Double parton scattering in the ultraviolet
addressing the double counting problem

M. Diehl

Deutsches Elektronen-Synchroton DESY

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Reminder: single vs. double hard scattering (SPS vs. DPS)

- example: prod’n of two gauge bosons, transverse momenta $q_1$ and $q_2$

  ![Diagram](image)

  single scattering:
  $$|q_1| \text{ and } |q_2| \sim \text{hard scale } Q^2$$
  $$|q_1 + q_2| \ll Q^2$$

  for transv. momenta $\sim \Lambda \ll Q$

  $$\frac{d\sigma_{SPS}}{d^2q_1 \, d^2q_2} \sim \frac{d\sigma_{DPS}}{d^2q_1 \, d^2q_2} \sim \frac{1}{Q^4 \Lambda^2}$$

  but single scattering populates larger phase space:

  $$\sigma_{SPS} \sim \frac{1}{Q^2} \gg \sigma_{DPS} \sim \frac{\Lambda^2}{Q^4}$$
Reminder: single vs. double hard scattering (SPS vs. DPS)

▶ example: prod’n of two gauge bosons, transverse momenta $q_1$ and $q_2$

Single scattering:

$|q_1|$ and $|q_2| \sim$ hard scale $Q^2$

$|q_1 + q_2| \ll Q^2$

▶ for small parton mom. fractions $x$

Double scattering enhanced by parton luminosity

▶ depending on process: enhancement or suppression

from parton type (quarks vs. gluons), coupling constants, etc.
Double parton scattering

\[
\frac{d\sigma_{\text{DPS}}}{dx_1 \, d\bar{x}_1 \, dx_2 \, d\bar{x}_2} = \frac{1}{C} \, \hat{\sigma}_1 \hat{\sigma}_2 \int \, \! d^2 y \, F(x_1, x_2, y) \, F(\bar{x}_1, \bar{x}_2, y)
\]

- \( C = \) combinatorial factor
- \( \hat{\sigma}_i = \) parton-level cross sections
- \( F(x_1, x_2, y) = \) double parton distribution (DPD)
- \( y = \) transv. distance between partons

- at higher orders in \( \alpha_s \) get usual convolution integrals over \( x_i \) in \( \hat{\sigma}_i \) and \( F \)
- analogous formulation for measured \( q_1 \) and \( q_2 \)
  \( \sim \) transverse-momentum dependent DPDs
- for \( y \ll 1/\Lambda \) can compute

\[
F(x_1, x_2, y) \sim \frac{1}{y^2} \, \text{splitting fct} \otimes \text{usual PDF}
\]
Double parton scattering: ultraviolet problem

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\frac{d\sigma_{\text{DPS}}}{dx_1 \, d\bar{x}_1 \, dx_2 \, d\bar{x}_2} = \frac{1}{C} \hat{\sigma}_1 \hat{\sigma}_2 \int d^2y \, F(x_1, x_2, y) F(\bar{x}_1, \bar{x}_2, y)
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gives \textbf{UV divergent} cross section \( \propto \int d^2y / y^4 \)

in fact, formula \textbf{not valid} for \( |y| \sim 1/Q \)
...and more problems

- double counting problem between double scattering with splitting and single scattering at loop level

  MD, Ostermeier, Schäfer 2011; Gaunt, Stirling 2011; Gaunt 2012
  Blok, Dokshitzer, Frankfurt, Strikman 2011; Ryskin, Snigirev 2011, 2012
  already noted by Cacciari, Salam, Sapeta 2009

- also have graphs with splitting in one proton only: “1 vs 2”

\[ \sim \int d^2 y / y^2 \times F_{\text{non-split}}(x_1, x_2, y) \]

B Blok et al 2011-13
J Gaunt 2012
B Blok, P Gunnellini 2015
A consistent solution

MD, J. Gaunt work in progress

- regulate DPS: \( \sigma_{\text{DPS}} \propto \int d^2y \Phi(\nu y) F(x_1, x_2, y) F(\bar{x}_1, \bar{x}_2, y) \)
  - \( \Phi \rightarrow 0 \) for \( u \rightarrow 0 \) and \( \Phi \rightarrow 1 \) for \( u \rightarrow \infty \), e.g. \( \Phi(u) = \theta(u - 1) \)
  - cutoff scale \( \nu \sim Q \)
  - \( F(x_1, x_2, y) \) has both splitting and non-splitting contributions

- analogous regulator for transverse-momentum dependent DPDs

- full cross section: \( \sigma = \sigma_{\text{DPS}} - \sigma_{\text{sub}} + \sigma_{\text{SPS}} \)
  - subtraction \( \sigma_{\text{sub}} \) to avoid double counting:
    - \( = \sigma_{\text{DPS}} \) with \( F \) computed for small \( y \) in fixed order perturb. theory
    - (much simpler computation than \( \sigma_{\text{SPS}} \) at given order)
A consistent solution

MD, J. Gaunt work in progress

regulate DPS:  \[ \sigma_{\text{DPS}} \propto \int d^2 y \ \Phi(\nu y) \ F(x_1, x_2, y) \ F(\bar{x}_1, \bar{x}_2, y) \]

- \( \Phi \rightarrow 0 \) for \( u \rightarrow 0 \) and \( \Phi \rightarrow 1 \) for \( u \rightarrow \infty \), e.g. \( \Phi(u) = \theta(u - 1) \)
- cutoff scale \( \nu \sim Q \)
- \( F(x_1, x_2, y) \) has both splitting and non-splitting contributions

analogous regulator for transverse-momentum dependent DPDs

full cross section:  \[ \sigma = \sigma_{\text{DPS}} - \sigma_{\text{sub}} (1\text{vs1} + 1\text{vs2}) + \sigma_{\text{SPS}} + \sigma_{\text{tw2}\times\text{tw4}} \]

- subtraction \( \sigma_{\text{sub}} \) to avoid double counting:
  = \( \sigma_{\text{DPS}} \) with \( F \) computed for small \( y \) in fixed order perturb. theory
  (much simpler computation than \( \sigma_{\text{SPS}} \) at given order)
- can also include twist 2 \( \times \) twist 4 contribution
  and double counting subtraction for “1 vs 2” term
Subtraction formalism at work

\[
\sigma = \sigma_{\text{DPS}} - \sigma_{\text{sub}} + \sigma_{\text{SPS}}
\]

- for \( y \lesssim 1/Q \) have \( \sigma_{\text{DPS}} \approx \sigma_{\text{sub}} \)
  because pert. computation of \( F \) gives good approx. at considered order
  \( \Rightarrow \sigma \approx \sigma_{\text{SPS}} \)
  dependence on \( \Phi(\nu y) \) cancels between \( \sigma_{\text{DPS}} \) and \( \sigma_{\text{sub}} \)

- for \( y \gg 1/Q \) have \( \sigma_{\text{sub}} \approx \sigma_{\text{SPS}} \)
  because DPS approximations work well in box graph
  \( \Rightarrow \sigma \approx \sigma_{\text{DPS}} \)
  with regulator fct. \( \Phi(\nu y) \approx 1 \)

- same argument for 1 vs 2 term and \( \sigma_{\text{tw2} \times \text{tw4}} \) (were neglected above)

- subtraction formalism works order by order in perturb. theory

  J. Collins, Foundations of Perturbative QCD, Chapt. 10
Added benefit: DGLAP logarithms

- define DPDs as matrix elements of renormalised twist-two operators:

\[ F(x_1, x_2, y; \mu_1, \mu_2) \sim \langle p|O_1(0; \mu_1) O_2(y; \mu_2)|p \rangle \]
\[ f(x; \mu) \sim \langle p|O(0; \mu)|p \rangle \]

⇒ separate DGLAP evolution for partons 1 and 2:

\[ \frac{d}{d \log \mu_i} F(x_i, y; \mu_i) = P \otimes x_i \ F \]

for \( i = 1, 2 \)

- for \( Q_1 \ll Q_2 \) higher orders in box graph give logarithms \( \alpha_s^n \log^n(Q_2/Q_1) \) of DGLAP type from region \( Q_1 \ll |k_1| \ll \cdots \ll |k_n| \ll Q_2 \)

  - resummed by DPD evolution if take \( \nu \sim \mu_1 \sim Q_1, \mu_2 \sim Q_2 \)

  and appropriate initial conditions, e.g. \( F = F_{\text{split}} + F_{\text{non-split}} \)

\[ F_{\text{split}}(x_1, x_2, y; 1/y^*, 1/y^*) = F_{\text{perturb.}}(y^*) e^{-y^2 \Lambda^2} \quad \text{with} \quad 1/y^2 = 1/y^2 + 1/y_{\text{max}}^2 \]

\[ F_{\text{non-split}}(x_1, x_2, y; \mu_0, \mu_0) = f(x_1; \mu_0) f(x_2; \mu_0) \Lambda^2 e^{-y^2 \Lambda^2 / \pi} \]
Added benefit: DGLAP logarithms

- define DPDs as matrix elements of renormalised twist-two operators:
  \[ F(x_1, x_2, y; \mu_1, \mu_2) \sim \langle p | O_1(0; \mu_1) O_2(y; \mu_2) | p \rangle \]
  \[ f(x; \mu) \sim \langle p | O(0; \mu) | p \rangle \]
  \[ \Rightarrow \text{separate DGLAP evolution for partons 1 and 2:} \]
  \[ \frac{d}{d \log \mu_i} F(x_i, y; \mu_i) = P \otimes x_i F \]
  \[ \text{for } i = 1, 2 \]

- lowest order 1 vs 2 term \( \propto \log(Q/\Lambda) \)
  additional logs \( \alpha_s^n \log^{n+1}(Q/\Lambda) \) from \( \Lambda \ll |k_1| \ll \cdots \ll |k_n| \ll Q \)
  - again resummed by DPD evolution if take \( \nu \sim \mu_1 \sim \mu_2 \sim Q \)
  - with \( \nu \sim Q \) have no \( \log(Q/\Lambda) \) in \( \sigma_{tw2 \times tw4} - \sigma_{sub (1vs2)} \)
    provides justification to omit this term while keeping 1 vs 2 in \( \sigma_{DPS} \)
  - after Fourier trf. our \( \sigma_{DPS} \) is very similar to M Ryskin, A Snigirev 2011, 2012
DPS parton luminosities for illustration, model parameters not tuned

- plot $\int d^2 y \Phi(\nu y) F(x_1, x_2, y) F(\bar{x}_1, \bar{x}_2, y)$ vs. rapidity of $q_1$
- bands for 2 vs 2 (violet), 1 vs 2 (blue) and 1 vs 1 (yellow)
- 1 vs 1 term has strong cutoff dependence $\propto \nu^2$
  if is important must add $-\sigma_{\text{sub (1vs1)}} + \sigma_{\text{SPS}}$
DPS parton luminosities for illustration, model parameters not tuned

- plot $\int d^2 y \, \Phi(\nu y) F(x_1, x_2, y) F(\bar{x}_1, \bar{x}_2, y)$ vs. rapidity of $q_1$
  with $q_2$ central and $Q_1 = Q_2 = M_W$ at $\sqrt{s} = 14$ TeV

- bands for 2 vs 2 (violet), 1 vs 2 (blue) and 1 vs 1 (yellow)
  with scales $\nu = \mu_1 = \mu_2 = 0.5 M_W \ldots 2 M_W$

- 1 vs 1 important, but not as much as for $u\bar{u}$

very preliminary
DPS parton luminosities for illustration, model parameters not tuned

- plot $\int d^2y \, \Phi(\nu y) F(x_1, x_2, y) \, F(\bar{x}_1, \bar{x}_2, y)$ vs. rapidity of $q_1$
  with $q_2$ central and $Q_1 = Q_2 = M_W$ at $\sqrt{s} = 14$ TeV
- bands for 2 vs 2 (violet), 1 vs 2 (blue) and 1 vs 1 (yellow)
  with scales $\nu = \mu_1 = \mu_2 = 0.5 M_W \ldots 2 M_W$

$u \bar{d}$ induced by splitting at $\mathcal{O}(\alpha_s^2)$, e.g. by $u \rightarrow ug \rightarrow udd\bar{d}$
A comment on sum rules

- $F(x_1, x_2, y)$ follows homogeneous DGLAP equation
  no splitting term $\Rightarrow$ does not conserve sum rules for $\int d^2 y \, F(x_1, x_2, y)$
  J Gaunt, J Stirling 2009

- is irrelevant if cannot satisfy sum rules at some scale $\mu$
  - if def. $F(x_1, x_2, y)$ by min. subtraction of UV divergences
    $\Rightarrow \int d^2 y \, F(x_1, x_2, y) = \infty$
    due to splitting at short distances
    i.e. same physics that would provide inhomogenous term in evolution

- to use sum rules as constraint for DPD modelling
  must subtract infinite splitting contribution such that result
  - fulfills sum rule
  - enters in factorisation formula for cross section

  This is not the case in any known scheme
  $\Rightarrow$ at present sum rules have no theory justification
Summary

- double parton scattering important in specific kinematics/for specific processes
- recent progress: towards a systematic formulation of factorisation in QCD
- solution for UV problem of DPS ↔ double counting with SPS
  - simple UV regulator for DPS using distance $y$ between partons
  - simple subtraction term to avoid double counting order by order in perturbation theory
  naturally includes “1 vs 2” contributions and correctly resums DGLAP logarithms
- distinction between “splitting” and “non-splitting” in DPD necessary in ansatz for DPD (inevitable model dependence) but not in formulation of factorisation