Application and calculation of amplitudes with off-shell partons

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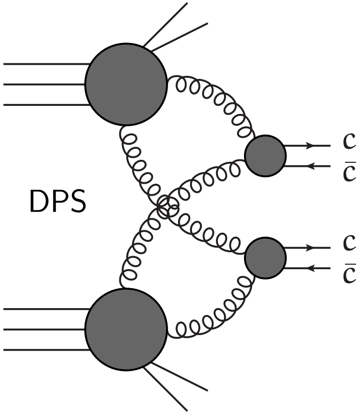
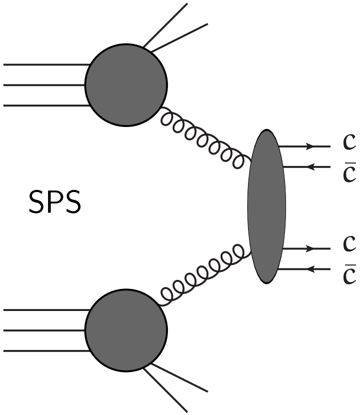
Outline

- DPS vs SPS for $pp \longrightarrow c\bar{c}\,c\bar{c}$
- k_T-factorization vs collinear factorization for $pp \longrightarrow c\bar{c}\,c\bar{c}$
- Off-shell amplitudes
- BCFW recursion for off-shell amplitudes
- Conclusions

DPS vs SPS for $pp \to c\bar{c}\, c\bar{c}$

- production of $c\bar{c}\, c\bar{c}$ is a good place to study DPS effects Łuszczak, Maciuła, Szczurek 2012
- DPS $c\bar{c}\, c\bar{c}$ cross section approaches $c\bar{c}$ cross section for large energies
- DPS $c\bar{c}\, c\bar{c}$ cross section is orders of magnitude larger than LO SPS $c\bar{c}\, c\bar{c}$ cross section Schäfer, Szczurek 2012, Maciuła, Szczurek, AvH 2014
- LHCb measured a surprisingly large cross section for the production of D-meson pairs JHEP 06 141 (2012)





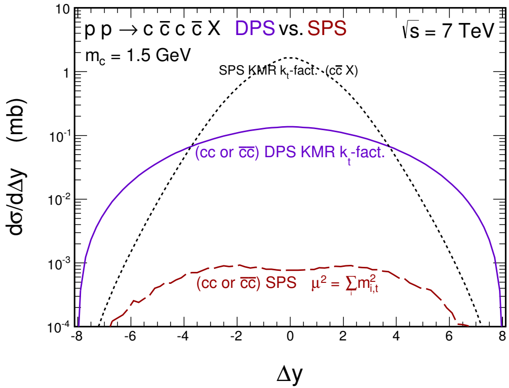
DPS vs SPS for $pp \longrightarrow c\bar{c}\,c\bar{c}$

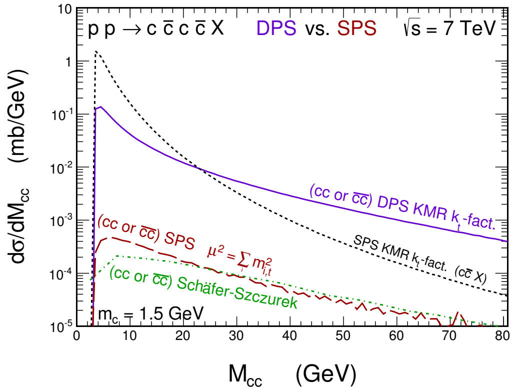
Maciuła, Szczurek, AvH 2014

Simple factorized model

$$d\sigma^{DPS}(pp \to c\bar{c}\,c\bar{c}X) = \frac{1}{2\sigma_{eff}}\, d\sigma^{SPS}(pp \to c\bar{c}X_1)\, d\sigma^{SPS}(pp \to c\bar{c}X_2)$$

with $\sigma_{eff} = 15\text{mb}$.





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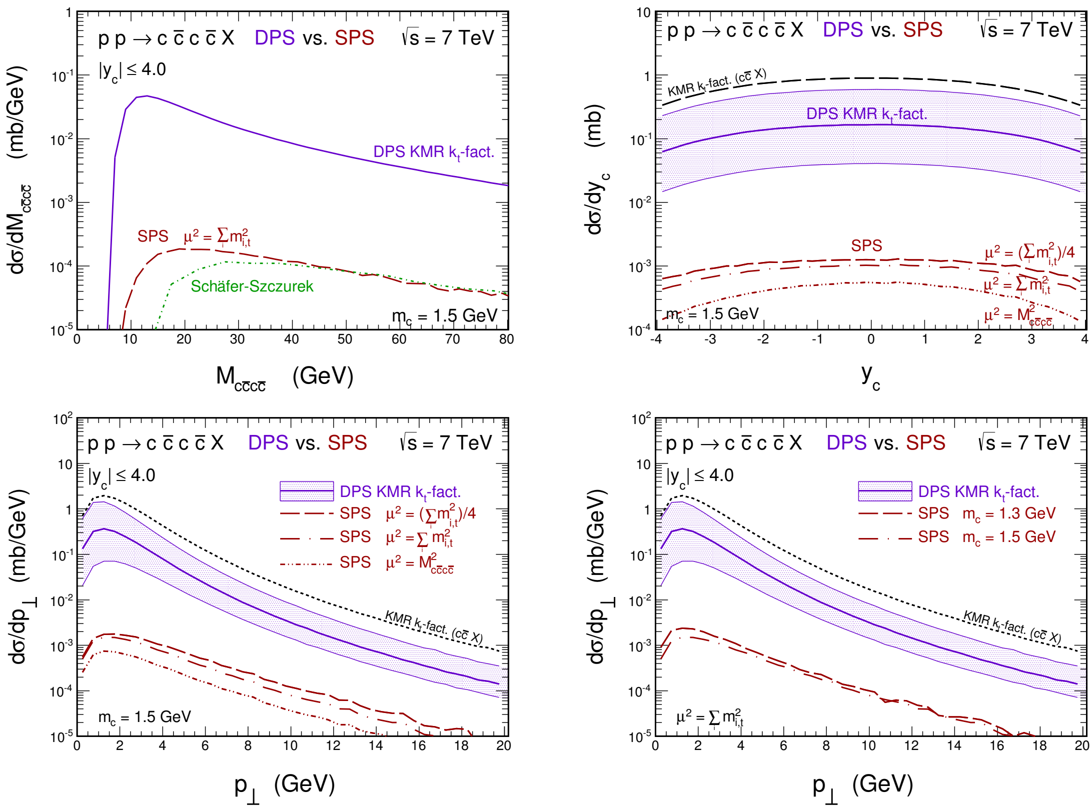


High Energy Factorization a.k.a. k_T-factorization

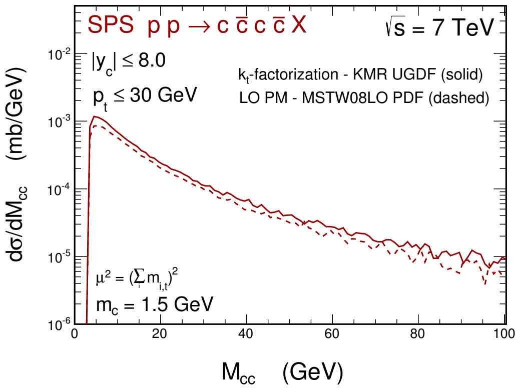
Catani, Ciafaloni, Hautmann 1991 Collins, Ellis 1991

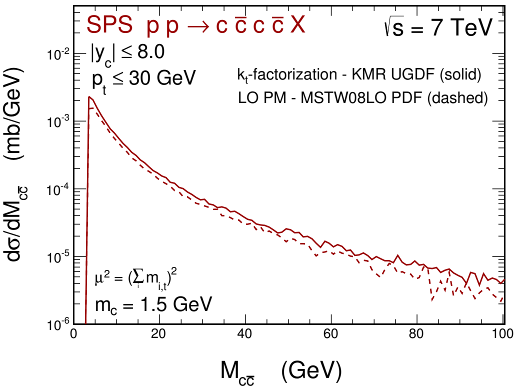
$$\sigma_{h_1,h_2\to QQ} = \int d^2k_{1\perp}\,\frac{dx_1}{x_1}\,\mathcal{F}(x_1,k_{1\perp})\,d^2k_{2\perp}\,\frac{dx_2}{x_2}\,\mathcal{F}(x_2,k_{1\perp})\,\hat{\sigma}_{gg}\left(\frac{m^2}{x_1x_2s},\frac{k_{1\perp}}{m},\frac{k_{2\perp}}{m}\right)$$

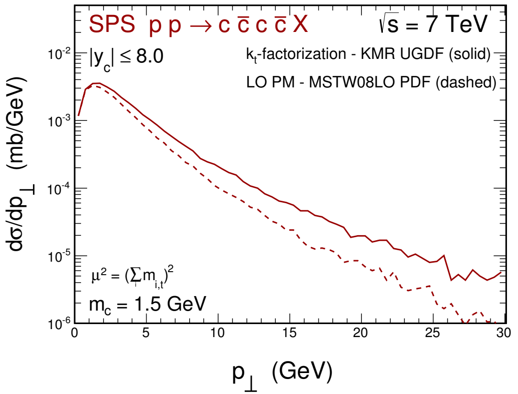
- to be applied in the 3-scale regime $s \gg m^2 \gg \Lambda^2_{QCD}$
- reduces to collinear factorization for $s \gg m^2 \gg k_\perp^2$, but holds al so for $s \gg m^2 \sim k_\perp^2$
- typically associated with small-x physics
- relevant for forward physics, saturation physics, heavy-ion physics...
- $k_\perp$ gives a handle on the size of the proton
- allows for higher-order kinematical effects at leading order
- requires matrix elements with *off-shell* initial-state partons with $k_i = x_ip_i + k_{i\perp}$
- $k_\perp$-dependent $\mathcal{F}$ may satisfy BFKL-eqn, CCFM-eqn, BK-eqn, KGBJS-eqn, ...
- in particular KMR-type unintegrated pdfs contain essential hard scale dependence via Sudakov resummation

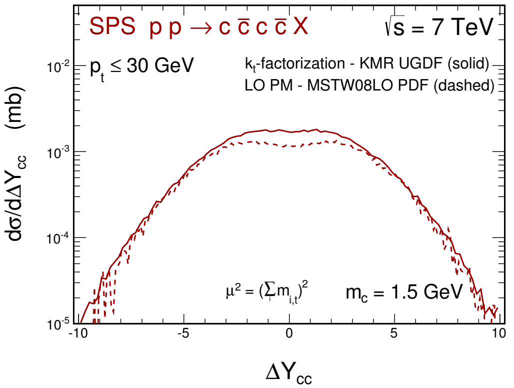
k_T vs collinear for $pp \to c\bar{c}\,c\bar{c}$

Maciuła, Szczurek, AvH 2015



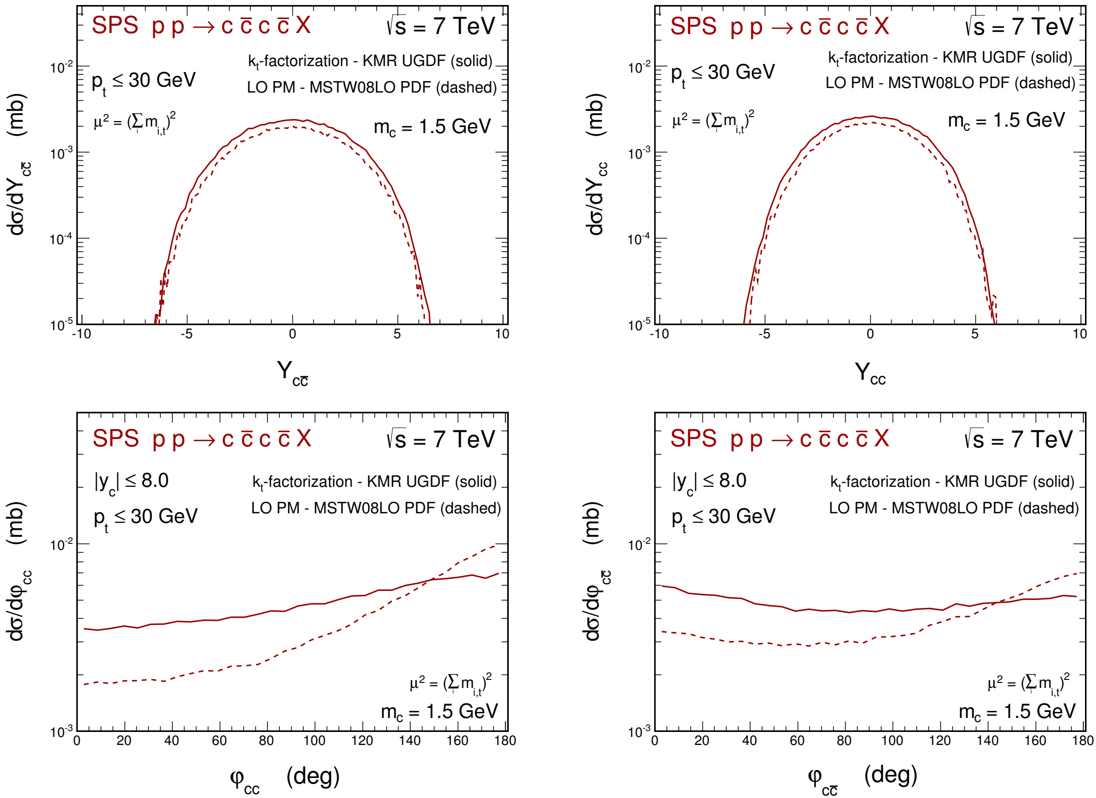






k_T vs collinear for $pp \longrightarrow c\bar{c}\,c\bar{c}$

Maciuła, Szczurek, AvH 2015



Scattering amplitude

The scattering amplitude

is the residue of the connected Green function obtained for the kinematical situation in which all external momenta are on-shell, and satisfy momentum conservation.

Off-shell currents,

or Green functions with both virtual and on-shell external lines, satisfy the recursive *Dyson-Schwinger equations* :

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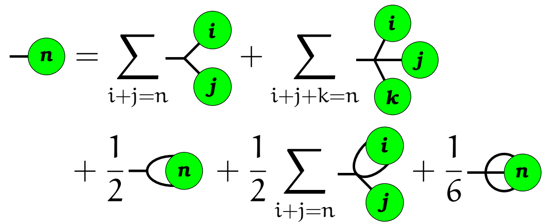
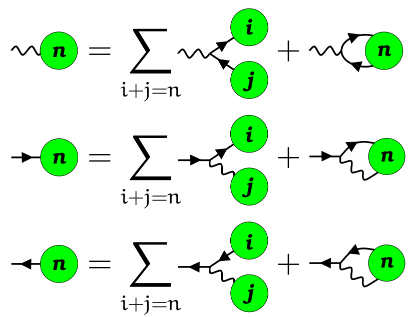
Off-shell currents,
or Green functions with both virtual and on-shell external lines, satisfy the recursive *Dyson-Schwinger equations*:

- Sums are over partitions of on-shell particles over the blobs, and over possible flavors for virtual particles.
- Current with $n = \#\text{externalparticles} - 1$ is completely on-shell and gives the amplitude.
- Solution can be represented as a sum of Feynman graphs,
- but recursion can also be used to construct amplitude directly.

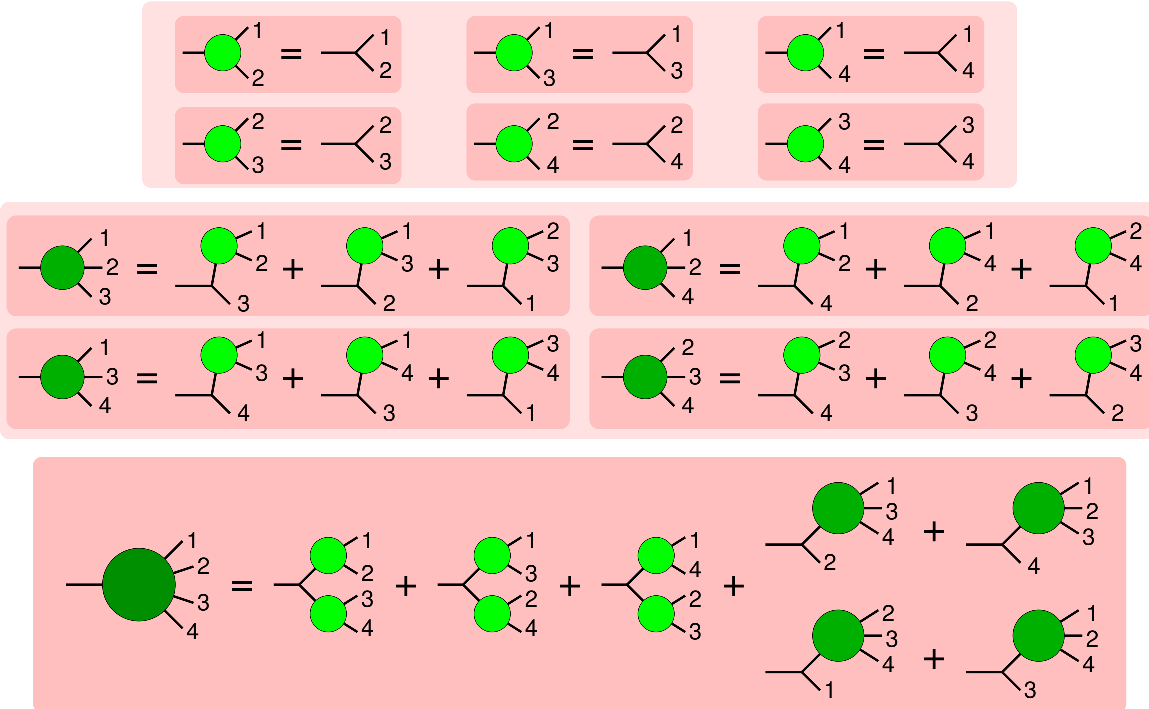
Theories with four-point vertices:



Theories with more types of currents:



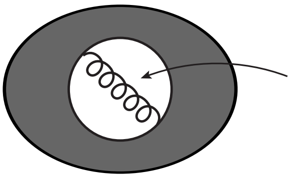
Dyson-Schwinger at tree-level



Gauge invariance

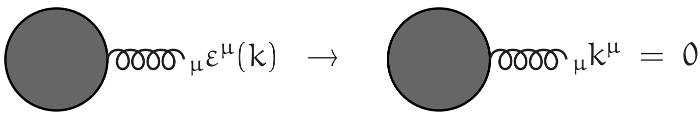
In order to be physically relevant, any scattering amplitude following the constructive definition given before must satisfy the following

Freedom in choice of gluon propagator:



$$\begin{cases} \dfrac{-i}{k^2}\left[g^{\mu\nu} - (1-\xi)\dfrac{k^\mu k^\nu}{k^2}\right] \\[2ex] \dfrac{-i}{k^2}\left[g^{\mu\nu} - \dfrac{k^\mu n^\nu + n^\mu k^\nu}{k\cdot n} + (n^2+\xi k^2)\dfrac{k^\mu k^\nu}{(k\cdot n)^2}\right] \end{cases}$$

Ward identity:

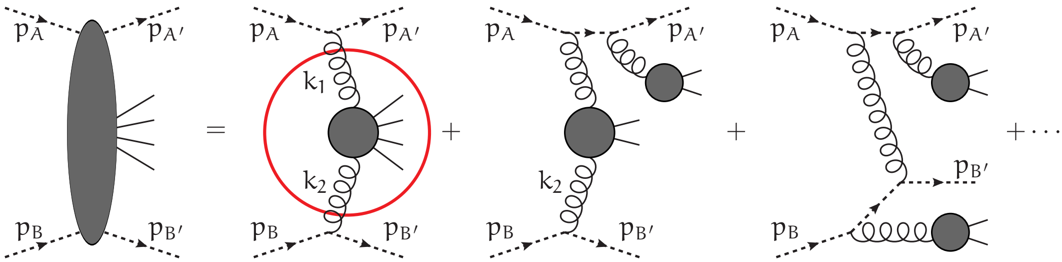


- Only holds if all external particles are on-shell.
- k_T-factorization requires off-shell initial-state momenta $k^\mu = p^\mu + k_T^\mu$.
- How to define amplitudes with off-shell intial-state momenta?

Amplitudes with off-shell partons

AvH, Kutak, Kotko 2013:

Embed the process in an on-shell process with auxiliary partons



Hadron momenta p_1, p_2:

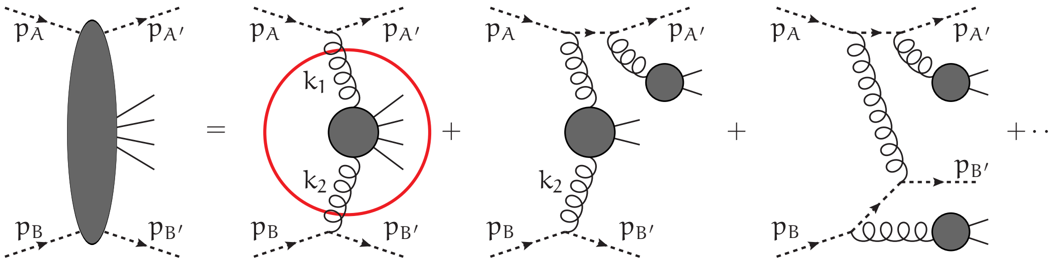
$p_1 \cdot p_A = p_1 \cdot p_{A'} = p_1 \cdot k_1 = 0$

$p_2 \cdot p_B = p_2 \cdot p_{B'} = p_2 \cdot k_2 = 0$

Amplitudes with off-shell partons

AvH, Kutak, Kotko 2013:

Embed the process in an on-shell process with auxiliary partons and eikonal Feynman rules.



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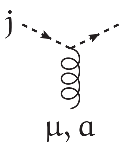
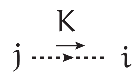


$= -i\, T^a_{i,j}\, p_1^\mu$



$= \delta_{i,j} \dfrac{i}{p_1 \cdot K}$

Amplitudes with off-shell partons

AvH, Kutak, Kotko 2013, AvH, Kutak, Salwa 2013:
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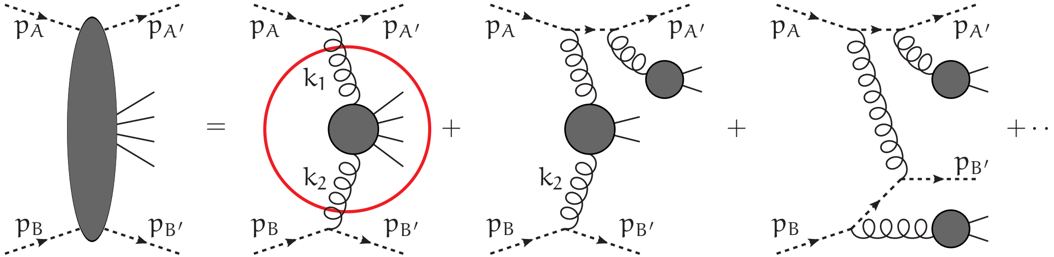

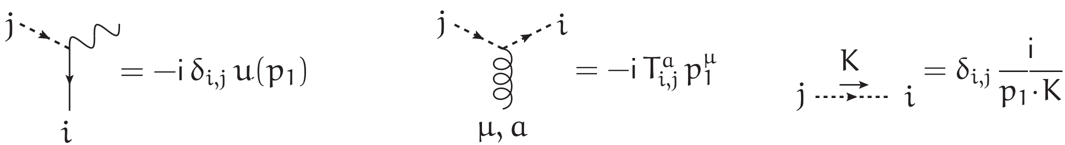

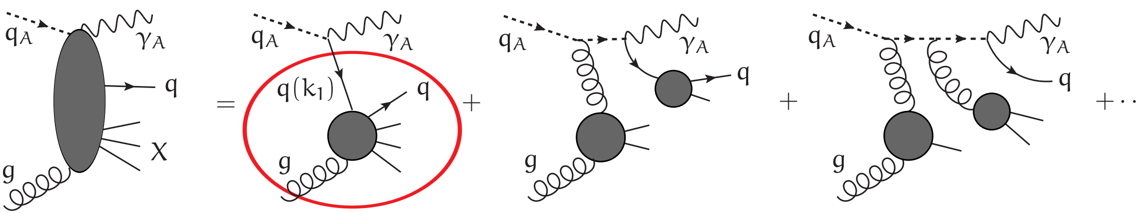

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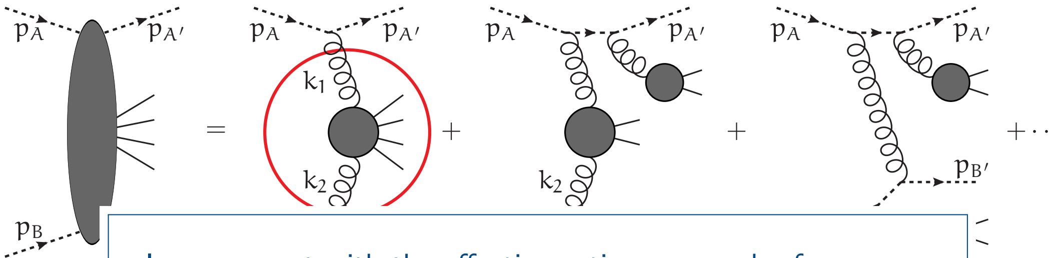
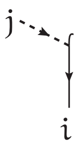
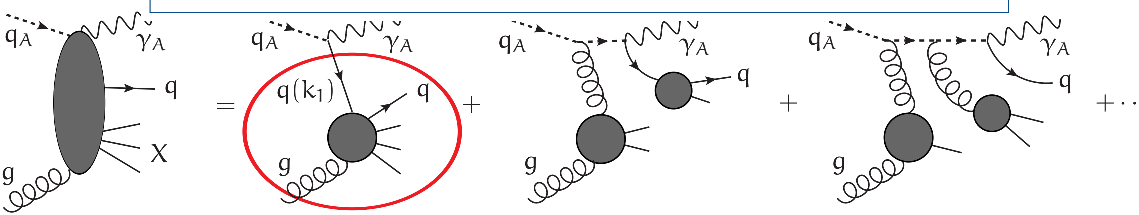




$$\delta_{i,j}\,\frac{i}{p_1\cdot K}$$

In agreement with the effective action approach of
Lipatov 1995, Antonov, Lipatov, Kuraev, Cherednikov 2005
Lipatov, Vyazovsky 2000, Nefedov, Saleev, Shipilova 2013
and the Wilson-line approach of
Kotko 2014



BCFW recursion for on-shell amplitudes

$$\mathcal{A}(i^-, j^-, (\text{the rest})^+) = \frac{\langle p_i p_j \rangle^4}{\langle p_1 p_2 \rangle \langle p_2 p_3 \rangle \cdots \langle p_{n-2} p_{n-1} \rangle \langle p_{n-1} p_n \rangle \langle p_n p_1 \rangle}$$

Color decomposition of multi-gluon amplitudes in terms of color-ordered amplitudes

$$\mathcal{M}\big(\{p_i, \lambda_i, a_i\}\big) = \sum_{\sigma \in S_{n-1}} \mathrm{Tr}\big(T^{a_{\sigma(1)}} T^{a_{\sigma(2)}} \cdots T^{a_{\sigma(n-1)}} T^{a_n}\big)\, \mathcal{A}\big(\sigma(1), \sigma(2), \ldots, \sigma(n-1), n\big)$$

The *color-ordered* or *dual* amplitudes $\mathcal{A}$ are functions of the momenta and helicities only.

This decomposition also holds for off-shell gluons.

BCFW recursion for on-shell amplitudes

Multi-gluon amplitudes have much simpler expressions than one would expect from the Feynman graphs, in particular the MHV amplitudes:

$$\mathcal{A}(i^-, j^-, (\text{the rest})^+) = \frac{\langle p_i p_j \rangle^4}{\langle p_1 p_2 \rangle \langle p_2 p_3 \rangle \cdots \langle p_{n-2} p_{n-1} \rangle \langle p_{n-1} p_n \rangle \langle p_n p_1 \rangle}$$

BCFW recursion (Britto, Cachazo, Feng, Witten 2005) allows for easy construction of such simple expressions

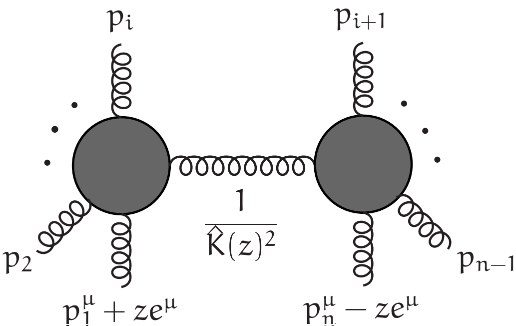
- it is a recursion of *on-shell amplitudes*, rather than off-shell Green functions
- it is most efficiently applied as a recursion of *expressions*
- it is easily proven using Cauchy's theorem

For a rational function f of a complex variable z which vanishes at infinity, we have

$$\oint_R \frac{dz}{2\pi i} \frac{f(z)}{z} \stackrel{R\to\infty}{=} 0 \qquad \Rightarrow \qquad f(0) = \sum_i \frac{\text{Residue}(f \ @ \ z = z_i)}{-z_i}$$

This is applied to amplitudes by turning them into functions of a complex variable by analytical continuation of the momenta to complex values.

BCFW recursion for on-shell amplitudes



$$\hat{K}^\mu(z) = p_1^\mu + \cdots + p_i^\mu + ze^\mu$$
$$= -p_{i+1}^\mu - \cdots - p_n^\mu + ze^\mu$$

$$e^\mu = \tfrac{1}{2}\langle p_1|\gamma^\mu|p_n]$$

$$\hat{K}(z)^2 = 0 \quad \Leftrightarrow \quad z = -\frac{(p_1 + \cdots + p_i)^2}{2(p_2 + \cdots + p_i)\cdot e}$$

$$\mathcal{A}(1^+, 2, \ldots, n-1, n^-) = \sum_{i=2}^{n-1} \sum_{h=+,-} \mathcal{A}(\hat{1}^+, 2, \ldots, i, -\hat{K}_{1,i}^h)\, \frac{1}{K_{1,i}^2}\, \mathcal{A}(\hat{K}_{1,i}^{-h}, i+1, \ldots, n-1, \hat{n}^-)$$

$$\mathcal{A}(1^+, 2^-, 3^-) = \frac{\langle 23\rangle^3}{\langle 31\rangle\langle 12\rangle} \quad , \quad \mathcal{A}(1^-, 2^+, 3^+) = \frac{[32]^3}{[21][13]}$$

Amplitudes with off-shell gluons

n-parton amplitude is a function of n momenta $k_1, k_2, \ldots, k_n$ and n *directions* $p_1, p_2, \ldots, p_n$, satisfying the conditions

$$k_1^\mu + k_2^\mu + \cdots + k_n^\mu = 0 \qquad \text{momentum conservation}$$
$$p_1^2 = p_2^2 = \cdots = p_n^2 = 0 \qquad \text{light-likeness}$$
$$p_1 \cdot k_1 = p_2 \cdot k_2 = \cdots = p_n \cdot k_n = 0 \qquad \text{eikonal condition}$$

With the help of an auxiliary four-vector q^μ with $q^2 = 0$, we define

$$k_T^\mu(q) = k^\mu - x(q) p^\mu \quad \text{with} \quad x(q) \equiv \frac{q \cdot k}{q \cdot p}$$

Construct k_T^μ explicitly in terms of p^μ and q^μ:

$$k_T^\mu(q) = -\frac{\kappa}{2}\frac{\langle p|\gamma^\mu|q]}{[pq]} - \frac{\kappa^*}{2}\frac{\langle q|\gamma^\mu|p]}{\langle qp\rangle} \quad \text{with} \quad \kappa = \frac{\langle q|\not{k}|p]}{\langle qp\rangle} \;,\; \kappa^* = \frac{\langle p|\not{k}|q]}{[pq]}$$

$k^2 = -\kappa\kappa^*$ is independent of q^μ, but also individually κ and κ^* are independent of q^μ.

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$$\frac{\langle q|\not{k}|p]}{\langle qp\rangle} = \frac{\langle q|\not{k}|p]\langle pr\rangle}{\langle qp\rangle\langle pr\rangle} = \frac{\langle q|\not{k}\not{p}|r\rangle}{\langle qp\rangle\langle pr\rangle} = \frac{\langle q|2k\cdot p - \not{p}\not{k}|r\rangle}{\langle qp\rangle\langle pr\rangle} = -\frac{\langle qp\rangle[p|\not{k}|r\rangle}{\langle qp\rangle\langle pr\rangle} = \frac{\langle r|\not{k}|p]}{\langle rp\rangle}$$

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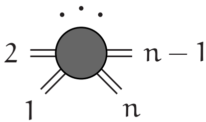
$k^2 = -\kappa\kappa^*$ is independent of q^μ, but also individually κ and κ^* are independent of q^μ.

Besides the spinors of directions and light-like momenta, κ and κ^* will show up in expressions for off-shell amplitudes.

BCFW recursion for off-shell amplitudes

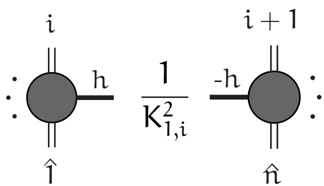
The BCFW recursion formula becomes



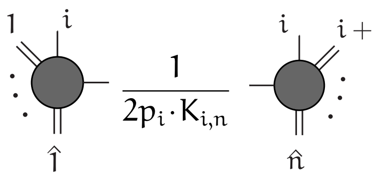
$$= \sum_{i=2}^{n-2} \sum_{h=+,-} A_{i,h} + \sum_{i=2}^{n-1} B_i + C + D \,,$$

where

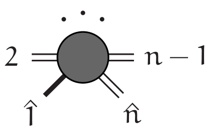
$A_{i,h} =$



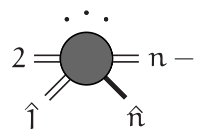
$B_i =$



$C = \dfrac{1}{\kappa_1}$



$D = \dfrac{1}{\kappa_n^*}$



The hatted numbers label the shifted external gluons.

Example of a 4-gluon amplitude

$$\mathcal{A}(1^*,2^-,3^*,4^+) = \frac{\langle 13\rangle^3[13]^3}{\langle 34\rangle\langle 41\rangle\langle 1|\not{k}_3+\not{p}_4|3]\langle 3|\not{k}_1+\not{p}_4|1][32][21]}$$
$$+\frac{1}{\kappa_1^*\kappa_3}\frac{\langle 12\rangle^3[43]^3}{\langle 2|\not{k}_3|4]\langle 1|\not{k}_3+\not{p}_4|3](k_3+p_4)^2}+\frac{1}{\kappa_1\kappa_3^*}\frac{\langle 23\rangle^3[14]^3}{\langle 2|\not{k}_1|4]\langle 3|\not{k}_1+\not{p}_4|1](k_1+p_4)^2}$$

- Eventual matrix element needs factor $k_1^2k_3^2 = |\kappa_1|^2|\kappa_3|^2$. This *must not* be included at the amplitude level not to spoil analytic structure.
- Last two terms dominate for $|k_1|\to 0$ and $|k_3|\to 0$, and give the on-shell helicity amplitudes in that limit.

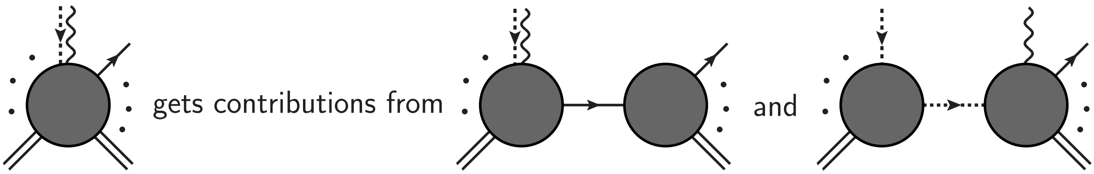
$$\mathcal{A}(1^*,2^-,3^*,4^+) \xrightarrow{|k_1|,|k_3|\to 0} \frac{1}{\kappa_1^*\kappa_3}\mathcal{A}(1^-,2^-,3^+,4^+)+\frac{1}{\kappa_1\kappa_3^*}\mathcal{A}(1^+,2^-,3^-,4^+)$$

- Coherent sum of amplitudes becomes incoherent sum of squared amplitudes via angular integrations for $\vec{k}_{1T}$ and $\vec{k}_{3T}$.

BCFW recursion with (off-shell) quarks

AvH, Serino 2015

- on-shell case treated in Luo, Wen 2005
  - only gluons are shifted
  - restrictions on allowed combinations of helicities and shift vectors
  - not always possible to have minimal number of terms by shifting adjacent gluons
- any off-shell parton can be shifted: propagators of "external" off-shell partons give the correct power of z in order to vanish at infinity
- different kinds of contributions in the recursion



- many of the MHV amplitudes come out as expected
- some more-than-MHV amplitudes do not vanish, but are sub-leading in k_T

$$\mathcal{A}(1^+, 2^+, \ldots, n^+, \bar{q}^*, q^-) = \frac{-\langle \bar{q} q \rangle^3}{\langle 12 \rangle \langle 23 \rangle \cdots \langle n \bar{q} \rangle \langle \bar{q} q \rangle \langle q 1 \rangle}$$

Conclusions

- Double-parton scattering gives an important contribution to the cross section for the process $pp \to c\bar{c}\, c\bar{c}$.
- This is confirmed by comparing with single-parton scattering at tree-level both in collinear factorization and k_T-factorization.
- Factorization prescriptions with explicit k_T dependence in the pdfs ask for hard matrix elements with off-shell initial-state partons.
- The necessary amplitudes can be defined in a manifestly gauge invariang manner that allows for Dyson-Schwinger recursion and BCFW recursion, both for off-shell gluons and off-shell quarks.
- Besides $pp \to c\bar{c}\, c\bar{c}$ another recent application is $pp \to 4j$ AvH, Kutak, Serino 2015.

Public programs http://bitbucket.org/hameren/

AVHLIB (A Very Handy LIBrary)

- complete Monte Carlo program for tree-level calculations
- any process within the Standard Model
- any initial-state partons on-shell or off-shell
- employs numerical Dyson-Schwinger recursion to calculate helicity amplitudes
- automatic phase space optimization
- flexibility at the cost of user-friendliness

AMP4HEF (AvH, M.Bury, K.Bilko, H.Milczarek)

- only provides tree-level matrix elements (or color-ordered helicity amplitudes)
- available processes (plus those with fewer on-shell gluons):

$$\emptyset \to \mathrm{g}\,\mathrm{g} + 4\mathrm{g} \qquad \emptyset \to \bar{\mathrm{q}}\,\mathrm{q} + 3\mathrm{g} \qquad \emptyset \to \bar{\mathrm{q}}^*\,\mathrm{q} + 3\mathrm{g}$$

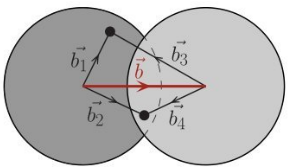
$$\emptyset \to \mathrm{g}^*\,\mathrm{g} + 4\mathrm{g} \qquad \emptyset \to \mathrm{g}^* + \bar{\mathrm{q}}\,\mathrm{q} + 2\mathrm{g} \qquad \emptyset \to \bar{\mathrm{q}}\,\mathrm{q}^* + 3\mathrm{g}$$

$$\emptyset \to \mathrm{g}^*\,\mathrm{g}^* + 4\mathrm{g}$$

- employs BCFW recursion to calculate color-ordered helicity amplitudes
- easy to use, both in Fortran and C++

Backup

Factorized Ansatz and double-parton distributions (DPDFs)



DPDF - emission of parton *i* with assumption that second parton *j* is also emitted:

$$\Gamma_{i,j}(b, x_1, x_2; \mu_1^2, \mu_2^2) = F_i(x_1, \mu_1^2)\ F_j(x_2, \mu_2^2)\ F(b; x_1, x_2, \mu_1^2, \mu_2^2)$$

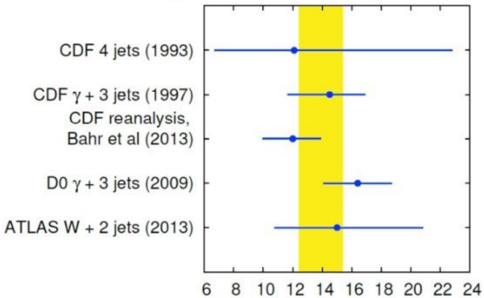
- correlations between two partons
  C. Flensburg et al., JHEP 06, 066 (2011)

in general:

$$\sigma_{eff}(x_1, x_2, x_1', x_2', \mu_1^2, \mu_2^2) = \left(\int d^2b\ F(b; x_1, x_2, \mu_1^2, \mu_2^2)\ F(b; x_1', x_2', \mu_1^2, \mu_2^2) \right)^{-1}$$

factorized Ansatz:

- additional limitations: $x_1 + x_2 < 1$ oraz $x_1' + x_2' < 1$
- DPDF in multiplicative form: $F_{ij}(b; x_1, x_2, \mu_1^2, \mu_2^2) = F_i(x_1, \mu_1^2) F_j(x_2, \mu_2^2) F(b)$
- $\sigma_{eff} = \left[\int d^2b\ (F(b))^2 \right]^{-1}$, $F(b)$ - energy and process independent



phenomenology: $\sigma_{eff} \Rightarrow$ nonperturbative quantity with a dimension of cross section, connected with transverse size of overlaping protons

$\sigma_{eff} \approx$ **15 mb** ($p_\perp$-independent)

a detailed analysis of σ_{eff}:
Seymour, Siódmok, JHEP 10, 113 (2013)