Circuit-QED
from qubits to **networks**

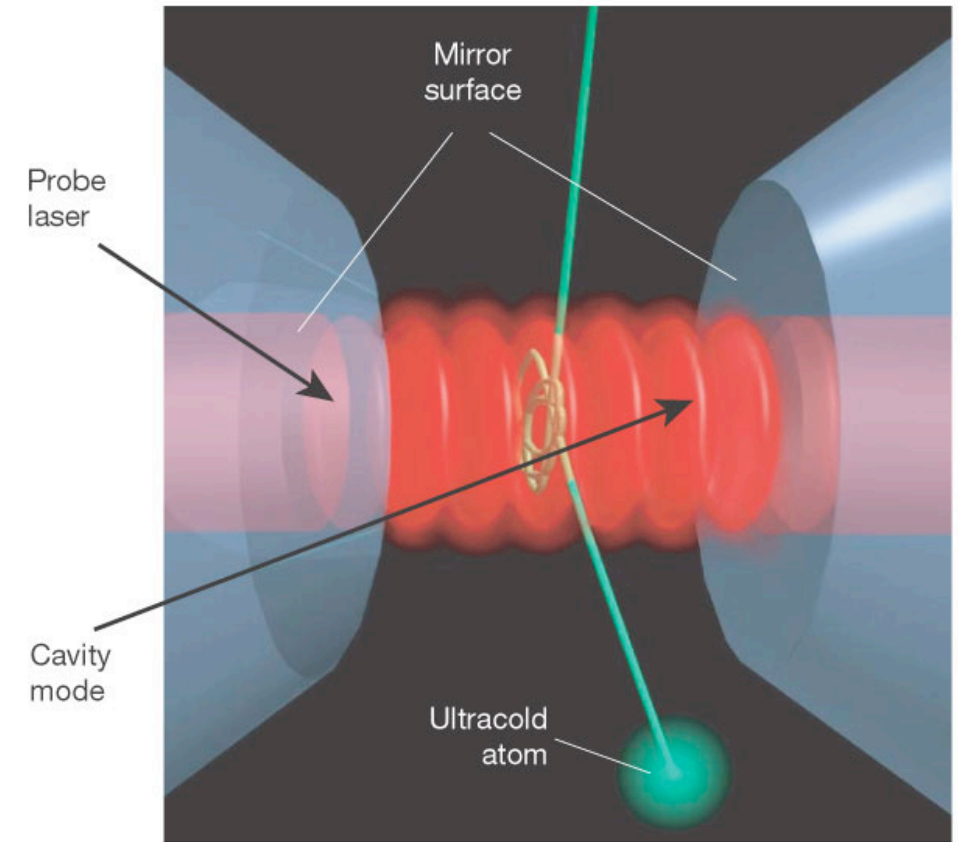
Rosario Fazio

NEST

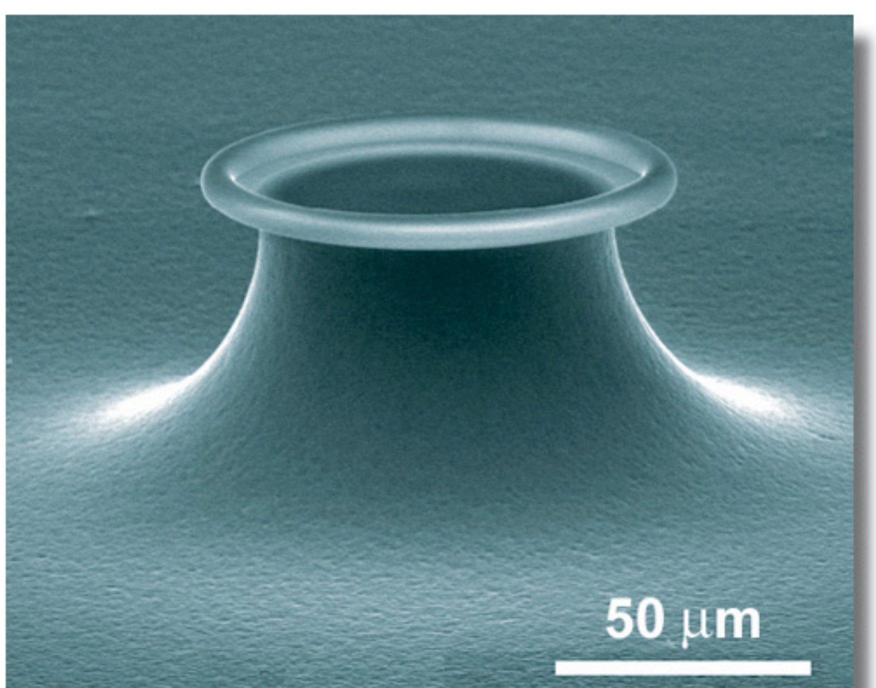
SCUOLA NORMALE SUPERIORE PISA

cavity-QED

J.M. Raimond *et al* Rev. Mod. Phys **73**, 565 (2001)



It studies the interaction of single atoms with single photons



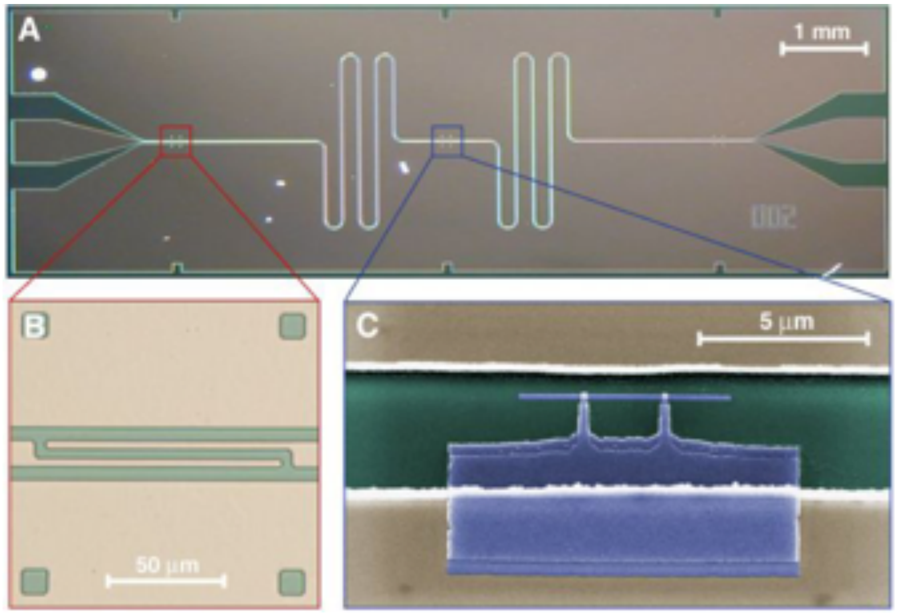
D.K. Armani *et al* Nature **421**, 925 (2003)

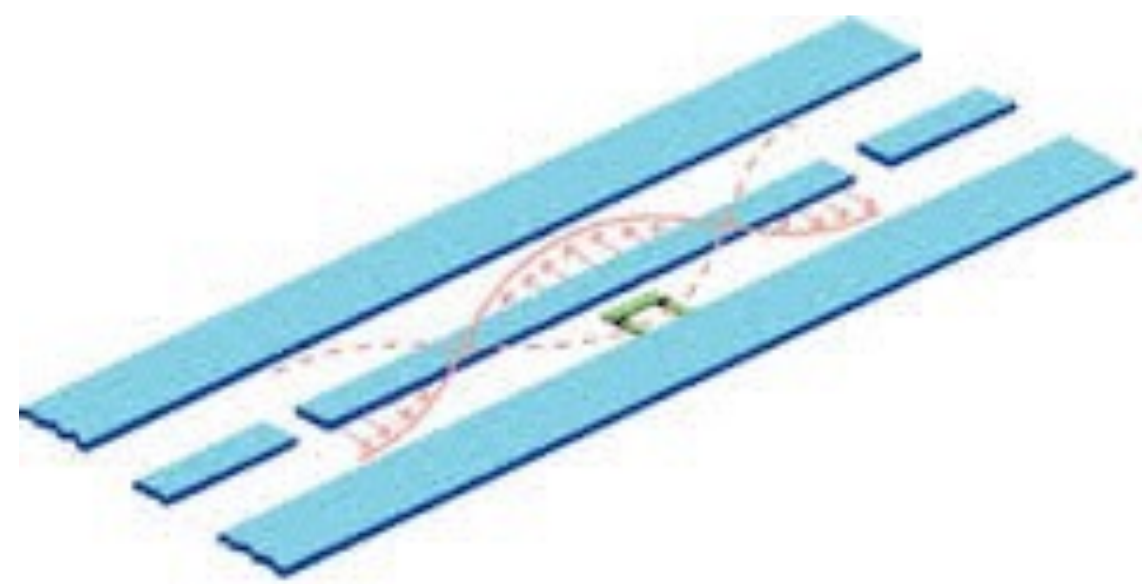
- Test for fundaments of Quantum Mechanics

- Quantum Information Processing

circuit-QED

A. Wallraff *et al*, Nature **431**, 162 (2004)



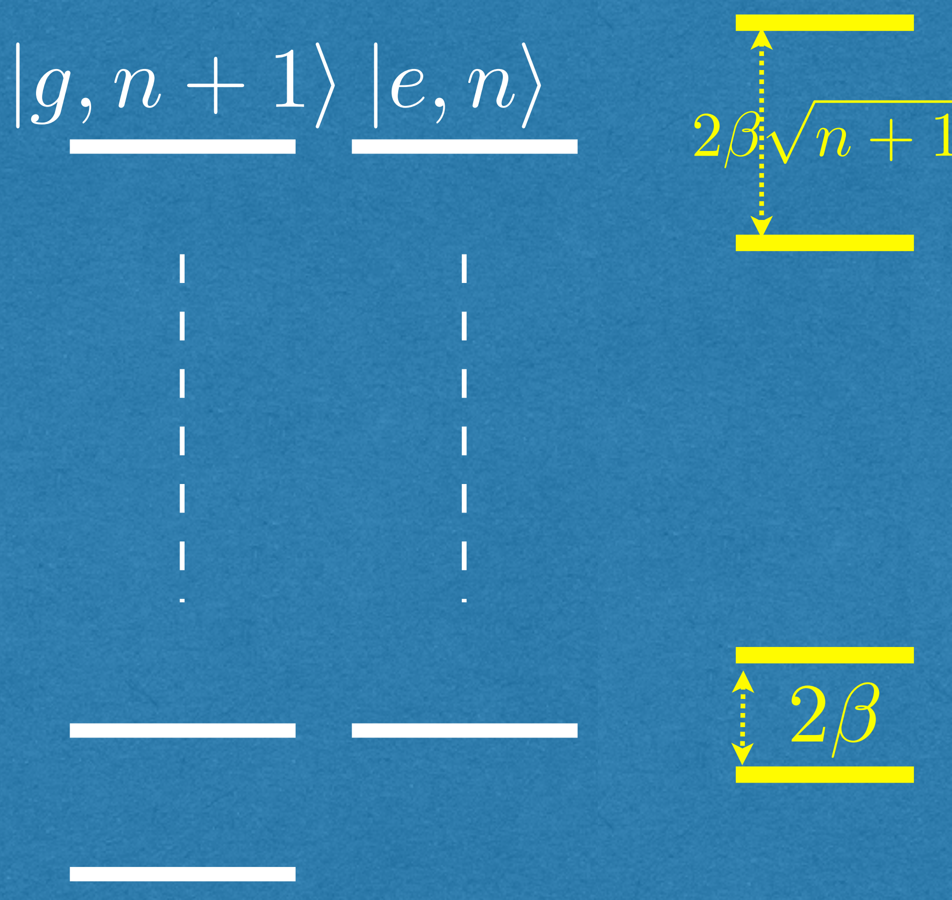


Jaynes-Cummings model

$$\mathcal{H} = \epsilon\sigma_z + \omega a^\dagger a + \beta(a^\dagger\sigma_- + a\sigma_+)$$

$$\epsilon = \omega$$

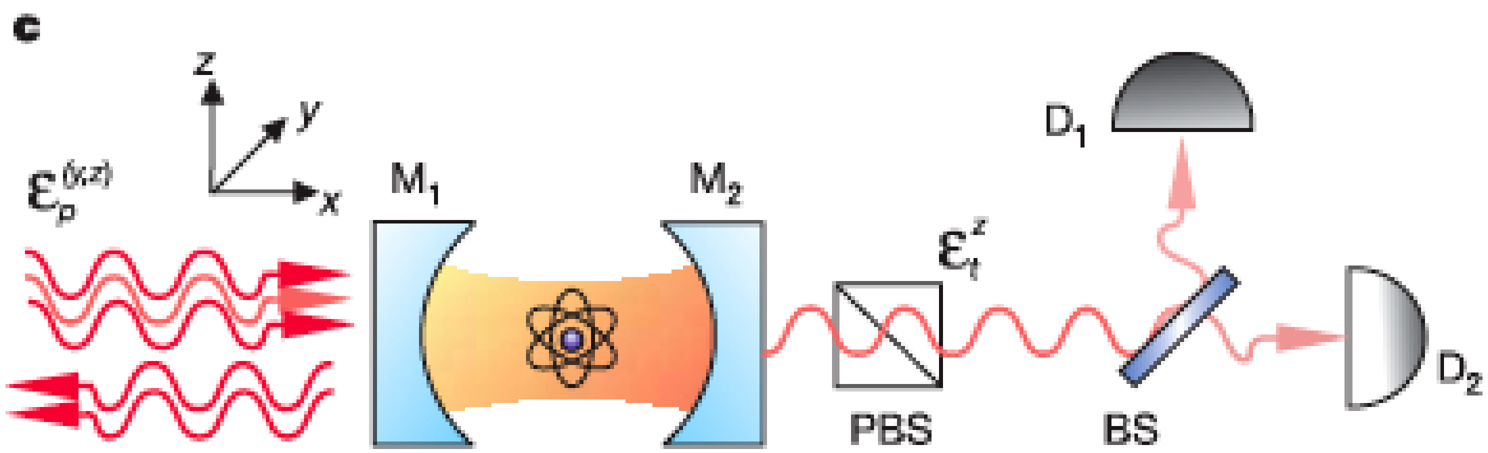
The degeneracy between
$|e,n\rangle$ and $|g,n+1\rangle$
is lifted by the coupling

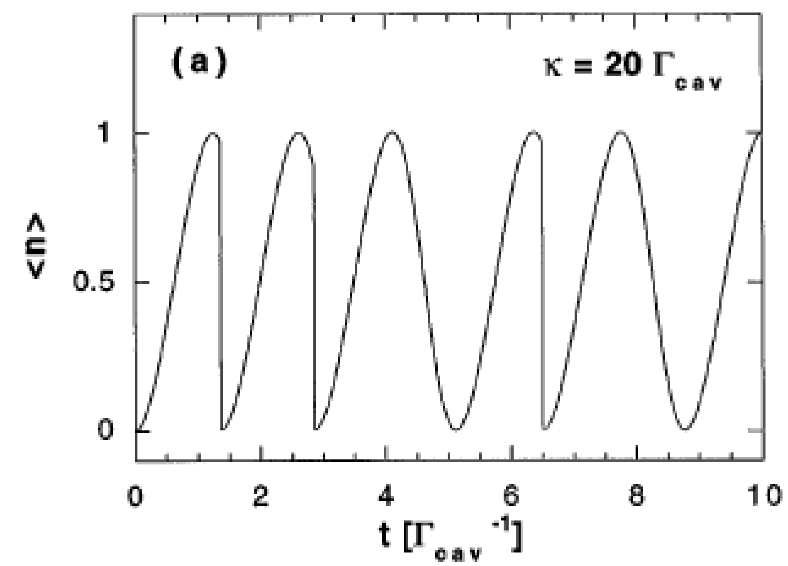


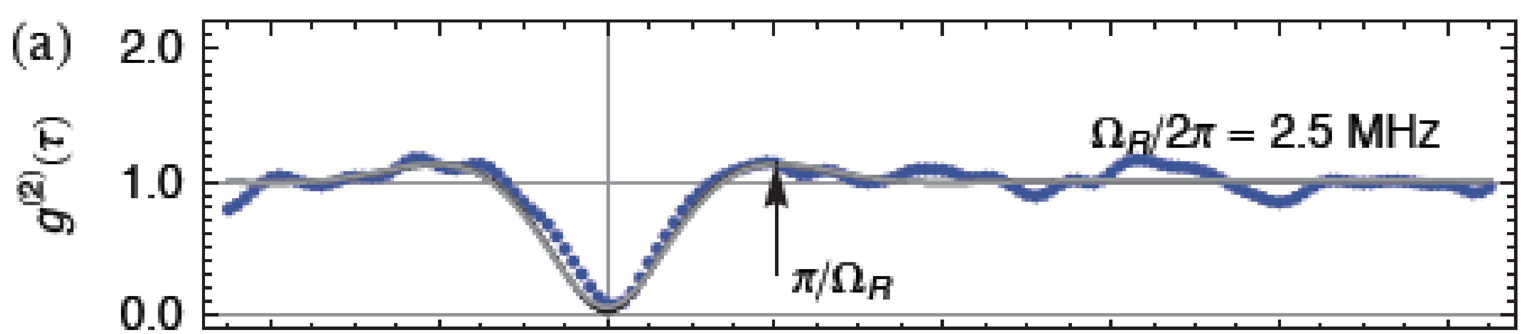
Photon blockade

A. Imamoglu *et al* 97, S. Rebic *et al* 99, J. Kim *et al* 99, K. Birnbaum *et al* 05

$$\mathcal{H} = \epsilon\sigma_z + \omega a^\dagger a + \beta(a^\dagger\sigma_- + a\sigma_+)$$







Lang *et al* 2011

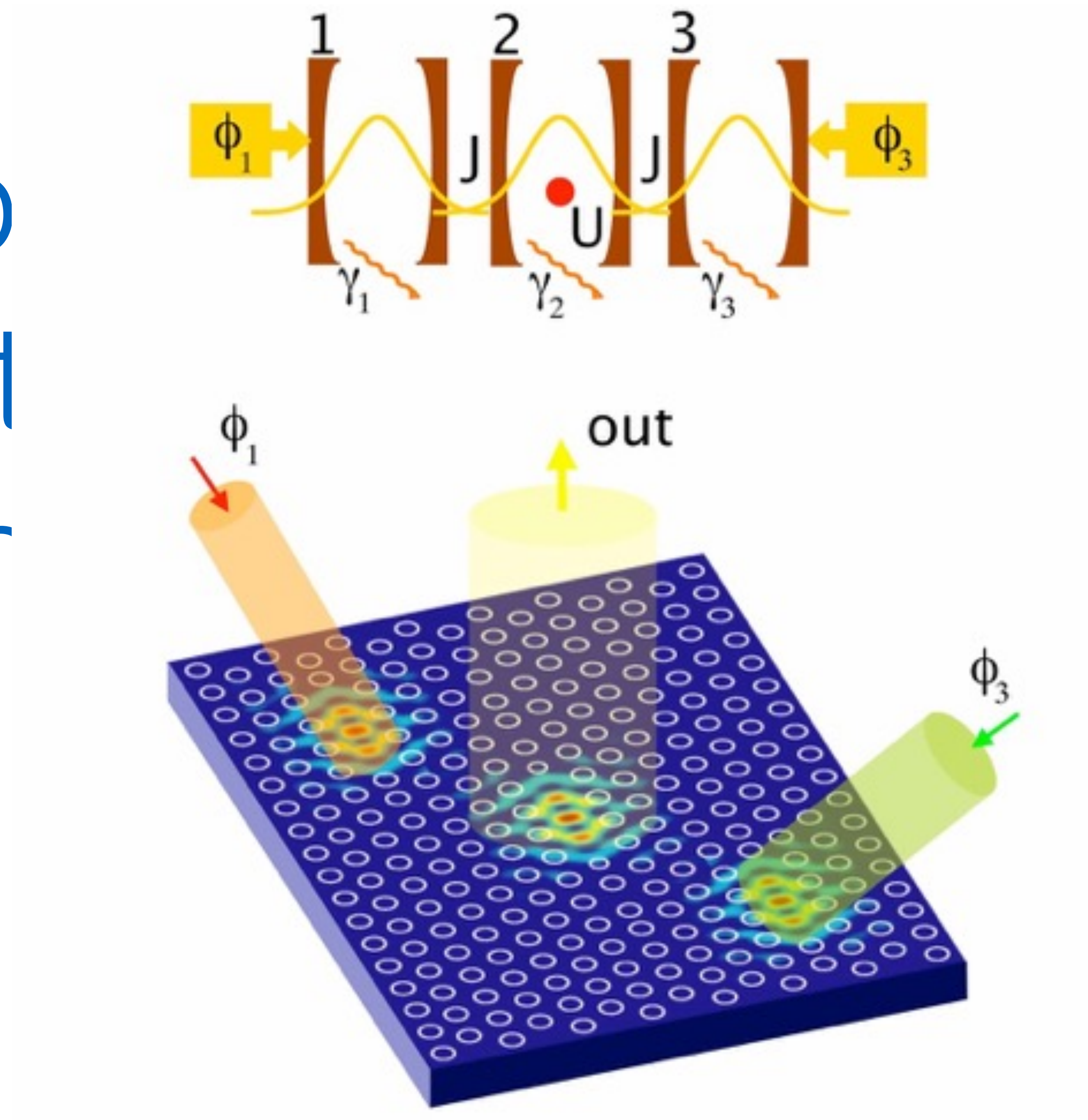
Coupled cavities

(under non-equilibrium conditions)

Competition of lo

wi

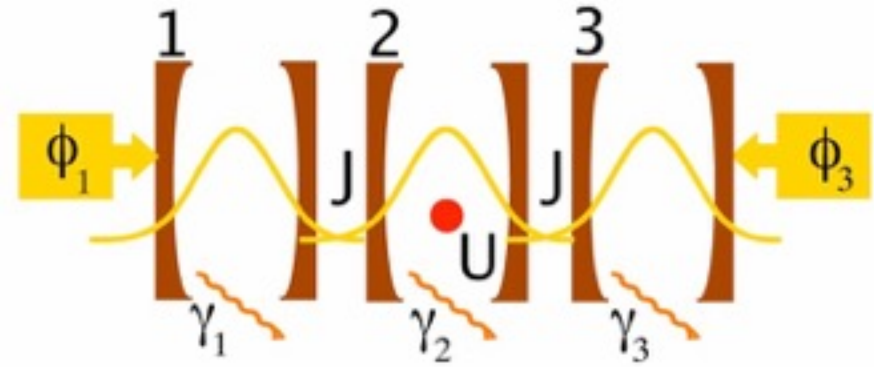
photon h



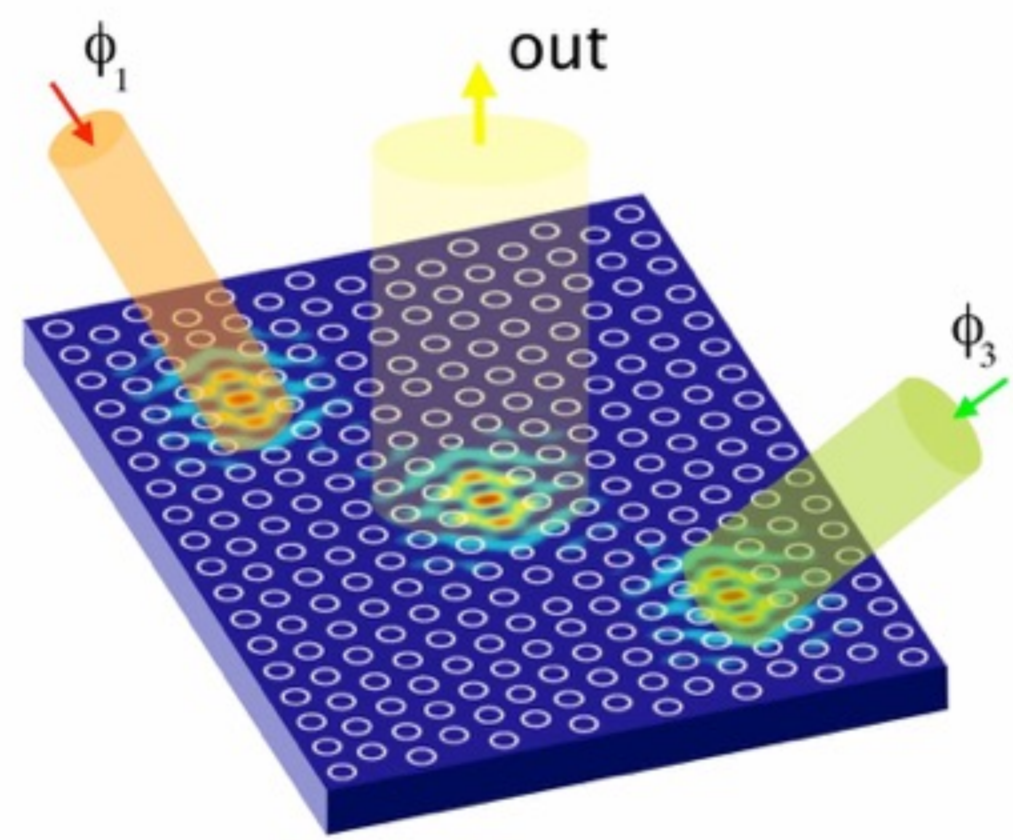
The quantum optical Josephson interferometer

D. Gerace, H. Tureci, A. Imamoglu and R.F. Nat. Phys 5, 281 (2009)

The interplay of tunnelling and interactions is analysed in the steady state of the system, when a dynamical equilibrium between driving and losses is established.



Strong photon correlations can be identified clearly in the suppression of Josephson-like oscillations of the light emitted from the central cavity as the nonlinearity is increased.



The quantum optical Josephson interferometer

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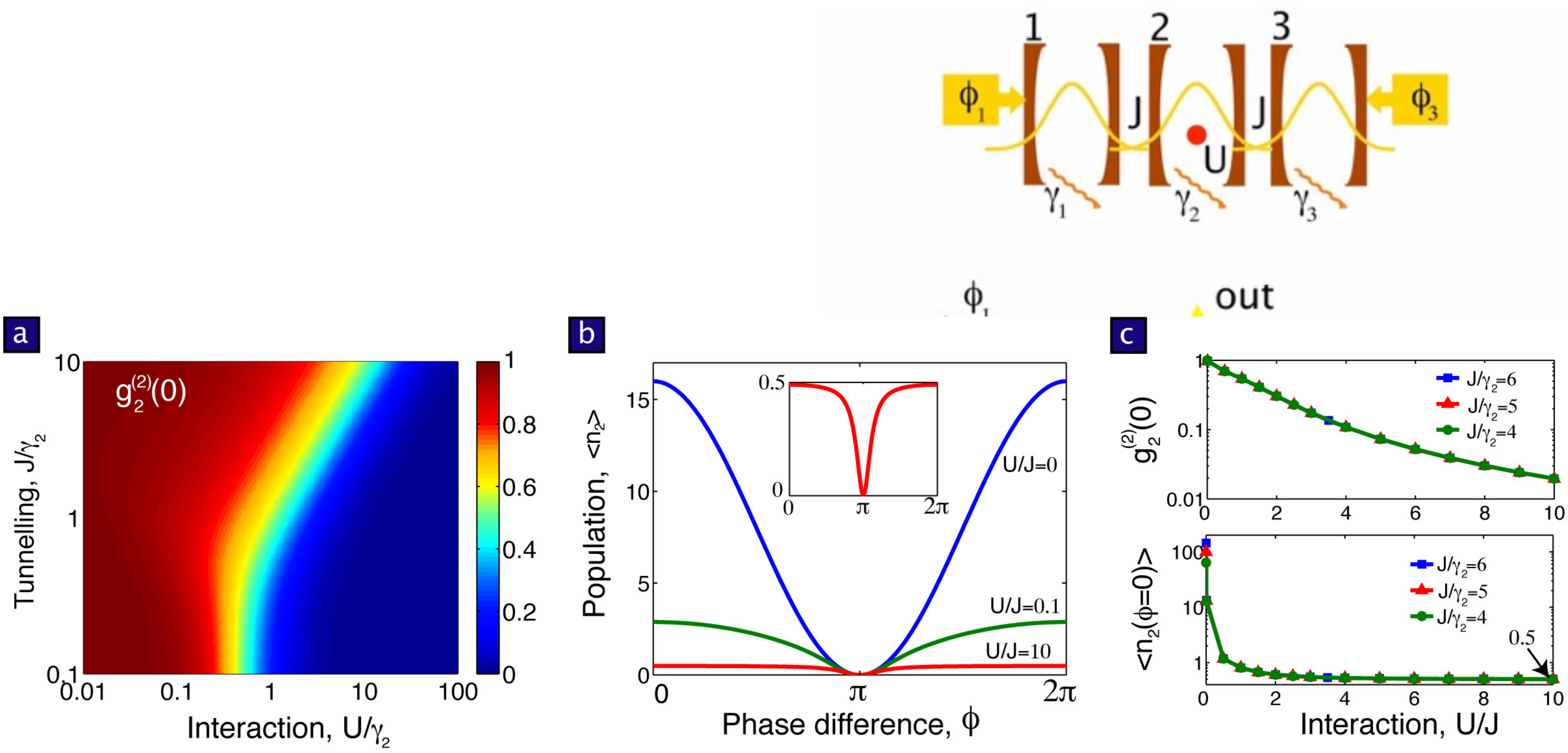
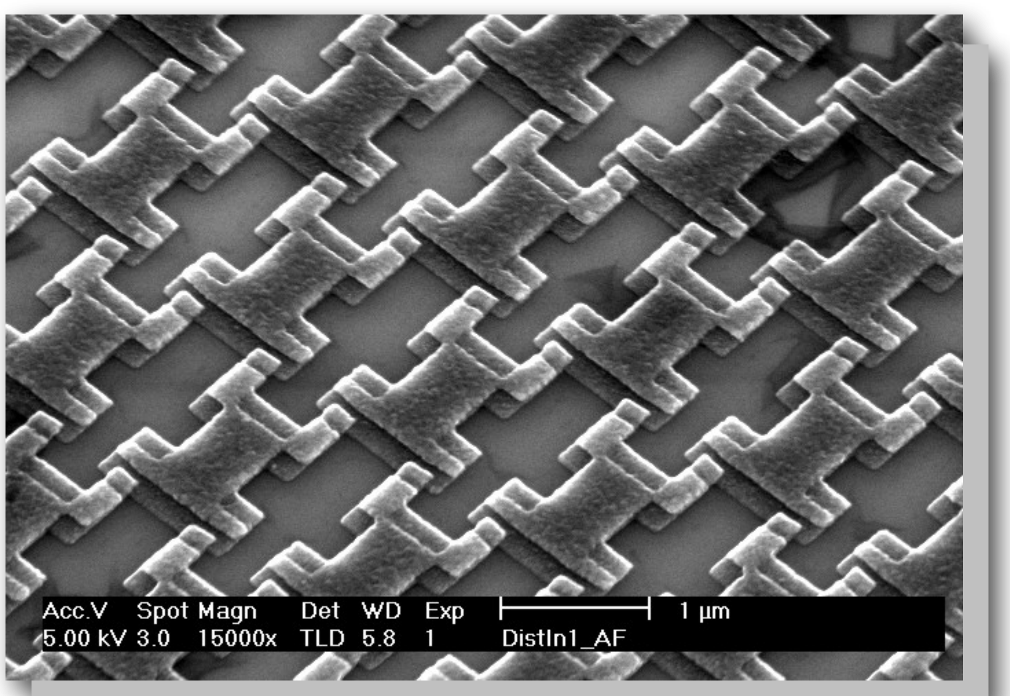
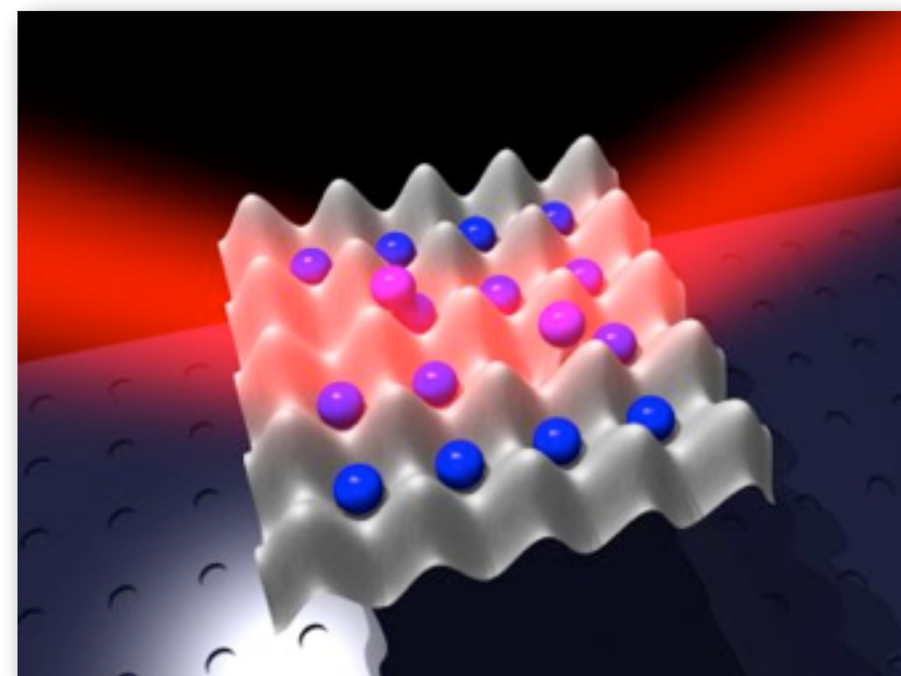


From small networks
to large arrays…

Models with competition of strong local correlation and delocalisation

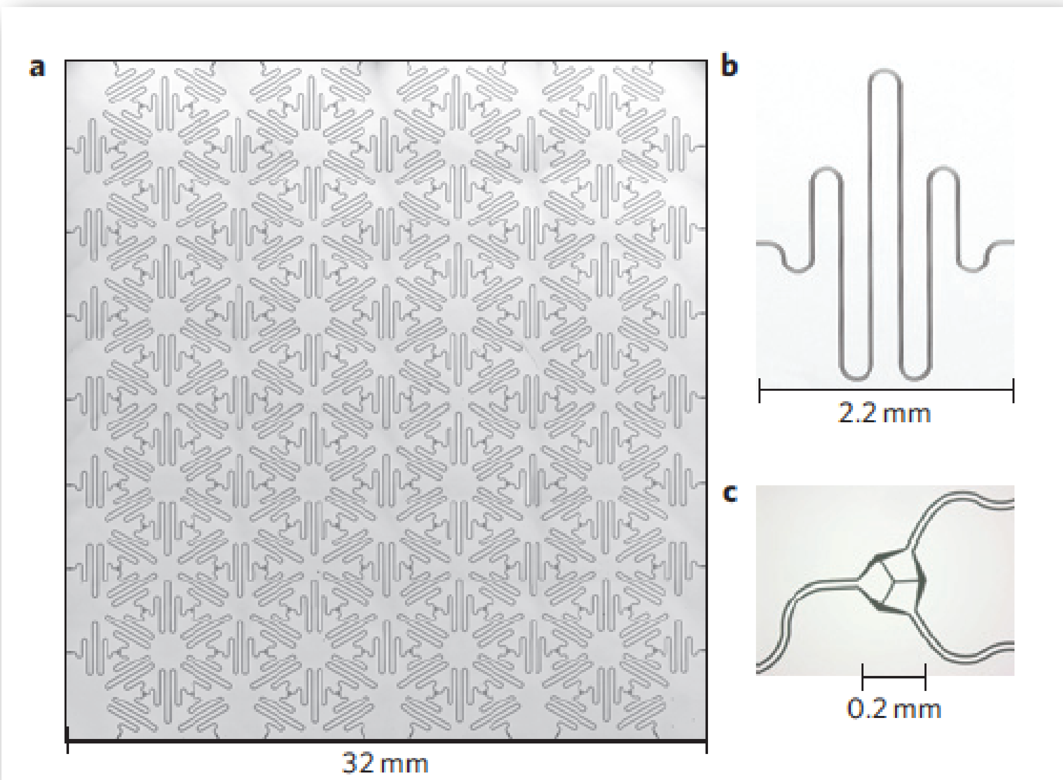
Josephson arrays

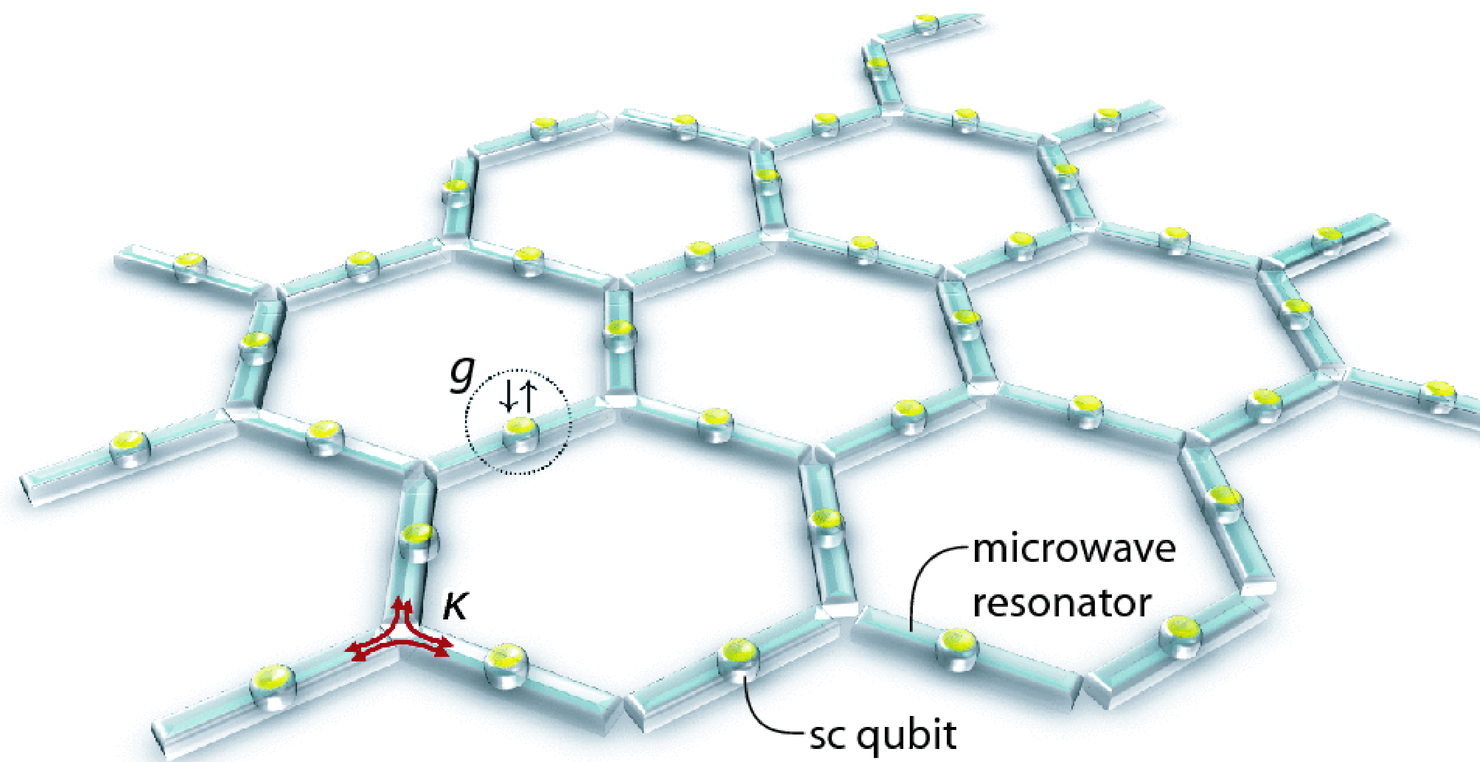




Optical lattices

Coupled cavity arrays



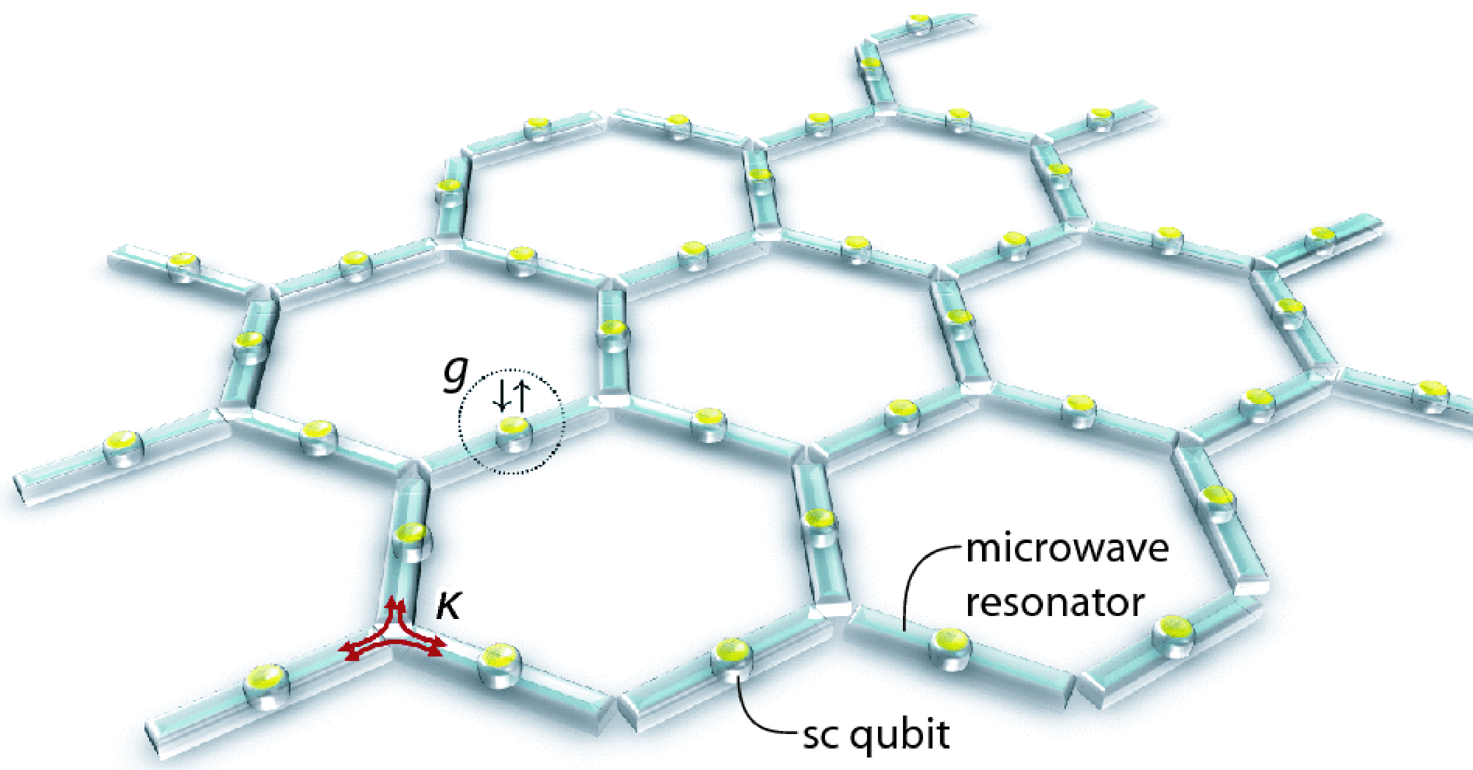


Picture from : J. Koch *et al* (2010)

Andrew Houck group

M. J. Hartmann, F.G. Brandao, and M. B. Plenio, Nat. Phys. **2**, 849 (2006)
A.D. Greentree *et al*, Nat. Phys. **2**, 856 (2006)
D.G. Angelakis, M.F. Santos, and S. Bose, Phys. Rev. A (RC) **76**, 031805 (2007)

Coupled cavity arrays



Picture from : J. Koch *et al* (2010)

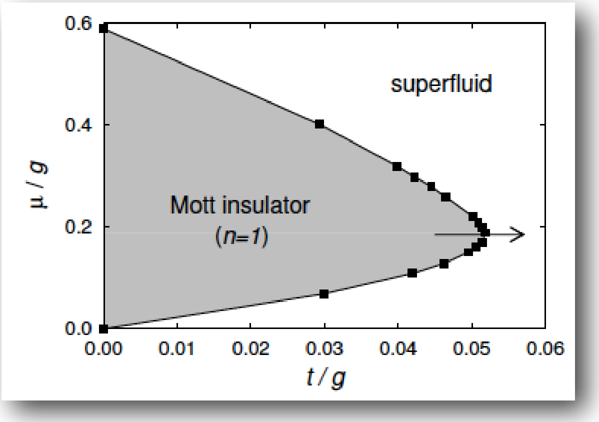
Promising platform to study dissipative transitions

Coupled cavity arrays

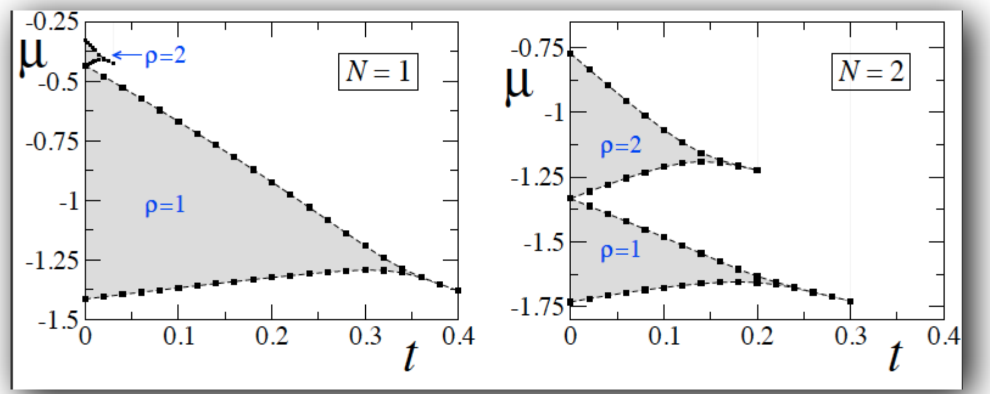
$$\mathcal{H} = \sum_i h_i - t \sum_{<ij>} (a_i^\dagger a_j + h.c.)$$

h_i "=" Jaynes-Cummings model

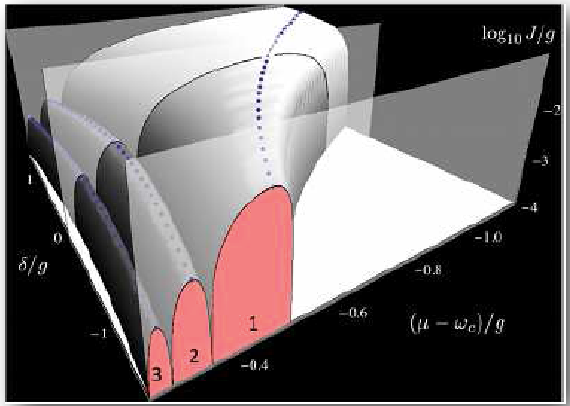
$$h_i = \epsilon\sigma_z + \omega a^\dagger a + \beta(a^\dagger \sigma_- + a\sigma_+)$$



Hohenadler, et al 2011



Rossini and Fazio 2007



Schmidt and Blatter 2009

Dissipative Phase Transitions

B. QI.

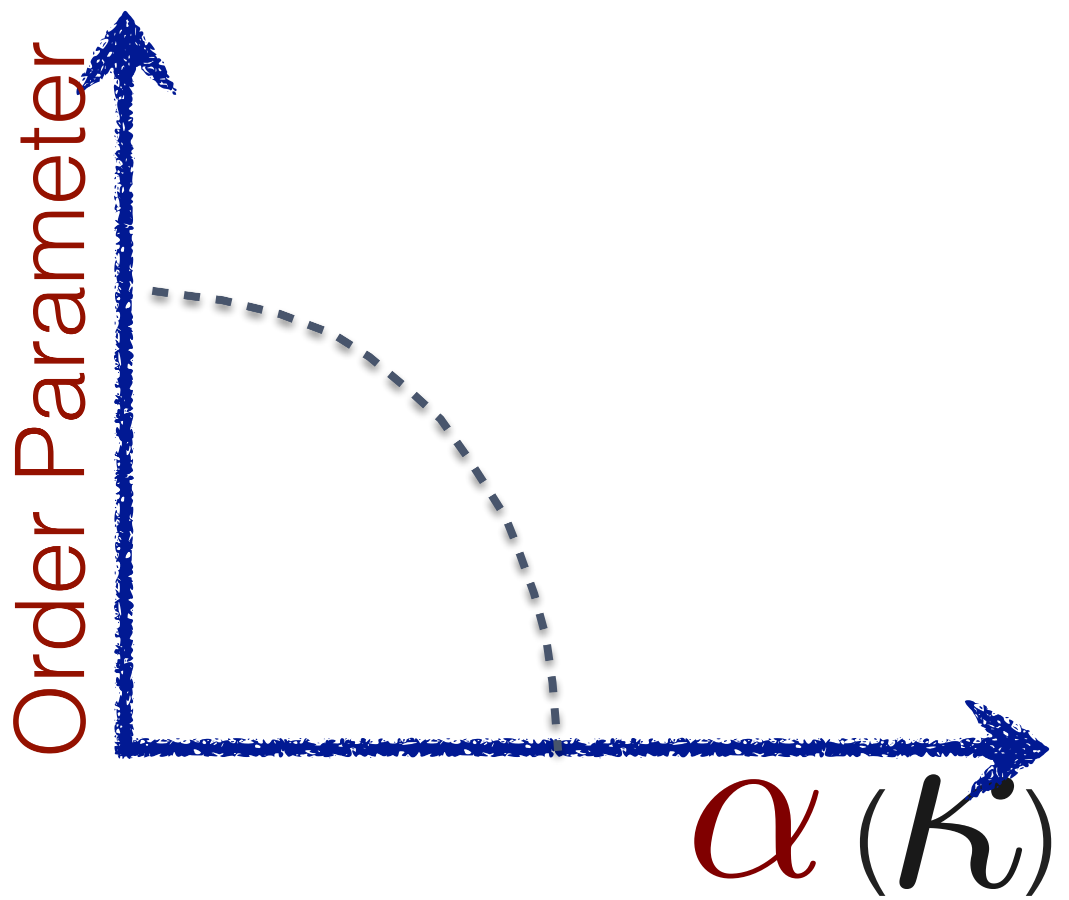
- Effect of dissipation and macroscopic quantum dynamics
- Josephson junction arrays

A. QI.

- Engineered baths
- Non-equilibrium effects
- Optical lattices, cavity arrays, trapped ions, ...

Dissipative phase transitions

System + environment
(in equilibrium)



Josephson junction arrays

VOLUME 63, NUMBER 3 PHYSICAL REVIEW LETTERS 17 JULY 1989

Charging Effects and Quantum Coherence in Regular Josephson Junction Arrays

L. J. Geerligs, M. Peters, L. E. M. de Groot,[a] A. Verbruggen,[a] and J. E. Mooij
Department of Applied Physics, Delft University of Technology, P.O. Box 5046, 2600 GA Delft, The Netherlands

(Received 17 April 1989)

al arrays of very-small-capacitance Josephson junctions have been studied. At low
arrays show a transition from superconducting to insulating behavior when the ratio of
o Josephson-coupling energy exceeds the value 1. Insulating behavior coincides with
a charging gap inside the BCS gap, with an S-shaped *I-V* characteristic. This so far
pe is predicted to arise from macroscopic quantum coherent effects including Bloch os-

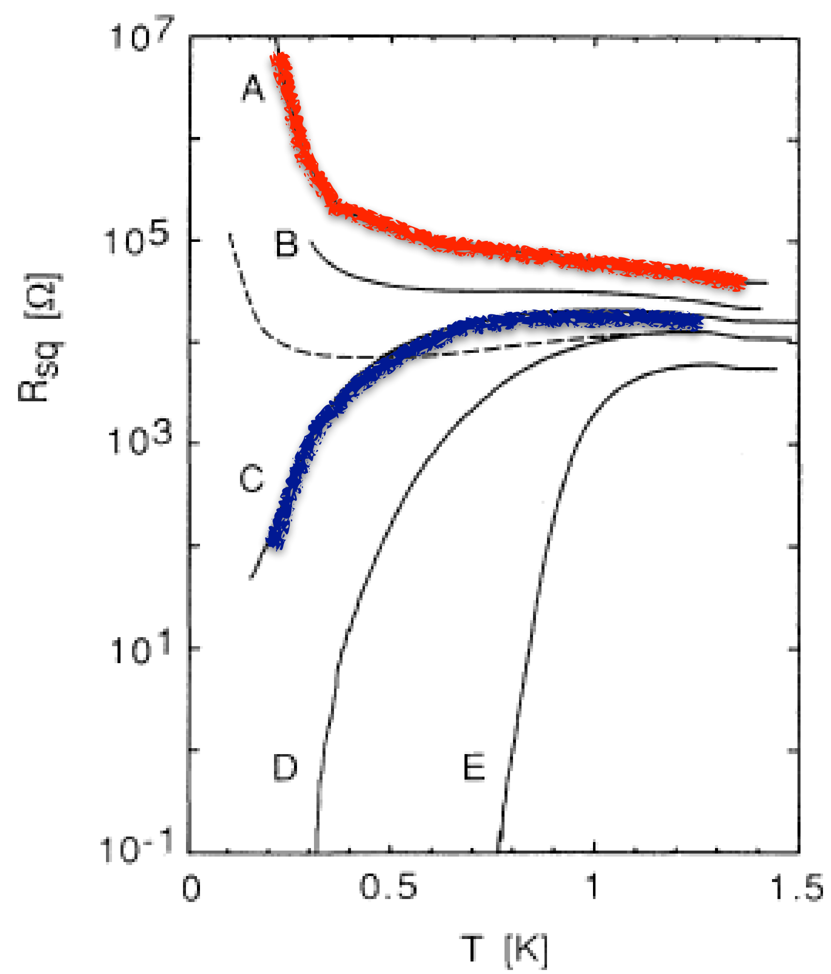


FIG. 1. $R(T)$ curves for arrays of 0.01-μm^2 junctions ($E_C \approx 0.84$ K). R_{sq} is the resistance divided by the length/width ratio 3.14. Each solid curve corresponds to an array with a particular normal-state resistance R_n in zero field. The dashed curve is for array D with $f \approx \frac{1}{2}$. Values of R_n in kΩ, E_J/k_B in K, and $x = E_j/E_C$ are, sample A: 36, 0.22, 3.9; B: 15.3, 0.51, 1.8; C: 14.1, 0.55, 1.5; D: 9.7, 0.80, 1.0; E: 4.8, 1.6, 0.53.

Josephson junction arrays

VOLUME 85, NUMBER 9 PHYSICAL REVIEW LETTERS 28 AUGUST 2000

Superconductor-Insulator Transition in a Two-Dimensional Array of Resistively Shunted Small Josephson Junctions

Yamaguchi Takahide,[1,2] Ryuta Yagi,[1] Akinobu Kanda,[1,2] Youiti Ootuka,[1,2] and Shun-ichi Kobayashi[3]

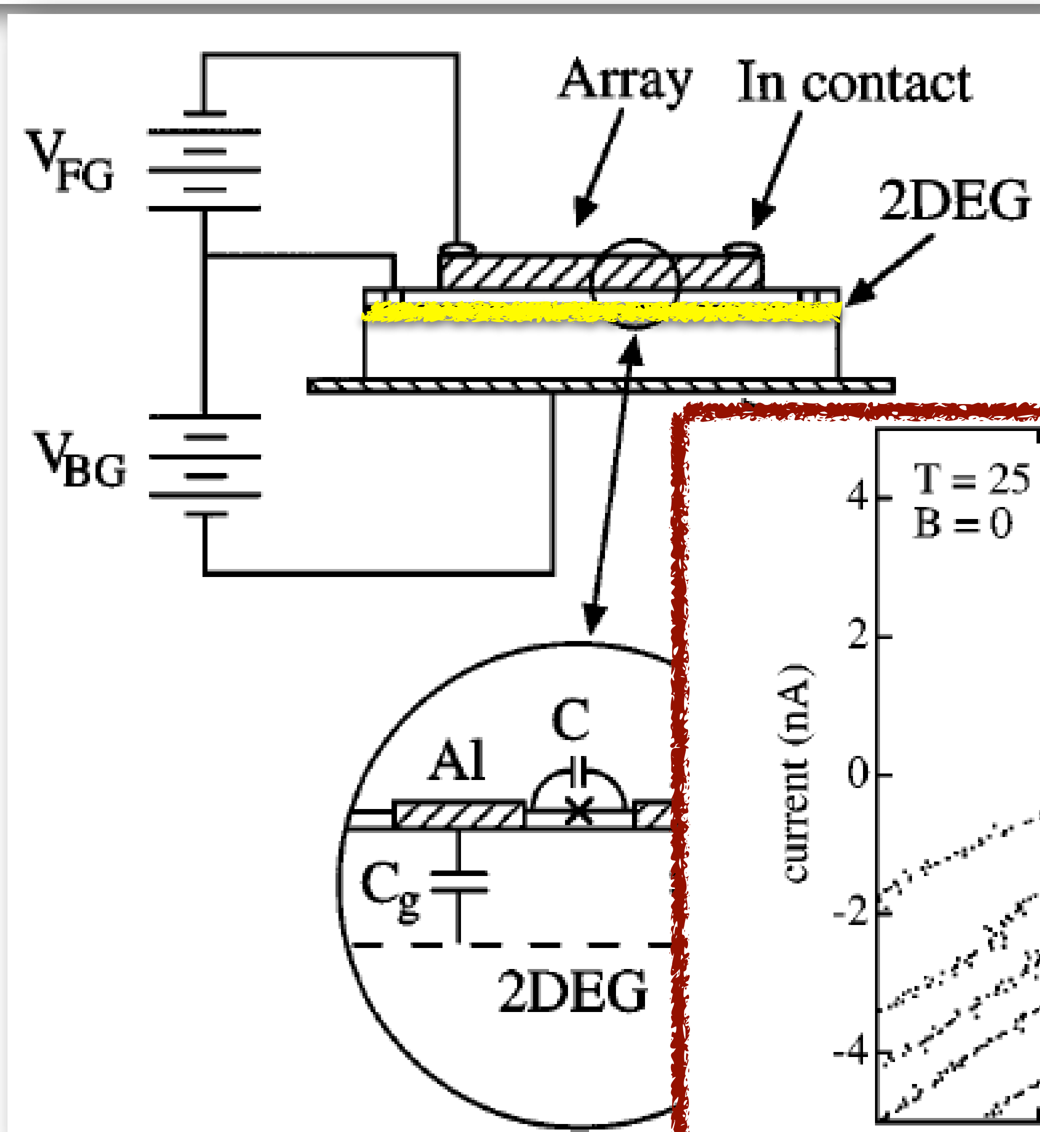
VOLUME 78, NUMBER 13 PHYSICAL REVIEW LETTERS 31 MARCH 1997

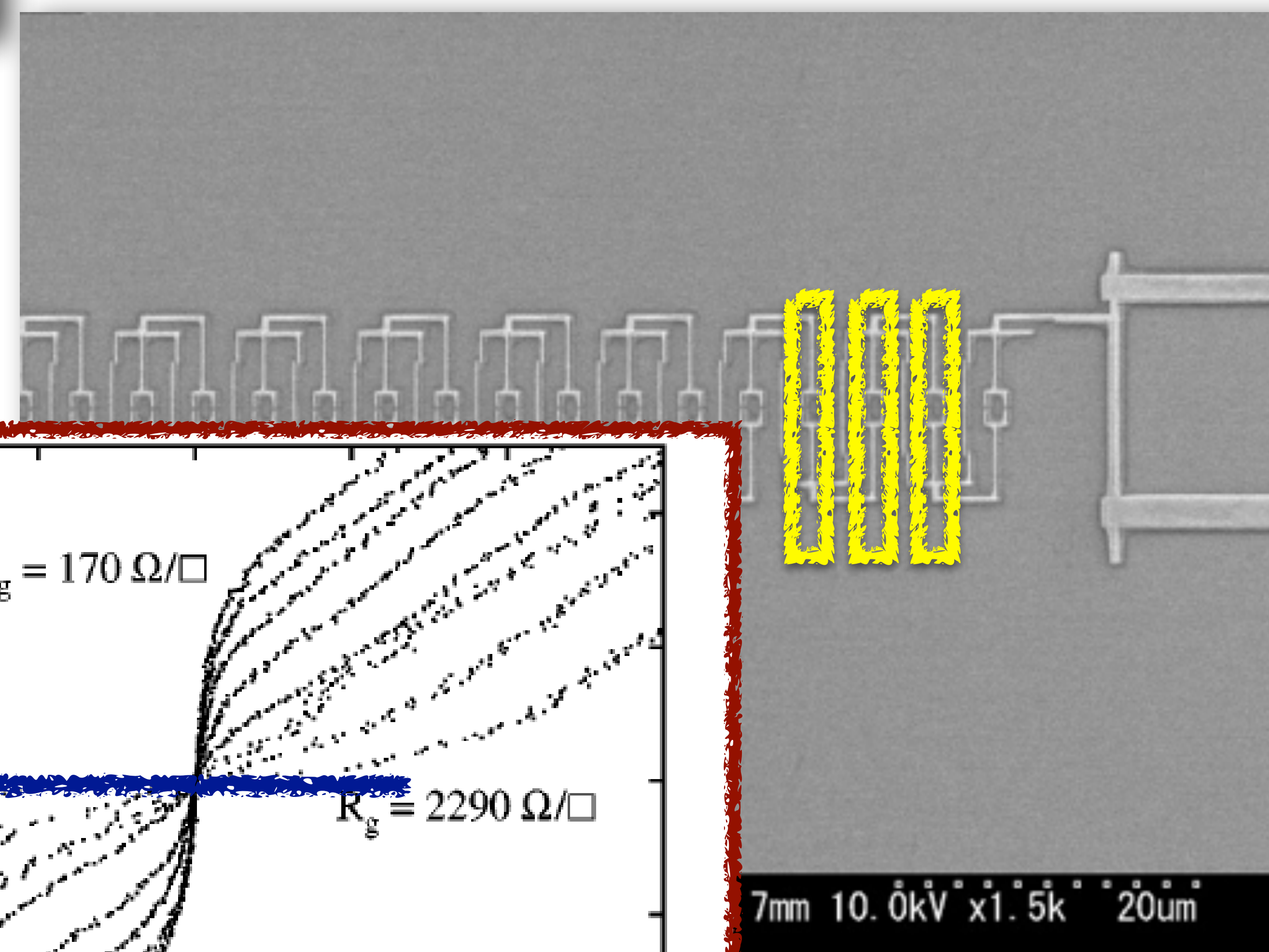
Dissipation-Driven Superconductor-Insulator Transition in a Two-Dimensional Josephson-Junction Array

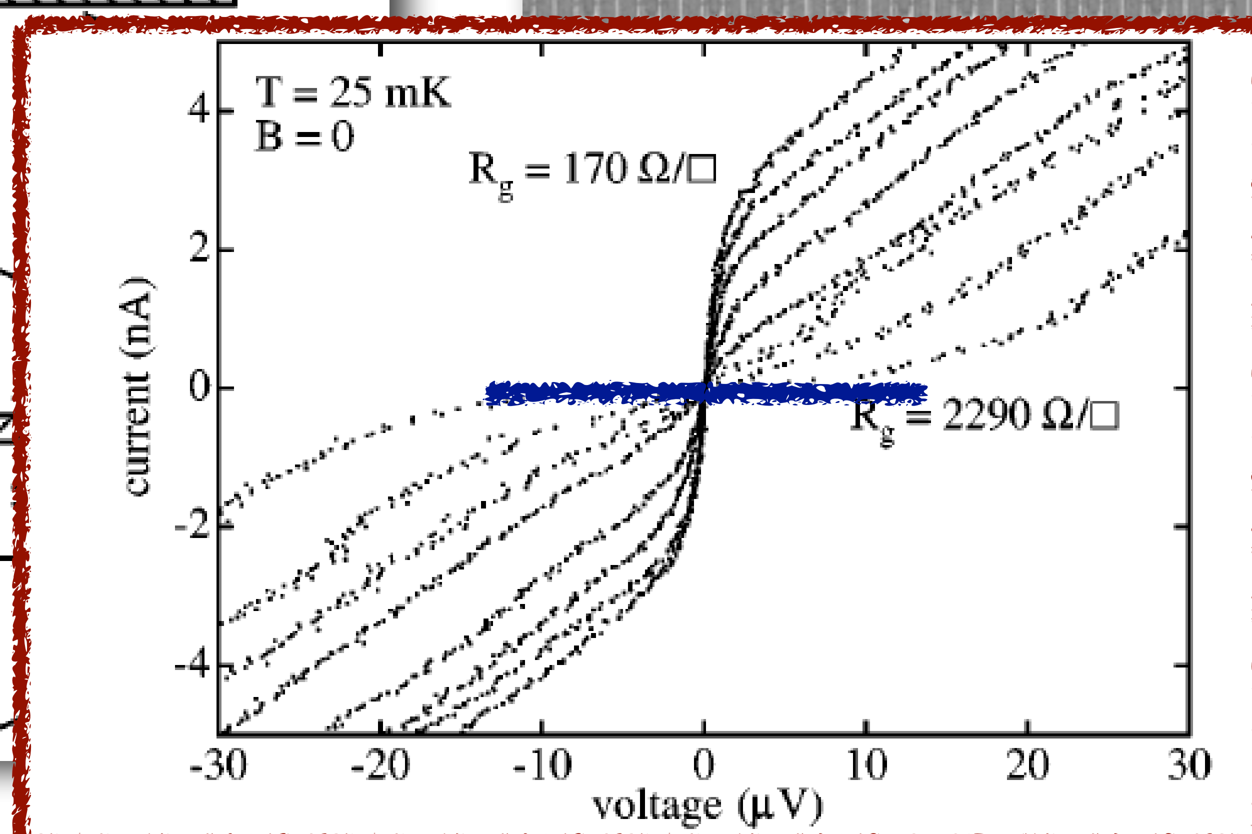
A. J. Rimberg,* T. R. Ho, Ç. Kurdak, and John Clarke
Department of Physics, University of California, Berkeley, California 94720
and Materials Sciences Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720

K. L. Campman and A. C. Gossard
Materials Department, University of California, Santa Barbara, California 93106

(Received 26 November 1996)

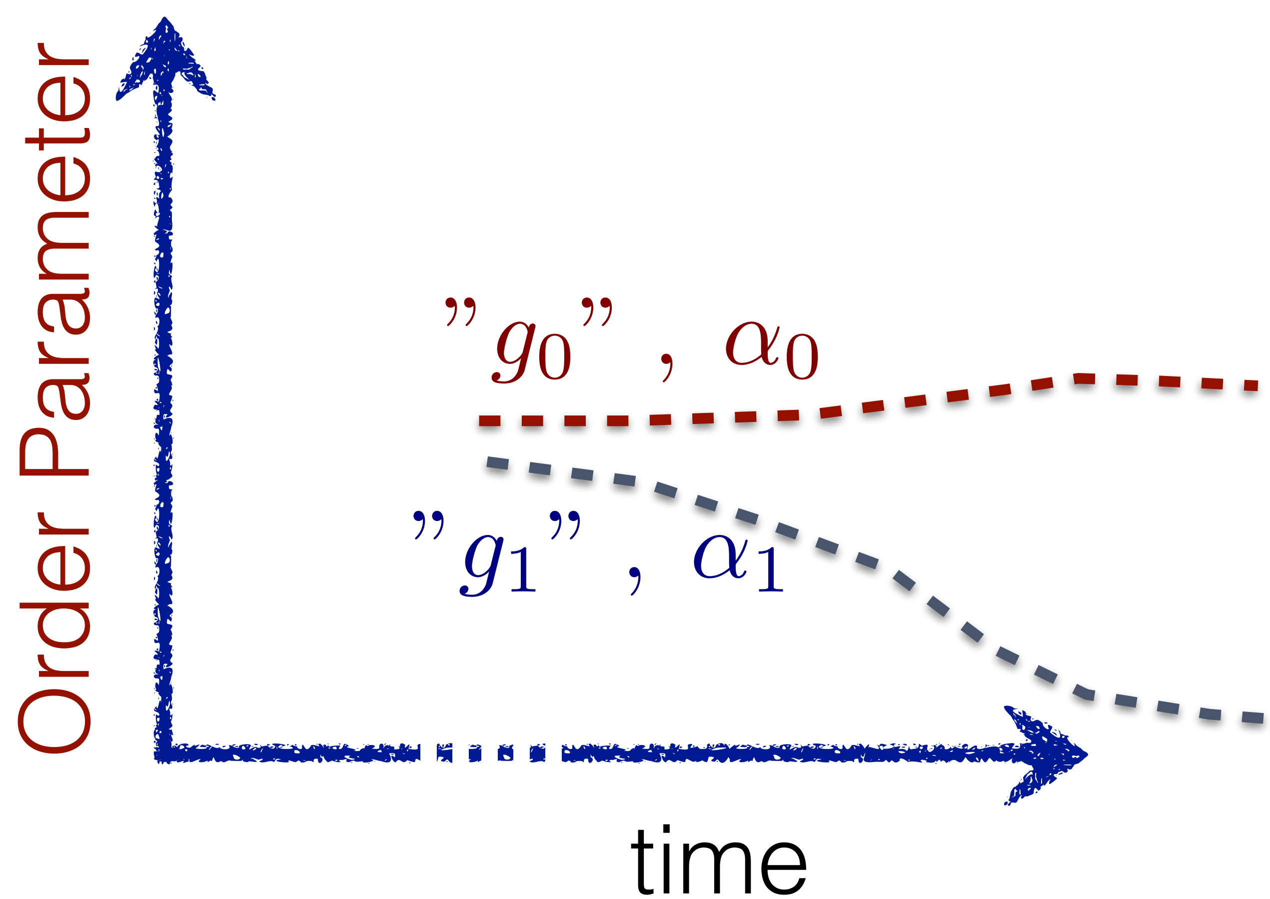






Dissipative phase transitions

$$\dot{\rho} = -i[\overset{g}{\mathcal{H}}, \rho] + \overset{\alpha}{\mathcal{L}}[\rho]$$



Dissipative phase transitions

- Rich(er) steady-state phase diagram: *(symmetry broken phases, incommensurate phases, limit cycles,…)*

J. Keeling *et al*, Carusotto & Ciuti, Lee et al, Boitè *et al*, Ludwig & Marquardt, Chan *et al*, …

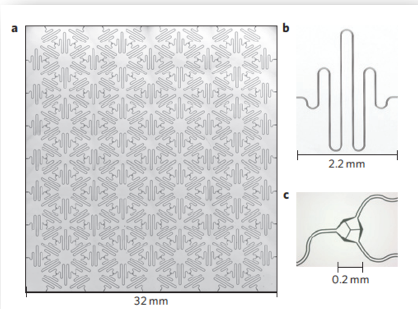
- Modified critical behaviour: *(coupling to the environment may change the universality class)*

Della Torre *et al*, Diehl *et al*, Öztop *et al,* Domokos *et al* …

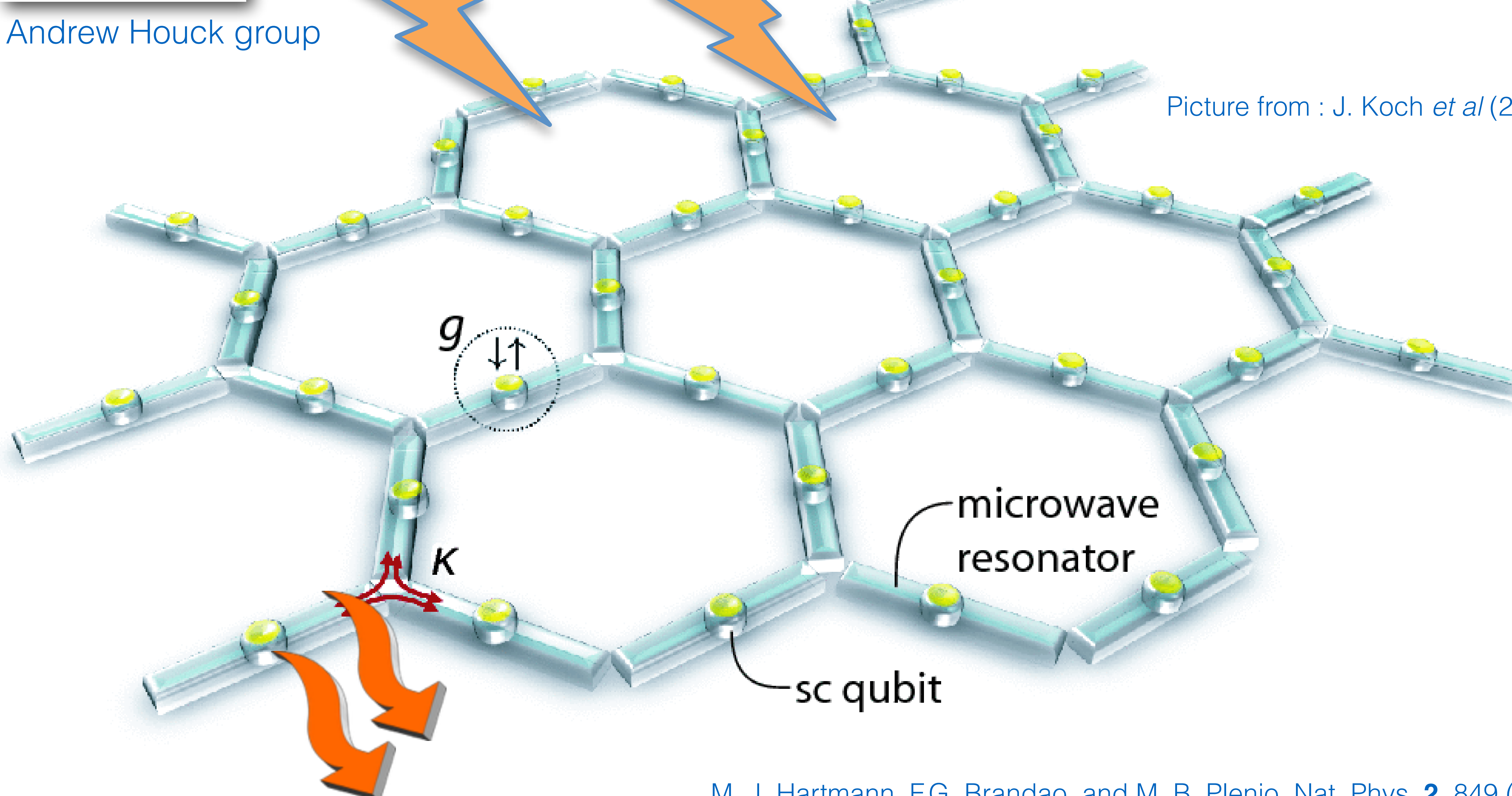
- Equilibrium vs non-equilibrium: *(the steady-state does not need to describe an equilibrium state)*

- Control on properties of the bath: *(Cavity arrays, BEC in cavities, Rydberg atoms, trapped ions,…)*

Coupled cavity arrays



Andrew Houck group



Picture from : J. Koch *et al* (2010)

M. J. Hartmann, F.G. Brandao, and M. B. Plenio, Nat. Phys. **2**, 849 (2006)
A.D. Greentree *et al*, Nat. Phys. **2**, 856 (2006)
D.G. Angelakis, M.F. Santos, and S. Bose, Phys. Rev. A (RC) **76**, 031805 (2007)

Coupled cavity arrays

- Competition of strong local correlation and delocalisation

$$\mathcal{H} = \sum_i h_i + \sum_{<ij>} h_{ij}^{(coupl)}$$

h_i "=" Jaynes-Cummings model

$$h_i = \epsilon\sigma_z + \omega a^\dagger a + \beta(a^\dagger\sigma_- + a\sigma_+)$$

or the Bose-Hubbard (Kerr) non-linearity

- Competition of photon leakage and external driving

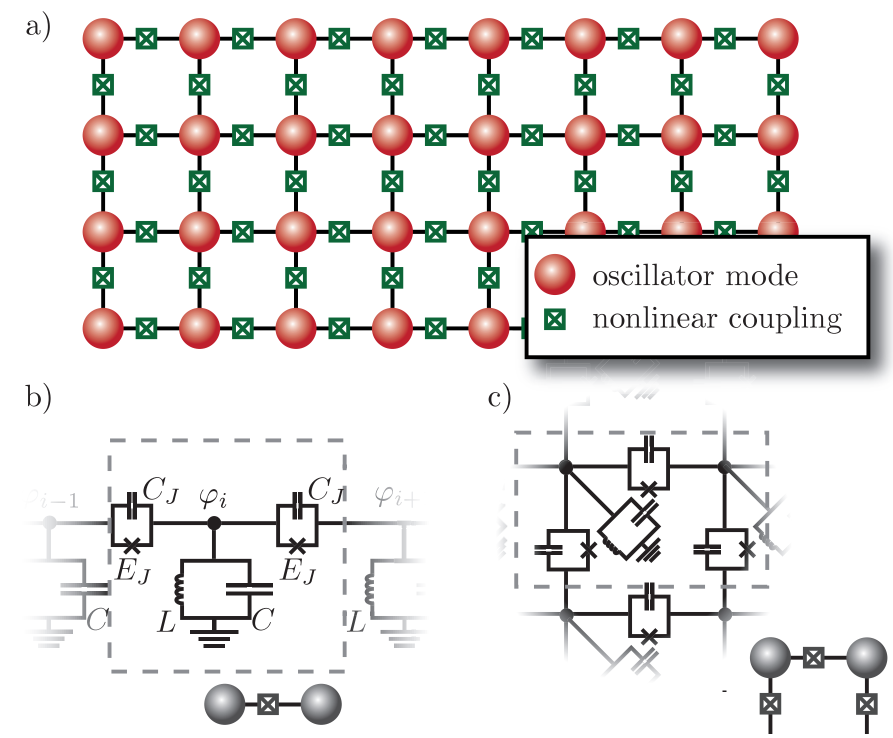
$$\mathcal{H} = \Omega \sum_i (a_i + a_i^\dagger)$$

$$\mathcal{L}[\rho] = \frac{\gamma}{2} \sum_i (2a_i\rho a_i^\dagger - \{n_i, \rho\})$$

How to reach a “non-trivial” steady state?

Dissipative transition in coupled cavity arrays

- Engineer non-linear couplings between cavities



$$h_{ij}^{(coupl)}$$

J. Jin, D. Rossini, R. F., M. Leib, and M. J. Hartmann, Phys. Rev. Lett. **110**, 163605 (2013)
J. Jin, D. Rossini, M. Lieb, M.J. Hartmann, and R. F., Phys. Rev. A **90**, 023827 (2014)

- Go to the ultra-strong regime (Rabi-Hubbard model) M.Schiro´,M.Bordyuh,B.Öztop, and H.E.Türeci, Phys. Rev. Lett. **109**, 053601 (2012).

$$h_i = \epsilon\sigma_{z,i} + \omega a_i^\dagger a_i + g(a_i^\dagger \sigma_{-,i} + h.c.) + \boxed{g'(a_i^\dagger \sigma_{+,i} + h.c.)}$$

M. Schiró, C. Joshi, M. Bordyuh, R. F., J. Keeling, H. E. Türeci, arXiv:1503.04456

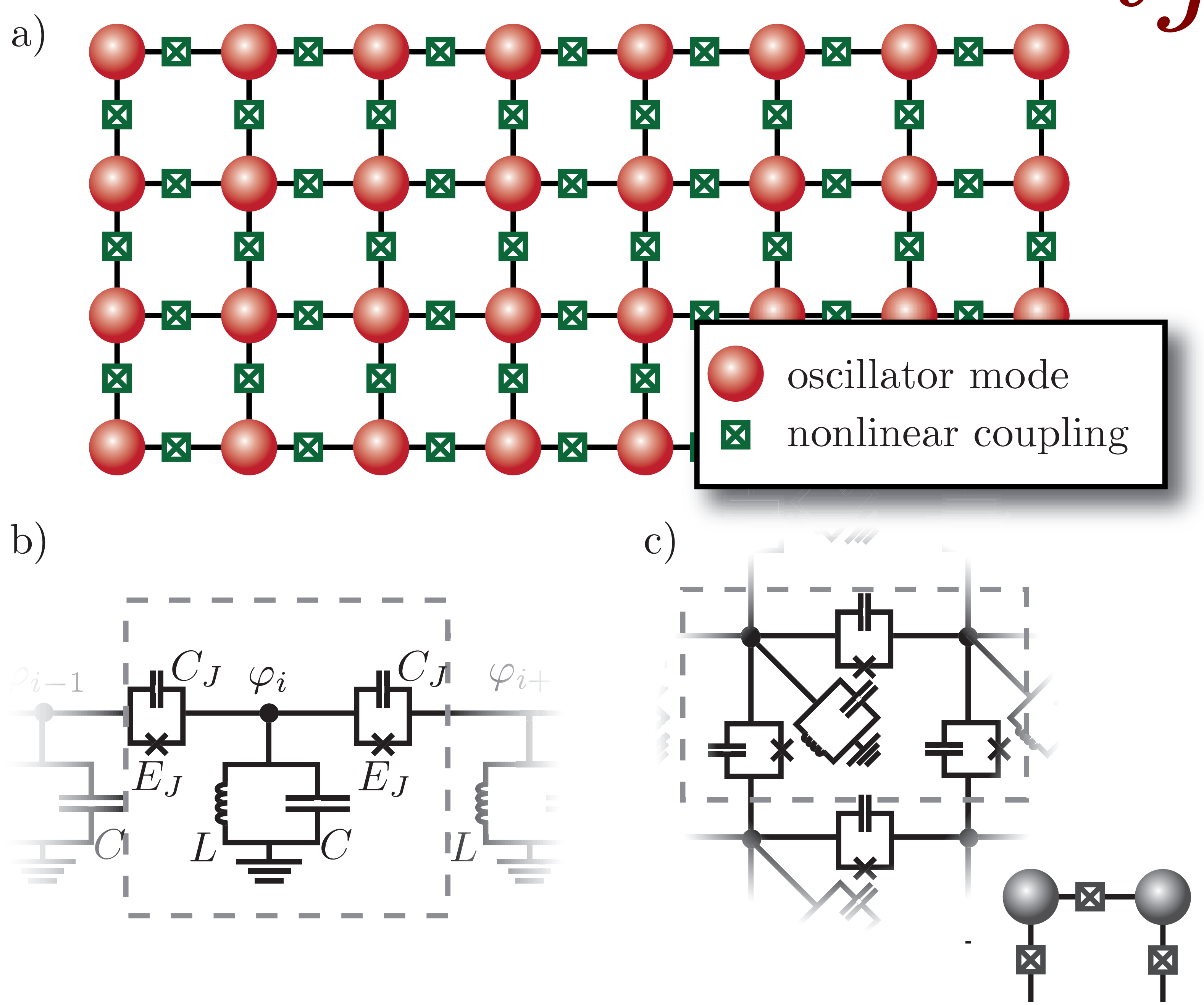
Dissipative transition in coupled cavity arrays

Non-linear coupled array			
Rabi-Hubbard model (g=g')			

Dissipative transition in coupled cavity arrays

- Engineer non-linear couplings between cavities

$$h_{ij}^{(coupl)}$$



Cross-Kerr nonlinearity

$$\mathcal{H} = -\overline{J}\sum_{\langle ij\rangle}(a_i^\dagger a_j + \mathrm{h.c.})$$

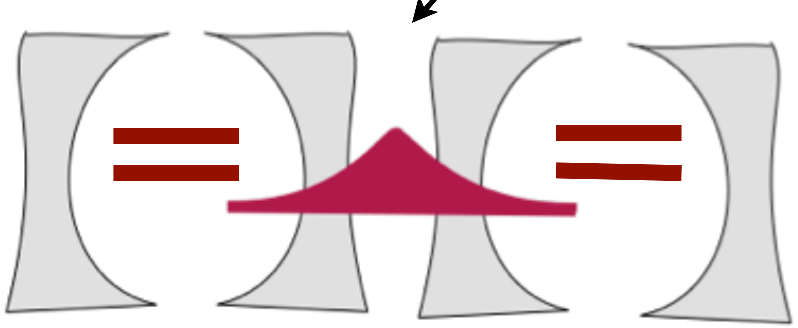
photon hopping

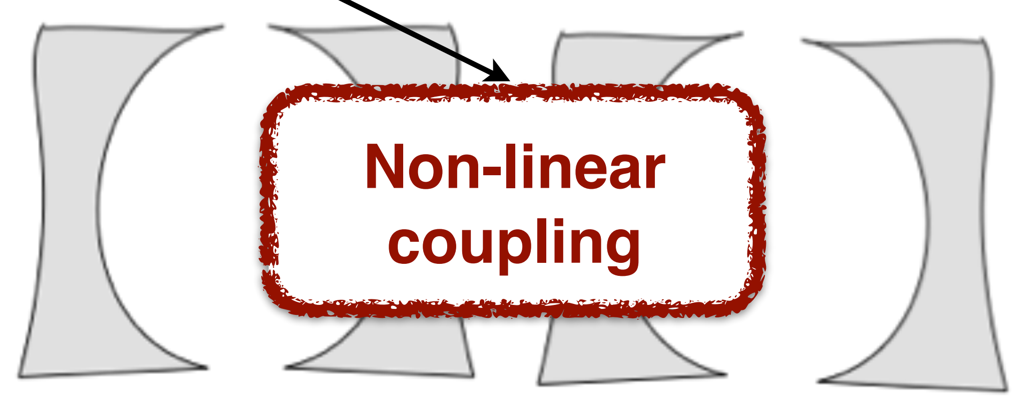
$$-\,\delta\sum_i a_i^\dagger a_i + \Omega\sum_i (a_i + a_i^\dagger)$$

driving

$$+\,U\sum_i n_i(n_i-1) + \overline{V}\sum_{\langle ij\rangle} n_i n_j$$

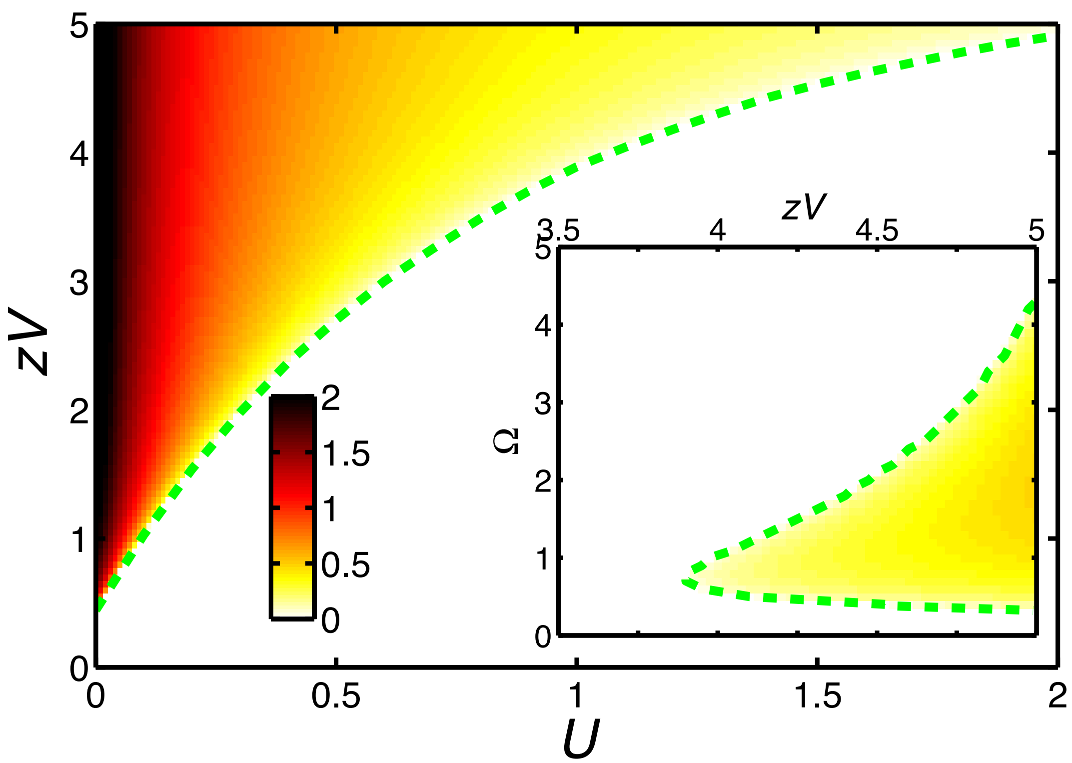
non-linearities

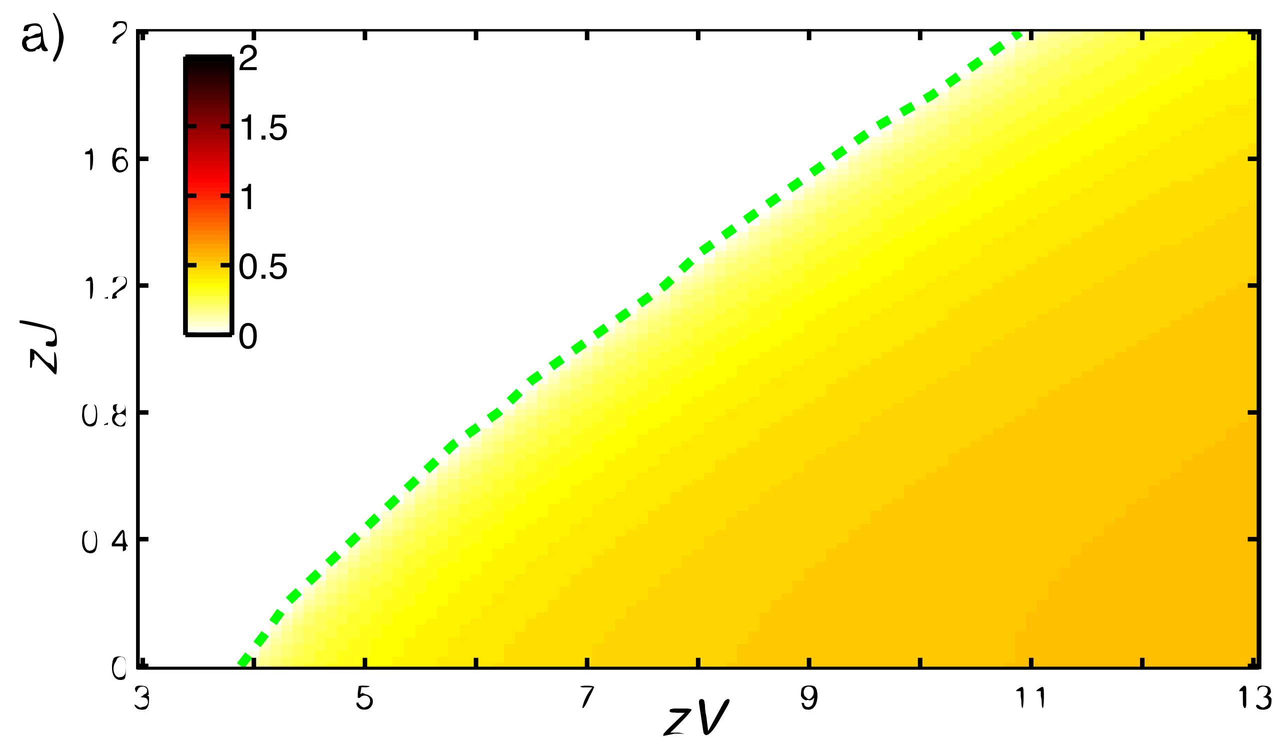




Steady state (mean-field) phase diagram

$$\langle n_A \rangle \neq \langle n_B \rangle$$ Photon crystal





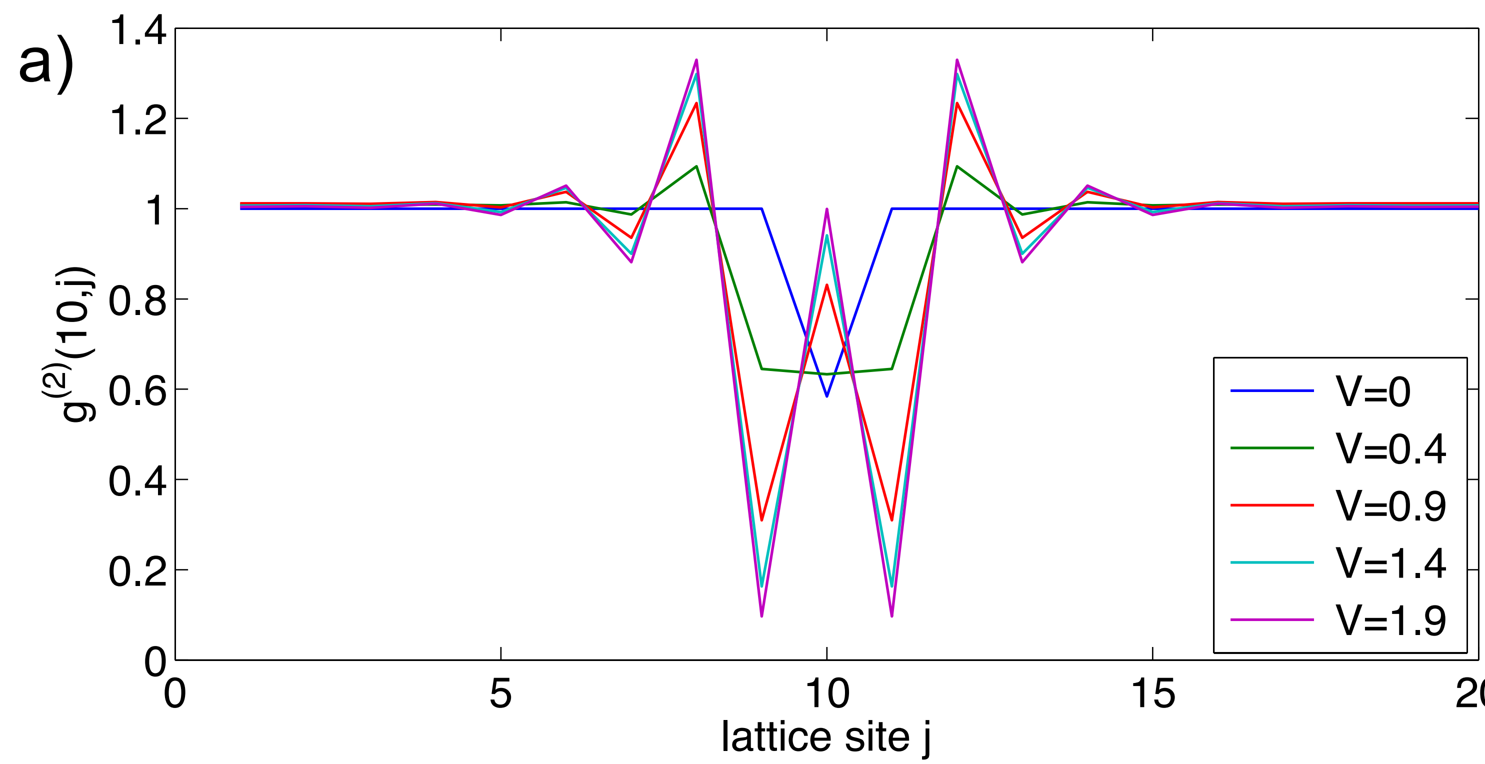
Antiferromagnetic photon blockade

“Crystalline” correlation in cavity chains

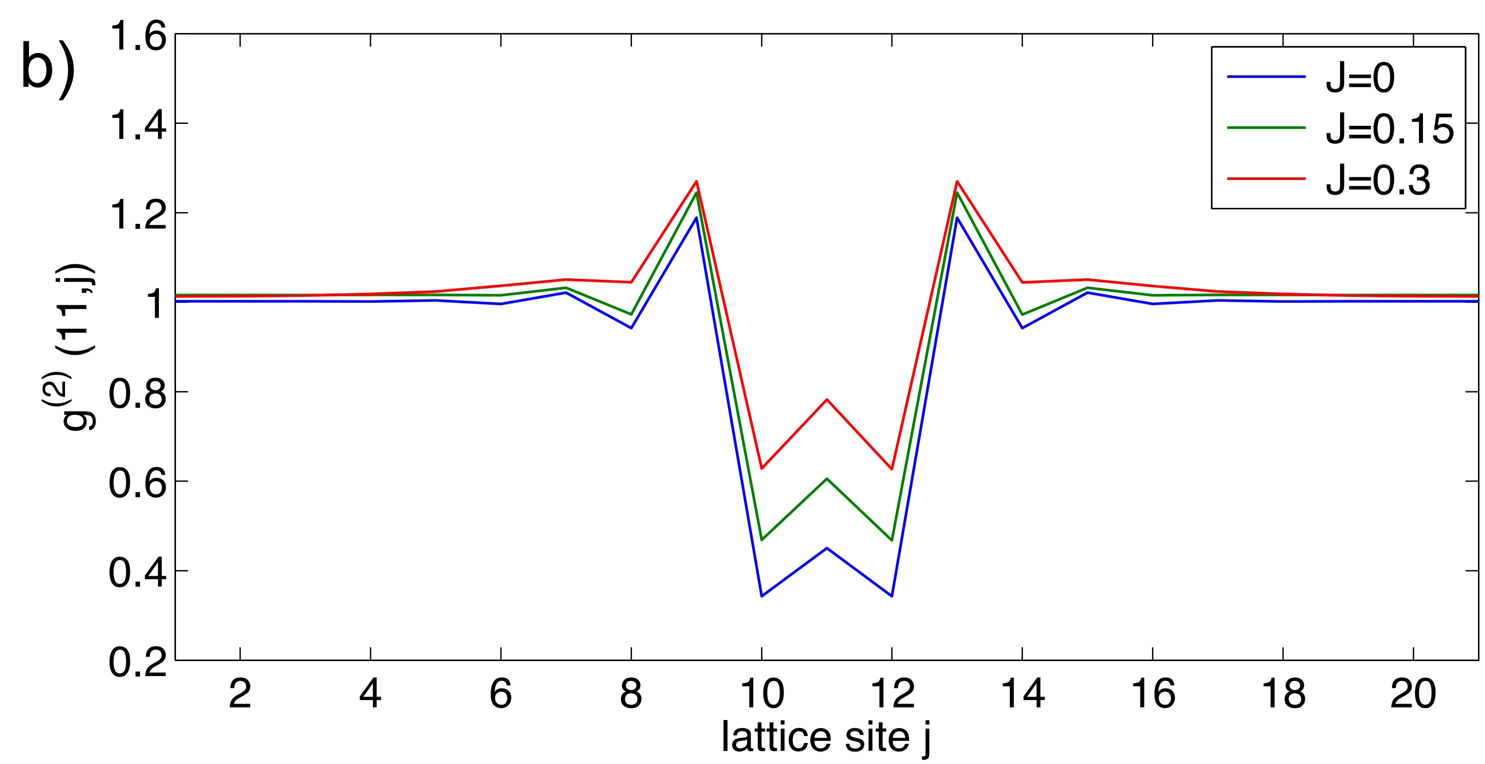
“Exact”-MPO numerical simulations in 1d

Density-density correlator:

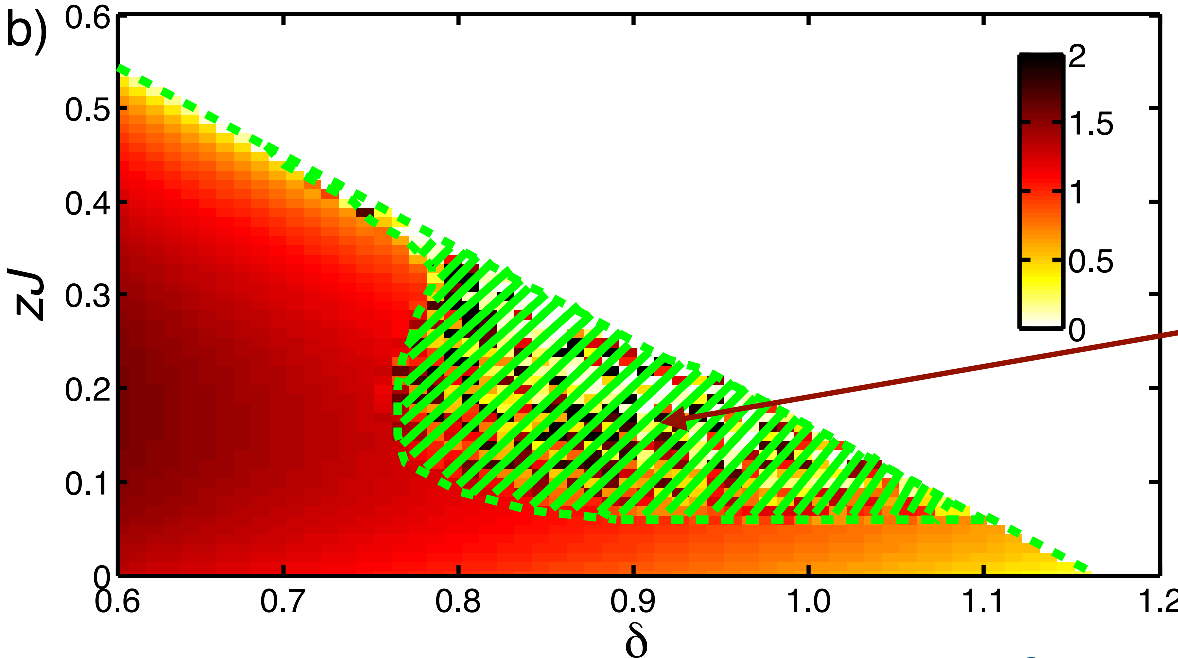
$$g^{(2)}(i,j) = \frac{\langle n_i^\dagger n_j \rangle}{\langle n_i \rangle \langle n_j \rangle}$$



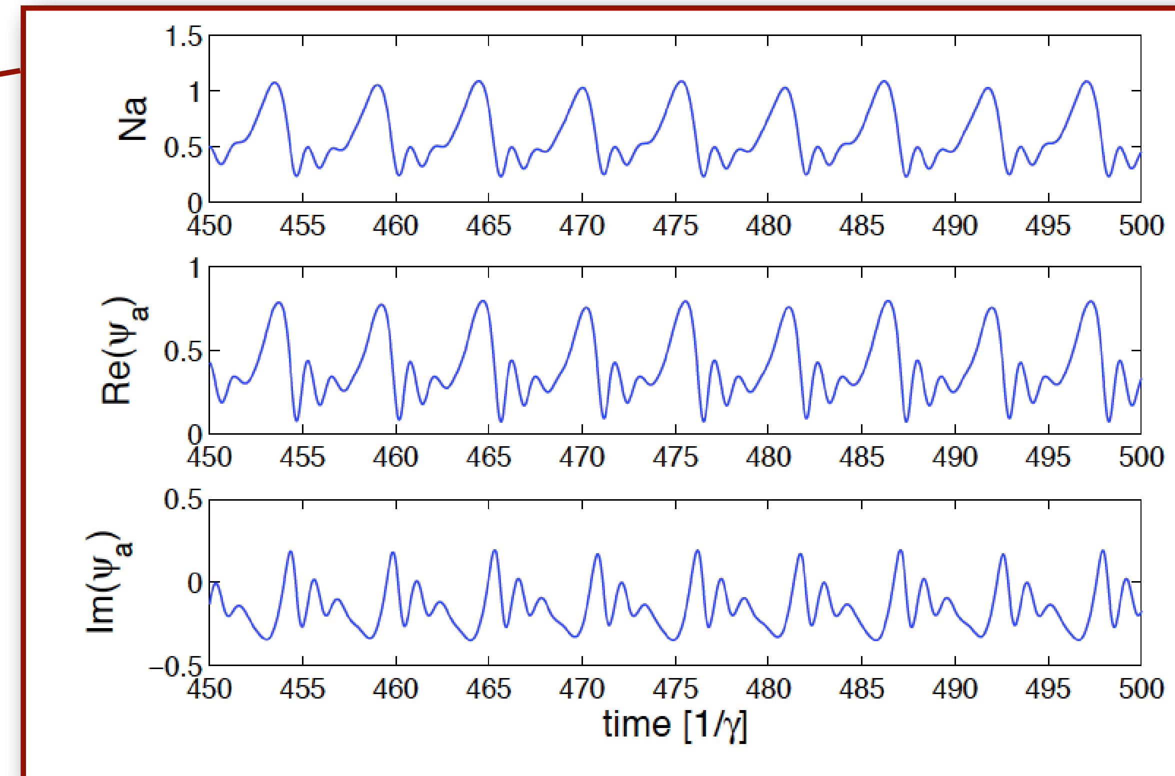
Short-range correlations



Spontaneous breaking of time-translational invariance



$$\psi_A = \langle a_A \rangle$$



"Time-crystals" out of equilibrium

See F. Wilczek

Existence of a spatio-temporal Goldstone mode
Chan *et al* arXiv:1501.0979

$$\psi_A \neq \psi_B$$

Sublattice phase synchronization (superfluidity)

See Ludwig and Marquardt for quantum synchronisation in opto-mechanical arrays(2012)

Dissipative transition in coupled cavity arrays

- Go to the ultra-strong regime (Rabi-Hubbard model)

M.Schiro´,M.Bordyuh,B.Öztop, and H.E.Türeci, Phys. Rev. Lett. 109, 053601 (2012).

$$h_i = \epsilon\sigma_{z,i} + \omega a_i^\dagger a_i + g(a_i^\dagger \sigma_{-,i} + h.c.) + g'(a_i^\dagger \sigma_{+,i} + h.c.)$$

$$\mathcal{H} = \sum_i h_i - J \sum_{<ij>} (a_i^\dagger a_j + h.c.)$$

The dissipative Rabi-Hubbard model displays various exotic attractors, including ferroelectric, antiferroelectric, and incommensurate fixed points, as well as regions of persistent oscillations

$$\mathcal{L}[\rho] = \frac{\gamma}{2}\sum_i (2a_i\rho a_i^\dagger - \{n_i, \rho\})$$

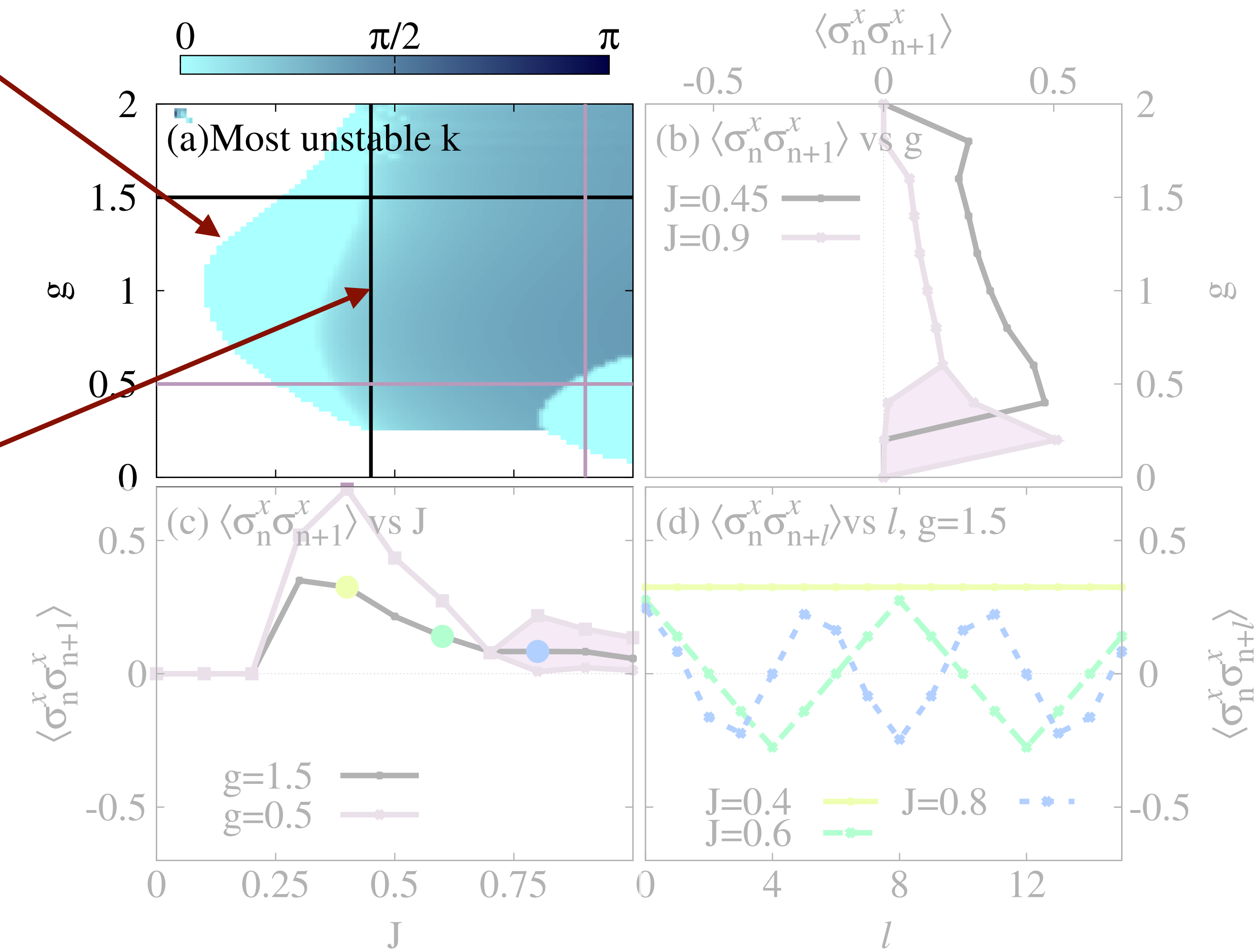
M. Schiró, C. Joshi, M. Bordyuh, R. Fazio, J. Keeling, H. E. Türeci, arXiv:1503.04456

Steady state phase diagram

Spontaneous Z_2 symmetry breaking in the steady state signalled by $\langle \sigma_n^x \rangle$ or $\langle a_n + a_n^\dagger \rangle \neq 0$

Instability of the normale state has a reentrant shape

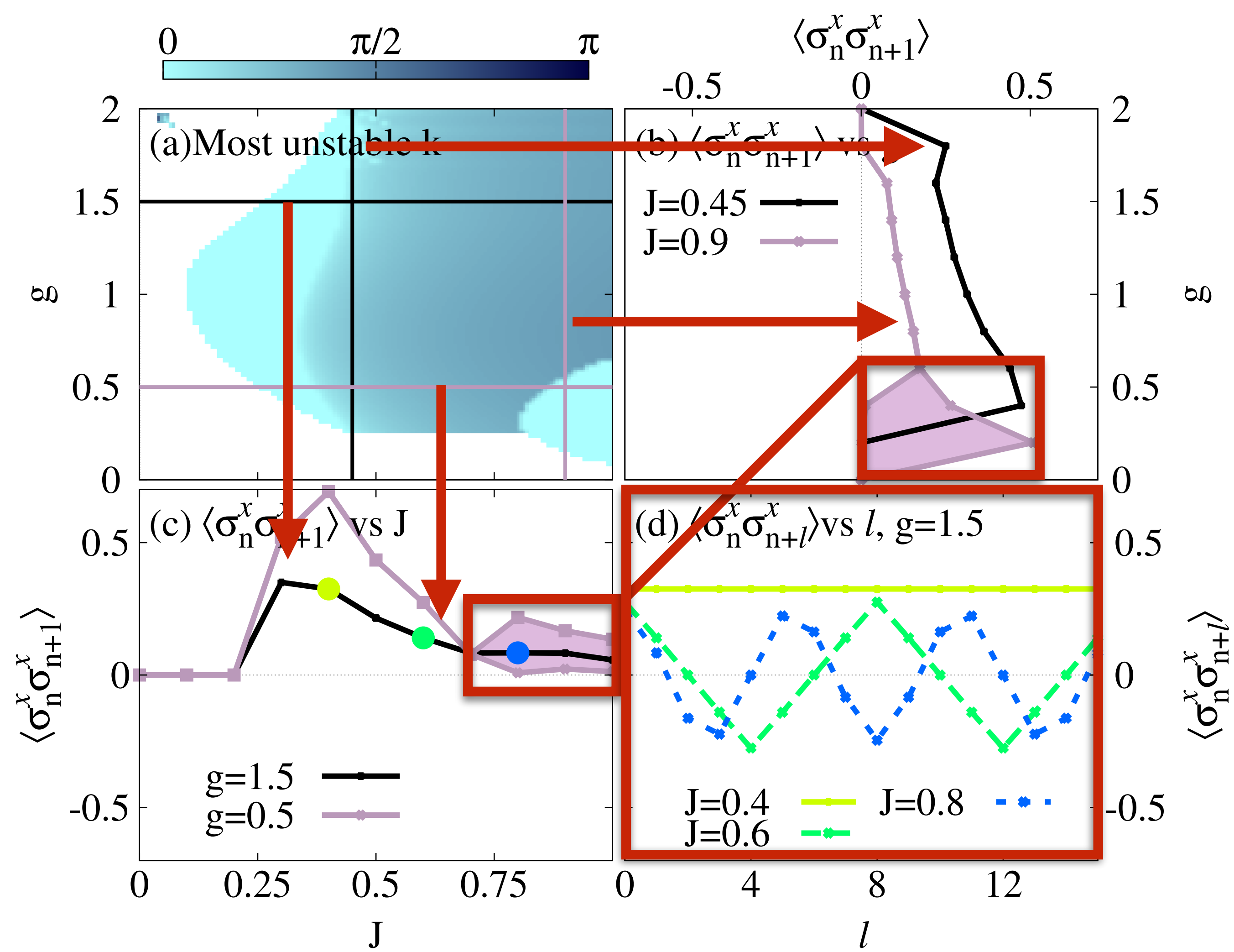
most unstable wavevector evolves smoothly from k=0 toward k= π/2, passing through irrational values, corresponding to an instability towards incommensurate order.



Steady state phase diagram

Appearance of a density wave, with a period decreasing as hopping increases.

The appearance of **incommensurate order** is **not** a feature **present in** the **equilibrium** phase diagram: the minimum free energy state always has a constant phase across the array.



The shaded region shows the envelope of the **limit cycle oscillations** of the correlation function

Photon transport & many-body states

To which extent is it possible to detect many-body states and phase transitions in photon transport?

"Spectroscopy" of many-body states

- At large interaction strength Tonks gas or photon crystal

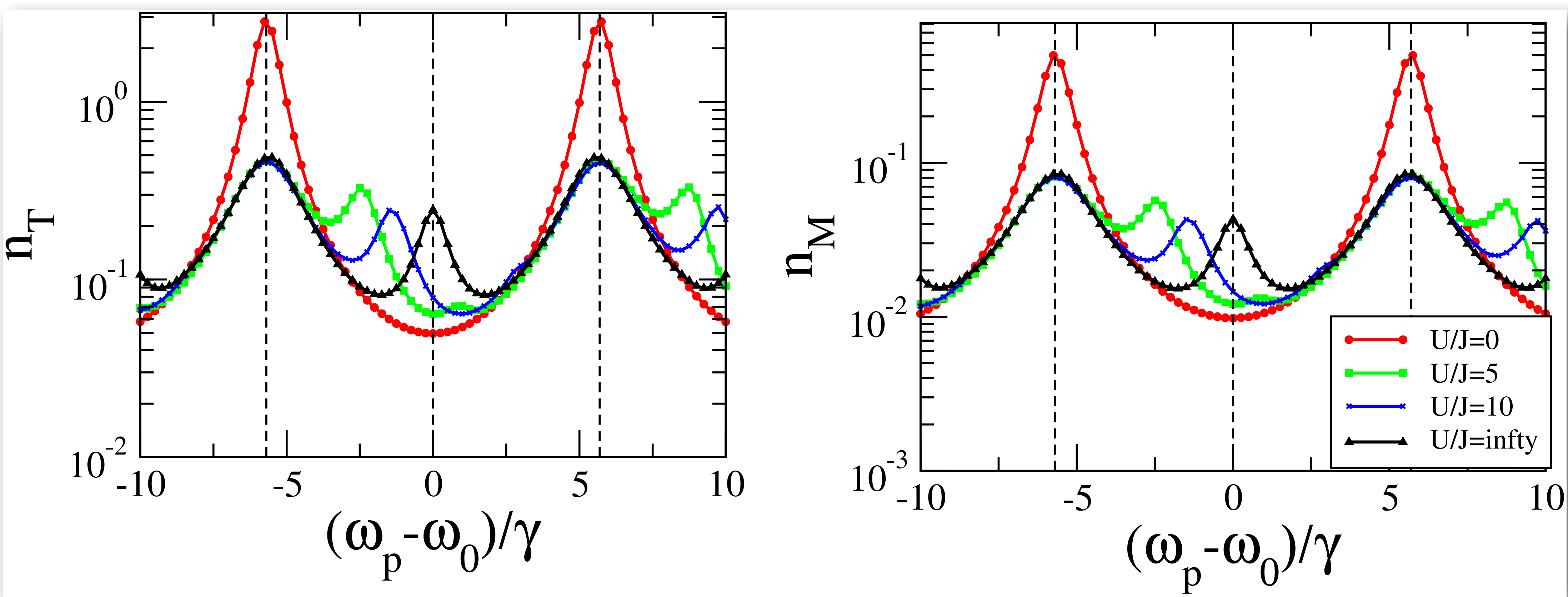
  see I. Carusotto *et al* Phys. Rev. Lett. **103**, 033601 (2009) for uniform pumping.

- In the opposite regime superfluidity? [detection of sound modes?] [Effect of impurities?]

Photon transport & many-body states

MPO approach to simulate
driven cavities
N~ 40

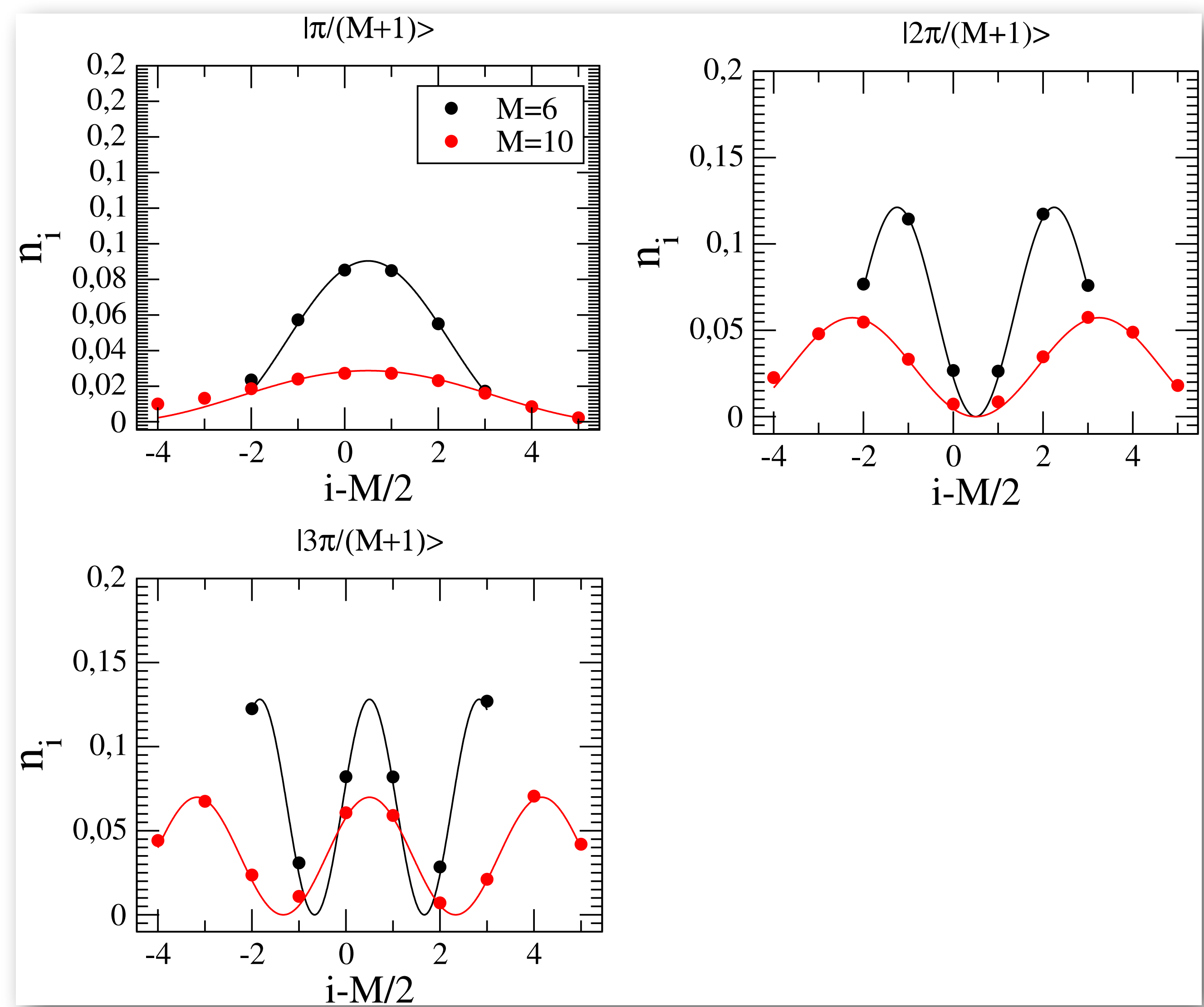
Peaks in the spectra relative to the many-body state



Photon transport & many-body states

MPO approach to simulate
driven cavities
N~ 40

The density profile resembles a plane wave



Conclusions

- Cavity arrays are very promising systems to study dissipative phase transitions
- Very rich scenario with features not present in equilibrium
- Signatures of ordering already present in "small" networks