Colour Dipole Cascades
Saturation, Correlations, and Fluctuations
in \( pp \) and \( pA \) collisions

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1. Introduction

**DIS at Hera:** High parton density at small $x$ grows $\sim 1/x^{1.3}$

Predicted by pert. BFKL pomeron

**$pp$ coll.:** Models based on multiple pert. parton-parton subcollisions very successful at high energies

*PYTHIA* (Sjöstrand–van Zijl 1987)
*HERWIG* also dominated by semihard subcollisions, although with soft underlying event

May be understood from unitarity constraints:

**CGC:** Suppression of partons with $k_\perp < Q_s^2$

When perturbative physics dominates, can the result be calculated from basic principles, without input pdf’s?
2. Unitarity constraints

Saturation most easily described in impact parameter space

Rescattering $\Rightarrow$ convolution in $k_\perp$-space $\rightarrow$ product in $b$-space

Unitarity $\Rightarrow$ Optical theorem: $\text{Im} A_{el} = \frac{1}{2} \{|A_{el}|^2 + \sum_j |A_j|^2\}$

$\text{Re} A_{el}^{pp}$ small $\Rightarrow$ interaction driven by absorption

Rescattering exponentiates in impact param. space:

Absorption probability in Born approx. $= 2F(b)$ $\Rightarrow$

$$\frac{d\sigma_{inel}}{d^2b} = 1 - e^{-2F(b)}$$
a) Eikonal approximation

\[ d\sigma_{inel}/d^2b = 1 - e^{-2F(b)} \]

If NO diffractive excitation:

Optical theorem \( \Rightarrow \) \( \text{Im} A_{el} \equiv T(b) = 1 - e^{-F} \)

\[ d\sigma_{el}/d^2b = T^2 = (1 - e^{-F})^2 \]
\[ d\sigma_{tot}/d^2b = 2T = 2(1 - e^{-F}) \]
\[ d\sigma_{inel}/d^2b = (2T - T^2) = 1 - e^{-2F} \]
b) Diffractive excitation

Example: A photon in an optically active medium:

Righthanded and lefthanded photons move with different velocity; they propagate as particles with different mass.

Study a beam of righthanded photons hitting a polarized target, which absorbs photons linearly polarized in the x-direction.

The diffractively scattered beam is now a mixture of right- and lefthanded photons.

If the righthanded photons have lower mass:

The diffractive beam contains also photons excited to a state with higher mass.
Good–Walker formalism:

Projectile with a substructure:

Mass eigenstates $\Psi_k$ can differ from eigenstates of diffraction $\Phi_n$ (eigenvalues $T_n$)

Elastic amplitude $= \langle \Psi_{in} | T | \Psi_{in} \rangle$

$\Rightarrow \quad d\sigma_{el} / d^2 b = \langle T \rangle^2$

Total diffractive cross section (incl. elastic):

$d\sigma_{diff \ tot} / d^2 b = \sum_k \langle \Psi_{in} | T | \Psi_k \rangle \langle \Psi_k | T | \Psi_{in} \rangle = \langle T^2 \rangle$

Diffractive excitation determined by the fluctuations:

$d\sigma_{diff \ ex} / d^2 b = d\sigma_{diff} - d\sigma_{el} = \langle T^2 \rangle - \langle T \rangle^2$
Scattering against a fluctuating target

Total diffractive excitation:

\[ d\sigma_{\text{tot.diffr.exc.}}/d^2b = \langle T^2 \rangle_{p,t} - \langle T \rangle_{p,t}^2 \]

\[ d\sigma_{\text{el}}/d^2b = \langle T \rangle_{p,t}^2 \]

Averageing over target states before squaring

⇒ the probability for an elastic interaction for the target.

Subtract \( \sigma_{\text{el}} \) → single diffr. excit.:

\[ d\sigma_{SD,p}/d^2b = \langle \langle T \rangle_t^2 \rangle_p - \langle T \rangle_{p,t}^2 \]

\[ d\sigma_{SD,t}/d^2b = \langle \langle T \rangle_p^2 \rangle_t - \langle T \rangle_{p,t}^2 \]

\[ d\sigma_{DD}/d^2b = \langle T^2 \rangle_{p,t} - \langle \langle T \rangle_t^2 \rangle_p - \langle \langle T \rangle_p^2 \rangle_t + \langle T \rangle_{p,t}^2. \]
Relation Good–Walker vs triple-pomeron

Diffractive excitation in $pp$ coll. commonly described by Mueller’s triple-pomeron formalism

Stochastic nature of the BFKL cascade $\Rightarrow$

Good–Walker and Triple-pomeron describe the same dynamics (PL B718 (2013) 1054)

But: Saturation is easier treated in the Good–Walker formalism; in particular for collisions with nuclei
3. BFKL evolution in impact param. space

a) Mueller’s Dipol model:

LL BFKL evolution in transverse coordinate space

Gluon emission: dipole splits in two dipoles:

\[ \frac{dP}{dy} = \frac{\alpha_s}{2\pi} d^2r_2 \frac{r_{01}^2 r_{12}^2}{r_{02}^2} \]
Dipole-dipole scattering

Single gluon exchange \( \Rightarrow \) Colour reconnection between projectile and target

\[
\begin{align*}
\text{Born amplitude:} & \quad f_{ij} = \frac{\alpha_s^2}{2} \ln^2 \left( \frac{r_{13} r_{24}}{r_{14} r_{23}} \right)
\end{align*}
\]

BFKL stochastic process with independent subcollisions:

Multiple subcollisions handled in eikonal approximation
b) The Lund cascade model, DIPSY

(E. Avsar, GG, L. Lönnblad, Ch. Flensburg)

Based on Mueller’s dipole model in transverse space

Includes also:

- Important non-leading effects in BFKL evol.
  (most essential rel. to energy cons. and running $\alpha_s$)
- Saturation from pomeron loops in the evolution
  (Not included by Mueller or in BK)
- Confinement $\Rightarrow t$-channel unitarity
- MC DIPSY; includes also fluctuations and correlations
- Applicable to collisions between electrons, protons, and nuclei
Saturation within evolution

Multiple interactions $\Rightarrow$ colour loops $\sim$ pomeron loops

Gluon scattering is colour suppressed $cf$ to gluon emission $\Rightarrow$
Loop formation related to identical colours.

Multiple interaction in one frame $\Rightarrow$
colour loop within evolution in another frame
Colour loop formation in a different frame

Same colour $\Rightarrow$ quadrupole

May be better described by recoupled smaller dipoles

$\Rightarrow$ smaller cross section: fixed resolution $\Rightarrow$ effective

$2 \rightarrow 1$ and $2 \rightarrow 0$ transitions

Is a form of colour reconnection

Not included in Mueller’s model or in BK equation
4.  \textit{pp} scattering
DIPSY results

Total and elastic cross sections

\[ \sigma_{\text{tot}} \text{ and } \sigma_{\text{el}} \]
Final states

Comparisons to ATLAS data at 7 TeV

Min bias
Charged particles
\( \eta \)-distrib. \( p_T \)-distrib.

Underlying event
\( N_{ch} \) in transv. region vs \( p_{\perp}^{\text{lead}} \)

Transverse \( N_{ch} \) density vs. \( p_{\perp}^{\text{clus.}} \), \( \sqrt{s} = 7 \text{ TeV} \)

Charged particle \( p_{\perp} \) at 7 TeV, track \( p_{\perp} > 500 \text{ MeV}, \) for \( N_{ch} \geq 6 \)

Charged particle \( \eta \) at 7 TeV, track \( p_{\perp} > 500 \text{ MeV}, \) for \( N_{ch} \geq 6 \)
Correlations
Double parton distributions

Correlation function $F(b)$. Depends on both $x$ and $Q^2$

Spike (hotspot) develops for small $b$ at larger $Q^2$

Spike for small $b \Rightarrow$ tail for large momentum imbalance $\Delta$, in transverse momentum space
Fluctuations and diffraction

What are the diffractive eigenstates?
Parton cascades, which can come on shell through interaction with the target.

BFKL dynamics $\Rightarrow$ Large fluctuations,
Continuous distrib. up to high masses
(Also Miettinen–Pumplin (1978), Hatta et al. (2006))
Single diffraction in $pp$ 1.8 TeV

$$\int dM_X^2 \frac{d\sigma_{SD}}{dM_X^2} \quad \text{for} \quad M_X < M_X^{(cut)}$$

Shaded area: Estimate of CDF result

Note: Tuned only to $\sigma_{tot}$ and $\sigma_{el}$. No new parameter
5. Collisions with nuclei

Initial state:

DIPSY gives full partonic picture, dense gluon soup.

Ex.: $Pb - Pb$ 200 GeV/N

Accounts for:

- saturation within the cascades,
- correlations and fluctuations in partonic state,
- finite size effects
Understanding the initial state essential for interpretation of collective final state effects

Models for initial state in AA collisions can be tested in pA

Study coherence effects in total, elastic, and diffractive cross sections
$pA$ collisions

Test: DIPSY agrees with CMS and LHCb inelastic cross section

General features

Scaling:

If \( pp \) interaction transparent

\[ \sigma_{tot}^{pA} \approx A \sigma_{tot}^{pp} \]

If black limit absorber

\[ \sigma_{tot}^{pA} \propto (A^{1/3} + 1)^2 \]

\( pp \) interaction rather close to black
b. Colour interference between different nucleons

Ratio: \( \text{no colour interference between different nucleons} \)

\( \text{include colour interference} \)

Small effect for \( pA \), which is close to black
\(~ 10\% \) effect for \( \gamma^* Au \), which is more transparent

Approximately independent of energy
6. Problems with the Glauber model

The Glauber model is frequently used in analyses of experimental data, in particular for estimating number of wounded nucleons and number of binary $NN$ collisions.

Note: A projectile in a state, $k$, penetrating the target and not absorbed, may contribute to diffractive excitation.

A projectile nucleon is wounded, or absorbed, when it has exchanged colour with the target.

Wounded nucleons correspond to the inelastic NON-diffractive cross section.
Glauber model, general formalism

Study a projectile proton at impact param. $b$, hitting a nucleus with $A$ nucleons at positions $b_\nu (\nu = 1, \ldots, A)$

Rescattering corresponds to a product in $b$-space:

$\Rightarrow$ S-matrix factorizes: $S^{(pA)}(b) = \prod_{\nu=1}^{A} S^{(pp,\nu)}(b - b_\nu)$

$\Rightarrow$ Elastic amplitude:

$T^{(pA)}(b) = 1 - \prod_{\nu=1}^{A} S^{(pp,\nu)}(b - b_\nu) = 1 - \prod_\nu \{ 1 - T^{(pp,\nu)}(b - b_\nu) \}$
Gribov corrections

A proton may fluctuate between different diffractive eigenstates

⇒ diffractive excitation

- The projectile is frozen in the same state, $k$, during the passage through the nucleus

- The target nucleons are in different, uncorrelated states $l_\nu$.

⇒ Elastic $pA$ scattering amplitude:

$$\langle T^{(pA)}(b) \rangle = 1 - \langle \prod_\nu \langle \{ 1 - T^{(pp,\nu)}_{k,l_\nu}(b - b_\nu) \} \rangle_{l_\nu} \rangle_{b_\nu} \rangle_k$$

with

$$d\sigma^{pA}_{tot}/d^2b = 2 \langle T^{(pA)}(b) \rangle,$$

$$d\sigma^{pA}_{el}/d^2b = \langle T^{(pA)}(b) \rangle^2$$
These averages involve higher moments \( \langle \langle T^{(pp)} \rangle^n_{\text{targ}} \rangle_{\text{proj}} \)

Can be calculated if the full distribution

\[
dP/d \langle T^{(pp)}(b) \rangle_{\text{targ}}
\]

is known, for all possible projectile states
Absorptive cross section and wounded nucleons

Absorption probability: \( d\sigma_{\text{abs}}/d^2b = 1 - S^2 \)

\( S^2 \) also factorizes

Absorptive (inelastic non-diffractive) cross section:
\[
d\sigma^{pA}_{\text{abs}}/d^2b = \langle 1 - \prod_\nu (S^{pp,\nu})^2 \rangle
\]

This involves also higher powers \( \langle (\langle T^{pp} \rangle_{\text{targ}})^n \rangle_{\text{proj}} \)

Wounded nucleons

\( (S^{pp,\nu})^2 = \) probability that target nucleon \( \nu \) is not absorbed

Average prob. for nucleon \( \nu \) to be wounded:
\[
1 - \langle (S_{k,l,\nu}^{pp,\nu})^2 \rangle_{k,l,\nu} = \langle 2T_{k,l,\nu}^{pp,\nu} - (T_{k,l,\nu}^{pp,\nu})^2 \rangle
\]
b) The model by Strikman and coworkers

(Blättel et al. 1993, also called Glauber–Gribov–colour–fluctuation model (GGCF))

**Total cross section:**

Notation: \( \hat{\sigma}_{tot} = 2 \int d^2 b \langle T^{(pp)}(b) \rangle_{targ} \)

= fluctuating \( pp \) total cross section, averaged over target states

Average also over projectile states \( \Rightarrow \sigma^{(pp)}_{tot} = \langle \hat{\sigma}_{tot} \rangle_{proj} \)

**Ansatz:**

\[ \frac{dP}{d\hat{\sigma}_{tot}} = \rho \frac{\hat{\sigma}_{tot}}{\hat{\sigma}_{tot} + \sigma_{0}^{tot}} \exp \left\{ -\frac{(\hat{\sigma}_{tot}/\sigma_{0}^{tot} - 1)^2}{\Omega^2} \right\} \]

\( \Omega \) is a parameter determining the fluctuations, related to \( \sigma_{SD}^{(pp)} \)

\( \sigma_{0}^{tot} \) is fixed from \( \sigma^{(pp)}_{tot} \); \( \rho \) is a normalization constant.
Absorptive cross section:

Notation: Fluctuating $pp$ absorptive cross section, averaged over target states:

$$\hat{\sigma}_{\text{abs}} = \int d^2 b \left\langle \left\{ 2T^{(pp)}(b) - T^{(pp)^2}(b) \right\} \right\rangle_{\text{targ}}$$

(Strikman et al. use the notation $\sigma_{\text{in}}$)

$$\sigma^{(pp)}_{\text{abs}} = \langle \hat{\sigma}_{\text{abs}} \rangle_{\text{proj}}$$

The same form is used, but $\Omega$ need not be the same:

$$\frac{dP}{d\hat{\sigma}_{\text{abs}}} = \rho' \frac{\hat{\sigma}_{\text{abs}}}{\hat{\sigma}_{\text{abs}} + \sigma^{\text{abs}}_0} \exp \left\{ -\left(\frac{\hat{\sigma}_{\text{abs}}/\sigma^{\text{abs}}_0 - 1}{\Omega^2}\right)^2 \right\}$$

Note: $\sigma^{\text{abs}}_0$ ought to be adjusted to $\sigma^{pp}_{\text{inel ND}}$, but is often tuned to $\sigma^{pp}_{\text{inel tot}}$!
c) DIPSY results

Distribution in $\hat{\sigma}_{tot}$

Compared with GGCF (tuned to the DIPSY $\sigma_{tot} = 89.6$ mb)
Distribution in $\hat{\sigma}_{abs}$

DIPSY compared with GGCF

GGCF normalized to $\sigma_{abs}$

GGCF normalized to $\sigma_{inel}$ as used by ATLAS
Distribution in no. of wounded nucleons

$pPb$ at 5 TeV

DIPSY (with $\sigma_{abs} = 58 \text{ mb}$)  

ATLAS using $\sigma_{tot \text{ inel}} = 70 \text{ mb}$
7. Conclusions

Saturation suppresses low-$p_{\perp}$ gluons
$\Rightarrow$ high energy hadronic collisions dominantly perturbative

Can therefore the initial state properties be understood from basic principles, without input pdf’s?

The DIPSY dipole cascade model is based on BFKL dynamics with non-leading corrections and saturation. It reproduces HERA structure fcns, and gives a fair description of $pp$ data, with no input pdf’s

MC implementation gives also correlations and fluctuations (diffraction)
$pA$ scattering intermediate step between $pp$ and $AA$
Possible to test models for initial state properties
via total, elastic, and diffractive cross sections

Glauber model frequently used in experimental analyses
Gribov pointed out importance of diffractive scattering (1955)
Frequently not treated in a proper way

A projectile nucleon in a diffractive eigenstate may pass
unharmed through the target, and yet contribute to the inelastic
(diffractive) scattering

Wounded nucleons determined by the non-diffractive inelastic
cross section
Extra slides
\( \gamma^* A \) collisions

(Note: \( \gamma^* \rightarrow q\bar{q} \) frozen during passage through nucleus)

\[ \gamma^* O/A \cdot \gamma^* p \]

\[ \gamma^* O/\gamma^* p \text{ total cross section ratios} \]

\[ \gamma^* Au/A \cdot \gamma^* p \]

\[ \gamma^* Au/\gamma^* p \text{ total cross section ratios} \]

\( \gamma^* p \) scaling closer to \( \sim A \sigma_{\gamma^*} \).

More transparent (and more so for high \( Q^2 \))

\( \Rightarrow \) dynamic effects more visible
### Results for $pPb$ at 5 TeV

<table>
<thead>
<tr>
<th>Model</th>
<th>DIPSY ((\sigma_{\text{tot}}))</th>
<th>Black disc ((\sigma_{\text{in}}))</th>
<th>Black disc ((\sigma_{\text{in,ND}}))</th>
<th>Black disc ((\sigma_{\text{tot}}, \sigma_{\text{el}}))</th>
<th>Grey disc ((\sigma_{\text{tot}}, \sigma_{\text{el}}))</th>
<th>New disc ((\sigma_{\text{tot}}, \sigma_{\text{el}}, \sigma_{\text{DD}}, \sigma_{\text{SD}}))</th>
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</thead>
<tbody>
<tr>
<td>(\sigma_{\text{tot}}) (b)</td>
<td>3.54</td>
<td>3.50</td>
<td>3.88</td>
<td>3.73</td>
<td>3.69</td>
<td>3.54</td>
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<tr>
<td>(\sigma_{\text{in}}) (b)</td>
<td>2.04</td>
<td>1.95</td>
<td>2.14</td>
<td>2.06</td>
<td>2.07</td>
<td>2.02</td>
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<tr>
<td>(\sigma_{\text{in,ND}}) (b)</td>
<td>1.89</td>
<td>1.75</td>
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<td>1.86</td>
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<tr>
<td>(\sigma_{\text{el}}) (b)</td>
<td>1.51</td>
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<td>1.73</td>
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<tr>
<td>(\sigma_{\text{SD},A}) (b)</td>
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<td>(\sigma_{\text{el}^*}) (b)</td>
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<td>1.86</td>
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<tr>
<td>(\sigma_{\text{el}^*/\sigma_{\text{in}}})</td>
<td>0.78</td>
<td>0.90</td>
<td>0.91</td>
<td>0.90</td>
<td>0.82</td>
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<tr>
<td>(\sigma_{\text{in,ND}}/\sigma_{\text{tot}})</td>
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<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.53</td>
</tr>
</tbody>
</table>

GG, L Lönnblad, A Ster, T Csörgő, JHEP 1510 (2015) 022
Final state saturation, Ropes


Old problem: $s/u$ ratio higher in $pp$ than in $e^+e^-$
LHC: Higher fractions of strange particles and baryons

Old proposal (Biro-Nielsen-Knoll 1984):
Many strings close in transverse space may form “ropes”
DIPSY: Extension of strings in \((r_\perp, y)\)-space in \(pp\) at 7 TeV

Radius set to 0.1 fm for more clear picture
String diameter \(\sim 1\) fm \(\Rightarrow\) a lot of overlap
Assume strings within a radius $R$ interact coherently

Ex.: 3 uncorrelated triplets

$\{3, 0\} = 10$: 

rope tension $4.5\kappa_0$; decays in 3 steps

$\{1, 1\} = 8$: 

rope tension $2.25\kappa_0$; decays in 2 steps

$\{0, 0\} = 1$: 

no force field
Results

Ratios $p/\pi$ and $\Lambda/K^0_S$ vs $p_\perp$ at 200 GeV. Data from STAR.
$\Lambda/K_s^0$ ratio vs rapidity at 0.9 and 7 TeV. Data from CMS
Impact parameter profile

Saturation $\Rightarrow$ Fluctuations suppressed in central collisions

Diffr. excit. largest in a circular ring,
expanding to larger radius at higher energy

Factorization broken between $pp$ and DIS
Exclusive final states in diffraction

If gap events are analogous to diffraction in optics ⇒
Diffractive excitation fundamentally a quantum effect
Different contributions interfere destructively,
no probabilistic picture
Still, different components can be calculated in a MC,
added with proper signs, and squared
Possible because opt. th. ⇒ all contributions real
(Makes it also possible to take Fourier transform and get $d\sigma/dt$.
JHEP 1010, 014, arXiv:1004.5502)
Early results for DIS and $pp$

**H1:** $W = 120$, $Q^2 = 24$

$dn_{ch}/d\eta$ in 2 $M_X$-bins

**UA4:** $W = 546$ GeV

$\langle M_X \rangle = 140$ GeV

Too hard in proton fragmentation end. Due to lack of quarks in proton wavefunction

Has to be added in future improvements

**Note:** Based purely on fundamental QCD dynamics