Astrophysical signals of dark matter:
II. Theoretical framework, WIMP freeze-out (& beyond)

Pasquale Dario Serpico

First ICTP Advanced School on Cosmology - 18-29/05/2016

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If "The Boss" said so...

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RECAP & PLAN

Recent determination (Planck 2015, 68% CL)

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[Main] Goal: *compute value of number to entropy density ratio, Y_0*

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[Main] Goal: *compute value of number to entropy density ratio, Y_0*

- We shall first provide a heuristic argument for the simplest (yet powerful!) toy-model evolution equation for Y
- We shall use this equation in different regimes to elucidate a couple of classes (not all!) of DM candidates
- We'll come back to sketch a "microscopic" derivation/interpretation of the equation we started with
- Some generalizations will be briefly discussed.

Caveat: *matching Ω_X is one condition for a good DM candidate, not the only one! Remember lecture I (collisionless, right properties for LSS structures...)*

BOLTZMANN EQ. FOR DM DENSITY CALCULATION

Assume that binary interactions of our particle *X* are present with species of the thermal bath

$$X\,X \leftrightarrow (\text{thermal bath particles})$$

If interaction rate **Γ=n σ v** very slow wrt Hubble rate **H**, # of particles conserved covariantly, i.e.

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The following equation has the right limiting behaviours

$$\frac{dn}{dt} + 3H\,n = -\langle\sigma v\rangle[n^2 - n_{\text{eq}}^2]$$

must be quadratic, for binary processes

for now, symbolic only

REWRITING IN TERMS OF Y AND x

$$\frac{dn}{dt} + 3H\,n = -\langle\sigma v\rangle[n^2 - n_{\rm eq}^2] \qquad \frac{dY}{dt} = -s\langle\sigma v\rangle[Y^2 - Y_{\rm eq}^2]$$

$$\frac{dY}{dt} = \frac{d}{dt}\left(\frac{n}{s}\right) = \frac{d}{dt}\left(\frac{n\,a^3}{s\,a^3}\right) = \frac{1}{s\,a^3}\frac{d}{dt}\left(n\,a^3\right) = \frac{1}{s\,a^3}\left(a^3\frac{dn}{dt} + 3a^2\dot{a}\,n\right) = \frac{1}{s}\left(\frac{dn}{dt} + 3H\,n\right)$$

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Define *x*=*m*/*T* (*m arbitrary mass, either* M_X *or not*); for an iso-entropic expansion one has

$$\frac{d}{dt}(a^3 s) = 0 \Longrightarrow \frac{d}{dt}(a\,T) = 0 \Longrightarrow \frac{d}{dt}(a/x) = \frac{\dot{a}}{x} - \frac{a}{x^2}\dot{x} = 0 \Longrightarrow \frac{dx}{dt} = H\,x$$

$$\frac{dY}{dx} = -\frac{x\,s\langle\sigma v\rangle}{H(T=m)}[Y^2 - Y_{\rm eq}^2]$$

radiation-dominated period

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radiation-dominated period

More in general (arbitrary *s(t)* and *H(t)*):

$$\frac{dY}{dx} = -\sqrt{45\pi}M_{\rm Pl}\,m\frac{h_{\rm eff}(x)\langle\sigma v\rangle}{\sqrt{g_{\rm eff}(x)}\,x^2}\left(1 - \frac{1}{3}\frac{d\log h_{\rm eff}}{d\log x}\right)(Y^2 - Y_{\rm eq}^2)$$

M. Srednicki, R. Watkins and K.A. Olive, "Calculations of Relic Densities in the Early Universe," Nucl. Phys. B 310, 693 (1988)

P. Gondolo and G. Gelmini, "Cosmic abundances of stable particles: Improved analysis," Nucl. Phys. B 360, 145 (1991).

FREEZE-OUT CONDITION

The previous equation is a *Riccati equation*: no closed form solution exist!

Approximate analytical solutions exist for different hypotheses/regimes

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For h_{eff} ~ const., we can re-write

$$\frac{x}{Y_{\rm eq}}\frac{dY}{dx} = -\frac{\Gamma_{\rm eq}}{H}\left[\left(\frac{Y}{Y_{\rm eq}}\right)^2 - 1\right] \quad \text{with} \quad \Gamma_{\rm eq} = \langle\sigma v\rangle n_{\rm eq}$$

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If Γ_{eq} >> H the particle starts from equilibrium condition at sufficiently small x (high-T), when relativistic. Crucial variable to determine the Y_{final} is the freeze-out epoch x_F from condition

$$\Gamma_{\rm eq}(x_F) = H(x_F)$$

RELATIVISTIC FREEZE-OUT

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If the solution to this condition yields x_F<<1, then *(Lecture 1)*

$$n = g\frac{\zeta(3)}{\pi^2}T^3 \times \left\{1(\text{B}), \frac{3}{4}(\text{F})\right\}$$

comoving abundance stays constant, and independent of *x (if dof do not change)*

$$Y(x_F) = 0.28\frac{g \times \{1(\text{B}), 3/4(\text{F})\}}{h_{\rm eff}(x_F)}$$

Today's abundance of such a relativistic freeze-out relic is thus

$$\Omega_X h^2 = 0.0762 \times \left(\frac{M_X}{\text{eV}}\right) \frac{g \times \{1(\text{B}), 3/4(\text{F})\}}{h_{\rm eff}(x_F)}$$

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For the neutrino case, $h_{\rm eff}$=10.75, g×{ }=3/2, thus

$$\Omega_\nu h^2 \simeq \frac{\sum m_\nu}{94\,\rm eV}$$

Inconsistent with DM for current upper limits!

NON-RELATIVISTIC FREEZE-OUT

to determine x_F

$$\Gamma_{\rm eq}(x_F) = H(x_F)$$

$$\frac{g\langle\sigma v\rangle}{(2\pi)^{3/2}} M_X^3\, x_F^{-3/2} e^{-x_F} = \sqrt{\frac{4\pi^3}{45} g_{\rm eff}}\, \frac{M_X^2}{x_F^2\, M_{\rm Pl}}$$

$$x_F^{1/2} e^{-x_F} = \sqrt{\frac{4\pi^3}{45} g_{\rm eff}}\, \frac{(2\pi)^{3/2}}{M_{\rm Pl} M_X\, g\,\langle\sigma v\rangle}$$

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Thus one obtains

$$Y(x_F) = \frac{n(x_F)}{s(x_F)} = \frac{g}{h_{\rm eff}} \frac{45}{2\pi^2 (2\pi)^{3/2}} x_F^{3/2} e^{-x_F}$$

which also writes

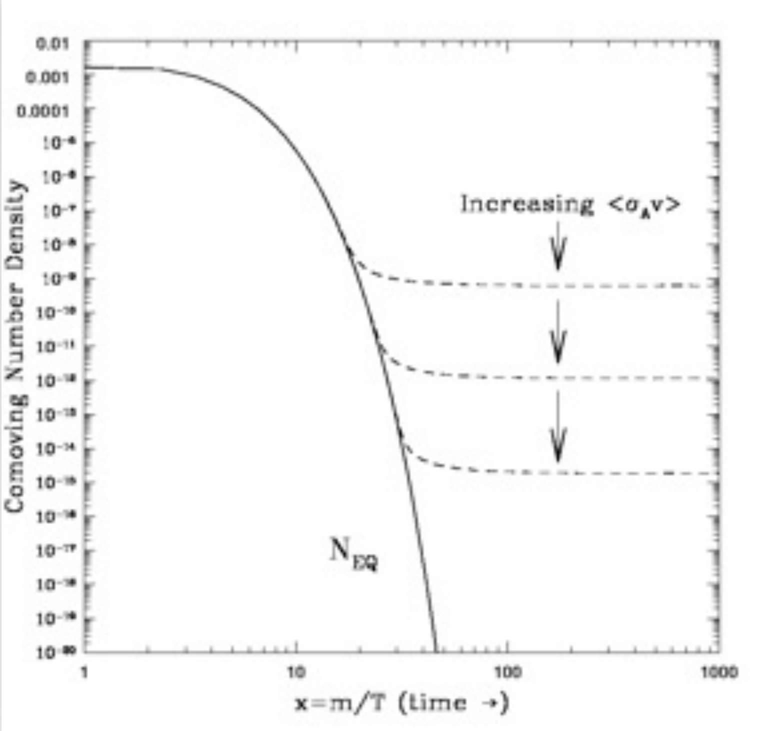
(Note the important result ***Y(x$_F$)~ 1/<σv>***)

$$Y(x_F) = \sqrt{\frac{45\, g_{\rm eff}}{\pi}} \frac{x_F}{h_{\rm eff} M_{\rm Pl}\, M_X\, \langle\sigma v\rangle} = \mathcal{O}(1) \frac{x_F}{M_{\rm Pl}\, M_X\, \langle\sigma v\rangle}$$

NON-RELATIVISTIC FREEZE-OUT: INTERPRETATION

$$Y(x_F) \simeq \mathcal{O}(1)\,\frac{x_F}{M_{\rm Pl}\, M_X\, \langle \sigma v \rangle}$$

makes sense, in the Boltzmann suppressed tail: The more it interacts, the later it decouples, the fewer particles around.



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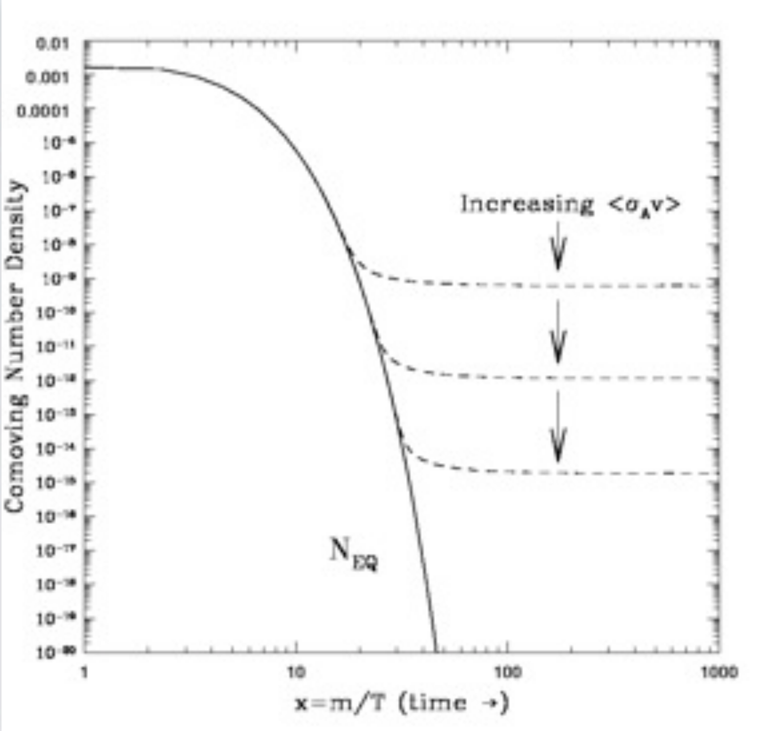

Also, plugging numbers (typically x_F~30), one has

$$\Longrightarrow \Omega_X h^2 \simeq \frac{0.1\,\text{pb}}{\langle\sigma v\rangle}$$

dimensionally, for electroweak scale masses and couplings, one gets the right value!

$$\langle\sigma v\rangle \sim \frac{\alpha^2}{m^2} \simeq 1\,\text{pb}\left(\frac{200\,\text{GeV}}{m}\right)^2$$

But the pre-factor depends from widely different cosmological parameters (Hubble parameter, CMB temperature) and the Planck scale. Is this match simply a coincidence?

Dubbed sometimes “Weakly Interacting Massive Particle” (WIMP) Miracle

EXERCISE

Apply the previous formalism to baryons, with
m_b~1 GeV & $\langle\sigma v\rangle \sim 1/m_\pi^2$
(*for the latter, you can also perform a more accurate search e.g. on PDB***)**

What is the current energy density of baryons?
(*$\Omega_b h^2 \sim 1/5\,\Omega_{CDM}\, h^2 \sim 0.02$, or look at recent Planck publication...***)**

Is freeze-out of a symmetric universe made of protons/ antiprotons a plausible mechanism behind their abundance?

CARE SHOULD BE TAKEN WHEN DEALING WITH...

- cohannihilations with other particle(s) close in mass
- resonant annihilations*
- thresholds*

K. Griest and D. Seckel,
"Three exceptions in the calculation of relic abundances,"
Phys. Rev. D 43, 3191 (1991).

* i.e., whenever $\boldsymbol{\sigma(s)}$ is a strongly varying function of the center-of-mass energy $\boldsymbol{s}$
(one recently popular example is the "Sommerfeld Enhancement")

For a pedagogical overview of generalization in presence of coannihilations (and decays), see

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Nowadays, relic density calculations have reached a certain degree of sophistication and are automatized with publicly available software. But if you have a theory with "unusual" features... better to check!

MicrOMEGAs: a code for the calculation of Dark Matter Properties including the relic density, direct and indirect rates in a general supersymmetric model and other models of New Physics

http://lapth.cnrs.fr/micromegas/

DarkSUSY

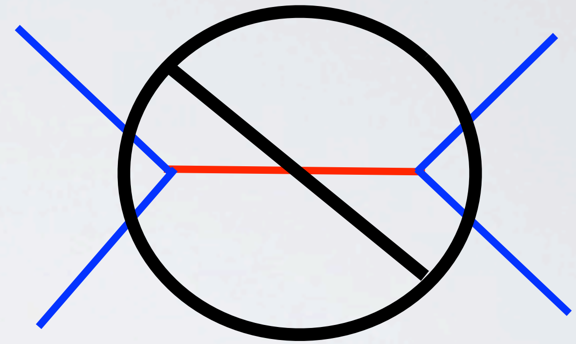
http://www.physto.se/~edsjo/darksusy/

LINK WITH COLLIDERS

- If one has a strong prior for new TeV scale physics (~with ew. strength coupling) due to the hierarchy problem, precision ew data (e.g. from LEP) suggest that tree-level couplings SM-SM-BSM should be avoided!

we want it!

we want to avoid!



- Straightforward solution (not unique!) is to impose a discrete "parity" symmetry e.g.: SUSY R-parity, K-parity in ED, T-parity in Little Higgs. New particles only appear in pairs!

➡ Automatically makes lightest new particle stable!
➡ May have other benefits (e.g. respect proton stability bounds...)

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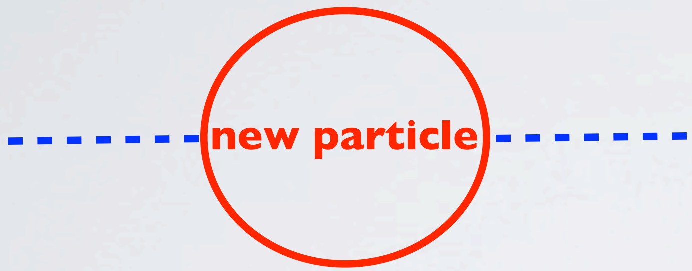

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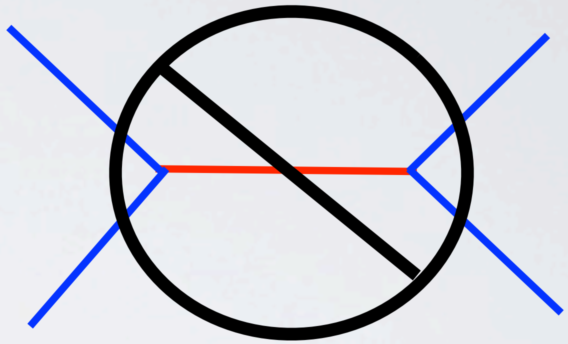

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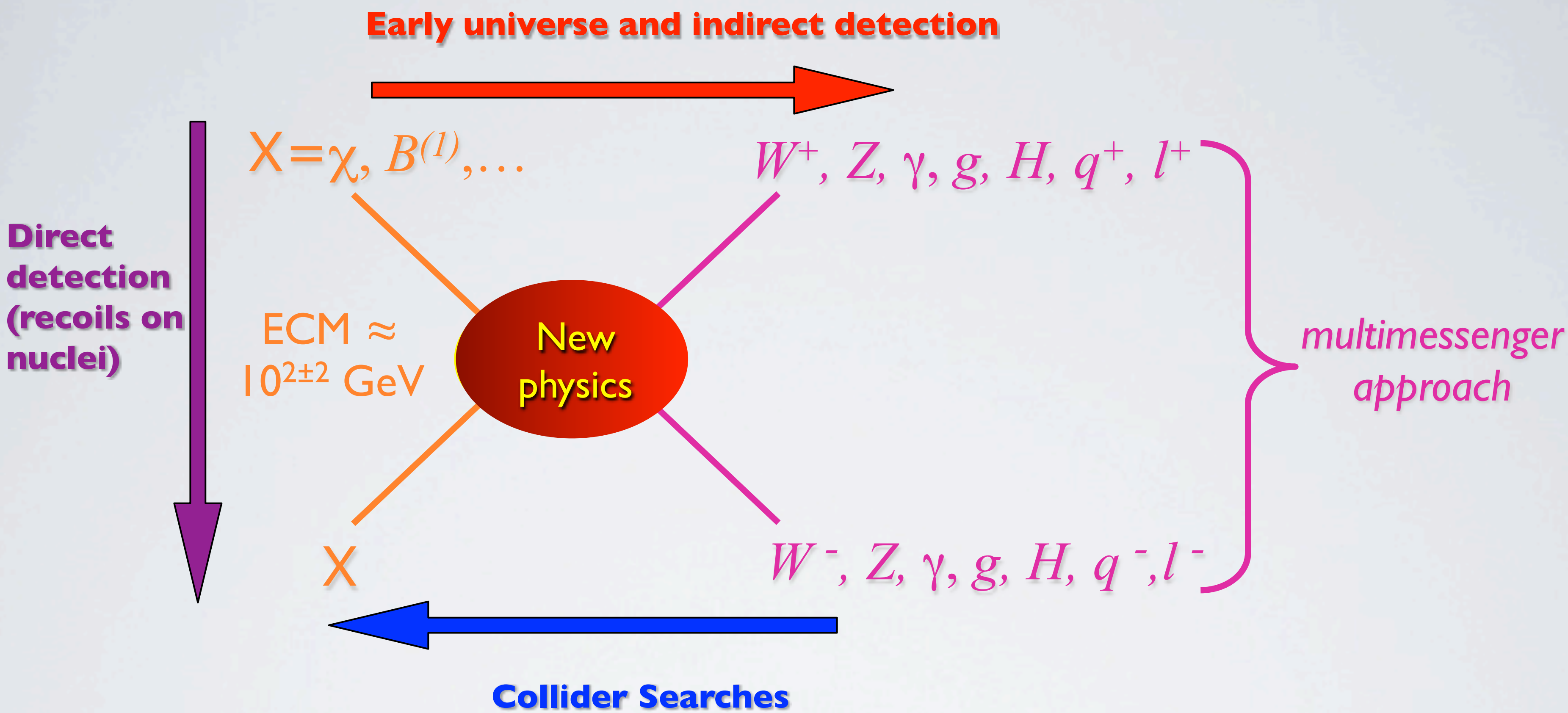
In a sense, some WIMP DM (too few? too much?) is "naturally" expected for consistency of the currently favored framework for BSM physics at EW scale.

Beware of the reverse induction:

LHC is now testing this paradigm, but if no new physics is found at EW scale it is at best the WIMP scenario to be disfavored, not the "existence of DM"

WIMP (NOT GENERIC DM!) SEARCH PROGRAM



- ✓ demonstrate that astrophysical DM is made of particles (locally, via DD; remotely, via ID)
- ✓ Possibly, create DM candidates in the controlled environments of accelerators
- ✓ Find a consistency between properties of the two classes of particles. Ideally, we would like to calculate abundance and DD/ID signatures → link with cosmology/test of production

FREEZE-IN

- We assumed that at small x ($T >> m$), RHS$\rightarrow$ 0, i.e. Y follows it's equilibrium value
- If, however, DM extremely weakly coupled, some production can take place via $ff \rightarrow XX$ but Y may never attain equilibrium. In this case:

$$\frac{dY}{dx} = \frac{x s(x)\langle\sigma v\rangle}{H(m)} Y_{\mathrm{eq},f}^2$$

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$$\frac{dY}{dx} = \frac{x s(x) \langle \sigma v \rangle}{H(m)} Y_{\mathrm{eq},f}^2$$

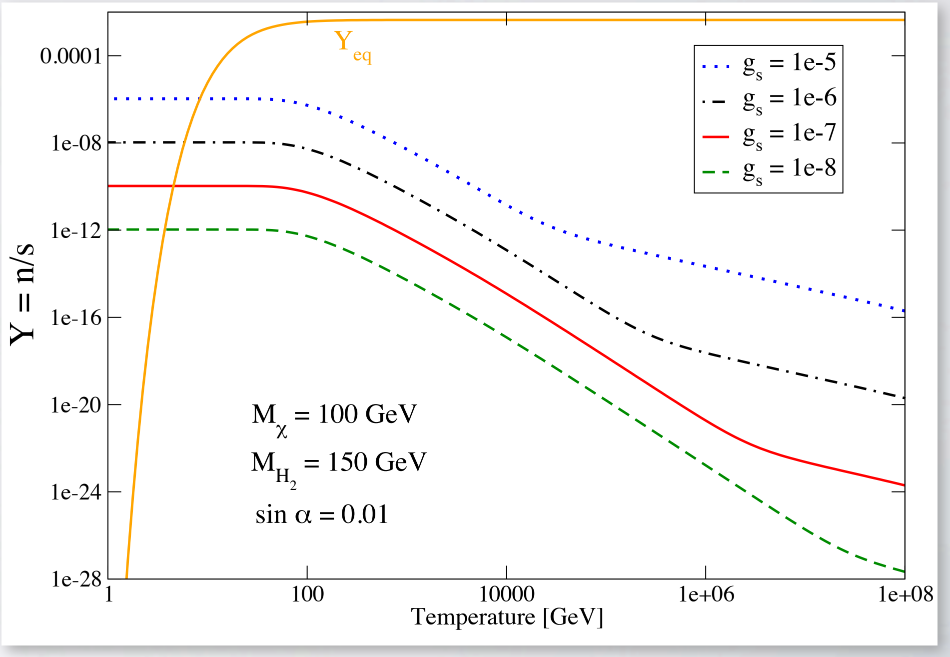
Assuming negligible initial abundance (otherwise it's not produced via freeze-in!)

$$Y_\infty \simeq \int_{x_0}^{\infty} dx' \frac{x' s(x') \langle \sigma v \rangle}{H(m)} Y_{\mathrm{eq},f}^2$$

Note that now

$$Y_\infty \propto \langle \sigma v \rangle$$

- Requires typically small couplings (harder to test...)
- It is more model dependent



M. Klasen and C. E. Yaguna, "Warm and cold fermionic dark matter via freeze-in," JCAP 1311, 039 (2013)

BOLTZMANN EQUATION

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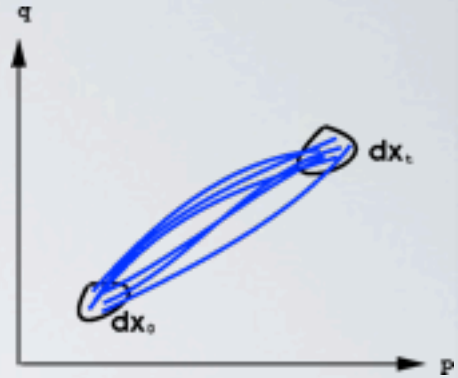
Start from Liouville equation for the phase-space distr. function *f*

along trajectories of hamiltonian flow

$$\frac{\partial f}{\partial t} + \dot{\mathbf{x}} \cdot \frac{\partial f}{\partial \mathbf{x}} + \dot{\mathbf{p}} \cdot \frac{\partial f}{\partial \mathbf{p}} = 0$$



In absence of collision, volume in phase space preserved, otherwise some non-vanishing RHS, depending on f-only under some assumption (molecular chaos...)

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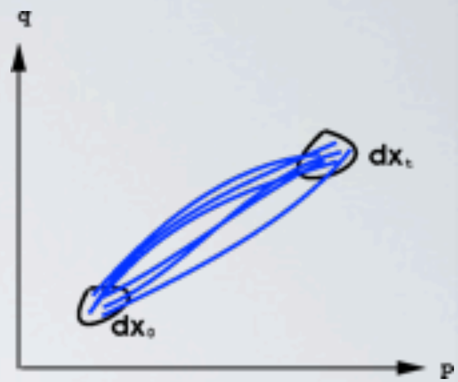

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Using the EOM, this is equivalent to:

$$m\frac{\partial f}{\partial t} + \mathbf{p} \cdot \frac{\partial f}{\partial \mathbf{x}} + \mathbf{F} \cdot \frac{\partial f}{\partial \mathbf{p}} = 0$$

which we can rewrite symbolically as (Liouville operator acting at the LHS)

$$\hat{L}[f] = \hat{C}[f]$$

At RHS, the Collisional operator accounts for sources or sinks of particles in phase space. Since these are typically quantum phenomena, most likely you rather encountered it written down in "relativistic/quantum realm" courses

BOLTZMANN EQUATION IN GR

In relativistic case, similar relation along geodesics

Liouville operator

$$\hat{L}[f] = \hat{C}[f]$$

Collisional operator

$$\hat{L}[f] = \frac{df}{d\lambda}(x^\mu(\lambda), p^\mu(\lambda))$$

in general, affine parameter λ to parametrize world-line

$$\hat{L}[f] = \frac{\partial f}{\partial x^\mu}\frac{d x^\mu}{d\lambda} + \frac{\partial f}{\partial p^\mu}\frac{d p^\mu}{d\lambda} = \hat{C}[f]$$

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Just like in classical theory the derivative of momentum is proportional to the "Force" (~ gradient of potential) in GR it can be expressed in terms of first-derivative of the metric $g_{\mu\nu}$, via the Christoffel symbols

$$\hat{L} \to p^\mu \frac{\partial}{\partial x^\mu} - p^\alpha p^\beta \Gamma^\mu_{\alpha\beta} \frac{\partial}{\partial p^\mu}$$

THE FORMULAE... JUST IN CASE

Dependence on gravitational background through affine connection

$$\Gamma^{\sigma}_{\lambda\mu} = \frac{1}{2}\left\{\frac{\partial g_{\mu\nu}}{\partial x^{\lambda}} + \frac{\partial g_{\lambda\nu}}{\partial x^{\mu}} - \frac{\partial g_{\mu\lambda}}{\partial x^{\nu}}\right\}$$

$$ds^2 = dt^2 - a^2(t)\left\{\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right\}$$

RW metric

non-zero terms:

$$\Gamma^{i}_{jk} = \frac{1}{2}h^{il}\left(\frac{\partial g_{lj}}{\partial x^{k}} + \frac{\partial g_{lk}}{\partial x^{j}} - \frac{\partial g_{jk}}{\partial x^{l}}\right)$$

$$\Gamma^{0}_{ij} = \frac{\dot{a}}{a}h_{ij} = Hh_{ij}$$

$$\Gamma^{i}_{0j} = \frac{\dot{a}}{a}\delta^{i}_{j}$$

$$h_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

BOLTZMANN EQUATION IN GR

thanks to homogeneity and isotropy in FLRW (cosmological principle)

$$f(x^\mu, p^\mu, t) = f(E, t) \qquad \hat{L} \to E\left(\frac{\partial}{\partial t} - \frac{\dot{a}}{a} p \frac{\partial}{\partial p}\right)$$

compare with the classical operator

$$\hat{L}[f] = m\frac{\partial f}{\partial t} + \mathbf{p} \cdot \frac{\partial f}{\partial \mathbf{x}} + \mathbf{F} \cdot \frac{\partial f}{\partial \mathbf{p}} = 0$$

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Now, let us take, for the specific case of FLRW metric:

$$\frac{\hat{L}[f]}{E} = \frac{\hat{C}[f]}{E}$$

And let's check that we obtain our "heuristic" equation for relic calculations, when we integrate over the energy.

This will also provide a "microscopic" expression for the C

LEFT-HAND SIDE...

Integrate over phase space

$$g\int\frac{d^3\vec{p}}{(2\pi)^3E}\hat{L}[f]=\frac{g}{(2\pi)^3}\int d^3\vec{p}\left(\frac{\partial f}{\partial t}-\frac{\dot{a}}{a}p\frac{\partial f}{\partial p}\right)=\frac{dn}{dt}+3H\,n=s\,\frac{dY}{dt}$$

recognize perhaps (twice) the relativistic invariant phase-space

integrate 2nd term by parts: *f* vanishes at boundary, deriving p^3 get factor 3...

$$Y\equiv n/s$$

where we introduced as customary the comoving density & entropy density

$$a^3\,s=const.$$

if relativistic d.o.f. do not change (isoentropic expansion)

$$\frac{dY}{dt}=\frac{d}{dt}\left(\frac{n}{s}\right)=\frac{d}{dt}\left(\frac{n\,a^3}{s\,a^3}\right)=\frac{1}{s\,a^3}\frac{d}{dt}\left(n\,a^3\right)=\frac{1}{s\,a^3}\left(a^3\frac{dn}{dt}+3a^2\dot{a}\,n\right)=\frac{1}{s}\left(\frac{dn}{dt}+3H\,n\right)$$

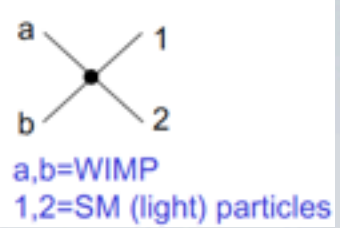
RIGHT-HAND SIDE...

$$\frac{g}{(2\pi)^3}\int \hat{C}[f_a]\frac{d^3\vec{p}_a}{E_a} =$$

assumes T-invariance

+ bosons
- fermions

$$d\Pi_a \equiv g_a \frac{d^3\vec{p}_a}{2E_a(2\pi)^3}$$



factor 1/2 to avoid double counting when we integrate over all momenta

$$-\int d\Pi_a d\Pi_b d\Pi_1 d\Pi_2 (2\pi)^4 \delta^{(4)}(p_a + p_b - p_1 - p_2)|\mathcal{M}|^2[f_a f_b(1\pm f_1)(1\pm f_2) - f_1 f_2 (1\pm f_a)(1\pm f_b)]$$

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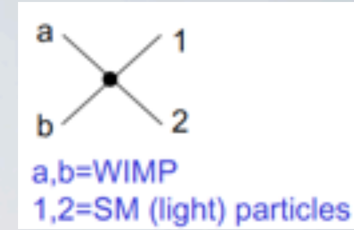

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$$\simeq -\int d\Pi_a d\Pi_b d\Pi_1 d\Pi_2 (2\pi)^4 \delta^{(4)}(p_a + p_b - p_1 - p_2)|\mathcal{M}|^2[f_a f_b - f_1 f_2]$$

~ ok for non-relativistic particles (in absence of bose cond. or degeneracy)

$$f_{1,2} = f_{1,2}^{\rm eq} \approx \exp(-E_{1,2}/T)$$

Thermal equilibrium & ~Maxwell-Boltzmann distributions

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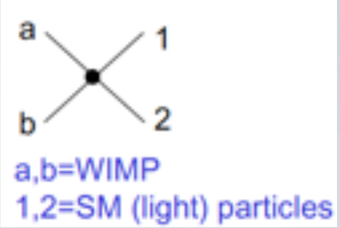

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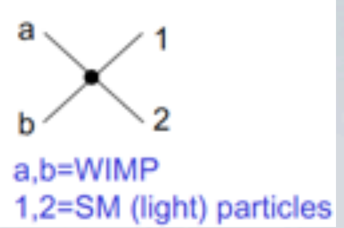

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Thermal equilibrium & ~Maxwell-Boltzmann distributions

$$f_1^{\rm eq} f_2^{\rm eq} = f_a^{\rm eq} f_b^{\rm eq}$$

detailed balance (enforces E-conservation)

thermally averaged annihilation cross section

$$\langle \sigma v\rangle \equiv \frac{1}{n_{\rm eq}^2}\int d\Pi_a d\Pi_b d\Pi_1 d\Pi_2 |\mathcal{M}|^2 (2\pi)^4 \delta^{(4)}(p_a + p_b - p_1 - p_2) f_a^{\rm eq} f_b^{\rm eq}$$

DO WE EVER NEED FULL BOLTZMANN EQ.?

I mean, apart from microscopic formula to compute relevant cross-sections?

Depending on the DM candidate, retaining the full dependence from the momentum can be crucial. Notable example: **sterile neutrinos**

We saw that neutrinos "almost work" as DM candidate.

A better candidate would:

- contribute more to energy density
- be "colder"

↓

Add a ***more massive neutrino with weaker than weak interaction***
(decouples earlier/more "non-relativistic")

PRELIMINARY: 1 SLIDE ON SEE-SAW...

Add at least 1 SM singlet, mixing with at least 1 active ν, plus its Majorana mass term

$$\delta\mathcal{L} = \bar{N} i \partial_\mu \gamma^\mu N - \lambda_\ell H \bar{N} L^\ell - \frac{M}{2} \bar{N}^c N + h.c.$$

after EW breaking can write mass matrix for L,R components in the compact form

$$\begin{pmatrix} 0 & \lambda_\ell v \\ \lambda_\ell v & M \end{pmatrix}$$

whose eigenvalues are $\mu_\pm = \dfrac{M \pm \sqrt{M^2 + 4\lambda_\ell^2 v^2}}{2}$

If $M \gg \lambda_\ell v$ ⟶

seesaw mechanism

$$\mu_+ \simeq M$$

$$\mu_- \simeq -\frac{(\lambda_\ell v)^2}{M}$$

DODELSON-WIDROW WARM STERILE NEUTRINO

S. Dodelson and L. M. Widrow, "Sterile-neutrinos as dark matter," PRL 72, 17 (1994) [hep-ph/9303287]

In the previous framework, for a small mixing and keV masses, say

$$\lambda_\ell v/M \sim 10^{-5} \qquad\qquad M \sim 10\,\text{keV}$$

The lightest active neutrino has sub-eV mass (Ok) and the "heavy" one is produced *via oscillations*, suppressed by the small mixing.

$$\left[\frac{\hat{C}}{E}\right] \sim \Gamma_{\text{int}} \sim \Gamma_w \times \theta^2$$

Remarkable that parameters can be chosen "right"!

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Remarkable that parameters can be chosen "right"!

$$\left(\frac{\partial}{\partial t} - HE\frac{\partial}{\partial E}\right) f_S(E,t) = \left[\frac{1}{2}\sin^2(2\theta_M(E,t))\,\Gamma(E,t)\right] f_A(E,t)$$

under some approx., one can compute the non-thermal spectrum analytically

$$\frac{f_S}{f_A} = \frac{7.7}{g_*^{1/2}}\left(\frac{\mu}{\mathrm{eV}}\right)^2\left(\frac{\mathrm{keV}}{M}\right)\, y\int_x^\infty \frac{dx'}{(1+y^2x'^2)^2}$$

$x \sim T^3/M$
$y \sim E/T$

EXTRA COMPLICATIONS & FEATURES

- The mixing matrix gets modified in the medium ("mixing in matter").
- The spectrum can be "quasi-thermal" or relatively far from equilibrium one. ν_s's are "relatively warmer" candidates, free-streaming length comparable with dwarf-Galaxies Jeans mass length: can suppress **non-linear** structures at sub-kpc scales
- With ν/anti-ν asymmetry, *resonant* production can happen (enhancement of lower-energy part) on their self-refraction potential. Corresponding DM "closer to cold DM".

X.-D. Shi and G. M. Fuller, "A New dark matter candidate: Nonthermal sterile neutrinos," PRL 82, 2832 (1999)

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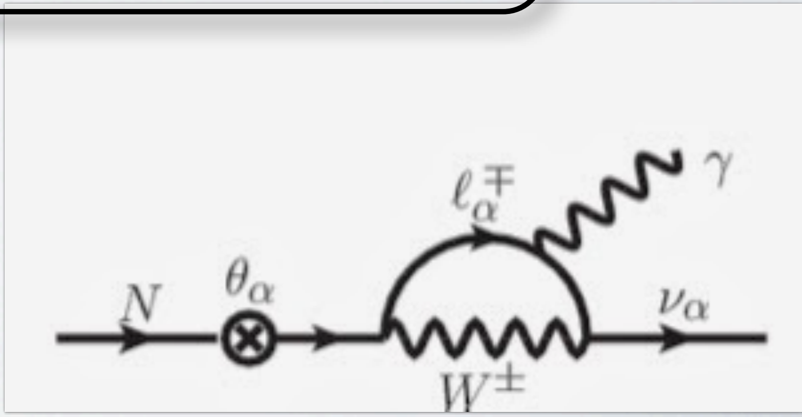
Some features:

- can be searched for via X-ray line (rare loop-suppressed decay)
- can be embedded in a "minimal extension" of the SM with 3 right-handed neutrinos (two GeV-ish ones explaining baryon asymmetry...)

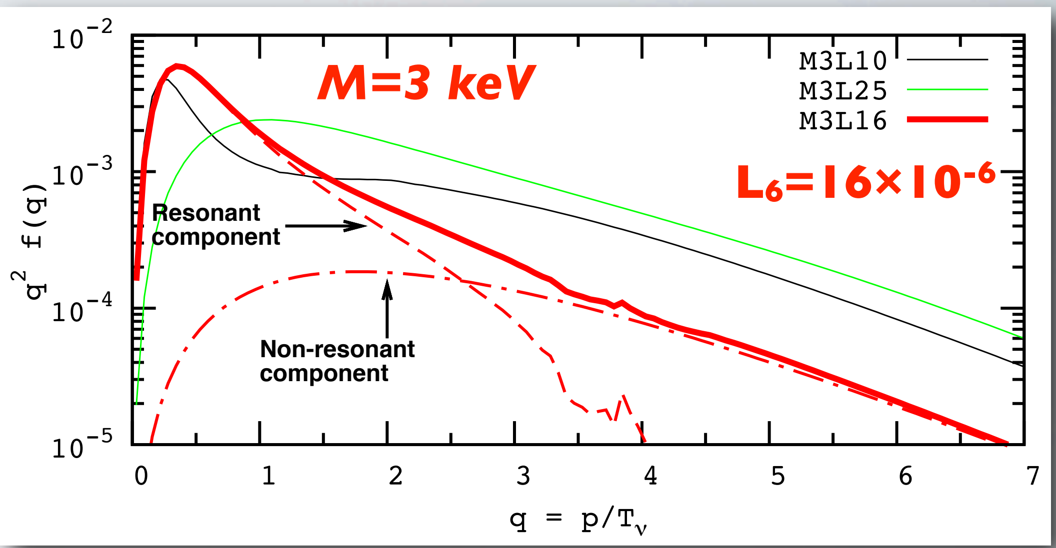
Note: no physics above the electroweak scale is required
(at least to address the DM problem, in this model...)

for a review, A. Boyarsky, O. Ruchayskiy and M. Shaposhnikov, Ann. Rev. Nucl. Part. Sci. 59, 191 (2009)

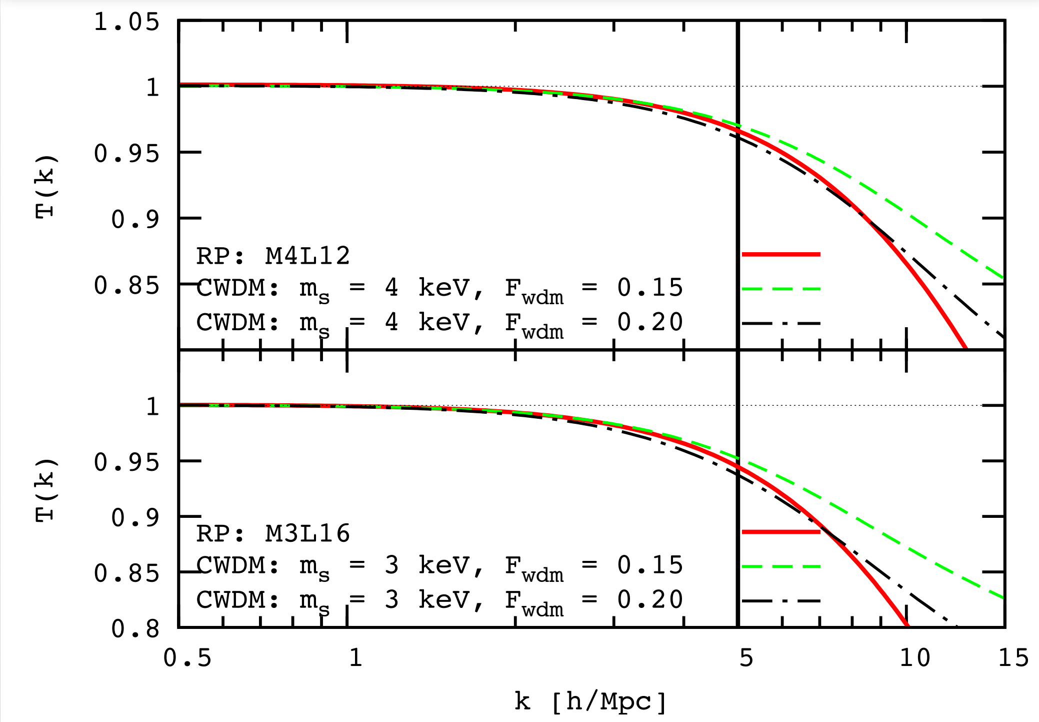


THOSE DETAILS MATTER!

A. Boyarsky, J. Lesgourgues, O. Ruchayskiy and M.Viel, "Realistic sterile neutrino dark matter with keV mass does not contradict cosmological bounds," PRL 102, 201304 (2009) [arXiv:0812.3256].



Momentum distribution should be calculated for different choices of particle parameters (mixing, asymmetry, mass...)



The *momentum shape* influences the *spatial* power-spectrum, again computed numerically.

$$T = \sqrt{P_{\nu_s}(k)/P_{\Lambda CDM}(k)}$$

Main feature: cutoff beyond some k ("free-streaming" effect)

EXERCISE: FREE-STREAMING LENGTH ESTIMATE

$$\lambda_{FS} = a(t) \int_{t_F}^{t} \frac{dt'}{a(t')} \sqrt{\langle v^2 \rangle}$$

Divide integral in pieces, with key times:

t_{NR}: time at which the particle becomes non relativistic, i.e. $3\,T_X \sim M_X$, before which $v \sim 1$; after that, it scales as $1/a$

t_{EQ}: time of matter-radiation equality, a(t) changes regime.

What comes first depends on the model details. If we assume **$t_{NR} < t_{EQ}$**

If I did not make mistakes:

$$\lambda_{FS}^{\rm com} = \frac{\lambda_{FS}}{a} = \frac{2\,c\,t_{NR}}{a_{NR}} \left[\frac{5}{2} + \log \frac{a_{\rm EQ}}{a_{NR}} \right]$$

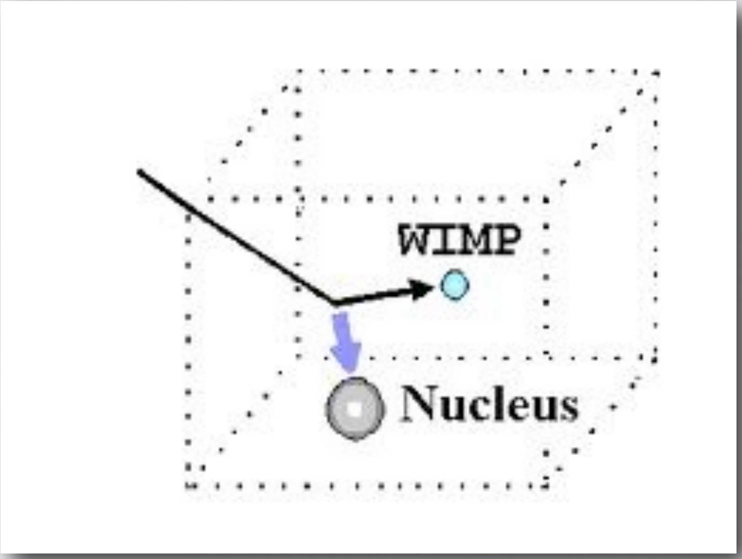
or, numerically:

$$\lambda_{FS}^{\rm com} = \frac{\lambda_{FS}}{a} \simeq {\rm Mpc} \left(\frac{\rm keV}{m_\nu} \right)$$

But one has a "mix" of species, actual observable is *P(k) ... one needs to solve Boltzmann eq.*

DIRECT DETECTION STRATEGY (FOR WIMPS!)

Strategy: measure recoil energy from elastic scattering of local DM WIMPs with detectors underground (to shield them from cosmic-rays & their induced "activation").



Observables:

- Rate and spectrum of the recoils (possibly different channels!)
- Time-dependence (modulation)
- Event Directionality (for future! At R&D stage, requires gaseous detectors...)

Issue: separate WIMP-induced recoils from backgrounds (radioactive, cosmic rays...)

KEY FORMULAS

J. D. Lewin and P. F. Smith, "Review of mathematics, numerical factors, and corrections for dark matter experiments based on elastic nuclear recoil," Astropart. Phys. 6, 87 (1996).

differential rate of events on a target containing N_T target nuclei given by

$$R \sim N_T \frac{\rho_X}{m_X} \sigma\, v$$

(check dimensionally! Same type of formula entering at the RHS of Boltzmann equation)

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in differential form, with respect to recoil energy E_R

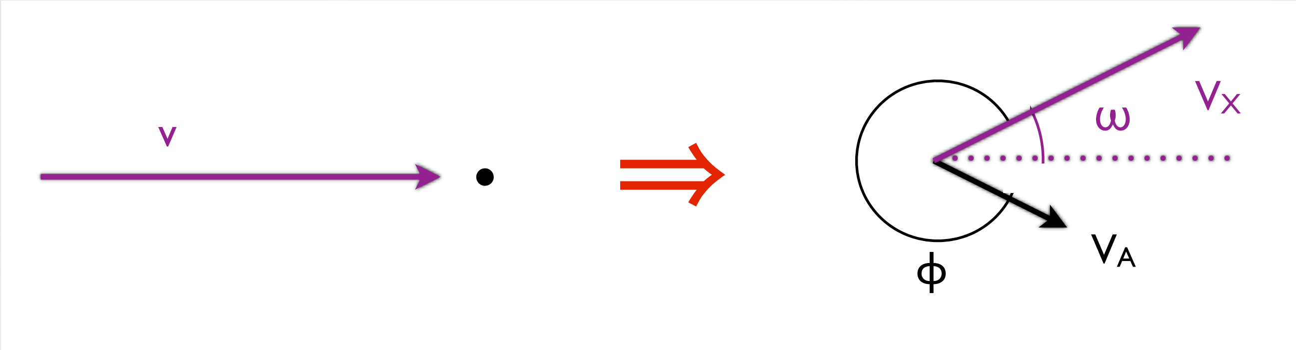
$$\frac{dR}{dE_R} = N_T \frac{\rho_X}{m_X} \int_{v_{min}}^{v_{max}} d^3\vec{v}\, f(\vec{v})\, |\vec{v}|\, \frac{d\sigma}{dE_R}$$

Need to know link between velocities and recoil energy, dependence of cross-section on the relevant variables, and specify v_{min}, v_{max}

CALCULATION OF RECOIL ENERGY - I

DM energy in the Lab $$T_X = \frac{1}{2} m_X v^2$$

DM momentum in the Lab $$p_X = m_X v$$



$$\frac{1}{2} m_X v^2 = \frac{1}{2} m_X V_X^2 + \frac{1}{2} m_A V_A^2$$
Energy conservation

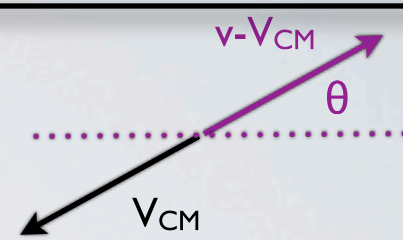
$$m_X v = m_X V_X \cos\omega + m_A V_A \cos\phi$$
momentum conservation (incoming DM direction)

$$0 = m_X V_X \sin\omega + m_A V_A \sin\phi$$
momentum conservation (perpendicular direction)

Since $V_X^2 \cos^2\omega + V_X^2 \sin^2\omega = V_X^2$ One immediately derives

$$E_R = \left(\frac{1}{2} m_X v^2\right) \frac{4 m_X m_A}{(m_X + m_A)^2} \cos^2\phi$$

CALCULATION OF RECOIL ENERGY - II



p-conservation gives CM velocity in the Lab

$$m_X(v - V_{CM}) = m_A V_{CM} \qquad V_{CM} = \frac{\mu_{XA}}{m_A} v$$

From the definition of ϕ

$$\tan\phi = \frac{V_A^y}{V_A^x}$$

But no boost takes place perpendicularly

$$V_A^y = W_A^y$$

while one has

$$V_A^x = W_A^x + V_{CM}$$

hence

$$\frac{W_A^y}{W_A^x + V_{CM}} = \frac{-V_{CM}\sin\theta}{-V_{CM}\cos\theta + V_{CM}}$$

$$\tan\phi = \frac{\sin\theta}{1 - \cos\theta} \qquad \tan\phi = \cot\theta/2 \qquad \phi = \left(\frac{\pi}{2} - \frac{\theta}{2}\right)$$

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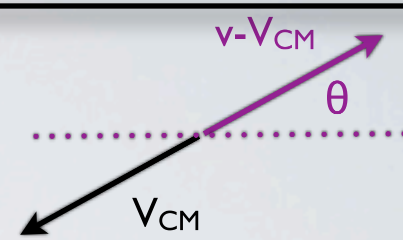

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Recoil energy related to the **WIMP kinetic energy**, **mass mismatch** and **relative angle of the recoil** (here param. by angle of WIMP outgoing vs incoming dir, θ)

$$E_R = \frac{1}{2} m_X v^2 \frac{4 m_X m_A}{(m_X + m_A)^2} \frac{1-\cos\theta}{2} = \frac{\mu_{XA}^2 v^2}{m_A}(1-\cos\theta) \leq \frac{2\mu_{XA}^2 v^2}{m_A} \equiv E_R^{\max}$$

Not larger than ~O(100) keV! Also implies $$\frac{dE_R}{d\cos\theta} = -\frac{E_R^{\max}}{2}$$

STANDARD EXPRESSION FOR SIGMA

In general, only simplification is that $d\sigma/dE_R$ describes a non-rel. scattering via 4-field operators (DM DM N N) which allows one to reduce it into a finite number of invariant structures and a handful of variables (exchanged momentum, spins...)

A. L. Fitzpatrick, W. Haxton, E. Katz, N. Lubbers, Y. Xu, "The Effective Field Theory of Dark Matter Direct Detection," JCAP 1302, 004 (2013) [arXiv:1203.3542]

Only a few of these terms present at leading orders, once spin of DM and nature of mediators is fixed. E.g., for SUSY neutralinos $d\sigma/dE_R$ ~ axial vector + scalar terms yielding respectively:

- spin-dep. term, depends on nuclear spin, only nucleon(s) outside complete shells matter
- spin-indep. term, often dominant: ~A^2 times bigger for coherence, vs A for incoherent sum

Assuming negligible cross section anisotropy *[ok, suppressed by $(v/c)^2 \sim 10^{-6}$]*, we can rewrite σ in the "point-like" target limit (~A^2 times the one with nucleon)

$$\frac{d\sigma}{dE_R} = \frac{d\sigma}{d\cos\theta}\frac{d\cos\theta}{dE_R} = \frac{\sigma}{2}\frac{2}{E_R^{max}}$$

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form-factor correction needed for large nuclei/large E, for which the WIMP "resolves" the nucleus

$$\lambda \sim (m_N\, E_R)^{-1/2} \lesssim R_A \simeq 1.2 A^{1/3}\,\mathrm{fm}$$

EMERGENCE OF THE FORM FACTOR

Scattering amplitude: Born approximation $\mathbf{q} = \mathbf{k}' - \mathbf{k}$

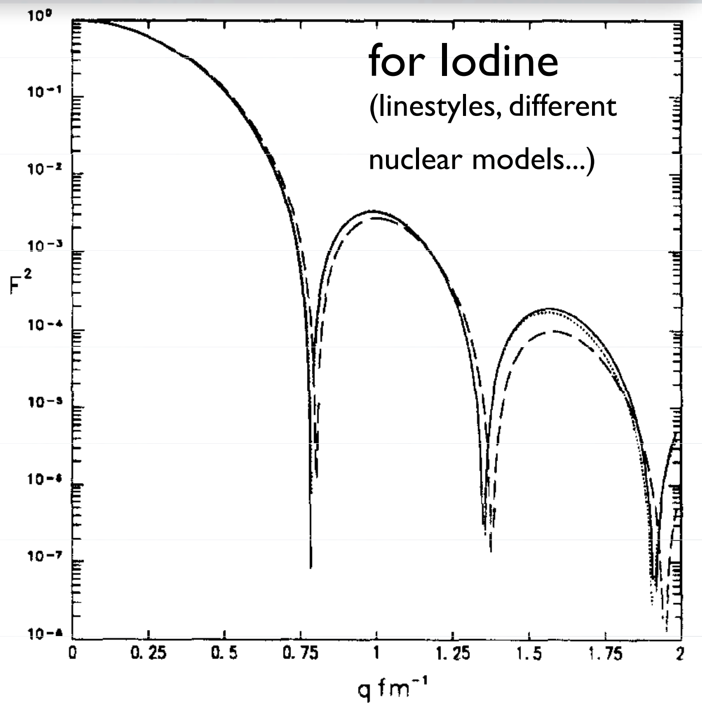
Spin-independent scattering is coherent over $\lambda = \hbar/q \sim \text{few fm}$

$$M(q) = f_N A \int d^3x \rho(\mathbf{x}) e^{i\mathbf{q}\cdot\mathbf{x}}$$

fundamental coupling to nucleon → f_N

mass number → A

mass profile of nucleus (norm. to 1) → $\rho(\mathbf{x})$



For large nuclei, DM "resolves" partially the nucleus, and the coherence is only partial...

$$\sigma \propto |M(q)|^2 \propto A^2 F^2(q)$$

STANDARD EXPRESSION FOR RATE

Apart for "nuclear complications", one ends up with "standard" expression

$$\frac{dR}{dE_R} = N_T \frac{\rho_X}{m_X} \frac{\sigma\, m_A}{2\,\mu_{AX}^2} \mathcal{I}(v_{min})$$

with $$\mathcal{I}(v_{min}) \equiv \int_{v_{min}} d^3\vec{v}\, \frac{f(\vec{v})}{v}$$

contains all "astrophysical" dependence from the velocity distribution

$$v_{min} = \sqrt{\frac{E_R\, m_A}{2\,\mu_{XA}}}$$

The role of v_{min} is especially important close to threshold

Upper end dominated by escape velocity (truncates f(v)), itself a complicated, global function of the halo potential ...

If distribution *f* is assumed "maxwellian", this allows one to understand the typical exclusion plots shapes

UNDERSTANDING DD EXCLUSION PLOTS

- For a given bound on the rates, the exclusion curve in the m_X-σ plane follows from simple considerations.

$$\frac{dR}{dE_R} = N_T \frac{\rho_X}{m_X} \frac{\sigma\, m_A}{2\,\mu_{AX}^2} \mathcal{I}(v_{min})$$

- Note that for

$$f(\vec{v}) \propto e^{-v^2/V^2} \Longrightarrow \mathcal{I}(v_{min}) \propto e^{-v_{min}^2/V^2}$$

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$$\frac{dR}{dE_R} = N_T \frac{\rho_X}{m_X} \frac{\sigma\, m_A}{2\,\mu_{AX}^2} \mathcal{I}(v_{min})$$

- Note that for

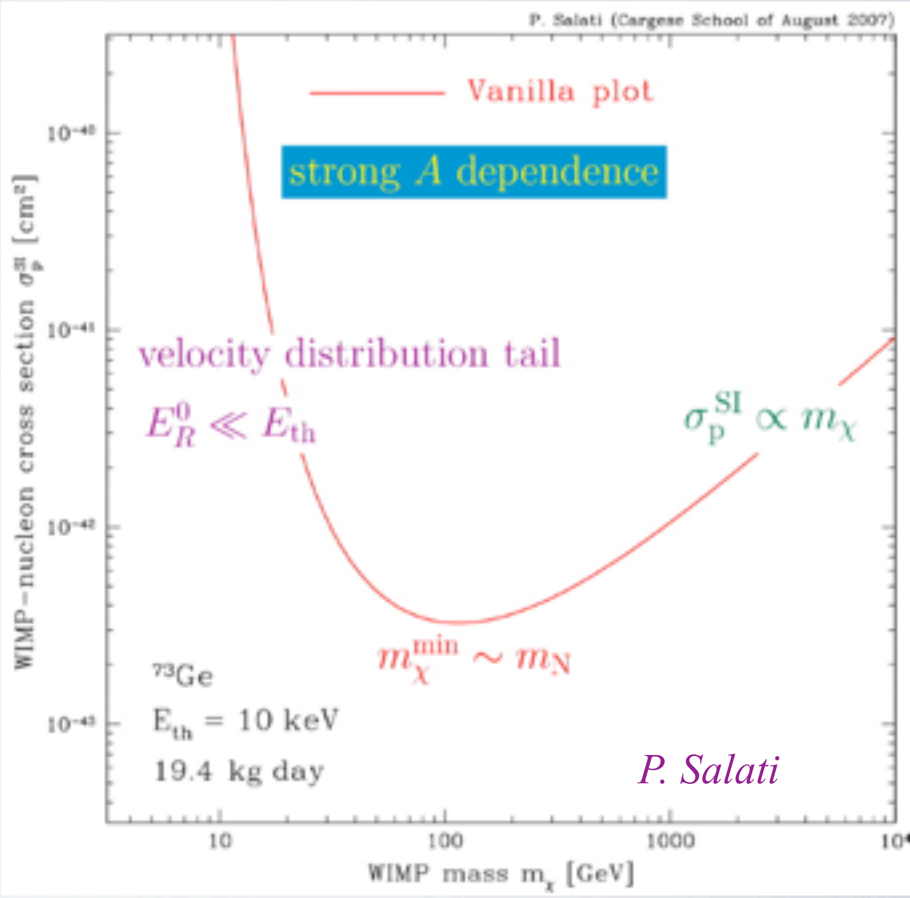
$$f(\vec{v}) \propto e^{-v^2/V^2} \Longrightarrow \mathcal{I}(v_{min}) \propto e^{-v_{min}^2/V^2}$$

$$v_{min} = \sqrt{\frac{E_R\, m_A}{2\,\mu_{XA}}}$$

- At very large masses, the above integral is ~indep. of the mass. The mass sensitivity depends on the prefactor, hence the excluded curve follows $\sigma \propto m_X$

- At small masses, the expected rate exponentially decreases as $\exp(-1/m_X)$

- Peak of sensitivity ~ target mass



COMPLICATION 1: FROM PARTONS TO NUCLEI

Difficulty: Theory provides (in the best case...) WIMP-parton couplings and amplitudes. In order to compare with experiments, one need WIMP-nucleon amplitudes! We need to know the values of the quark currents inside the nucleon...

example:

$$\sigma^{SI} = \frac{4\,\mu^2_{XA}}{\pi}[\lambda_p\,Z + \lambda_n(A-Z)]^2$$

effective coupling with nucleons expressed in terms of the coupling with quarks, λ^q

$$\frac{\lambda_N}{m_N} = \sum_{q=1,6} f_q^N \frac{\lambda^q}{m_q}$$

The proportionality coefficient is the contribution of quark q to the nucleon mass, m_N

$$m_N\,f_q^N = \langle N|m_q\,\bar{q}\,q|N\rangle \equiv m_q\,B_q^N$$

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A contribution from heavy quarks (c,b,t) is induced via gluon exchange with the nucleon

$$f_Q^N = \frac{2}{27}\left(1 - \sum_{q=u,d,s} f_q^N\right)$$

Light quark contributions deduced from nuclear/hadronic physics and/or lattice QCD

$$f_d^p = 0.033, \quad f_u^p = 0.023, \quad f_s^p = 0.26$$
$$f_d^n = 0.042, \quad f_u^n = 0.018, \quad f_s^n = 0.26$$

Larger than 50% uncertainty due to error on the "strange content of the nucleon"!

fiducial values in MicrOMEGAs, G. Belanger et al. 0803.2360

COMPL. II: VELOCITY DISTRIBUTION FUNCTION

In the **galactic frame,** WIMPs are usually assumed to be statistically at rest and with a Maxwellian distribution

$$f_G(\vec{v}_G) = \left(\frac{3}{2\pi v_{rms}^2}\right)^{3/2} \exp\left(-\frac{3\, v_G^2}{2\, v_{rms}^2}\right) d^3\vec{v}_G$$

Why DM has a thermal-like distribution if it's non-interacting?

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Why DM has a thermal-like distribution if it's non-interacting?

Heuristic view: "Violent relaxation" paradigm

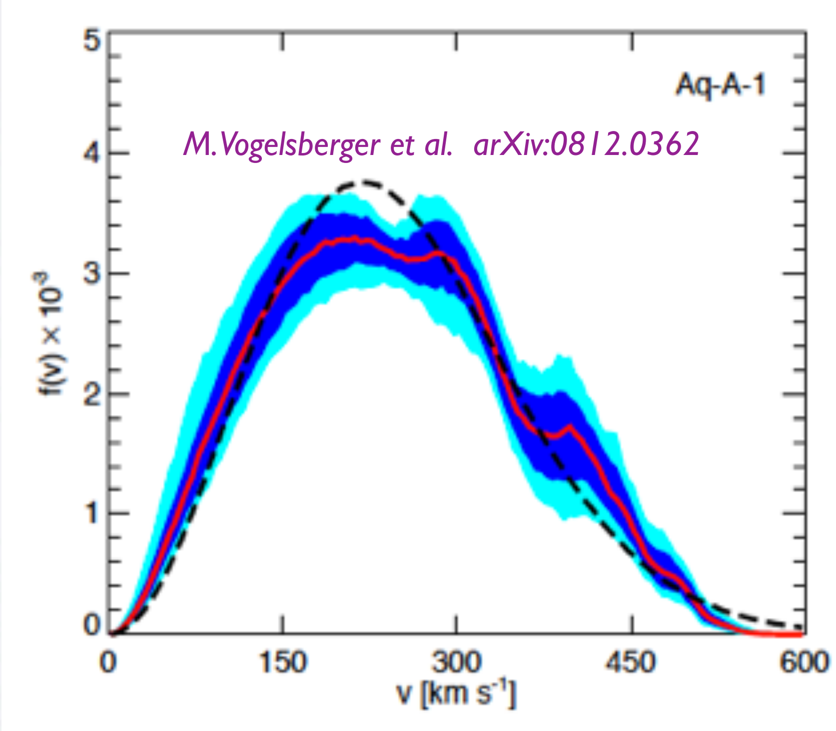
Lynden-Bell, Mon.Not.Roy.Astron.Soc. 136 (1967) 101:

sudden variations of the potential (like mergers) lead to fast mixing of the phase-space elements *coarse grained f* into highest-entropy configuration (at fixed energy) ~ Maxwell-Boltzmann-like

- Much later, this has been *roughly* confirmed by N-body simulations: loosely fit by a multivariate gaussian distribution in v_i^2: deviations remain due to the *assembly history* of the halo (non-deterministic, irreducible "uncertainty")

- Alternatively, *f(v)* can be linked to "observable" DM density distribution under some symmetry conditions

For interested students, see e.g. Binney & Tremaine's book (Galactic Dynamics)



INFERRING THE VELOCITY DISTRIBUTION FUNCTION

Would need inverting the eq. below, evaluating *F* at solar position

$$\rho_{DM}(\vec{r}) = m_{DM} \int d^3\vec{v}\, F(\vec{r}, \vec{v})$$

symmetry conditions needed to bypass degeneracies!

For example, under spherical approximation DM density inferred from rotation curve

$$v_{rot}^2(r) = \frac{G_N}{r} \int_0^r d\xi\, 4\pi\, \xi^2 [\rho_{vis}(\xi) + \rho_{DM}(\xi)]$$

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Jean's theorem: steady-state solutions of collisionless Boltzmann Eq. (Vlasov) depend on phase space only through integrals of motion I_j ↔ $F(I_j)$ is a solution of CBE

In the spherical approx, *F=F(E)*. Introduce a constant energy scale ϕ_0 and 2 new variables (relative potential and relative energy, redefinition of zero point energy)

$\psi = -\phi + \phi_0$
$\varepsilon = -E + \phi_0$

in such a way that:
$\varepsilon \leq 0$, $\varepsilon = 0$ when $v \to \infty$: $\varepsilon = \psi - 1/2\, v^2$

$m_{DM} = 1$

The potential ψ is a monotonic function of *r*, so it is possible to express ρ as a function of ψ, $\rho(\psi)$, invertible

EDDINGTON EQUATION

By analytical manipulations (see BT) one arrives at **Eddington's equation**

$$F(\epsilon) = \frac{1}{\sqrt{8}\pi^2} = \frac{d}{d\epsilon}\int_0^\epsilon \frac{d\rho}{d\psi}\frac{d\psi}{\sqrt{\epsilon - \psi}}$$

From the density profile input one may obtain the velocity distribution function... Provided one knows how to get ρ(ψ)!!!

Note: it is not necessarily true that a density profile ρ*(r)* is associated to a valid *F(ε)>0* (not every density profile is actually consistent to a steady-state solution)!

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Example: isothermal sphere

$$\rho(r) = \rho_0 (r_0/r)^2$$

suggested by rotation curves when DM dominates

by plugging into Poisson eq.

$$\nabla^2\psi = \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\psi}{dr}\right) = -4\pi G_N\,\rho(r)$$

$$\psi = \sigma^2 \log\left(\frac{\rho}{\rho_0}\right)$$

i.e., inverting:

$$\rho(\psi) = \rho_0\, e^{\frac{\psi}{\sigma^2}}$$

EMERGENCE OF THE MAXWELLIAN

Plugging the above solution $\rho(\psi)$ into Eddington's equation

$$F(\epsilon) = \frac{\rho_0}{\sqrt{8}\pi^2\sigma^2}\frac{d}{d\epsilon}\int_0^{\epsilon} e^{\psi/\sigma^2}\frac{d\psi}{\sqrt{\epsilon-\psi}}$$

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$$t = \sqrt{\epsilon - \psi}, \;\; d\psi = -2\,t\,dt$$

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For reference, note that

$$\sigma^2 = \frac{3}{2} v_{rot}^2$$

$$v_{rot} \simeq 220\,\mathrm{km/s} \qquad \sigma = v_{rms} \simeq 270\,\mathrm{km/s}$$

V DISTRIBUTION FUNCTION... AT THE EARTH

We really need $f(\vec{v})$ i.e. distribution of WIMP particles at the Earth, as function of their velocity wrt the Earth

$$\vec{v} = \vec{v}_G - \vec{w}$$

$\leftarrow$ velocity of Earth wrt DM halo (Sun in halo+Earth wrt Sun)

In terms of auxiliary variables $x^2 = \frac{3\,v^2}{2\,v_{rms}^2}$ $\eta^2 = \frac{3\,w^2}{2\,v_{rms}^2}$ $erf(x) \equiv \frac{2}{\sqrt{\pi}} \int_0^x e^{-\xi^2} d\xi$

one can write $\mathcal{I}(v_{min}) = \sqrt{\frac{3}{8}} \frac{1}{v_{min}\eta} [erf(x_{min} + \eta) - erf(x_{min} - \eta)]$

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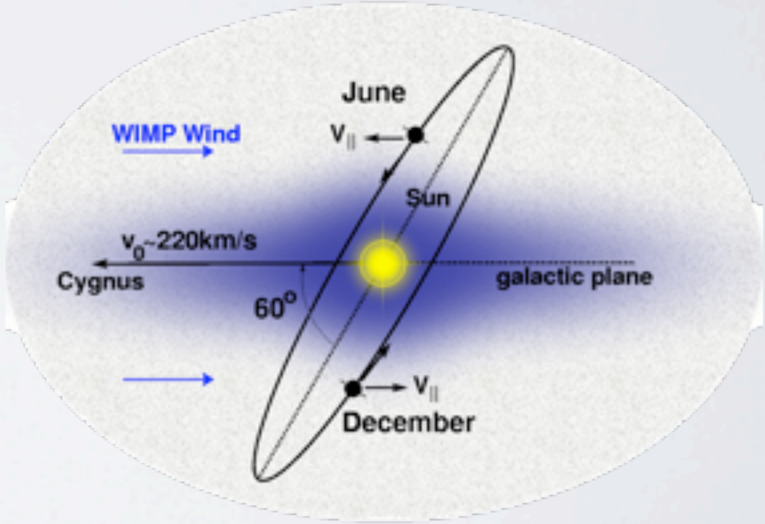
Earth rotation around the Sun causes a **modulation**

$$\eta = \eta_0 + \Delta\eta \cos[\omega(t - t_0)]$$

$$\eta_0 \simeq 1 \quad \Delta\eta \simeq 0.07$$

$$\omega = (2\pi/365)\,\mathrm{day}^{-1}$$

$$t_0 = \mathrm{day}\,156 \simeq 2\,\mathrm{June}$$

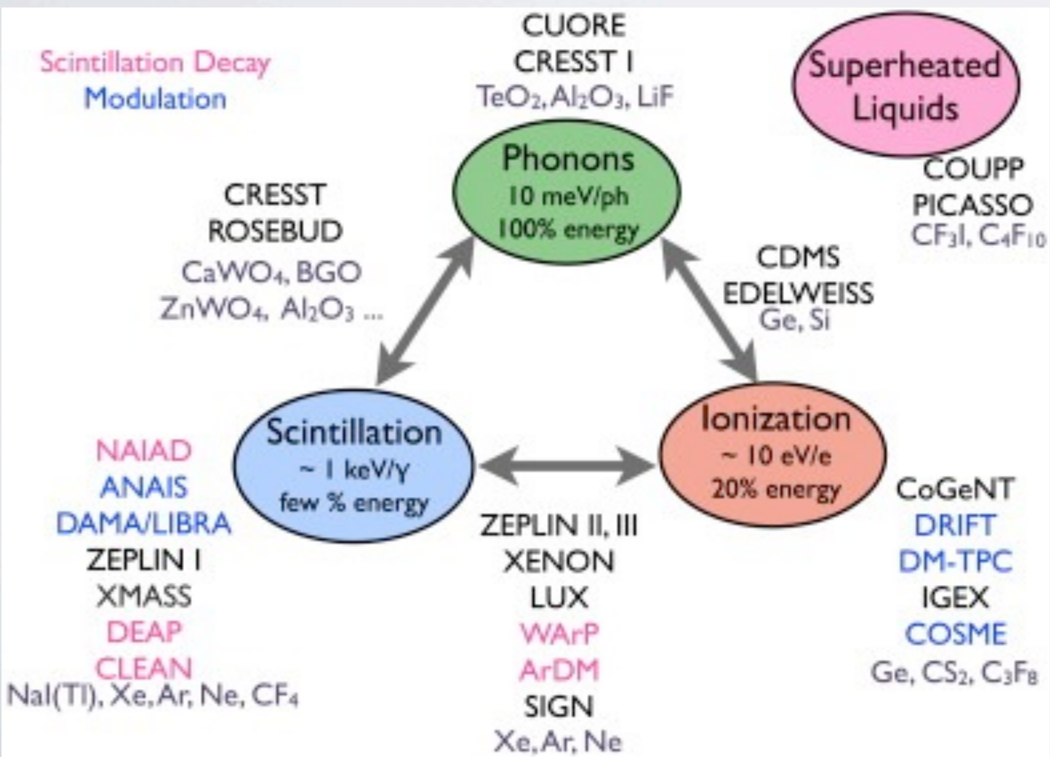


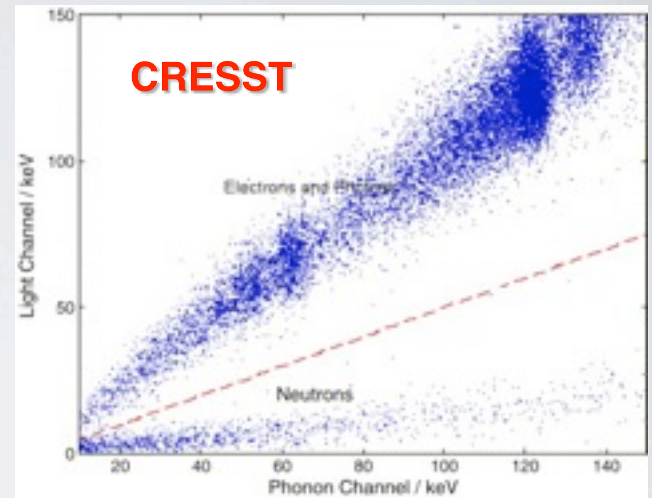
O(5%) time modulated signal is expected, but exact properties depend from the v-distribution (universal!) and the detector (material, threshold...)

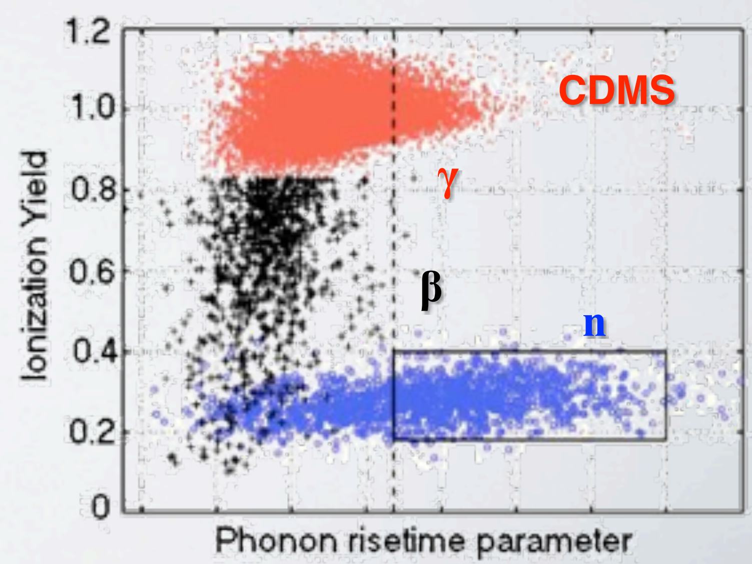
THE RACE: BACKGROUND REJECTION TECHNIQUES

despite low "noise" (high purity materials, low cosmic ray rate...), many phenomena can cause energy deposition (e.g. radioactive decays); largest worry is to separate "e.m.-like" recoils from "nucleon-like" recoils (like expected from WIMPs)

Strategy: event by event, measure different channels energy is deposited into (+e.g. position in the detector, for surface vs. bulk events). Select region where expecting <1 fake event leakage (based on known backgrounds)







THE RACE: BACKGROUND REJECTION TECHNIQUES

Letters to Nature

Nature **422**, 876-878 (24 April 2003) | doi:10.1038/nature01541; Received 20 November 2002; Accepted 10 March 2003

Experimental detection of α-particles from the radioactive decay of natural bismuth

Pierre de Marcillac, Noël Coron, Gérard Dambier, Jacques Leblanc & Jean-Pierre Moalic*

1. Institut d'Astrophysique Spatiale, CNRS & Université Paris Sud, UMR 8617, Bât. 121, 91405 Orsay Cedex, France

Correspondence to: Noël Coron Correspondence and requests for material should be addressed to P.d.M. (e-mail: Email: pierre.demarcillac@ias.u-psud.fr) or N.C. (e-mail: Email: noel.coron@ias.u-psud.fr).

The only naturally occurring isotope of bismuth, ^{209}Bi, is commonly regarded as the heaviest stable isotope. But like most other heavy nuclei abundant in nature and characterized by an

Top

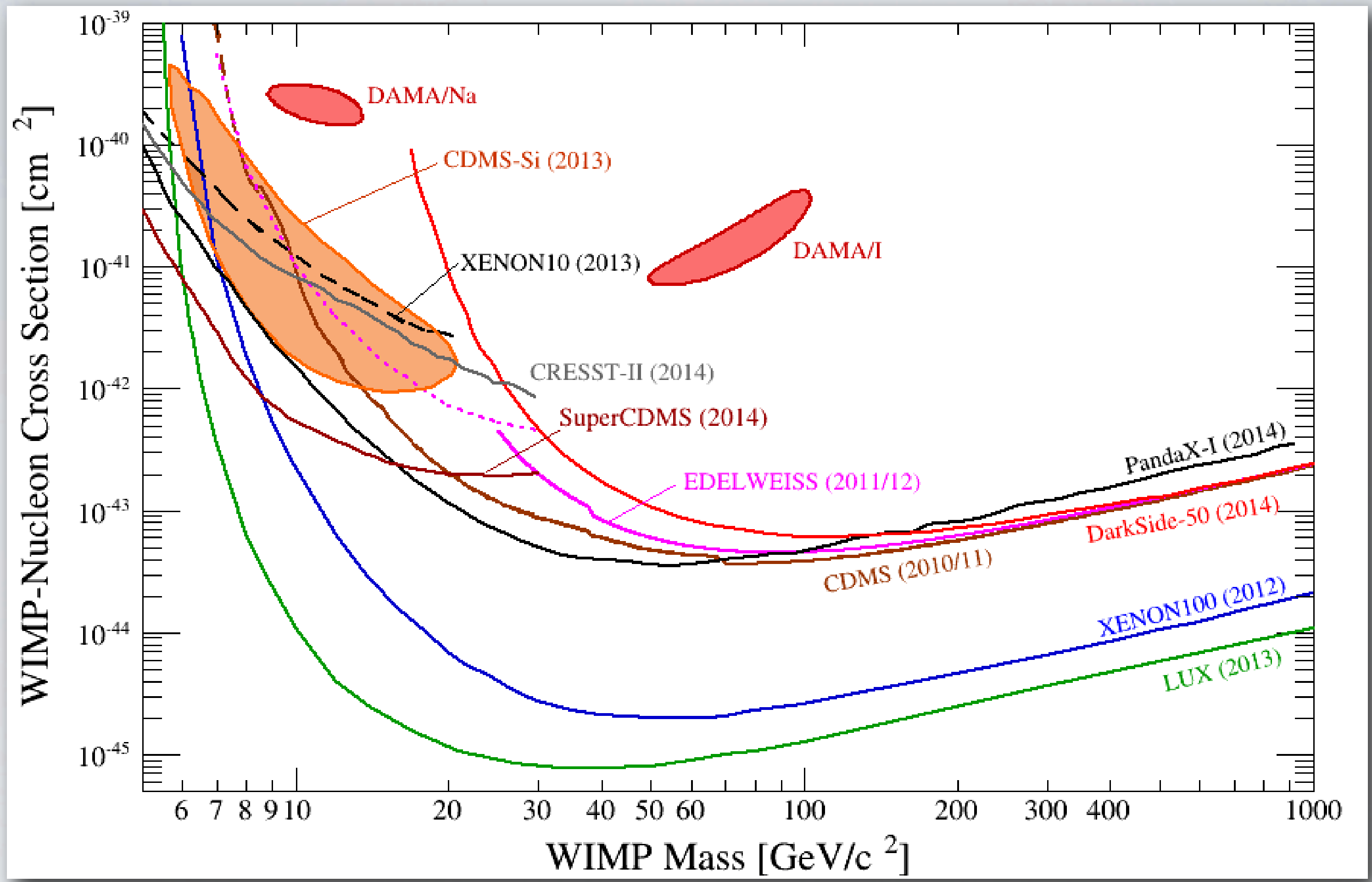
*French group developing low-temperature bolometers for dark matter direct detection...

message: don't be surprised if DM researchers should hit "new", unexpected backgrounds...

Bismuth breaks half-life record for alpha decay

Apr 23, 2003

CURRENT CONSTRAINTS ON ELASTIC SCATTERING



M. Schumann, arXiv:1501.01200

SUMMARY OF WHAT WE LEARNED

- We described heuristically how to derive the relic abundance via freeze-out mechanism

- We saw why non-relativistic relics seem to work... WIMP cold DM paradigm.

- WIMPs rich in collider, direct and indirect signatures and thus extremely well studied.

- We saw at least one alternative to WIMP freeze-out: freeze-in (harder to detect!)

- We returned to the "Boltzmann Eq." tool, which in its integrated form coincides with the above heuristic eq. (and tells how to compute RHS)

- For most DM applications its integrated form is sufficient.

- In some cases, momentum-dependent equations are needed: case of sterile neutrino, which in many respects is one of the minimal scenarios extending the SM capable of obtaining a DM candidate.

- We described the basic physics ingredients entering the WIMP direct detection strategy

- **Tomorrow's menu:** indirect detection strategies