Constraining the double parton distributions

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Plan

- Single versus double parton scattering
- QCD evolution equations in collinear limit
- Sum rules for single and double PDFs
- Initial conditions for evolution of gluons
- Perspective of adding quarks to initial conditions

Review M. Diehl, D. Ostermeier, A. Schafer, JHEP 1203 (2012) 089.

Talk based on K.Golec-Biernat, E.Lewandowska, M.Serino, Z.Snyder, A.Staśto, Phys. Lett. B750(2015)559.

Single versus double parton scattering

- Single parton scattering - single PDFs

$$\frac{d\sigma_{AB}^{SPS}}{dx\, d\overline{x}} = \sum_{ff'} D_f(x)\, \sigma_{ff'}^{\mathrm{AB}}\, D_{f'}(\overline{x})$$

- Double parton scattering - double PDFs

$$\frac{d\sigma_{AB}^{DPS}}{dx_1\, dx_2\, dx_1'\, dx_2'} = \sum_{flav} \int_{\mathbf{q}} D_{f_1 f_2}(x_1, x_2, \mathbf{q})\, \sigma_{f_1 f_1'}^{A}\, \sigma_{f_2 f_2'}^{B}\, D_{f_1' f_2'}(x_1', x_2', -\mathbf{q})$$

- Lowest order results.

- Collinear factorization cross sections in QCD introduce hard scales into PDFs.

QCD evolution equations for single PDFs

- Quark-gluon emissions with $k_\perp \in (\Lambda_{QCD}, Q)$ make single PDFs scale dependent

$$D_f(x,Q), \qquad f \in \{q_i, \overline{q}_i, g\}$$

- DGLAP evolution equations:

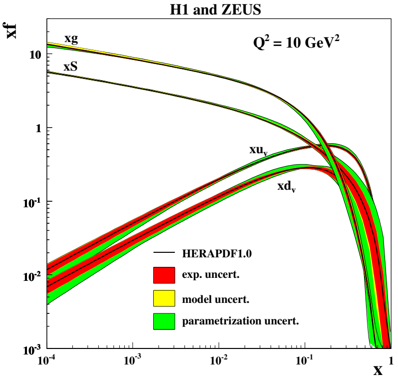
$$\frac{\partial}{\partial \ln Q^2} D_f(x,Q) = \frac{\alpha_s}{2\pi} \sum_{f'} \int_x^1 du\, \mathcal{P}_{ff'}(x/u)\, D_{f'}(u,Q)$$

- Initial conditions, $D_f(x,Q_0)$, are very well known from global fits.

- For example, Durham group fits (MSTW, etc.)

$$D_f(x,Q_0) = \sum_i A_f^i\, x^{\alpha_f^i}\, (1-x)^{\beta_f^i}$$

PDFs from global fits



- Non-homogeneous DGLAP evolution equations in LLA $(Q_1 = Q_2 \equiv Q, \mathbf{q} = 0)$

$$\begin{aligned}
\frac{\partial}{\partial \ln Q^2} D_{f_1 f_2}(x_1, x_2, Q) &= \frac{\alpha_s}{2\pi} \sum_{f'} \Bigg\{ \int_{x_1}^{1-x_2} du\, \mathcal{P}_{f_1 f'}(x_1/u)\, D_{f' f_2}(u, x_2, Q) \\
&+ \int_{x_2}^{1-x_1} du\, \mathcal{P}_{f_2 f'}(x_2/u)\, D_{f_1 f'}(x_1, u, Q) \\
&+ \mathcal{P}_{f' \to f_1 f_2}\left(\frac{x_1}{x_1 + x_2}\right) D_{f'}(x_1 + x_2, Q) \Bigg\}
\end{aligned}$$

- Additional splitting terms

(plots borrowed from M. Diehl, et al. review)

Sum rules

- We need initial conditions at initial scale Q_0, $D_f(x)$ and $D_{f_1 f_2}(x_1, x_2)$.
- Evolution equations preserve valence quark number and momentum sum rules once imposed on initial conditions.
- For single PDFs:

$$\sum_{f\in\{q,\overline{q},g\}} \int_0^1 dx\, x\, D_f(x) = 1$$

$$\int_0^1 dx\, (D_q(x) - D_{\overline{q}}(x)) = N_q$$

- For double PDFs:

$$\sum_{f_1} \int_0^{1-x_2} dx_1\, x_1\, D_{f_1 f_2}(x_1, x_2) = (1 - x_2)\, D_{f_2}(x_2)$$

$$\int_0^{1-x_2} dx_1\, \{D_{qf_2}(x_1, x_2) - D_{\bar{q}f_2}(x_1, x_2)\} = (N_q - \delta_{qf_2} + \delta_{\bar{q}f_2})\, D_{f_2}(x_2)$$

and similar for the second parton.

Existing initial conditions

- Most popular (J. Gaunt, W. J. Stirling, JHEP 1106, 048 (2011))

$$D_{f_1 f_2}(x_1, x_2) = D_{f_1}(x_1)\, D_{f_2}(x_2)\, \frac{(1 - x_1 - x_2)^2}{(1 - x_1)^{2+n_1}(1 - x_2)^{2+n_2}}$$

  do not exactly obey sum rules (especially quark number one).

- Proposed in (E. Lewandowska, K.G-B, PRD 90, 094032 (2014))

$$D_{f_1 f_2}(x_1, x_2) = \frac{1}{1 - x_2} D_{f_1}\left(\frac{x_1}{1 - x_2}\right) D_{f_2}(x_2)$$

$$D_{q(q/\overline{q})}(x_1, x_2) = \frac{1}{1 - x_2} \left\{ D_q\left(\frac{x_1}{1 - x_2}\right) \mp \frac{1}{2} \right\} D_{(q/\overline{q})}(x_2)$$

  obey sum rules for one parton only and D_{qq} is negative for large x.

- For valence quarks only from valon model (W. Broniowski, E. Ruiz Arriola, Few Body Syst. 55 (2014) 381).

Pure gluon case

- MSTW08 parameterization at $Q_0 = 1\ \mathrm{GeV}$ with known parameters A_k, α_k, η

$$D_g(x) = \sum_{k=1}^{3} A_k\, x^{\alpha_k}\, (1-x)^{\eta} \qquad (1)$$

- Solve momentum sum rule

$$\int_0^{1-x_2} dx_1\, x_1\, D_{gg}(x_1, x_2) = (1-x_2)\, D_g(x_2)$$

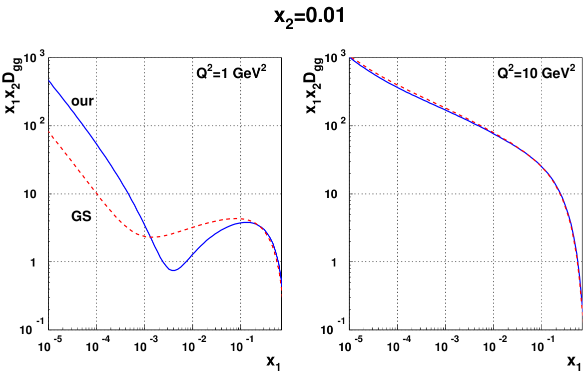
for the following form of the double gluon

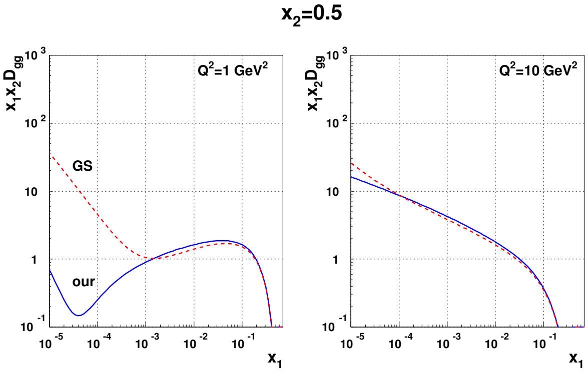
$$\int_0^{1-x_2} dx_1\, x_1 \left[\sum_{k=1}^{3} \bar{N}_k\, (x_1\, x_2)^{a_k}\, (1-x_1-x_2)^{b_k}\right] = (1-x_2) \sum_{k=1}^{3} \bar{A}_k\, {x_2}^{a_k}\, (1-x_2)^{a_k+b_k+1}$$

- From the comparison with (1)

$$\bar{A}_k = A_k \qquad a_k = \alpha_k \qquad b_k = \eta - \alpha_k - 1$$

Numerical results for $x_2 = 10^{-2}$





- Evolution to relatively low scale washes out the difference at the initial scale!

Adding quarks

- Light-cone Fock expansion of the proton state in terms of partonic states

$$|P\rangle = \sum_N \sum_{f_1 \ldots f_N} \int dx_1 \ldots dx_N \, \delta(1 - \sum_{k=1}^{N} x_k) \, \Psi_N(x_1 \ldots x_n; f_1 \ldots f_N) \, |x_1 \ldots x_N; f_1 \ldots f_N\rangle$$

  is the origin of the sum rules. Now $f_i \in \{q_i, \overline{q}_i, g\}$.

- Try to model wave functions Ψ_N

$$D_f(x) = \sum_N \sum_{f_1 \ldots f_N} \int d\Pi_n \, |\Psi_N|^2 \left\{ \sum_{i=1}^{N} \delta(x - x_i) \, \delta_{ff_i} \right\}$$

$$D_{fh}(x, y) = \sum_N \sum_{f_1 \ldots f_N} \int d\Pi_n \, |\Psi_N|^2 \left\{ \sum_{i=1}^{N} \sum_{j \neq i}^{N} \delta(x - x_i)\delta(y - x_j) \, \delta_{ff_i} \delta_{hf_j} \right\}$$

- What Ψ_N leads to MSTW08 parameterization?

A model

- Assume

$$|\Psi_N(x_1 \dots x_N; f_1 \dots f_N)|^2 = A^N_{f_1 \dots f_N}\, \phi_N(x_1, f_1) \, \dots \, \phi_N(x_N, f_N)$$

with

$$\phi_N(x_i, f_i) = x_i^{\alpha^N_{f_i} - 1}$$

- Single PDFs

$$D_f(x) = \sum_N \sum_{(f_1 \dots f_N)'} \bar{A}^N \, x^{\alpha^N_f - 1} \, (1-x)^{\alpha^N_{(f_1} + \dots + \alpha^N_{f_N)'} - 1}$$

- The small x powers $\alpha^N_{f_i}$ determine the large x powers

$$\eta^N_f = \alpha^N_{(f_1} + \dots + \alpha^N_{f_N)'}$$

- Such a relation is not built in MSTW08.

Summary

- In pure gluon case, we constructed initial double gluon distribution which fulfills momentum sum rule and gives the MSTW08 single gluon distribution.
- The difference between initial double gluon distributions is very quickly washed out by evolution to higher scales.
- Adding quarks, e.g. through modeling partonic wave functions Ψ_N, is still a challenge.
- What Ψ_N are behind single PDF parameterisations fitted to data?