Building Theoretical Models from Observational Parameters

P. Baki

Department of Physics and Space Sciences

Technical University of Kenya

P.O Box 52428-00200 nairobi

Email: paulbaki@gmail.com

Outline

1. **What are the issues?**
2. **Methodologies**

   (a) Dimensional Analysis

   -The π-Theorem

-The Szirtes Algorithm

-Applications to Ionspheric Physics

(b) Deducing the Physics from Observations to build equations.

3. Concluding Remarks

1. What are the issues?

Researchers are able to obtain data from websites, use free software to process and obtain plots or write programs to process the data from some equipment that they are hosting. The problem, however, is that some researchers/students:

- cannot figure out what features of the graphs to look for that may be attributable to a particular phenomenon of interest to them.

- Fail to provide sound theoretical explanations for their results.
- Know parameters that possibly describe the phenomenon but are unable to develop empirical models for the observed phenomenon.

2. Methodologies

We shall use the following methods:

(a) Dimensional Analysis

(b) Deducing the underlying dynamics from observations in order to build equations

(a) Dimensional Analysis

- **Developed in 1914 by Buckingham**
- In the field of physics proper, it is most commonly used as a tool for checking equations for dimensional correctness i.e . the requirement that all the terms in an equation describing a physical law should have the same dimensions

- Given certain inputs based on experimental observations or data, dimensional analysis can be used to derive empirical laws which would otherwise be quite difficult, if not impossible, to arrive at.
- very essential to construct and study a scaled down model of the system to be investigated or a machine to be constructed.
- important that one determines the 'scaling factors' which relate the parameters or the variables of the model to those of the prototype.

Example e.g Fluid flow in a pipe

- Best described by:
  - fluid density ρ ,
  - pipe radius ***a***,
  - flow velocity ***u*** and;
  - fluid viscosity η.
- Then, the quantity $R = \frac{\rho a u}{\eta}$ (called the Reynolds number) is dimensionless and plays a very important role in determining the transition of the flow from the laminar to the turbulent state.

The π- Theorem

- The theorem is applicable to any dimensionally homogeneous equation which relates, say, n physical quantities π defined in terms of *r* reference dimensions (such as *M*,*L* and T).

- A physical equation is said to be dimensionally homogeneous if every term in the equation reduces to the same algebraic quantity when expressed in terms of the reference dimensions.
- According to the central result of the 1t-theorem, it is always possible to reduce the equation to a relationship between *(n-r)* independent dimensionless quantities provided the reference dimensions themselves are considered as independent of one another.
- The minimal set of such dimensionless quantities for a given systemconstitutes the fundamental or the complete set.
- There is no unique or universally applicable method to actually construct such quantities explicitly.
- practical difficulties in its actual implementation for finding out a minimal set of dimensionless.

The Szirtes Algorithm

According to Szirtes, the following steps are involved in the construction of a minimal set of dimensionless quantities:

Step 1

- Given the system, identify the variables, parameters and constants

which govern its behavior. It is most essential that all the relevant quantities are included in the list since if anyone of them is omitted, the outcome of the dimensional analysis could be erroneous.

- However, if any irrelevant or superfluous quantity

is included, it does not influence the outcome but only makes the analysis more complicated. Thus, when in doubt about the suitability of a quantity, it is best to include it in the list!

Taking the case of fluid flow, we have

Table 1

	α	ρ	a	u	η
M	1	1	0	0	1
L	1	-3	1	1	-1
T	-2	0	0	-1	-1

The entries in *Table* 1 are just the exponents of the corresponding dimensions for each of the variables. For example, viscosity has the dimensions *ML* -1 *T* -1, hence the corresponding values 1, -1,and -1 in the η column in *Table 1*.

Step 2

- Form the square matrix **A** by taking the right most elements of *Table* 1. We have three reference dimensions, namely,M, L and *T* and therefore, the matrix *A* is of the order 3 x 3
- Make sure that the matrix *A*is non-singular, that is, $\det A \neq 0$. If det *A* = 0, rearrange the columns in *Table* 1 so as to obtain a new square matrix with nonzero determinant.
- Call the matrix formed by the remaining elements in *Table* 1 as matrix B which need not be square.

Thus, for the example of fluid flow, we have

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & -1 \\ 0 & -1 & -1 \end{bmatrix} \quad , \quad B = \begin{bmatrix} 1 & 1 \\ 0 & -3 \\ -2 & 0 \end{bmatrix}$$

Step 3

- Calculate the inverse of the matrix *A* and find the product matrix **C** defined by $\mathbf{C=-(A^{-1}B)^{T}}$ where *T* denotes the transpose operation (that is, the interchange of the rows with the corresponding columns) and A^{-1} is the inverse of the matrix A.
- One can easily show that

$$A^{-1} = \begin{bmatrix} 2 & 1 & 1 \\ -1 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 1 & -1 \\ & & \end{bmatrix}$$

Step 4

Extend *Table* 1 as described in the following:

- Place the matrix C below the matrix *A.* Place an identity (square) matrix I of appropriate size below the matrix *B.*
- Extend the column containing M,L, *T* by the dimensionless quantities (to be constructed) denoted by, say π_i , i = 1,2,3, ...
- Thus, if the number of physical variables listed is *n* and the number of reference dimensions used is *m* with *n* > *m,* then the dimension of the matrix A is $m \times m$ that of *B* is $m \times (n-m)$ that of C is $m \times (n-m)$ and that of I is *(n - m)* X *(n - m).*

- The total matrix formed *by A*,*B*, C, and I

Table 2

MATRIX DIMENSIONS	V $[1 \times n]$	
D $[m \times 1]$ [mx1]	B $[m \times (n-m)]$	A $[m \times m]$
π $[(n-m) \times 1]$	$[(n-m) \times (n-m)]$	C $[(n-m) \times m]$

Table 2