Factorisation in Double Parton Scattering: Glauber Gluons

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Based on [arXiv:1510.08696],
Markus Diehl, JG, Daniel Ostermeier, Peter Plössl and Andreas Schäfer

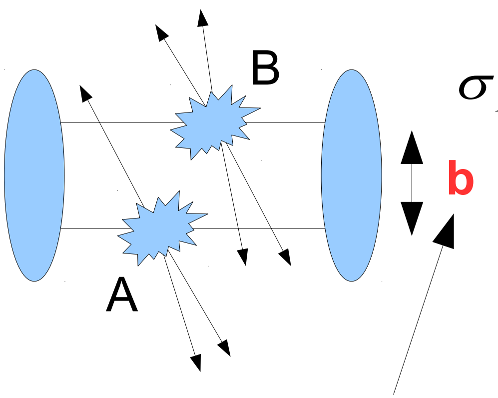
Outline

- Proposed factorisation formulae for DPS.
- Ingredients for proving a factorisation formula, a la Collins-Soper-Sterman (CSS). Necessity for the cancellation of so-called Glauber gluons to achieve factorisation.
- Demonstration of the cancellation of Glauber gluons in double Drell-Yan at the one-gluon level in a simple model, to show the principles.
- Brief discussion (only) of all-order proof

Double Parton Scattering

We know that in order to make a prediction for any process at the LHC, we need a factorisation formula (always hadrons/low energy QCD involved).

It's the same for double parton scattering. Postulated form for double parton scattering cross section based on analysis of lowest order Feynman diagrams:



Symmetry factor

Collinear double parton distribution (DPD)

$$\sigma_D^{(A,B)} = \frac{m}{2} \sum_{i,j,k,l} \int \Gamma_h^{ik}(x_1, x_2, \mathbf{b}; Q_A, Q_B) \Gamma_h^{jl}(x_1', x_2', \mathbf{b}; Q_A, Q_B)$$
$$\times \hat{\sigma}_{ij}^{A}(x_1, x_1') \hat{\sigma}_{kl}^{B}(x_2, x_2') \mathrm{d}x_1 \, \mathrm{d}x_1' \mathrm{d}x_2 \, \mathrm{d}x_2' \mathrm{d}^2\mathbf{b}$$

Parton level cross sections

N. Paver, D. Treleani, Nuovo Cim. A70 (1982) 215.
M. Mekhfi, Phys. Rev. D32 (1985) 2371.
Diehl, Ostermeier and Schafer (JHEP 1203 (2012))

b = separation in transverse space between the two partons

Further assumptions

(DPD factorises)

$$\sigma_D^{(A,B)} = \frac{\sigma_S^{(A)} \sigma_S^{(B)}}{\sigma_{eff}}$$

Factorisation formulae for DPS: $q_T \ll Q$

For small final state transverse momentum ($\mathbf{q}_i \ll Q$), differential DPS cross section postulated to have the following form: Diehl, Ostermeier and Schafer (JHEP 1203 (2012))

k_T dependent DPD

$$\frac{d\sigma_D^{(A,B)}}{d^2\mathbf{q}_1 d^2\mathbf{q}_2} = \frac{m}{2}\sum_{i,j,k,l}\int \Gamma_h^{ik}\left(x_1,x_2,\mathbf{k}_1,\mathbf{k}_2,\mathbf{b}\right)\Gamma_h^{jl}\left(x_1',x_2',\bar{\mathbf{k}}_1,\bar{\mathbf{k}}_2,\mathbf{b}\right)$$
$$\times\hat{\sigma}_{ij}^{A}\left(x_1,x_1'\right)\hat{\sigma}_{kl}^{B}\left(x_2,x_2'\right)\mathrm{d}x_1\,\mathrm{d}x_1'\mathrm{d}x_2\,\mathrm{d}x_2'\mathrm{d}^2\mathbf{b}$$
$$\times\prod_{i=1,2}\int d^2\mathbf{k}_i d^2\bar{\mathbf{k}}_i\delta\left(\mathbf{k}_i+\bar{\mathbf{k}}_i-\mathbf{q}_i\right)$$

(Neglecting a possible soft factor + dependence of the k_T-DPDs on rapidity regulator)

To what extent we prove these formulae hold in full QCD? Let's focus on the double Drell-Yan process to avoid complications with final state colour.

Establishing factorisation – the CSS approach

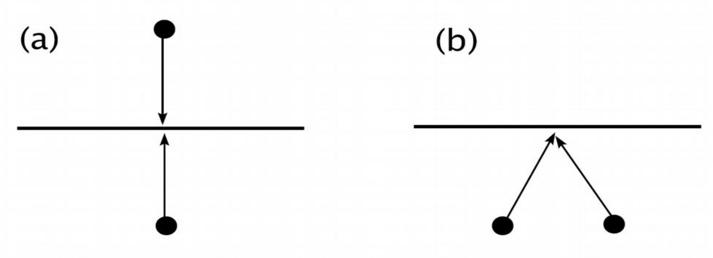
How does one establish a leading power factorisation for a given observable?

Here I review the original Collins-Soper-Sterman (CSS) method that has already been used to show factorisation for single Drell-Yan

CSS Nucl. Phys. B261 (1985) 104,
Nucl. Phys. B308 (1988) 833
Collins, pQCD book

To obtain a factorisation formula, need to identify IR leading power regions of Feynman graphs – i.e. small regions around the points at which certain particles go on shell, which despite being small are leading due to propagator denominators blowing up.

More precisely, need to find regions around pinch singularities – these are points where propagator denominators pinch the contour of the Feynman integral.



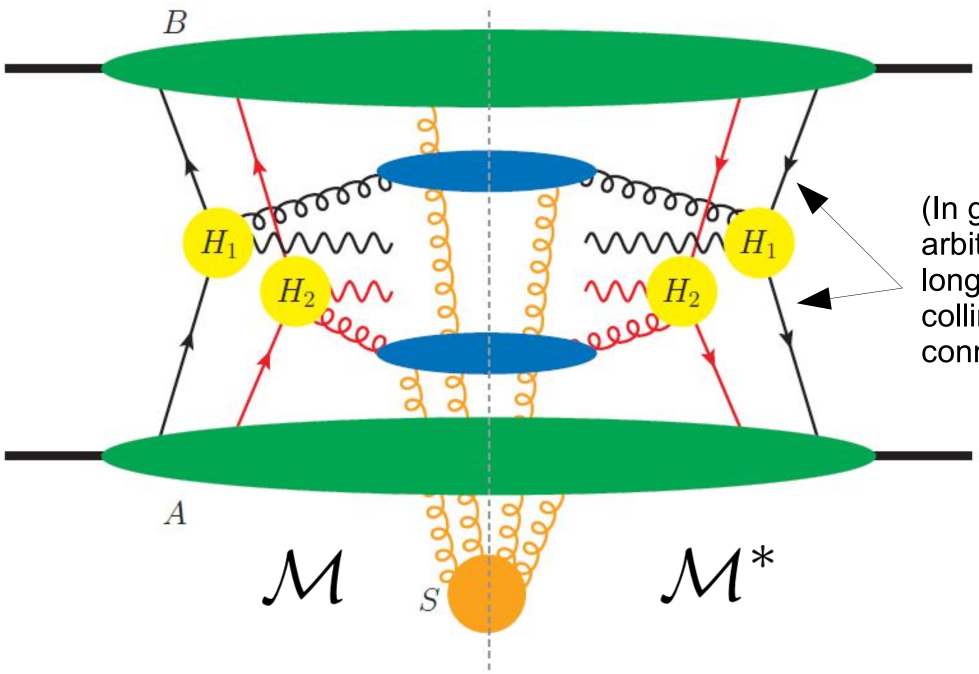
Pinched Non-pinched

CSS Factorisation Analysis

Pinch singularities in Feynman graphs correspond to physically (classically) allowed processes.

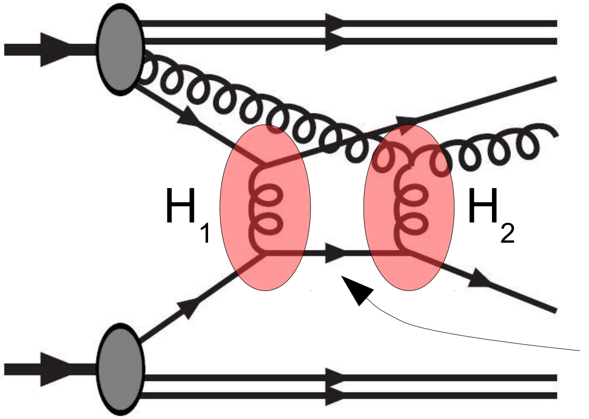
Coleman-Norton theorem

Double Drell-Yan (collinear factorisation)



(In general, also arbitrarily many longitudinally polarised collinear gluon connections to hard)

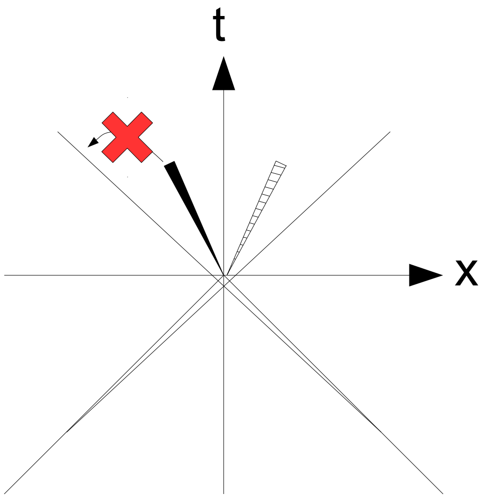
Side Note: Rescattering



It has been proposed that aside from double (or multiple) parton scattering, parton rescattering might be an interesting process to consider.

N. Paver, D. Treleani, Z. Phys. C28 (1985) 187
R. Corke, T. Sjöstrand, JHEP 1001 (2010) 035

The trouble is that this sort of graph does not have a pinch singularity corresponding to the rescattering process, if two processes are hard. No classical process corresponding to rescattering.



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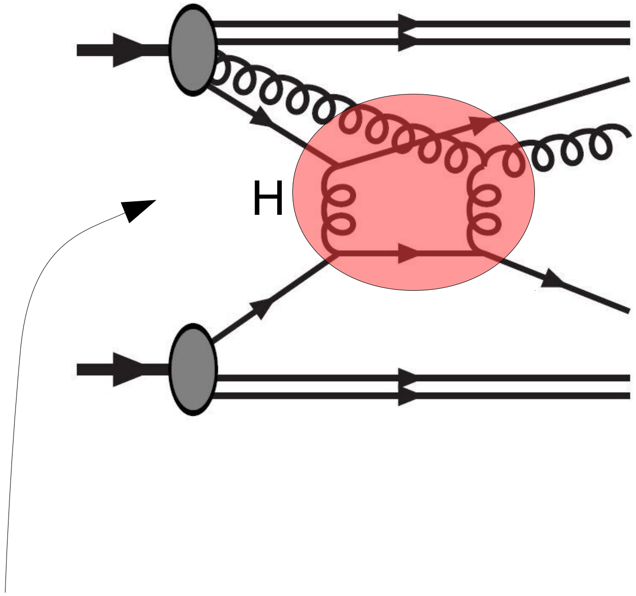

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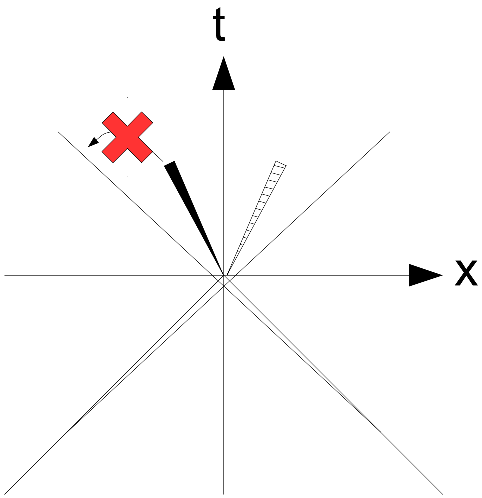
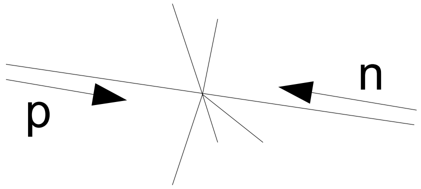
This graph should be computed as 2 parton vs. 1 parton “twist 4 x twist 2” process



Momentum Regions

Also need to do a power-counting analysis to determine if region around a pinch singularity is leading

Scalings of loop momenta that can give leading power contributions:



n/- component

p/+ component

transverse component

1) Hard region – momentum with large virtuality (order *Q*)

$$k \sim Q\,(1, 1, 1)$$

2) Collinear region – momentum close to some beam/jet direction

$$k \sim Q\,\left(1, \lambda^2, \lambda\right)$$ (for example)

3) (Central) soft region – all momentum components small and of same order

$$k \sim Q\,(\lambda^n, \lambda^n, \lambda^n)$$

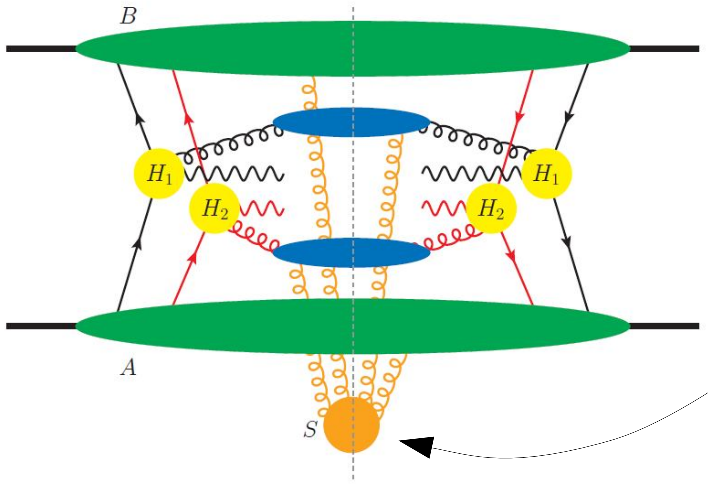
Momentum Regions

AND...

4) Glauber region – all momentum components small, but transverse components much larger than longitudinal ones

$$|k^+k^-| \ll \mathbf{k}_T^2 \ll Q^2$$

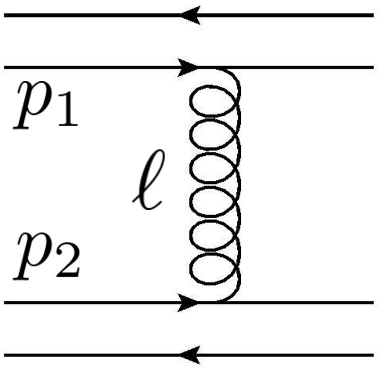
Canonical example: $k \sim Q\left(\lambda^2, \lambda^2, \lambda\right)$



Soft + Glauber particles

Side note: Glauber Gluons

Note that Glauber gluons are actually the momentum mode responsible for low x physics/Regge behaviour. First example low x calculation in 'Quantum Chromodynamics at High Energy' by Kovchegov and Levin:



$$0 = (p_1 + l)^2 = p_1^+ l^- + l^2$$

$$l^- = -l^2/p^+ = -l^2/P^+ \simeq 0$$

$$l^+ = l^2/p_2^- = l^2/P^- \simeq 0$$

$$l^2 \simeq -l_T^2$$

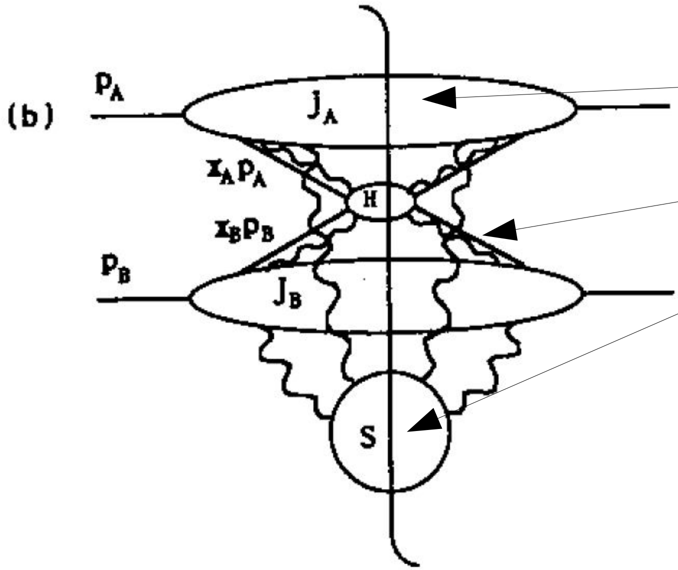
$\ell^+, \ell^- \simeq 0$

ℓ mainly transverse

“We see that in the high energy approximation the exchanged gluon has no longitudinal momentum: we will refer to it as an instantaneous or Coulomb gluon.”

Glauber Gluons and Factorisation

Deriving a factorisation formula that includes Glauber gluons is problematic.

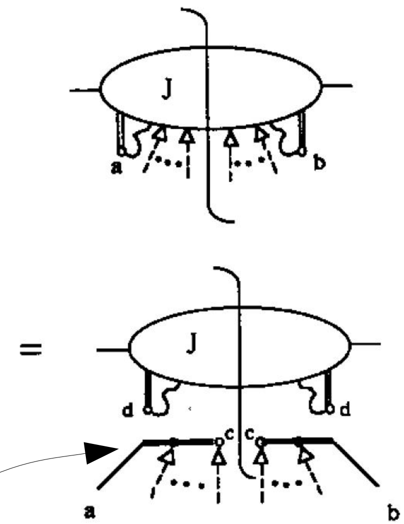


Starting picture (colourless V)

Collinear to proton A

Single parton + extra longitudinally polarised gluon attachments into hard

Soft + Glauber particles

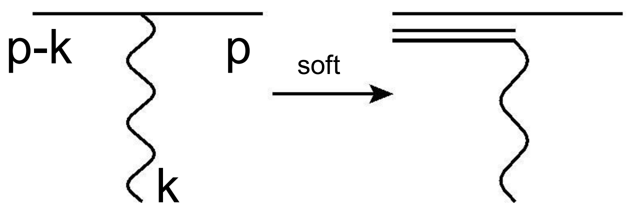


If blob S only contained central soft, then we could strip soft attachments to collinear J blobs using Ward identities, and factorise soft factor from J blobs.

Eikonal line in direction of J

Glauber Gluons and Factorisation

Simple example:



Propagator denominator:

$$(p-k)^2 = -2p\cdot k + k^2 \xrightarrow{\text{soft}} -2p\cdot k$$

Eikonal piece

This manipulation is NOT POSSIBLE for Glauber gluons – two terms in denominator are of same order in Glauber region

How do we get around this problem?

Only established way at present: try and show that that contribution from the Glauber region cancels (already used by CSS in the single Drell-Yan case)

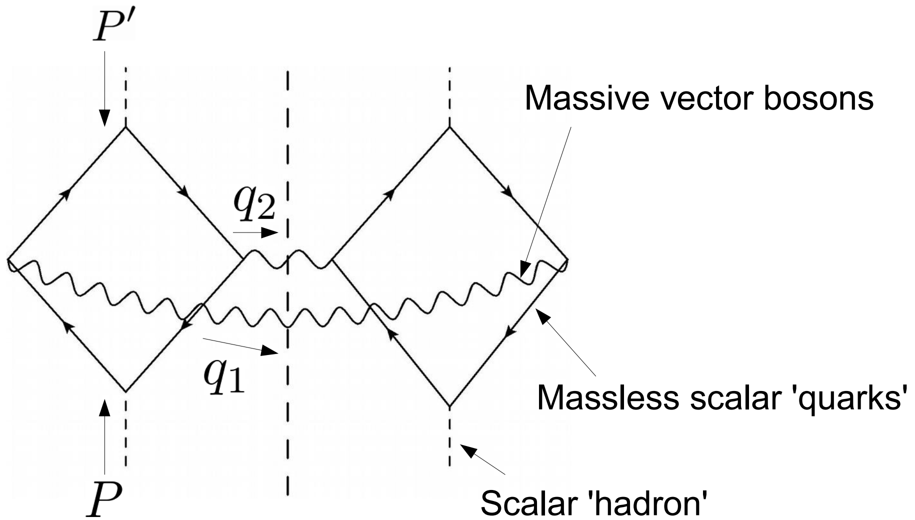
'Cancels' here means that there is no remaining 'distinct' Glauber contribution – may be contributions from Glauber modes that can be absorbed into soft or collinear.

Let's see if the Glauber modes cancel for double Drell-Yan.

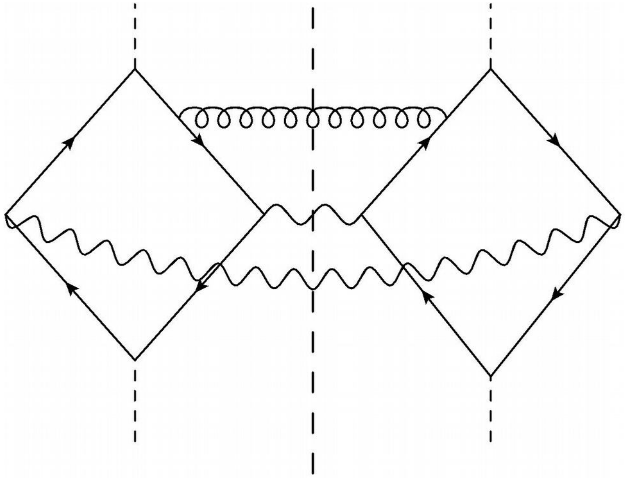
One-gluon model calculation: Lowest-order diagrams

One loop model calculation

'Parton-model' process:

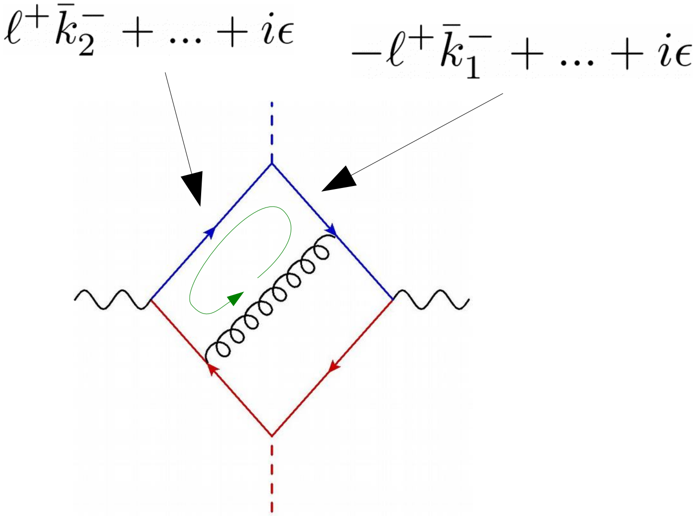


Real corrections:



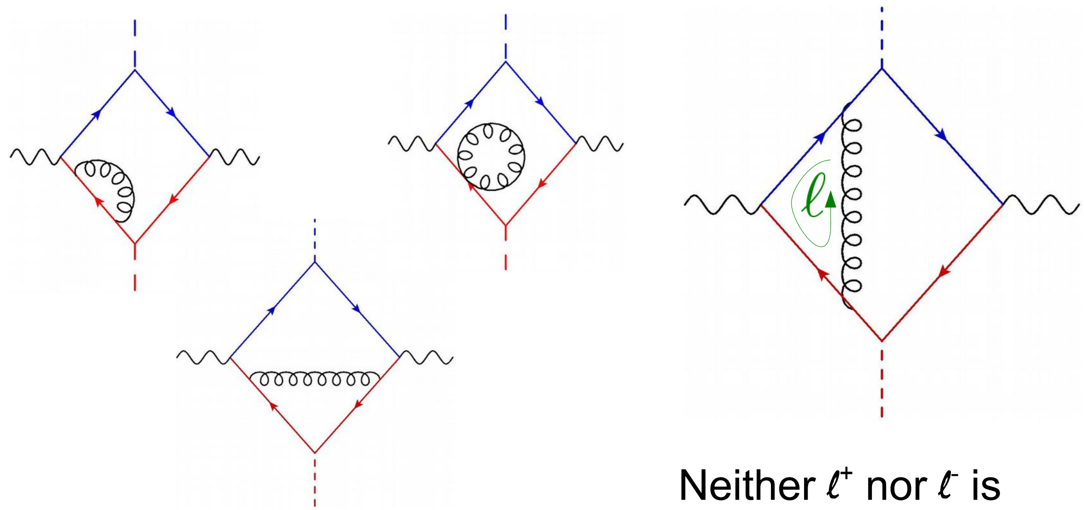
$$\propto [Tr(t^A)]^2 = 0$$

One-gluon model calculation: Lowest-order diagrams



ℓ^+ only is trapped small – ℓ^- can be freely deformed away from origin (into region where ℓ is collinear to P').

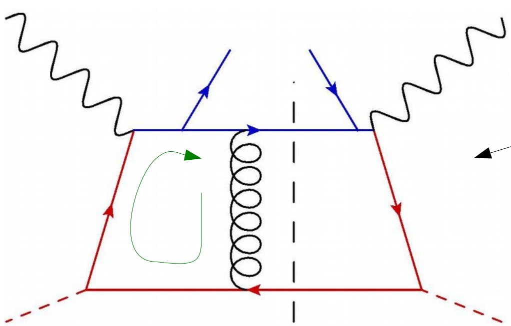
Virtual corrections:



'Topologically factored graphs'

Neither ℓ^+ nor ℓ^- is trapped small

Very similar to situation in SIDIS – no Glauber contribution there too. Collins, Metz, Phys.Rev.Lett. 93 (2004) 252001



More detailed checks that Glauber contributions are absent in the one-loop calculation are in the paper.

One-gluon model calculation: More complex diagrams

Can extend this to arbitrarily complex one-gluon diagrams in the model. Most of the time we can route ℓ^+ and ℓ^- such that at least one of these components is not pinched.

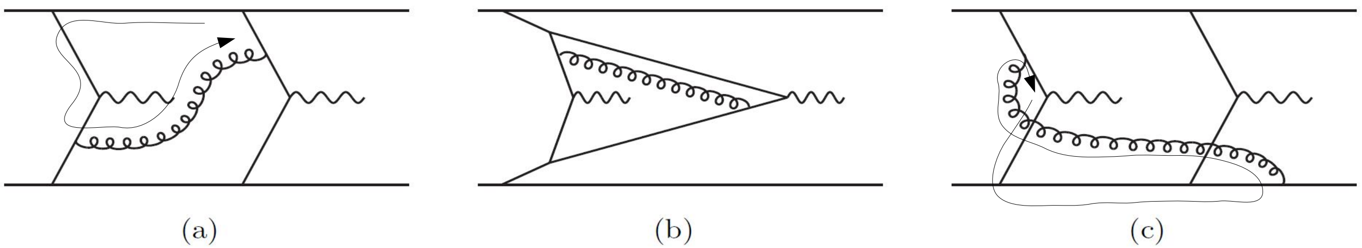
No ℓ^- pinch

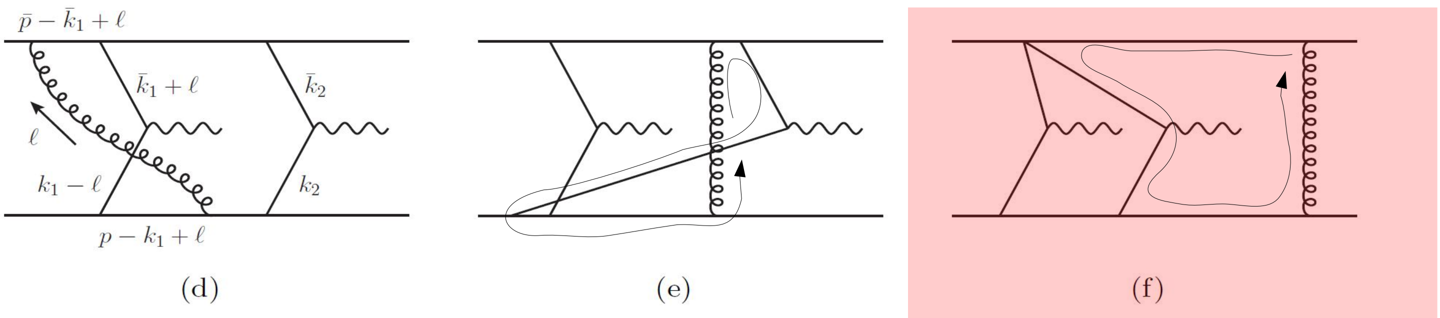
Simplest diagram embedded in more complex structure

No ℓ^+ pinch

Mainly -

Mainly +





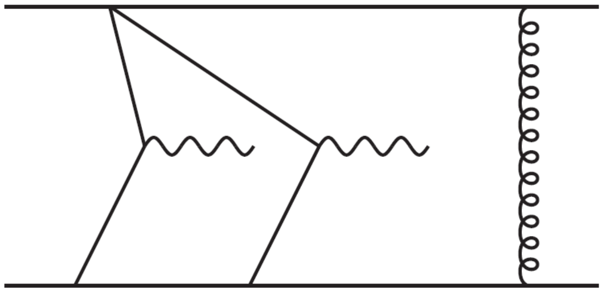
No ℓ^+ pinch

No ℓ^+ pinch

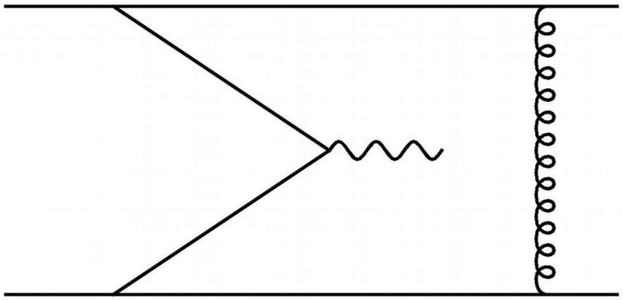
Both ℓ^-,ℓ^+ pinched!

Spectator-spectator interactions

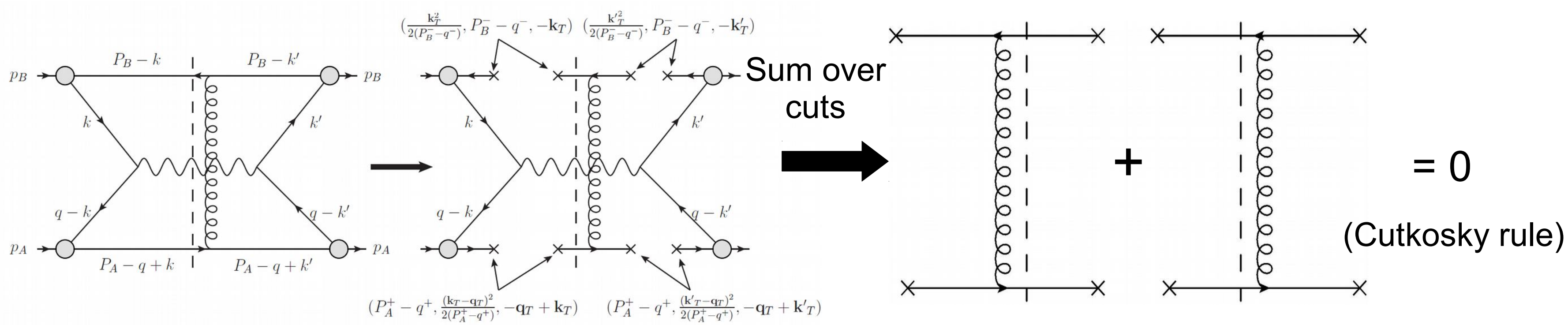
Only type of exchange that is pinched in Glauber region is this 'final state' interaction between spectator partons.



But we also have this type of pinched exchange in single Drell-Yan:



We can show that this Glauber exchange cancels after a sum over possible cuts of the graph, using exactly the same technique that is used for single scattering.

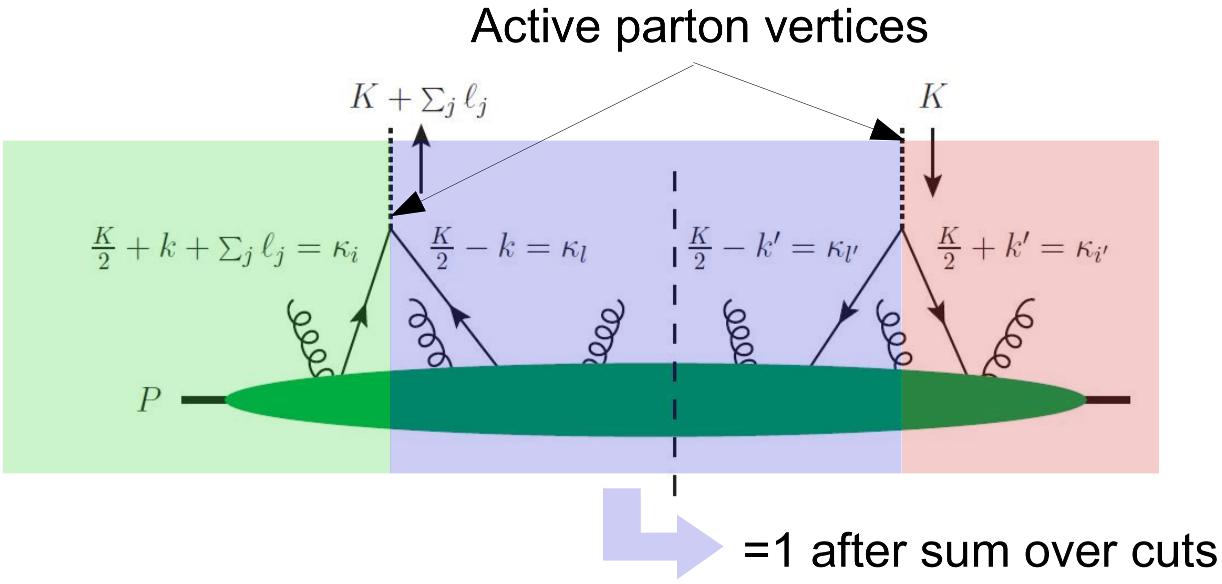


See e.g. Collins, pQCD book
JG, JHEP 1407 (2014) 110

All-order analysis

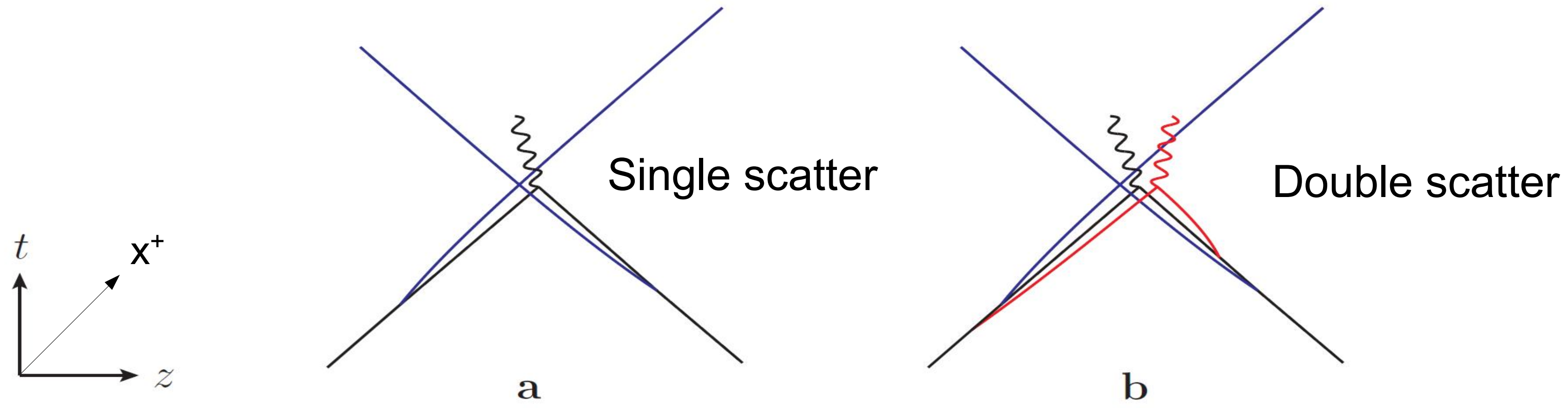
This methodology is not really suitable to be extended to all-orders – for the all-order proof of Glauber cancellation in double Drell-Yan, we use a different technique based on light-cone perturbation theory.

This is rather technical, so I won't go over this today. The principle is the same as the one-loop proof though – troublesome 'final state' poles obstructing deformation out of the Glauber region cancel after the sum over cuts, given that the observable is completely insensitive to all other (soft) scatterings except the two hard ones of interest.



Glauber in DPS – space-time structure

Basic reason why Glauber modes cancels for double Drell-Yan, just as it does for single Drell-Yan – spacetime structure of pinch surfaces for single and double scattering are rather similar:

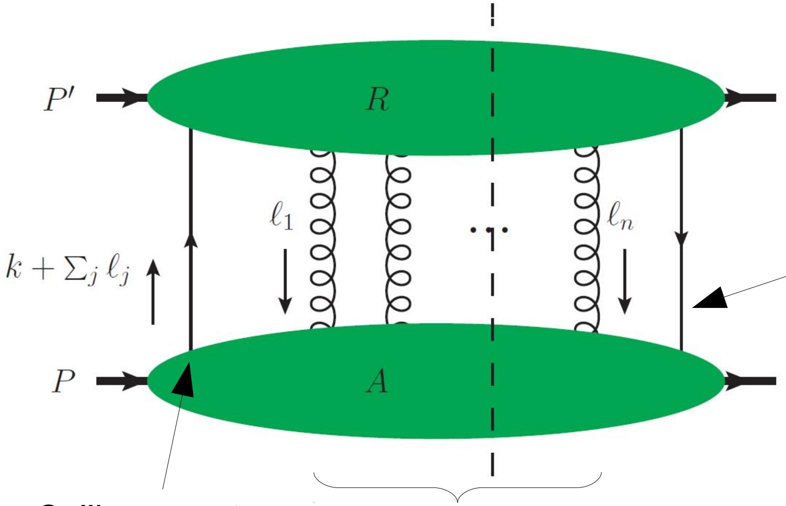


Diehl, Ostermeier and Schäfer (JHEP 1203 (2012))

Conclusions

- A proof of cancellation of Glauber gluons is an important step towards the factorisation proof for an observable.
- I discussed the cancellation of Glauber gluons for double Drell-Yan at the one-loop level in this talk. In the paper there is also an all-order proof using light-cone perturbation theory.
- Much more detail on this Glauber cancellation argument, and its interplay with the rest of the factorisation proof, may be found in the paper.

Glauber in SPS – all-order analysis



Collinear parton

Soft/Glauber attachments

Steps of the proof (schematic):

1) Partition leading order region into one collinear factor A and the remainder R

Partioning of soft vertex attachments in A between amplitude and conjugate

All compatible cuts of A

In A can approximate

$\ell_j \to \tilde{\ell}_j = (0, \ell_j^-, \boldsymbol{\ell}_j)$

even if this momentum is in the Glauber region

$$G_R = \int \frac{dk^+ \, d^{d-2}\boldsymbol{k}}{(2\pi)^{d-1}} \int \left[\prod_j \frac{d\ell_j^- \, d^{d-2}\boldsymbol{\ell}_j}{(2\pi)^{d-1}}\right] \sum_V \sum_{F_A \in \mathcal{A}(V)} \int \frac{dk^-}{2\pi} A_{F_A}^{\mu_1 \cdots \mu_n}(k, \tilde{\ell}_j)$$

$$\times \sum_{F_R \in \mathcal{R}(V)} \int \left[\prod_j \frac{d\ell_j^+}{2\pi}\right] R_{F_R, \mu_1 \cdots \mu_n}(k^+, \boldsymbol{k}, \ell_j) \, .$$

All compatible cuts of R

Glauber in SPS – all-order analysis

$$G_R = \int \frac{\mathrm{d}k^+\,\mathrm{d}^{d-2}\boldsymbol{k}}{(2\pi)^{d-1}} \int \Big[\prod_j \frac{\mathrm{d}\ell_j^-\,\mathrm{d}^{d-2}\boldsymbol{\ell}_j}{(2\pi)^{d-1}}\Big] \sum_V \sum_{F_A \in \mathcal{A}(V)} \int \frac{\mathrm{d}k^-}{2\pi} A_{F_A}^{\mu_1\cdots\mu_n}(k,\tilde{\ell}_j)$$
$$\times \sum_{F_R \in \mathcal{R}(V)} \int \Big[\prod_j \frac{\mathrm{d}\ell_j^+}{2\pi}\Big] R_{F_R,\,\mu_1\cdots\mu_n}(k^+,\boldsymbol{k},\ell_j)\,.$$

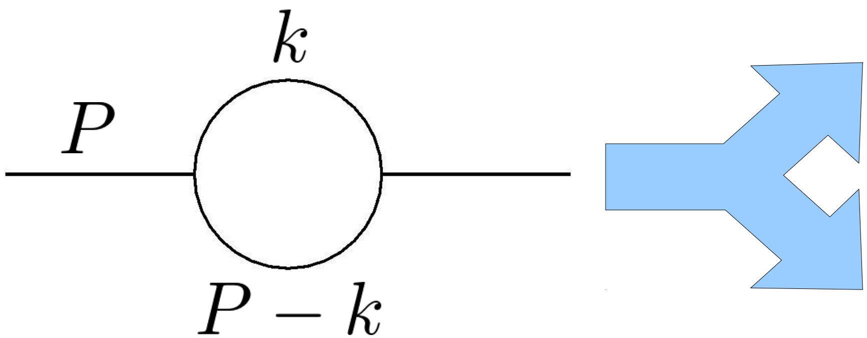
2) Let us assume R is independent of the partitioning V (will come back to this)

Then sum over V then acts only on A:

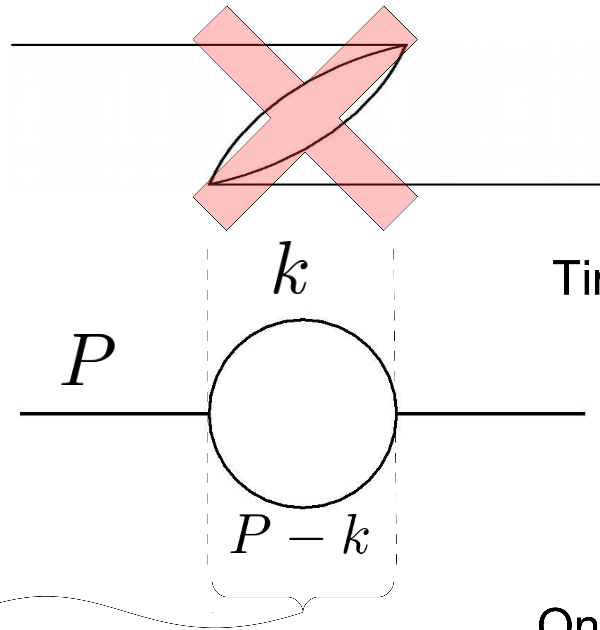
$$\sum_V \sum_{F_A \in \mathcal{A}(V)} \int \frac{\mathrm{d}k^-}{2\pi} A_{F_A}(k,\tilde{\ell}_j) = \sum_{\text{all } F_A} \int \frac{\mathrm{d}k^-}{2\pi} A_{F_A}(k,\tilde{\ell}_j)$$

Glauber in SPS – all-order analysis

3) Consider this factor in lightcone ordered perturbation theory (LCPT) – this is like old-fashioned time ordered perturbation theory except ordered along the direction of the P-jet.



Feynman graph



Time orderings

Denominator associated with state ξ:

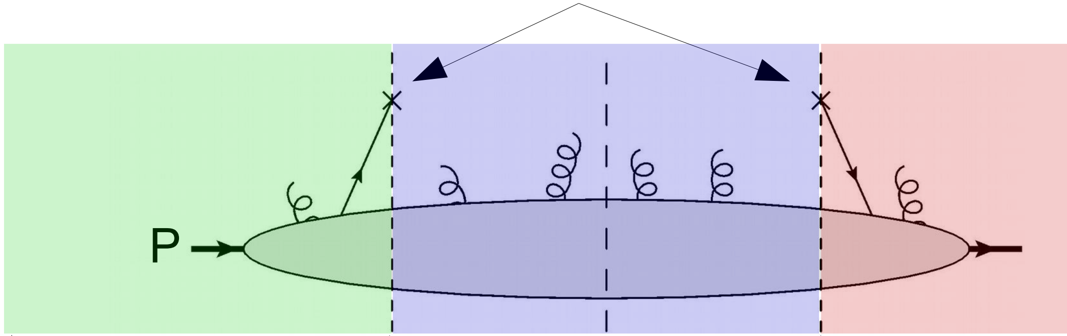
On-shell minus momenta of lines in state

Total minus momentum entering state from left

$$P^- - \frac{\mathbf{k}^2+m^2}{2k^+} - \frac{\mathbf{k}^2+m^2}{2(P^+-k^+)} + i\epsilon$$

Glauber in SPS – all-order analysis

Active parton vertices



$$\prod_{\substack{\text{states } \xi\\ \xi<H}} \frac{1}{P^- + \sum\limits_{\substack{\text{vertices } j\\ j<\xi}} \ell_j^- - \sum\limits_{\substack{\text{lines } L\\ L\in\xi}} \kappa_L + i\epsilon}$$

$$\prod_{\substack{\text{states } \xi\\ H'<\xi}} \frac{1}{P^- - \sum\limits_{\substack{\text{vertices } j\\ j>\xi}} \ell_j^- - \sum\limits_{\substack{\text{lines } L\\ L\in\xi}} \kappa_L - i\epsilon}$$

$$\sum_{F_A} \int_{-\infty}^{\infty} \frac{dk^-}{2\pi} F_T(k, \tilde{\ell}_j)$$

$$= \int_{-\infty}^{\infty} \frac{dk^-}{2\pi} \sum_{c=1}^{N} \left\{ \prod_{f=c+1}^{N} \frac{1}{P^- - k^- - \sum\limits_{j>f} \ell_j^- - D_f - i\epsilon} \right\} (2\pi)\delta\left(P^- - k^- - \sum_{j>c} \ell_j^- - D_c\right) \left\{ \prod_{f=1}^{c-1} \frac{1}{P^- - k^- - \sum\limits_{j>f} \ell_j^- - D_f + i\epsilon} \right\}$$

$$= \int_{-\infty}^{\infty} \frac{dk^-}{2\pi} \left\{ i \prod_{f=1}^{N} \frac{1}{P^- - k^- - \sum\limits_{j>f} \ell_j^- - D_f - i\epsilon} - i \prod_{f=1}^{N} \frac{1}{P^- - k^- - \sum\limits_{j>f} \ell_j^- - D_f + i\epsilon} \right\}$$

$$= \begin{cases} 1 & \text{if } N = 1 \\ 0 & \text{otherwise} \end{cases}$$

(LCPT version of Cutkosky rules)

Glauber in DPS – all-order analysis

Now let's study double Drell-Yan using the same method. Assume again that R is independent of V, and study A.

Change variables from 'default' DPS ones →

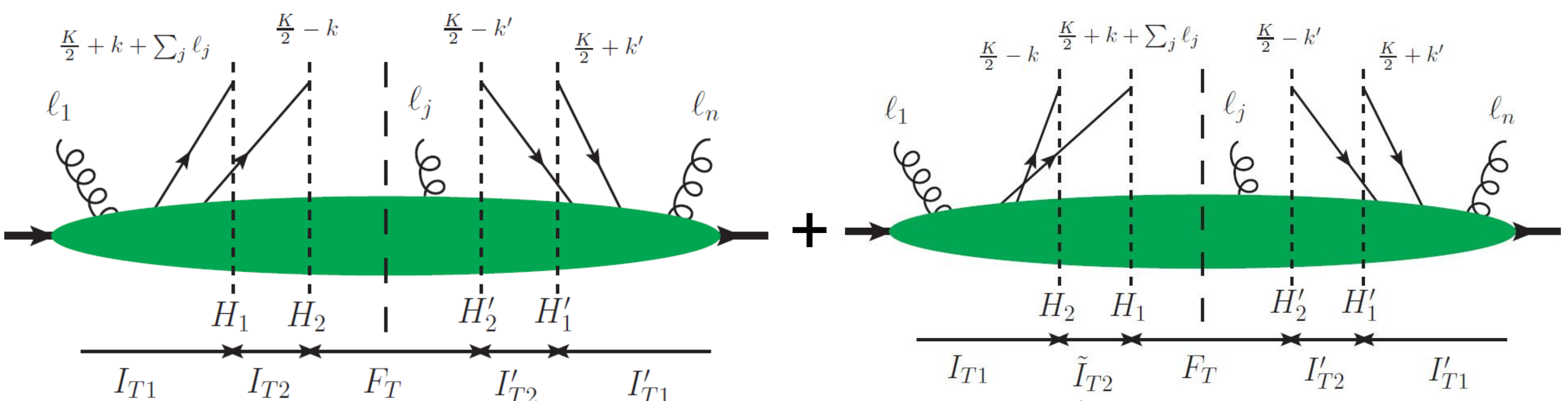
$$K = k_1 + k_2$$ Total coll mtm from $\mathcal{M}$ or $\mathcal{M}^*$

$$k = \tfrac{1}{2}(k_1 - k_2 - r)$$ Mtm diff in $\mathcal{M}$

$$k' = \tfrac{1}{2}(k_1 - k_2 + r)$$ Mtm diff in $\mathcal{M}^*$

In A we have integrals over k^-, k'^-, K^-

LCPT graphs for A in DPS:



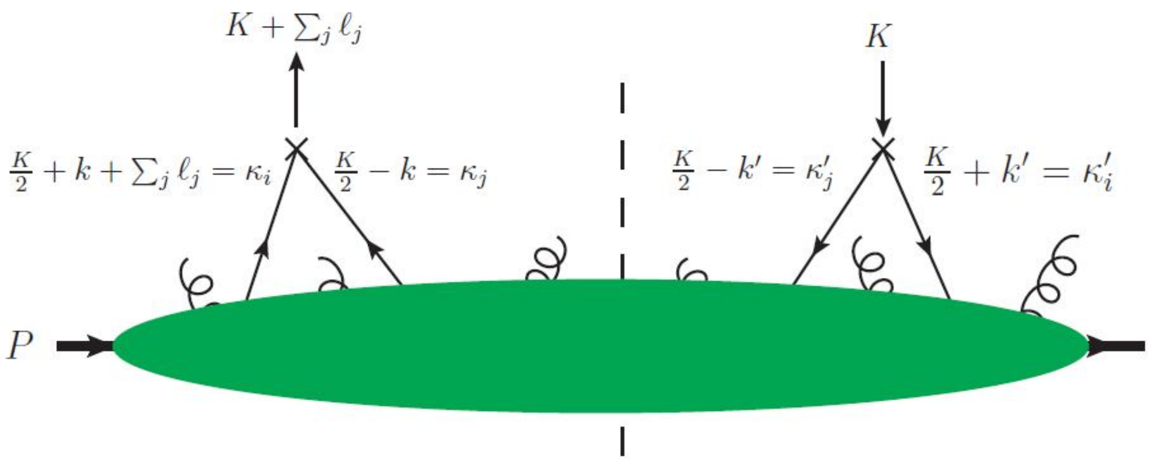
$$\int \frac{\mathrm{d}k^-}{2\pi}\left[I_{T2} + \tilde{I}_{T2}\right] = \int \frac{\mathrm{d}k^-}{2\pi}\left[\frac{1}{p^- - k^- - K^-/2 - \sum_{j>H_1}\ell_j - D_f + i\epsilon} + \frac{1}{p^- + k^- - K^-/2 + \sum_{j<H_1}\ell_j - \tilde{D}_f + i\epsilon}\right] = -i$$

k^- integration used here

Glauber in DPS – all-order analysis

Repeat for k' in conjugate – end up with the following picture:

k'⁻ integration used here



Just one external vertex in amplitude and conjugate – diagram looks essentially identical to SPS *A* and cancellation of Glaubers proceeds as for SPS.

K^- integration used here

More direct demonstration of this is in the paper

Glauber in DPS – all-order analysis

How can we show independence of *R* on *V*?

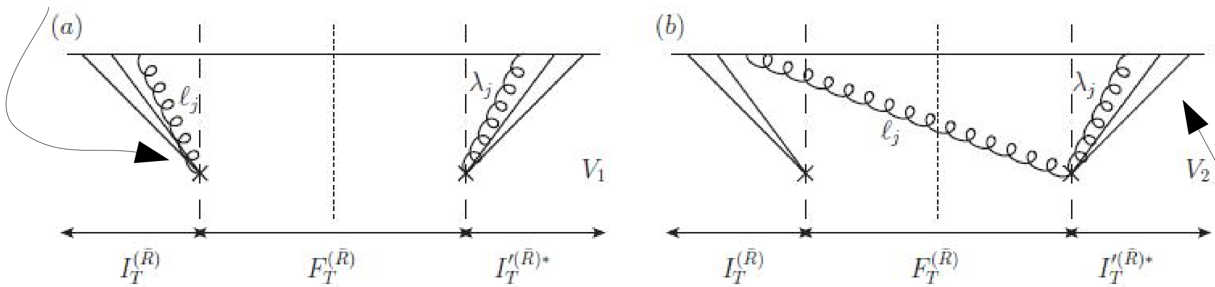
Separate R into hard factor H and remainder $\hat{R}$

$$R = \hat{R} \times H$$

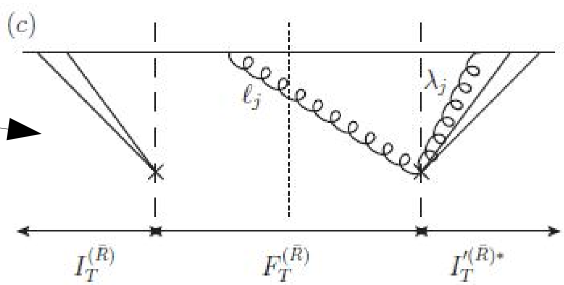
Note integral over all ℓ_j^+

$$\widehat{R}(\bar{k}_1, \bar{k}_2, \bar{r}, \bar{\lambda}_l, \ell_j) = \sum_{F_{BS} \in \mathcal{R}(V)} \int \frac{\mathrm{d}\bar{k}_1^+}{2\pi} \frac{\mathrm{d}\bar{k}_2^+}{2\pi} \frac{\mathrm{d}\bar{r}^+}{2\pi} \left[\prod_l \frac{\mathrm{d}\bar{\lambda}_l^+}{2\pi} \right] \left[\prod_j \frac{\mathrm{d}\ell_j^+}{2\pi} \right] (BS)_{F_{BS}}(\bar{k}_1, \bar{k}_2, \bar{r}, \bar{\lambda}_l, \ell_j) \Big|_{\bar{r}^- = 0}$$

Then can tie ends of all soft lines + one/two partons entering hard scatterings together in amplitude/conjugate



Then no attachments into final state allowed (give zero)...



...and considering two partitionings, we can always find graphs with matching initial state factors