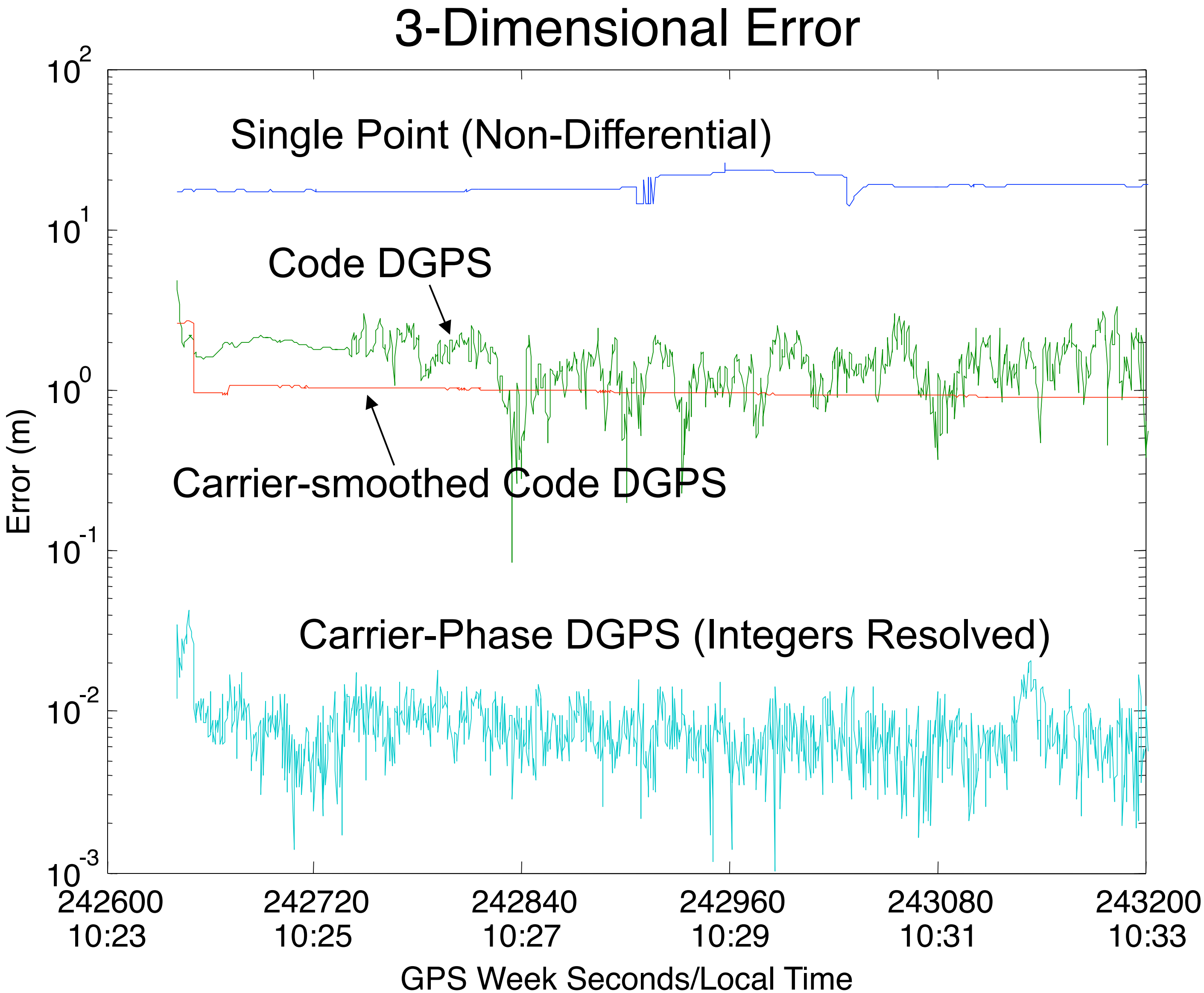
DGPS - Measurements Used (2 of 4)

- **Type of measurements (continued)**
  - Carrier-phase
    - GPS receiver can track exact phase of incoming GPS carrier
      - Can determine “where” in the cycle
      - Cannot determine “which” cycle
      - Results in an unknown integer ambiguity
    - If carrier-phase integer ambiguities can be determined, then the carrier-phase measurement will yield the most precise (and accurate) positioning possible
    - Fairly complex to implement
    - Difficult to resolve integer ambiguities over long reference/mobile receiver baselines
    - Normally requires some period of time to resolve ambiguities
      - 1-3 minutes typical
      - Depends upon baseline distance, algorithm
    - Extremely sensitive to loss of carrier-lock (or cycle slips)
    - Often, code measurements will be used to initially aid in determining the integer ambiguities
      - Final solution normally based primarily on carrier-phase measurements
    - Typical accuracy: 0.01-0.05 m

DGPS - Measurements Used (3 of 4)
Sample Comparison of Horizontal Error

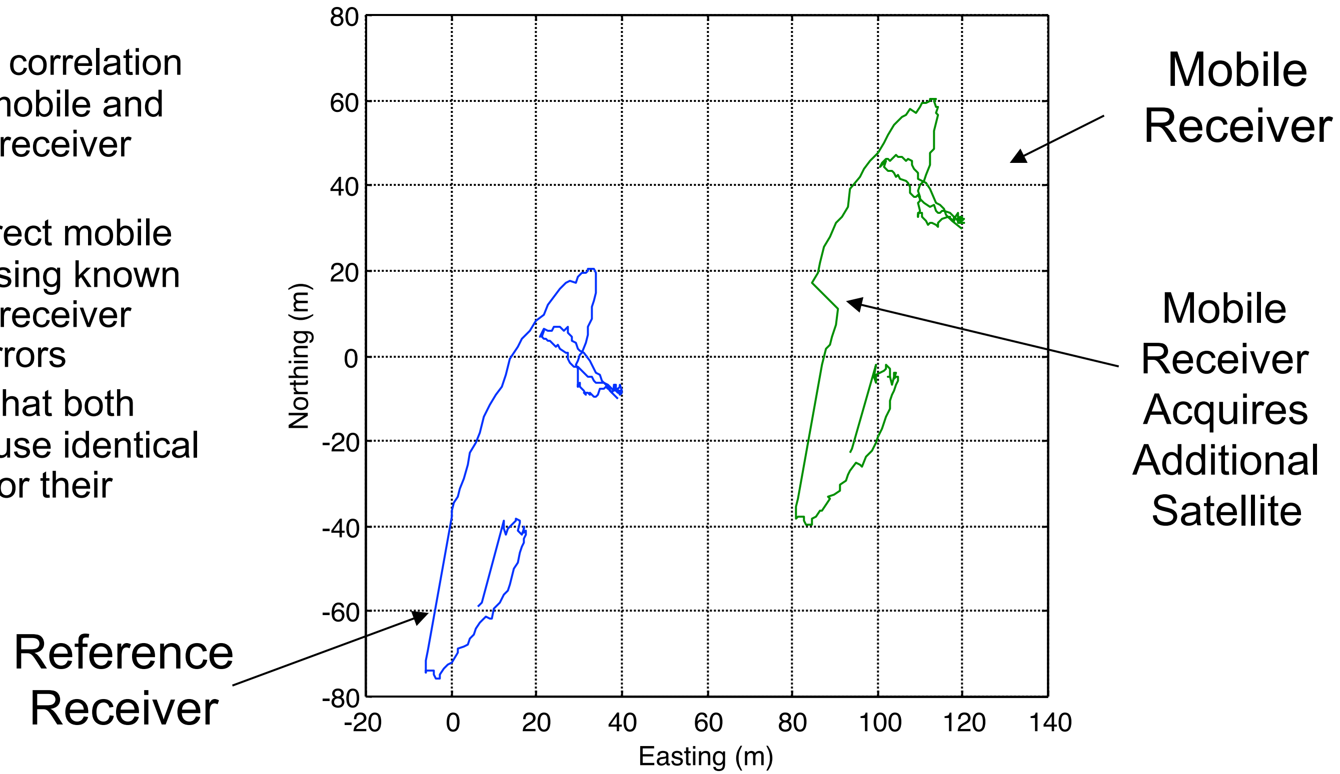


DGPS - Measurements Used (4 of 4)
Sample Comparison of 3-D Error



DGPS - Position vs. Measurement Domain (1 of 2)

- **Position Domain**
  - Reference receiver at known point (origin of plot)
  - Mobile receiver located to the northeast
  - Horizontal position of both receivers plotted on local coordinate system
    - Note error correlation between mobile and reference receiver errors
    - Could correct mobile receiver using known reference receiver position errors
    - Requires that both receivers use identical satellites for their solutions!



DGPS - Position vs. Measurement Domain (2 of 2)

- **Measurement domain**
  - Differential corrections are given for each *measurement*
  - These corrections are then applied to the mobile receiver measurements
    - Results in corrected measurements
    - Position calculated using corrected measurements
  - Advantages
    - Doesn't require same satellite coverage at mobile and reference receivers
      - Reference receiver can only generate corrections for measurements that it can see
    - Standardized formats are defined
      - RTCM SC-104 messages
    - Makes it possible to detect individual measurement errors
  - Disadvantages
    - Requires that more data be transmitted to mobile user than position domain approach
    - Not generally a large problem with modern radio data modems
    - Insignificant for non-real-time applications

DGPS - Type of Correction

- **Two ways to give corrections in measurement domain**
  - Corrections to measurements
    - Actual correction values to be applied to each individual measurement
    - Simple, easy to implement
  - Explicit representation of errors
    - DGPS corrections describe all of the errors in a particular measurement
    - Sometimes, error functions or data are transmitted
      - Different error sources can then be combined to generate a correction for a single measurement
      - Example
        - » Precise ephemeris (to remove satellite position error)
        - » Ionospheric grid (to remove ionospheric error)
        - » Tropospheric model parameters (to improve tropospheric model)
    - Advantages
      - Generally valid for wider area of coverage
      - More flexible
    - Disadvantages
      - More complex
      - Requires more differential data to be transmitted

DGPS - Differencing Methods - Pseudorange Measurement Errors

- **Two types of differencing methods are common**
  - Single differencing
  - Double differencing
- **Choice of method depends upon application. Typically**
  - Code differential → single differencing
  - Carrier-phase differential → double differencing
- **Pseudorange errors**
  - Original representation

neglect

$$\rho = r + c(\delta t_u - \delta t_{sv} + \delta t_D)$$

$$\delta t_D = \delta t_{trop} + \delta t_{iono} + \delta t_{noise\&res} + \delta t_{mp} + \cancel{\delta t}_{hw} + \delta t_{SA}$$

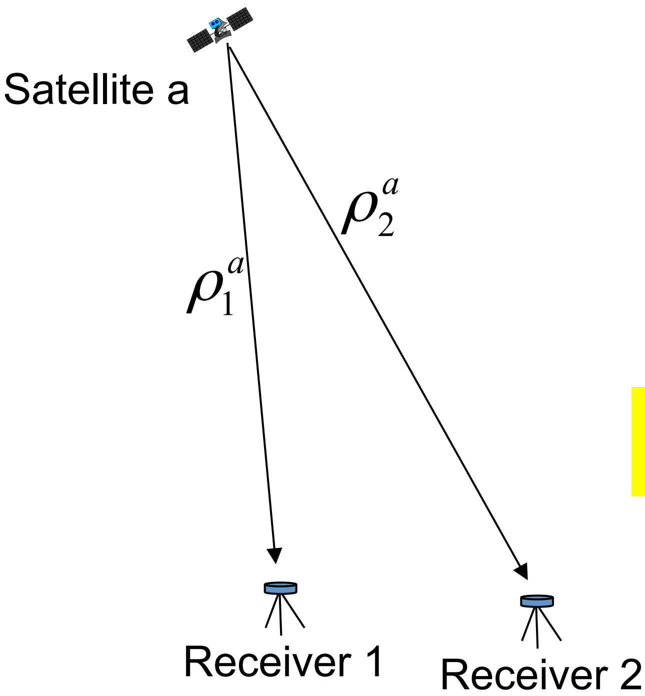
  - Simplification

$$\rho = r + c\delta t_u - c\delta t_{sv} + c\delta t_{trop} + c\delta t_{iono} + c\delta t_{noise\&res} + c\delta t_{mp} + c\delta t_{SA}$$

$$\rho = r + c\delta t_u - c\delta t_{sv} + T + I + \upsilon + m + SA$$

DGPS - Differencing Methods (1/2)

- **Single differencing**
  - Difference measurements between one satellite and two receivers



$$\Delta\rho_{12}^a \equiv \rho_1^a - \rho_2^a$$

$$= r_1^a + c\delta t_{u_1} - c\delta t_{sv}^a + T_1^a + I_1^a + \upsilon_1^a + m_1^a + SA^a$$

$$- r_2^a - c\delta t_{u_2} + c\delta t_{sv}^a - T_2^a - I_2^a - \upsilon_2^a - m_2^a - SA^a$$

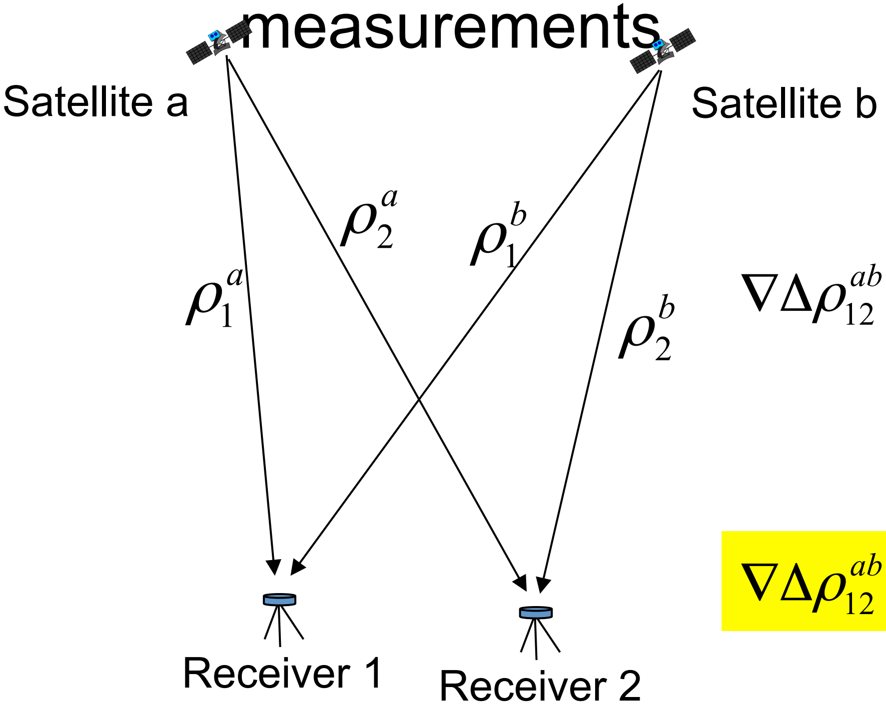
$$\Delta\rho_{12}^a = \Delta r_{12}^a + \Delta c\delta t_{u_{12}} + \Delta T_{12}^a + \Delta I_{12}^a + \Delta \upsilon_{12}^a + \Delta m_{12}^a$$

  - SV clock error and SA *cancelled*[1]
  - Tropospheric, ionospheric errors *reduced* $\sqrt{2}$
  - Multipath and noise *amplified* (by factor of)

[1]Assuming that only the dither portion of SA is utilized (if SA is on at all!)

DGPS - Differencing Methods (2/2)

- **Double differencing**
  - Difference between two single difference measurements



$$\nabla\Delta\rho_{12}^{ab} \equiv \Delta\rho_{12}^{a} - \Delta\rho_{12}^{b} = \rho_1^a - \rho_2^a - (\rho_1^b - \rho_2^b)$$

$$= \Delta r_{12}^{a} + \Delta c\delta t_{u_{12}} + \Delta T_{12}^{a} + \Delta I_{12}^{a} + \Delta \upsilon_{12}^{a} + \Delta m_{12}^{a}$$
$$- \Delta r_{12}^{b} - \Delta c\delta t_{u_{12}} - \Delta T_{12}^{b} - \Delta I_{12}^{b} - \Delta \upsilon_{12}^{b} - \Delta m_{12}^{b}$$

$$\nabla\Delta\rho_{12}^{ab} = \nabla\Delta r_{12}^{ab} + \nabla\Delta T_{12}^{ab} + \nabla\Delta I_{12}^{ab} + \nabla\Delta \upsilon_{12}^{ab} + \nabla\Delta m_{12}^{ab}$$

[1]Assuming that only the dither portion of SA is utilized (if SA is on at all!)

DGPS Errors

- **Errors completely cancelled by DGPS**
  - Receiver clock error
  - Satellite clock error
- **DGPS errors can be grouped into two classes**
  - Uncorrelated errors
    - Errors that are not spatially related
    - Do not increase with reference/mobile baseline distance
    - Include multipath and measurement noise
    - DGPS actually increases these errors

Typical Multipath + Noise Error Standard Deviation Values

	Single Meas (non-DGPS)	Single Difference	Double Difference
Code	0.5-1.5 m	0.7-2.1 m	1-3 m
Carrier-Phase	0.2 - 1 cm	0.3 - 1.4 cm	0.4 - 2 cm

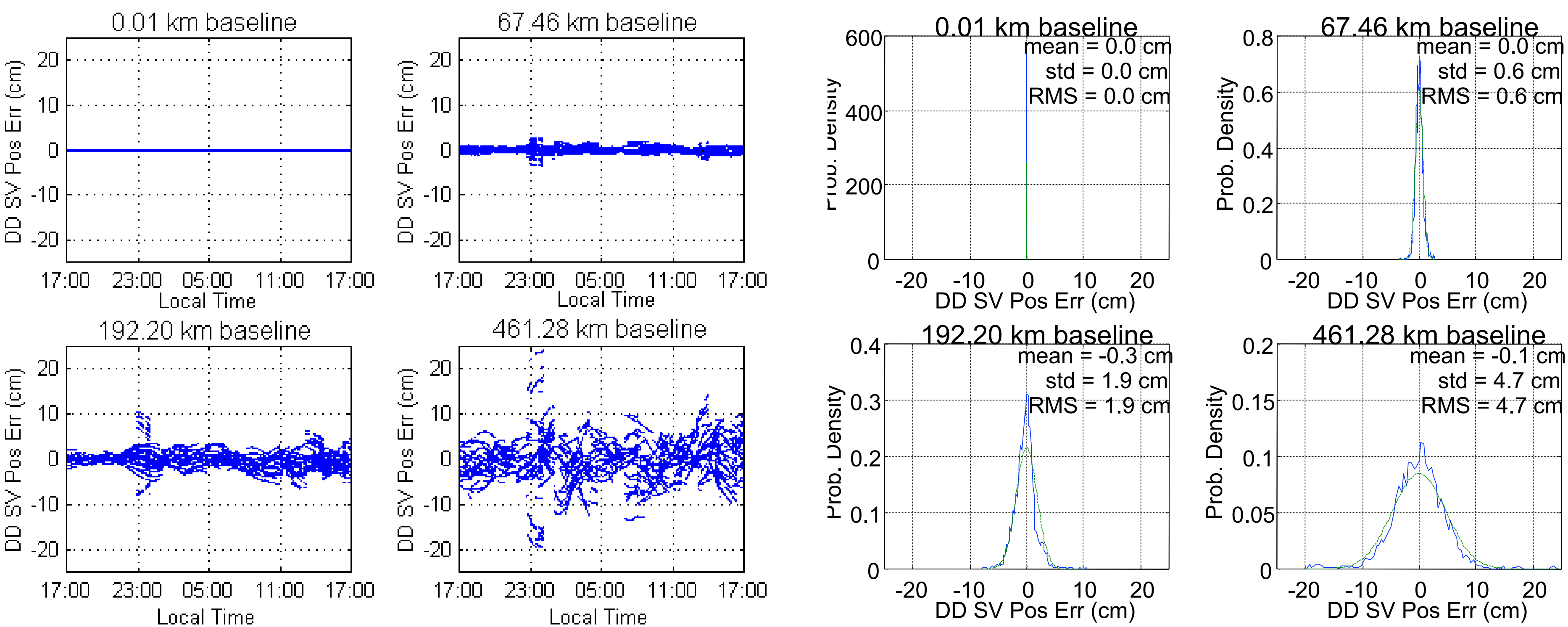
  - Correlated errors
    - Are spatially related
    - Increase with baseline distance
    - Include satellite position (ephemeris), ionospheric, and tropospheric errors

Differential Satellite Position Errors

- **Satellite position errors are errors in ephemeris that cause calculated SV position to differ from true SV position**
  - Absolute (non-DGPS) error
    - Zenith: ~1 m (1-σ)
    - Non-zenith axes: ~3 m (1-σ)
  - For a given measurement, it is the projection of the 3-D SV position error onto the measurement line-of-sight vector that counts
    - With DGPS, line-of-sight vectors converge as reference/mobile baseline distance goes to zero
  - Satellite position error can be determined using precise ephemeris as truth
    - Precise ephemeris accurate to ~10 cm
  - Differential satellite position errors typically less than 5 cm (1-σ), except for very long baselines (> 500 km)
    - True as long as same set of ephemeris is used for both reference and mobile receivers

Sample SV Position DGPS Error (Double Difference)

Data collected in Norway on Sep 30th, 1998

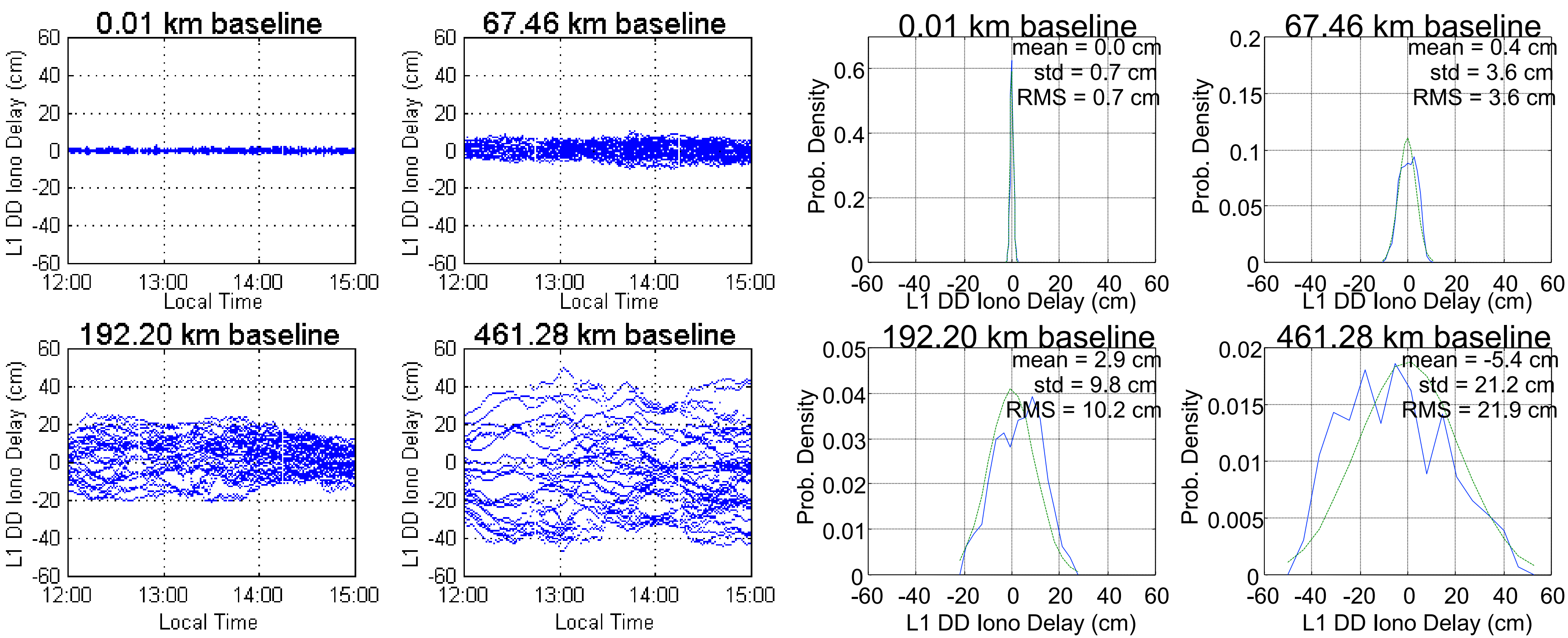


Differential Ionospheric Errors

- **Ionospheric errors are spatially correlated**
  – Signal from same satellite to two nearby receivers passes through approximately same ionosphere
    - Exception: scintillation
      – Highly local effects
      – Can affect one receiver but not another (unless receivers are collocated)
  – DGPS ionospheric error follows same general trends as overall (non-DGPS) ionospheric error
    - Maximum at ~14:00 local time
    - Minimum at night
    - Varies with solar cycle
  – Ionospheric delay (or phase advance) can be precisely measured using linear combination of phase measurements
    - Requires successful resolution of L1 and L2 carrier-phase ambiguities
    - Accurate to ~1 cm (includes effects of carrier-phase multipath and noise)

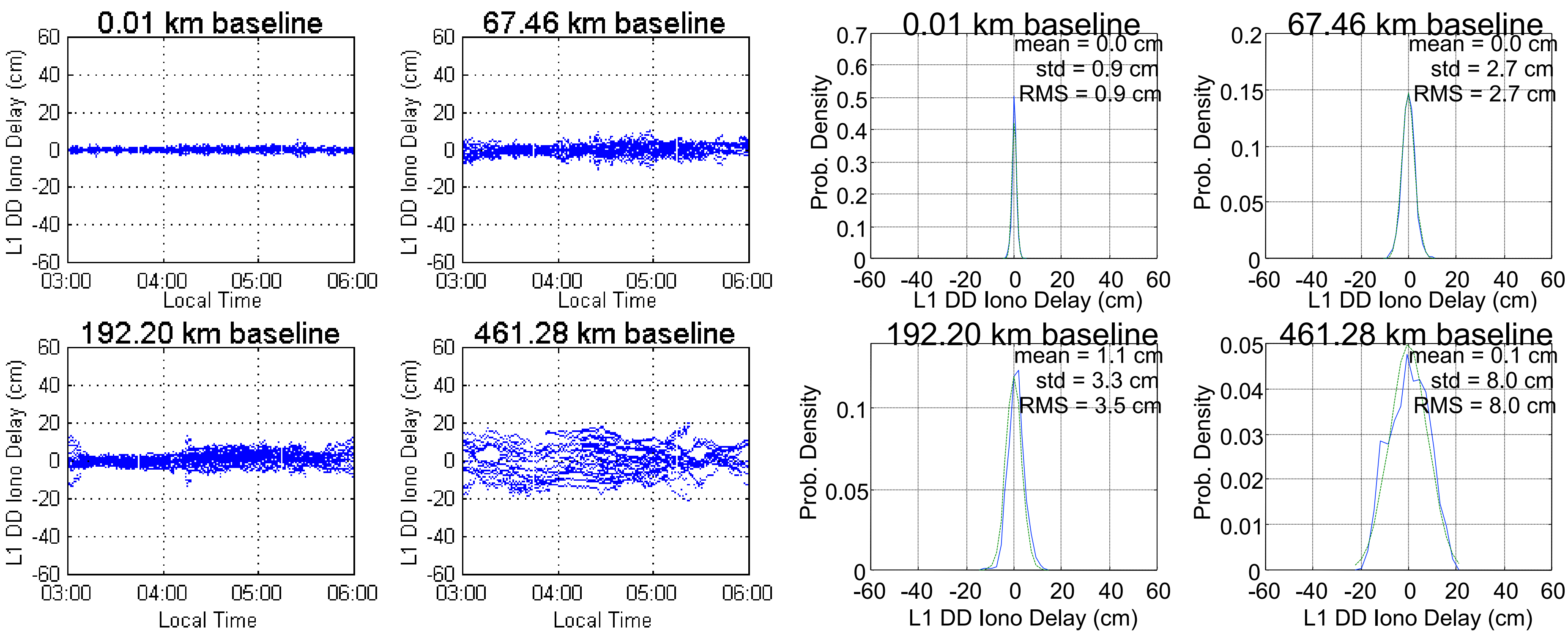
Sample Afternoon Ionospheric DGPS Errors (Double Difference)

Data collected in Norway on Sep 30th, 1998
(between minimum and mid-point of solar cycle)



Sample Nighttime Ionospheric DGPS Errors (Double Difference)

Data collected in Norway on Sep 30th, 1998
(between minimum and mid-point of solar cycle)

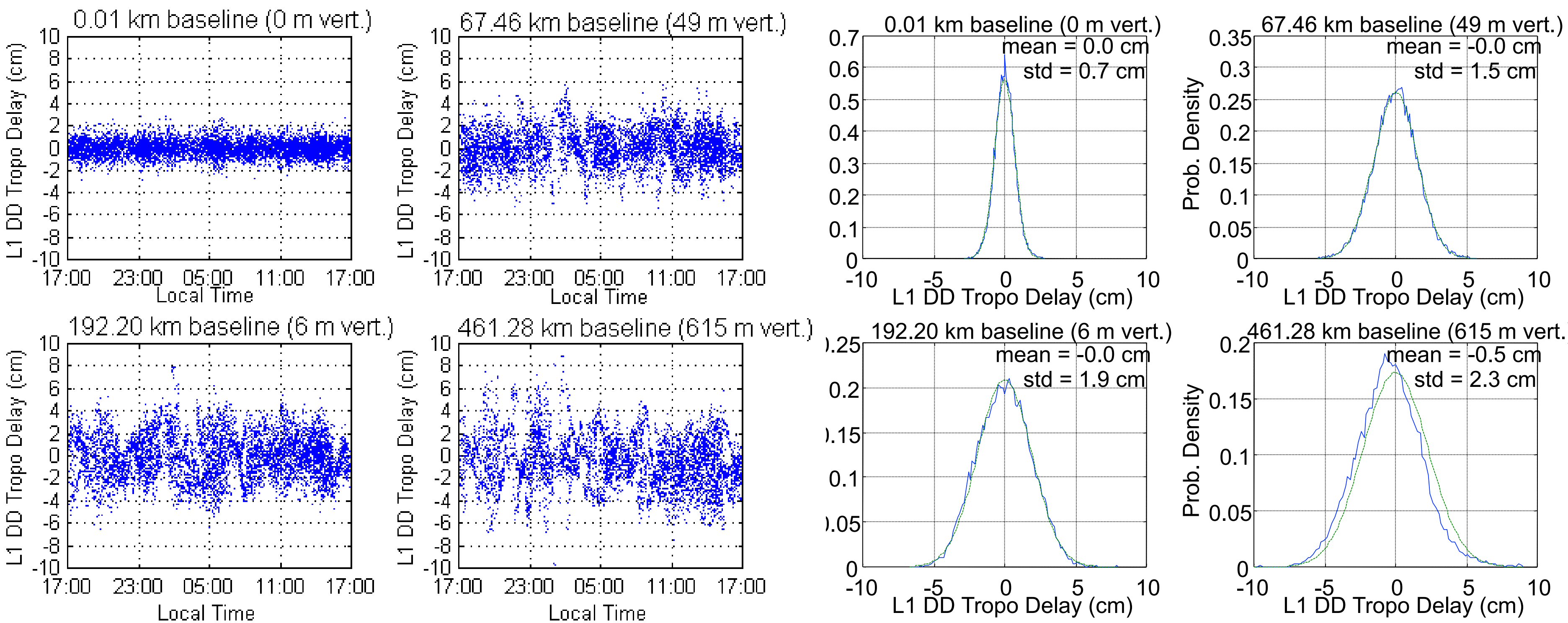


Differential Tropospheric Errors

- **Tropospheric errors highly sensitive to altitude of receiver and elevation of satellite**
  - Most of the error can be effectively modeled
  - Important to always apply tropospheric model for DGPS
    - If don't apply, then can introduce differential errors on order of meters for receivers at different altitudes
    - Should use same tropospheric model (if possible)
- **With a good model, differential tropospheric errors are relatively small**
  - Under normal conditions don't exceed ~3 cm (1-σ) for baselines < 500 km
  - Can be worse under extreme conditions (e.g., high humidity)
- **Differential tropospheric error can be calculated from carrier-phase measurements**
  - Use ionospheric-free combination with precise orbits to remove other errors
  - All that remains is tropospheric error (plus multipath and noise)

Sample Tropospheric DGPS Errors (Double Difference)

Data collected in Norway on Sep 30th, 1998
Modified Hopfield Tropospheric Error Model Applied



Application: Differential GPS Processing Software

- There's nothing like seeing how this actually works itself out in practice!

GNSS Fundamentals: What We've Covered

Section 1: GPS History

Section 2: Background—Time of Arrival (TOA) Positioning

Section 3: GPS System Overview

Section 4: Receiver measurements

Section 5: Signal Structure

Section 6: Measurement Errors

Section 7: Differential GPS

U.S. Air Force photo by Carleton Bailie,
http://www.af.mil/weekinphotos/040625-04.html