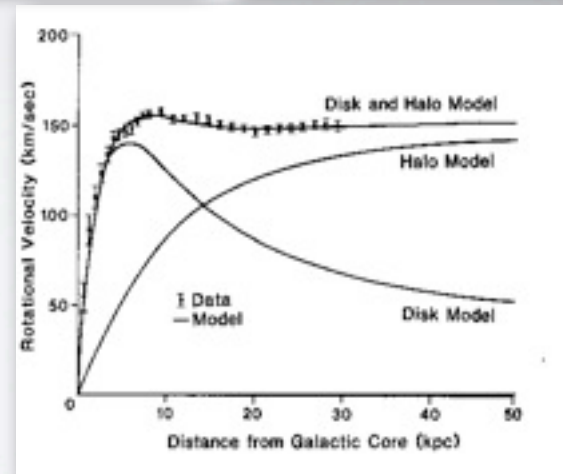
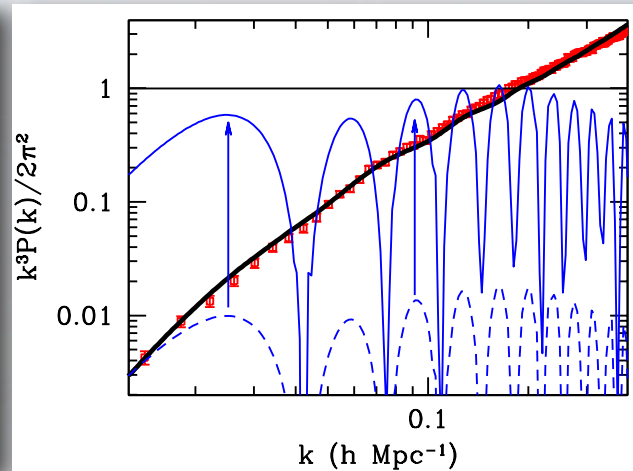
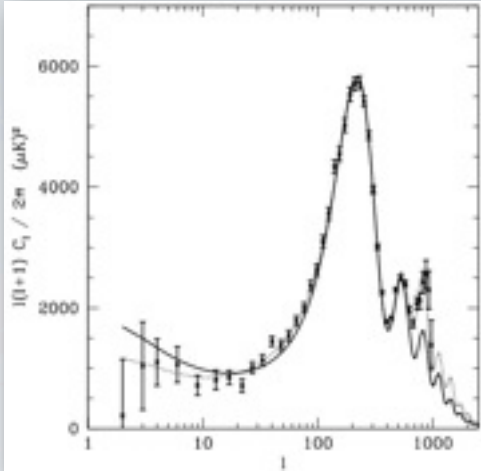


# Astrophysical signals of dark matter:

## I. Gravitational evidence & basic properties



# OUTLINE & GOALS

## **Lecture one**

- **basic notions of cosmology for “particle astrophysics”**
- **Gravitational evidence for Dark Matter (why are we so sure? Are we?)**
- **A few ‘particle physics’ constraints from astro/cosmo observations**

## **Lecture two**

- **freeze-out (hot, cold), “WIMPs & their relatives”**
- **Heuristic and more formal introduction of the Boltzmann eq. and its applications to DM-related problems**
- **notions on direct detection**

## **Lecture three**

- **Indirect detection of dark matter, mostly focused on WIMPs (different channels, strategies, challenges)**

## **My Goals**

*To those who have basic notions, manual towards “working knowledge” of the problems  
To those who have none, at least the key physical ideas and the tools needed to attack the problems*

# SOME REFERENCES

## *General references*

- ❖ **The Early Universe”, E. W. Kolb & M. S. Turner**
- ❖ **“Physical Foundations of Cosmology”, V. Mukhanov**

...

## *Specific monographs*

- ❖ **“Kinetic Theory in the expanding Universe”, J. Bernstein**
- ❖ **“Neutrino Cosmology”, J. Lesgourgues, G. Mangano, G. Miele, Pastor**
- ❖ **“Particle Dark Matter” Edited by Gianfranco Bertone (chapters on different particle physics candidates and probes)**

...

*others will be introduced along the course*

# BASIC NOTIONS OF (SMOOTH) COSMOLOGY

*Minimum you need to know to follow the rest of the lectures. Cannot replace a proper knowledge in cosmology needed to work on this subject!*

*Extra details in D.Weinberg's and M. Zaldarriaga's lectures*

# PILLARS OF STANDARD COSMOLOGICAL MODEL

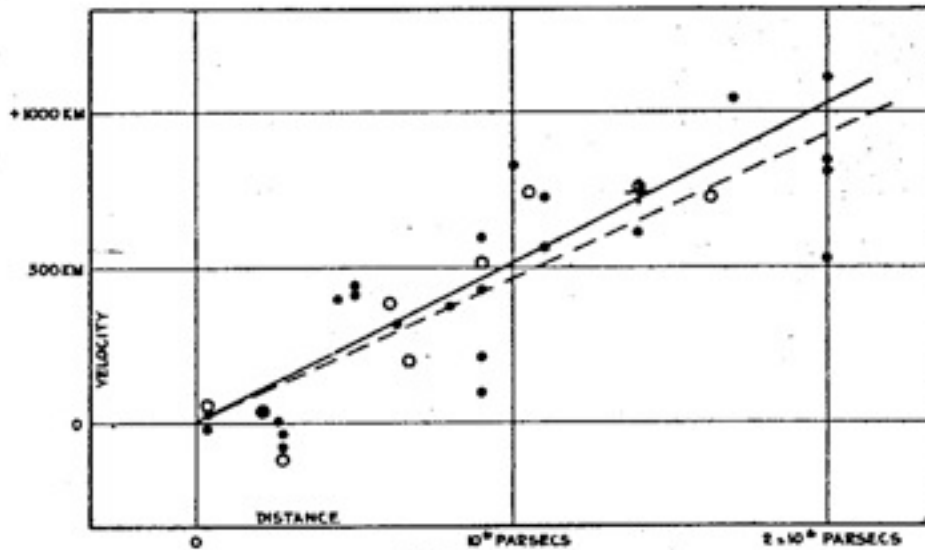
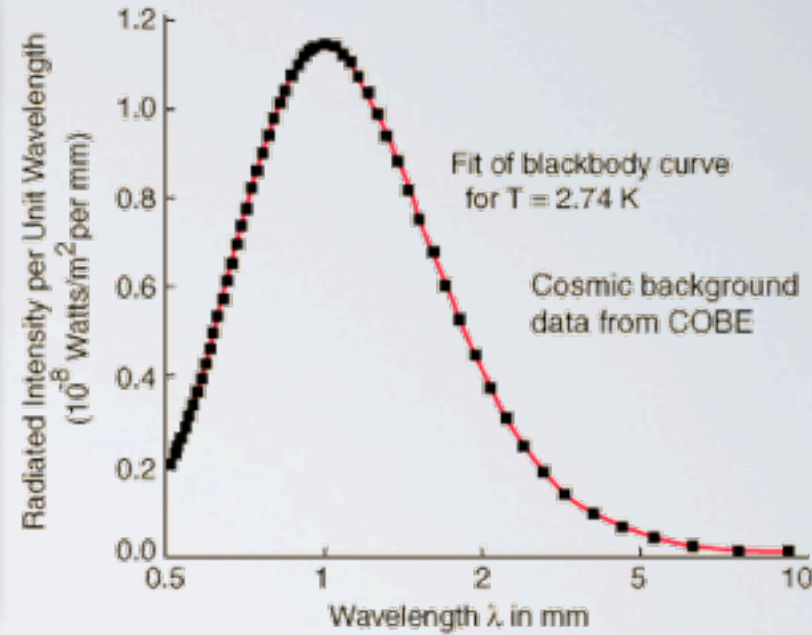


FIGURE 1



- Galaxies sufficiently far away from us recede with  $\mathbf{v=Hd}$  (Hubble law)
- The Universe is permeated by an almost perfect blackbody radiation, with  $T \sim 2.73$  K (Cosmic Microwave Background)
- Yields of light elements (notably Deuterium and Helium) way larger than what expected from “stellar” phenomena.

# STANDARD COSMOLOGICAL MODEL

Based on:

- General Relativity (GR): metric theory of gravitation
- Cosmological Principle (spatial homogeneity & isotropy on large scales)
- “Standard Physics”, in particular Kinetic Theory of Fluids, Particle & Nuclear Physics, Plasma Physics, Atomic Physics.

# STANDARD COSMOLOGICAL MODEL

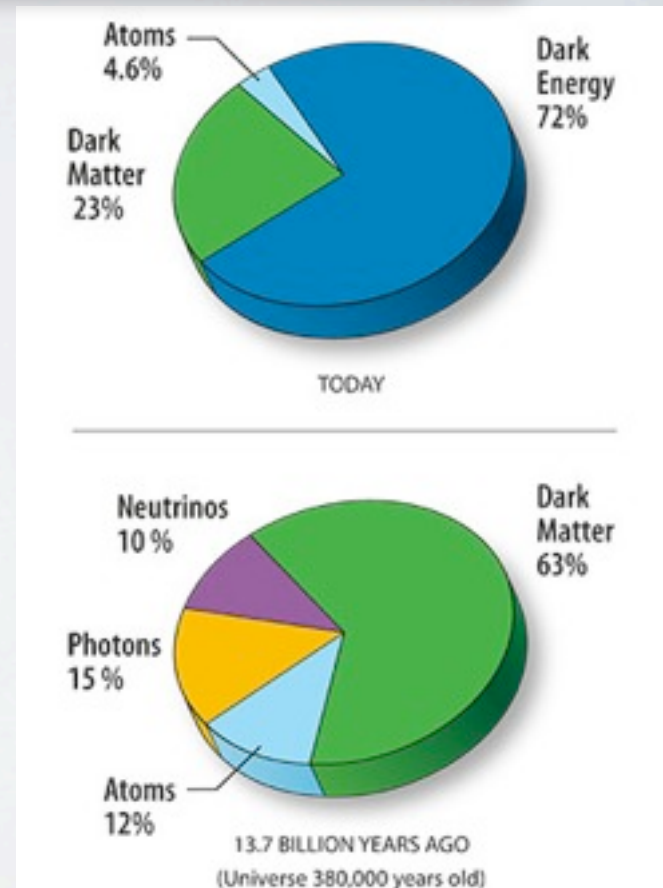
Based on:

- General Relativity (GR): metric theory of gravitation
- Cosmological Principle (spatial homogeneity & isotropy on large scales)
- “Standard Physics”, in particular Kinetic Theory of Fluids, Particle & Nuclear Physics, Plasma Physics, Atomic Physics.

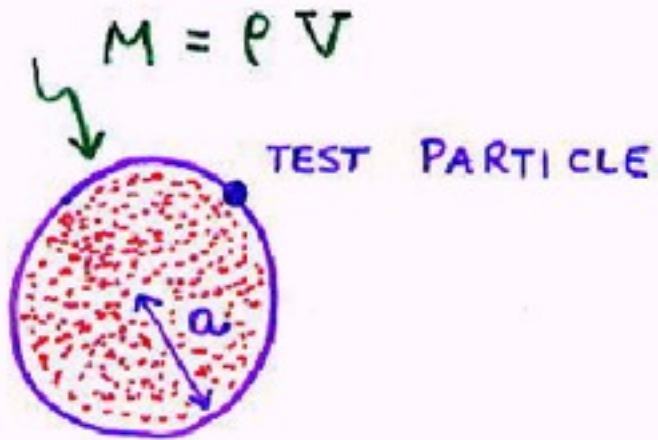
Evolving the expanding universe backwards in time  
→ picture of hot Early Universe, made of a “gas”  
which has been cooling while expanding

Basic (not unique!) task of cosmology: to understand  
what the universe is made of, now & in the past (the  
“mixture” can and does evolve with time...)

Natural units :  $c = \hbar = k_B = 1$



# EXAMPLES



Consider the Newtonian toy model of a sphere of dust. The acceleration is

$$\ddot{a} = -\frac{G_N M}{a^2}$$

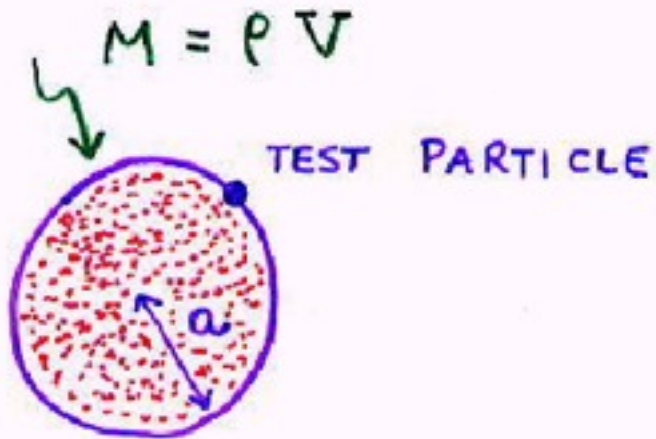
$$M = \frac{4\pi}{3} \rho a^3$$

by integration

$$\frac{\dot{a}^2}{2} = \frac{G_N M}{a} - \frac{k}{2}$$



# EXAMPLES



Consider the Newtonian toy model of a sphere of dust. The acceleration is

$$\ddot{a} = -\frac{G_N M}{a^2}$$

$$M = \frac{4\pi}{3} \rho a^3$$

by integration

$$\frac{\dot{a}^2}{2} = \frac{G_N M}{a} - \frac{k}{2}$$

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3} \rho - \frac{k}{a^2}$$

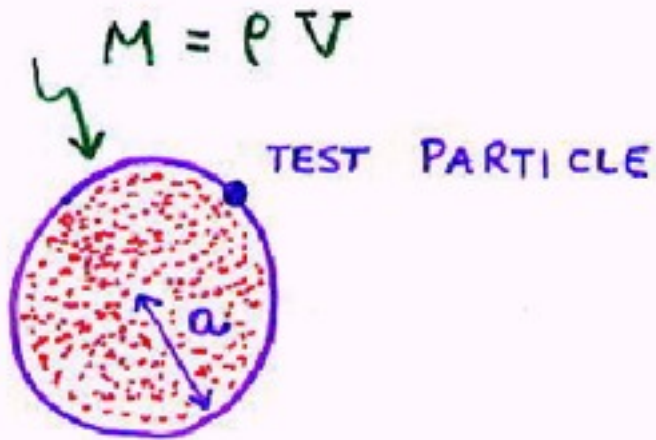
This naïve model reproduces correctly one of the 2 independent GR equations in the FLRW metric=(implementing the Cosm. Pr.)

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + P) = 0$$

The additional independent equation implements “energy conservation” and contains a peculiar GR term

**closed system if an Equation Of State  $P=P(\rho)$  is provided**

# EXAMPLES



Consider the Newtonian toy model of a sphere of dust. The acceleration is

$$\ddot{a} = -\frac{G_N M}{a^2}$$

$$M = \frac{4\pi}{3} \rho a^3$$

by integration

$$\frac{\dot{a}^2}{2} = \frac{G_N M}{a} - \frac{k}{2}$$

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3} \rho - \frac{k}{a^2}$$

This naïve model reproduces correctly one of the 2 independent GR equations in the FLRW metric=(implementing the Cosm. Pr.)

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + P) = 0$$

The additional independent equation implements “energy conservation” and contains a peculiar GR term

**closed system if an Equation Of State  $P=P(\rho)$  is provided**

Compositions usually expressed in  $\Omega_i$ 's, ratios of density of i-species to “critical density”

$$\rho_c = \frac{3}{8\pi G_N} H_0^2$$

# SOME GENERIC SOLUTIONS ( $K=0$ )

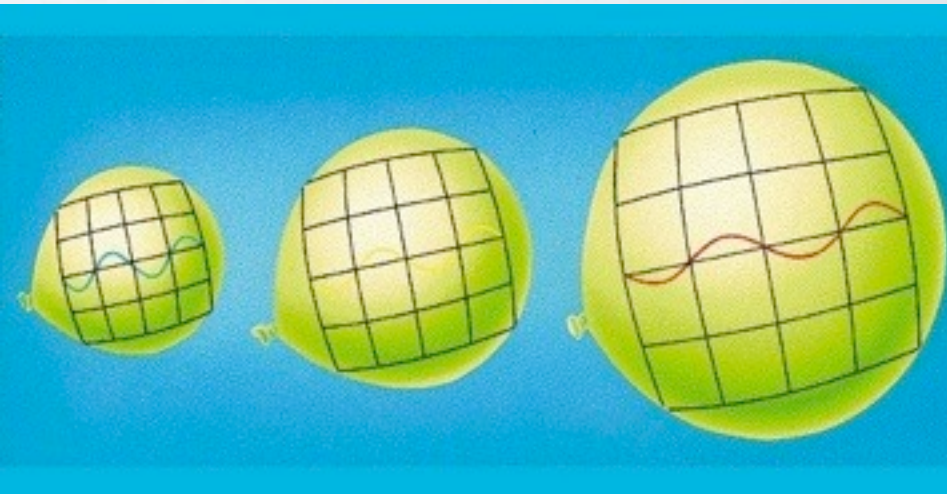
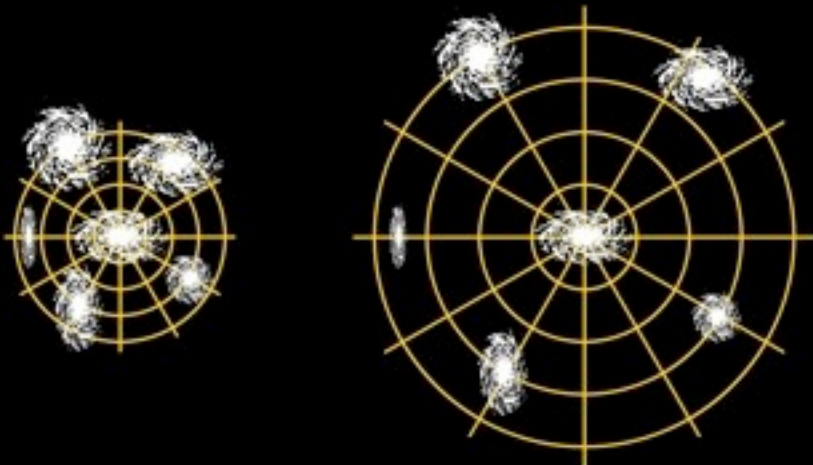
	Equation of State	Behaviour of $\rho$	Scale Factor
Matter	$P \simeq 0$ ( $T \ll m$ )	$\rho \propto a^{-3}$	$a \propto t^{2/3}$
Radiation	$P = \rho/3$	$\rho \propto a^{-4}$	$a \propto t^{1/2}$
Cosm. constant	$P = -\rho$	$\rho = \text{const.}$	$a \propto e^{H_0 t}$

# SOME GENERIC SOLUTIONS ( $K=0$ )

	Equation of State	Behaviour of $\rho$	Scale Factor
Matter	$P \simeq 0$ ( $T \ll m$ )	$\rho \propto a^{-3}$	$a \propto t^{2/3}$
Radiation	$P = \rho/3$	$\rho \propto a^{-4}$	$a \propto t^{1/2}$
Cosm. constant	$P = -\rho$	$\rho = \text{const.}$	$a \propto e^{H_0 t}$

conservation of particles per comoving volume  
 For radiation, further a-factor due to wavelength stretching, also called "redshift"

$$1 + z = \frac{\lambda_{\text{today}}}{\lambda_{\text{then}}} = \frac{a_{\text{today}}}{a_{\text{then}}}$$



# “THERMODYNAMICS”

Let's introduce the phase space density  $f$  describing the occupation number of microstates of different energies.

**The Universe is not a system in equilibrium with an external bath, need nonequilibrium system tools.**

However, for sufficiently fast processes (*wrt expansion rate*) exchanging both energy & particles, locally the entropy gets maximized & “local equilibrium conditions” hold

$$f(E) = \frac{1}{\exp[(E - \mu)/T] \pm 1}$$

T and  $\mu$ : parameters maximizing the entropy under given constraints on the energy and number of particles present per unit volume, respectively.

# “THERMODYNAMICS”

Let's introduce the phase space density  $f$  describing the occupation number of microstates of different energies.

**The Universe is not a system in equilibrium with an external bath, need nonequilibrium system tools.**

However, for sufficiently fast processes (*wrt expansion rate*) exchanging both energy & particles, locally the entropy gets maximized & “local equilibrium conditions” hold

$$f(E) = \frac{1}{\exp[(E - \mu)/T] \pm 1}$$

$T$  and  $\mu$ : parameters maximizing the entropy under given constraints on the energy and number of particles present per unit volume, respectively.

- If energy is exchanged rapidly, different species share the the same  $T$
- Similarly, if particle changing reactions of the type



are fast enough

a conservation rule holds

$$\mu_A + \mu_B = \mu_C + \mu_D$$

⇒ chemical potential  $\mu$  vanishes for particles that can be freely created/annihilated,

like photons; particles and antiparticles have opposite  $\mu$

# USEFUL RECIPE FOR LTE

To know if LTE holds, compare

Rate of process  
of interest  $\Gamma = n \sigma v$  VS.  $H$  Hubble expansion  
rate

*Most of the interesting cosmological processes happen when those quantities become comparable (“freeze-out”): departures from equilibria!*

- $T \sim 1 \text{ eV}$  (@  $t \sim 10^{13} \text{ s}$ )



freezes-out: recombination, photons nowadays forming CMB decouple

- $T \sim 0.1 \text{ MeV}$  (@  $t \sim 10^2 \text{ s}$ )



freezes-out: the “nuclear statistical equilibrium” ends, BBN takes place

# TD IN THE EXPANDING UNIVERSE

If  $f$  is the phase space distribution function, homogeneity and isotropy imply that it can only depend on  $t$  and  $|\mathbf{p}|=p$

*“Kinetic theory” demands a dynamical equation for  $f$  (Boltzmann Eq.)  
However, in most applications the whole energy spectrum is not needed and one can work with moments of  $f$  (and corresponding equations)*



# TD IN THE EXPANDING UNIVERSE

If  $f$  is the phase space distribution function, homogeneity and isotropy imply that it can only depend on  $t$  and  $|\mathbf{p}|=p$

“Kinetic theory” demands a dynamical equation for  $f$  (Boltzmann Eq.)  
However, in most applications the whole energy spectrum is not needed and one can work with moments of  $f$  (and corresponding equations)

## current density of particles

$$n^\mu = g \int f \frac{p^\mu}{p^0} \frac{d\vec{p}}{(2\pi)^3} \Rightarrow n = \int f \frac{d\vec{p}}{(2\pi)^3}$$

internal (spin) dof

due to isotropy, only  $n^0 \neq 0$

# TD IN THE EXPANDING UNIVERSE

If  $f$  is the phase space distribution function, homogeneity and isotropy imply that it can only depend on  $t$  and  $|\mathbf{p}|=p$

“Kinetic theory” demands a dynamical equation for  $f$  (Boltzmann Eq.)  
However, in most applications the whole energy spectrum is not needed and one can work with moments of  $f$  (and corresponding equations)

## current density of particles

$$n^\mu = g \int f \frac{p^\mu}{p^0} \frac{d\vec{p}}{(2\pi)^3} \Rightarrow n = \int f \frac{d\vec{p}}{(2\pi)^3}$$

internal (spin) dof      due to isotropy, only  $n^0 \neq 0$

can be proven that the covariant conservation of particle number

$$\nabla_\mu n^\mu = 0 \Rightarrow \nabla_\mu n^\mu = \frac{1}{a^3} \frac{\partial}{\partial t} (a^3 n) = 0$$

OK with physical intuition of previous cartoon       $n \propto a^{-3} \propto V^{-1}$

# SECOND MOMENT

In GR, the Einstein tensor depends on second moments of  $f$

## Stress-energy Tensor

$$T^{\mu\nu} = g \int f \frac{p^\mu p^\nu}{p^0} \frac{d\vec{p}}{(2\pi)^3}$$

(note the isotropy assumption)  $\longrightarrow$

**Energy density**  
 $\rho = T^{00} = g \int f p^0 \frac{d\vec{p}}{(2\pi)^3}$

**Pressure**  
 $-P\delta^{ij} = T^{ij} = -\delta^{ij} g \int f \frac{|\vec{p}|^2}{3E} \frac{d\vec{p}}{(2\pi)^3}$

# SECOND MOMENT

In GR, the Einstein tensor depends on second moments of  $f$

## Stress-energy Tensor

$$T^{\mu\nu} = g \int f \frac{p^\mu p^\nu}{p^0} \frac{d\vec{p}}{(2\pi)^3}$$

(note the isotropy assumption)  $\longrightarrow$   $-P\delta^{ij} = T^{ij} = -\delta^{ij} g \int f \frac{|\vec{p}|^2}{3E} \frac{d\vec{p}}{(2\pi)^3}$

**Energy density**  
 $\rho = T^{00} = g \int f p^0 \frac{d\vec{p}}{(2\pi)^3}$

**Pressure**

## Bianchi identities (1 ind. eq.), “energy conservation”

$$\nabla_\mu T^{\mu\nu} = 0 \quad \longrightarrow \quad \frac{d\rho}{dt} = -3H(\rho + P)$$

**We recover the second Friedmann equation!**

If we express  $f$  in terms of “temperature”, this equation provides a time-temperature relation!

# EQUILIBRIUM EXPRESSIONS ( $\mu=0$ )

## Relativistic species

$$n = g \frac{\zeta(3)}{\pi^2} T^3 \times \left\{ 1(-), \frac{3}{4}(+) \right\}$$

$$\rho = g \frac{\pi^2}{30} T^4 \times \left\{ 1(-), \frac{7}{8}(+) \right\} \quad P = \rho/3$$

applying comoving particle number conservation law we obtain a simple  $t(T)$

$$a^3 T^3 = \text{const.} \rightarrow T \propto a^{-1}$$

we can use e.g. CMB photon “temperature” as “clock variable” for the epoch of the universe, at least after recombination when the # of photons does not change...

# EQUILIBRIUM EXPRESSIONS ( $\mu=0$ )

## Relativistic species

$$n = g \frac{\zeta(3)}{\pi^2} T^3 \times \left\{ 1(-), \frac{3}{4}(+) \right\}$$

$$\rho = g \frac{\pi^2}{30} T^4 \times \left\{ 1(-), \frac{7}{8}(+) \right\} \quad P = \rho/3$$

applying comoving particle number conservation law we obtain a simple  $n(T)$

$$a^3 T^3 = \text{const.} \rightarrow T \propto a^{-1}$$

we can use e.g. CMB photon “temperature” as “clock variable” for the epoch of the universe, at least after recombination when the # of photons does not change...

## Non-relativistic species at LTE

$$n = g \left( \frac{m T}{2\pi} \right)^{3/2} \exp \left( -\frac{m}{T} \right) \quad \rho = m n \quad P = n T \ll \rho$$

# ENTROPY

**Remember Boltzmann's formula? It naturally suggests the following formula for the entropy density/current (classical limit)**

$$s^\mu = -g \int f(\ln f - 1) \frac{p^\mu}{p^0} \frac{d\vec{p}}{(2\pi)^3} \Rightarrow s^0 = -g \int f(\ln f - 1) \frac{d\vec{p}}{(2\pi)^3}$$

**Exercise:** using  $f \sim \exp[(\mu - E)/T]$  in the parenthesis, check that at equilibrium & for a perfect fluid, this gives

$$s = \frac{\rho + P - \mu n}{T}$$

# ENTROPY

**Remember Boltzmann's formula? It naturally suggests the following formula for the entropy density/current (classical limit)**

$$s^\mu = -g \int f(\ln f - 1) \frac{p^\mu}{p^0} \frac{d\vec{p}}{(2\pi)^3} \Rightarrow s^0 = -g \int f(\ln f - 1) \frac{d\vec{p}}{(2\pi)^3}$$

**Exercise:** using  $f \sim \exp[(\mu - E)/T]$  in the parenthesis, check that at equilibrium & for a perfect fluid, this gives

$$s = \frac{\rho + P - \mu n}{T}$$

**For relativistic species (the entropy is dominated by relativistic species)**

$$s \simeq \frac{4}{3} \frac{\rho}{T} \qquad s = \frac{2\pi^2}{45} h_{\text{eff}}(T) T^3$$

$$h_{\text{eff}}(T) = \sum_{i=\text{rel. bos.}} g_i \left( \frac{T_i}{T} \right)^3 + \frac{7}{8} \sum_{j=\text{rel. ferm.}} g_j \left( \frac{T_j}{T} \right)^3$$



# ENERGY & ENTROPY IN RELATIV. ERA

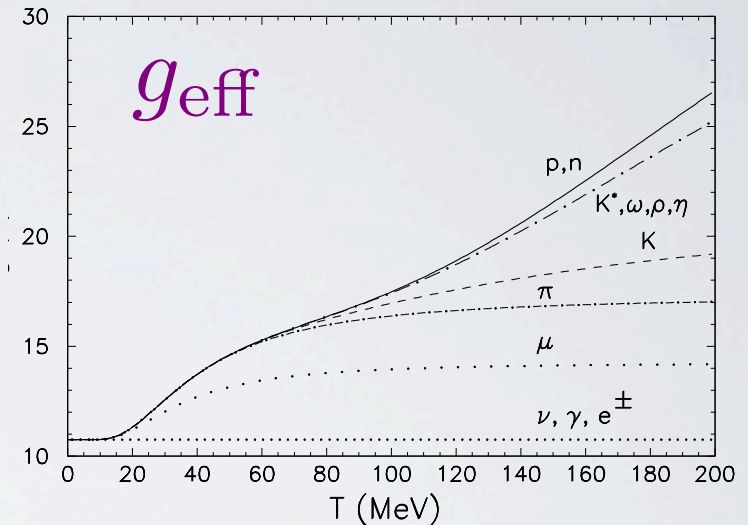
similarly 
$$g_{\text{eff}}(T) = \sum_{i=\text{rel.bos.}} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8} \sum_{j=\text{rel.ferm.}} g_j \left(\frac{T_j}{T}\right)^4$$

entering

$$\rho_{\text{tot}} = \frac{\pi^2}{30} g_{\text{eff}}(T) T^4$$

&

$$H^2 = \frac{8\pi}{3 M_P^2} \rho_{\text{tot}} = \frac{4\pi^3}{45 M_P^2} g_{\text{eff}} T^4$$

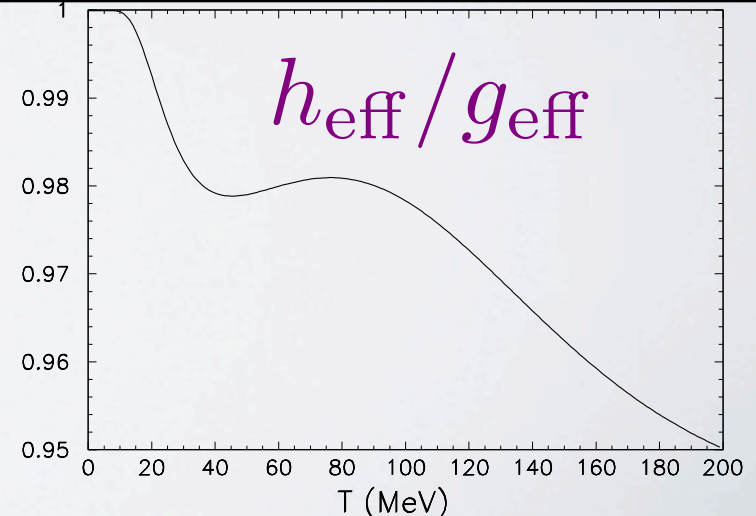


compare with 
$$s = \frac{2\pi^2}{45} h_{\text{eff}}(T) T^3$$

**they vary when species annihilate!**

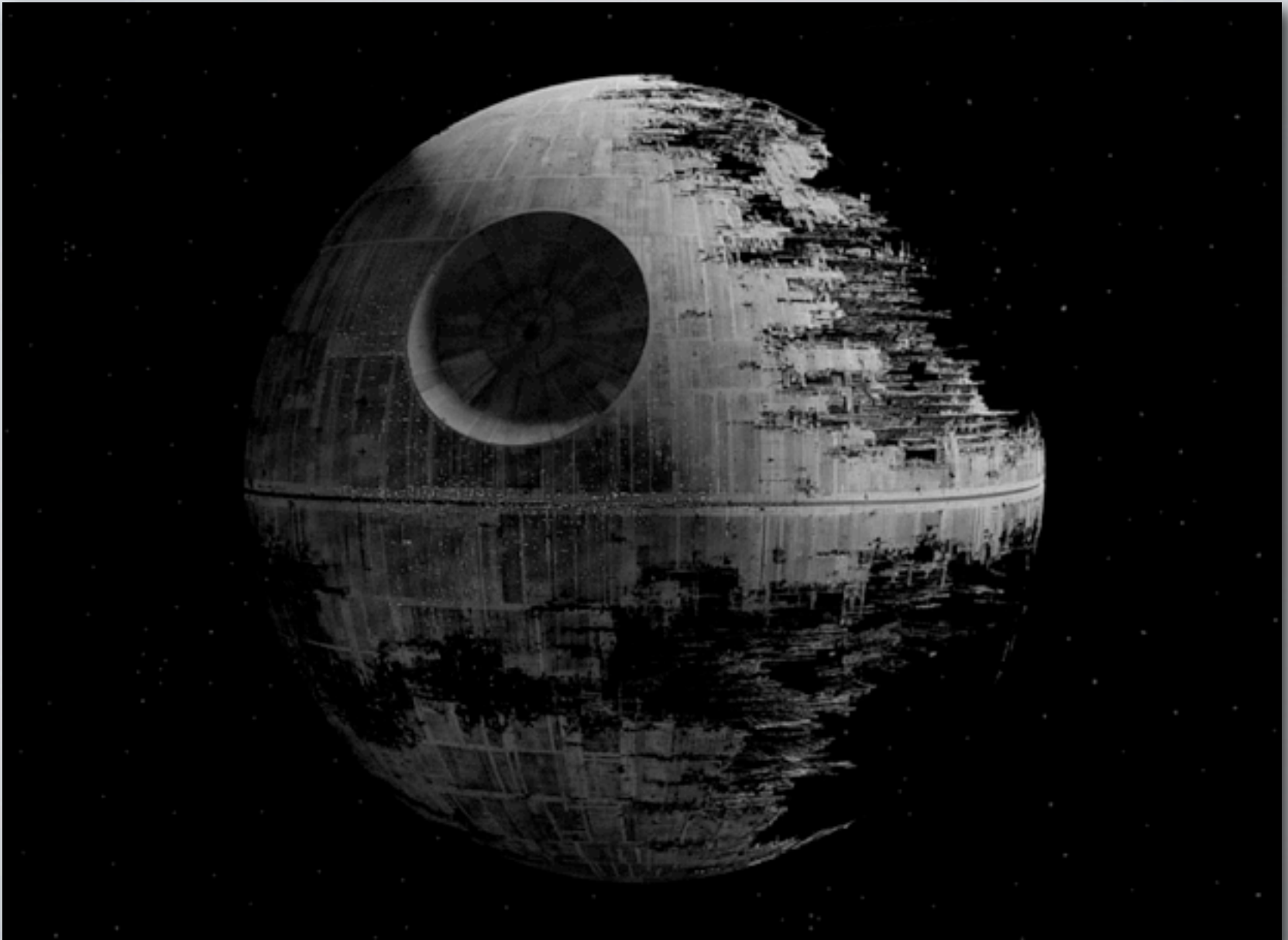
for reference, currently-accounting for photons and neutrinos-one has

$$h_{\text{eff}} \sim 2 + 3 \cdot 2 \cdot (4/11) \cdot 7/8 \sim 3.91, \quad T \sim 2.73 \text{ K}$$



DARK MATTER ENTERS THE SCENE...

DARK MATTER ENTERS THE SCENE...



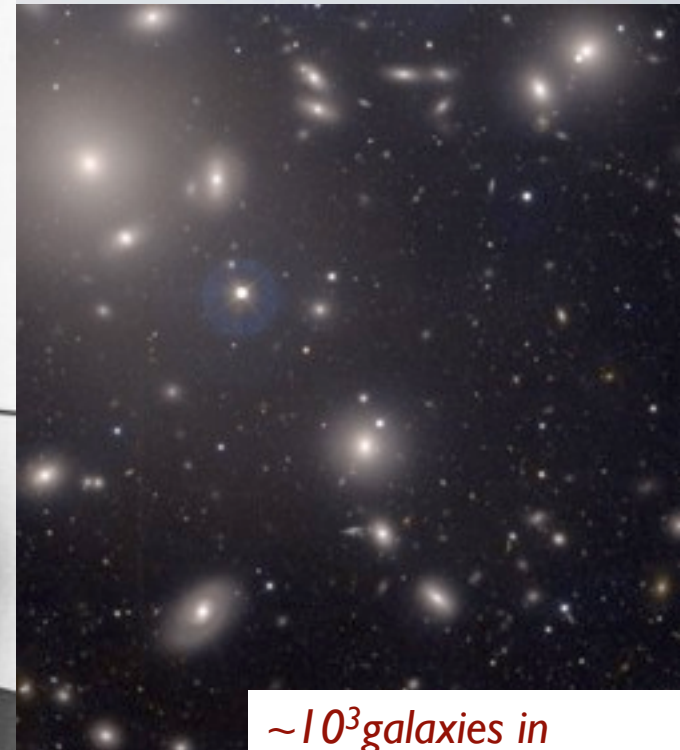
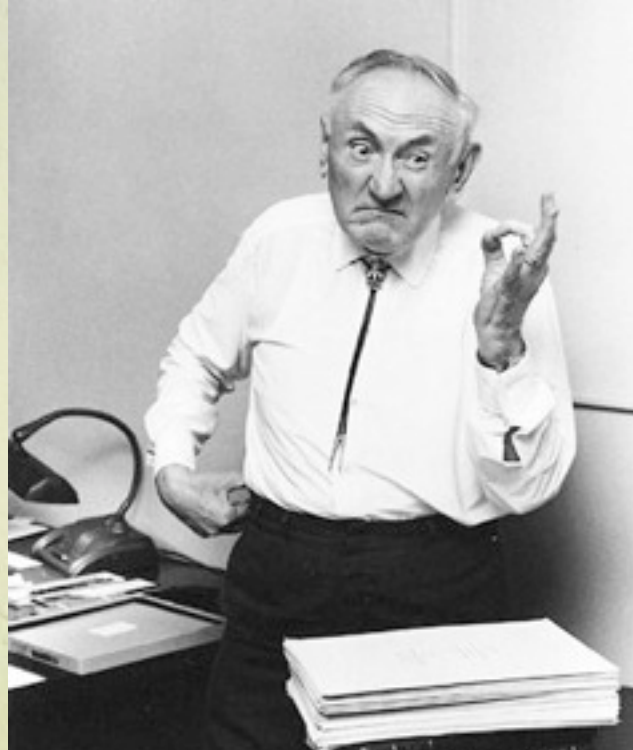
# DM “DISCOVERY” IN COMA CLUSTER (~1933)

Varna, Bulgaria

В ТОЗИ ДОМ Е РОДЕН  
ФРИЦ ЦВИКИ - АСТРОНОМЪТ,  
КОЙТО ОТКРИ  
НЕУТРОННИТЕ ЗВЕЗДИ  
И ТЪМНАТА МАТЕРИЯ  
ВЪВ ВСЕЛЕНАТА.

---

IN THIS HOME  
WAS BORN FRITZ ZWICKY -  
THE ASTRONOMER  
WHO DISCOVERED  
NEUTRON STARS  
AND THE DARK MATTER  
IN THE UNIVERSE.

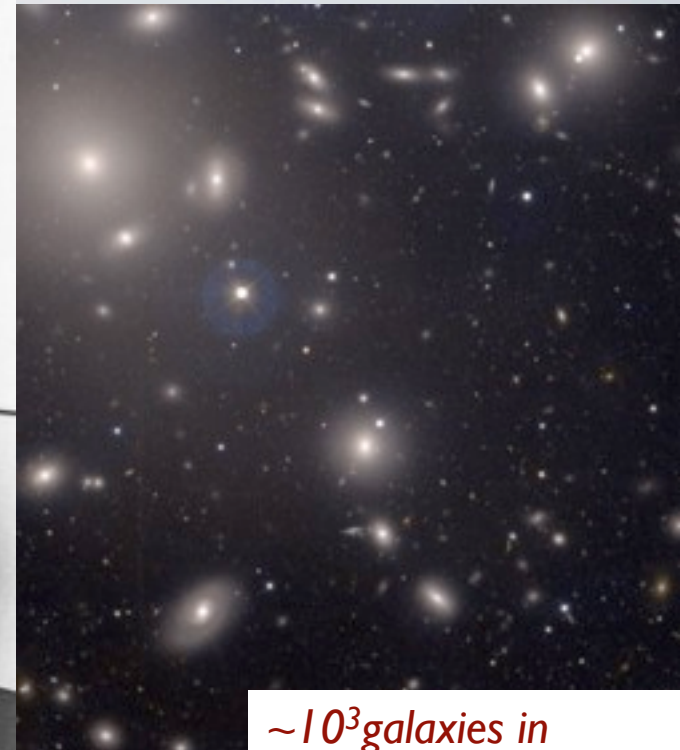
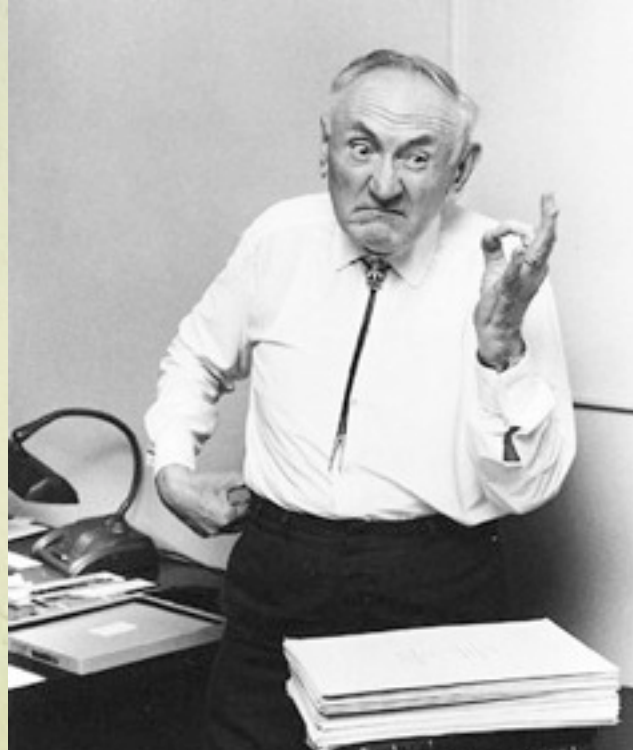


We remember F. Zwicky here for two important discoveries:

*~10<sup>3</sup> galaxies in  
~1 Mpc radius region*

# DM “DISCOVERY” IN COMA CLUSTER (~1933)

Varna, Bulgaria



*~10<sup>3</sup> galaxies in  
~1 Mpc radius region*

We remember F. Zwicky here for two important discoveries:

- “Astronomers are spherical bastards. No matter how you look at them they are just bastards.”
- Inferred the mass of the Coma cluster from the proper motion of the Galaxies, finding that the required mass is much larger than what could be accounted for

*Die Rotverschiebung von extragalaktischen Nebeln\**, *Helvetica Physica Acta* (1933) **6**, 110–127.

*"On the Masses of Nebulae and of Clusters of Nebulae"*, *Astrophysical Journal* (1937) **86**, 217

\*Nebula=Early XXth century name for what we call now galaxy

**I. No “BSM” implications (yet)**

**II. How did he do it? Clever & original application of Virial Theorem**

# SKETCH OF THE METHOD

Expression of time average of total kinetic energy  
 $T$  of  $N$  particles bounded by conservative forces  $F$

$$2\langle T \rangle = - \sum_{k=1}^N \langle \mathbf{r}_k \cdot \mathbf{F}_k \rangle$$

Average total  
potential energy  $\langle U \rangle$

$$U(r) = A r^n \implies - \sum_{k=1}^N \langle \mathbf{r}_k \cdot \mathbf{F}_k \rangle = n \langle U_{tot} \rangle$$

For Gravity,  $U \sim r^{-1}$

$$2\langle T \rangle + \langle U_{tot} \rangle = 0$$

# SKETCH OF THE METHOD

Expression of time average of total kinetic energy  
 $T$  of  $N$  particles bounded by conservative forces  $F$

$$2\langle T \rangle = - \sum_{k=1}^N \langle \mathbf{r}_k \cdot \mathbf{F}_k \rangle$$

Average total  
potential energy  $\langle U \rangle$

$$U(r) = A r^n \implies - \sum_{k=1}^N \langle \mathbf{r}_k \cdot \mathbf{F}_k \rangle = n \langle U_{tot} \rangle$$

For Gravity,  $U \sim r^{-1}$

$$2\langle T \rangle + \langle U_{tot} \rangle = 0$$

$$\langle T \rangle = N \frac{\langle m v^2 \rangle}{2}$$

$N^2/2$  pairs  
of Galaxies

$$\langle U_{tot} \rangle \simeq - \frac{N^2}{2} G_N \frac{\langle m^2 \rangle}{\langle r \rangle}$$

# SKETCH OF THE METHOD

Expression of time average of total kinetic energy  
 $T$  of  $N$  particles bounded by conservative forces  $F$

$$2\langle T \rangle = - \sum_{k=1}^N \langle \mathbf{r}_k \cdot \mathbf{F}_k \rangle$$

Average total  
potential energy  $\langle U \rangle$

$$U(r) = A r^n \implies - \sum_{k=1}^N \langle \mathbf{r}_k \cdot \mathbf{F}_k \rangle = n \langle U_{tot} \rangle$$

For Gravity,  $U \sim r^{-1}$

$$2\langle T \rangle + \langle U_{tot} \rangle = 0$$

$$\langle T \rangle = N \frac{\langle m v^2 \rangle}{2}$$

$N^2/2$  pairs  
of Galaxies

$$\langle U_{tot} \rangle \simeq - \frac{N^2}{2} G_N \frac{\langle m^2 \rangle}{\langle r \rangle}$$

doppler shifts in galactic spectra

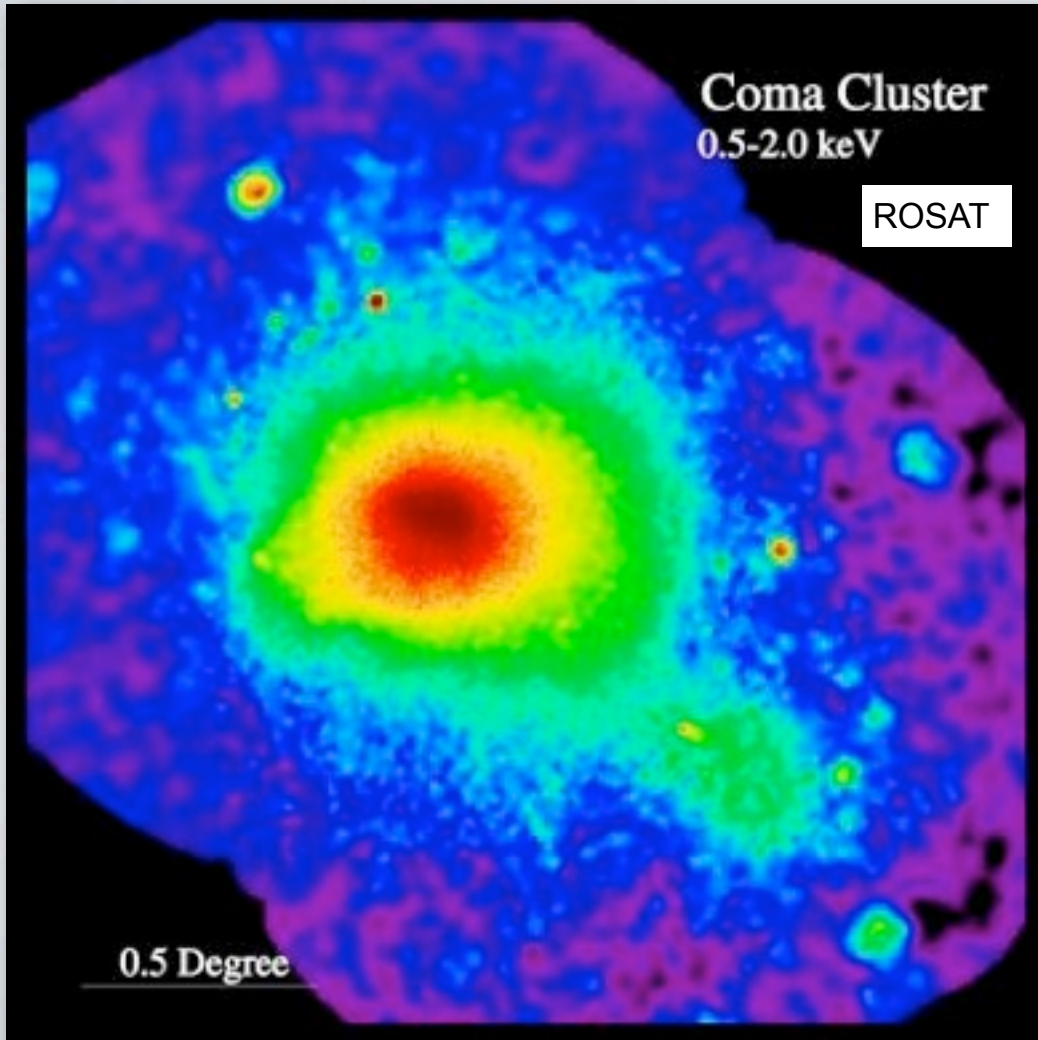
$$M_{tot} \simeq N \langle m \rangle \simeq - \frac{2 \langle v^2 \rangle \langle r \rangle}{G_N} \xrightarrow{\text{inferred geometrically}}$$

found a factor  $\sim 400$  larger mass than the one from converting luminosity into mass!



# MODERN PROOFS FROM CLUSTERS: X-RAYS

We know today that most of the mass in clusters (not true for galaxies!) is in the form of hot, intergalactic gas, which can be traced via X rays: bolometric X-luminosity can be eventually converted into gas density maps, spectral info into pressure information (or potential depth)



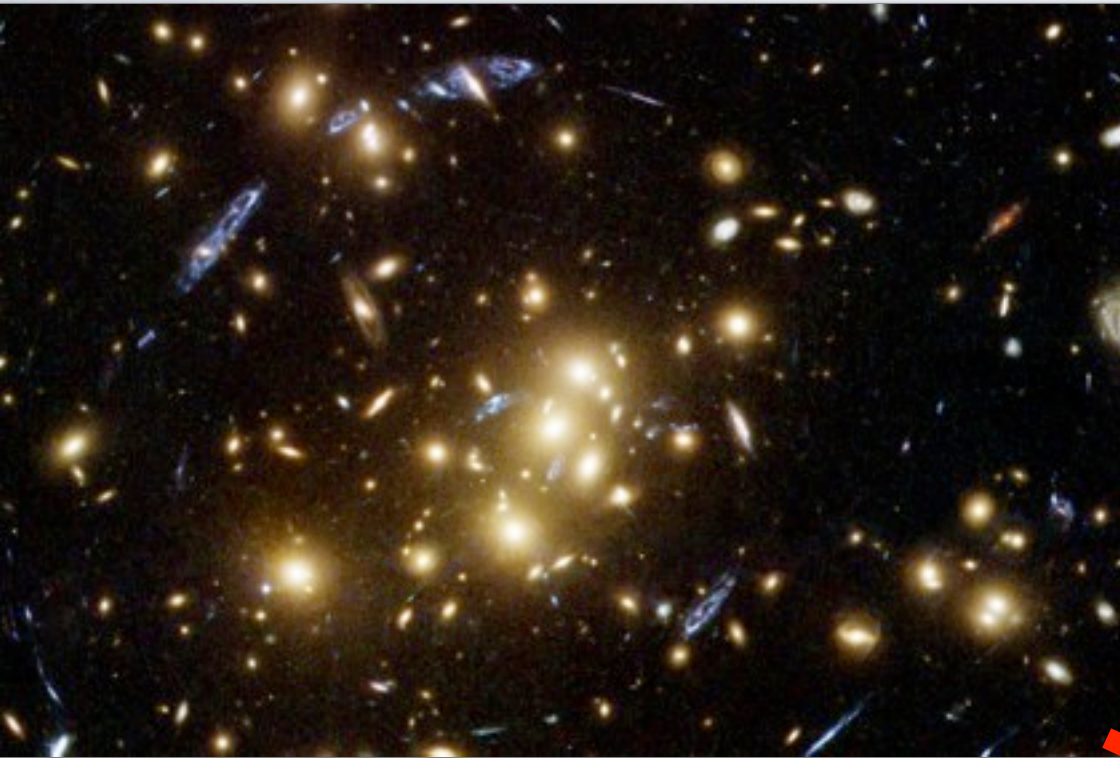
$$\frac{dP_{gas}}{dr} = G_N \frac{M(< r) \rho_{gas}}{r^2}$$

See for example

Lewis, Buote, and Stocke, *ApJ* (2003), 586, 135

**Again, a factor ~7 more mass than those in gas form is inferred (also its profile can be traced...)**

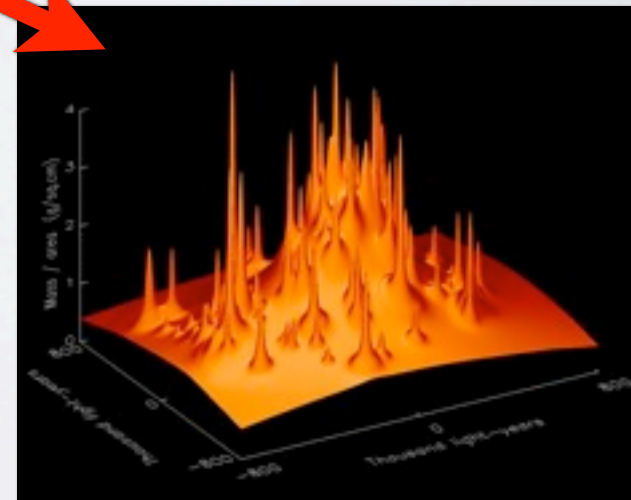
# MODERN PROOFS FROM CLUSTERS: LENSING



*CL0024+1654,  
Hubble space telescope*

*its gravitating mass distribution  
inferred from lensing tomography*

Consistent inference done from clusters of Galaxies:  
Presence of Dark Matter smoothly distributed in-  
between galaxies is required  
(and actually must dominate total potential)



# MORE SPECTACULAR: SEGREGATION!

Baryonic gas gets “shocked” in the collision and stays behind. The mass causing lensing (as well as the subdominant galaxies) pass through each other (non-collisional)

**(most of the) Mass is not in the collisional gas, as would happen if law of gravity had been altered!**

Galaxy Cluster MACS J0025.4–1222  
Hubble Space Telescope ACS/WFC  
Chandra X-ray Observatory

1.5 million light-years  
460 kiloparsecs



**bullet cluster**

# FLAT GALAXY ROTATION CURVES

- observed (equate centripetal acc. & Newton's law)

$$v_{rot}^2 = \frac{G M(R)}{R} \simeq const. \quad M(R) = \int_0^R 4\pi r^2 \rho(r) dr$$

- predicted based on visible light

$$v_{rot}^2 \propto \frac{1}{R}$$

# FLAT GALAXY ROTATION CURVES

- observed (equate centripetal acc. & Newton's law)

$$v_{rot}^2 = \frac{GM(R)}{R} \simeq const. \quad M(R) = \int_0^R 4\pi r^2 \rho(r) dr$$

- predicted based on visible light

$$v_{rot}^2 \propto \frac{1}{R}$$

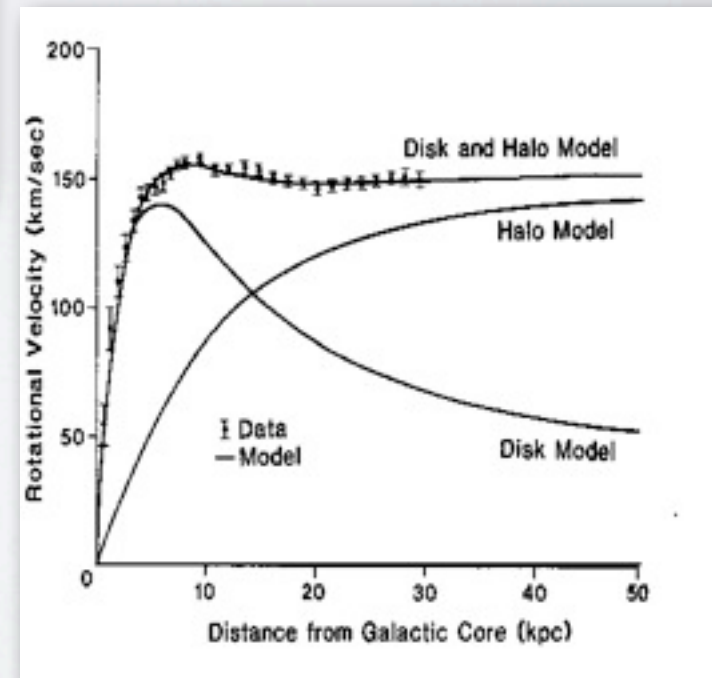
Data are well described by an additional component extending to distance  $\gg$  visible mass scale, with a profile

$$\rho(r) \propto r^{-2} \quad (\text{clearly not valid at asymptotically large } r!)$$

Historically, only after these studies (in the ~'70-'80) people started to take the dark matter problem seriously



**Vera Rubin**



# FLAT GALAXY ROTATION CURVES

- observed (equate centripetal acc. & Newton's law)

$$v_{rot}^2 = \frac{GM(R)}{R} \simeq const. \quad M(R) = \int_0^R 4\pi r^2 \rho(r) dr$$

- predicted based on visible light

$$v_{rot}^2 \propto \frac{1}{R}$$

Data are well described by an additional component extending to distance  $\gg$  visible mass scale, with a profile

$$\rho(r) \propto r^{-2} \quad (\text{clearly not valid at asymptotically large } r!)$$

Historically, only after these studies (in the ~'70-'80) people started to take the dark matter problem seriously

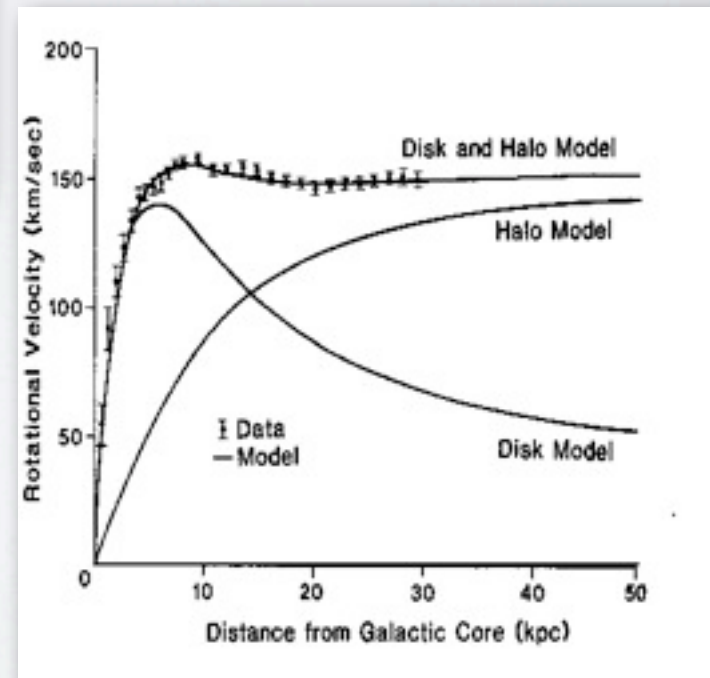
The determination of "local" (Galactic) DM properties require a multi-parameter fit including parameterizations for stellar disk, gas, bulge...

$$\rho_{\odot} \simeq 0.4 \text{ GeV}/\text{cm}^3$$

Important for direct and indirect searches of DM, not so important/robust to infer its existence and properties

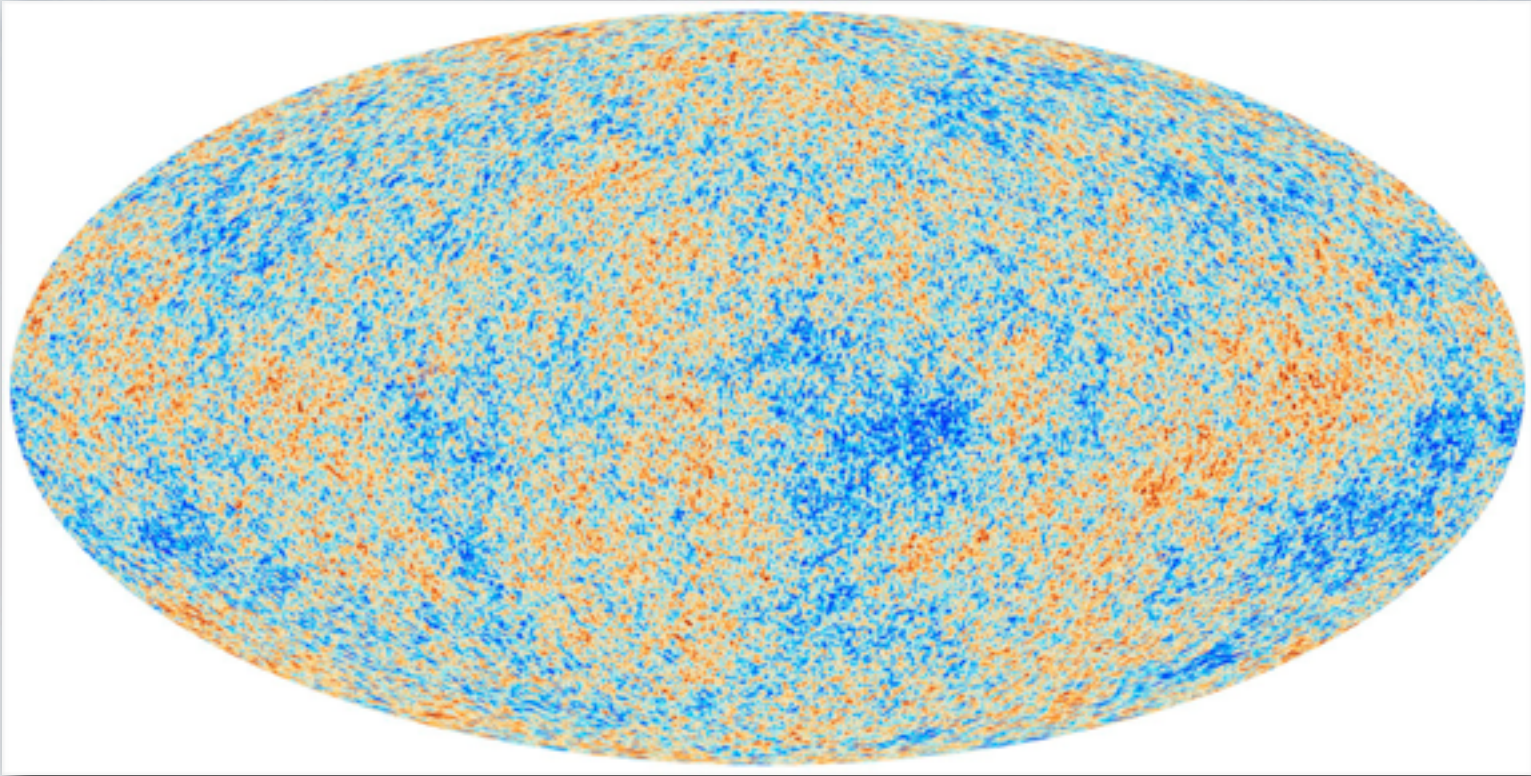


**Vera Rubin**



# GROWTH OF STRUCTURES

This picture, plus some (linear) theory is a robust proof for the existence of DM!



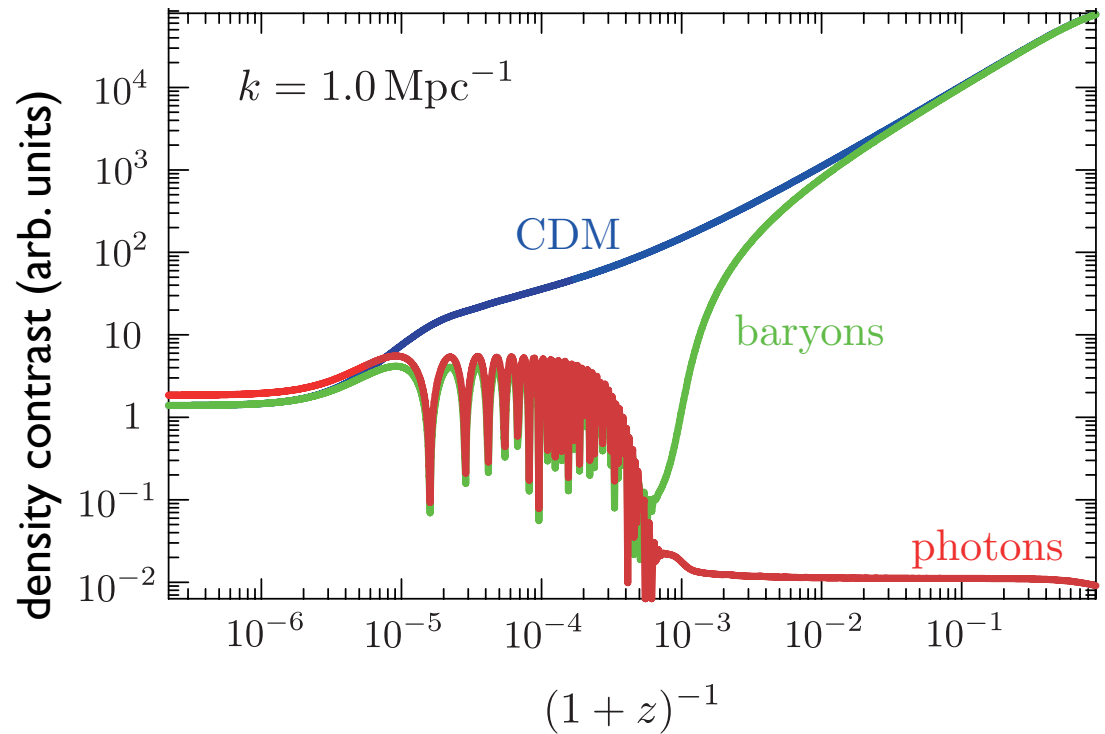
## Key argument

- ▶ Before recombination: baryons & photons coupled, “share perturbations”
- ▶ We measure amplitude  $\sim 10^{-5}$  at recombination (*picture above*)
- ▶ Evolving forward in time, insufficient to achieve collapsed structures as we see nowadays, unless lots of gravitating matter (not coupled to photons) creates deeper potential wells!

# IN GRAPHICAL TERMS

$$\delta = \frac{\rho}{\langle \rho \rangle} - 1$$
$$= \sum_{\mathbf{k}} \tilde{\delta}_{(\mathbf{k})} e^{i\mathbf{k} \cdot \mathbf{x}}$$

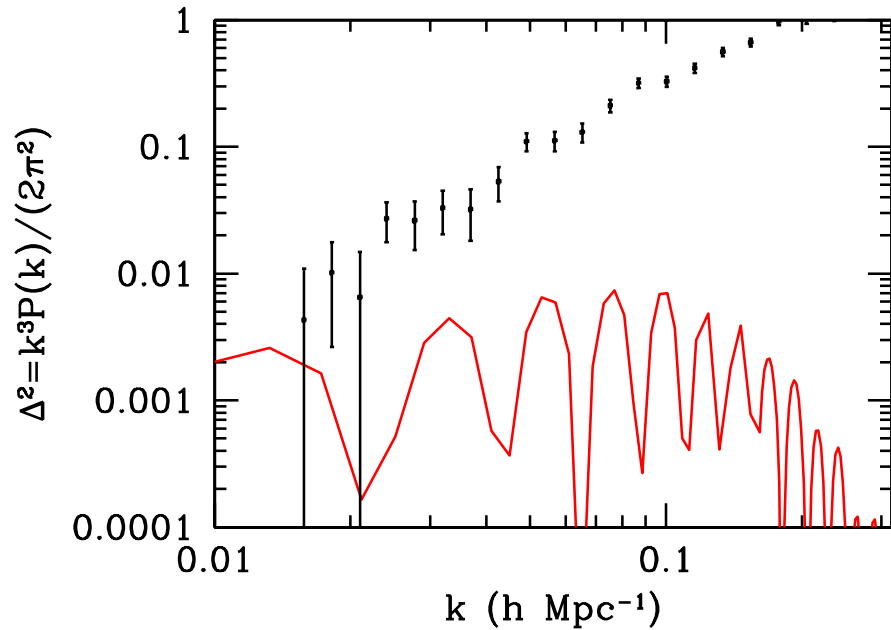
Density contrast for a “mode” (in Fourier space).  
Indep. evolution in linear theory, its “variance” is the power spectrum  $P(k)$



- Ignore evolution at very early times (before entering the Hubble horizon, gauge dependent).
- Upon horizon entry, as long as the baryonic gas is ionized, it is coupled to radiation & oscillates, as pressure prevents overdensities from growing. The (uncoupled, pressureless) CDM mode instead grows, first logarithmically during radiation domination, then linearly in the matter era.
- After recombination, baryons behave as CDM, quickly fall in their “deep” potential wells... but, had not been for CDM, they would need much longer to reach the same density contrast!



# WHAT IF ONLY BARYONS PRESENT?

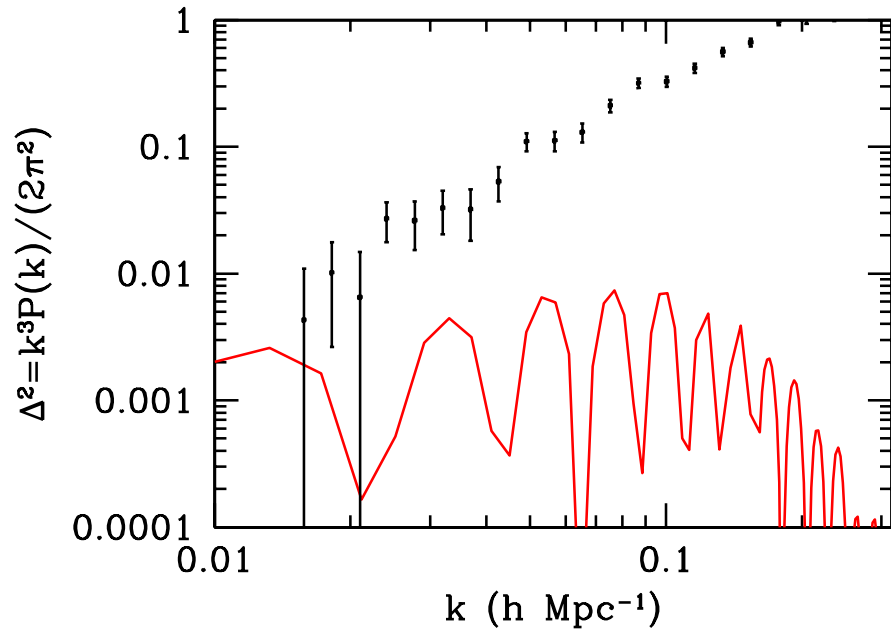


No structure non-linear by now & pattern of “clumpiness” would be very different!

Even putative models of modified gravity that could “boost” growth (e.g. TeVeS...) have hard time to get the right shape!

*See pedagogical discussion in  
Scott Dodelson, arXiv:1112.1320*

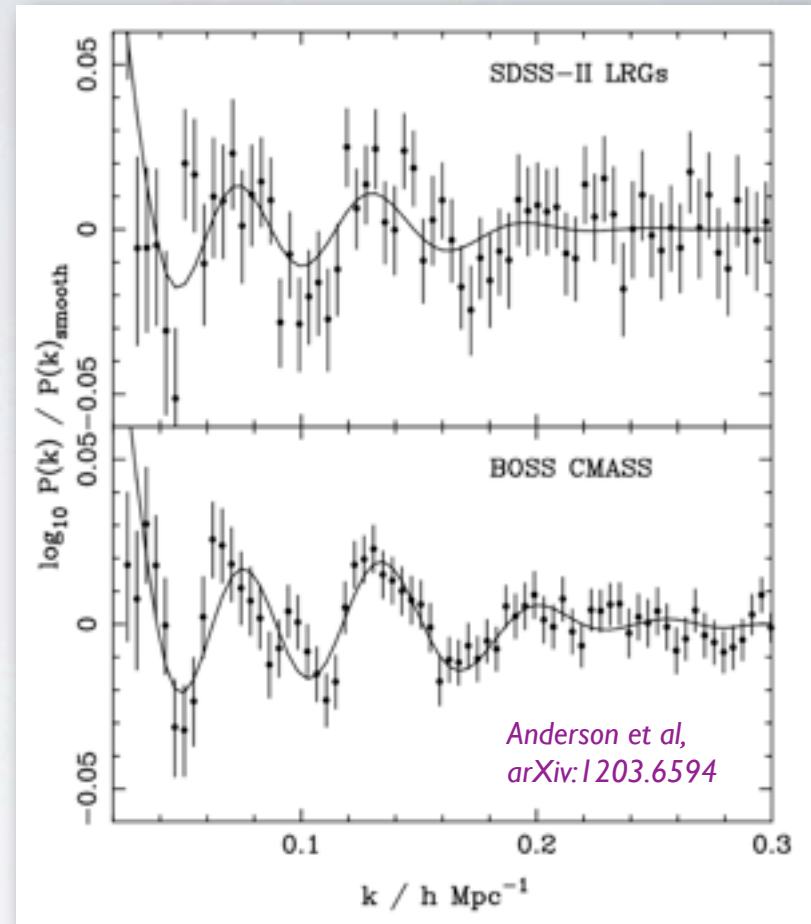
# WHAT IF ONLY BARYONS PRESENT?



No structure non-linear by now & pattern of “clumpiness” would be very different!

Even putative models of modified gravity that could “boost” growth (e.g. TeVeS...) have hard time to get the right shape!

*See pedagogical discussion in Scott Dodelson, arXiv:1112.1320*



Credibility of our understanding reinforced since we see the residual “oscillations” due to coupling of subleading baryons with photons (BAO)!

# EXERCISE (OR MINI-PROJECT)

Previous considerations can be easily made more quantitative (although quite advanced notions needed to justify rigorously some statements...)

I. Write down (Newtonian physics!) continuity, Euler equation & Poisson Equation

$$\frac{\partial \rho}{\partial t} + \partial_{\alpha}(\rho v^{\alpha}) = 0$$

continuity (mass conservation)

$\alpha=1,2,3$

$$\frac{\partial v_{\alpha}}{\partial t} + v^{\beta} \partial_{\beta} v_{\alpha} + \frac{1}{\rho} \partial_{\alpha} p + \partial_{\alpha} \Phi = 0$$

Euler/Newton's law  
(momentum conservation)

$$\partial^2 \Phi - 4\pi G \rho = 0$$

Poisson Eq. (source Grav. potential)

Follow any cosmology perturbation theory course, or refs. such as C. G. Tsagas, [astro-ph/0201405](https://arxiv.org/abs/astro-ph/0201405)  
D. Baumann's Cosmology Lectures, <http://www.damtp.cam.ac.uk/user/db275/Cosmology/Lectures.pdf>

# EXERCISE

2. Consider expanding background case, previous equation write

$$\frac{\partial \rho}{\partial t} + 3H\rho + \frac{1}{a} \partial_\alpha (\rho u^\alpha) = 0$$

$$\frac{d^2 a}{dt^2} x_\alpha + \frac{\partial u_\alpha}{\partial t} + H u_\alpha + \frac{1}{a} u^\beta \partial_\beta u_\alpha + \frac{1}{a\rho} \partial_\alpha p + \frac{1}{a} \partial_\alpha \Phi = 0$$

$$\partial^2 \Phi - 4\pi G a^2 \rho = 0$$

where I defined a comoving set of coordinates ( $x^\alpha$ ) as opposed to “physical” ones ( $r^\alpha$ )

$$r^\alpha = a(t) x^\alpha$$

physical  
velocity  
 $dr^\alpha/dt$

$$v^\alpha = H r^\alpha + u^\alpha$$

“Hubble flow”

“peculiar”  
velocity  
 $dx^\alpha/dt$

derivative wrt  $r$  related to derivative wrt  $x$

$$(\partial_\alpha)_{\text{phys}} = (1/a) (\partial_\alpha)_{\text{com}}$$

$t$ -derivative at fixed  $r$  and fixed  $x$  related by

$$(\partial/\partial t)_{\text{phys}} = (\partial/\partial t)_{\text{com}} - H x^\alpha \partial_\alpha$$

**Proof:**  $(\partial/\partial t)_r = (\partial/\partial t)_x + (\partial x/\partial t)_r (\partial/\partial x) = (\partial/\partial t)_x + (\partial a^{-1} r/\partial t)_r (\partial/\partial x)$

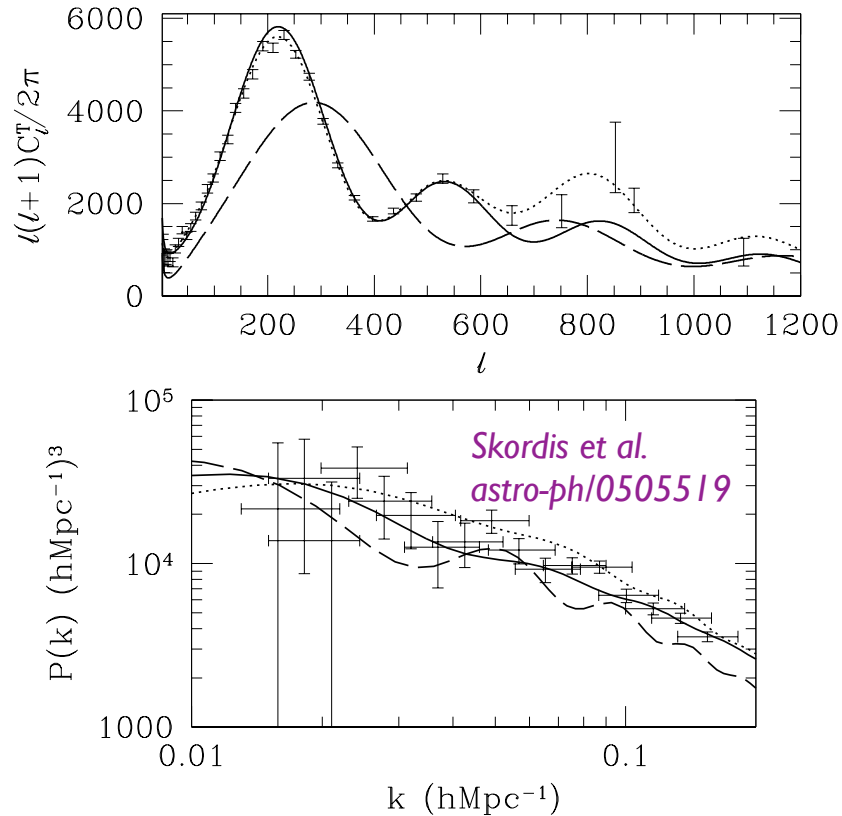
# EXERCISE

3. solve the cases of a “smooth” background
4. Linearize these equations for small perturbation around the smooth solutions.
5. Write them down also in Fourier space.
6. Extension to multi-fluid case is also possible.
7. Which perturbation grow? (*Concept of Jeans length*)
8. How do perturbation grow in the radiation-dominated era?
9. How do they grow in the matter-dominated era?
10. How do they grow in the cosmological constant-dominated era?

Follow any cosmology perturbation theory course, or refs. such as C. G. Tsagas, [astro-ph/0201405](https://arxiv.org/abs/astro-ph/0201405)  
D. Baumann’s *Cosmology Lectures*, <http://www.damtp.cam.ac.uk/user/db275/Cosmology/Lectures.pdf>

# SIMILAR ISSUES WITH CMB...

A few years ago, modified gravity models could still accommodate data (with large  $\Omega_v$ )

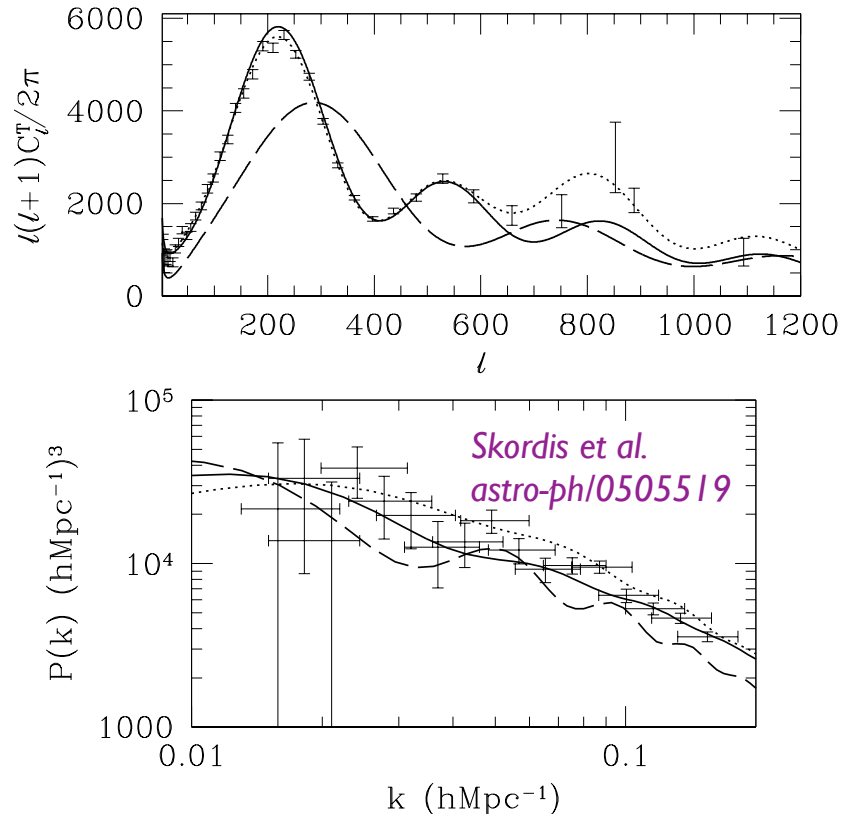


MOND universe (with  $a_0 \approx 4.2 \times 10^{-8} \text{cm/s}^2$ ) with  $\Omega_\Lambda = 0.78$  and  $\Omega_v = 0.17$  and  $\Omega_b = 0.05$  (solid line), for a MOND universe  $\Omega_\Lambda = 0.95$  and  $\Omega_b = 0.05$  (dashed line) and for the  $\Lambda$ CDM model (dotted line)

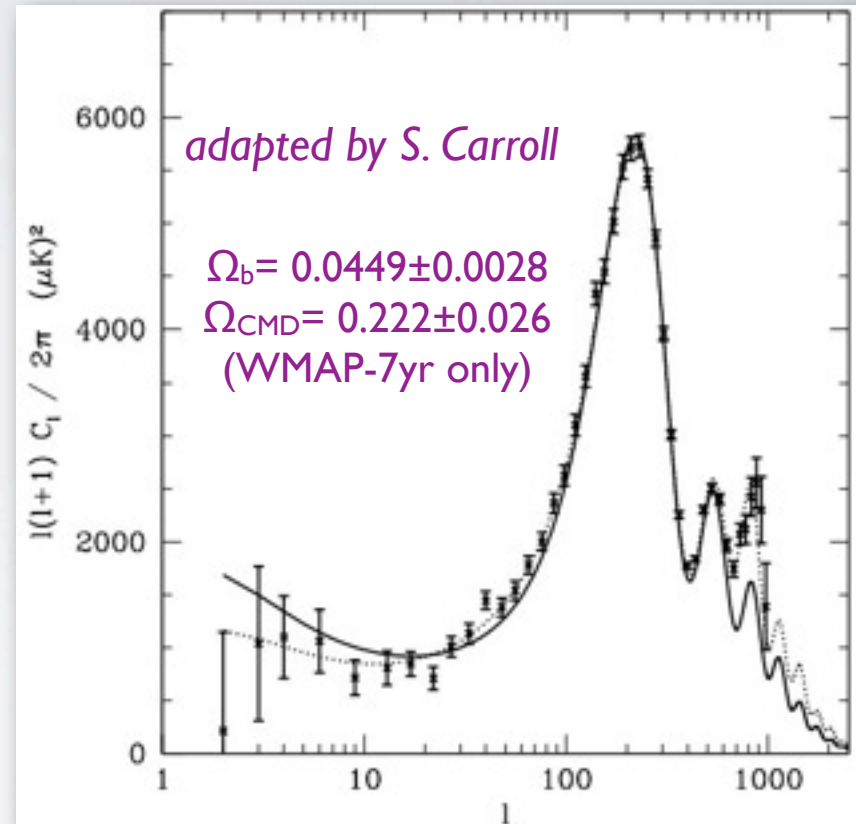
# SIMILAR ISSUES WITH CMB...

A few years ago, modified gravity models could still accommodate data (with large  $\Omega_\nu$ )

recent data inconsistent with these “old” proposals:  
e.g. CMB 3<sup>rd</sup> peak, baryon acoustic oscillations...



MOND universe (with  $a_0 \approx 4.2 \times 10^{-8} \text{cm/s}^2$ ) with  $\Omega_\Lambda = 0.78$  and  $\Omega_\nu = 0.17$  and  $\Omega_b = 0.05$  (solid line), for a MOND universe  $\Omega_\Lambda = 0.95$  and  $\Omega_b = 0.05$  (dashed line) and for the  $\Lambda$ CDM model (dotted line)



$\Omega_b^{\text{CMB}}$  (from atomic physics) is also in agreement with  $\Omega_b^{\text{BBN}}$ , sensitive to total number of nucleons in the plasma at  $T \sim 0.01 - 1$  MeV (nuclear physics)  
Great success of cosmology!

# WHY COSMO EVIDENCE IS IMPORTANT

***I. It is essentially based on exact solutions or linear perturbation theory applied to simple physical systems (gravity, atomic physics...): credible and robust!***

***II. It suggests additional species, rather than a modification of gravity.***

***III. Because it tells us that the largest fraction of required dark matter is non-baryonic, rather than brown dwarf stars, planets, etc.***

*Only (even more radical) way out: modify cosmology to allow “collapsed” objects at very early times (e.g. primordial Black Holes, [But very constrained/on the verge of exclusion, see e.g. F. Capela, M. Pshirkov, P. Tinyakov, PRD 90, 083507 \(2014\) \[arXiv:1403.7098\]. and refs. therein](#))*



# WHY COSMO EVIDENCE IS IMPORTANT

**I. It is essentially based on exact solutions or linear perturbation theory applied to simple physical systems (gravity, atomic physics...): credible and robust!**

**II. It suggests additional species, rather than a modification of gravity.**

**III. Because it tells us that the largest fraction of required dark matter is non-baryonic, rather than brown dwarf stars, planets, etc.**

Only (even more radical) way out: modify cosmology to allow “collapsed” objects at very early times (e.g. primordial Black Holes, *But very constrained/on the verge of exclusion, see e.g. F. Capela, M. Pshirkov, P. Tinyakov, PRD 90, 083507 (2014) [arXiv:1403.7098]. and refs. therein*)

The only possible SM candidate are **neutrinos** (which are also stable).  
But neutrinos (at least known ones) do not work!

# WHY COSMO EVIDENCE IS IMPORTANT

- I. It is essentially based on exact solutions or linear perturbation theory applied to simple physical systems (gravity, atomic physics...): credible and robust!**
- II. It suggests additional species, rather than a modification of gravity.**
- III. Because it tells us that the largest fraction of required dark matter is non-baryonic, rather than brown dwarf stars, planets, etc.**

Only (even more radical) way out: modify cosmology to allow “collapsed” objects at very early times (e.g. primordial Black Holes, *But very constrained/on the verge of exclusion, see e.g. F. Capela, M. Pshirkov, P. Tinyakov, PRD 90, 083507 (2014) [arXiv:1403.7098]. and refs. therein*)

The only possible SM candidate are **neutrinos** (which are also stable).  
But neutrinos (at least known ones) do not work!

**This implies that Dark Matter requires “new physics”, beyond the theories known today. Only a handful of similar indications: explains the interest of particle physicists!**

# NEUTRINOS AS DARK MATTER?

**Condition 1. Must be massive** (which is already a departure from SM...)

**Fulfilled!** Oscillations established, at least 2 massive states, measured splitting implies at least one state heavier than 0.05 eV

$$\Delta m_{\text{atm}}^2 \simeq 2.4 \times 10^{-3} \text{ eV}^2$$

# NEUTRINOS AS DARK MATTER?

**Condition 1. Must be massive** (which is already a departure from SM...)

**Fulfilled!** Oscillations established, at least 2 massive states, measured splitting implies at least one state heavier than 0.05 eV

$$\Delta m_{\text{atm}}^2 \simeq 2.4 \times 10^{-3} \text{ eV}^2$$

**Condition 2. Must match cosmological abundance**

**Failed!** Direct mass limits combined with splittings from oscillation experiments impose upper limit of about 7 eV to the sum (After KATRIN, potentially improved to ~0.7 eV)

$$\Omega_\nu = \frac{\rho_\nu}{\rho_c} \simeq \frac{\sum_i m_i}{45 \text{ eV}}$$

$$\Omega_{\text{DM}} \approx 0.3 (\text{Planck}) \Rightarrow \sum m_i \approx 15 \text{ eV}$$

*we will perform this computation in lecture 2.*

# NEUTRINOS AS DARK MATTER?

**Condition 1. Must be massive** (which is already a departure from SM...)

**Fulfilled!** Oscillations established, at least 2 massive states, measured splitting implies at least one state heavier than 0.05 eV

$$\Delta m_{\text{atm}}^2 \simeq 2.4 \times 10^{-3} \text{ eV}^2$$

**Condition 2. Must match cosmological abundance**

**Failed!** Direct mass limits combined with splittings from oscillation experiments impose upper limit of about 7 eV to the sum (After KATRIN, potentially improved to ~0.7 eV)

$$\Omega_\nu = \frac{\rho_\nu}{\rho_c} \simeq \frac{\sum_i m_i}{45 \text{ eV}}$$

$$\Omega_{\text{DM}} \approx 0.3 (\text{Planck}) \Rightarrow \sum m_i \approx 15 \text{ eV}$$

*we will perform this computation in lecture 2.*

**Condition 3. Must allow for structure formation (of the right kind)**

**Failed!** We will see shortly why it is so... which applies to more general classes of candidates.

# AN IMPORTANT NUMBER...

Recent determination (Planck 2015, 68% CL)

$$\Omega_c h^2 = 0.1188 \pm 0.0010, \text{ i.e. } \Omega_c \sim 0.26$$

# AN IMPORTANT NUMBER...

Recent determination (Planck 2015, 68% CL)

$$\Omega_c h^2 = 0.1188 \pm 0.0010, \text{ i.e. } \Omega_c \sim 0.26$$

$$\rho_{X,0} = M_X n_{X,0} = M_X s_0 Y_0$$

$$\rho_c = \frac{3H_0^2}{8\pi G_N} = 1.054 \times 10^{-5} h^2 \text{ GeV cm}^{-3}$$

$$s_0 = 2889 \left( \frac{T_{\gamma,0}}{2.725} \right)^3 \text{ cm}^{-3} \quad \text{where } \mathbf{h_{eff}} \sim 2 + 3 \times 2 \left( \frac{4}{11} \right) \times 7/8 \sim \mathbf{3.91}$$

comes from accounting for  $\gamma$ 's &  $\nu$ 's

$$\Omega_X h^2 = 2.74 \times 10^8 \left( \frac{M_X}{\text{GeV}} \right) Y_0$$

**[Main] Goal:** compute value of number to entropy density ratio,  $Y_0$

# IN ADDITION MUST BE SURE THAT DM...

**...also fulfills some basic requirements from astro/cosmo**

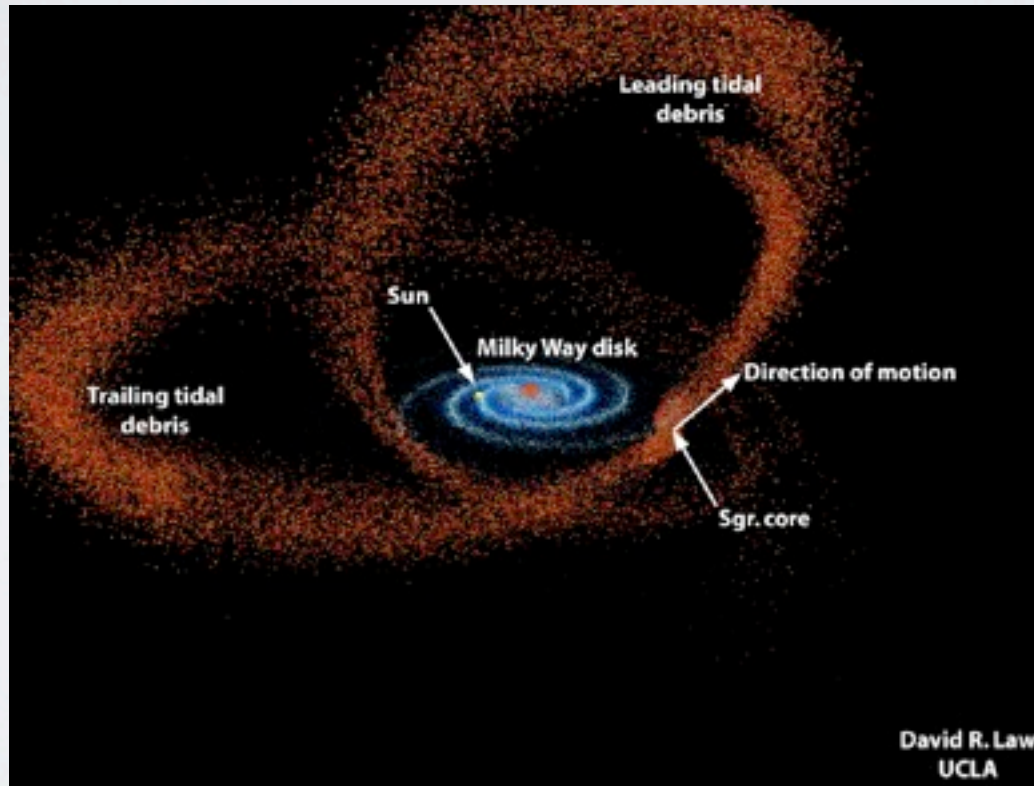
- ***Dark matter... is dark, and dissipationless***
- ***Dark matter is collisionless (or not very collisional)***
- ***Dark matter is smoothly distributed (at astrophysical scales)***
- ***Dark matter behaves as a classical fluid at astrophysical scales***
- ***Dark matter is not “hot” (non-relativistic velocity distribution)***

**Let's detail them one by one: they have more or less stringent particle physics implications**



# I) DM IS... DARK AND DISSIPATIONLESS

- DM must not couple “much” to photons (perturbation shape & amplitude argument, invisibility in e.m. channels...)
- DM forms extended, triaxial halos, while baryons “sink” in inner halo parts, form disks, etc. since they can dissipate energy by e.m. emission. At Galactic scale, evidence from tidal streams of satellite galaxies



e.g. D. R. Law, S. R. Majewski, K.V. Johnston, “Evidence for a Triaxial Milky Way Dark Matter Halo from the Sagittarius Stellar Tidal Stream” *Astrophys. J.* 703, L67 (2009)

## 2) DM IS... COLLISIONLESS (WRT BARYONIC GAS)

- if DM-DM interaction too strong, spherical structures would be obtained rather than triaxial. From actual clusters, one can derive  $\sigma/m < 0.02 \text{ cm}^2/\text{g}$

*Jordi Miralda-Escudé ApJ 564 60 (2002)*

**but different levels  
of robustness...**

- From Bullet cluster,  $\sigma/m < 0.7\text{-}1.3 \text{ cm}^2/\text{g}$ ,

*S. W. Randall et al. ApJ 679, 1173 (2008)*

- similar bounds from different arguments, for a compilation see e.g.

System	$v_0$ [km/s]	$\sigma/m_\chi$ [ $\text{cm}^2/\text{g}$ ]	References
Bullet Cluster	1000	1.25	[41, 43]
Galactic Evaporation	1000	0.3	[45]
Elliptic Cluster	1000	0.02	[46]
Dwarf Evaporation	100	0.1*	[45]
Black Hole	100	0.02*	[59]
Mean Free Path	44 – 2400	0.01 – 0.6	[57]
Dwarf Galaxies	10	0.1	[56]

*From M. R. Buckley and P. J. Fox,  
Phys. Rev. D 81, 083522 (2010)  
(\*= $v$ -dependent)*

- Very loose from particle physics standard (barn level!), but much less than atomic or molecular cross sections characteristic of gas.

$$\frac{\text{cm}^2}{\text{g}} = 1.78 \frac{\text{barn}}{\text{GeV}}$$

# NEW BOUNDS COMING OUT “EVERY DAY”...

Science 27 March 2015:  
Vol. 347 no. 6229 pp. 1462–1465  
DOI: 10.1126/science.1261381

[< Prev](#) | [Table of Contents](#) | [Next >](#)


 [Read Full Text to Comment \(0\)](#)

REPORT

## The nongravitational interactions of dark matter in colliding galaxy clusters

David Harvey<sup>1,2,\*</sup>, Richard Massey<sup>3</sup>, Thomas Kitching<sup>4</sup>, Andy Taylor<sup>2</sup>, Eric Tittley<sup>2</sup>

 [Author Affiliations](#)

 <sup>\*</sup>Corresponding author. E-mail: [david.harvey@epfl.ch](mailto:david.harvey@epfl.ch)

*arXiv:1503.07675*

ABSTRACT

EDITOR'S SUMMARY

Collisions between galaxy clusters provide a test of the nongravitational forces acting on dark matter. Dark matter's lack of deceleration in the “bullet cluster” collision constrained its self-interaction cross section  $\sigma_{\text{DM}}/m < 1.25$  square centimeters per gram ( $\text{cm}^2/\text{g}$ ) [68% confidence limit (CL)] ( $\sigma_{\text{DM}}$ , self-interaction cross section;  $m$ , unit mass of dark matter) for long-ranged forces. Using the Chandra and Hubble Space Telescopes, we have now observed 72 collisions, including both major and minor mergers. Combining these measurements statistically, we detect the existence of dark mass at  $7.6\sigma$  significance. The position of the dark mass has remained closely aligned within  $5.8 \pm 8.2$  kiloparsecs of associated stars, implying a self-interaction cross section  $\sigma_{\text{DM}}/m < 0.47 \text{ cm}^2/\text{g}$  (95% CL) and disfavoring some proposed extensions to the standard model.

### 3) DM IS... SMOOTHLY DISTRIBUTED

**DM has a “continuum” (fluid limit), rather than having “granular” structure.**

❖ Granular distribution would provide time-dependent gravitational potentials, disrupting bound systems of different sizes (function of “grain mass”)

- thickness of disks:  $M_X < 10^6 M_{\text{sun}}$   
satellites, globular clusters:  $M_X < 10^3 M_{\text{sun}}$
- Halo-wide binaries:  $M_X < 43 M_{\text{sun}}$

*H-W.Rix and G. Lake,  
astro-ph/9308022 & refs. therein*

*J.Yoo, J. Chaname and A. Gould,  
Astrophys.J. 601, 311 (2004)*

# 3) DM IS... SMOOTHLY DISTRIBUTED

**DM has a “continuum” (fluid limit), rather than having “granular” structure.**

❖ Granular distribution would provide time-dependent gravitational potentials, disrupting bound systems of different sizes (function of “grain mass”)

- thickness of disks:  $M_X < 10^6 M_{\text{sun}}$
- satellites, globular clusters:  $M_X < 10^3 M_{\text{sun}}$

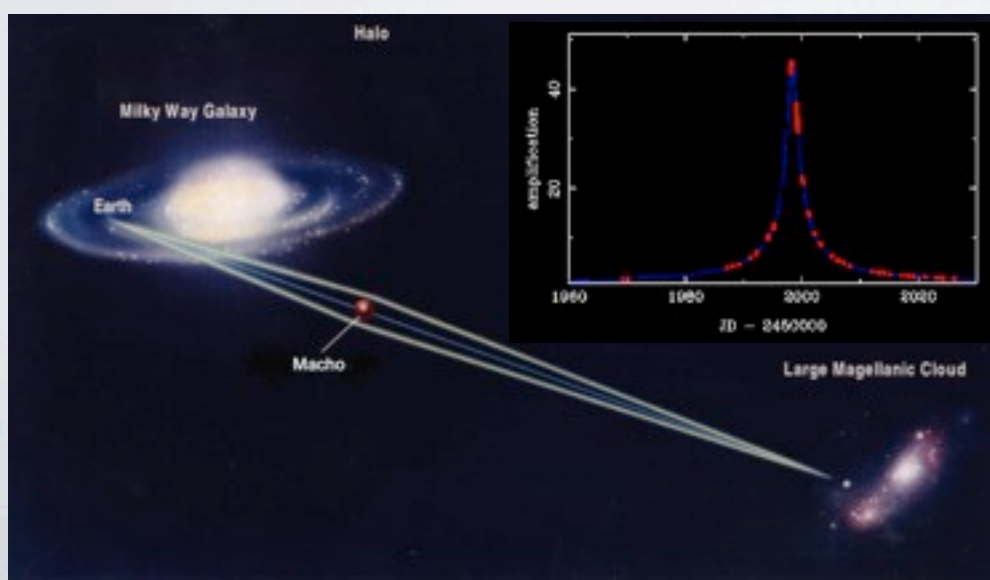
*H-W.Rix and G. Lake,  
astro-ph/9308022 & refs. therein*

- Halo-wide binaries:  $M_X < 43 M_{\text{sun}}$

*J.Yoo, J. Chaname and A. Gould,  
Astrophys.J. 601, 311 (2004)*

❖ Several searches (EROS, OGLE...) for  $\mu$ lensing events towards Magellanic Cloud exclude dominant MACHOs component as halo DM for  $10^{-7}$  to  $10 M_{\text{sun}}$

*e.g. L.Wyrzykowski et al.,  
arXiv:1106.2925 & refs. therein*



**idea:** constrain the frequency of a peculiar magnification pattern

$$A(u) = \frac{u^2 + 2}{u\sqrt{u^2 + 4}}$$

$$u = \theta/\theta_E$$

ang. distance source-lens

$$\theta_E = \sqrt{\frac{4GM}{c^2} \frac{d_S - d_L}{d_S d_L}}$$

depends on lens mass and Geometry

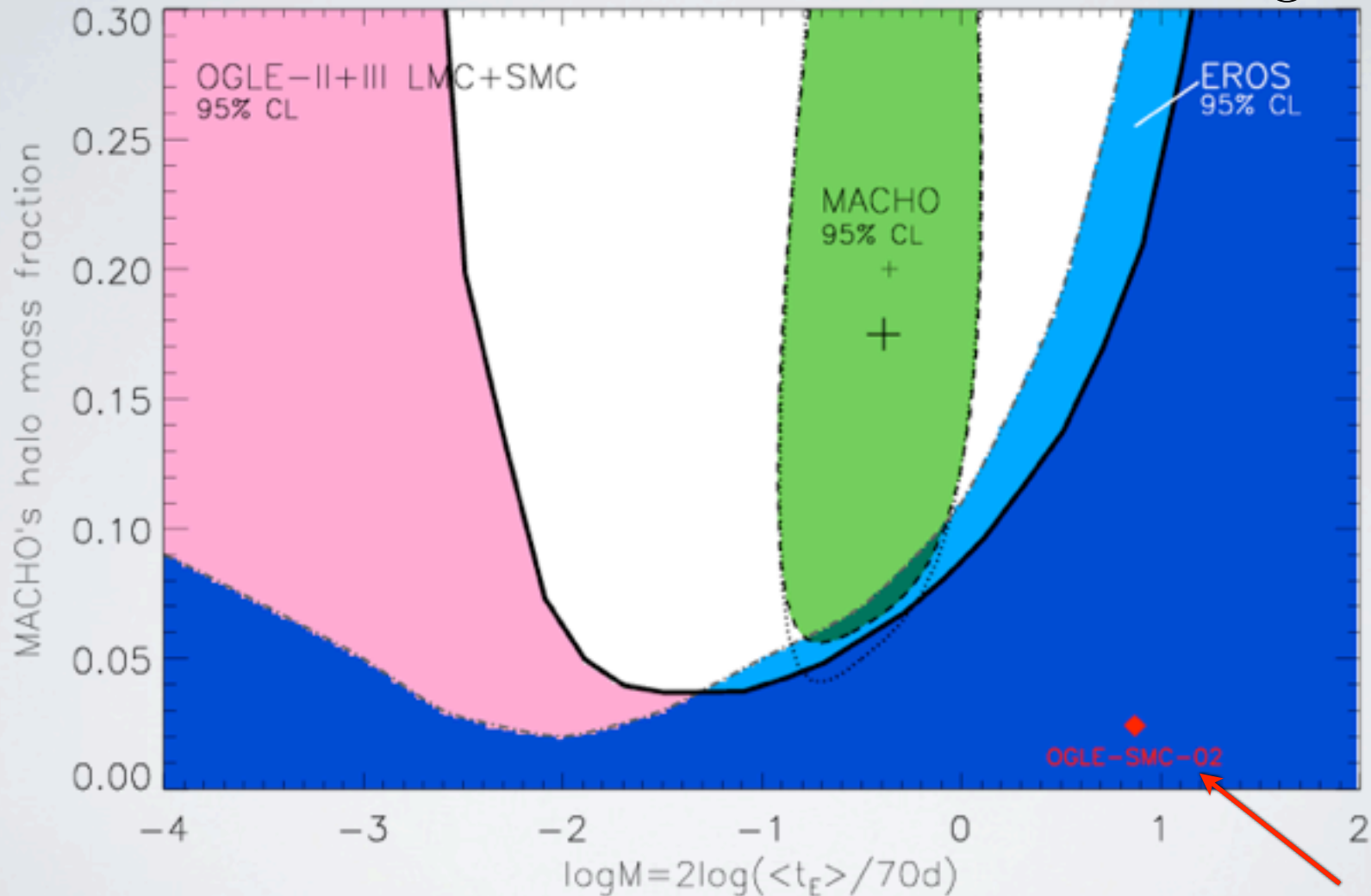
$$u(t) = \sqrt{u_{\text{min}}^2 + \left(\frac{t - t_0}{t_E}\right)^2}$$

$t_E$  = time to cross einstein angular size

# MICROLENSING CONSTRAINTS

← goes to  $\simeq 10^{26}$  g

$2 \times 10^{34}$  g



some events expected due to stellar BH

## 4) DM IS... CLASSICAL (AT GAL. SCALES, AT LEAST)

**dark matter is confined/detected at least at astrophysical scales, hence must be “localized” and behave classically there.**

$$\lambda_{De\ Broglie} = \frac{h}{m v} \lesssim \text{kpc} \implies m \gtrsim 10^{-22} \text{ eV} \quad (v \simeq 100 \text{ km/s})$$

## 4) DM IS... CLASSICAL (AT GAL. SCALES, AT LEAST)

**dark matter is confined/detected at least at astrophysical scales, hence must be “localized” and behave classically there.**

$$\lambda_{De\ Broglie} = \frac{h}{m v} \lesssim \text{kpc} \implies m \gtrsim 10^{-22} \text{ eV} \quad (v \simeq 100 \text{ km/s})$$

For *fermions* a much stronger bound holds, due to the fact that their quantum nature emerges more easily, so to speak, thanks to Pauli principle/Fermi-Dirac statistics

$$f \leq \frac{g}{h^3}$$

Conservation of phase space density of a non-interacting fluid (Liouville Eq.) + condition that any observable, coarse grained p.s. density must be lower than the real one, in turn lower than the above maximum, one derives

$$m > \mathcal{O}(10 - 100) \text{ eV}$$

*S.Tremaine and J. E. Gunn, Phys. Rev. Lett. 42, 407 (1979)*

updated lower limit around  $\sim 400 \text{ eV}$

*A. Boyarsky, O. Ruchayskiy and D. Iakubovskyi, JCAP 0903, 005 (2009)*



## 5) DM IS NOT “HOT” (IT IS NOT RELATIVISTIC)

***dark matter is not “hot”: cannot have a relativistic velocity distribution (at least from matter-radiation equality for perturbation to grow)***

# 5) DM IS NOT “HOT” (IT IS NOT RELATIVISTIC)

**dark matter is not “hot”:** cannot have a relativistic velocity distribution (at least from matter-radiation equality for perturbation to grow)

This is the more profound reason why neutrinos would not work as DM, even if they had the correct mass: they were born with relativistic velocity distribution which prevents structures below  $O(100 \text{ Mpc})$  to grow till late!

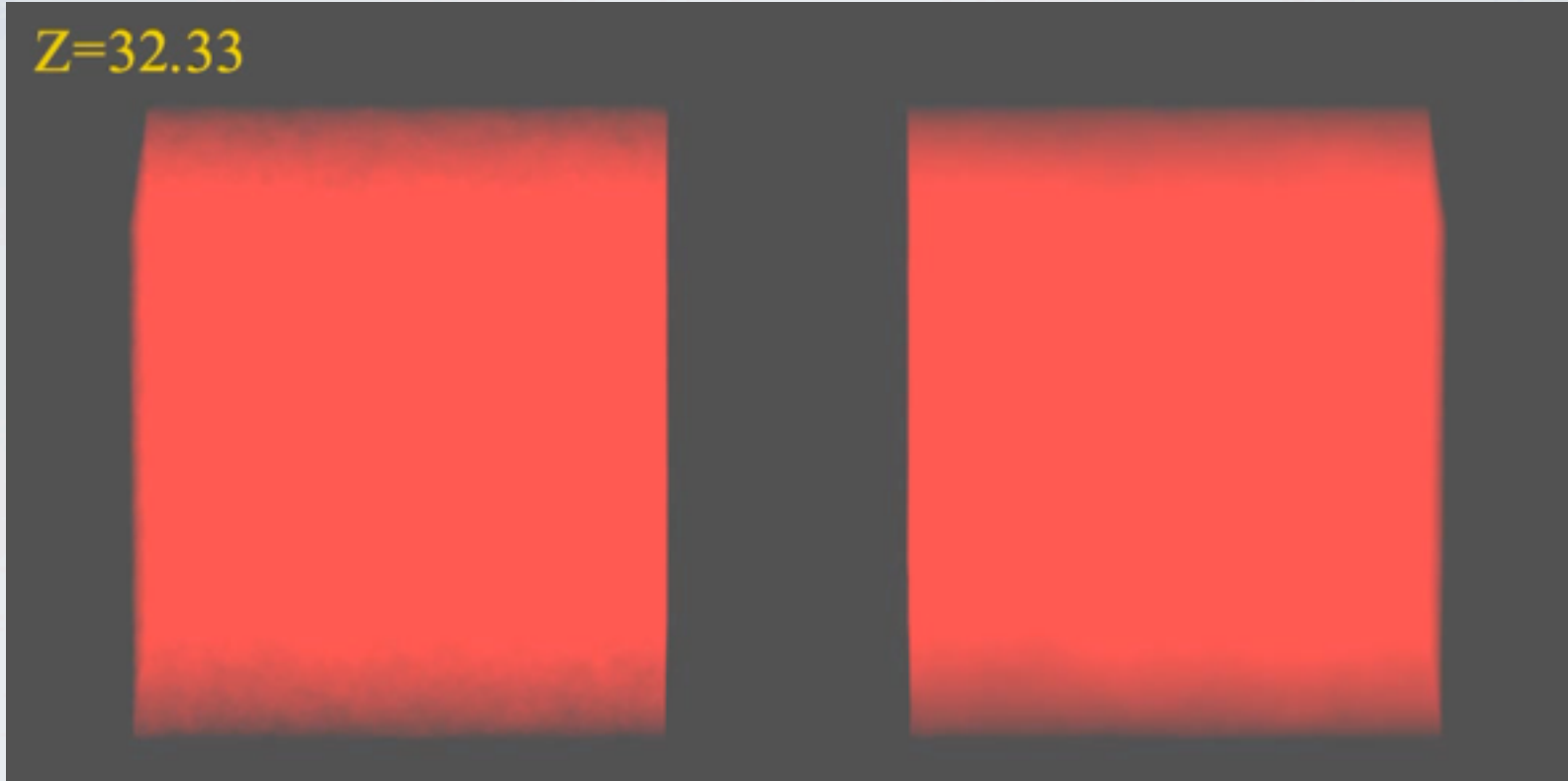


## Cartoon Picture:

$v$ 's “do not settle” in potential wells that they can overcome by their typical velocity: compared with CDM, they suppress power at small-scales

# THE NUMERICAL PROOF

$\Lambda$ CDM run vs. cosmology including neutrinos (total mass of 6.9 eV)



simulation by Troels Haugbølle, see

<http://users-phys.au.dk/haugboel/projects.shtml>

# SUMMARY OF WHAT WE LEARNED

- ❖ Numerous observations tell us that we need some degree of freedom, gravitating as ordinary matter but with otherwise suppressed couplings.
- ❖ It turns out that this requires **new physics**, with some specific properties. Justifies the enormous amount of attention particle physicists devote to it!
- ❖ Unfortunately, “gravity is universal” → it does not tell us what kind of physics it is.
- ❖ We need some “strategy” to identify what DM is. For that, first we need some extra input/constraint → must necessarily come from theory (at very least to conceive what we should be looking for!)
- ❖ Notice that I have not mentioned (yet!) neutralinos, nor “WIMPs”, these aspects belong to theoretical creativity... & prejudice. While defining some **theoretical context** (and will do it in **Lec. 2!**) is necessary to engage in **identification strategies** (see **Lec. 3**), please decouple the validation of specific particle physics theories (e.g. electroweak scale SUSY at the LHC) from the DM problem.