Introduction to Random Matrix Theory and its various applications
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Outline of the course:

(1) Reminder of basic mathematical tools: Fourier transform, Laplace transform, Saddle point method, large deviation functions etc.

(2) Independent and identically distributed (IID) random variables: laws of averages, laws of extremes. Limiting laws for the distribution of maximum: universality classes: Gumbel, Frechet and Weibull distributions

(3) Brief historical introduction to RMT: applications

Discussion of basic properties of magtrices, different random matrix ensembles, rotationally invariant ensembles such as Gaussian ensembles etc.

(4) Gaussian ensembles: derivation of the joint probability distribution of eigenvalues, starting from the joint distribution of matrix entries.

(5) Analysis of the spectral properties of eigenvalues: given the joint distribution of eigenvalues, how to calculate various observables such as:

(i) Average density of eigenvalues ----Wigner semi-circle law
(ii) Counting statistics, spacings between eigenvalues etc.
(iii) Distribution of the extreme (maximum or minimum eigenvalues)

(6) Two complementray approaches to study spectral statistics: (a) Large N (fora n NxN matrix) method by the Coulomb gas approach: saddle point method (b) finite N method: for Gaussian unitary ensemble: orthogonal polynomial method (essentially quantum mechanics of simple harmonic oscialltors)

(7) Tracy-Widom distribution: prob. distribution of the top eigenvalue. Its appearence in a large number of problems, universality and an assocaited third ordder phase transition

(8) Perspctives and summary.

Suggested readings/references:

(1) ``Random matrices"...book by M. L. Mehta (particularly the introduction)

(2) ``Log-gases and Random matrics" ...book by P.J. Forrester

(3) S.N. Majumdar, Les Houches lecture notes (Complex systems, 2006), arXiv/cond-mat/0701193

(4) ``Extreme Value statistics of correlated random variables", lecture notes for the GGI
(Florence, 2014) workshop by S.N. Majumdar (notes taken by a student A. Pal),
arXiv: 1406.6768

(5) S.N. Majumdar, a book chpater in ``Handbook of random matrix theory" ed. by G. Akemann
et.al. arXiv: 1005.4515

(6) A recent review: S.N. Majumdar and G. Schehr, ``Top eigenvalue of a random matrix: large
deviations and third order phase transition", J. Stat. Mech. P01012 (2014),
arXiv: 1311.0580

(7) Review article by Y. V. Fyodorov, arXiv: 0412017

(8) Review ``Random matrix theory of quantum transport" by C.W.J. Beenakker, Rev. of Mod.
Phys. 69, 731 (1997).

See also two recent polpular articles:

(1) ``Equivalence Principle" by M. Buchanan, Nature Phys. 10, 543 (2014)

http://www.nature.com/nphys/journal/v10/n8/full/nphys3064.html?WT.ec_id=NPHYS-201408

(2) ``At the far ends of a new universal law" by N. Wolchover, Quanta magazine (October,
2014)

https://www.quantamagazine.org/20141015-at-the-far-ends-of-a-new-universal-law/