Lecture 2: Ward Identities

General Ward Identities

Hinterbichler, Hui and Khoury, 1304.5527

Single-field inflation constrained by infinite number of symmetries, corresponding to an infinite number of consistency relations:

$$\lim_{\vec{q}\to 0} \frac{\partial^n}{\partial q^n} \left(\frac{\langle \zeta_{\vec{q}} \mathcal{O}_{\vec{k}_1,\ldots,\vec{k}_N} \rangle}{P_\zeta(q)} + \frac{\langle \gamma_{\vec{q}} \mathcal{O}_{\vec{k}_1,\ldots,\vec{k}_N} \rangle}{P_\gamma(q)} \right) \sim \frac{\partial^n}{\partial k^n} \langle \mathcal{O}_{\vec{k}_1,\ldots,\vec{k}_N} \rangle$$

- q^0 and q behavior completely fixed (KNOWN)
- q^n , $n \geq 2$, behavior partially fixed (NEW)
- These are physical statements (i.e., can be violated)
- Hold on any spatially-flat FRW background (no slow-roll)

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$\Longrightarrow$ Complete checklist for testing single-field mechanisms

$$\lim_{\vec{q}\to 0} M_{i\ell_0\ldots\ell_n}(\hat{q})\frac{\partial^n}{\partial q_{\ell_1}\cdots\partial q_{\ell_n}}\left(\frac{1}{P_\gamma(q)}\langle\gamma^{i\ell_0}(\vec{q})\mathcal{O}(\vec{k}_1,\ldots,\vec{k}_N)\rangle_c' + \frac{\delta^{i\ell_0}}{3P_\zeta(q)}\langle\zeta(\vec{q})\mathcal{O}(\vec{k}_1,\ldots,\vec{k}_N)\rangle_c'\right)$$

$$= -M_{i\ell_0\ldots\ell_n}(\hat{q})\Bigg\{\sum_{a=1}^{N}\left(\delta^{i\ell_0}\frac{\partial^n}{\partial k^a_{\ell_1}\cdots\partial k^a_{\ell_n}} - \frac{\delta_{n0}}{N}\delta^{i\ell_0} + \frac{k^i_a}{n+1}\frac{\partial^{n+1}}{\partial k^a_{\ell_0}\cdots\partial k^a_{\ell_n}}\right)\langle\mathcal{O}(\vec{k}_1,\ldots,\vec{k}_N)\rangle_c'$$

$$-\sum_{a=1}^{M}\Upsilon^{i\ell_0 i_a j_a}(\hat{k}_a)\frac{\partial^n}{\partial k^a_{\ell_1}\cdots\partial k^a_{\ell_n}}\langle\mathcal{O}^\zeta(\vec{k}_1,\ldots,\vec{k}_{a-1},\vec{k}_{a+1},\ldots\vec{k}_M)\gamma_{i_a j_a}(\vec{k}_a)\mathcal{O}^\gamma(\vec{k}_{M+1},\ldots,\vec{k}_N)\rangle_c'$$

$$-\sum_{b=M+1}^{N}\Gamma^{i\ell_0}{}_{i_b j_b}{}^{k_b\ell_b}(\hat{k}_b)\frac{\partial^n}{\partial k^b_{\ell_1}\cdots\partial k^b_{\ell_n}}\langle\mathcal{O}^\zeta(\vec{k}_1,\ldots,\vec{k}_M)\mathcal{O}^\gamma_{i_{M+1}j_{M+1},\ldots,k_b\ell_b,\ldots i_N j_N}(\vec{k}_{M+1},\ldots,\vec{k}_N)\rangle_c'\Bigg\} + \ldots$$

where

$$\Upsilon_{abcd}(\hat{k}) \equiv \frac{1}{4}\delta_{ab}\hat{k}_c\hat{k}_d - \frac{1}{8}\delta_{ac}\hat{k}_b\hat{k}_d - \frac{1}{8}\delta_{ad}\hat{k}_b\hat{k}_c\,;$$

$$\Gamma_{abijk\ell}(\hat{k}) \equiv -\frac{1}{2}\left(\delta_{ij}+\hat{k}_i\hat{k}_j\right)\left(\delta_{ab}\hat{k}_k\hat{k}_\ell - \frac{1}{2}\delta_{ak}\hat{k}_\ell\hat{k}_b - \frac{1}{2}\delta_{a\ell}\hat{k}_k\hat{k}_b\right) + \delta_{b(i}\delta_{j)(k}\delta_{\ell)a} - \delta_{a(i}\delta_{j)(k}\delta_{\ell)b}$$
$$- \delta_{b(i}\hat{k}_{j)}\delta_{a(k}\hat{k}_{\ell)} + \delta_{a(i}\hat{k}_{j)}\delta_{b(k}\hat{k}_{\ell)} - \delta_{a(k}\delta_{\ell)(i}\hat{k}_{j)}\hat{k}_b - \delta_{b(k}\delta_{\ell)(i}\hat{k}_{j)}\hat{k}_a + 2\delta_{ab}\hat{k}_{(i}\delta_{j)(k}\hat{k}_{\ell)}$$

Known Consistency Relations:

- $n = 0$ relations:

Maldacena (2002); Creminelli & Zaldarriaga (2004);
Cheung, Fitzpatrick, Kaplan & Senatore (2007);
Assassi, Baumann & Green (2012);
Goldberger, Hui & Nicolis (2013)

Dilation consistency relation $M_{i\ell_0}^{\text{dilation}} = \lambda\delta_{i\ell_0}$

$$\lim_{\vec{q}\to 0}\frac{1}{P_\zeta(q)}\langle\zeta(\vec{q})\mathcal{O}^\zeta(\vec{k}_1,\ldots,\vec{k}_N)\rangle'_c = -\left(3(N-1)+\sum_{a=1}^{N}\vec{k}_a\cdot\frac{\partial}{\partial\vec{k}_a}\right)\langle\mathcal{O}^\zeta(\vec{k}_1,\ldots,\vec{k}_N)\rangle'_c$$

Anisotropic scaling consistency relation $M_{i\ell_0}^{\text{anisotropic}} = \epsilon^s_{i\ell_0}$

$$\lim_{\vec{q}\to 0}\frac{1}{P_\gamma(q)}\langle\gamma^s(\vec{q})\mathcal{O}^\zeta(\vec{k}_1,\ldots,\vec{k}_N)\rangle'_c = -\frac{1}{2}\epsilon^s_{i\ell_0}(\hat{q})\sum_{a=1}^{N}\left\{k_a^i\frac{\partial}{\partial k_a^{\ell_0}}\langle\mathcal{O}^\zeta(\vec{k}_1,\ldots,\vec{k}_N)\rangle'_c - \frac{1}{2}\langle\mathcal{O}^\zeta(\vec{k}_1,\ldots,\vec{k}_{a-1},\vec{k}_{a+1},\ldots,\vec{k}_N)\gamma_{i\ell_0}(\vec{k}_a)\rangle'_c\right\}+\ldots$$

- $n = 1$ (linear-gradient) relations:

Creminelli, Norena & Simonovic, 1203.4595; Goldberger, Hui & Nicolis, 1303.1193; Creminelli, D'Amico, Musso & Norena, 1104.1462

SCT consistency relation $$M^{\rm SCT}_{i\ell_0\ell_1} = b_{\ell_1}\delta_{i\ell_0} + b_{\ell_0}\delta_{i\ell_1} - b_i\delta_{\ell_0\ell_1}$$

$$\lim_{\vec{q}\to 0}\frac{\partial}{\partial q^i}\left(\frac{1}{P_\zeta(q)}\langle\zeta(\vec{q})\mathcal{O}^\zeta(\vec{k}_1,\ldots,\vec{k}_N)\rangle'_c\right) = -\frac{1}{2}\sum_{a=1}^N\left(6\frac{\partial}{\partial k_a^i} - k_a^i\frac{\partial^2}{\partial k_a^j\partial k_a^j} + 2k_a^j\frac{\partial^2}{\partial k_a^j\partial k_a^i}\right)\langle\mathcal{O}^\zeta(\vec{k}_1,\ldots,\vec{k}_N)\rangle'_c + \ldots$$

Linear-gradient tensor relation $$M^{\rm tensor}_{i\ell_0\ell_1} = q_{\ell_1}\epsilon^s_{i\ell_0} + q_{\ell_0}\epsilon^s_{i\ell_1} - q_i\epsilon^s_{\ell_0\ell_1}$$

$$\begin{aligned}\lim_{\vec{q}\to 0} q^{\ell_1}\frac{\partial}{\partial q^{\ell_1}}\left(\frac{1}{P_\gamma(q)}\langle\gamma^s(\vec{q})\mathcal{O}^\zeta(\vec{k}_1,\ldots,\vec{k}_N)\rangle'_c\right) = &-\frac{1}{2}q^{\ell_1}\epsilon^s_{i\ell_0}(\vec{q})\sum_{a=1}^N\Bigg\{\left(k_a^i\frac{\partial}{\partial k_a^{\ell_1}} - \frac{k_a^{\ell_1}}{2}\frac{\partial}{\partial k_i^a}\right)\frac{\partial}{\partial k^a_{\ell_0}}\langle\mathcal{O}^\zeta(\vec{k}_1,\ldots,\vec{k}_N)\rangle \\ &-\left(2\Upsilon^{i\ell_0 i_a j_a}(\hat{k}_a)\frac{\partial}{\partial k^a_{\ell_1}} - \Upsilon^{\ell_1 i i_a j_a}(\hat{k}_a)\frac{\partial}{\partial k^a_{\ell_0}}\right) \\ &\times\langle\mathcal{O}^\zeta(\vec{k}_1,\ldots,\vec{k}_{a-1},\vec{k}_{a+1},\ldots,\vec{k}_N)\gamma_{i\ell_0}(\vec{k}_a)\rangle'_c\Bigg\} + \ldots\end{aligned}$$

Example of New Consistency Relation

- $n = 2$ tensor relation:

$$\lim_{\vec{q}\to 0} M^{\mathrm{T}}_{i\ell_0\ell_1\ell_2}(\hat{q})\frac{\partial^2}{\partial q_{\ell_1}\partial q_{\ell_2}}\left(\frac{1}{P_\gamma(q)}\langle\gamma^{i\ell_0}(\vec{q})\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\rangle'\right) = -M^{\mathrm{T}}_{i\ell_0\ell_1\ell_2}(\hat{q})\sum_{a=1}^{2}\frac{k_a^i}{3}\frac{\partial^3\langle\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\rangle'}{\partial k^a_{\ell_0}\partial k^a_{\ell_1}\partial k^a_{\ell_2}}$$

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Check using Maldacena's 3-pt function:

$$\frac{1}{P_\gamma(q)}\langle\gamma_{i\ell_0}(\vec{q})\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\rangle' = P^{\rm T}_{i\ell_0 j m_0}(\hat{q})\frac{H^2}{4\epsilon k_1^3k_2^3}k_1^j k_2^{m_0}\left(-K+\frac{(k_1+k_2)q+k_1k_2}{K}+\frac{qk_1k_2}{K^2}\right).$$

where $K = q + k_1 + k_2$

2-pt function: $\langle\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\rangle' = \dfrac{H^2}{4\epsilon k_1^3}$

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$$\lim_{\vec{q}\to 0} M^{\mathrm{T}}_{i\ell_0\ell_1\ell_2}(\hat{q}) \frac{\partial^2}{\partial q_{\ell_1}\partial q_{\ell_2}} \left(\frac{1}{P_\gamma(q)} \langle \gamma^{i\ell_0}(\vec{q}) \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \rangle' \right) = -M^{\mathrm{T}}_{i\ell_0\ell_1\ell_2}(\hat{q}) \sum_{a=1}^{2} \frac{k_a^i}{3} \frac{\partial^3 \langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \rangle'}{\partial k^a_{\ell_0} \partial k^a_{\ell_1} \partial k^a_{\ell_2}}$$

Check using Maldacena's 3-pt function:

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where $K = q + k_1 + k_2$

2-pt function: $\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \rangle' = \dfrac{H^2}{4\epsilon k_1^3}$

LHS: $$\lim_{\vec{q}\to 0} M^{\mathrm{T}}_{i\ell_0\ell_1\ell_2}(\hat{q}) \frac{\partial^2}{\partial q_{\ell_1}\partial q_{\ell_2}} \left(\frac{1}{P_\gamma(q)} \langle \gamma^{i\ell_0}(\vec{q}) \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \rangle'_c \right) = \frac{H^2}{4\epsilon k_1^3} \frac{35}{k_1^2} M^{\mathrm{T}}_{i\ell_0\ell_1\ell_2}(\hat{q}) \hat{k}_1^i \hat{k}_1^{\ell_0} \hat{k}_1^{\ell_1} \hat{k}_1^{\ell_2}$$

RHS: $$-M^{\mathrm{T}}_{i\ell_0\ell_1\ell_2}(\hat{q}) \sum_{a=1}^{2} \frac{k_a^i}{3} \frac{\partial^3 \langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \rangle'_c}{\partial k_a^{\ell_0} \partial k_a^{\ell_1} \partial k_a^{\ell_2}} = \frac{H^2}{4\epsilon k_1^3} \frac{35}{k_1^2} M^{\mathrm{T}}_{i\ell_0\ell_1\ell_2}(\hat{q}) \hat{k}_1^i \hat{k}_1^{\ell_0} \hat{k}_1^{\ell_1} \hat{k}_1^{\ell_2}$$

Explicit 3pt -> 2pt checks

Berezhiani, Khoury & Wang, 1401.7991

E.g.,

$$\lim_{\vec{q}\to 0} \frac{\partial^n}{\partial q^n}\left(\frac{\langle \zeta_{\vec{q}}\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\rangle}{P_\zeta(q)} + \frac{\langle \gamma_{\vec{q}}\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\rangle}{P_\gamma(q)}\right) \sim \frac{\partial^n}{\partial k^n} P_\zeta(k)$$

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Schematically,

$$\frac{\langle\zeta\zeta\zeta\rangle}{P_\zeta}\sim\mathcal{O}(q^2)+\ldots$$

$$\frac{\langle\gamma\zeta\zeta\rangle}{P_\gamma}\sim\frac{1}{\epsilon c_s}\left(1+Aq+\ldots\right).$$

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$$\frac{\langle \zeta\zeta\zeta\rangle}{P_\zeta} \sim \frac{1-c_s^2}{\epsilon c_s^3} q^2 + \ldots$$

$$\frac{\langle \gamma\zeta\zeta\rangle}{P_\gamma} \sim \frac{1}{\epsilon c_s}\left(1 + Aq + B\frac{q^2}{c_s^2} + Cq^2 \ldots\right) .$$

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Did all 3 -> 2 checks with $\zeta\zeta,\ \zeta\gamma,\ \gamma\gamma$ insertions up to (and including) q^3 order (new correlators!)

Multiple Soft Limits

Another probe of higher-q dependence.

Senatore & Zaldarriaga, 1203.6884
Chen, Huang & Shiu, hep-th/0610235
Joyce, JK & Simonovic, 1409.6318

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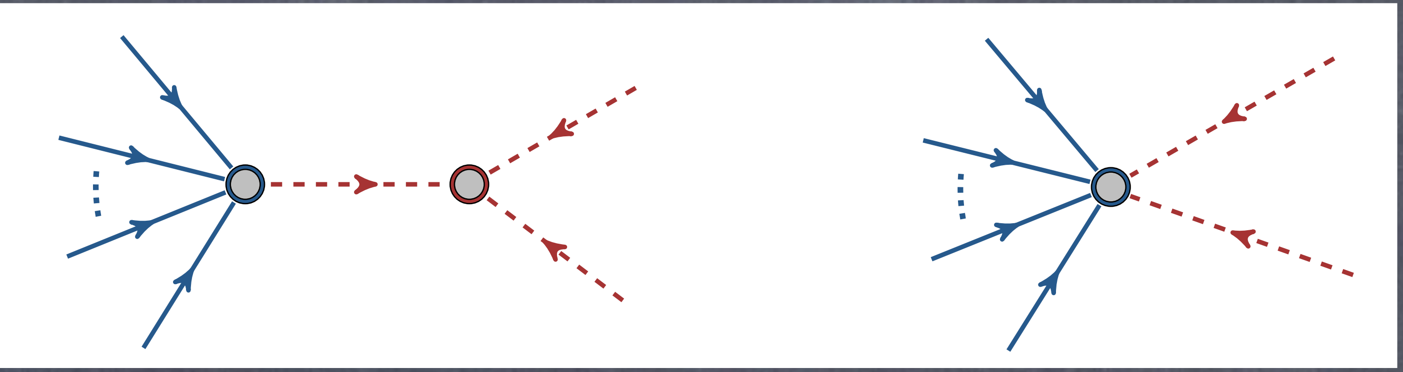
Double-soft result:



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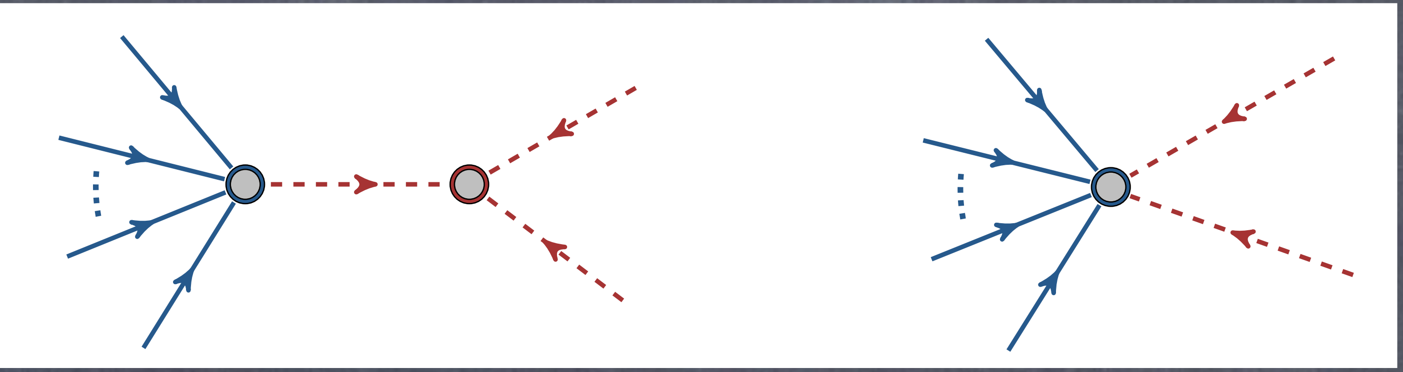


$$\lim_{\vec{q}_1,\vec{q}_2\to 0}\frac{\langle\zeta_{\vec{q}_1}\zeta_{\vec{q}_2}\zeta_{\vec{k}_1}\cdots\zeta_{\vec{k}_N}\rangle'}{P_\zeta(q_1)P_\zeta(q_2)} = \frac{\langle\zeta_{\vec{q}_1}\zeta_{\vec{q}_2}\zeta_{-\vec{q}}\rangle'}{P_\zeta(q_1)P_\zeta(q_2)}\left(\delta_{\mathcal{D}}+\frac{1}{2}\vec{q}_1\cdot\delta_{\vec{\mathcal{K}}}\right)\langle\zeta_{\vec{k}_1}\cdots\zeta_{\vec{k}_N}\rangle'$$
$$+\left(\delta_{\mathcal{D}}^2+\frac{1}{2}\vec{q}_1\cdot\delta_{\vec{\mathcal{K}}}\delta_{\mathcal{D}}+\frac{1}{4}q_1^i q_2^j\delta_{\mathcal{K}^i}\delta_{\mathcal{K}^j}\right)\langle\zeta_{\vec{k}_1}\cdots\zeta_{\vec{k}_N}\rangle'$$
$$+\lim_{\vec{q}\to 0}\left[\frac{1}{2}\left(\vec{q}^{\,2}\nabla_q^2-2q_iq_j\nabla_q^i\nabla_q^j\right)\langle\zeta_{\vec{q}}\zeta_{\vec{k}_1}\cdots\zeta_{\vec{k}_N}\rangle'+\frac{\langle\zeta_{\vec{q}_1}\zeta_{\vec{q}_2}\zeta_{-\vec{q}}\rangle'}{P_\zeta(q_1)P_\zeta(q_2)}q_iq_j\nabla_q^i\nabla_q^j\frac{\langle\zeta_{\vec{q}}\zeta_{\vec{k}_1}\cdots\zeta_{\vec{k}_N}\rangle'}{P_\zeta(q)}\right]$$

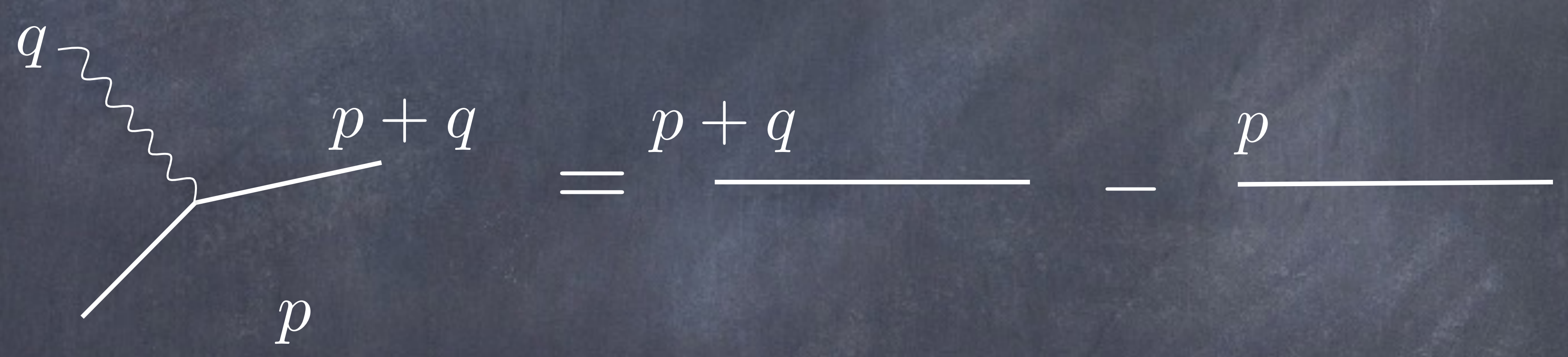
$\delta_{\mathcal{D}} \equiv \text{dilation}$ $\qquad \delta_{\mathcal{K}} \equiv \text{SCT}$

Master consistency relation

Berezhiani and Khoury, 1309.4461
(See also: Pimentel, 1309.1793)

Gauge invariance in EM implies Ward-Takahashi identity:

$$q^\mu \Gamma_\mu^{A\psi\psi}(q,p,p+q) = e\left(\Gamma^\psi(p+q) - \Gamma^\psi(p)\right) .$$



Similarly, spatial diffeomorphisms should give rise to a Slavnov-Taylor identity.

Cosmological Slavnov-Taylor Identity

Berezhiani & Khoury, 1309.4461

Following similar steps,

$$2\partial_j\left(\frac{1}{6}\delta_{ij}\frac{\delta\Gamma}{\delta\zeta}+\frac{\delta\Gamma}{\delta\gamma_{ij}}\right)=\partial_i\zeta\frac{\delta\Gamma}{\delta\zeta}+\mathrm{G.F.}$$

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Can vary this a number of times wrt the fields,

e.g. vary twice wrt ζ,

$$q^j \left(\frac{1}{3}\delta_{ij}\Gamma^{\zeta\zeta\zeta} + 2\Gamma^{\gamma\zeta\zeta}_{ij} \right) = q_i\Gamma_\zeta(p) - p_i\Big(\Gamma_\zeta(|\vec{q}+\vec{p}|) - \Gamma_\zeta(p)\Big)$$

(Exact in q)

Analogue of W-T identity in E&M

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Analogue of W-T identity in E&M

General schematic solution:

$$\frac{1}{3}\delta_{ij}\Gamma^{\zeta\zeta\zeta} + 2\Gamma_{ij}^{\gamma\zeta\zeta} = \sum_{n=0}^{\infty} q^n \frac{\partial^n}{\partial p^n} P_\zeta(p) + A_{ij}(\vec{p},\vec{q})$$

physical piece $q^j A_{ij}(\vec{p},\vec{q}) = 0$

Whether or not consistency relation holds hinges on model-dependent piece A_{ij} . Most general form:

$$A_{ij}(\vec{p},\vec{q}) = \epsilon_{ikm}\epsilon_{j\ell n}q^k q^\ell \left(a(\vec{p},\vec{q})\delta^{mn} + b(\vec{p},\vec{q})p^m p^n \right)$$

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<u>Key assumption:</u> Suppose a and b are analytic in q, such that

$$A_{ij} = \mathcal{O}(q^2)$$

(Locality condition)

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arbitrary scalar functions

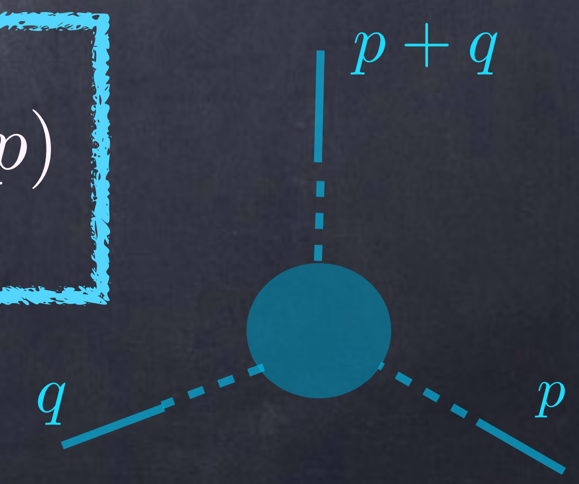
Key assumption: Suppose a and b are analytic in q, such that

$$A_{ij} = \mathcal{O}(q^2)$$

(Locality condition)

Then Maldacena's relation holds. Moreover, at each order in q can project out A_{ij} :

$$\lim_{\vec{q}\to 0}\frac{\partial^n}{\partial q^n}\left(\frac{\langle \zeta_{\vec{q}}\zeta_{\vec{p}}\zeta_{-\vec{q}-\vec{p}}\rangle}{P_\zeta(q)} + \frac{\langle \gamma_{\vec{q}}\zeta_{\vec{p}}\zeta_{-\vec{q}-\vec{p}}\rangle}{P_\gamma(q)} \right) \sim -\frac{\partial^n}{\partial p^n}P_\zeta(p)$$



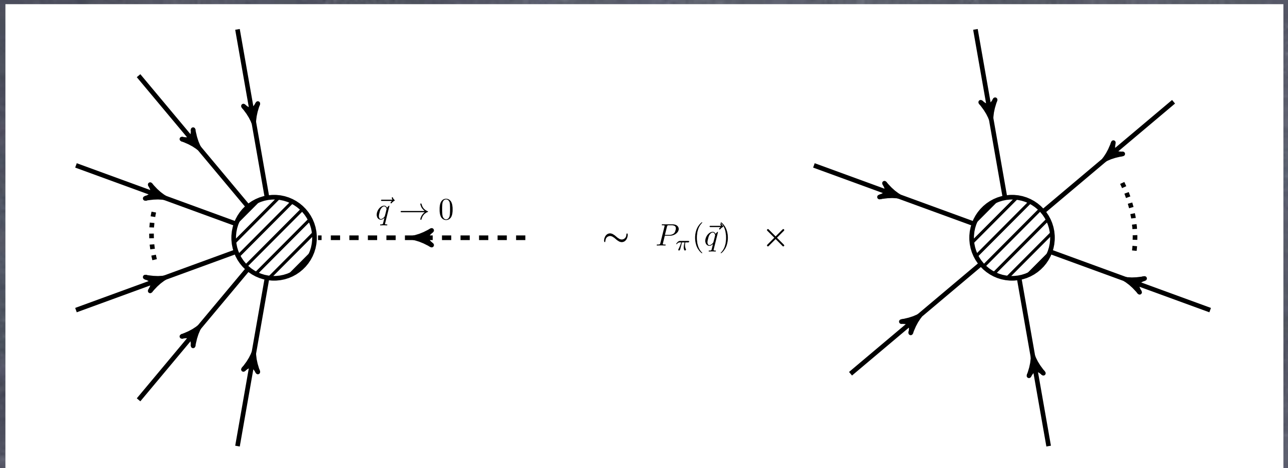
General consistency relations

Lecture 3: Conformal mechanism

Model-independent predictions

Creminelli, Joyce, Khoury & Simonovic, 1212.3329

- Have additional consistency relations (Ward identities) from the 5 broken symmetries $so(4,2) \to so(4,1)$



$$\lim_{\vec{q}\to 0} \frac{1}{P_\pi(q)} \langle \pi(\vec{q}) \mathcal{O}(\vec{k}_a) \rangle = -\left(1 + \frac{1}{N}\sum_a \vec{q}\cdot\frac{\partial}{\partial \vec{k}_a} + \frac{q^2}{6N}\sum_a \frac{\partial^2}{\partial k_a^2}\right) t \frac{\partial}{\partial t} \langle \mathcal{O}(\vec{k}_a) \rangle$$

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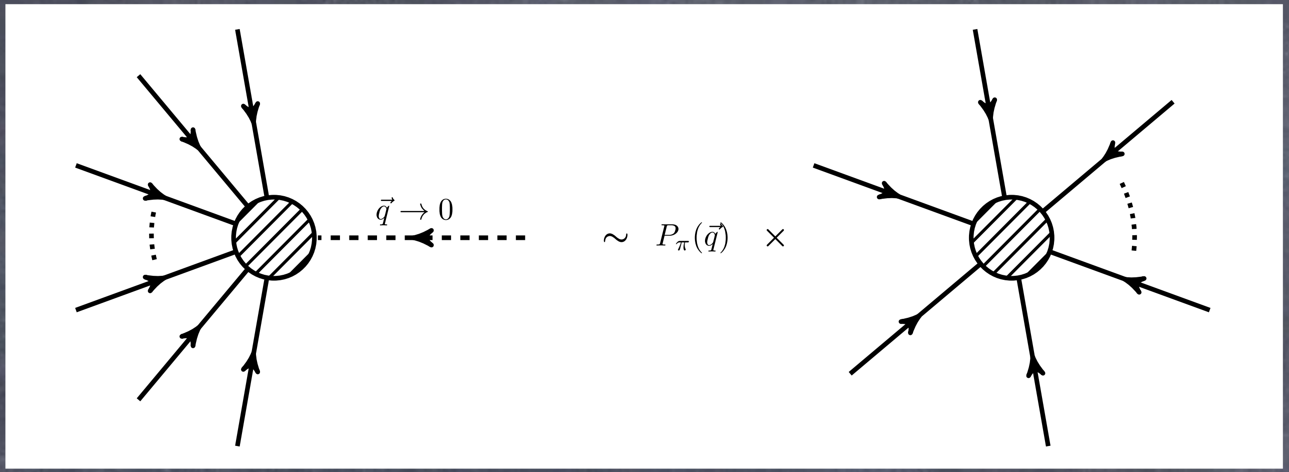

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- Goldstone spectrum is very red:

$$q^3 P_\pi(q) = \frac{A_\pi^2}{q^2 t^2}$$

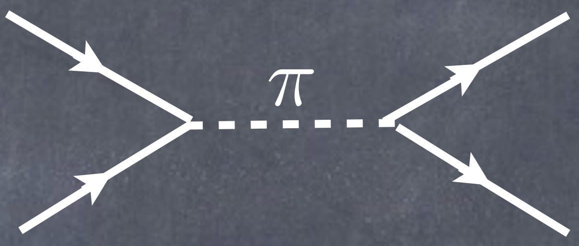
Observational Signatures

Creminelli, Joyce, Khoury & Simonovic, 1212.3329

- Soft internal lines: Libanov, Mironov & Rubakov (2011)

$$\langle \chi_{\vec{k}_1}\chi_{\vec{k}_2}\chi_{\vec{k}_3}\chi_{\vec{k}_4}\rangle_{q\to 0} = \frac{1}{P_\pi(q)}\langle \pi_{-\vec{q}}\chi_{\vec{k}_1}\chi_{\vec{k}_2}\rangle_{q\to 0}\langle \pi_{\vec{q}}\chi_{\vec{k}_3}\chi_{\vec{k}_4}\rangle_{q\to 0}$$

$$\sim \frac{1}{q}\left(3(\hat{k}_1\cdot\hat{q})^2 - 1\right)\left(3(\hat{k}_3\cdot\hat{q})^2 - 1\right) .$$



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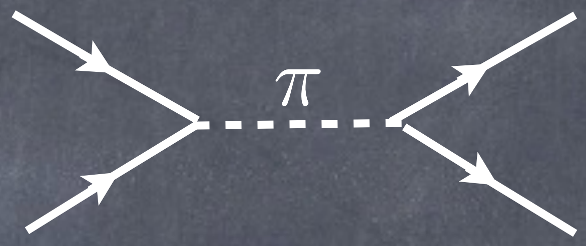

Diverges as $q \to 0$

(Vanishes as q^2 in inflation)

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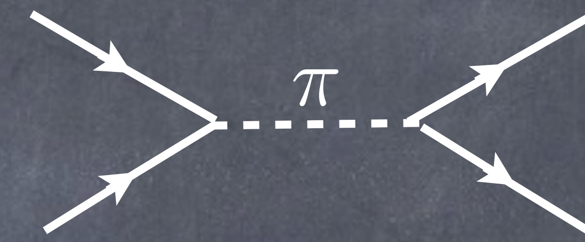

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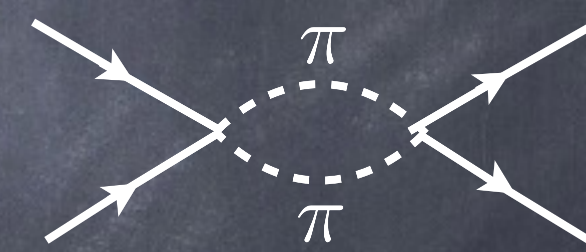
$$\tau_{\rm NL} \sim \log\frac{q}{\Lambda}$$



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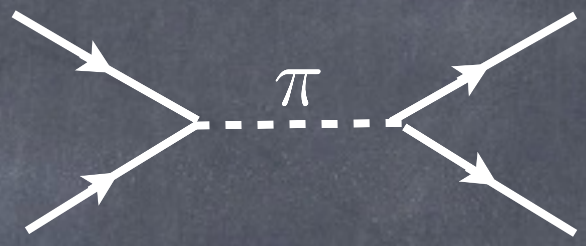

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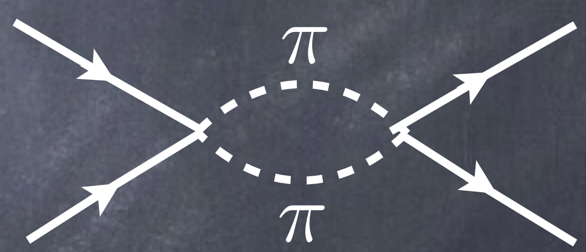
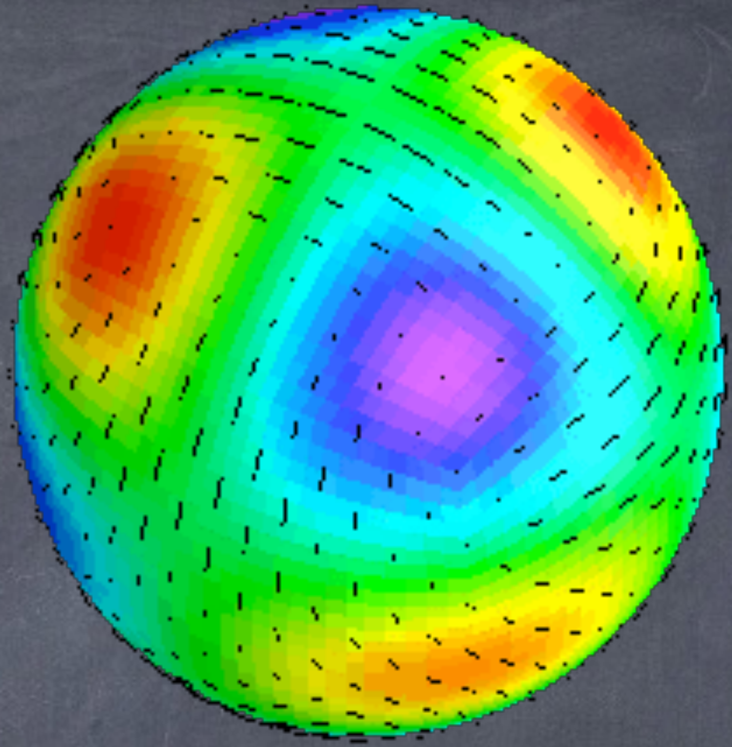


- Anisotropy: Realization-dependent from super-Hubble π mode

Libanov & Rubakov (2010)

$$\langle \chi_{\vec{k}}\chi_{-\vec{k}}\rangle_{\pi_{\vec{q}}} = \langle \chi_{\vec{k}}\chi_{-\vec{k}}\rangle\left(1 + c_1\frac{A_\pi}{2\pi}\frac{H_0}{k}\left(3\cos^2\theta - 1\right) + c_2\frac{3A_\pi^2}{4\pi^2}\cos^2\theta\log\frac{H_0}{\Lambda}\right)$$

Ultimate Smoking Gun



- Inflation: - Rapid background expansion

  - All light fields are excited, including gravitational waves

$\Longrightarrow$ scale invariant primordial gravity waves

- Conformal Scenario (and Ekpyrotic):

  - Very slow contraction/expansion

  - Graviton modes not appreciably excited

Brustein, Gasperini, Giovannini & Veneziano (1995)
Khoury, Ovrut, Steinhardt and Turok (2001)

Detection of primordial gravity waves, e.g. through CMB polarization, would rule out pre-big bang scenarios.