Can I make Andromeda with the axion field?

or...first stumbles to an Eqn of State for CDM from LSS data

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confusion in progress (+arXiv:1405.1139 , 1307.8024 with M Elmer)

1. the axion in Large Scale Structure (LSS) formation:
   classical field + bath of incoherent modes/particles

2. the classical field has extra pressures
   $might$ be relevant in non-linear structure formation?

3. (structure formation is a dynamical process...
   ⇒ hack gadget/AREPO/etc + run DM as fluid? could study many "interacting" DM candidates)

4. assume the galaxy is a stable solution
   ...but I have trouble to find a stable, cored, Andromeda with flat rotn curve, and made of QCD axion-field

Rindler-DallerShapiro
Chavanis
...

The QCD axion, A Bsm Curiosity

- boson from Beyond-the-Standard-Model, but
  - $light:\ 10^{-6}\mathrm{eV} \lesssim m_a \approx 10^{-5}\mathrm{eV} \lesssim 10^{-2}$ eV
  - weakly coupled: $\mathcal{L}_{eff} = \partial_\mu a \partial^\mu a - m^2 a^2 + \frac{m_a^2}{4! f^2} a^4$
  - one parameter model: couplings $\propto$ mass
  - and theoretically beloved

- $m_a \sim m_\nu$, but $COLD$ Dark Matter
  - for axion born after inflation, two contributions to DM:
    axion field from misalignment mechanism
    $incoherent$ cold bath of axion modes/particles
  - redshifts as $1/R(t)^3$
  - growth of linear density fluctuation like for WIMPs — Ratra, Hwang+Noh
  - ?non-linear epoch?

To distinguish axions vs WIMPs using Large Scale Structure data

There are many papers/words/analogies, 'tis a bit confusing.

But we are doing physics = "(shut up) and calculate". When you don't know what to calculate, ask the path integral, it knows everything.

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Consulting the path integral:

1. me: What are relevant variables and equations?
   PI: expectation values of n-pt functions ($\phi \equiv$ axion)

$$\langle \phi \rangle \leftrightarrow \text{ classical field} = \text{misalignment axions } \phi_{cl}$$

$$\langle \phi(x_1)\phi(x_2) \rangle \leftrightarrow \text{ (propagator)} + \text{distribution of particles } f(x,p)$$

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   get EqnofM for expectation values in Closed Time Path formulation

$$\text{Einsteins Eqns with } T^{\mu\nu}(\phi_{cl}, f) + \text{quantum corrections}(\lambda, G_N)$$

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⇒**leading order is simple:** Einsteins Eqns with $T^{\mu\nu}(\phi_{cl}, f)$. Q corr. from 2PI, CTP PI in CST?
(=saddle point of PI)

Using $T^{\mu\nu}{}_{;\nu} = 0$ vs Eqns of motion of the field ϕ

Both obtained from $T^{\mu\nu}{}_{;\nu} = 0$ and Poisson Eqn ($\rightarrow$ dynamics is equivalent?)

$$
\begin{aligned}
T^{\mu\nu}{}_{;\nu} &= \nabla_\nu[\nabla^\mu\phi\nabla^\nu\phi] - \nabla_\nu[g^{\mu\nu}\left(\frac{1}{2}\nabla^\alpha\phi\nabla_\alpha\phi - V(\phi)\right)] \\
&= (\nabla_\nu\nabla^\mu\phi)\nabla^\nu\phi + \nabla^\mu\phi(\nabla_\nu\nabla^\nu\phi) - g^{\mu\nu}\nabla_\nu\nabla^\alpha\phi\nabla_\alpha\phi + g^{\mu\nu}V'(\phi)\nabla_\nu\phi \\
0 &= \nabla^\mu\phi[(\nabla_\nu\nabla^\nu\phi) + V'(\phi)]
\end{aligned}
$$

1. For linear structure formation, eqns for $T_{\mu\nu} \sim \phi^2$ solvable Find $\delta \equiv \delta\rho(\vec{k}, t)/\overline{\rho}(t)$ in dust or axion field has same behaviour on LSS scales ($c_s \simeq \partial P/\partial\rho \rightarrow 0$):

   Ratra, Hwang+Noh

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_N\overline{\rho}\delta + c_s^2\frac{k^2}{R^2(t)}\delta = 0$$

2. For perturbative graviton scattering calns, $T_{\mu\nu}$ gives a better handle on IR divs: ensures that long-wave-length gravitons see large objects (like MeV photons see the proton, and not quarks inside)

3. For non-linear structure formation...??

Rediscovering...stress-energy tensors

non-rel axion particles are dust, like WIMPs:

$$T_{\mu\nu} = \begin{bmatrix} \rho & \rho\vec{v} \\ \rho\vec{v} & \rho v_i v_j \\ & \end{bmatrix}$$

compare to perfect fluid: $T_{\mu\nu} = (\rho + P)U_\mu U_\nu - P g_{\mu\nu}$. $P_{int} \propto \lambda^2 \to 0$, nonrel $\Rightarrow P \ll \rho, U = (1, \vec{v}), |\vec{v}| \ll 1$

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Classical field in non-relativistic limit $a \to \frac{1}{\sqrt{2m}}(\phi(x)e^{iS(x)}e^{-imt} + h.c.)$

$$T_{\mu\nu} = \begin{bmatrix} \rho & \rho\vec{v} \\ \rho\vec{v} & \rho v_i v_j + \Delta T_{ij} \end{bmatrix}$$

$$\rho = m|\phi|^2 \qquad \vec{v} = -\frac{\nabla S}{m}$$

$$\Delta T^i_j \sim \partial_i\phi\partial_j\phi \ , \ \lambda\phi^4$$

Sikivie

"extra" pressure with classical field! *(not need Bose Einstein condensation)*

Distinguishing axions vs WIMPs in structure formation?

- not during linear structure formation: pressure irrelevant — Ratra, Hwang+Noh
- ? non-linear dynamics: (black=eqns for dust)

Rindler-DallerShapiro

$$T^{\mu}{}_{\nu;\mu} = 0 \quad \Leftrightarrow \quad \begin{cases} \partial_t \rho + \nabla \cdot (\rho \vec{v}) = 0 \\ \partial_t \vec{v} + (\vec{v} \cdot \nabla)\vec{v} = -\rho \nabla V_N \pm \text{ extra pressures from field} \end{cases}$$

$\Rightarrow$ hack a structure formation code to run fluid DM and compare to dust code...

- Caveat: need to know — does gravity move axions between the field and particle bath? $\Leftrightarrow$ does it condense cold axion particles/evaporate the field?

not at $\mathcal{O}(G_N)$:

$$\langle n, \phi | \hat{T}_{\mu\nu}(X) | n, \phi \rangle = T^{(\phi c)}_{\mu\nu}(X) + T^{(part)}_{\mu\nu}(X)$$

$\Rightarrow$ at $\mathcal{O}(G_N^2)$?

NO, according to me (only person to calculate it, as far as I know).

Trying to learn something analytically...

From $T^{\mu}_{\nu;\mu} = 0$:

$$\begin{aligned} 0 &= \partial_t \rho + \nabla \cdot (\rho \vec{v}) \\ \rho \partial_t \vec{v} + \rho (\vec{v} \cdot \nabla) \vec{v} &= -\rho \nabla V_N + \rho \nabla \left(\frac{\nabla^2 \sqrt{\rho}}{2m^2 \sqrt{\rho}} - |g| \frac{\rho}{m^2} \right) \end{aligned}$$

$$a = \frac{1}{\sqrt{2m}} \left(\phi e^{-imt} + \phi^* e^{+imt} \right) \ , \ \phi(\vec{r}, t) = \sqrt{\frac{\rho}{m}} e^{-iS(\vec{r},t)} \ , \ \vec{v} = -\frac{1}{m} \nabla S \ , \ V_N = \frac{GM(r)}{r} \ , \ g = 1/(3! f^2)$$

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Some approaches (incomplete unrepresentative list):

1. CDM : Eqns of Motion are scale free , so power law scaling solutions... FillmoreGoldreich
2. ... ...
3. scalar fields: look for "static" /stable solutions ($\simeq$ equilibrium of forces on RHS Euler)
   - Rindler-Daller+Shapiro:
     include *positive* self-interaction pressure $+|g|$, rotation.
     variable m, g; fix to obtain solution with galactic mass/radius (not $\rho \propto 1/r^2$ at large r)
   - Chavanis:
     also *negative* self-interaction pressure, no rotation
     variable m, g...

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I want to fix m, g for QCD axion($g < 0$, pressure *inwards*); can I obtain Andromeda?

To make Andromeda with an axion field

Andromeda : core $\lesssim$ kpc $\simeq 3 \times 10^{21}$ cm

flat rotation curve for stars $\Rightarrow \rho_{DM} \propto 1/r^2$ out to 100s kpc.

$$\text{centrifugal :} \qquad \frac{v_{tang}^2}{r} \quad = \quad \frac{4\pi G}{r^2} \int^r \rho(r') {r'}^2 dr' \qquad \text{gravity}$$

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Neglect LHS (v constant?):

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$$V_N = -\frac{GM(r)}{r} \qquad g \simeq \frac{1}{3! f^2}$$

Neglect LHS (v constant?):

1. if gradient pressure balances gravity...
   $\frac{1}{m^2R^2} \simeq \frac{4\rho R^3}{m_{pl}^2 R} \quad \Rightarrow \quad \sqrt{\frac{m_{pl}}{m}}\frac{1}{\rho^{1/4}} \sim R_{Jeans} \sim .5 \times 10^{14}$ cm

2. if gradient pressure balances self-interactions...
   $\frac{1}{m^2R^2} \simeq \frac{\rho}{6m^2f^2} \quad \Rightarrow \quad R \sim \frac{f}{\sqrt{\rho}} \sim 10^{18}$ cm

Isothermal sphere $\rho = \frac{\rho_c r_c^2}{r^2+r_c^2}$ not a solution.

Approx soln: $\rho = \frac{\rho_c r_c^4}{(r^2+r_c^2)^2}$, core radius $r_c \ll$ kpc, core density $\rho_c \gg$ GeV/cm^3.

How to get $1/r^2$ at large r ... rotation?

If rotate halo with $\rho \propto 1/r^2 \Rightarrow M(r) \propto r$, at $v_{tang} \simeq$ constant, can balance gravity with centrifugal force:

$$\frac{v_{tang}^2}{r} \leftrightarrow \frac{GM(r)}{r^2}$$

Can I put $v_{tang} \simeq$ constant in the axion-field halo?

$$??no? \qquad v_{tang} \equiv \frac{1}{mr\sin\theta}\frac{\partial S}{\partial\varphi} \qquad\qquad (a \sim \frac{\sqrt{\rho}}{m}e^{-iS})$$

$S \propto \varphi \Rightarrow v_{tang} \propto 1/r$, $S \propto r\varphi \Rightarrow v_r$ discontinuous in ϕ.

How to get $1/r^2$ density with axion field? A DM candidate must make spiral galaxies...

Summary

The QCD axion is a motivated dark matter candidate. If the PQ transition is after inflation, there are two populations: the classical "misalignment" field, and cold particles radiated by strings

to distinguish axion from WIMP CDM: direct detection, axion effects on γ propagation, maybe the extra pressures from the axion field give differences during non-linear structure formation?
$\Rightarrow$ *numerical galaxy formation*

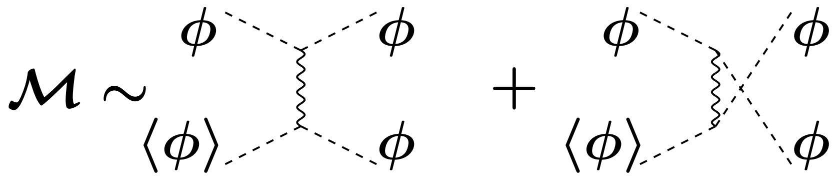
Can try looking for a stable/stationary solution of the field eqns, corresponding to a galaxy. I did not (yet) find a rotating spiral: how to obtain $\rho \sim 1/r^2$ out to 100s of kpc?

- maybe the $1/r^2$ tails are made of cold axion particles? (?but then they would form a cusp?)
- maybe spiral galaxies are not stationary solutions?
- maybe I did not try hard enough...

Backup

Moving axions between field and bath with gravity? (in galaxy today)

at $\mathcal{O}(G_N^2)$, quantized GR $(v \sim 10^{-3}$ in cm frame)



$$\sigma = \frac{G_N^2 m^2 \pi}{8v^4} \int \sin\theta d\theta \left(\frac{1}{\sin^2(\theta/2)} + \frac{1}{\cos^2(\theta/2)} \right)^2$$

Dewitt

IR cutoff of graviton momenta $\sim H$?

$$\sigma \sim \frac{G_N}{v^2}$$

...but this is for empty U containing two axions...

Moving axions between field and bath with gravity? (in galaxy today)

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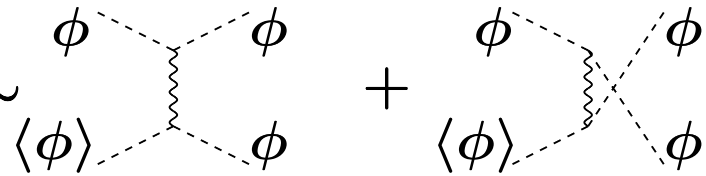
$\mathcal{M} \sim$



$$\sigma = \frac{G_N^2 m^2 \pi}{8v^4} \int \sin\theta d\theta \left(\frac{1}{\sin^2(\theta/2)} + \frac{1}{\cos^2(\theta/2)} \right)^2 \to 10^4 \frac{m^2}{m_{pl}^4} \qquad (m \sim 10^{-5} eV)$$

graviton couples to $T^{\mu\nu}$! Only sees single axion when can look inside box $\delta^3 \sim 1/(mv)^3 \Rightarrow$ IR cutoff of graviton momenta $\sim mv$.

$$\text{probability} = \left| \sum \text{indistinguisable amplitudes} \right|^2$$

graviton of 10 metre wavelength interacts coherently with all axions in 10 metre cube $\leftrightarrow T_{\mu\nu}$. (like MeV γ scatters off proton and not individual quarks inside).

To estimate rate, account for high axion occupation # (in galaxy today)

to estimate evaporation/condensation rate, must take into account high occupation number of axions:

$$\frac{\partial}{\partial t} n \quad = \quad \int \Pi_i \widetilde{d^3 p_i} \tilde{\delta}^4 |\mathcal{M}|^2 \Big[f_1 f_2 (1 + f_3)(1 + f_4) - f_3 f_4 (1 + f_1)(1 + f_2) \Big]$$

$[...] \sim f^3$, so rate for individual axion to evaporate/condense

$$\Gamma \sim n_\phi \sigma_G f \sim 10^{13} \left(\frac{\rho_{DM}}{\rho_c} \right)^2 \left(\frac{m}{m_{pl}} \right)^3 H_0 \ll H_0$$

is negligeable...

What is a Bose Einstein condensate? (I don't know. Please tell me if you do!)

Important characteristics of a BE condensate seem to be

1. a classical field,
2. carrying a conserved charge,
3. ? whose fourier modes are concentrated at a particular value — most of the "particles" who condense, should coherently do the same thing (but not necc the zero-momentum mode)

consistent with

- BE condensation in equilibrium stat mech, finite T FT, alkali gases.
- LO theory of BE condensates (Boguliubov $\rightarrow$ Pitaevskii) as a classical field

Are the misalignment axions a BE condensate?

1. a classical field yes
2. carrying a conserved charge, in the NR limit, $\approx$ yes
3. ? whose fourier modes are concentrated at a particular value — most of the "particles" who condense, should coherently do the same thing (but not necc the zero-momentum mode) ....umm?

Two approaches:

A: Maybe the axion field is a condensate? Or a superposition of BE condensates coupled via gravity? what I think now

B: Follow Sikivie = misalignment field is not a BE condensate $\Rightarrow$ does gravity put it there?

Saikawa+Yamaguchi+etal
Davidson+Elmer,...

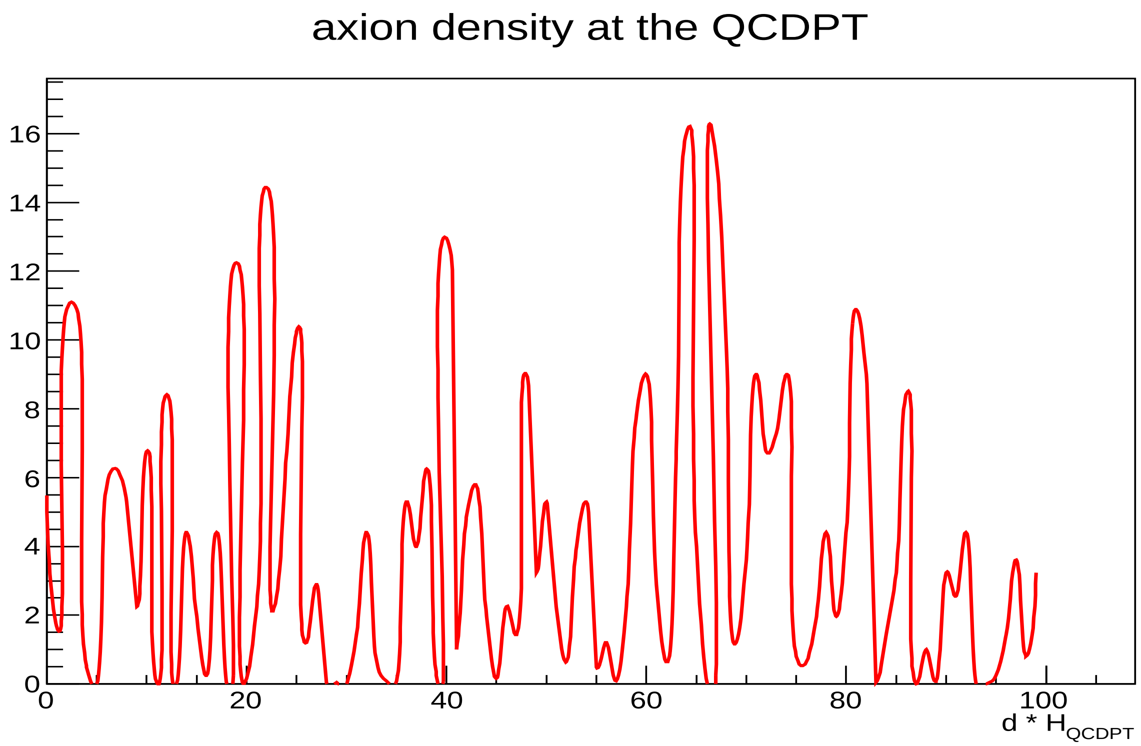
But what does vocabulary matter?

Just need right variables (field + particle density), and their EoM...

BE condensate analogy doubtful for axions, because familiar BE condensates have stronger self-interactions....

Inhomogeneities are $\mathcal{O}(1)$ on the QCD horizon scale

$a(\vec{x},t)$ random from one horizon(~ 5km) to next; $\rho_a(\vec{x},t) \simeq m_a^2 a^2(\vec{x},t)$



$\Rightarrow$ its *not* a spatially homogeneous distribution of particles various momenta

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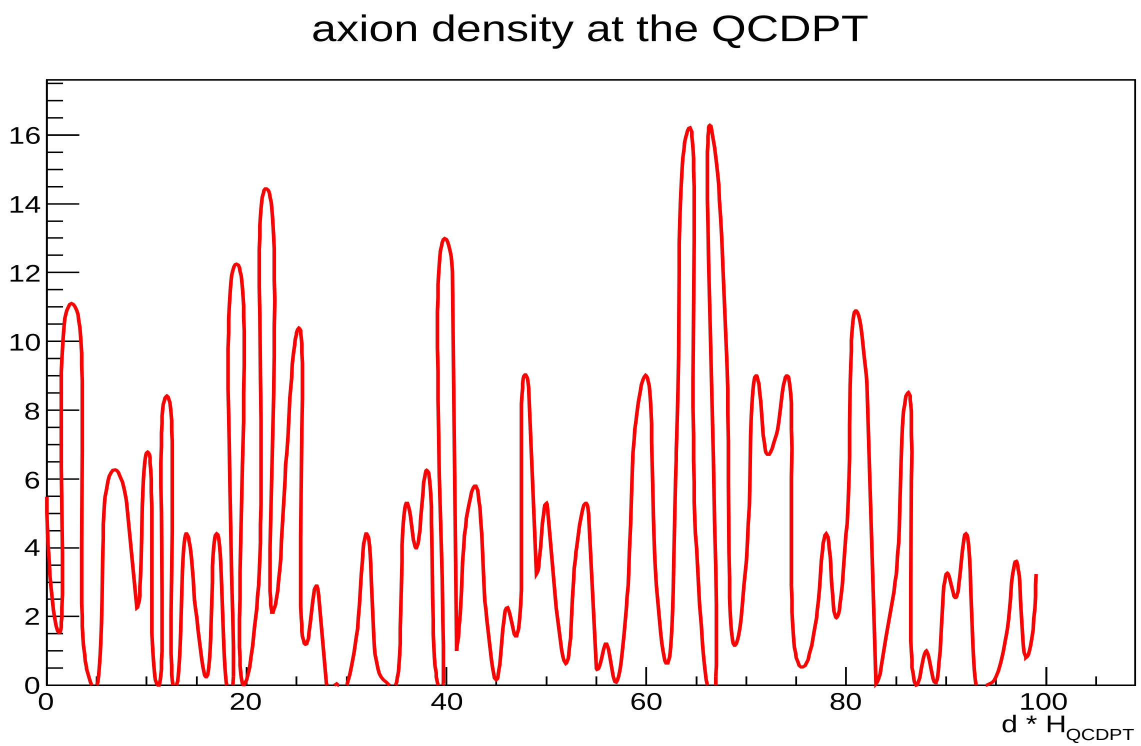

But how can axions form a *homogeneous-on-QCD-horizon-scale* bose-einstein condensate = zero mode of field? ??

$v = H_{QCDPT}/m_a \lesssim 10^{-6}c$...not "free-stream" QCD-horizon distance before t_{eq}:

$$d(t) = \int^t \frac{H_{QCDPT}}{m_a R(t')} dt' \sim \frac{H_{QCDPT}}{m_a} \frac{1}{H(t)R(t)} = \frac{R(t)}{m_a} \ll \frac{R(t)}{H_{QCDPT}}$$

(RD U, $R(t)$ =1@QCDPT)

thermalisation in closed unitary systems?

$$\text{entropy} = \sum_{states\ s} P_s \ln P_s \quad \text{increases}$$

- unitary evolution creates no entropy $\Leftrightarrow$ NO entropy generation in closed systems ... BUT... can calculate "effective" thermalisation: a subset of observables evolve towards equilibrium expectations
$\Rightarrow$ the "rest" of the system is the bath??

- ex: couple two SHOs. Solve one, substitute into Eqns of second, and find dissipation.

- ...$K - \bar{K}$ evolution is non-unitatry, because not also follow 2π 3π states...

? $\Rightarrow$ divide axions+gravity into

1. U expansion + structure growth
2. other fluctuations which are the bath?

gravity and the second law

1. undergraduate memories say that gravitational collapse of a gas cloud to a star respects the second law...

2. story of $\Omega_{baryon} = 1$ U
   (a) quasi-homogeneous dust clouds collapse
   (b) ...generations of stars, supernovae, black holes...
   (c) ... ... proton decays...
   (d) venerable homogeneous and isotropic U full of photons and gravitons

3. so gravitational thermalisation of axions will happen.
   But does it happen before the U a year old?

Particles vs fields

Develop field operator

$$\hat{a}(t,\vec{x}) = \frac{1}{[R(t)L]^{3/2}} \int \frac{d^3k}{(2\pi)^3} \left\{ \hat{b}_{\vec{k}} \frac{\chi(t)}{\sqrt{2\omega}} e^{i\vec{k}\cdot\vec{x}} + \hat{b}^{\dagger}_{\vec{k}} \frac{\chi^*(t)}{\sqrt{2\omega}} e^{-i\vec{k}\cdot\vec{x}} \right\}$$

then write the coherent state:

$$|a(\vec{x},t)\rangle \quad \propto \quad \exp\left\{ \int \frac{d^3p}{(2\pi)^3} a(\vec{p},t) b^{\dagger}_{\vec{p}} \right\} |0\rangle$$

which satisfies $\hat{b}_{\vec{q}}|a(\vec{x},t)\rangle = a(\vec{q},t)|a(\vec{x},t)\rangle$ (can check $\hat{b}_{\vec{q}}\{1 + \int \frac{d^3p}{(2\pi)^3} a(\vec{p},t) b^{\dagger}_{\vec{p}}\}|0\rangle = a(\vec{q},t)|0\rangle$)
where the classical field is

$$a(t,\vec{x}) = \frac{1}{[R(t)L]^{3/2}} \int \frac{d^3k}{(2\pi)^3} \left\{ a(\vec{k},t) \frac{\chi(t)}{\sqrt{2\omega}} e^{i\vec{k}\cdot\vec{x}} + a^*(\vec{q},t) \frac{\chi^*(t)}{\sqrt{2\omega}} e^{-i\vec{k}\cdot\vec{x}} \right\}$$

Brodsky+Heurer,Donoghue etal
Olive+Montonen...I+Z,C-T...

What is quantum?

Classical = saddle-point configurations of the path integral
$\Rightarrow$ attribute dimensions to fields/parameters $\ni$ [action]= E*t, and no $\hbar$ in selected classical limit (this is *not* unique)

Summary: particles or fields can be obtained in a "classical" (= no $\hbar$) limit. However, $\hbar$ is differently distributed in the Lagrangian in the two limits, so to get from one to another requires $\hbar$...

in particular, to define a number of quanta, in the field picture, requires $\hbar$.

ex 1: massive scalar electrodynamics

$$\mathcal{L} = (D_\mu \phi)^\dagger D^\mu \phi - \tilde{m}^2 \phi^\dagger \phi - \frac{1}{4} F F \qquad , \quad D_\mu = \partial_\mu - i\tilde{e} A_\mu$$

Classical field limit: $[\phi, A] = \sqrt{E/L}$, $[m] = 1/L$, $[\tilde{e}] = 1/\sqrt{EL}$.

No $\hbar$ in classical EoM. OK that $[m^2] = 1/L^2$ because gravity couples is the stress-energy tensor, function of the fields.

If in Maxwells Eqns, want $j^0 = i\tilde{e}(\dot{\phi}^\dagger \phi - \phi^\dagger \dot{\phi})$ to be eN/V, then need number of charge-carrying quanta $\Rightarrow e = \tilde{e}\hbar$.

De même, if classically m a particle mass, need $m = \tilde{m}\hbar$.

ex 2: the SHO Hamiltonian is (no $\hbar$)

$$H = \frac{1}{2m} P^2 + \frac{m\nu^2}{2} X^2$$

where ν is the oscillator frequency.

But to *quantise*, = introduce creation and annihilation ops, requires $\hbar$.
To write the total energy as $\omega(N + 1/2)$, requires $\hbar$ to convert frequency to energy $\omega = \hbar\nu$, and downstairs in the defn of N, because its the number of *quanta*.