FFLO strange metal and quantum criticality in two dimensions

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Strange metal: Fermi surface without electronic quasiparticles, typically 2d

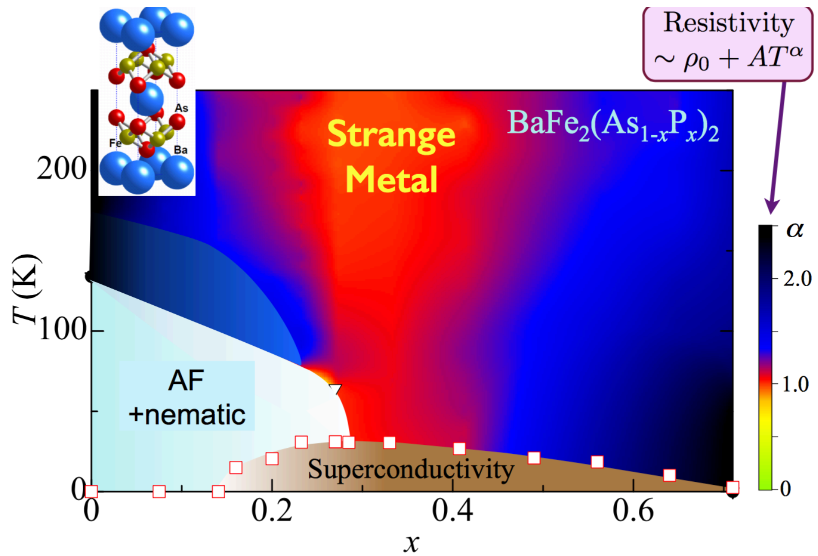
Independent electrons:

- Electrons move in crystal
- Conducting energy bands
- Electrical transport from quasiparticles
- “Weak dressing” of electrons from interactions/impurities, etc.

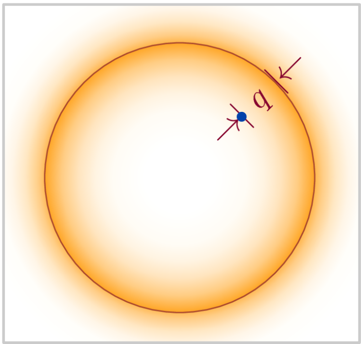
Many-body physics beyond quasiparticles:

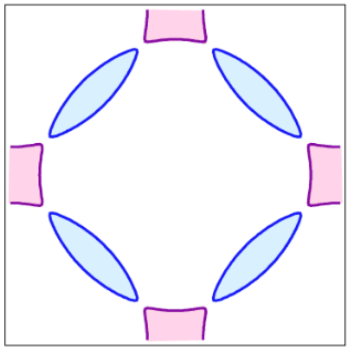
- Strong interactions/criticality/disorder
- Breakdown of independent electron approximations/Landau Fermi liquid
- Generally not amendable to numerics
- k_F breaks conformal symmetry

New ideas, techniques needed



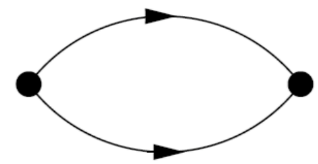
S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* 81, 184519 (2010)





Superconducting instability robust in time-reversal invariant metals

Infrared divergence in particle-particle bubble:



For vanishing total momentum (Cooper channel) at $T = 0$

$$\text{pp-bubble} \propto \int dk_0 \int d^d k \, \frac{1}{ik_0 - \xi_{\mathbf{k}}} \frac{1}{-ik_0 - \xi_{-\mathbf{k}}} \overset{\xi_{-\mathbf{k}} = \xi_{\mathbf{k}}}{=}$$

$$\int dk_0 \int d^d k \, \frac{1}{k_0^2 + \xi_{\mathbf{k}}^2} = \int dk_0 \int d\xi \, \frac{N(\xi)}{k_0^2 + \xi^2}$$

logarithmically divergent in *any* dimension if $N(0) \neq 0$

Note: Propagator divergent on $(d-1)$-dimensional manifold, embedded in $(d+1)$-dimensional space (spanned by k_0 and $\mathbf{k}$)

Frustrate superconductivity via:

(i) competing instabilities (e.g. antiferromagnetism),

(ii) suppressing density of states (e.g. semimetals)

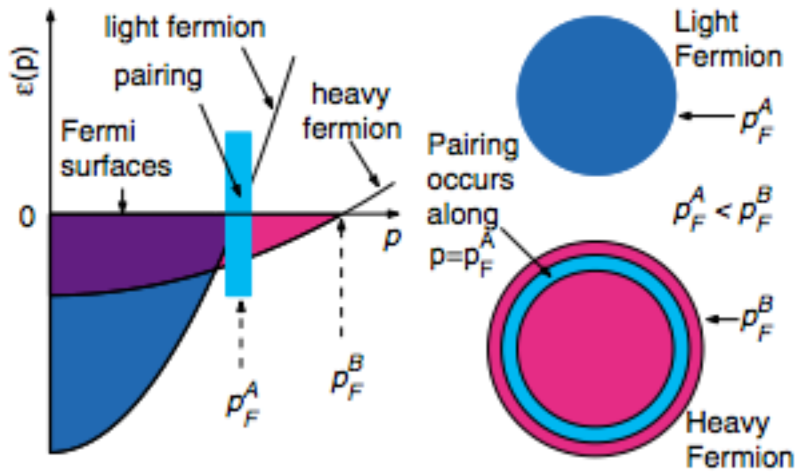
(iii) **This talk: including magnetic fields/spin imbalance so $\xi_k \neq \xi_{-k}$**

In isotropic, spin-imbalanced systems, breakdown of pairing via Sarma-Liu-Wilczek superfluid possible; unstable at mean-field…

- Generic Hamiltonian

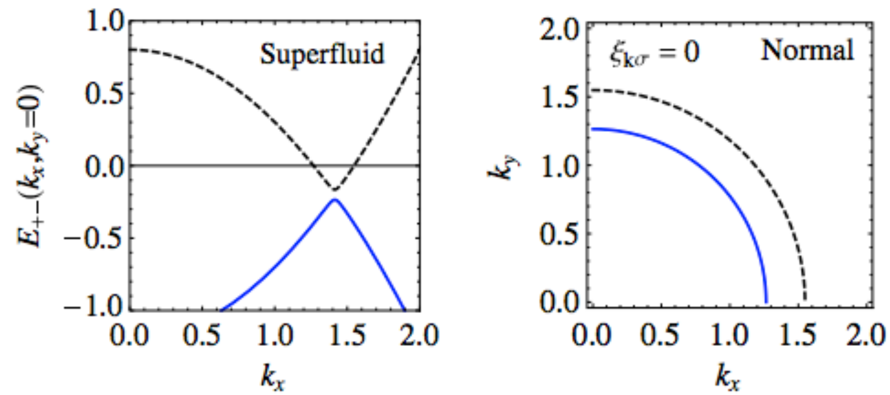
$$H = \sum_{\mathbf{k}\sigma} \xi_{\mathbf{k}\sigma} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \frac{g}{V} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} c^{\dagger}_{\mathbf{k}+\mathbf{q}/2,\uparrow} c^{\dagger}_{-\mathbf{k}+\mathbf{q}/2,\downarrow} c_{\mathbf{k}'+\mathbf{q}/2,\downarrow} c_{-\mathbf{k}'+\mathbf{q}/2,\uparrow}, \quad (1)$$

where the dispersion of the two spin components is $\xi_{\mathbf{k}\sigma} = (\mathbf{k}^2/2m_\sigma) - \mu_\sigma$, with $\sigma = \uparrow, \downarrow$, and $g < 0$ is an



- Pairing gap opens away from both Fermi surfaces

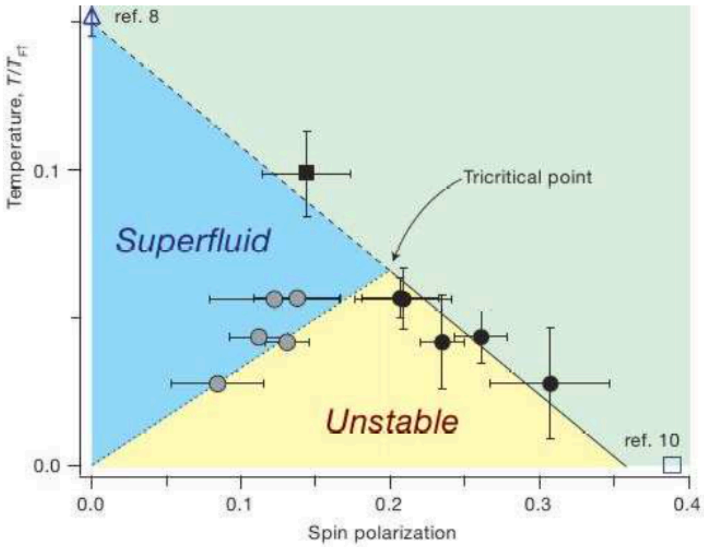
$$E_{\pm} = \frac{\xi_{\mathbf{k}\uparrow} - \xi_{-\mathbf{k}\downarrow}}{2} \pm \sqrt{\frac{\alpha^2}{2} + \left(\frac{\xi_{-\mathbf{k}\downarrow} + \xi_{\mathbf{k}\uparrow}}{2}\right)^2},$$



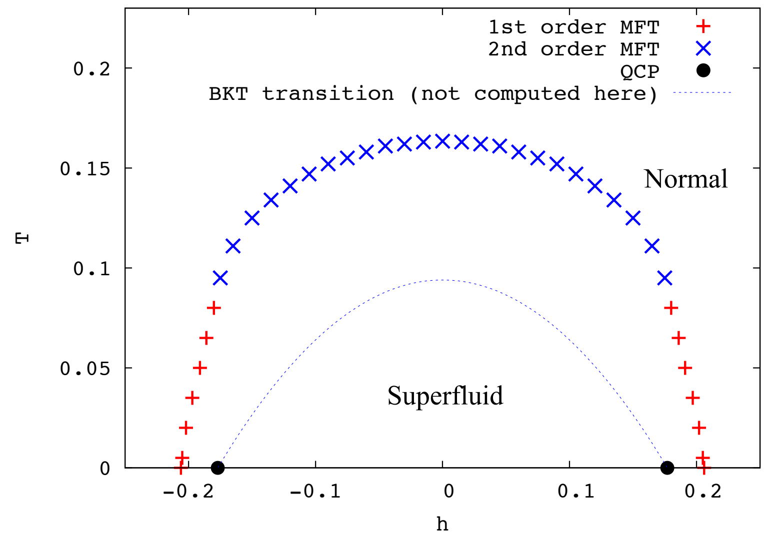
- Generically first order at mean-field level (as many magnetic metals)

…but quantum fluctuations may stabilize it in isotropic systems two dimensions

3d experiment ultracold fermionic atoms:



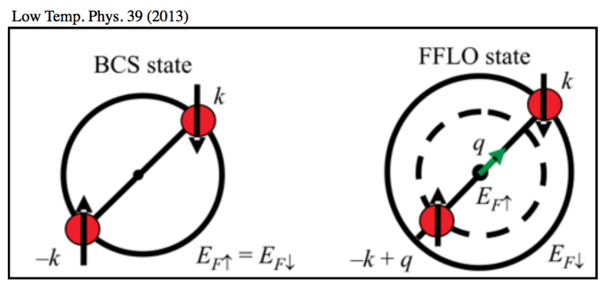
2d theory with quantum fluctuations (Strack, Jakubczyk, PRX 2014):



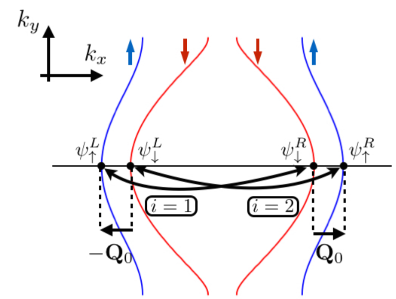
- Mean-field tri-critical points renormalized to T=0, h_{crit}
- Possibility of new quantum critical points to Sarma-Liu-Wilczek phase
- Second transition to fully gapped state at smaller h expected
- *Technique:* functional flow of effective potential with Goldstone and amplitude fluctuations

FFLO superconductivity: avoid pair-breaking by spatially modulating gap; anisotropy/Fermi surface shape can single out modulation vector(s)

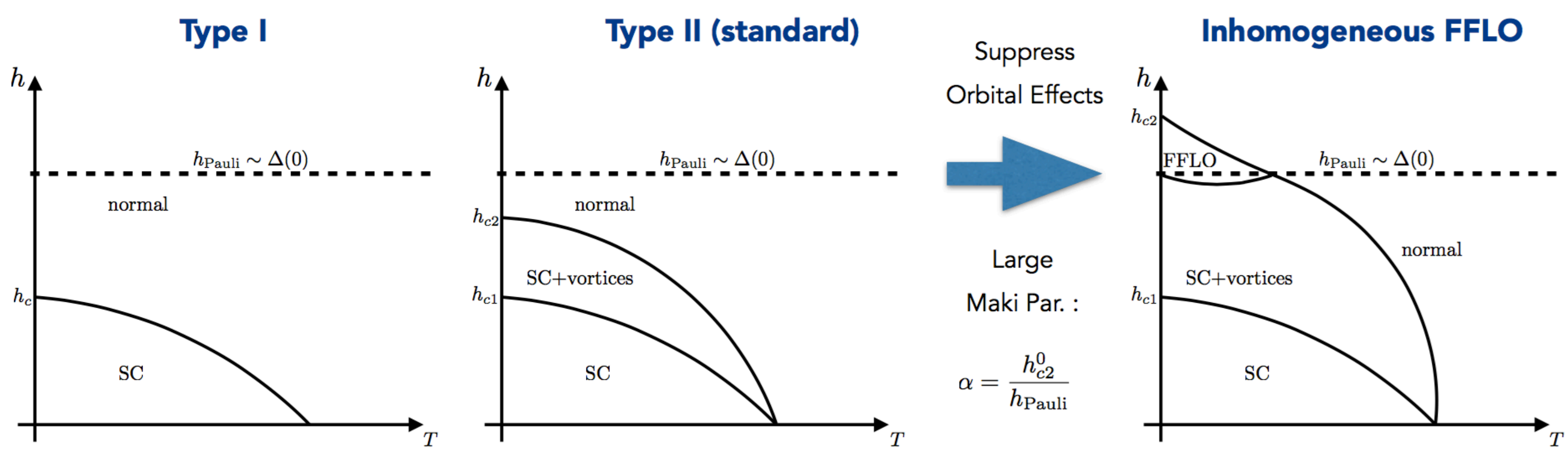
- Spatial modulation of gap, translation symmetry-breaking
- Pairing of high DOS regions; minimize Q's



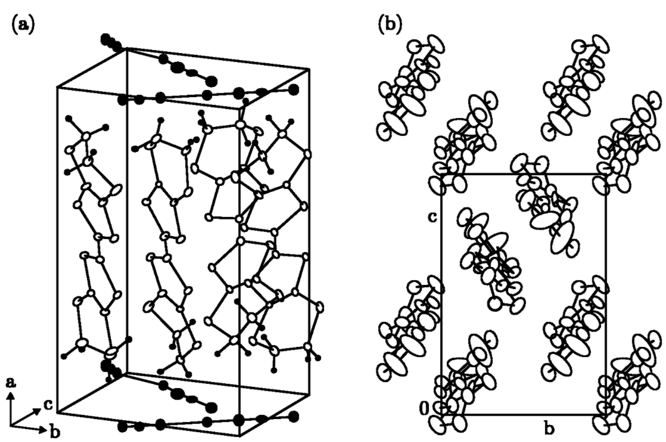
$$\Delta_{\mathrm{FFLO}} = \Delta_0 \cos(qr)$$

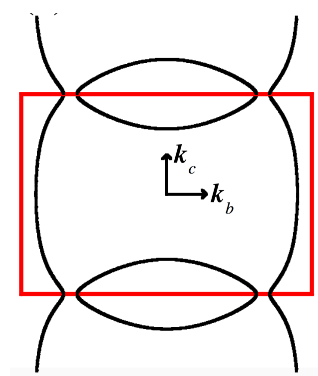


- Multiple Q's possible – *anisotropic Fermi surfaces* help single out Q's



Experimental puzzle: organic superconductors, κ-$(BEDT\text{-}TTF)_2Cu(NCS)_2$





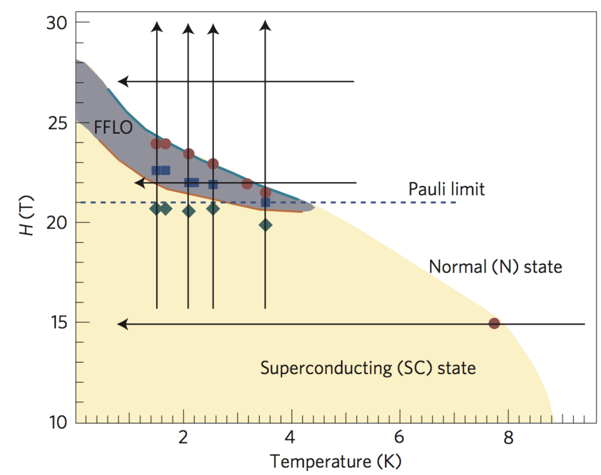


Figure 1 | (*H*, *T*) phase diagram of *κ*-$(BEDT\text{-}TTF)_2Cu(NCS)_2$. Curves and

- Quasi-2d (super-) conducting layers
- In-plane magnetic fields
- Closed and open Fermi sheets in layer
- Enhancement of NMR relaxation rate from polarized quasi-particles at nodes of FFLO superconducting order

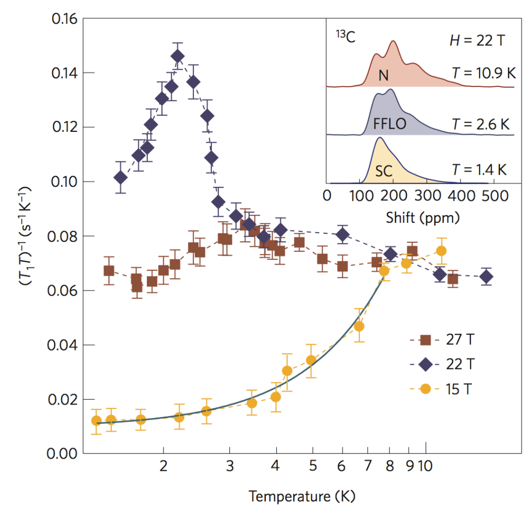
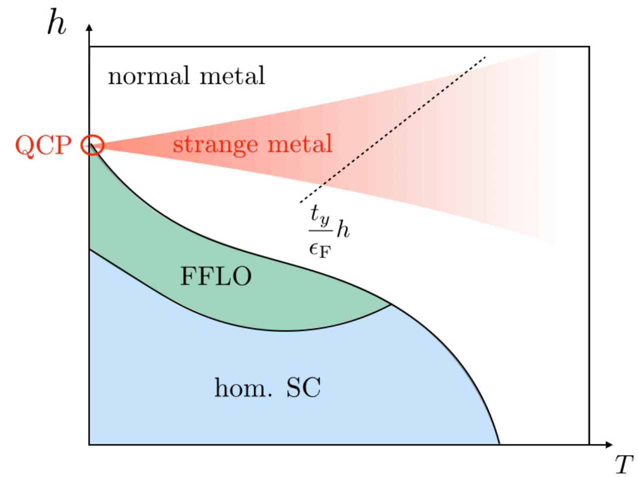


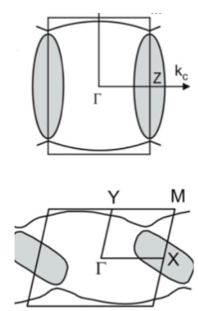
Figure 2 | NMR relaxation rate in the normal and superconducting states.

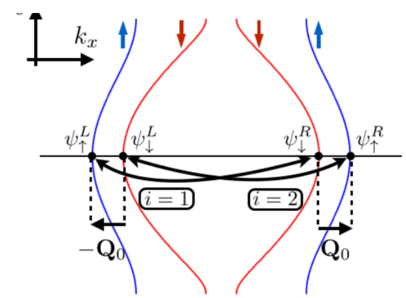
Main result: new FFLO strange metal in 2d anisotropic electron systems

- **Strange metal phase** extending to finite temperatures at onset of FFLO-SC
- **Genuine quantum critical point** at onset of FFLO superconductivity
- Strange metallic behavior due to **absence of proper electronic quasi-particles:**
  - Non-Fermi liquid electron self-energy
  - Anomalous power-laws in thermodynamics and NMR response
- Surprising point of view on FFLO data in organic 2d superconductors, **κ-(BEDT-TTF)$_2$Cu(NCS)$_2$**, (TMTSF)$_2$ClO$_4$
- Possibility of **unmasked quantum critical point** in pairing channel

Piazza, Zwerger, Strack;
arXiv:1506.08819 (2015)







Effective model for anisotropic organic superconductors

- Typically 5% mismatch:

$h \simeq 30\mathrm{T} \simeq \epsilon_{\mathrm{F}}/20$

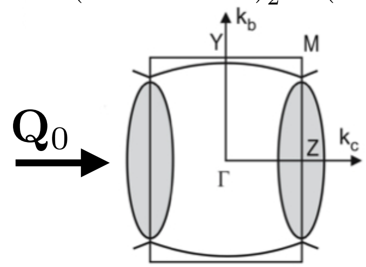
- Hole-pockets involved
- Hopping hierarchy:

$t_x \simeq 1340\mathrm{K}, t_y \simeq 134\mathrm{K}, t_z \simeq 2.6\mathrm{K}$

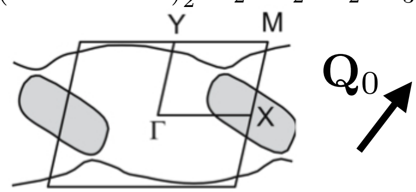
Unidirectional preferred modulation

At **low-energy** only **hot-spots** matter!

$\kappa-(\mathrm{BEDT\text{-}TTF})_2\mathrm{Cu(NCS)}_2$

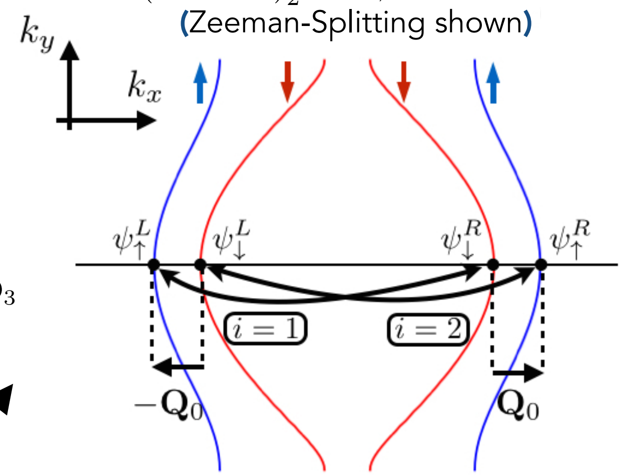


$\beta''-(\mathrm{BEDT\text{-}TTF})_2\mathrm{SF_2CH_2CF_2SO_3}$



$(\mathrm{TMTSF})_2\mathrm{ClO}_2,$ Fermi Gas

(Zeeman-Splitting shown)



- Dispersion for $(\mathrm{TMTSF})_2\mathrm{ClO}_2,$ Fermi Gas

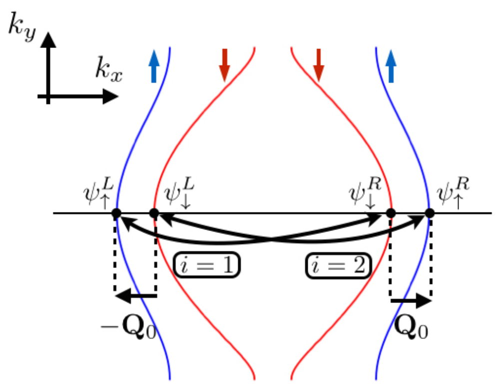
$$\xi_\sigma(\mathbf{k}) = k_x^2/2m - 2t_y\cos(dk_y) - \mu - \sigma h$$

- Effective short-range **attraction** in weak coupling (mechanism irrelevant)

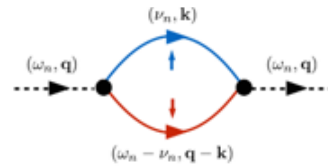
$$\hat{H}_{\mathrm{int}} = -g\int d^2\mathbf{r}\ \hat{\psi}_\uparrow^\dagger(\mathbf{r})\hat{\psi}_\downarrow^\dagger(\mathbf{r})\hat{\psi}_\downarrow(\mathbf{r})\hat{\psi}_\uparrow(\mathbf{r})$$

- Interlayer hopping is small and incoherent & in-plane magn. field: **no orbital effects**
- Triplet (p-wave) pairing excluded via knight-shift measurements: consider **singlet**
- We will consider **s-wave** pairing (d-wave might be important too)

Genuine quantum critical point at the onset of FFLO superconductivity



Pairing susceptibility

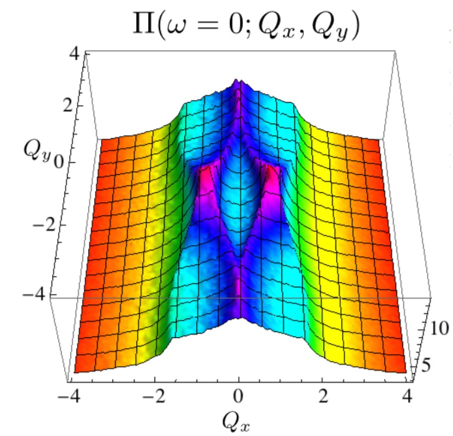


$$\Pi(\omega_n, \mathbf{q}) = \frac{g^2}{\beta} \sum_{\mathbf{k}} \frac{1 - n_F(\beta\xi_\downarrow(\mathbf{q}-\mathbf{k})) - n_F(\beta\xi_\uparrow(\mathbf{k}))}{\xi_\uparrow(\mathbf{k}) + \xi_\downarrow(\mathbf{q}-\mathbf{k}) - i\omega_n}$$

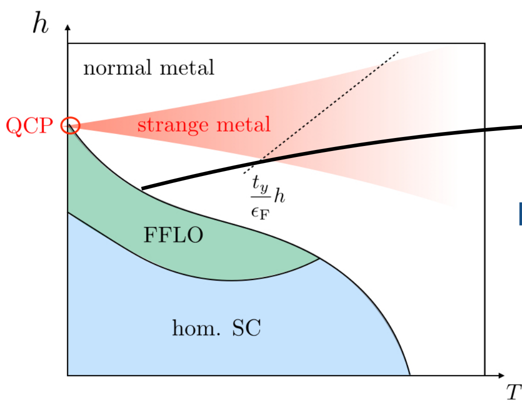
shows 2 equivalent dominant peaks:

preferred direction

Modulated Gap: $\Delta_{\mathrm{FFLO}} = \Delta_0 \cos(\mathbf{Q}_0 \cdot \mathbf{r})$



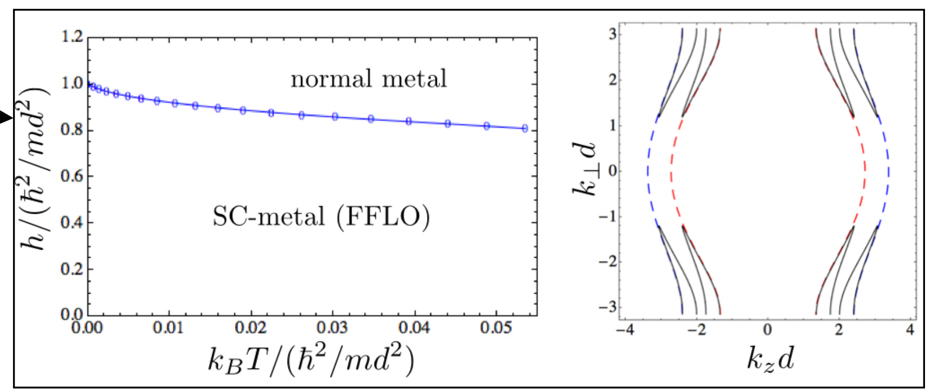
Mean-Field Ginzburg-Landau Theory $a_2(T, g, h) = \frac{1}{g} - (\Pi(0, +\mathbf{Q}_0) - \Pi(0, -\mathbf{Q}_0)) := 0$ and $\lim_{T \to 0} a_4 > 0$



Consider the

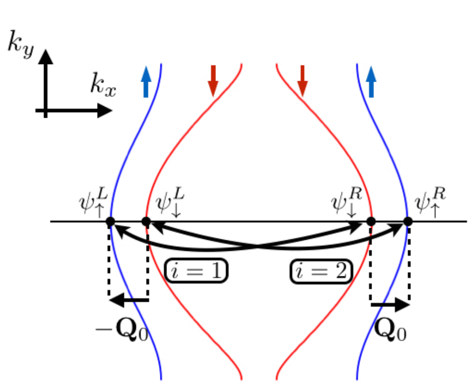
Normal to FFLO

transition



- **Continuous transition** at low temperatures [in agreement with Larkin, Ovchinikov (1965) and Parish, Huse PRL (2006)]
- Symmetry-broken state leaves metallic **Fermi "tongues"** (Superconducting Metal)

Capture quantum fluctuations via hot spot model in pairing channel



$$Z = \int D\{\bar\psi^{L,R}_{\uparrow,\downarrow}, \psi^{L,R}_{\uparrow,\downarrow}\} D\{\Delta^*_{1,2}\Delta_{1,2}\} \exp(-\mathcal{S})$$

Lagrangian: 4 hot-spot fermions coupled through two complex pairing bosons

$$\mathcal{L} = g \sum_{i=1,2} |\Delta_i|^2 + \sum_{\substack{\sigma=\uparrow,\downarrow \\ j=R,L}} \bar\psi^j_\sigma \left(\partial_\tau - i v^j_\sigma \partial_x + \frac{\partial_y^2}{2m_y} \right) \psi^j_\sigma$$

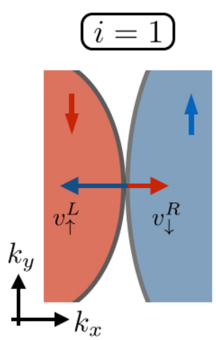
$$- g \left[\left(\Delta_1^* \psi^R_\downarrow \psi^L_\uparrow + \Delta_2^* \psi^L_\downarrow \psi^R_\uparrow \right) + \mathrm{h.c} \right]$$

No time-reversal symm.

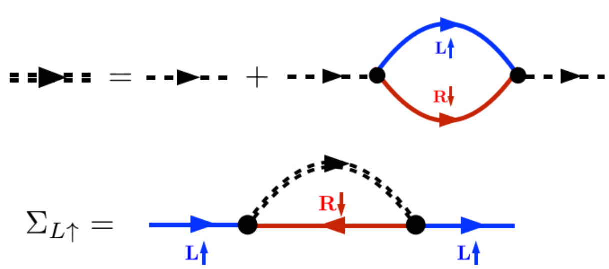
Hot-spot dispersion: $\xi^{R,L}_\sigma(\mathbf{k}) = v^{R,L}_\sigma k_x + k_y^2/2m_y$

Similar to patch-theories for fermions coupled to nematic fluctuations [Lee, PRB(2009); Metlitski, Sachdev, PRB (2010)] and also to incommensurate charge-density fluctuations [Altshuler, et al., PRB (1995); Holder, Metzner, PRB (2014)]
BUT here the **fluctuations** are in the **particle-particle channel and break time-reversal** (e.g. no vertex corr.@1-loop).

Single hot-spot



One-Loop Diagramatics:



Complex field: hot-spots uncoupled!

Propagator for bosonic pairing field

$$D_{i=1}(\tau, \mathbf{r}) = \langle \hat\Delta_1(\tau,\mathbf{r}) \hat\Delta_1^\dagger(0,\mathbf{0}) \rangle$$

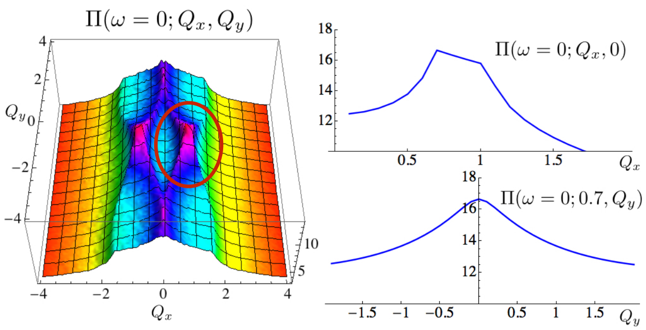
Propagator for fermionic field

$$G_{L\uparrow}(\tau,\mathbf{r}) = \langle \hat\psi_{L\uparrow}(\tau,\mathbf{r}) \hat\psi^\dagger_{L\uparrow}(0,\mathbf{0}) \rangle$$

Scattering off incommensurate FFLO waves destroys electronic quasi-particles at low T (1/2)

$$D_1^{-1}(\omega_n,\mathbf{k}) = \frac{V\sqrt{2m_y}}{4\pi v|\delta v/v|}\left[2\mathrm{Re}\sqrt{-\frac{k_y^2}{2m_y}+\delta v k_x+\frac{\delta v}{v}i\omega_n}+B\frac{k_y^2}{2m_y v}+C\frac{\delta v}{v}k_x\right]$$

Expanded about one of the peaks small $\mathbf{k}, \omega_n$



Mass is set to zero

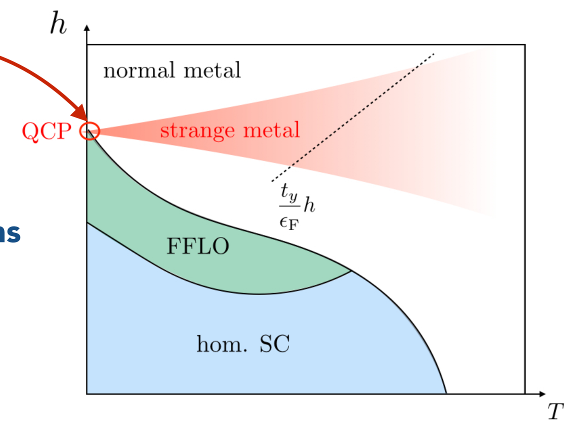
at the QCP

SquareRoot behaviour:

strong overdamping of fluctuations

Small-imbalance limit:

$$\delta v = v_\uparrow - v_\downarrow \ll v = (v_\uparrow + v_\downarrow)/2$$

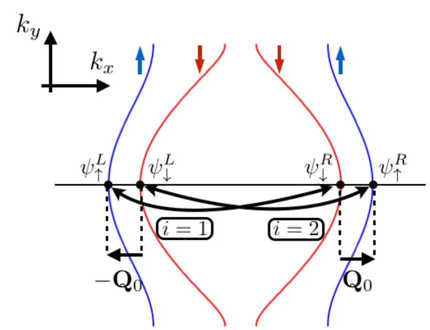


Quantum critical Fan: scaling behaviour implies that

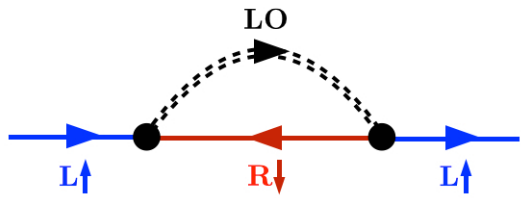
QCP properties extend to $k_B T > (h - h_{QCP})^{\nu_b z_b}$

Nesting energy scale: $\epsilon_{nest} \sim \dfrac{t_y}{\epsilon_F} h$

$k_B T > \epsilon_{nest}$ Fermi surfaces look fully nested via Q: **no hot-spot physics**



Scattering off incommensurate FFLO waves destroys electronic quasi-particles at low T (2/2)



- Non-Fermi liquid electron quasiparticle lifetime for small imbalance:

$$\mathrm{Im}\Sigma_{L\uparrow}(\omega, \mathbf{q}=0) = \frac{1}{\sqrt{3}}\left(\frac{|\delta v/v||\omega|}{B}\right)^{2/3}$$

- Electronic critical exponents:

Anomalous Dimensions (one-loop)

$$\eta_\tau = \frac{1}{3},\ \eta_k = 0$$

Dynamic Exponent (one-loop)

$$z^f_{\mathrm{FFLO}} = \frac{1-\eta_k}{1-\eta_\tau} = \frac{3}{2}$$

cfr. Fermi Liquid

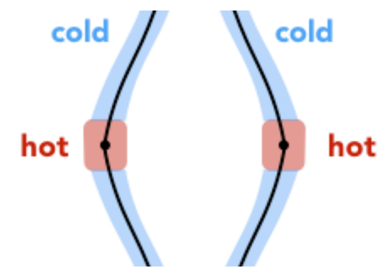
$$z^f_{\mathrm{FL}} = 1$$

- Scaling of spectral function: $A^{\mathrm{hot}}_\sigma(\omega, \mathbf{k}) = -\mathrm{Im}\frac{1}{\pi}G^{\mathrm{ret}}_{L\uparrow}(\omega, \mathbf{k}) \sim \frac{c_0}{|\omega|^{1-\eta_\tau}}\mathcal{F}_\sigma\left(\frac{c_1\omega}{(k_x+k_y^2)^{z_f}}, \frac{\omega}{T}\right)$

- Density of states has power-law component:

$$N^{\mathrm{hot}}_\sigma(\omega) = \int \frac{d^2\mathbf{k}}{(2\pi)^2} A^{\mathrm{hot}}_\sigma(\omega, \mathbf{k}) \sim \omega^{1/3}$$

Ansatz: $N_\sigma(\omega) = N^{\mathrm{cold}}_{0\sigma} + N^{\mathrm{hot}}_\sigma(\omega)$

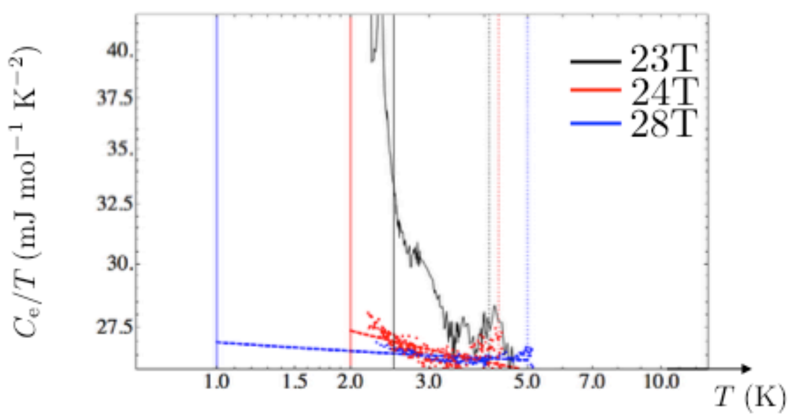


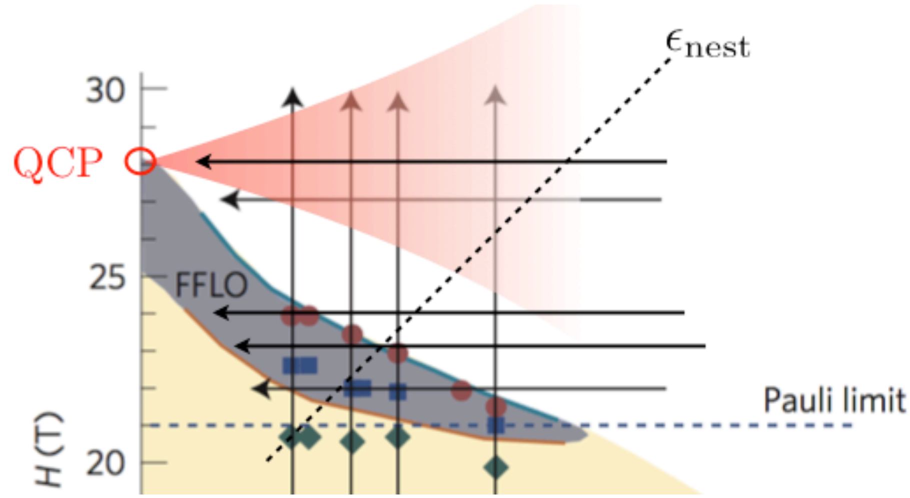
- Momentum resolved RG needed to check hyper-scaling, ω/T scaling

For different incommensurate charge order see: Holder, Metzner PRB (2014); Holder Diploma Thesis (2012)

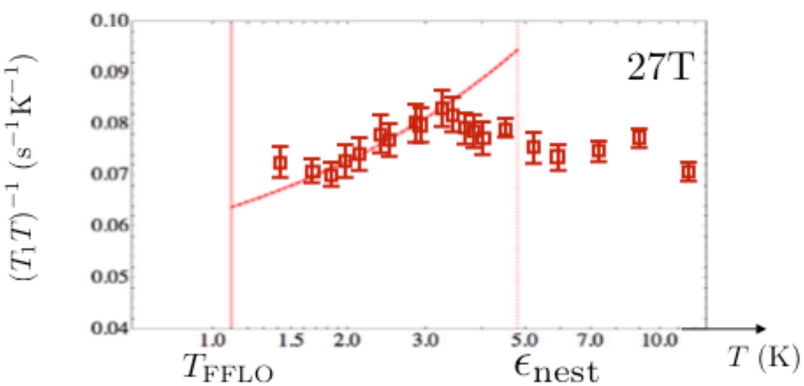
Hunt strange metal anomalies in experimental data

Electron specific heat [Lortz, et al., PRL (2007)]





NMR relaxation rate [Mayaffre, et al., Nat.Phys. (2014)]



- **Electron specific heat** from critical scaling

$$d = 2, \theta = 1$$

Hyperscaling violation (to be checked)

$$C_e/T \sim T^{\frac{d-\theta}{z^f}-1} = T^{1/z^f-1} = T^{-0.33}$$

- **NMR relaxation rate** from density of states

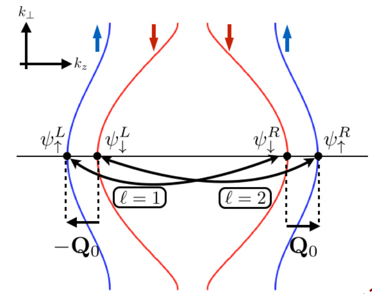
$$\frac{1}{T_1T} = \mathrm{R}^{(\text{cold})} + \frac{c_0}{T}\int d\omega N_\uparrow^{\text{hot}}(\omega,T)N_\downarrow^{\text{hot}}(\omega,T)n_F(\beta\omega)\left[1-n_F(\beta\omega)\right]$$
$$= \mathrm{R}^{(\text{cold})} + c_0T^{2/3}$$

- More data over extended field and temperature ranges needed

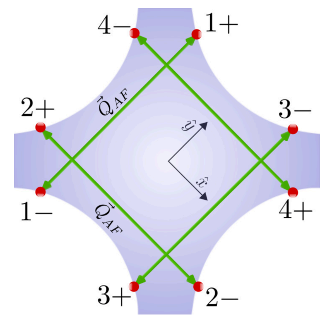
Summary – differences to (some) previous strange metals

Today's talk

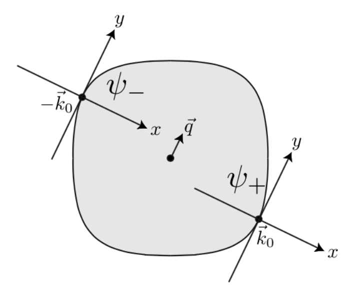
- **Inhomogeneous superconductor**:
  - Nested single hot spot pair in *pairing channel*
  - Different graphs (no 1-loop vertex corrections)
  - More "naked", broken time-reversal



- **Commensurate antiferromagnet**:
  - Collection of 4 hot spot pairs, no curvature
  - Dimensionally reduced, nested fixed point[1]
  - Likely masked by d-wave superconductivity



- **Ising-nematic**:
  - Two patch fermions and "tangential" boson[2]
  - Entire Fermi surface hot
  - Enhanced competition from superconductivity



+ Van-Hove criticality[3], incommensurate charge and spin order[4]

[1]Sur, Lee (PRB 2015), [2]Lee PRB (2009), Metlitski, Sachdev PRB (2011); [3] Giering, Salmhofer PRB (2012); Altshuler et al. PRB (1995); Holder, Metzner PRB (2014)

Back-up

Experimental puzzle (II): Bechgaard salt $(TMTSF)_2ClO_4$

Anomalous In-Plane Anisotropy of the Onset of Superconductivity in $(TMTSF)_2ClO_4$

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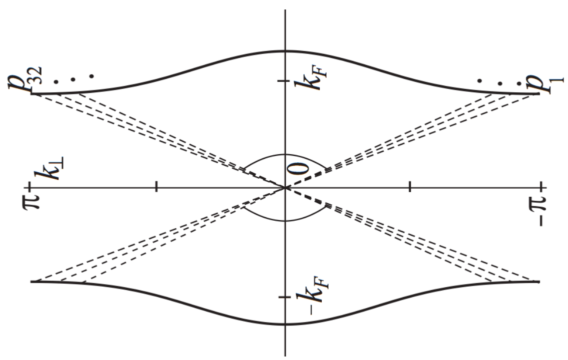
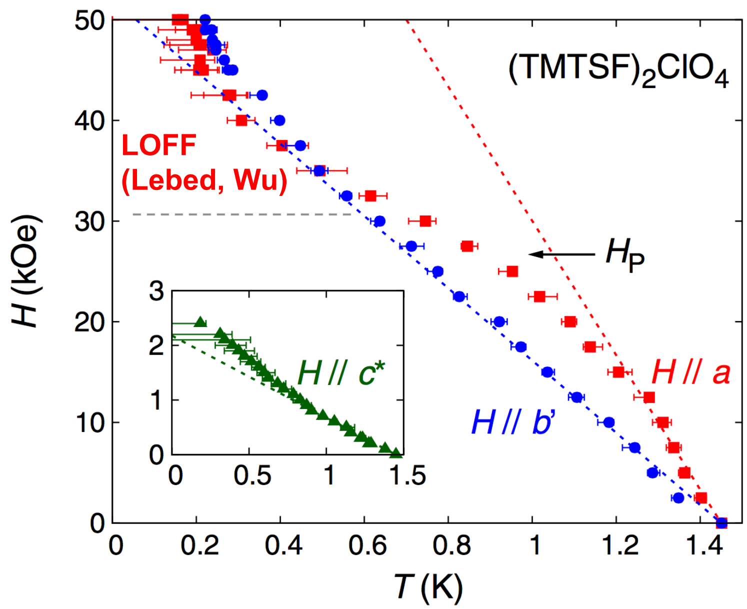
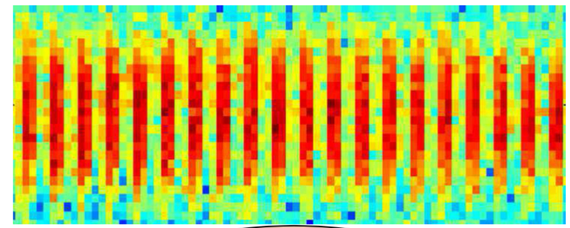
- B- field parallel to conducting chains
- T_c from resistance measurements
- Open Fermi sheets at zero field:

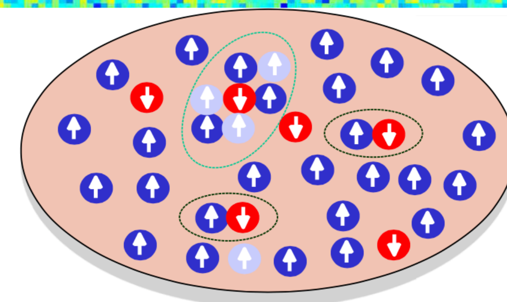




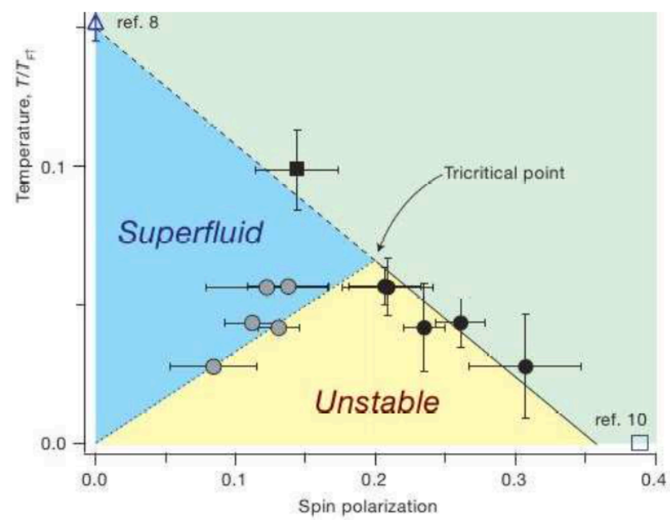
Experimental puzzle (III): imbalanced ^{6}Li atomic fermions in 2d traps

- Quantum degenerate Fermi gas ^{6}Li
- Coupled 1-dimensional tubes with tunable transversal hopping t_{perp}
- Tunable attraction via Feshbach resonance
- Superfluid “Smectics/liquid crystals”
- “Best-of-both-worlds” wire geometry:
  - Low-dimensionality to single out Fermi points for Q_{FFLO}
  - But 2d-system (LL unstable to t_{perp})
- Breakdown of homogeneous superfluid already studied in 3d

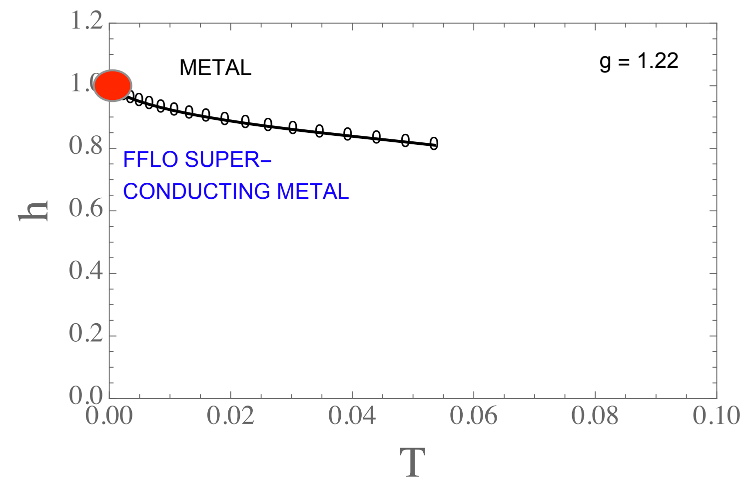




3d:



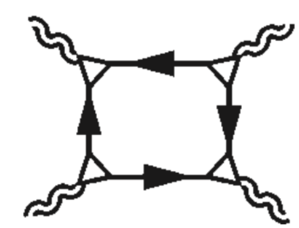
Mean-field theory yields quantum phase transition: $a_4>0$ for low T



- Infinite-dimensional quasi-momentum basis[1] due to incommensurate Q_0
- Quartic term in effective action:

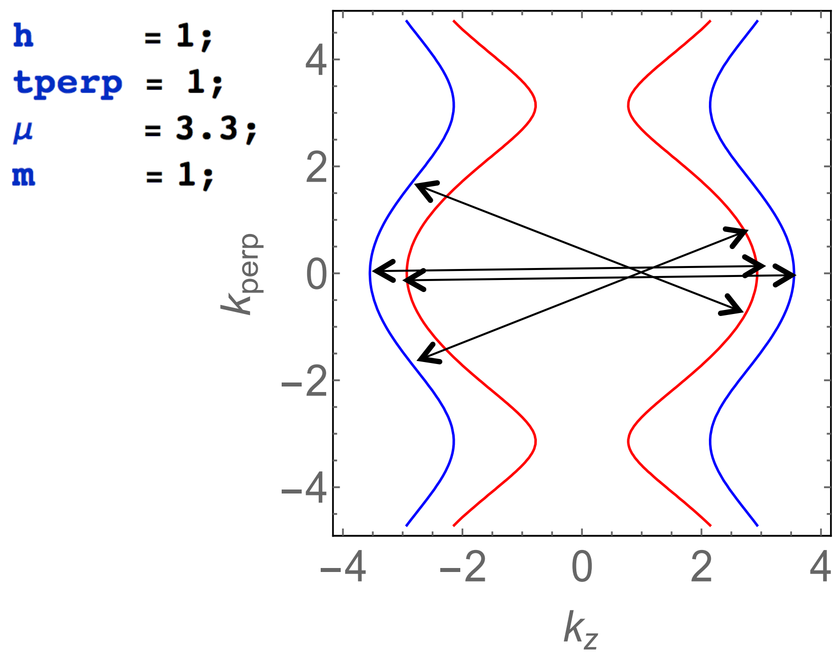
$$S_4 = (-1)(-1)\frac{1}{4}T^3 \sum_{\omega_{n_1}\ldots\omega_{n_4}} \delta\left(\omega_{n_1}+\omega_{n_2}-(\omega_{n_3}+\omega_{n_4})\right)$$
$$\int \frac{d^2\mathbf{q}_1}{(2\pi)^2}\ldots\int \frac{d^2\mathbf{q}_4}{(2\pi)^2}\delta\left(\mathbf{q}_1+\mathbf{q}_2-(\mathbf{q}_3+\mathbf{q}_4)\right)$$
$$\Delta^*(\omega_{n_1},\mathbf{q}_1)\Delta^*(\omega_{n_2},\mathbf{q}_2)\Delta(\omega_{n_3},\mathbf{q}_3)\Delta(\omega_{n_4},\mathbf{q}_4)A_4\left(\omega_{n_1}\ldots\omega_{n_4};\mathbf{q}_1\ldots\mathbf{q}_4\right)$$

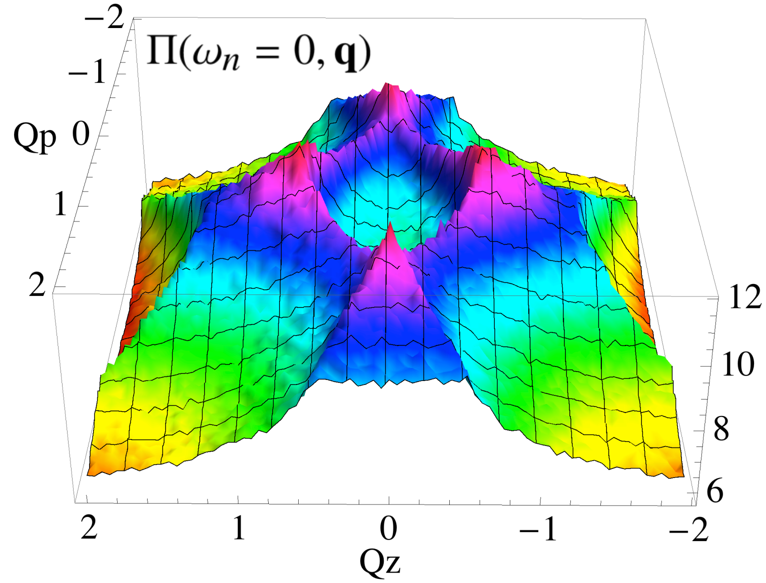
- Plugging in: $\Delta(\omega_n,\mathbf{q}) \to \Delta_0(0,\mathbf{Q}_0) = \frac{\delta_{\omega_n,0}}{T}\left[d_{+\mathbf{Q_0}}\delta_{\mathbf{q},\mathbf{Q}_0}(2\pi)^2 + d_{-\mathbf{Q_0}}\delta_{\mathbf{q},-\mathbf{Q}_0}(2\pi)^2\right]$
- 6 contractions invariant under: $\mathbf{Q}_0 \leftrightarrow -\mathbf{Q}_0, \uparrow\leftrightarrow\downarrow$

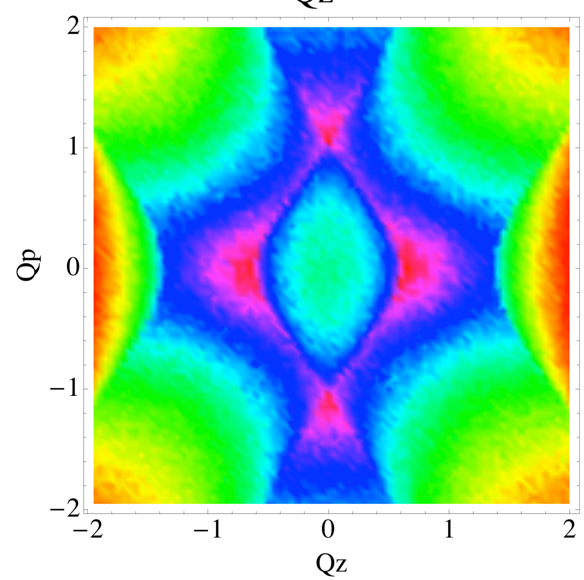


- Continuous transition at low temperatures: $\lim_{T\to 0} a_4 > 0$
- In agreement with Larkin, Ovchinikov (1965) and Parish, Huse PRL (2006)

Bi-directional spatial modulation, multiple competing hot spots possible

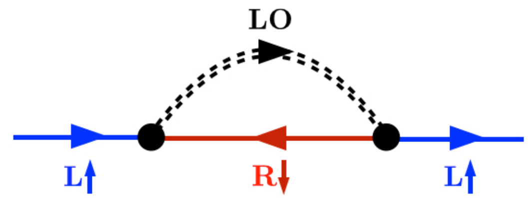






- Spatial modulation of superconducting order parameter depends on filling, band structure, and Zeeman field
- Mixture of closed and open Fermi surfaces also possible
- Proceed with single hot spot pair

Non-Fermi liquid behavior without quasiparticles at hot spot



- Evaluate electron quasiparticle lifetime for small imbalance:
  - Analytic continuation, frequency integral

$$\mathrm{Im}\Sigma_{L\uparrow}(\omega, \mathbf{q}=0) = \frac{\sqrt{\frac{\delta v}{v}}}{\pi} \int_{-\infty}^{\infty} dy_\perp \int_{-y_\perp^2-\omega}^{-y_\perp^2}$$

$$\mathrm{Im} \frac{dy_z}{\sqrt{-\frac{v}{\delta v} y_\perp^2 + 2y_z + \omega + i0^+} + \sqrt{-\frac{v}{\delta v} y_\perp^2 + 2y_z - \omega - i0^+} + \bar{B}\frac{v}{\delta v} y_\perp^2 + \bar{C} y_z}$$

  - For small ω, expand square-root on branch cut:

$$\sqrt{-\frac{v}{\delta v} y_\perp^2 + 2y_z + \omega + i0^+} + \sqrt{-\frac{v}{\delta v} y_\perp^2 + 2y_z - \omega - i0^+} \simeq -i \frac{\sqrt{\frac{\delta v}{v}}}{|y_\perp|}(s_z + \omega)$$

$$s_z = y_z + y_\perp^2$$

- Inverse quasi-particle lifetime vanishes at low frequencies:

$$\mathrm{Im}\Sigma_{L\uparrow}(\omega, \mathbf{q}=0) = \frac{1}{\sqrt{3}} \left(\frac{|\delta v / v||\omega|}{B} \right)^{2/3}$$

For different incommensurate charge order see: Holder, Metzner PRB (2014); Holder Diploma Thesis (2012)