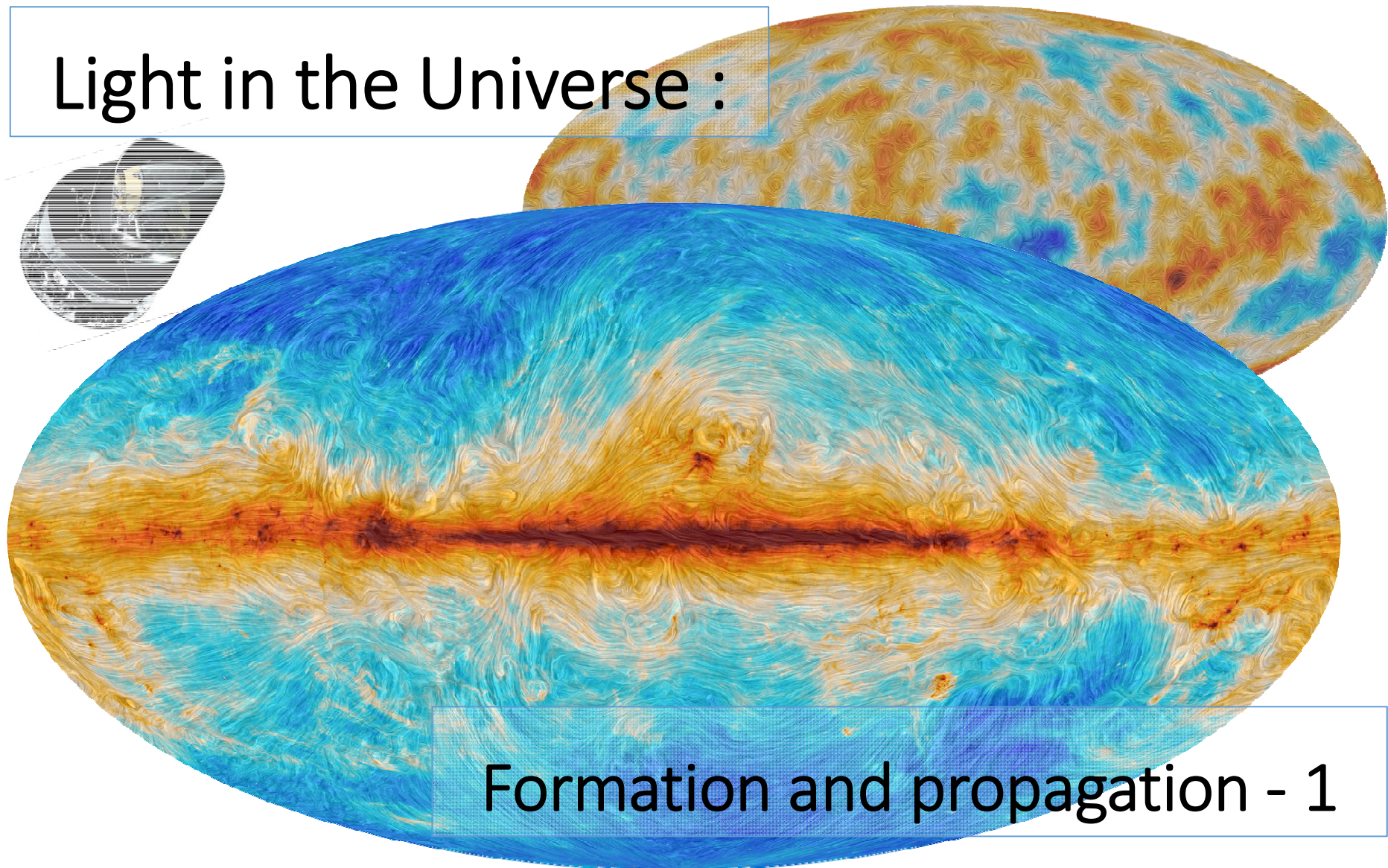
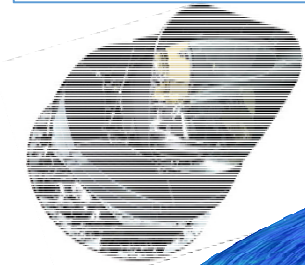


# Light in the Universe :



## Formation and propagation - 1

Paolo de Bernardis

Dipartimento di Fisica, Sapienza Università di Roma

Winter College on Optics 2015 - Trieste, 9-20 February

# Photons from the Universe

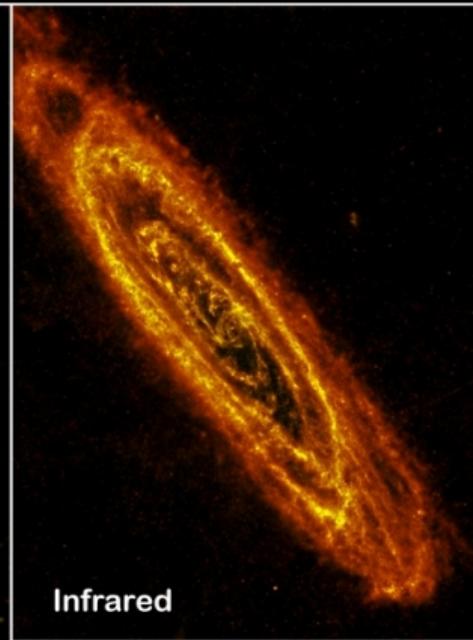
- Photons propagate for billions of light-years in the universe, carrying information on the distant, past cosmos.
- This very fact makes observational astronomy possible.
- 400 years of active astronomical observations have developed tools to collect and analyze photons from the universe in deep detail.
- What we know in Astrophysics and Cosmology today is based by far mainly on these measurements.
- Over the whole EM spectrum:
  - Direction (Brightness Maps)
  - Energy distribution (Spectroscopy)
  - Polarization state (Polarimetry)
- The fidelity of the information carried by photons from the universe depends on
  - The wavelength and the interaction with intervening matter (atmospheric, interstellar and intergalactic absorption and emission)
  - The expansion of the universe (cosmological redshift)
  - The interaction with intervening masses (gravitational lensing).



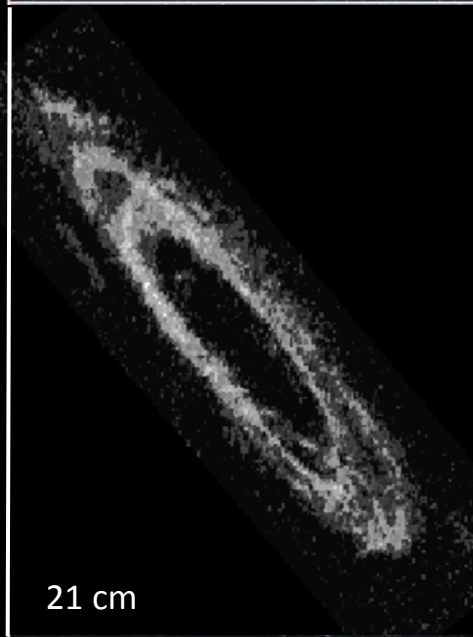
Optical



Infrared & X-rays



Infrared

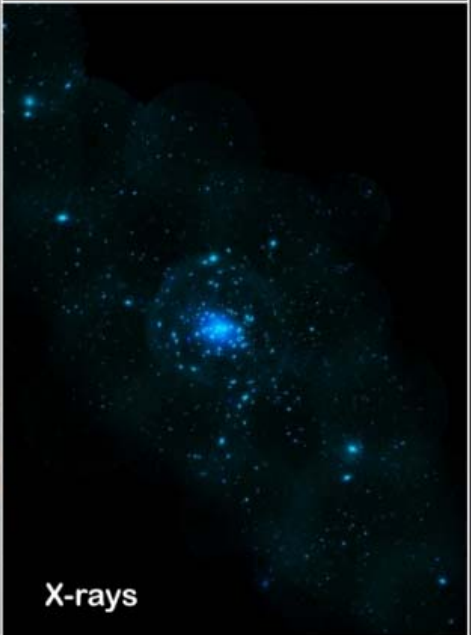
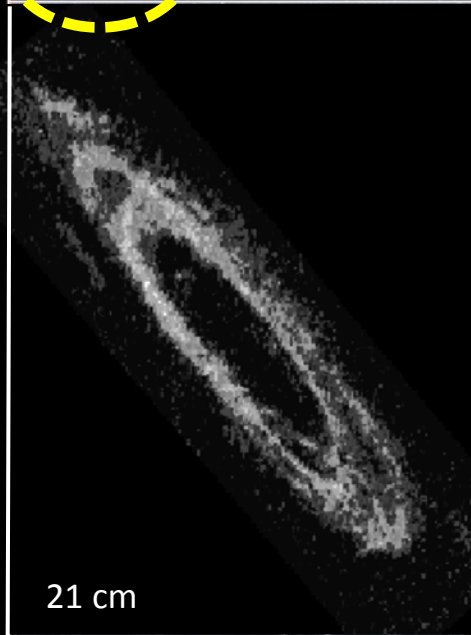
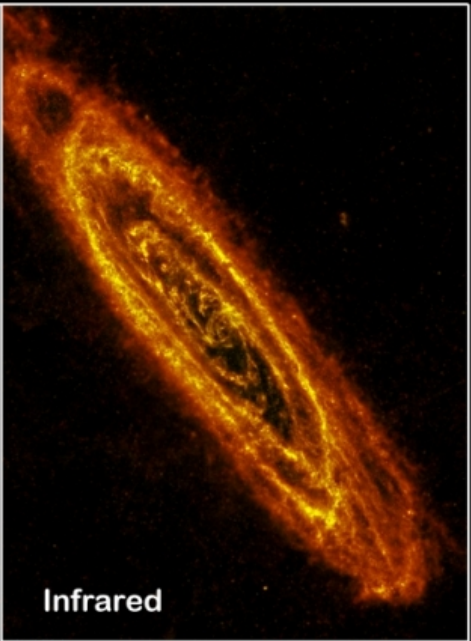


21 cm



X-rays

Understanding of a cosmic source requires observations across the whole EM spectrum



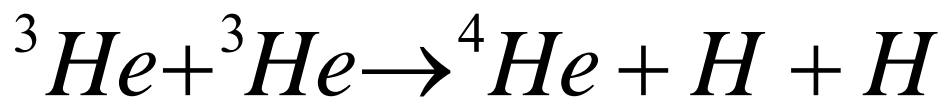
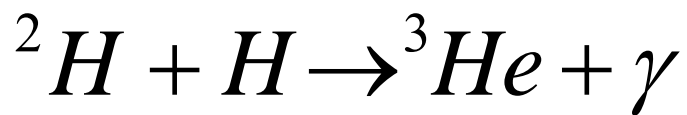
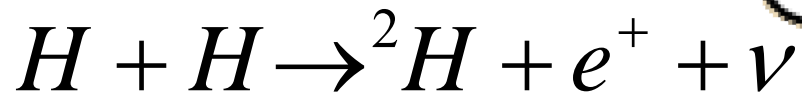
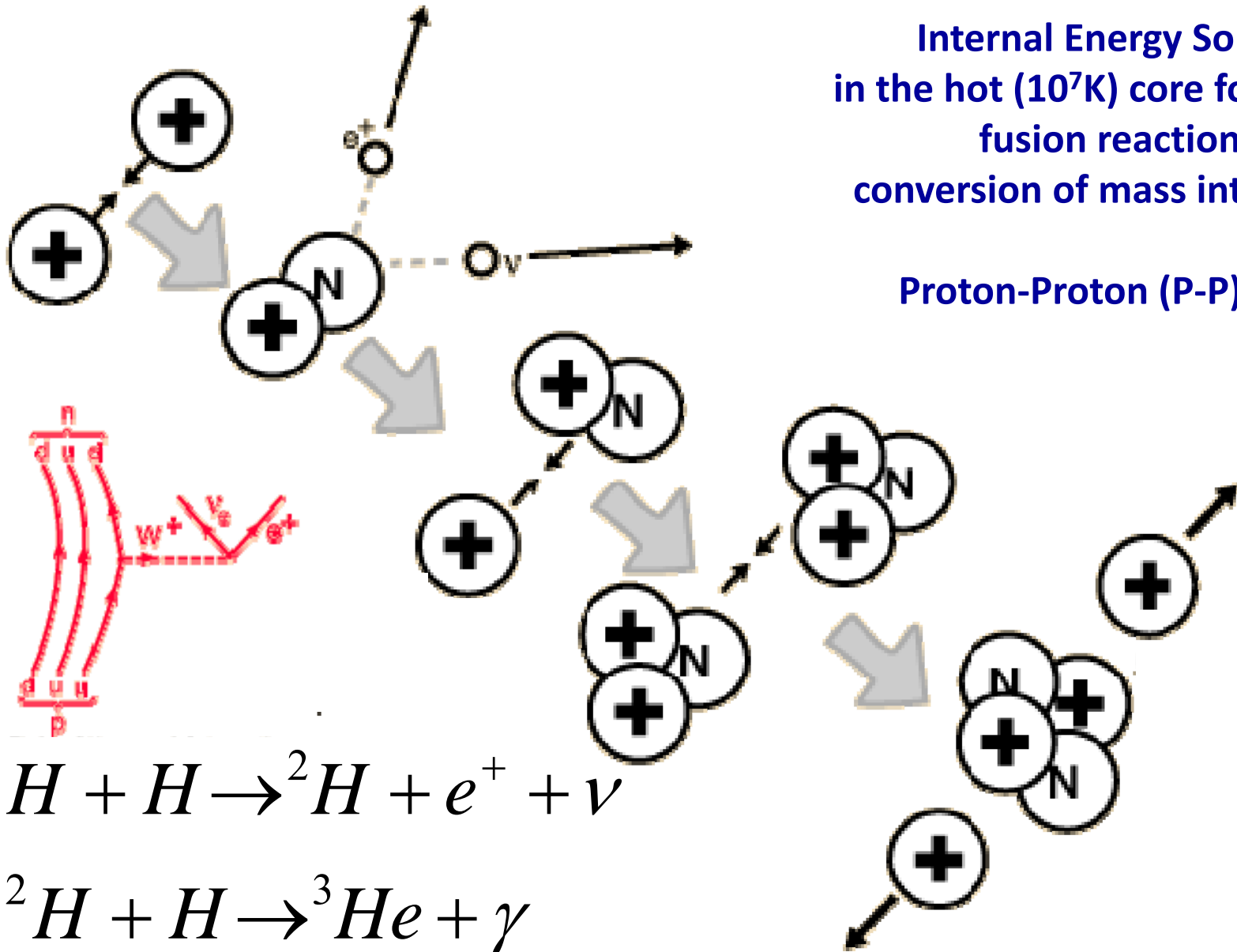
Understanding of a cosmic source requires observations across the whole EM spectrum

# Stellar radiation

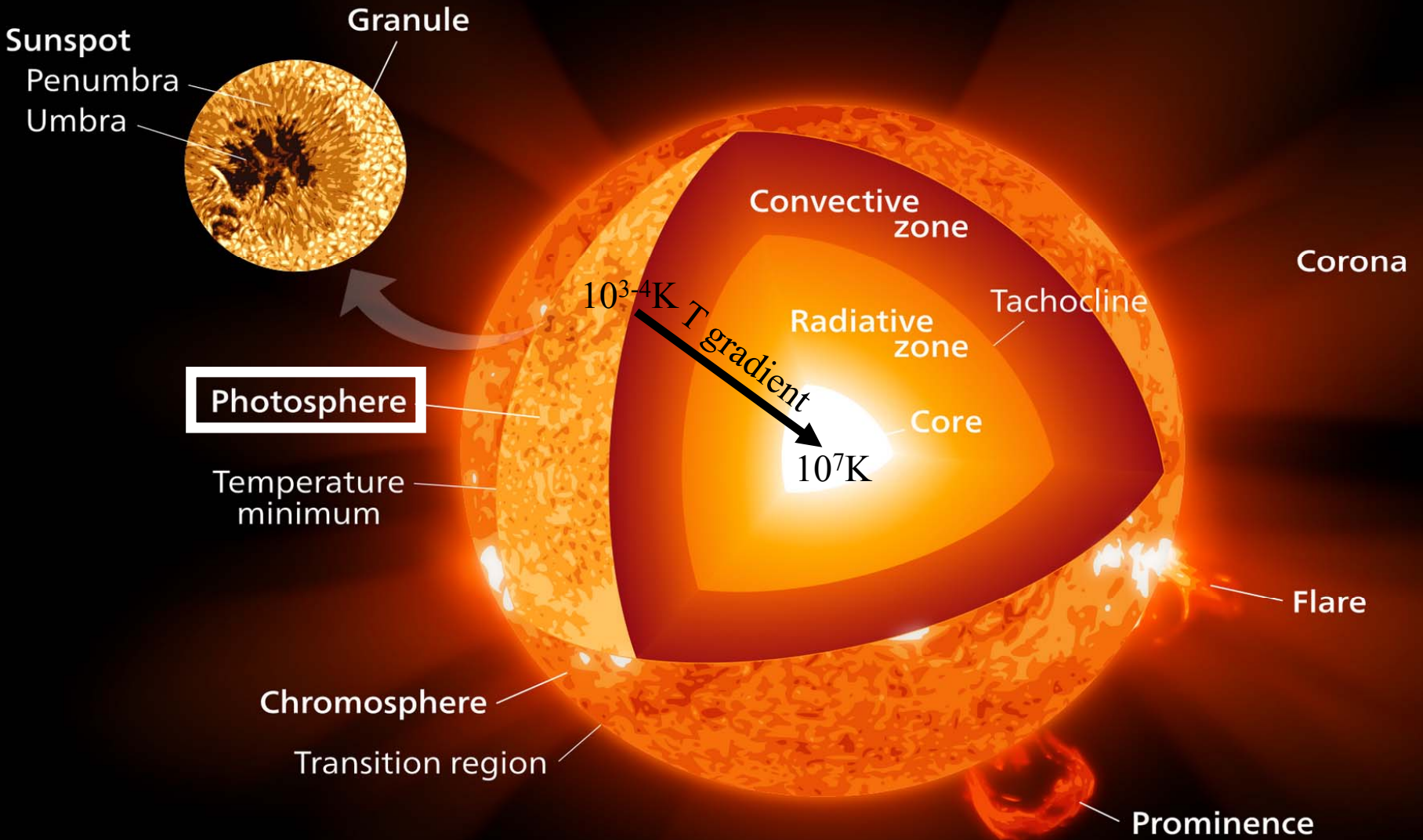


Internal Energy Source,  
in the hot ( $10^7\text{K}$ ) core fo the star :  
fusion reactions,  
conversion of mass into energy

Proton-Proton (P-P)-chain

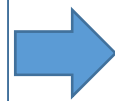


# Thermalization through scattering / absorption



Photosphere =  
**semitransparent shell**,  
 temperature  $T_s$ , optical  
 depth  $\tau(\nu)$

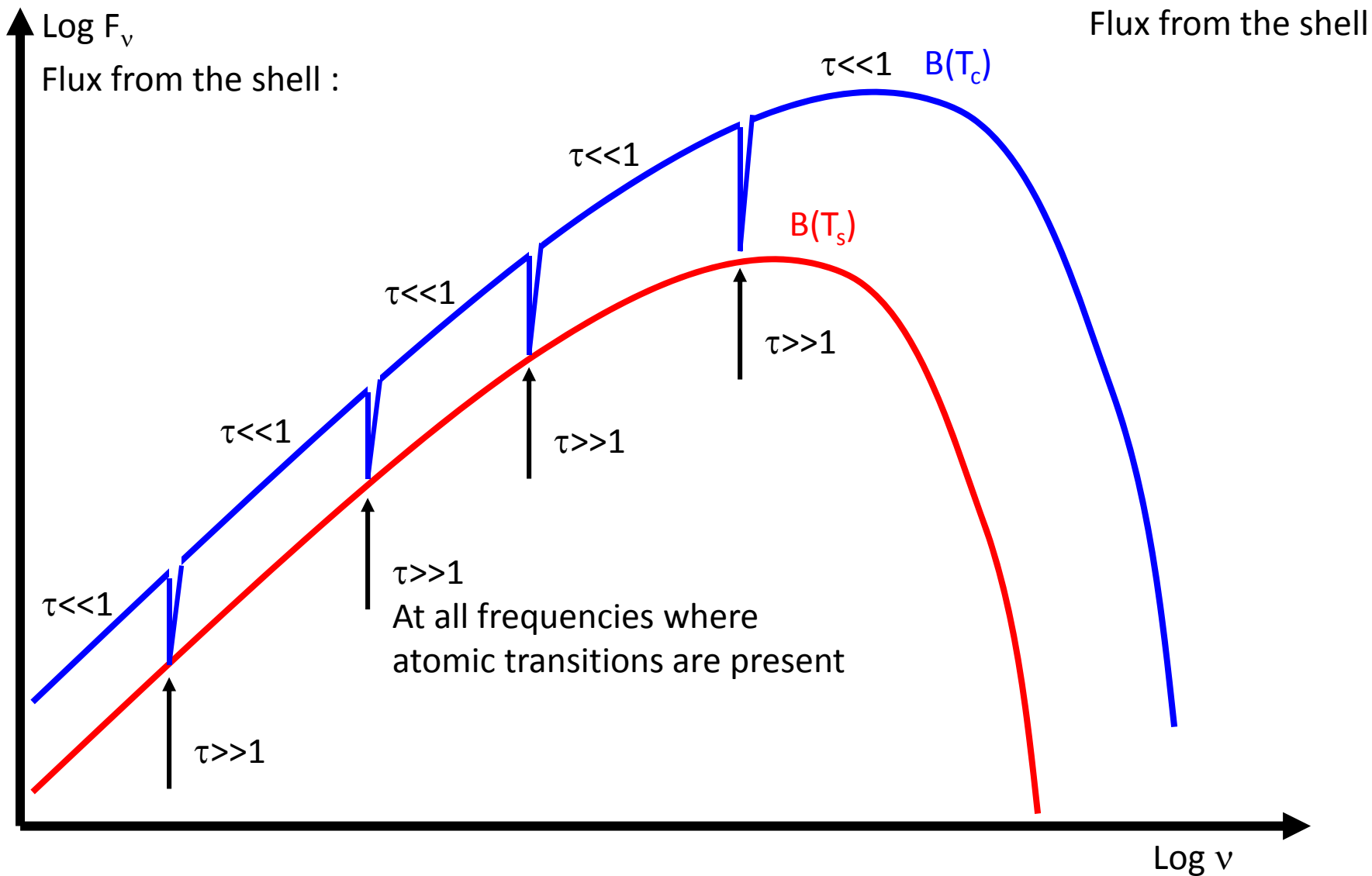
Internal = opaque core,  
 temperature  $T_c$  and  
 rising – high optical  
 depth, BB spectrum



$$I_A = B(\nu, T_c) e^{-\tau} + B(\nu, T_s) [1 - e^{-\tau}]$$

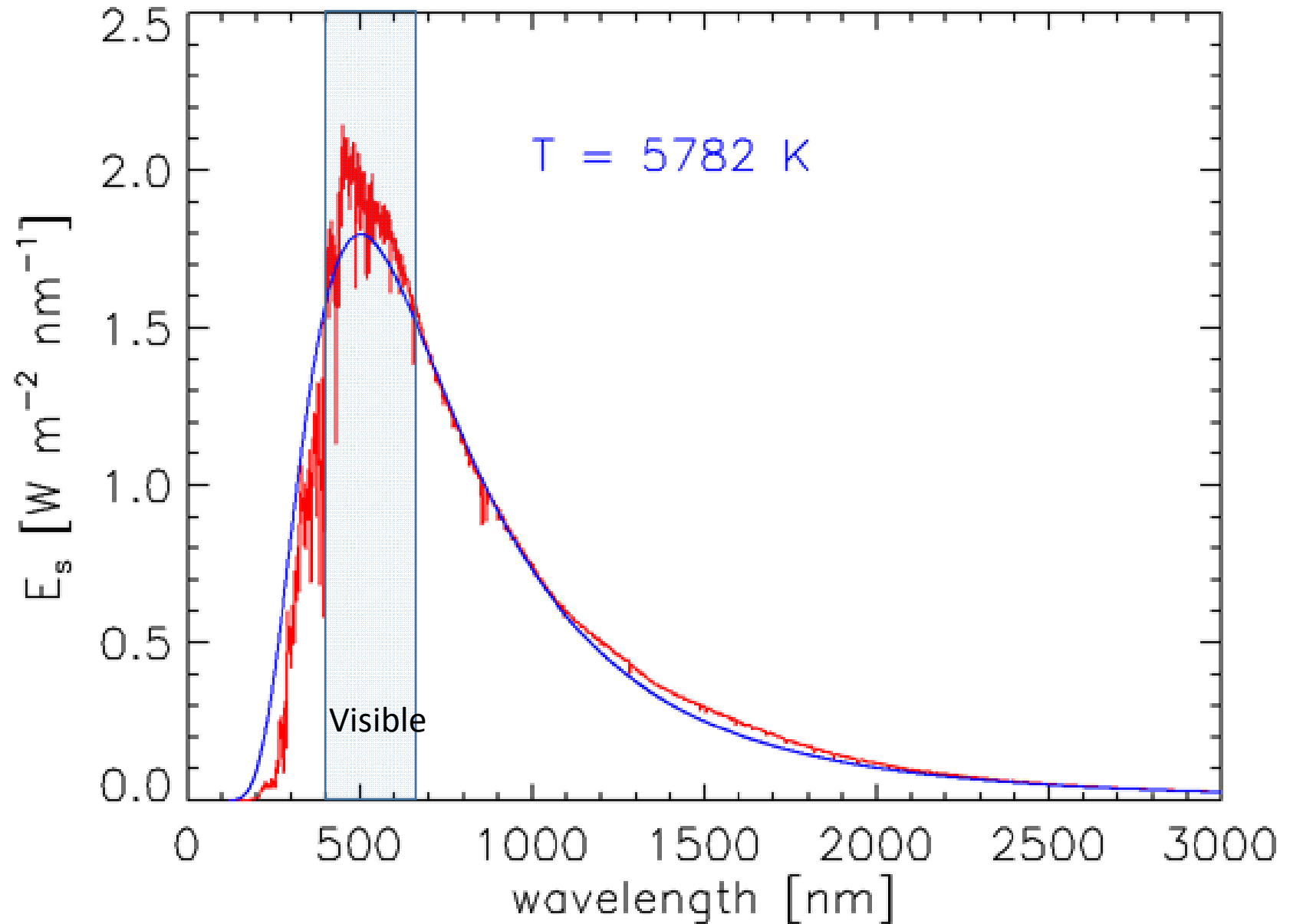
$$\tau \ll 1 \Rightarrow I_A = B(\nu, T_c) + \tau B(\nu, T_s)$$

$$\tau \gg 1 \Rightarrow I_A = B(\nu, T_s)$$





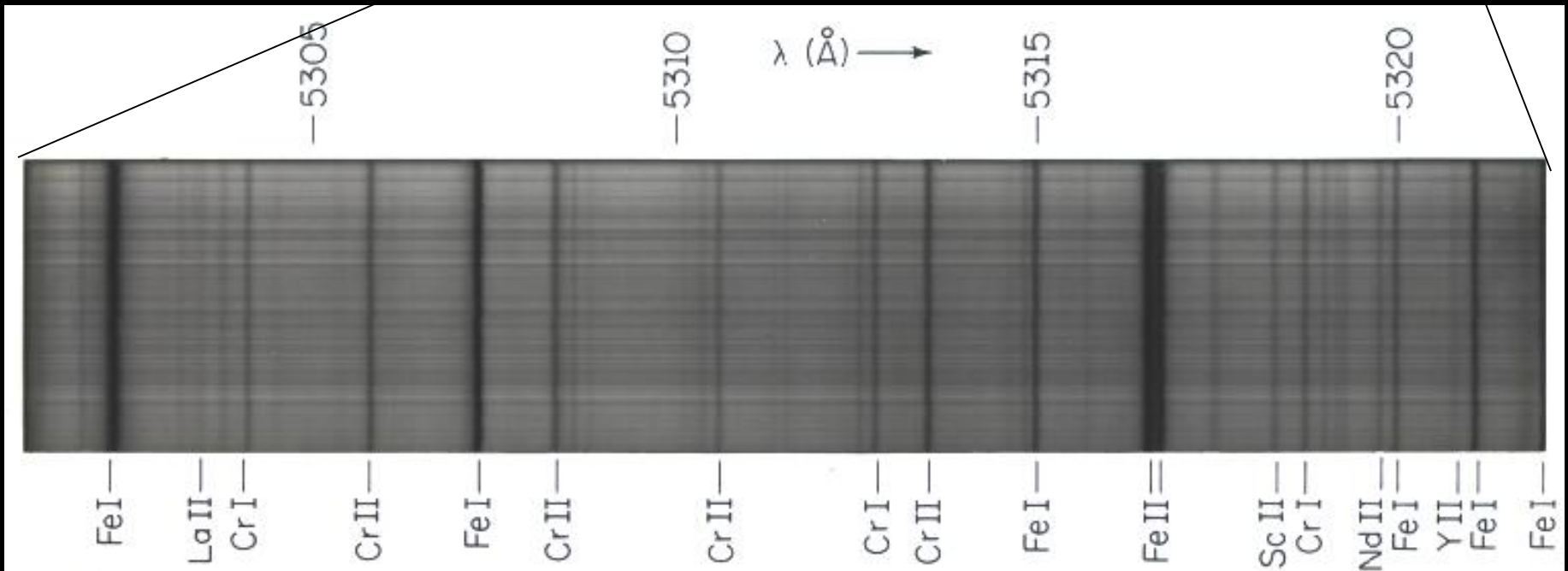
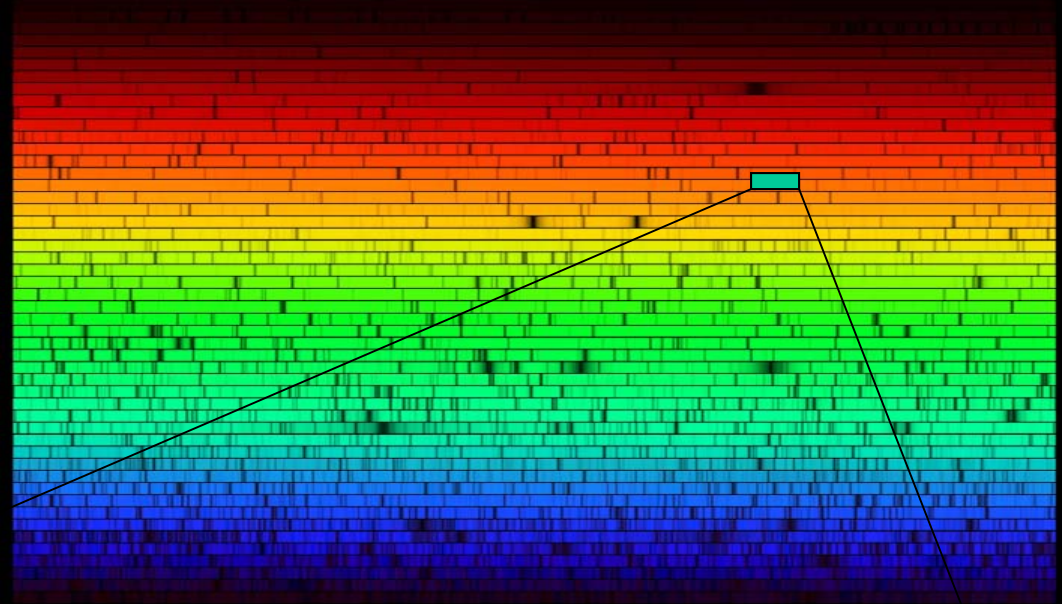
# Specific flux from the sun (red curve) vs 5782K blackbody

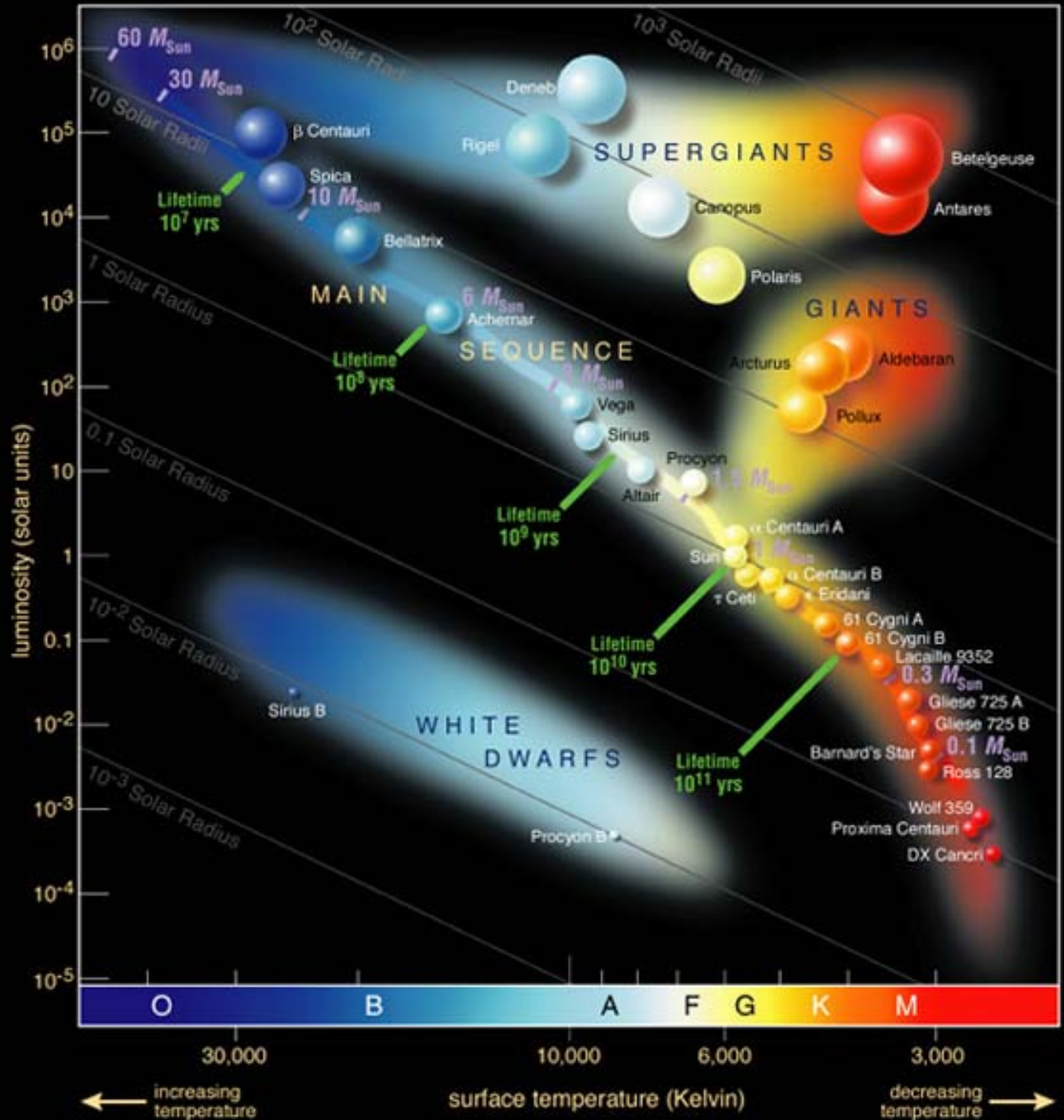


# Stellar spectra :

Continuum (Blackbody)  
+  
absorption lines

from many different elements in  
the photosphere





# Thermal Radiation

- There are many situations where matter is in local thermal equilibrium with radiation, or close to :
  - Photospheres of stars
  - Surface (or atmosphere) of planets
  - Clouds of interstellar dust
  - Early Universe
- In all these cases the emission is thermal: blackbody or similar.

# Blackbody Radiation

$$B(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

$$B(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{kT\lambda}} - 1}$$

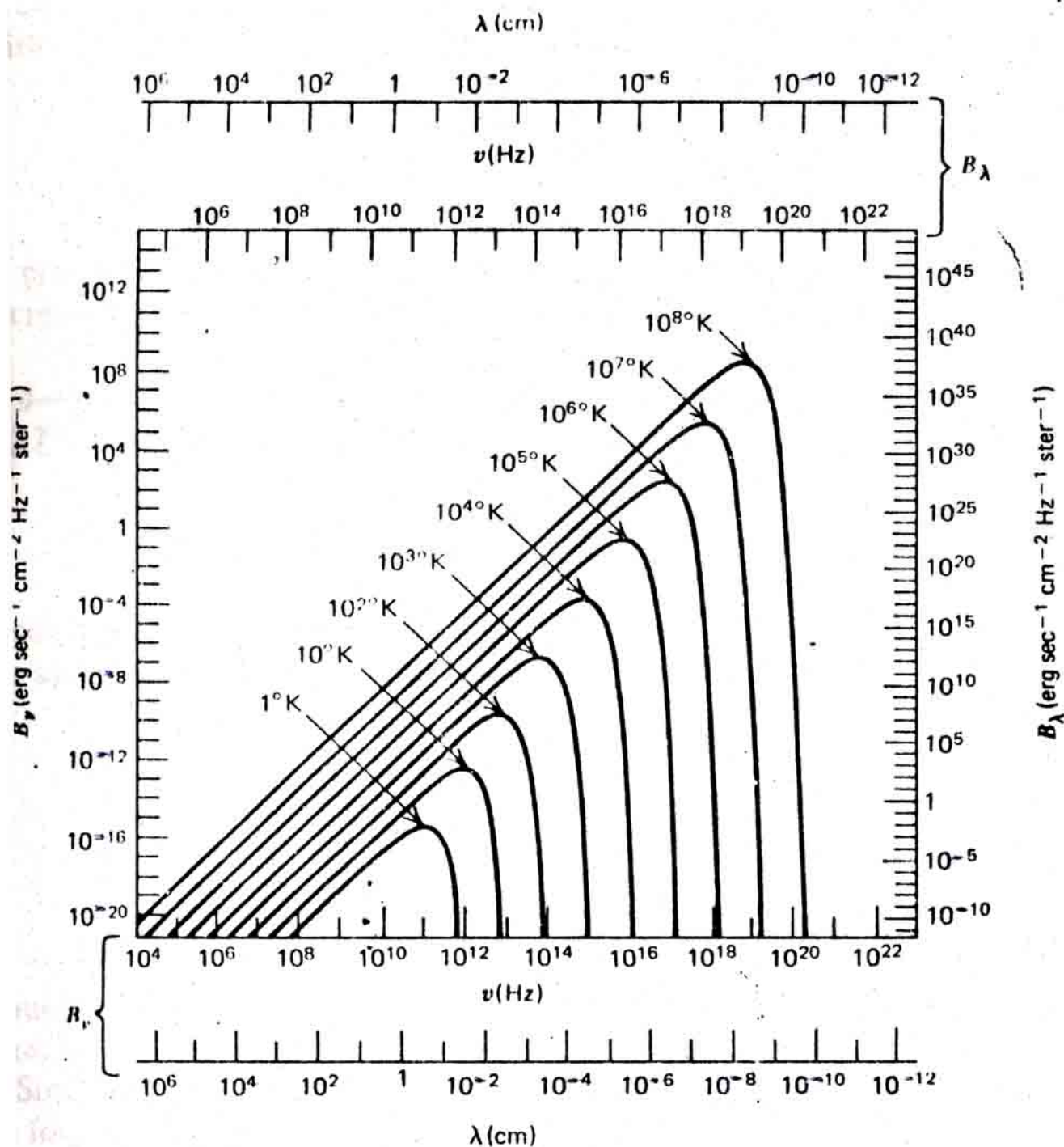
- Maximum brightness

$$\lambda_{\max} T = 0.290 \text{ cm K}$$

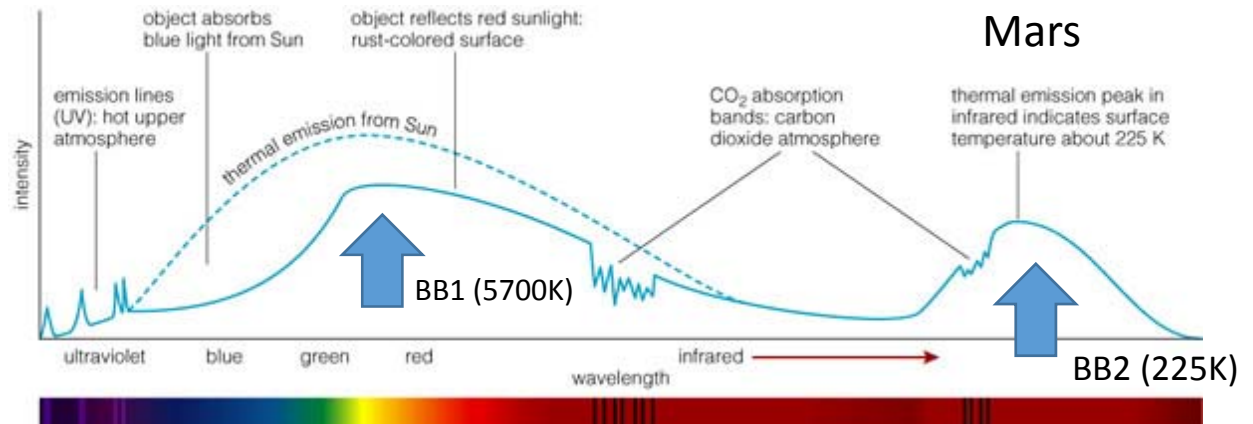
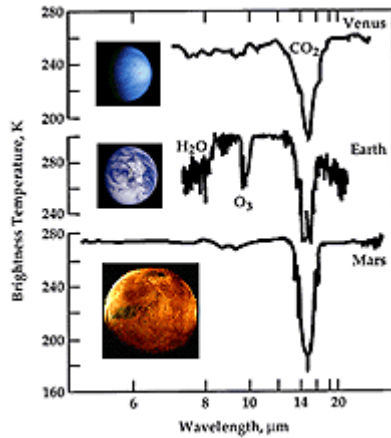
- Low frequency :

$$B(\nu, T) \approx \frac{2\nu^2}{c^2} kT$$

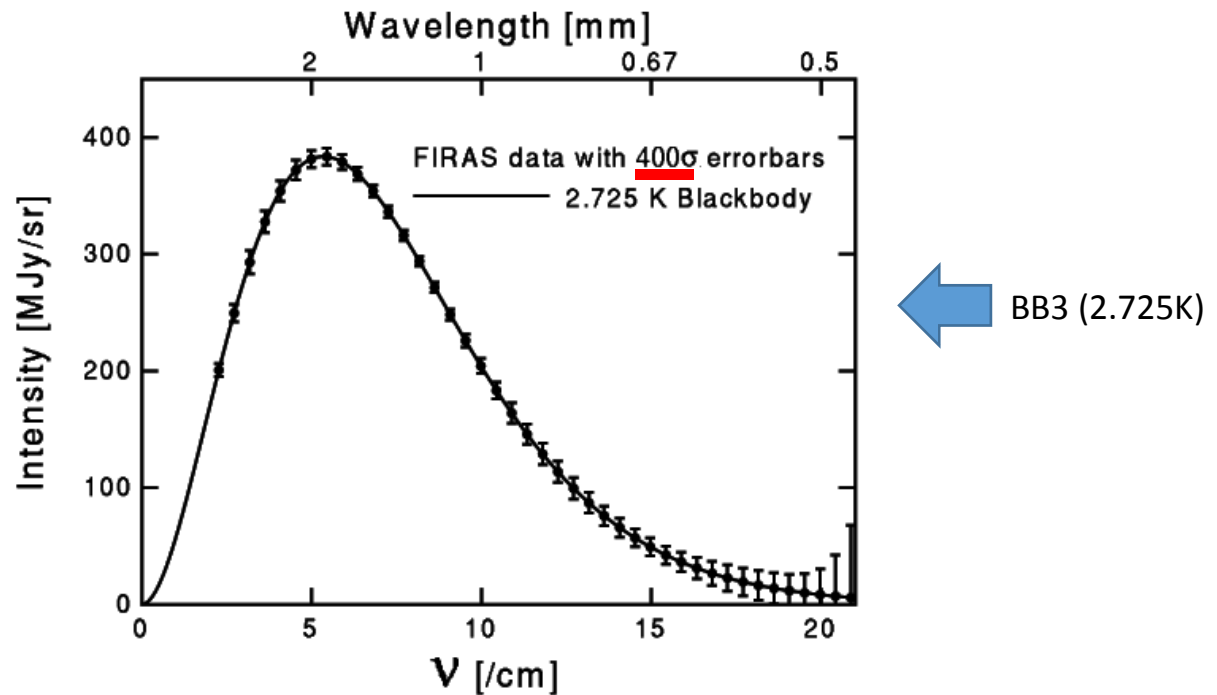
$$h\nu \ll kT$$



## Spectrum of planets



## Spectrum of the Cosmic Microwave Background



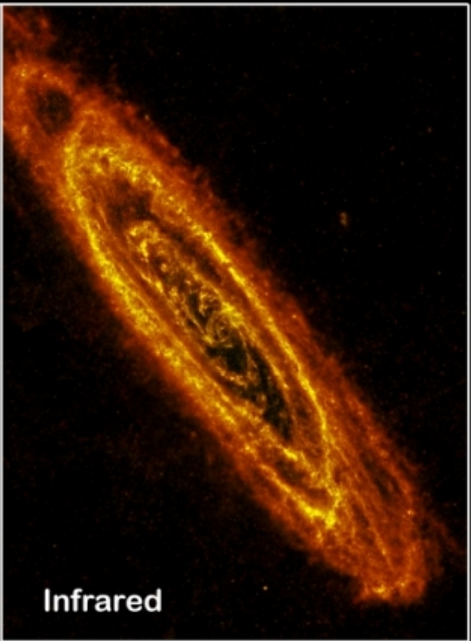


**Optical**

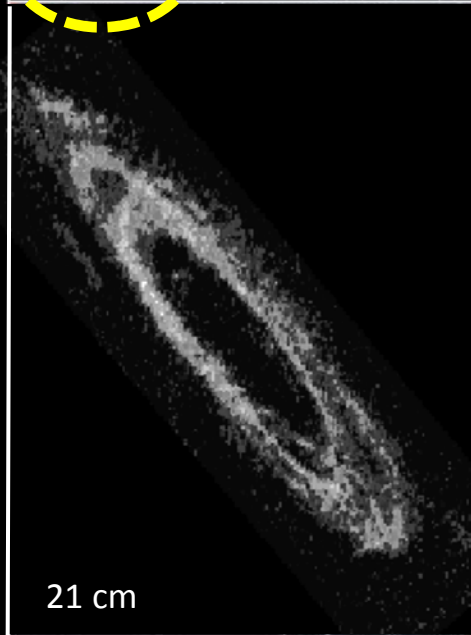
Stars in the bulge  
and in the spiral  
arms



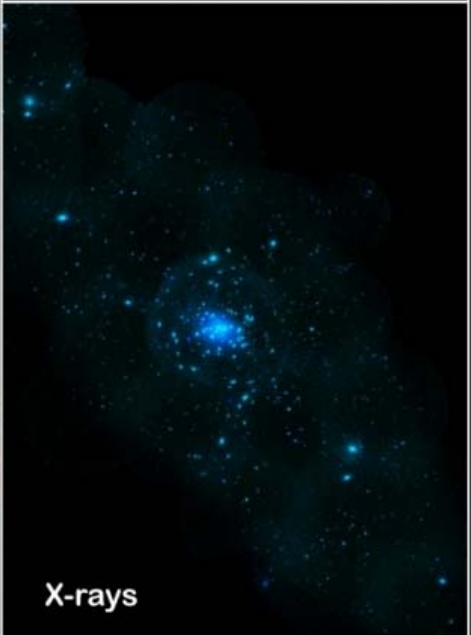
**Infrared & X-rays**



**Infrared**



**21 cm**



**X-rays**

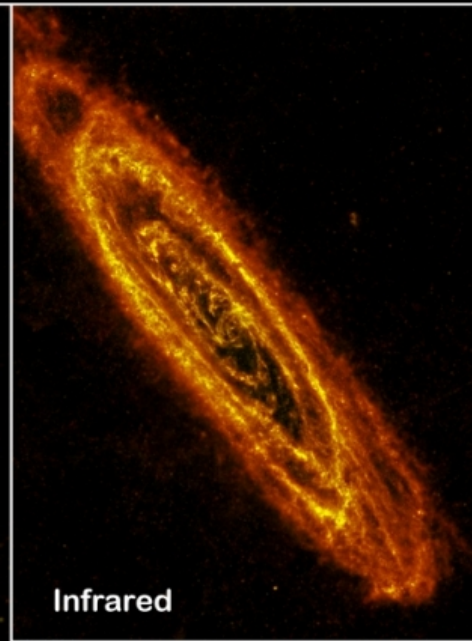
Understanding of a cosmic source requires observations across the whole EM spectrum



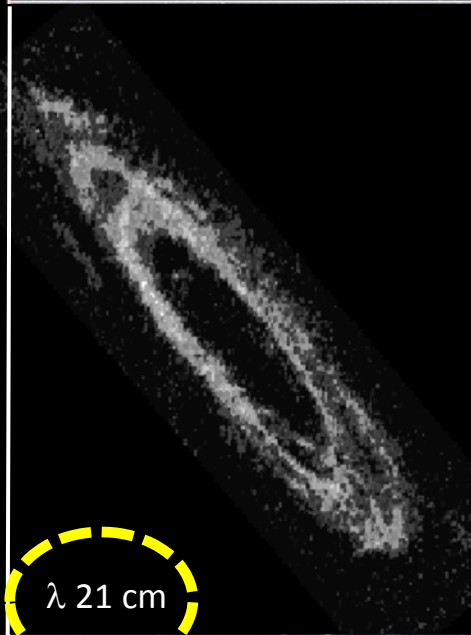
Optical



Infrared & X-rays



Infrared



$\lambda$  21 cm



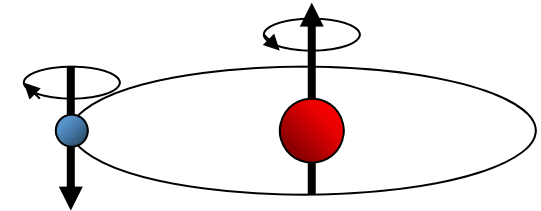
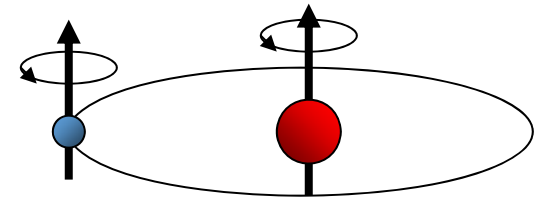
X-rays

Understanding of a cosmic source requires observations across the whole EM spectrum



- Hydrogen is the most abundant element in the universe
- $3 \times 10^9$  solar masses of HI in our Galaxy, in optically thin clouds, with T between 30 and 80K, density from 100 to 800  $\text{cm}^{-3}$ , masses from 1 to 100  $M_{\text{sun}}$ .
- The best way to study neutral H (HI) is the spin-flip line at 21.1 cm (1420.4 MHz).
- This is a forbidden line, the lifetime of the excited state is million years, and is very difficult to study it in the laboratory.
- Despite of this, there are so many H atoms in any line of sight through our Galaxy, that a significant amount of 21 cm photons is produced continuously.
- The intrinsic width of the line is negligible, and the measured width is due to Doppler broadening

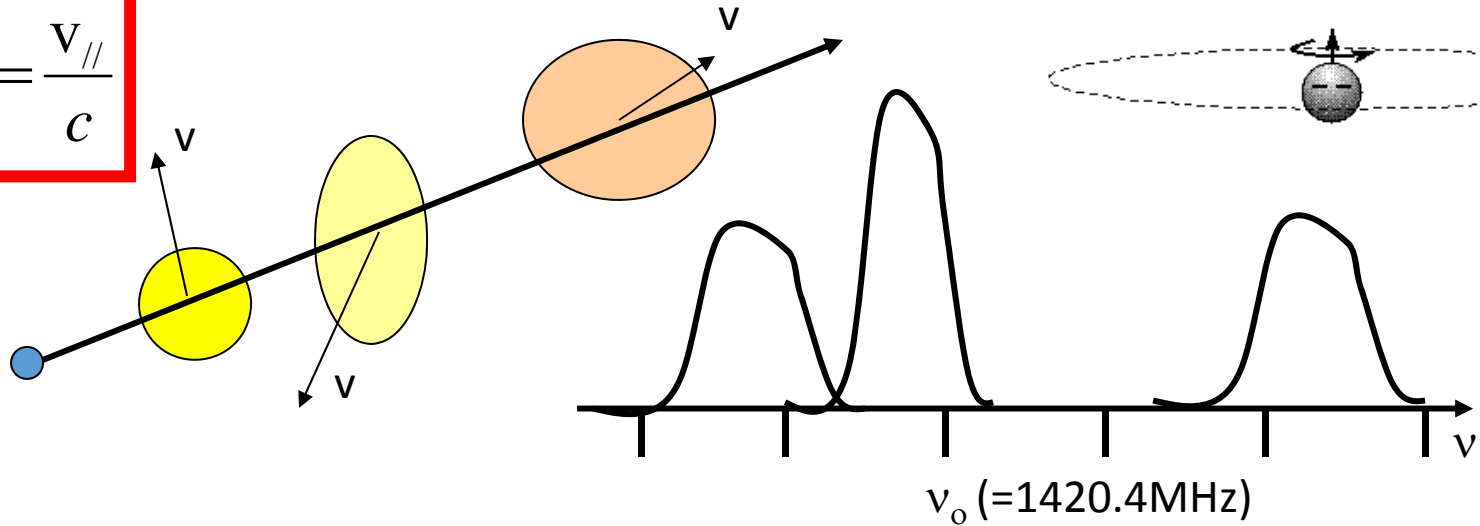
# Neutral H



Neutral atomic Hydrogen creates 21 cm radiation



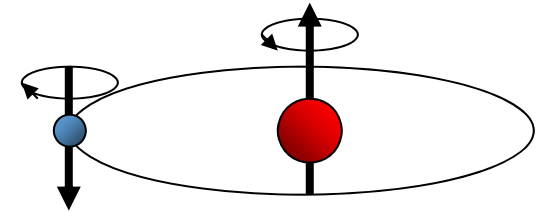
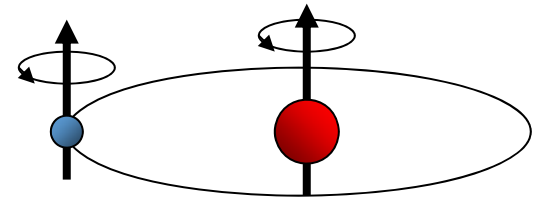
$$\frac{\Delta \nu}{\nu_0} = \frac{v_{\parallel}}{c}$$



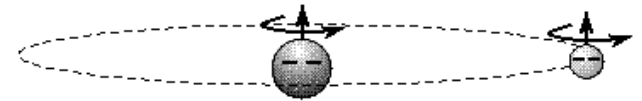
- Hydrogen is the most abundant element in the universe
- $3 \times 10^9$  solar masses of HI in our Galaxy, in optically thin clouds, with T between 30 and 80K, density from 100 to 800  $\text{cm}^{-3}$ , masses from 1 to 100  $M_{\text{sun}}$ .
- The best way to study neutral H (HI) is the spin-flip line at 21.1 cm (1420.4 MHz).
- This is a forbidden line, the lifetime of the excited state is million years, and is very difficult to study it in the laboratory.
- Despite of this, there are so many H atoms in any line of sight through our Galaxy, that a significant amount of 21 cm photons is produced continuously.
- The intrinsic width of the line is negligible, and the measured width is due to Doppler broadening

# Neutral H

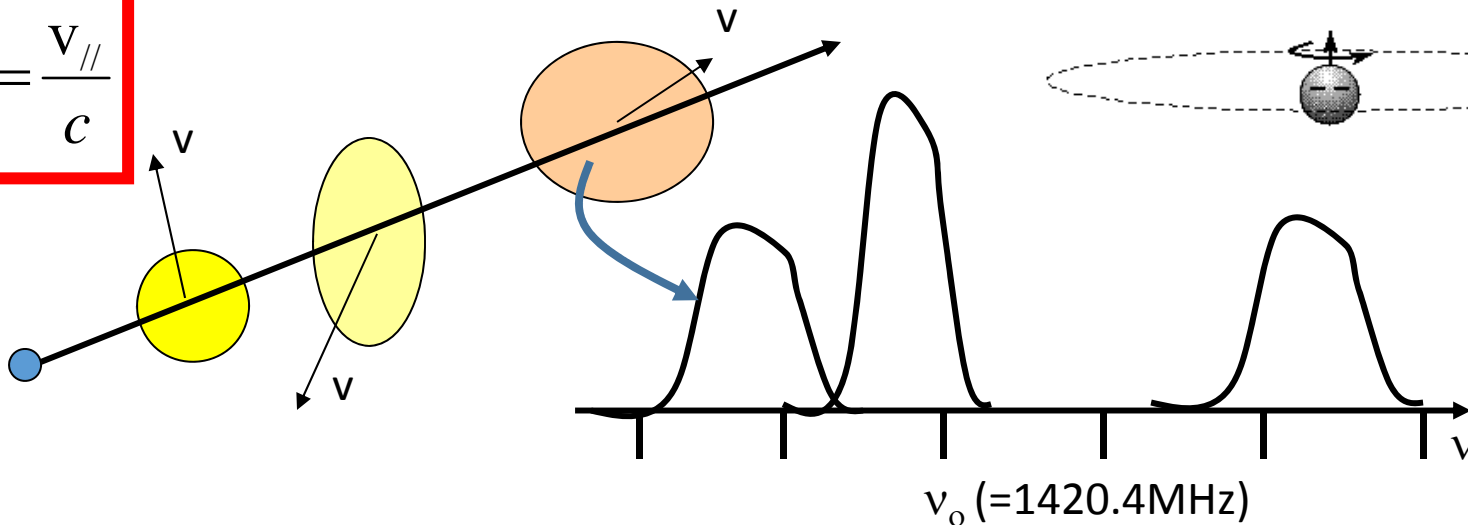
E



Neutral atomic Hydrogen creates 21 cm radiation



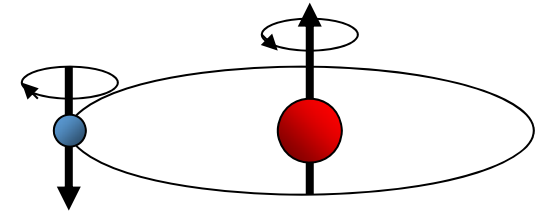
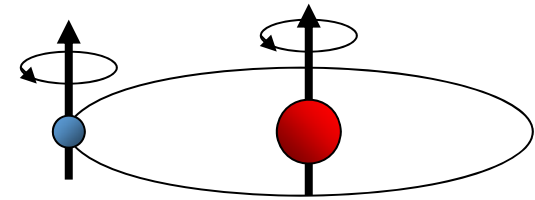
$$\frac{\Delta \nu}{\nu_o} = \frac{v_{\parallel}}{c}$$



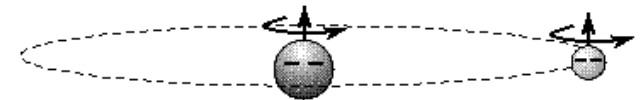
- Hydrogen is the most abundant element in the universe
- $3 \times 10^9$  solar masses of HI in our Galaxy, in optically thin clouds, with T between 30 and 80K, density from 100 to 800  $\text{cm}^{-3}$ , masses from 1 to 100  $M_{\text{sun}}$ .
- The best way to study neutral H (HI) is the spin-flip line at 21.1 cm (1420.4 MHz).
- This is a forbidden line, the lifetime of the excited state is million years, and is very difficult to study it in the laboratory.
- Despite of this, there are so many H atoms in any line of sight through our Galaxy, that a significant amount of 21 cm photons is produced continuously.
- The intrinsic width of the line is negligible, and the measured width is due to Doppler broadening

# Neutral H

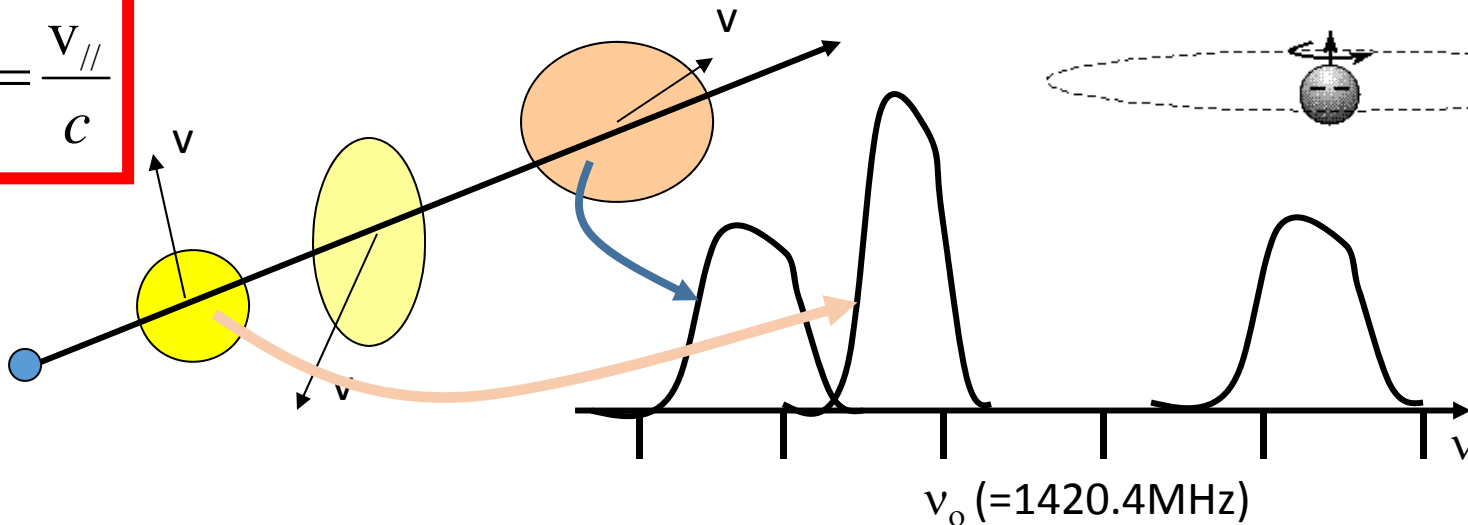
E



Neutral atomic Hydrogen creates 21 cm radiation



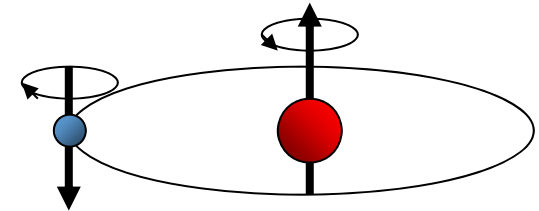
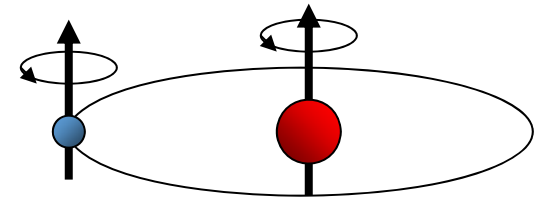
$$\frac{\Delta \nu}{\nu_0} = \frac{v_{\parallel}}{c}$$



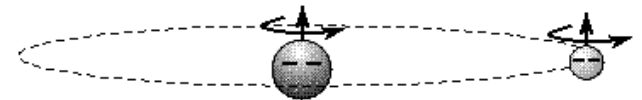
- Hydrogen is the most abundant element in the universe
- $3 \times 10^9$  solar masses of HI in our Galaxy, in optically thin clouds, with T between 30 and 80K, density from 100 to 800  $\text{cm}^{-3}$ , masses from 1 to 100  $M_{\text{sun}}$ .
- The best way to study neutral H (HI) is the spin-flip line at 21.1 cm (1420.4 MHz).
- This is a forbidden line, the lifetime of the excited state is million years, and is very difficult to study it in the laboratory.
- Despite of this, there are so many H atoms in any line of sight through our Galaxy, that a significant amount of 21 cm photons is produced continuously.
- The intrinsic width of the line is negligible, and the measured width is due to Doppler broadening

# Neutral H

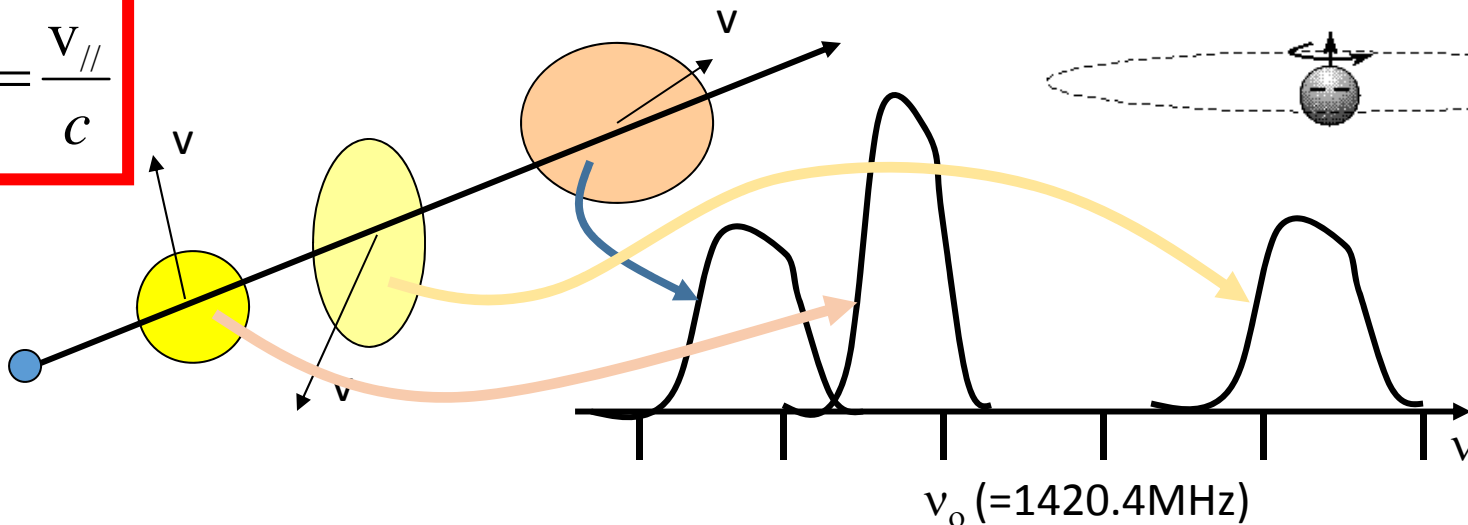
E



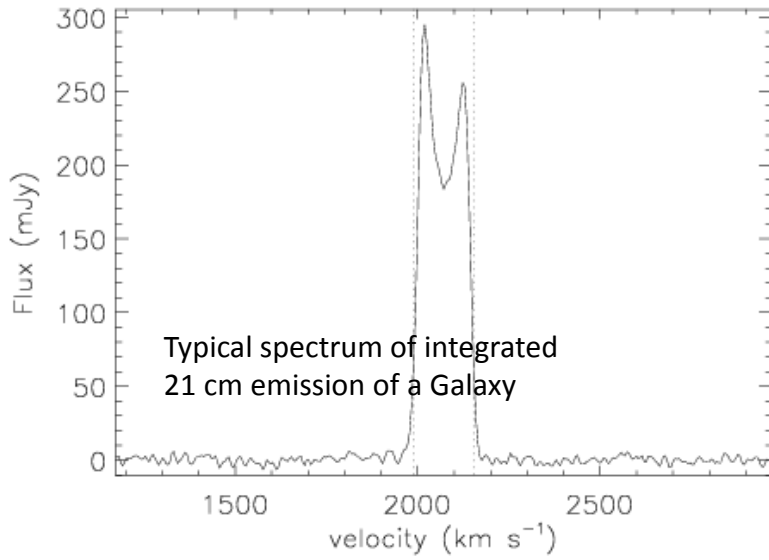
Neutral atomic Hydrogen creates 21 cm radiation



$$\frac{\Delta \nu}{\nu_0} = \frac{v_{\parallel}}{c}$$



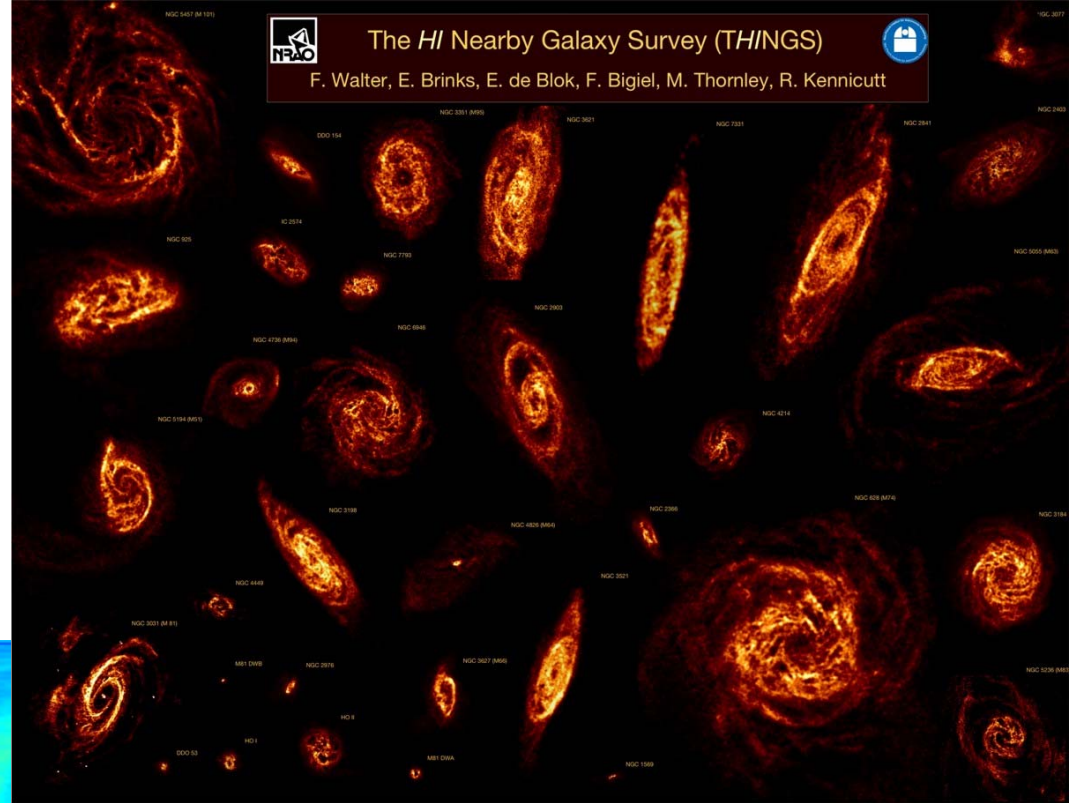
UGC4599



The *HI* Nearby Galaxy Survey (*THINGS*)



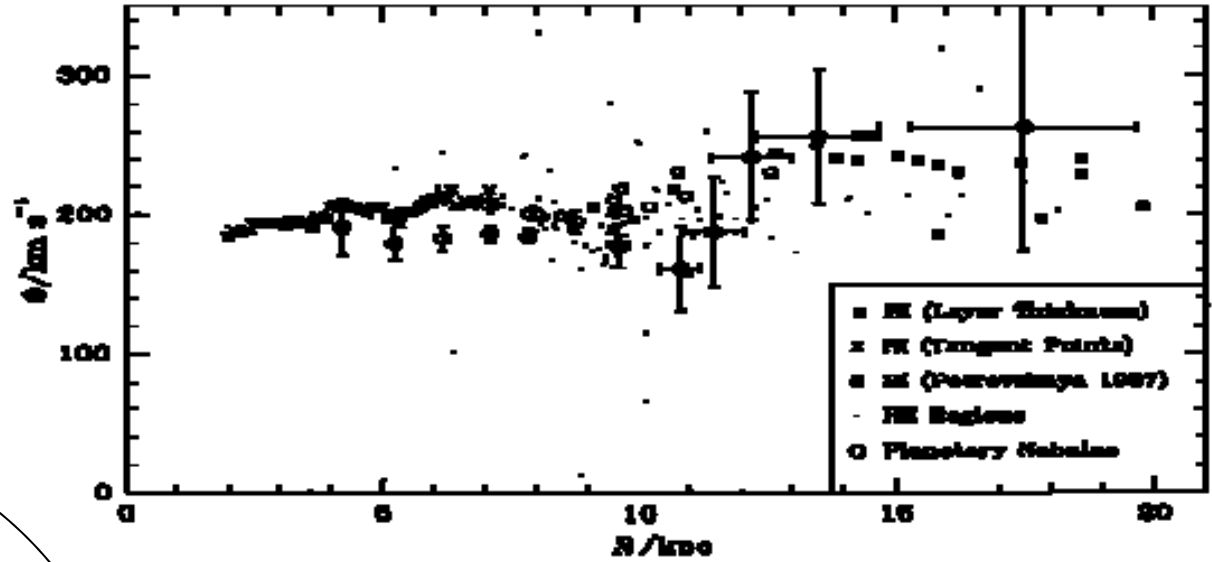
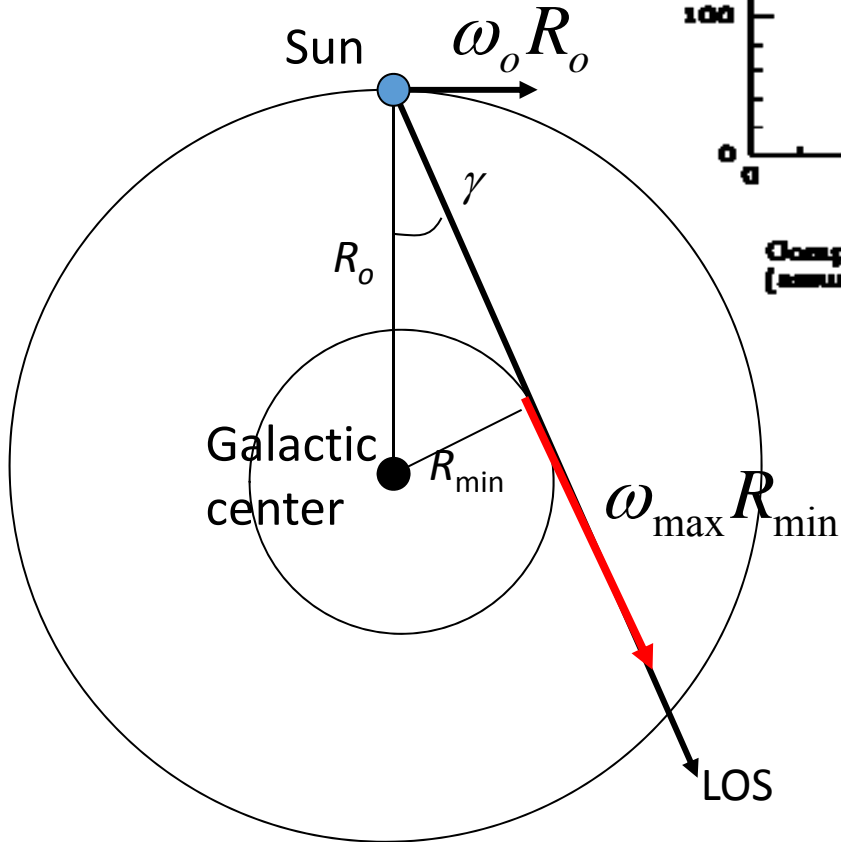
F. Walter, E. Brinks, E. de Blok, F. Bigiel, M. Thornley, R. Kennicutt



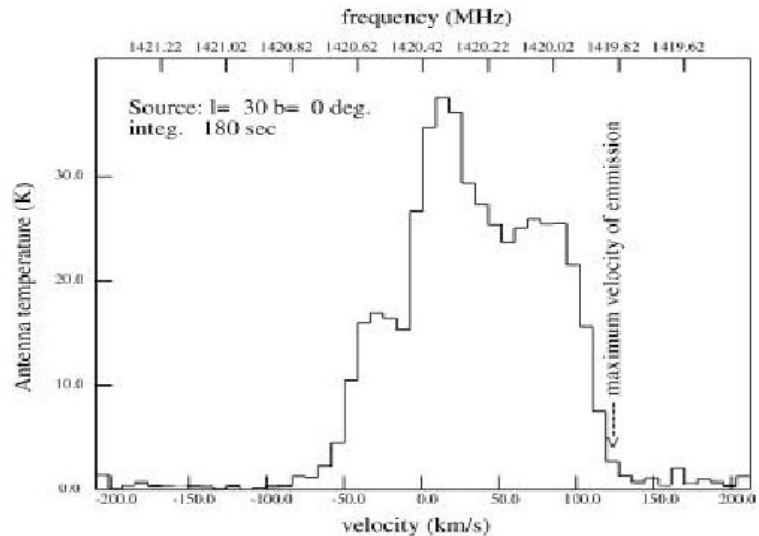
21 cm emission in our Galaxy

# Rotation curve of our galaxy from 21cm line etc.

Contrary to Keplerian expectation, orbital velocity of matter high even at large distance from the center.

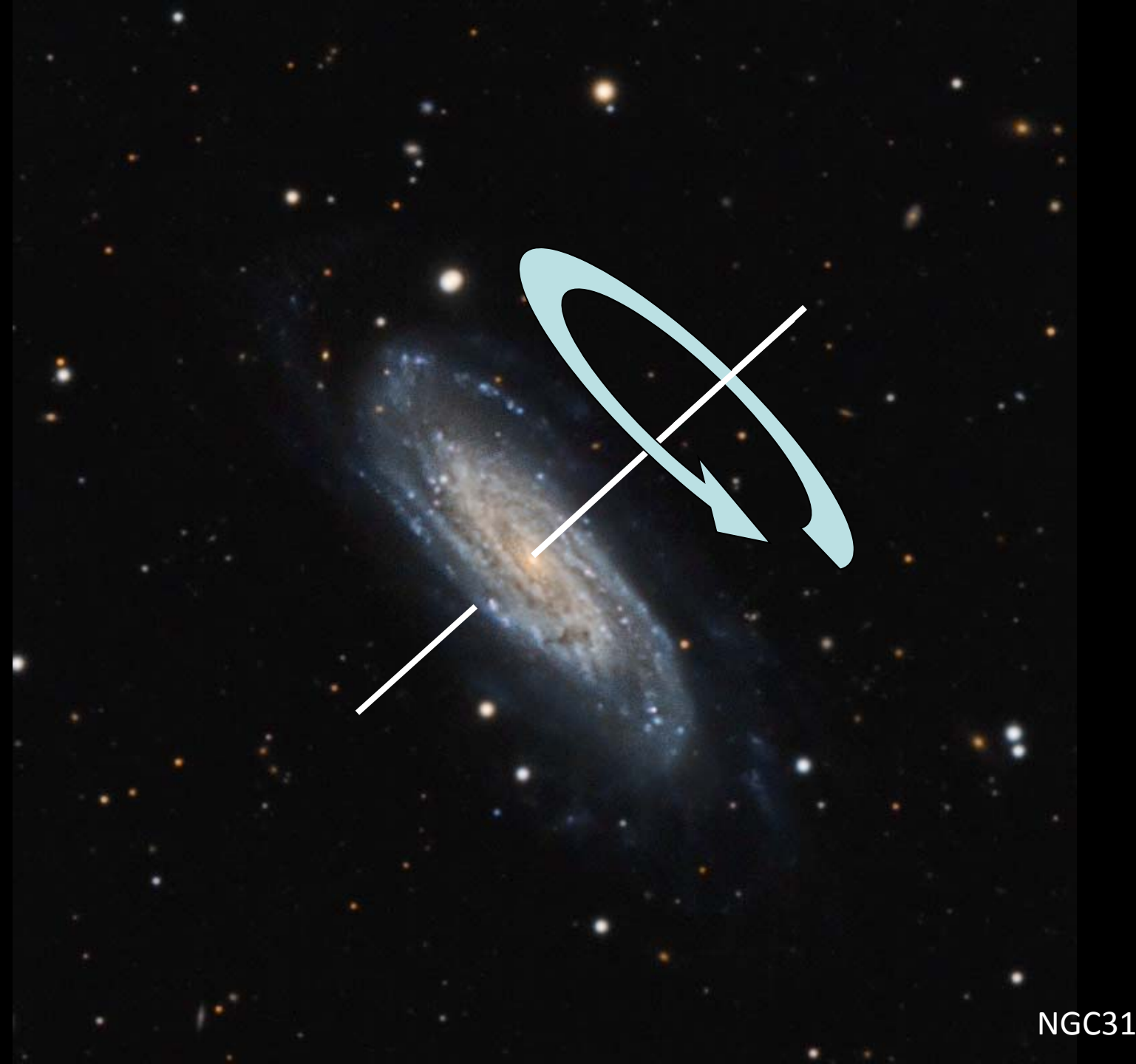


Comparison of the rotation curves calculated by all the methods discussed (assuming  $R_0 = 7.0$  kpc and  $\Theta_0 = 200$   $\text{km s}^{-1}$ ).



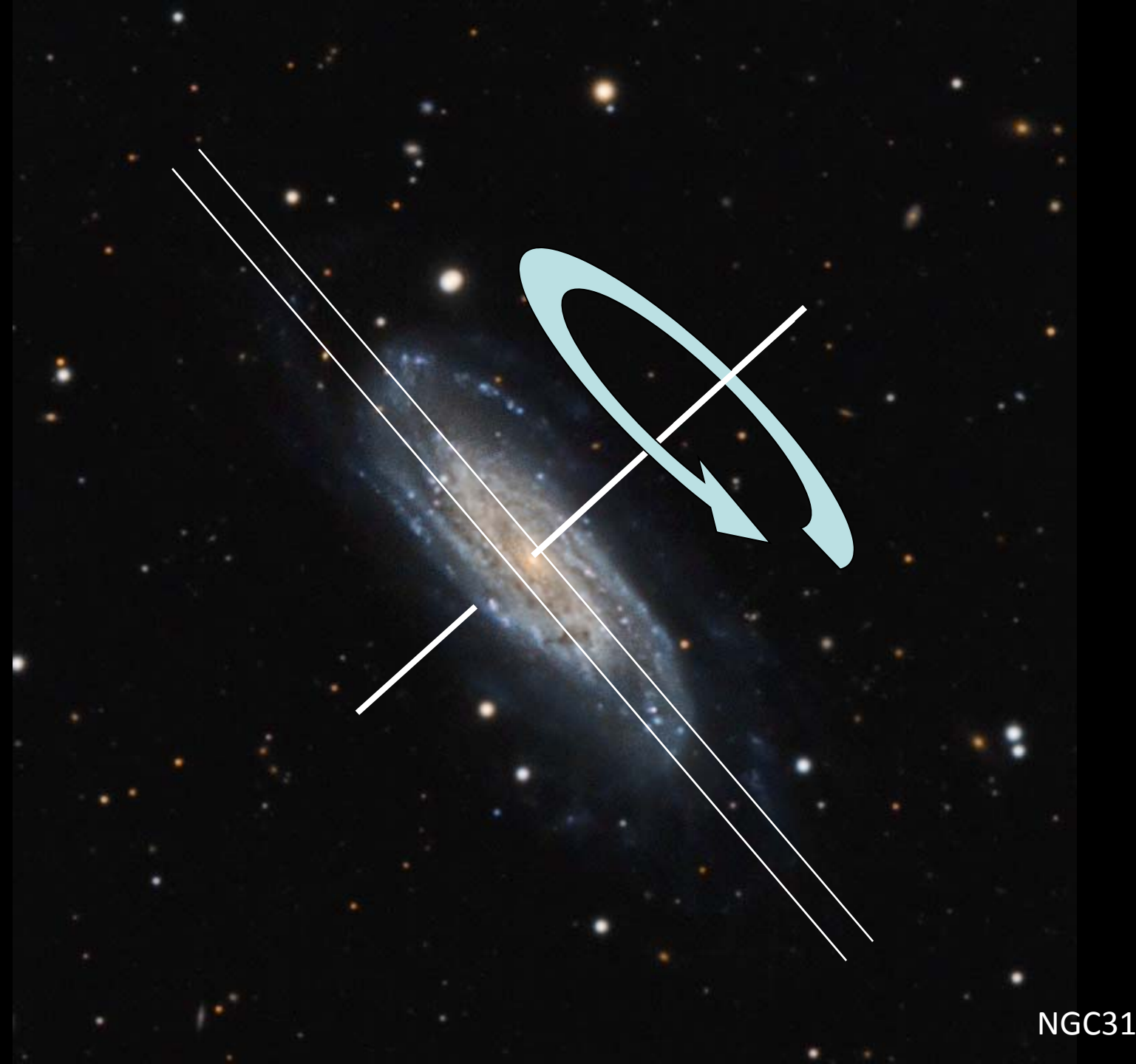


NGC3198

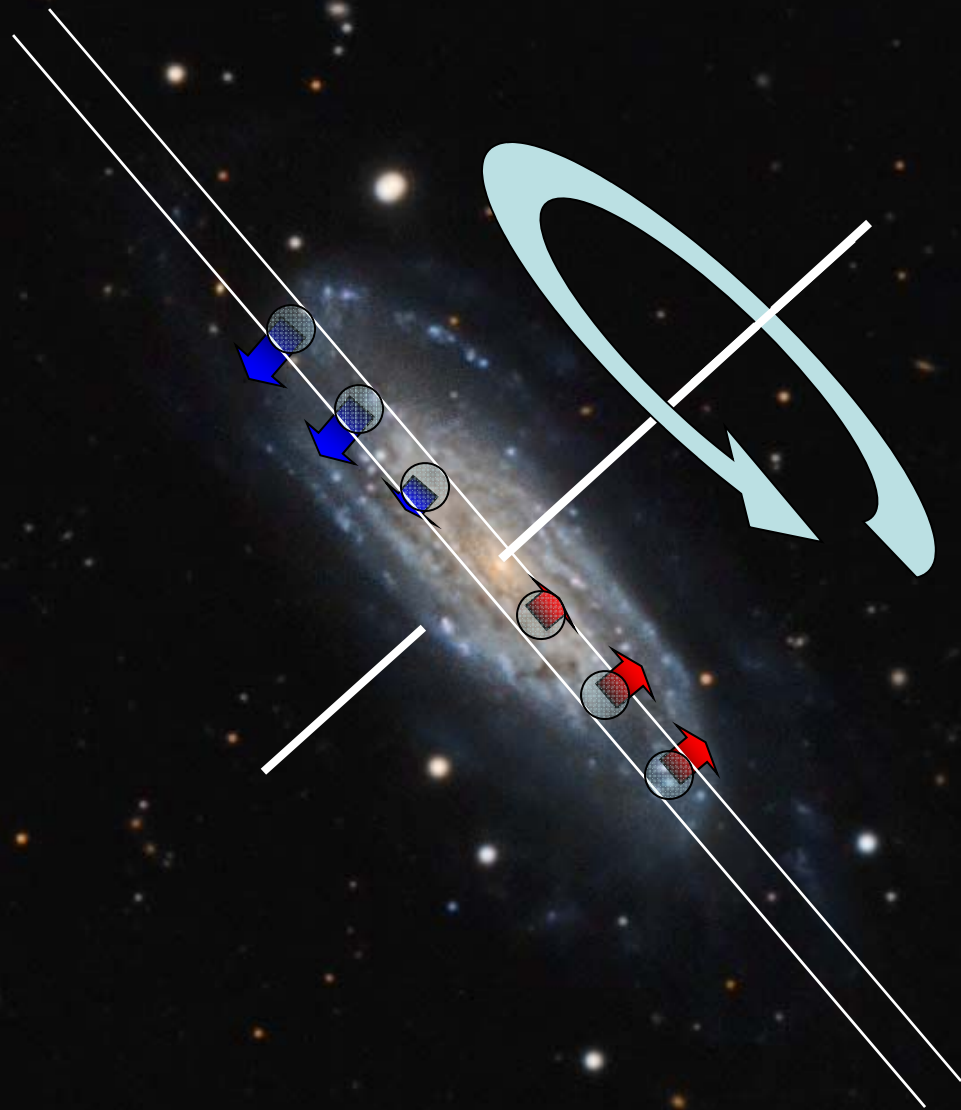


NGC3198





NGC3198



$$\frac{\Delta\lambda}{\lambda} = \frac{v_{//}}{c}$$

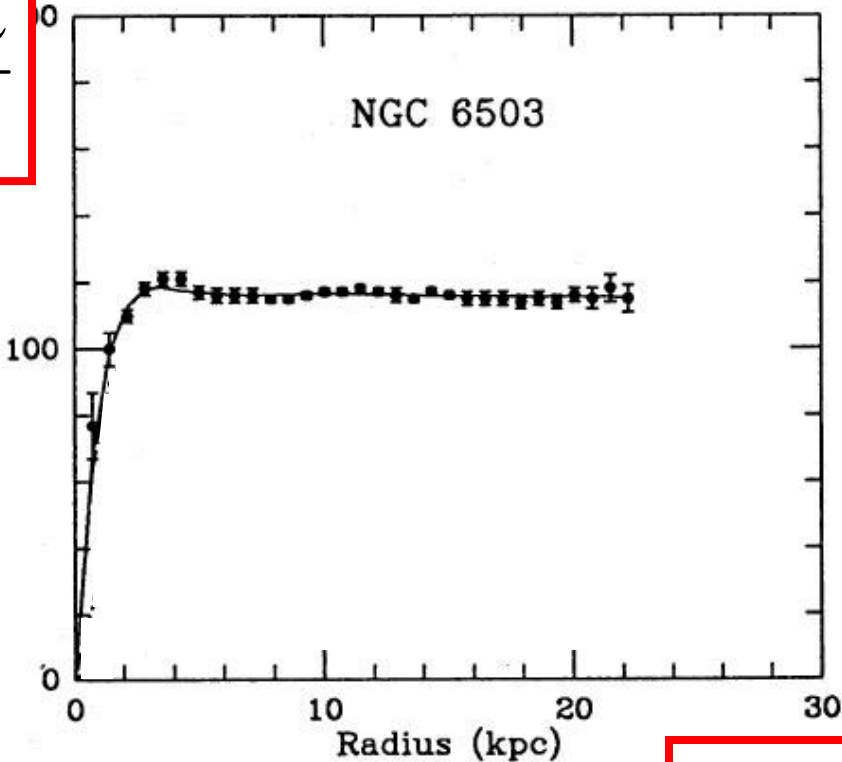
$$V_{\text{LOS}} = c \frac{\Delta\lambda}{\lambda}$$

Edge-on spiral galaxy :

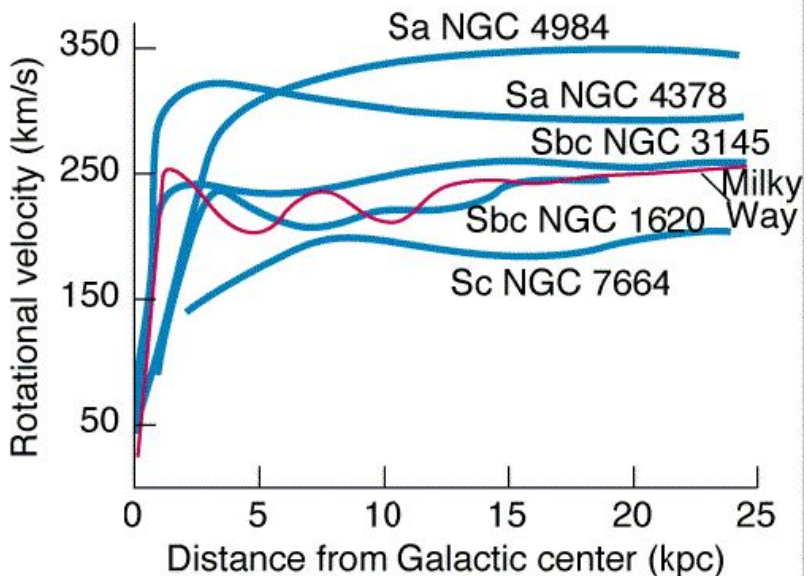
bulge

disk

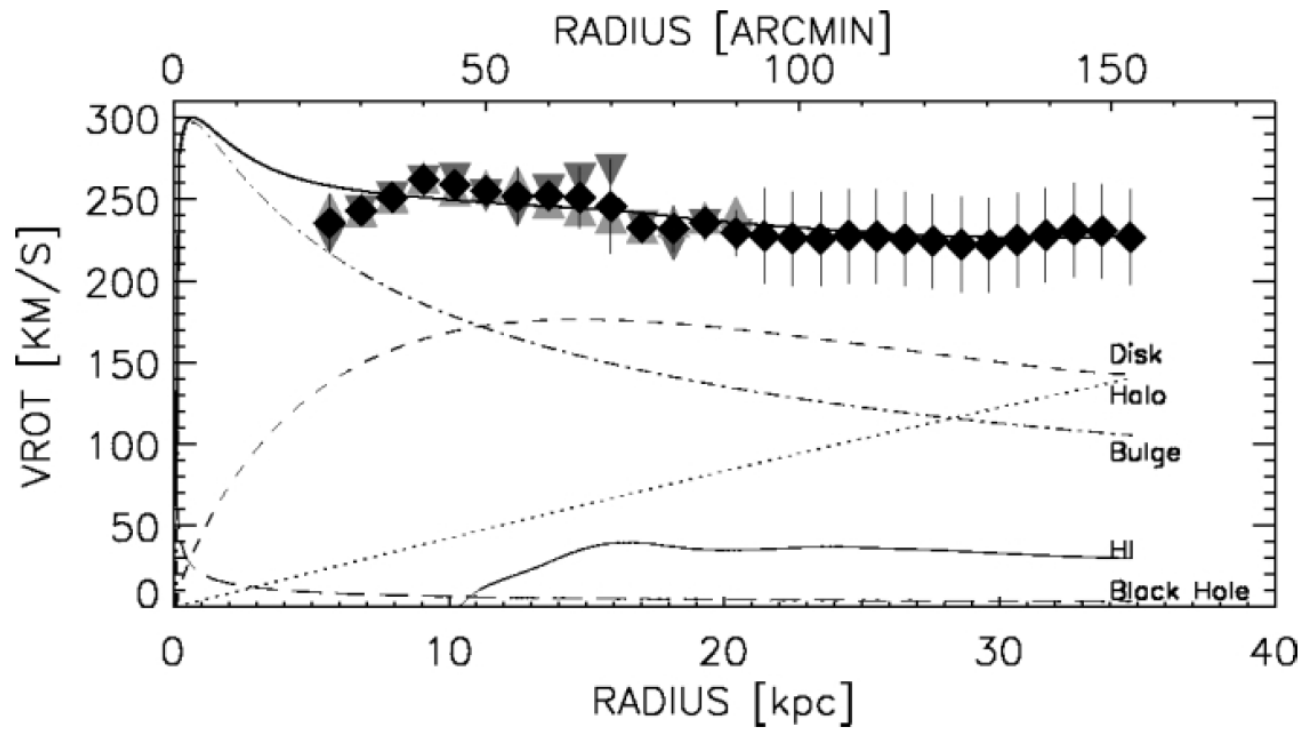
$V_{\text{cir}} \text{ (km s}^{-1}\text{)}$



$$r = \theta D$$



- All the rotation curves share the same basic trend (linear rise in the bulge region, approximately constant at larger distances)



M31

- Using 21 cm radiation the constant v trend at large distances is confirmed, at radii  $\gg$  than the optical radius.



Edge-on galaxy :

$$V = c \frac{\Delta\lambda}{\lambda}$$

bulge

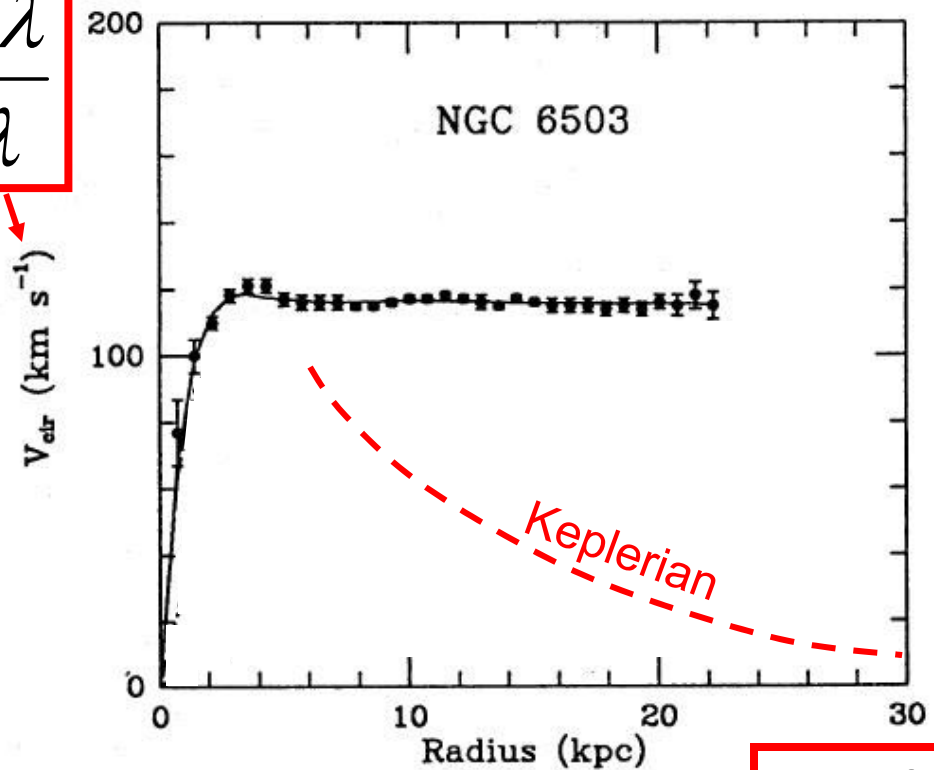
disk

If the mass density has a spherical symmetry (as in the bulge of the galaxy)

$$V = \sqrt{\frac{GM(< r)}{r}}$$

From the bulge only, the velocity of a test star at  $r > r_b$  would be Keplerian

$$V = \sqrt{\frac{GM_b}{r}}$$



$$r = \theta D$$

Edge-on galaxy :

$$V = c \frac{\Delta\lambda}{\lambda}$$

bulge

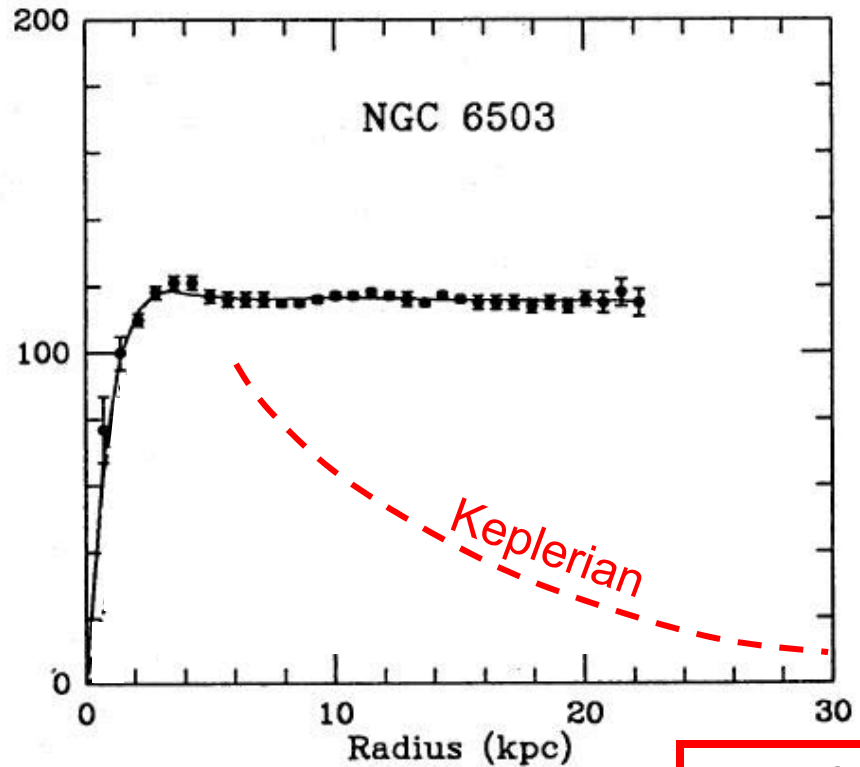
disk

If the mass density has a spherical symmetry (as in the bulge of the galaxy)

$$V = \sqrt{\frac{GM(< r)}{r}}$$

From the bulge only, the velocity of a test star at  $r > r_b$  would be Keplerian

$$V = \sqrt{\frac{GM_b}{r}}$$



$$r = \theta D$$

It can be shown that in the presence of an exponential disk with scale radius  $r_d$  the combined effect of bulge + disk is still Keplerian :

$$V \approx \frac{1}{\sqrt{r}} ; \quad r > 3r_d$$

Edge-on galaxy :

$$V = c \frac{\Delta\lambda}{\lambda}$$

bulge

disk

If the mass density has a spherical symmetry (as in the bulge of the galaxy)

$$V = \sqrt{\frac{GM(< r)}{r}}$$

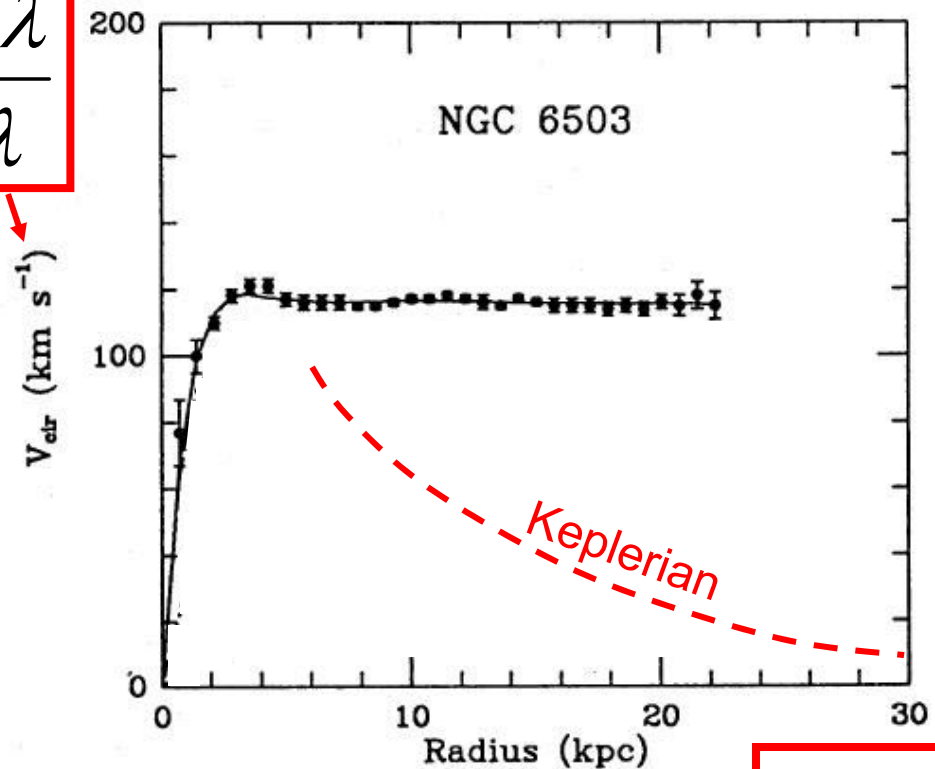
From the bulge only, the velocity of a test star at  $r > r_b$  would be Keplerian

$$V = \sqrt{\frac{GM_b}{r}}$$

It can be shown that in the presence of an exponential disk with scale radius  $r_d$  the combined effect of bulge + disk is still Keplerian :

**THIS DOES NOT FIT THE MEASURED DATA !**

$$V \approx \frac{1}{\sqrt{r}} ; \quad r > 3r_d$$



$$r = \theta D$$

- Two main interpretations:
  - The mass of the galaxy is dominated by an approximately spherical dark halo, much larger than the optical radius, and with density falling as  $1/r^2$  at large radii. In fact, in this case

$$M(< r) = \int_0^r 4\pi r^2 \rho(r) dr \propto r$$

$$\longrightarrow v(r) = \sqrt{\frac{GM(< r)}{r}} \propto \text{constant}$$

- Gravity deviates from Newton's law at extremely low accelerations (i.e. at large distances in galaxies) (Milgrom, MOND theory).



# Dark Matter

- The luminosity of spiral galaxies falls off exponentially with  $r$  ; the luminosity of elliptical galaxies falls off as  $1/r^4$ .
- So, luminous matter cannot produce the observed constant rotation curves at large radii.
- The total luminous mass is much less than the mass required from the relation

$$v = \sqrt{\frac{GM(< r)}{r}} \quad \rightarrow \quad M(< r_{\max}) = \frac{r_{\max} v_{\max}^2}{G}$$

- Typical numbers (confirmed also by the analysis of the motion of satellite galaxies):

$$M(< 200 \text{ kpc}) \approx 2 \times 10^{12} M_{\text{sun}} \quad \Rightarrow \quad \gamma = \frac{M_{\text{Sp}}}{L_{\text{Sp}}} \approx 100 \frac{M_{\text{sun}}}{L_{\text{sun}}} = 100 \gamma_{\text{sun}}$$

$$L \approx 2 \times 10^{10} L_{\text{sun}}$$

- Stars in spiral galaxies have an average mass to light ratio  $\gamma < 5 \gamma_{\text{sun}}$  . So the required mass has to be **dark**.
- Luminous matter can account for at most 5% of the total mass.

Blue = non-baryonic, dark matter halo  
White = baryonic matter

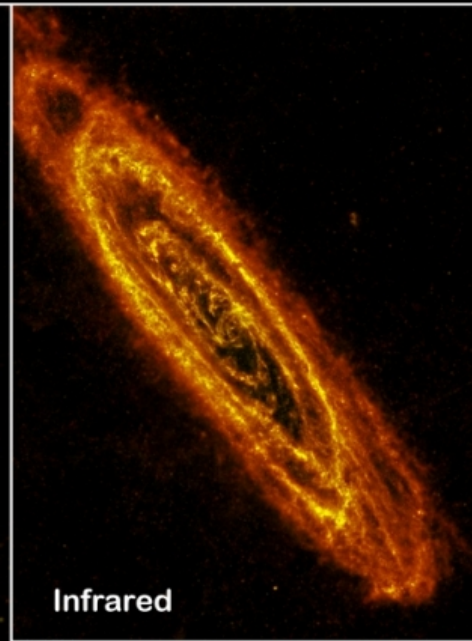




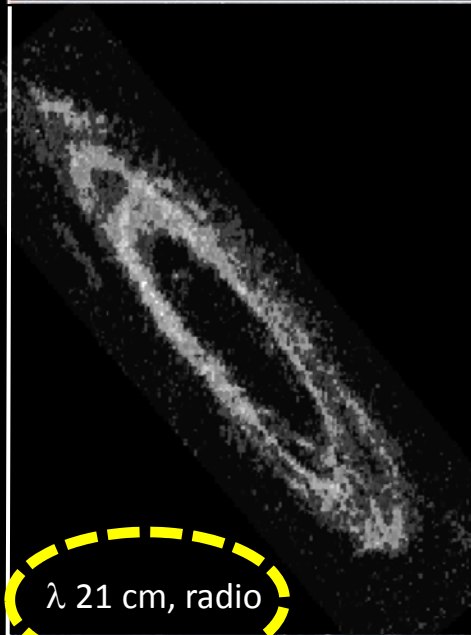
Optical



Infrared & X-rays



Infrared



$\lambda$  21 cm, radio



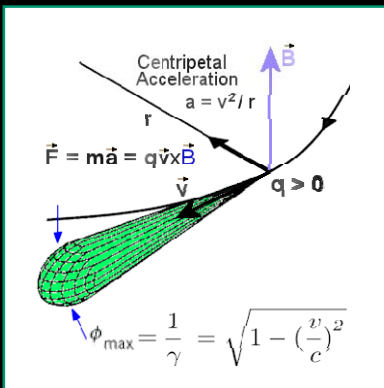
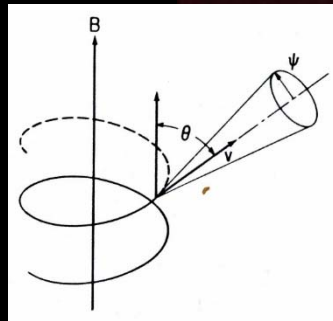
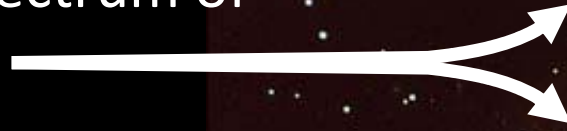
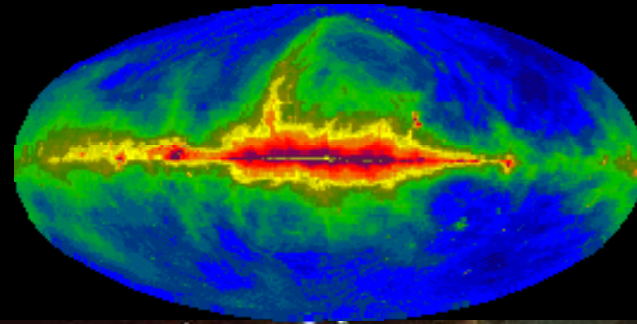
X-rays

Understanding of a cosmic source requires observations across the whole EM spectrum

# Galactic Synchrotron

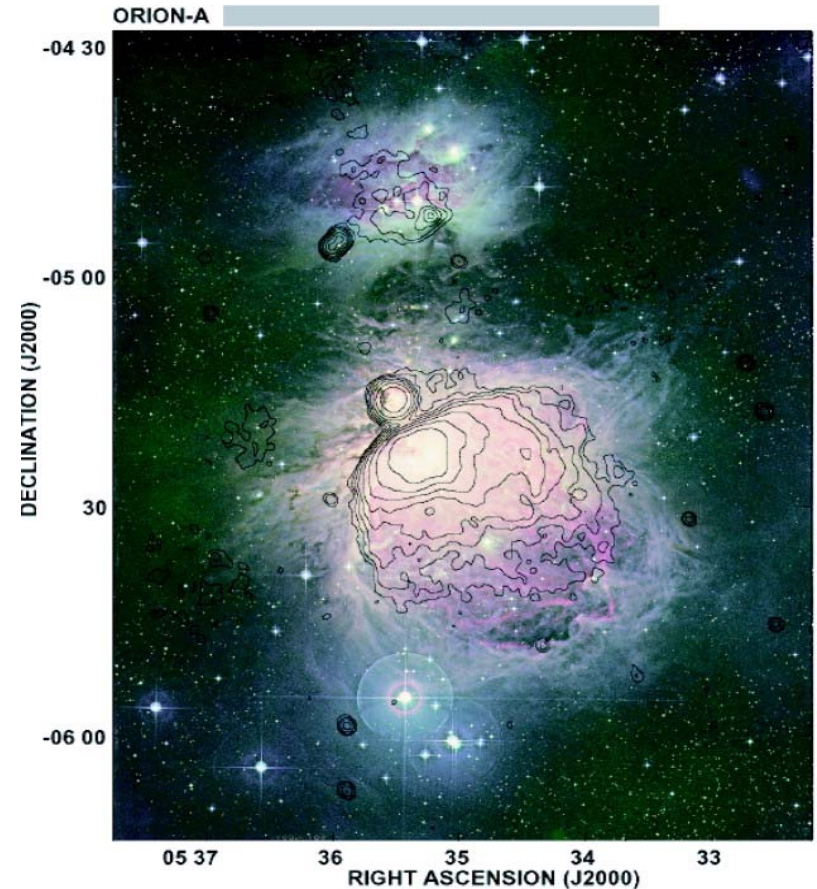
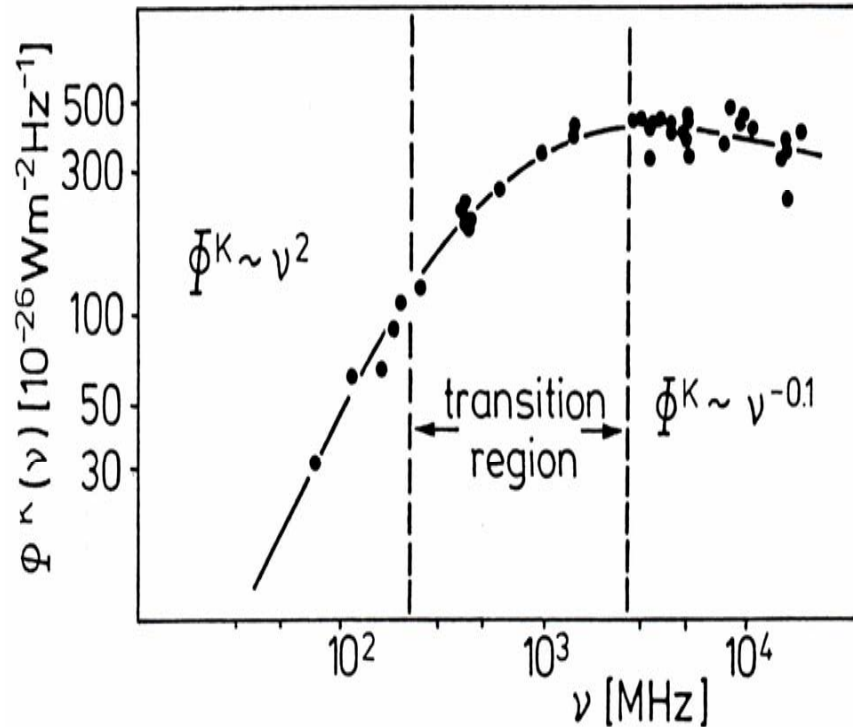
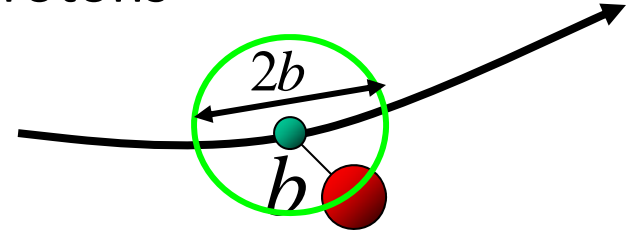
$$I(\nu) \approx \nu^\alpha$$
$$\alpha \approx -0.8 \pm 0.1$$

- Due to free electrons spiraling in the magnetic field of our Galaxy
- Continuum spectrum, depends on energy spectrum of electrons
- Significantly polarized (linear & circular)



# Free-Free Radiation

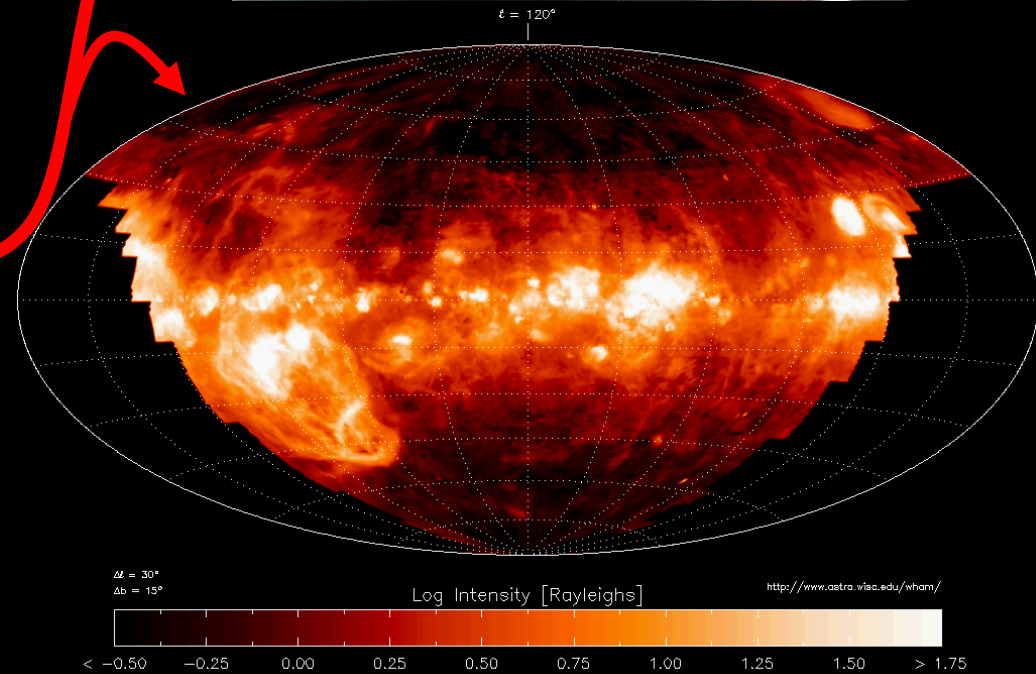
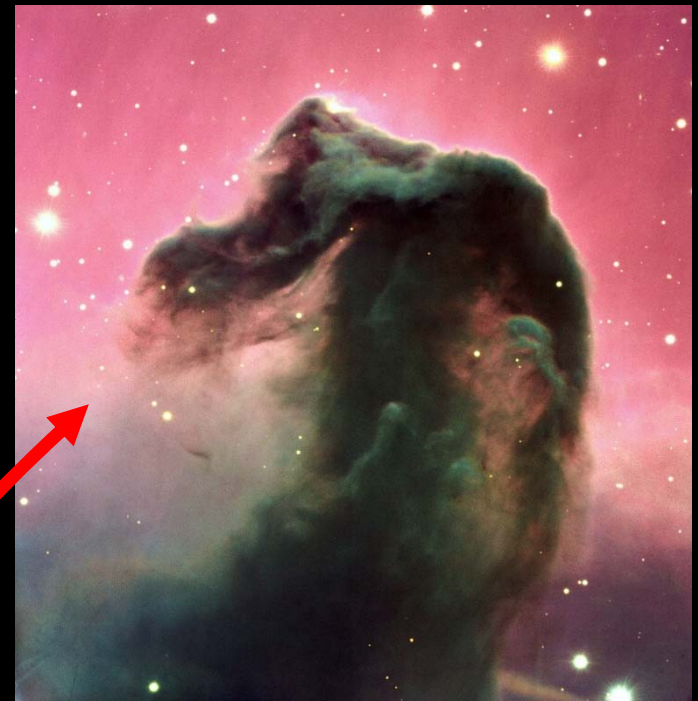
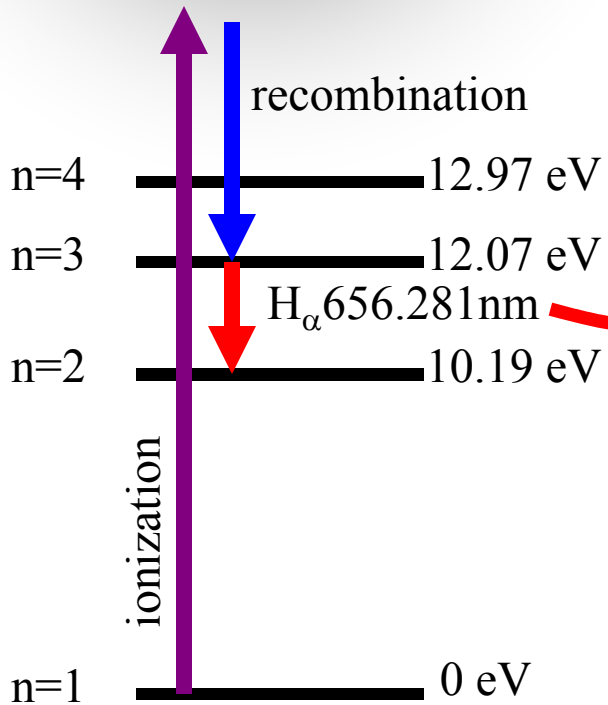
- Due to free electrons scattering against protons
- Continuum spectrum
- not polarized
- Evident in HII regions

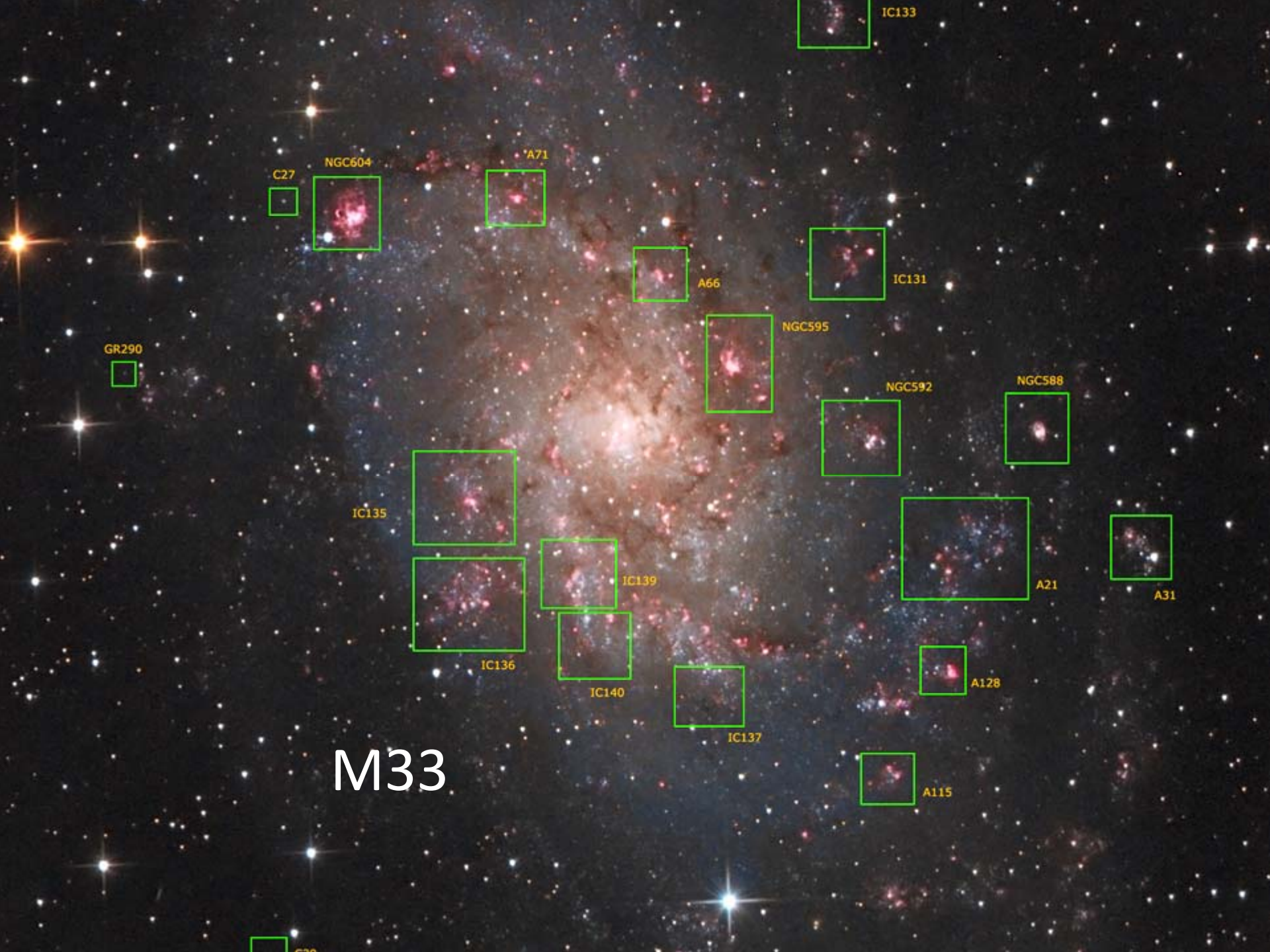


# HII regions



HI





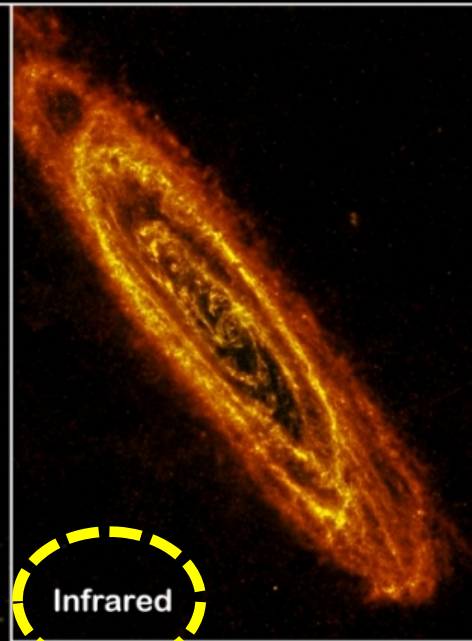
M33



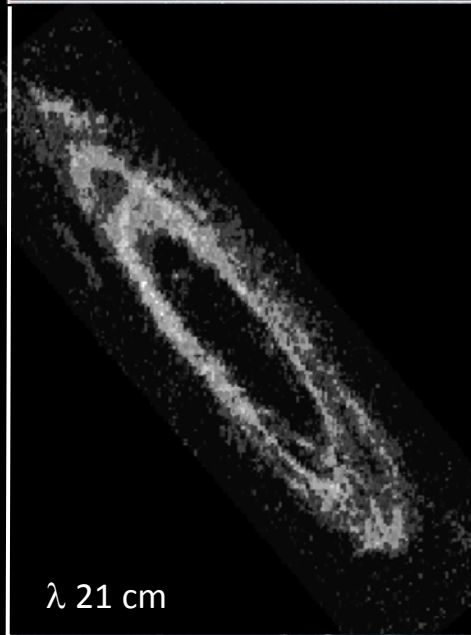
Optical



Infrared & X-rays



Infrared



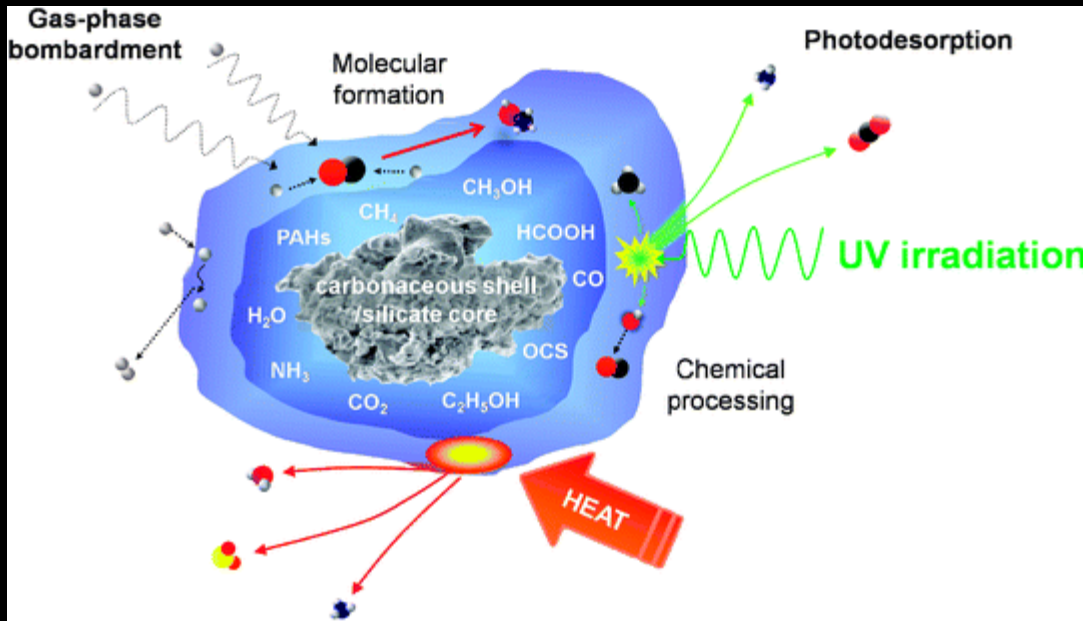
$\lambda$  21 cm



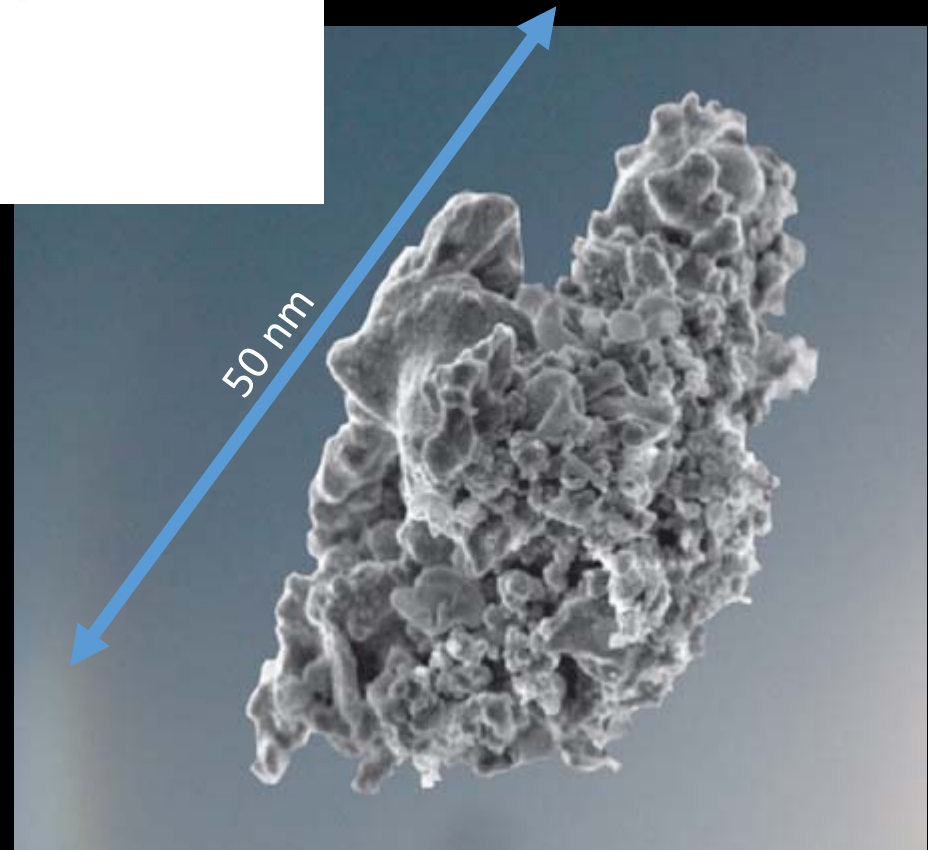
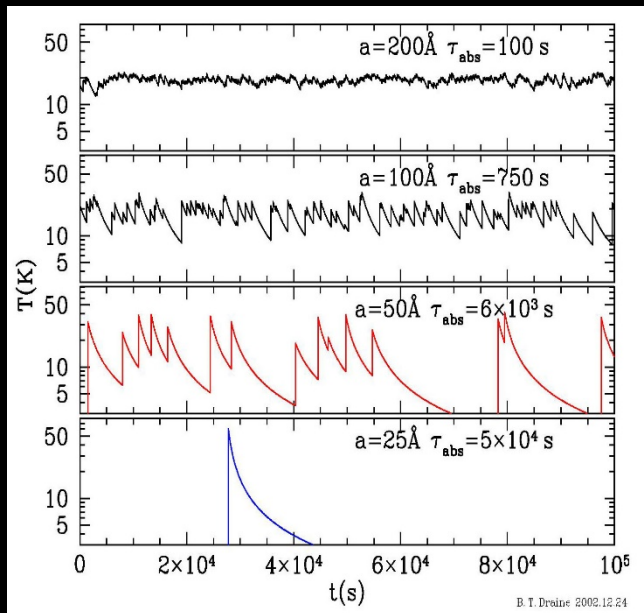
X-rays

Understanding of a cosmic source requires observations across the whole EM spectrum

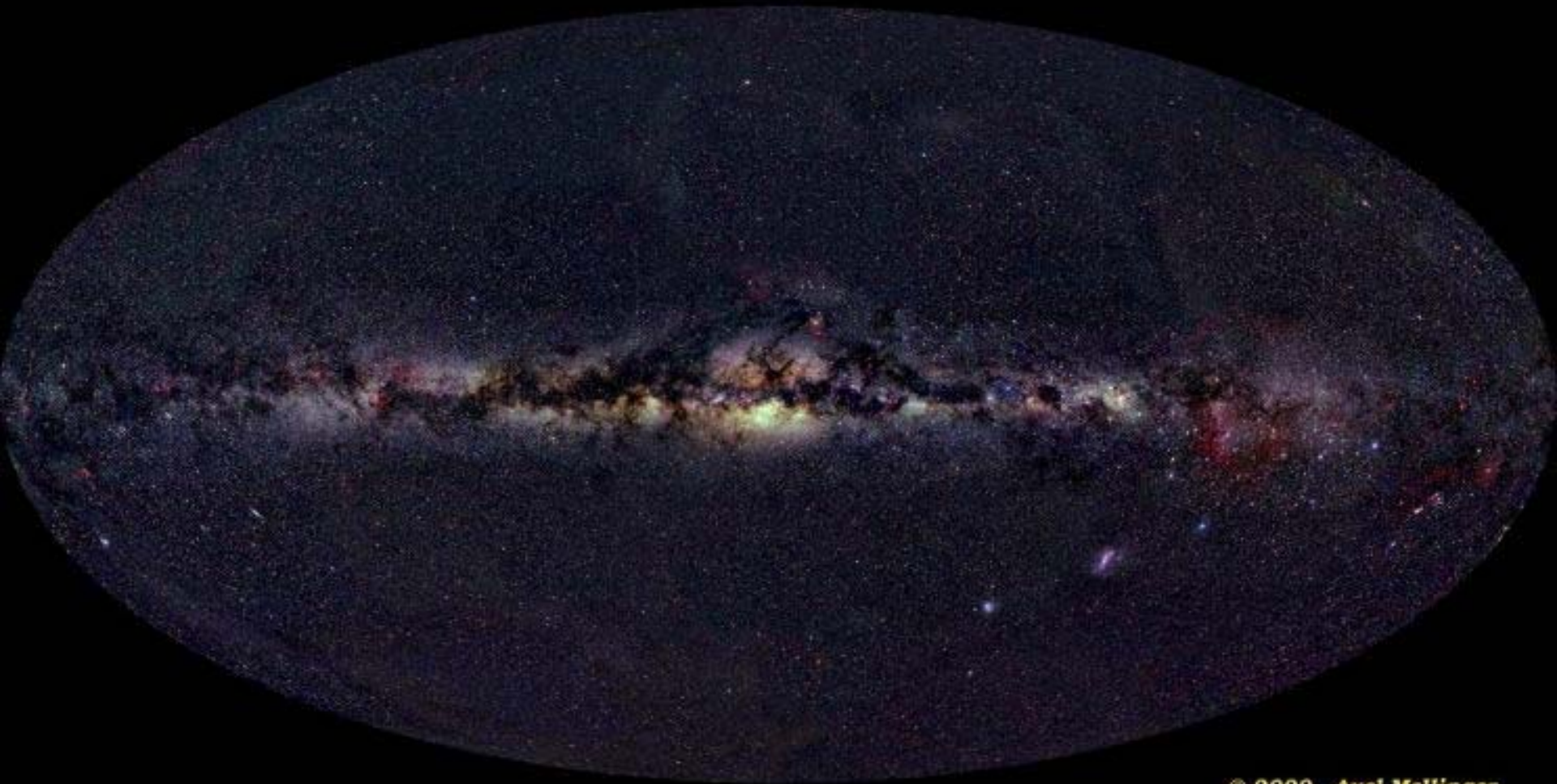




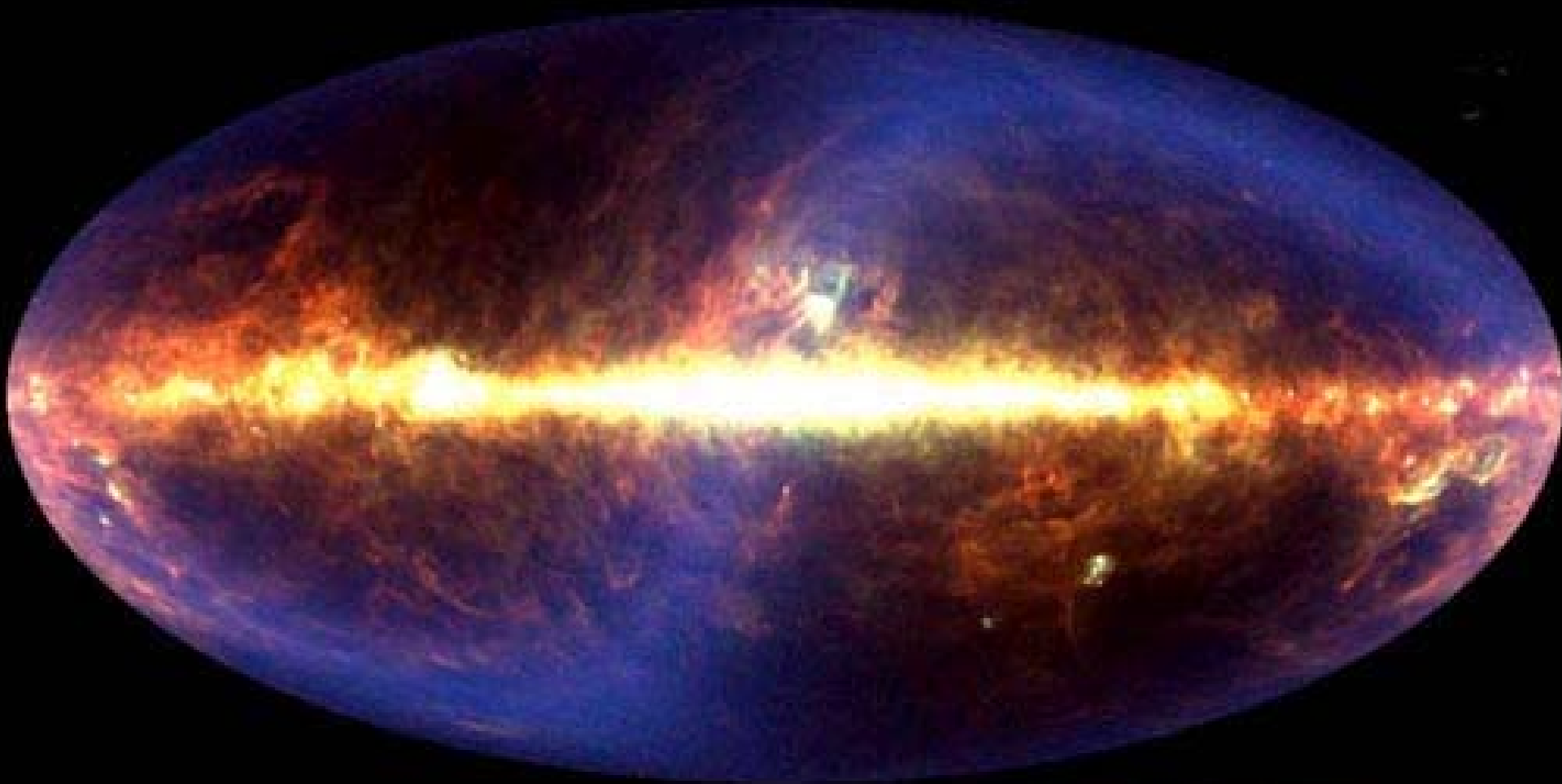
Interstellar dust grains



- Dust grains absorbing visible light ....



- ... and emitting thermal / far IR



- absorbing visible light ....



- ... and emitting thermal / far IR

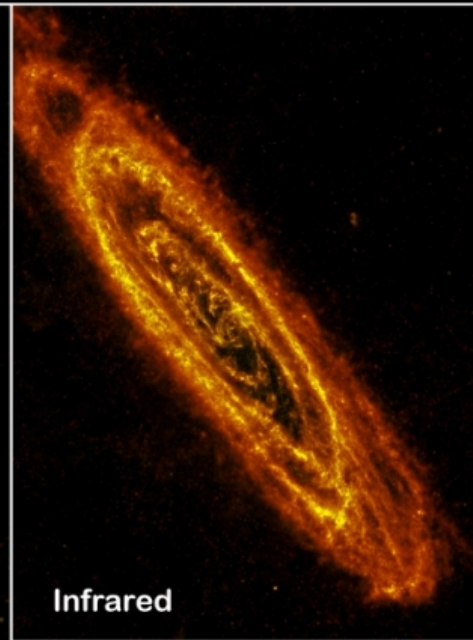




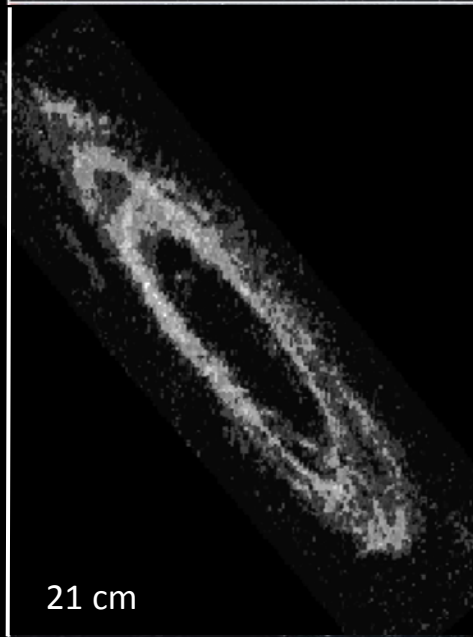
Optical



Infrared & X-rays



Infrared



21 cm

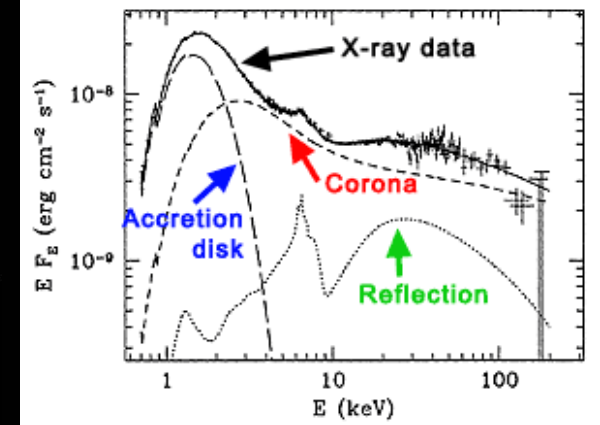
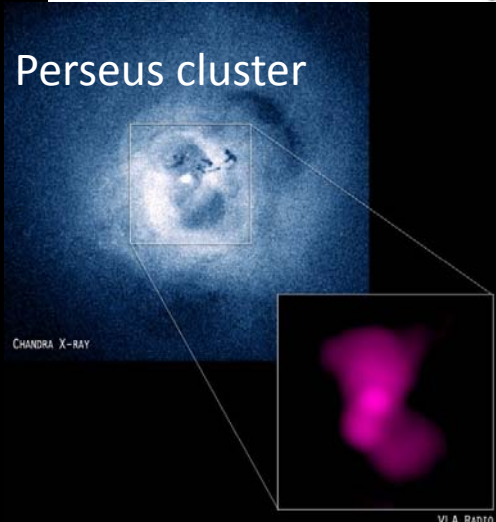
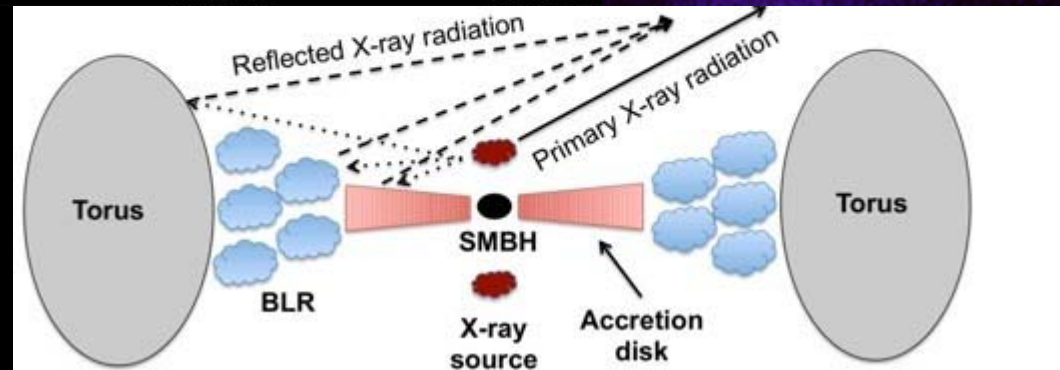
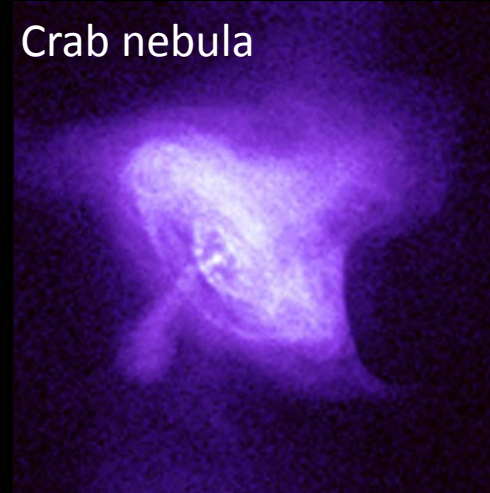
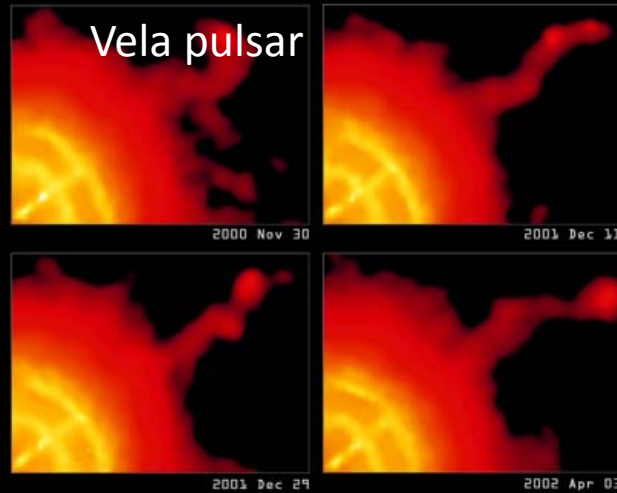


X-rays

Understanding of a cosmic source requires observations across the whole EM spectrum

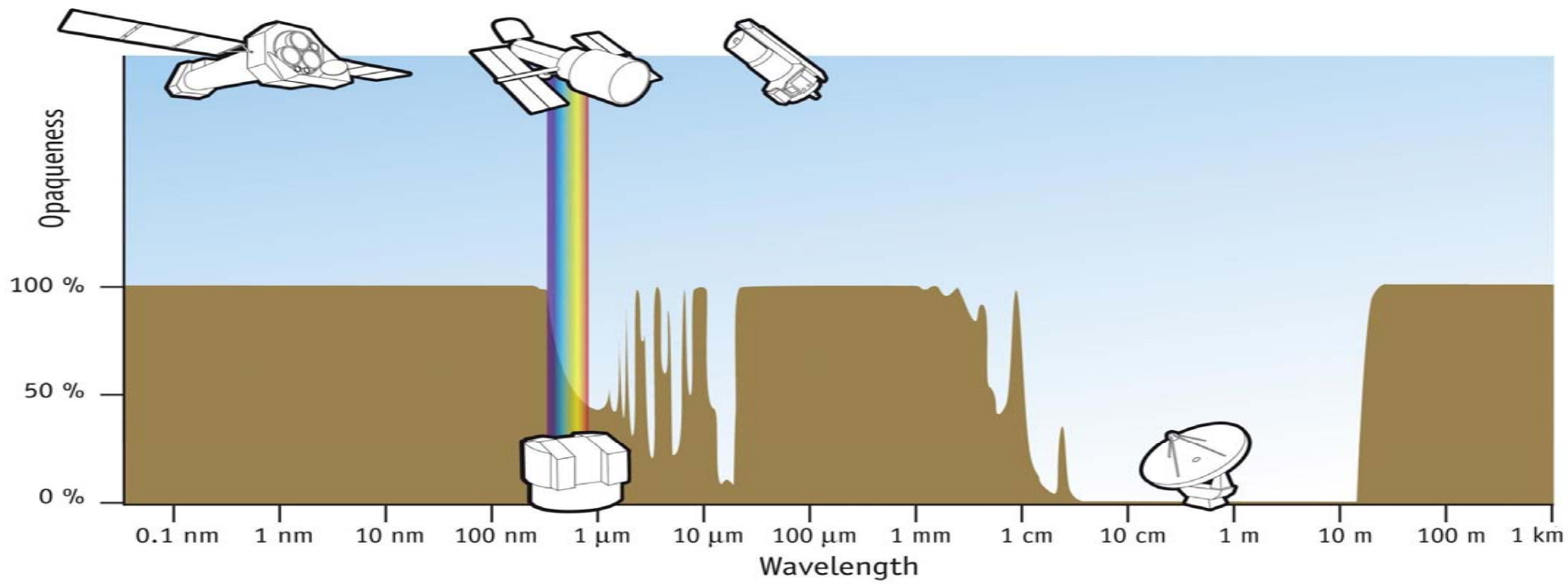
# X-rays

- X-rays come from extremely hot gasses,  $10^6$ - $10^8$ K.
- Mainly in compact stars, such as neutron stars or black holes : material falling into a black hole is heated by friction in the extremely strong gravitational field
- Also, intergalactic gas is heated to very high T in the deep gravitational potential well of galaxy clusters

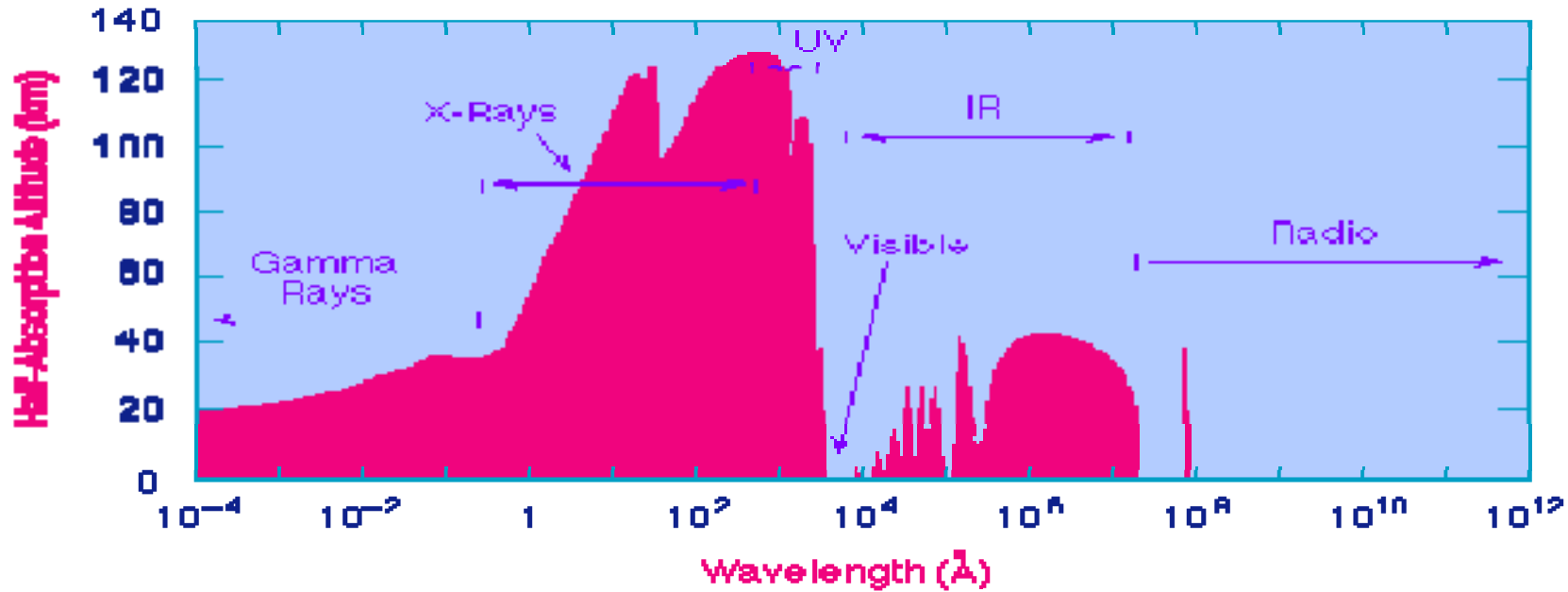


# Propagation effects

- Absorption / emission / scattering in the Earth atmosphere
- Absorption / scattering / emission in the universe
- Lensing
- Redshift

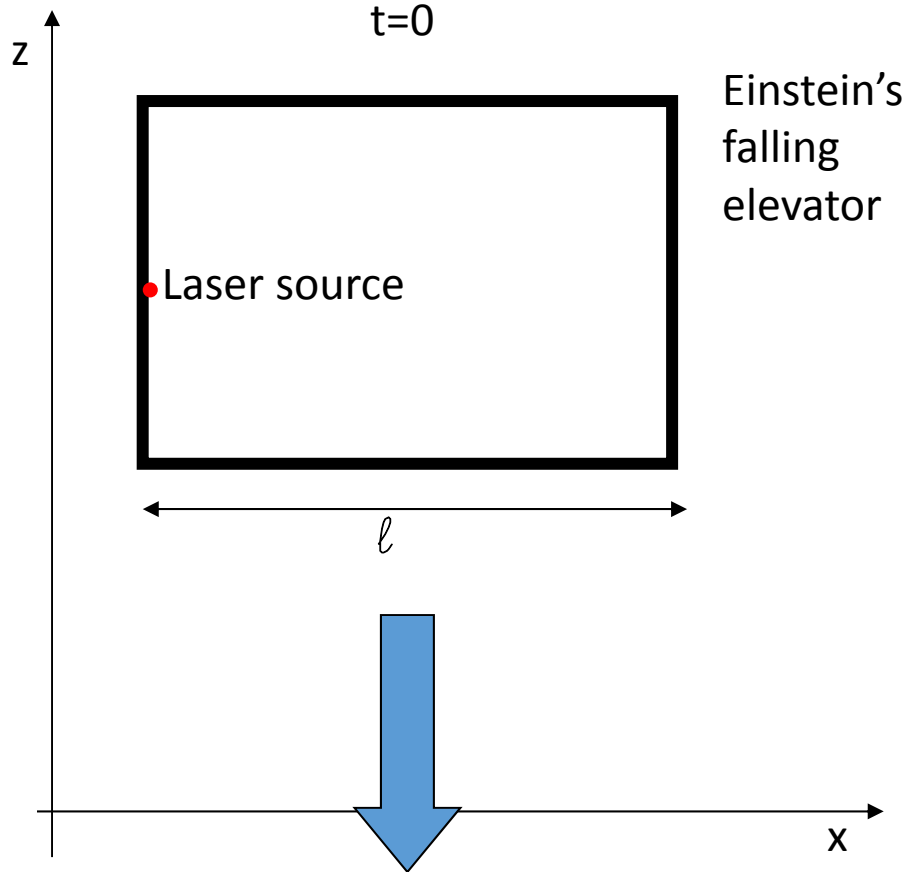


$\gamma$ -rays X-rays UV V NIR FIR  $\mu$ -waves radio radio



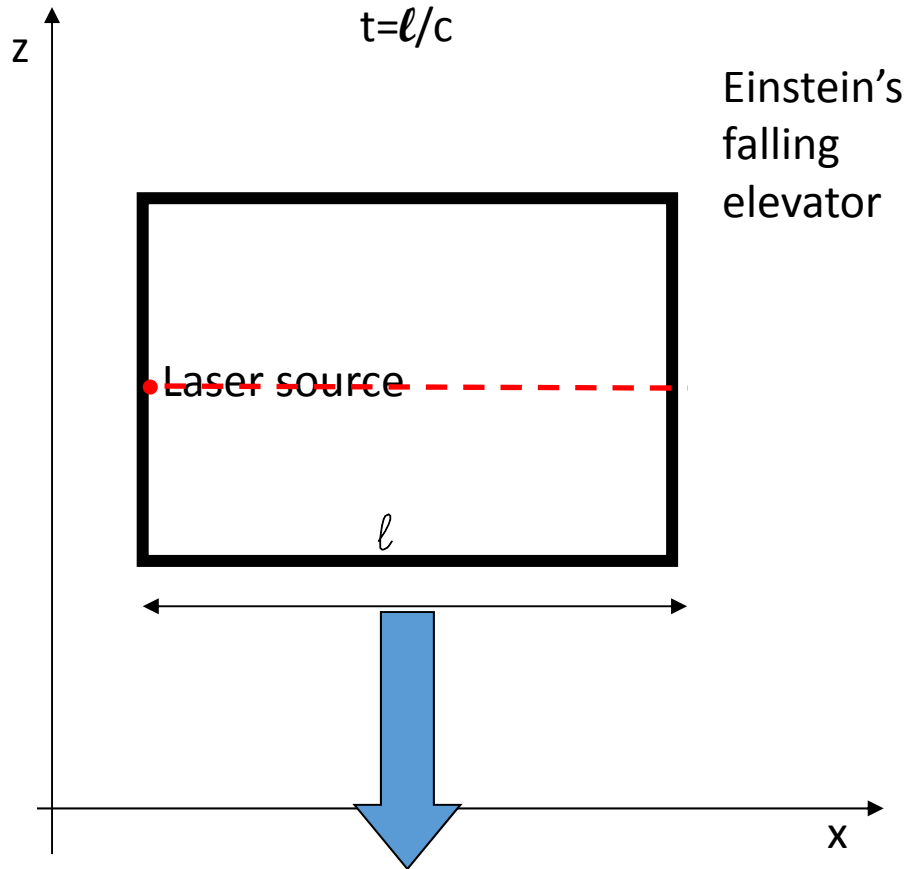


# Bending of light in a gravitational field



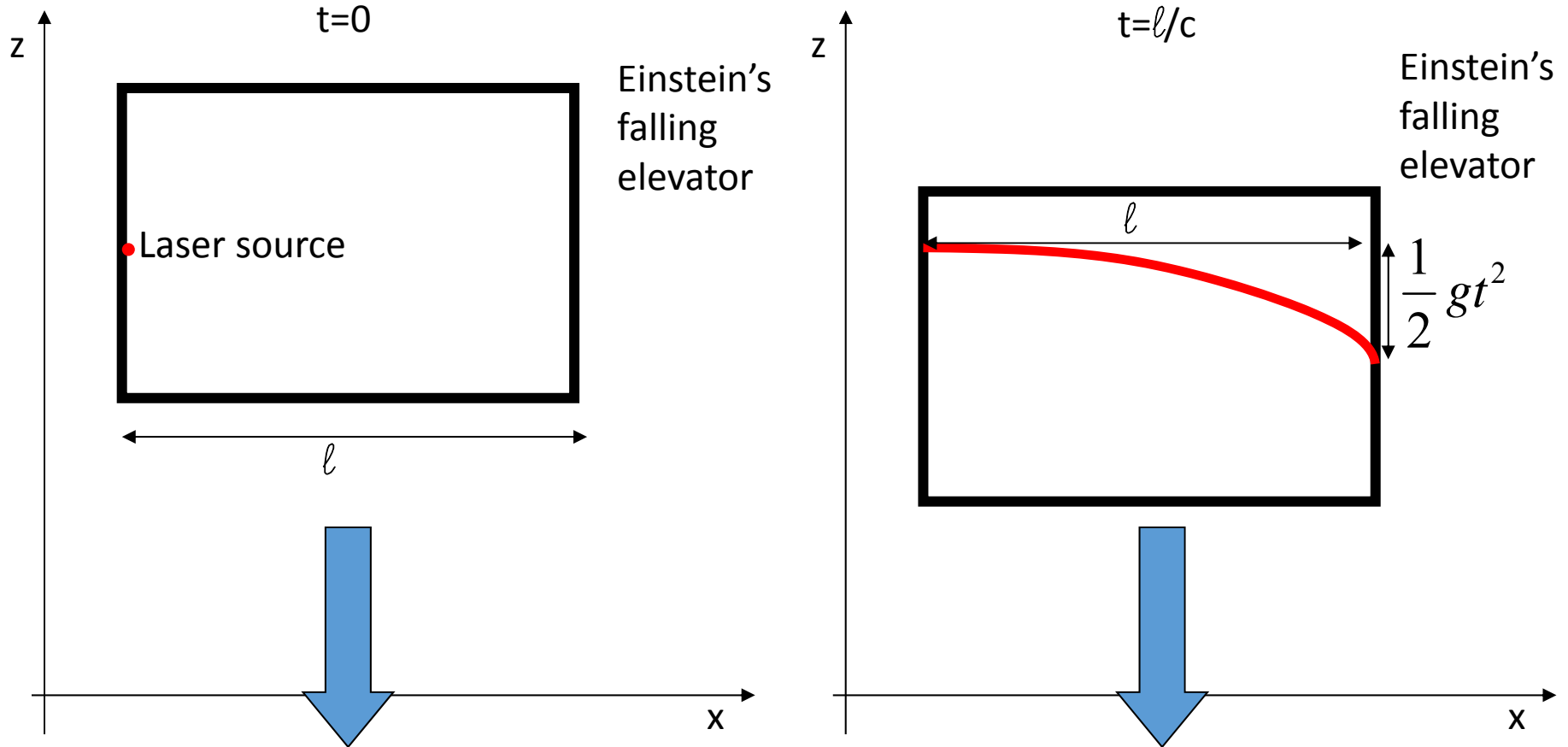
- The observer in the free-falling elevator does not feel gravity, and will see the laser beam propagating at constant height wrt the floor.

# Bending of light in a gravitational field



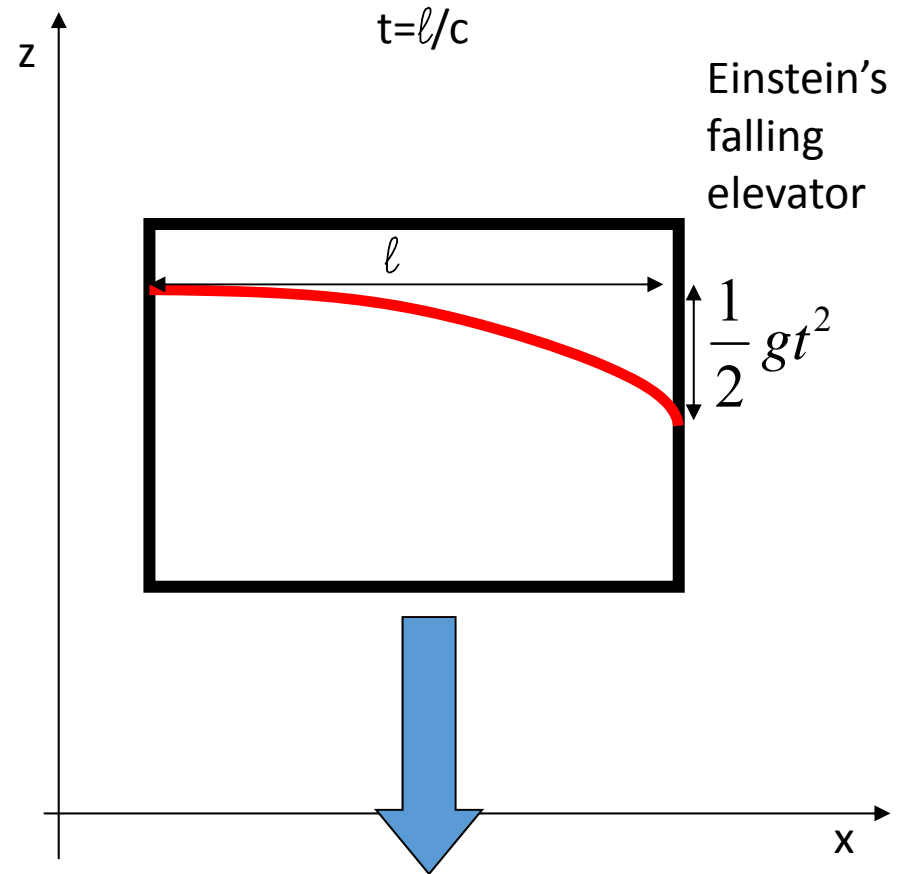
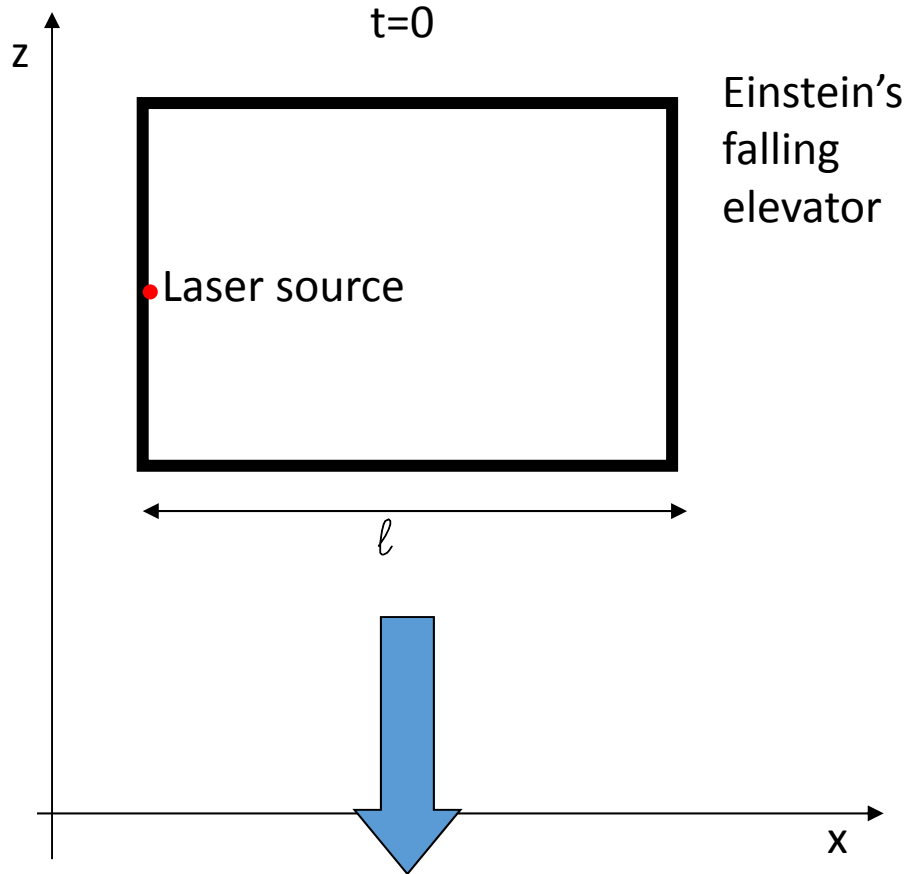
- The observer in the free-falling elevator does not feel gravity, and will see the laser beam propagating at constant height wrt the floor while falling.

# Bending of light in a gravitational field



- The observer in the free-falling elevator does not feel gravity, and will see the laser beam propagating at constant height wrt the floor.
- The observer at rest outside (feeling gravity) will see the same outcome of the experiment, which means that the laser beam is deflected down in his reference frame.

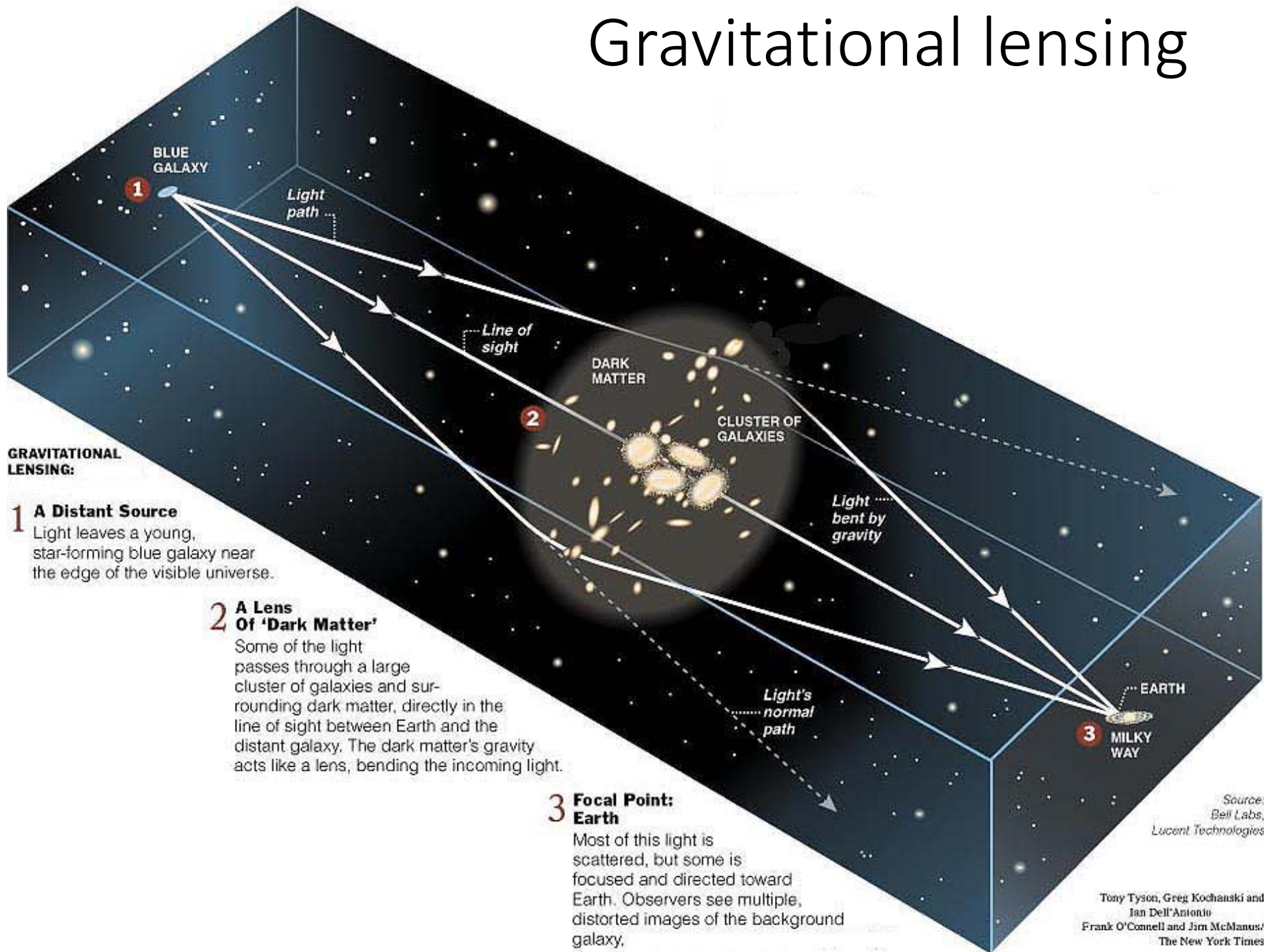
# Bending of light in a gravitational field



- The observer in the free-falling elevator does not feel gravity, and will see the laser beam propagating at constant height wrt the floor.
- The observer at rest outside (feeling gravity) will see the same outcome of the experiment, which means that the laser beam is deflected down in his reference frame: **the presence of gravity deflects light rays.**

$$\varphi \approx \frac{\frac{1}{2}gt^2}{l} \approx \frac{gl}{c^2} \approx \frac{10m/s^2 \times 10m}{(3 \times 10^8 m/s)^2} \approx 10^{-15} \text{ rad} \approx 2 \times 10^{-10} \text{ arcsec}$$

# Gravitational lensing



## GRAVITATIONAL LENSING:

### 1 A Distant Source

Light leaves a young, star-forming blue galaxy near the edge of the visible universe.

### 2 A Lens Of 'Dark Matter'

Some of the light passes through a large cluster of galaxies and surrounding dark matter, directly in the line of sight between Earth and the distant galaxy. The dark matter's gravity acts like a lens, bending the incoming light.

### 3 Focal Point: Earth

Most of this light is scattered, but some is focused and directed toward Earth. Observers see multiple, distorted images of the background galaxy.

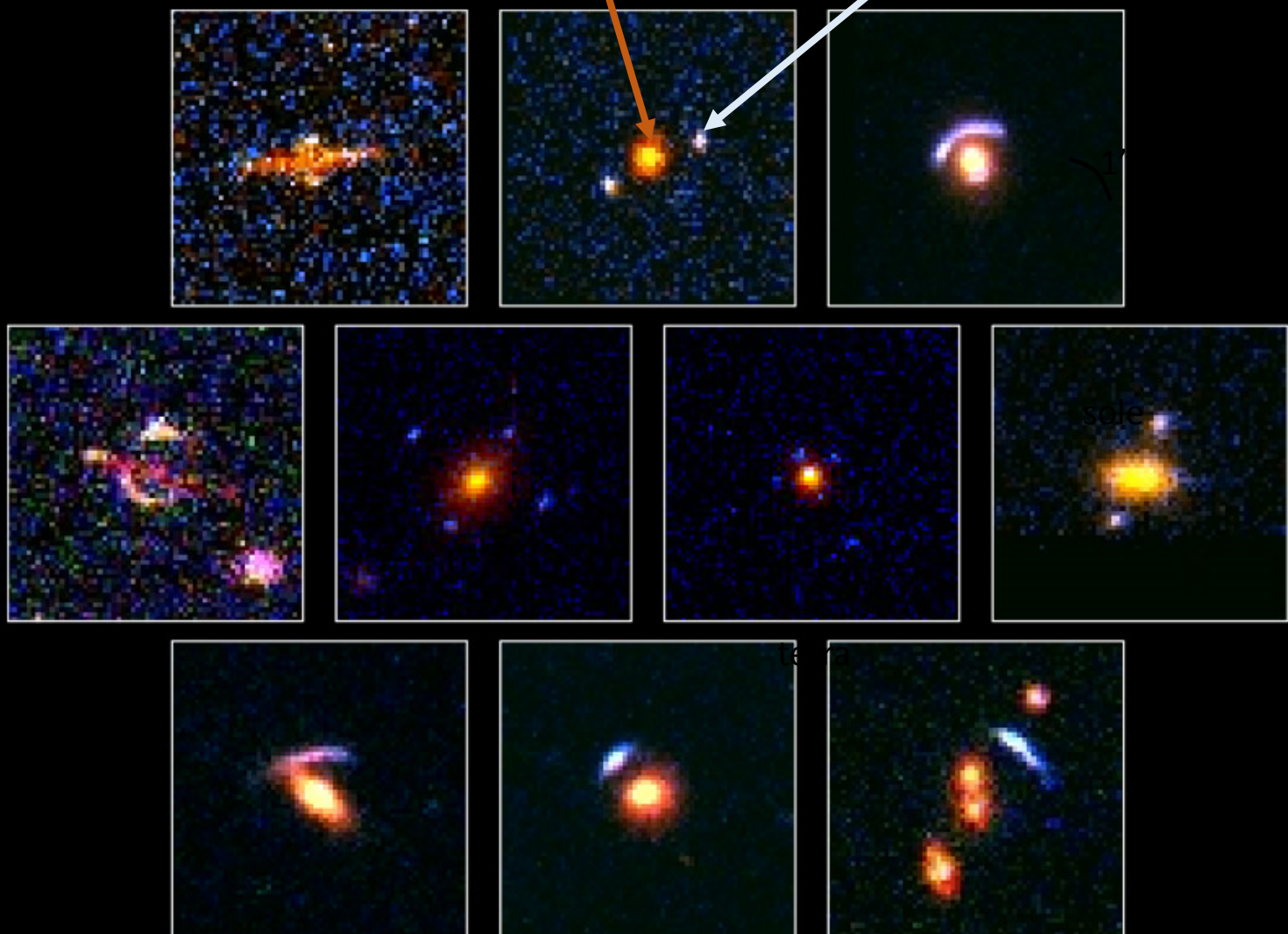
Source:  
Bell Labs,  
Lucent Technologies

Tony Tyson, Greg Kochanski and  
Ian Dell'Antonio  
Frank O'Connell and Jim McManus/  
The New York Times

Here

galaxy

QSO



### Gallery of Gravitational Lenses

HST • WFPC2

PRC99-18 • STScI OPO • K. Ratnatunga (Carnegie Mellon University) and NASA

# Gravitational deflection of light

LRG 3-757

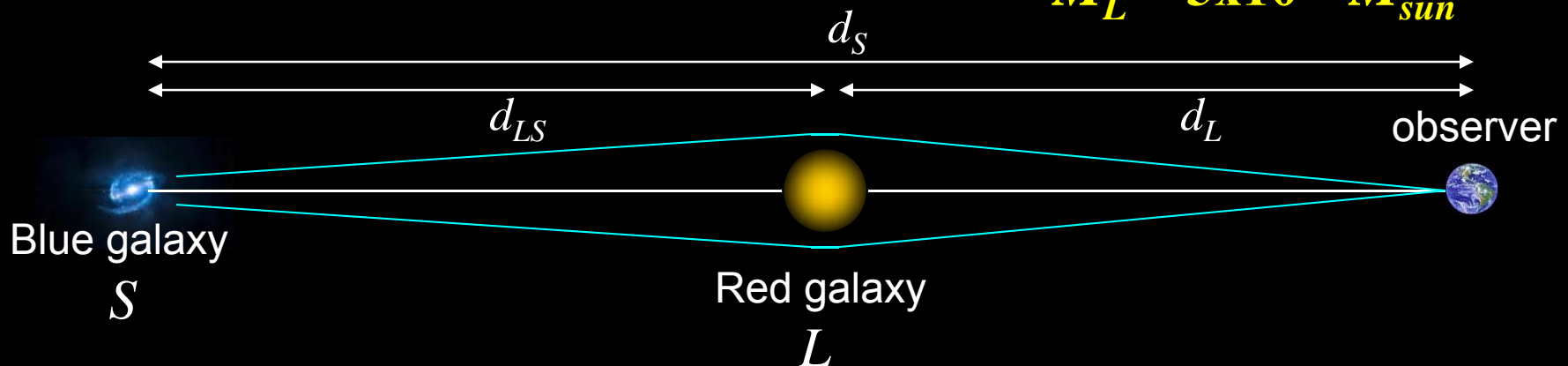
$$\theta_E = \sqrt{\frac{4GM_L}{c^2} \frac{d_{LS}}{d_L d_S}}$$

$M_L$  can be computed from measured data :

Angular diameter distances are estimated from the measured redshifts of  $L$  and  $S$

$\theta_E$  is estimated from the image

$$M_L = 5 \times 10^{12} M_{sun}$$



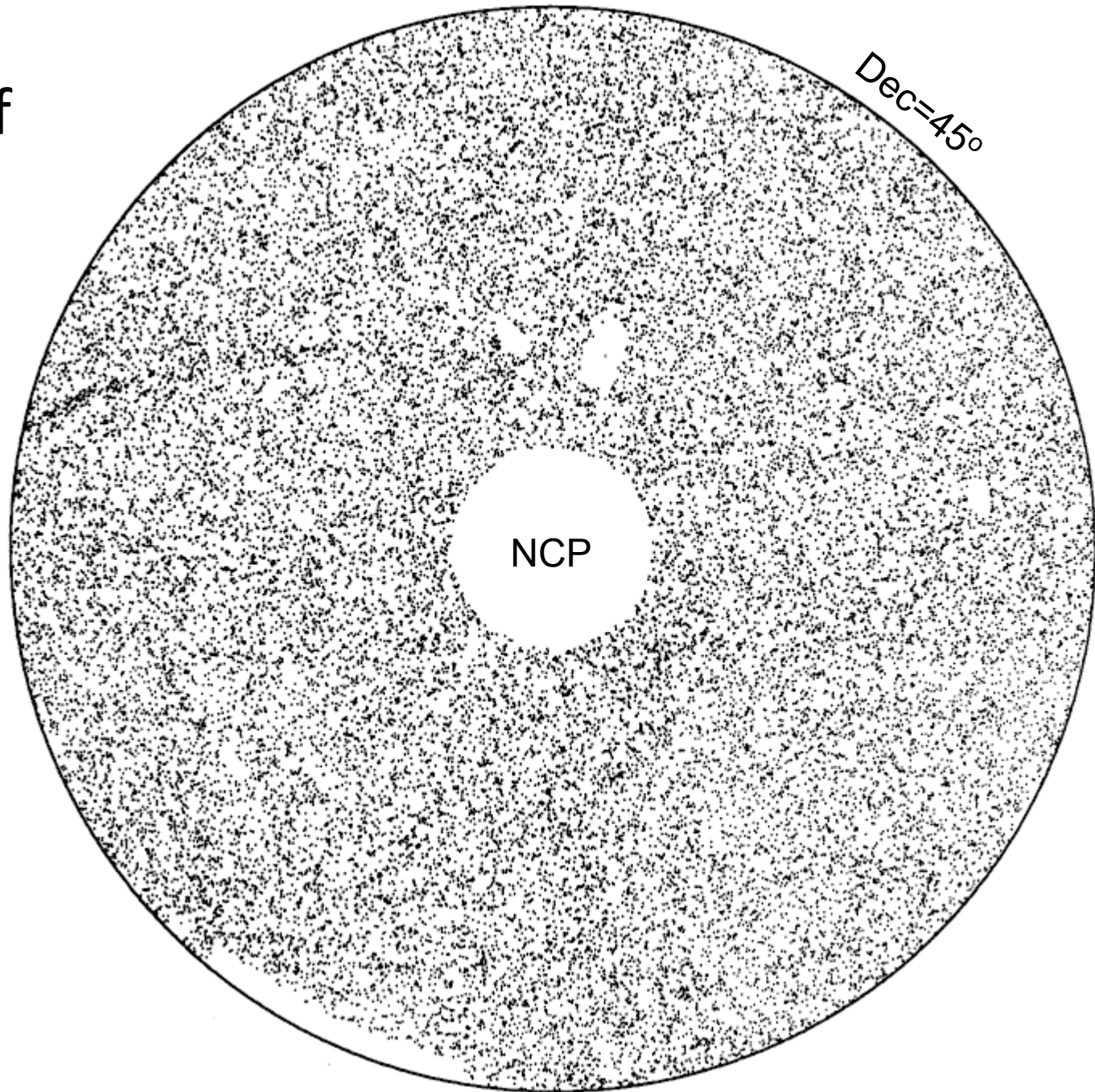
# Cosmology: the universe at the largest scales

- If we believe that
  - we do not occupy a special position in the Universe, i.e. the Universe at large scales is the same everywhere
  - The universe at large scales is isotropic around us
  - The correct description of gravity (the only force active at cosmological scales) is general relativity,
- then we get the FRW metric, describing the geometry of space in an isotropic homogenous medium, and allowing for expansion or contraction, described by a scalar scale factor, and we can study the evolution of the medium
- Empirical evidence for the above:

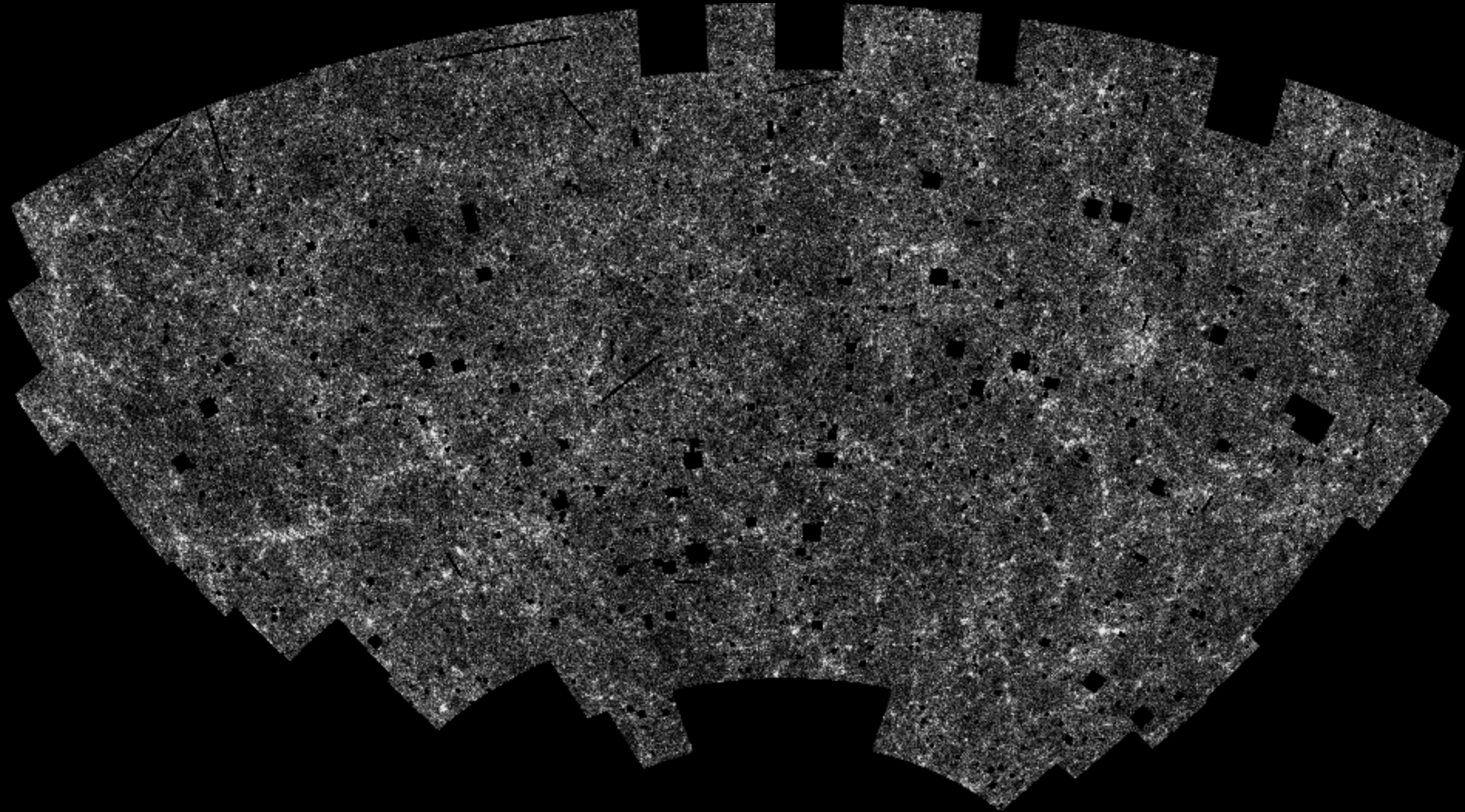


# Projected distribution of distant radiogalaxies

- From Gregory & Condon 1991
- 31000 strong radiosources at  $\lambda=6\text{cm}$
- Sampling rare objects one gets good Poissonian **isotropy**

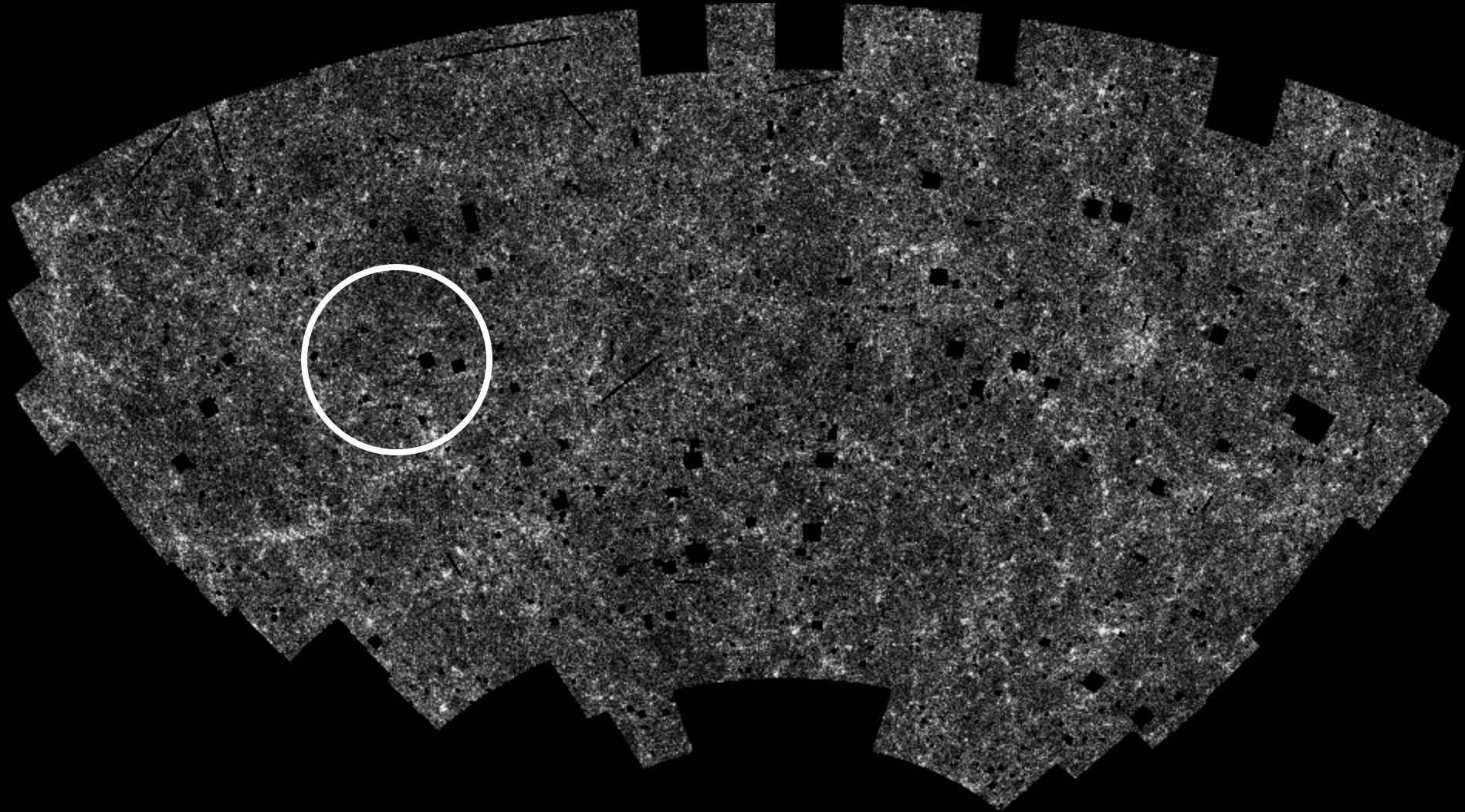


# Projected Distribution of Galaxies



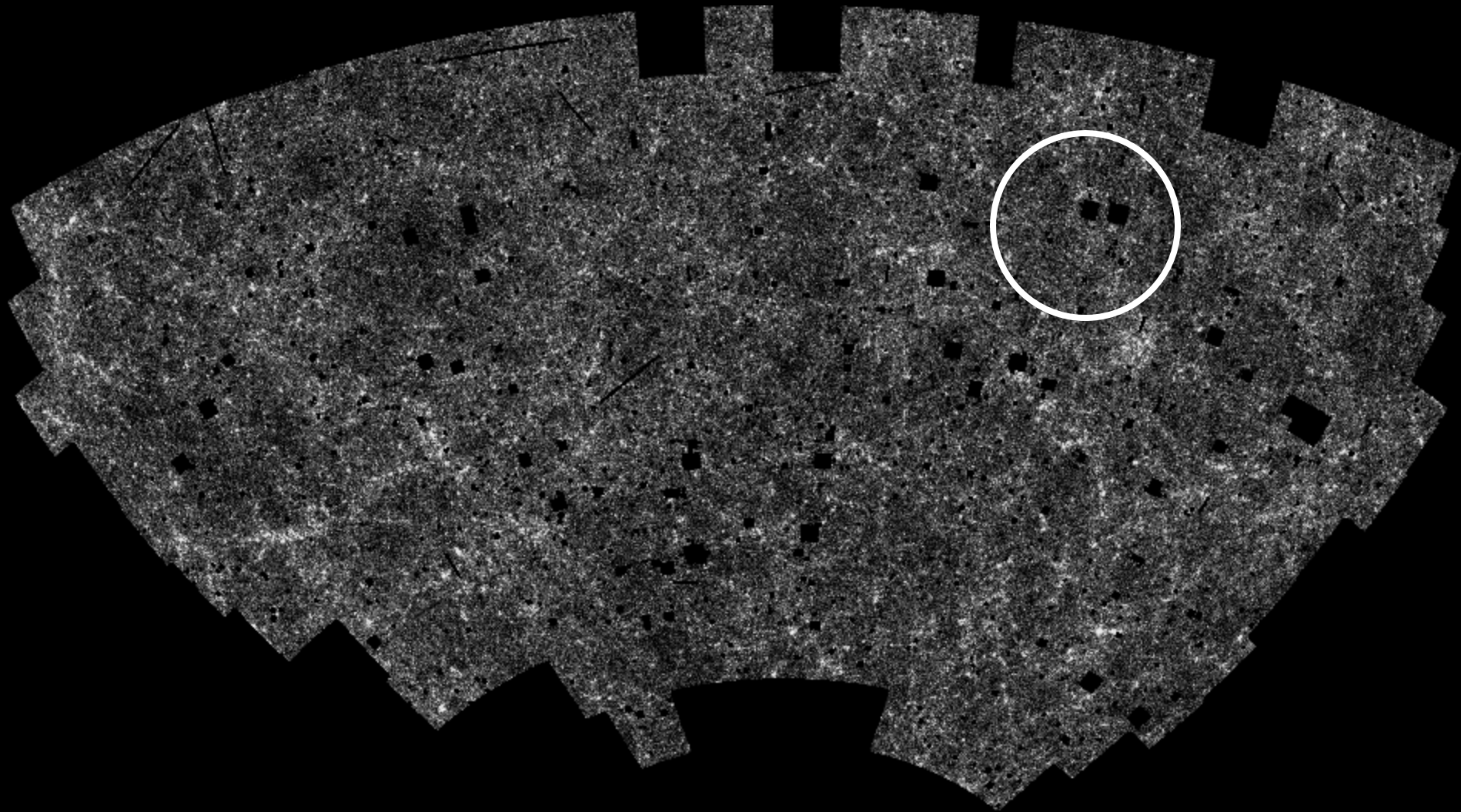
APM (Automatic Plate Machine) survey,  $10^6$  galaxies  
About 1/10 of the sky (circa 1985)

# Projected Distribution of Galaxies



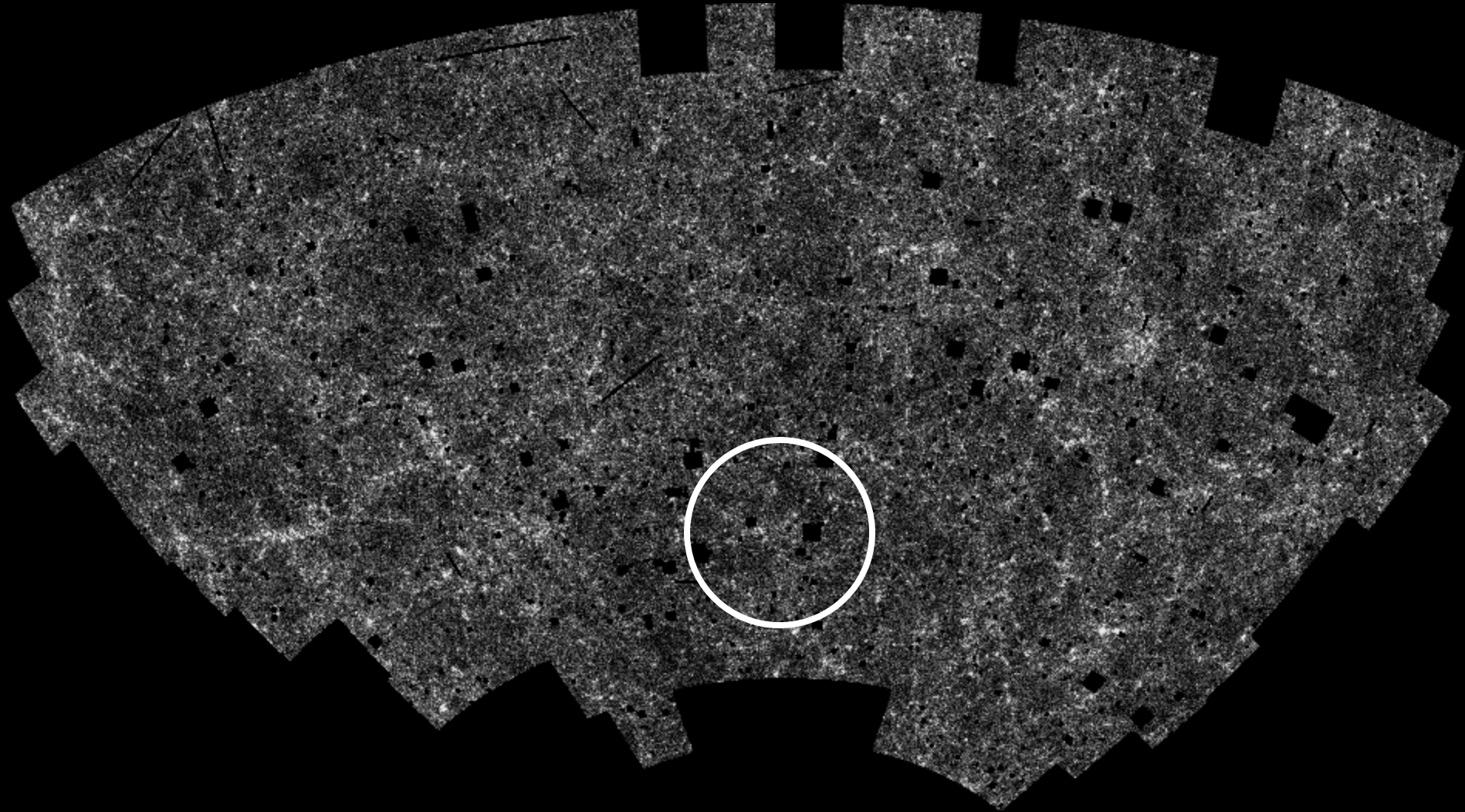
APM (Automatic Plate Machine) survey,  $10^6$  galaxies  
About 1/10 of the sky (circa 1985)

# Projected Distribution of Galaxies




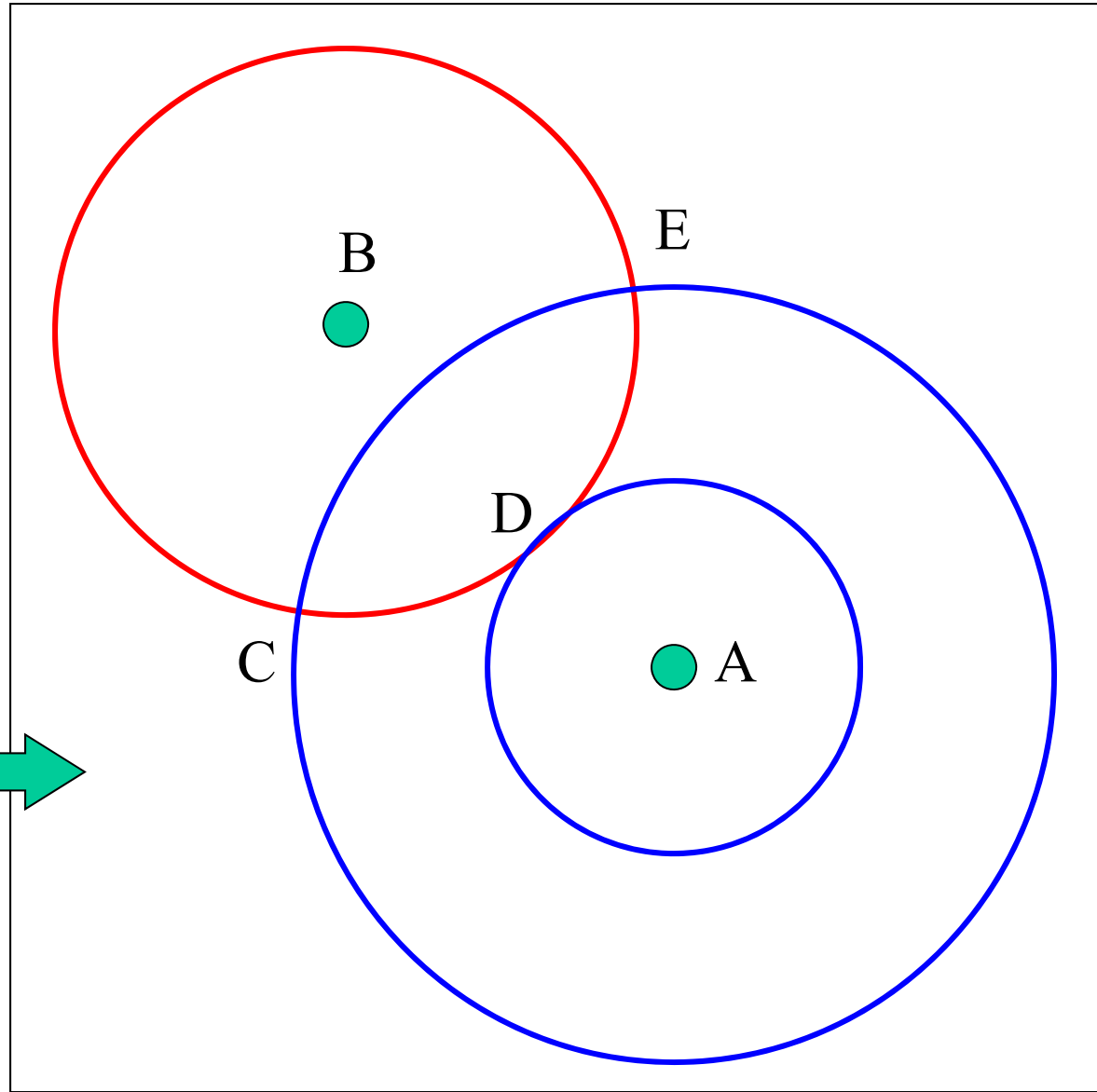
APM (Automatic Plate Machine) survey,  $10^6$  galaxies  
About 1/10 of the sky (circa 1985)

# Projected Distribution of Galaxies



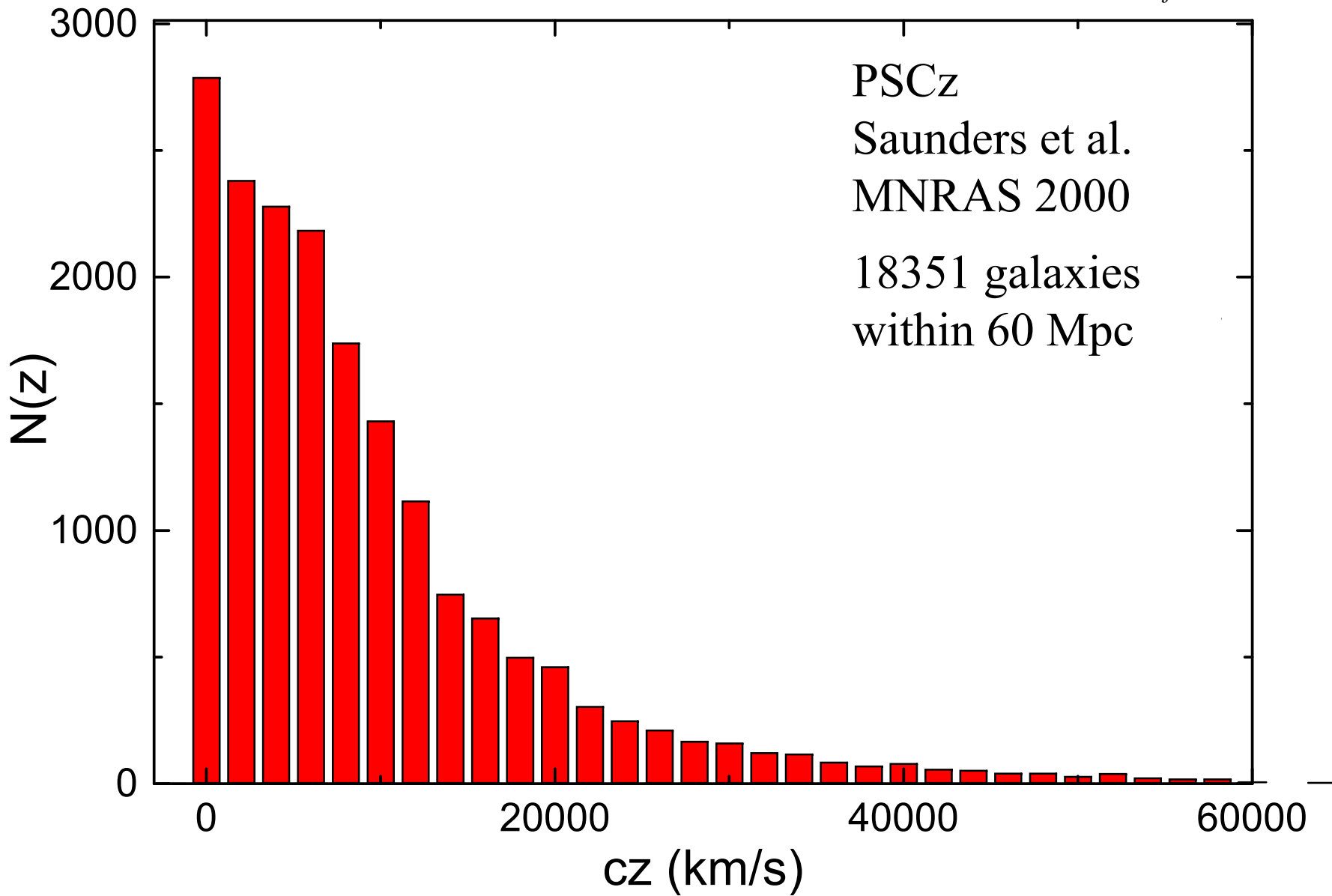
APM (Automatic Plate Machine) survey,  $10^6$  galaxies  
About 1/10 of the sky (circa 1985)

- The distribution of galaxies is isotropic
- We (A) cannot be at the center of the Universe (Copernican principle), so any other observer in the universe (B) should experience the same isotropy we see.
- This implies homogeneity : 



# Cosmological Redshift of Galaxies

$$z = \frac{\lambda_{\text{observed}} - \lambda_{\text{restframe}}}{\lambda_{\text{restframe}}}$$

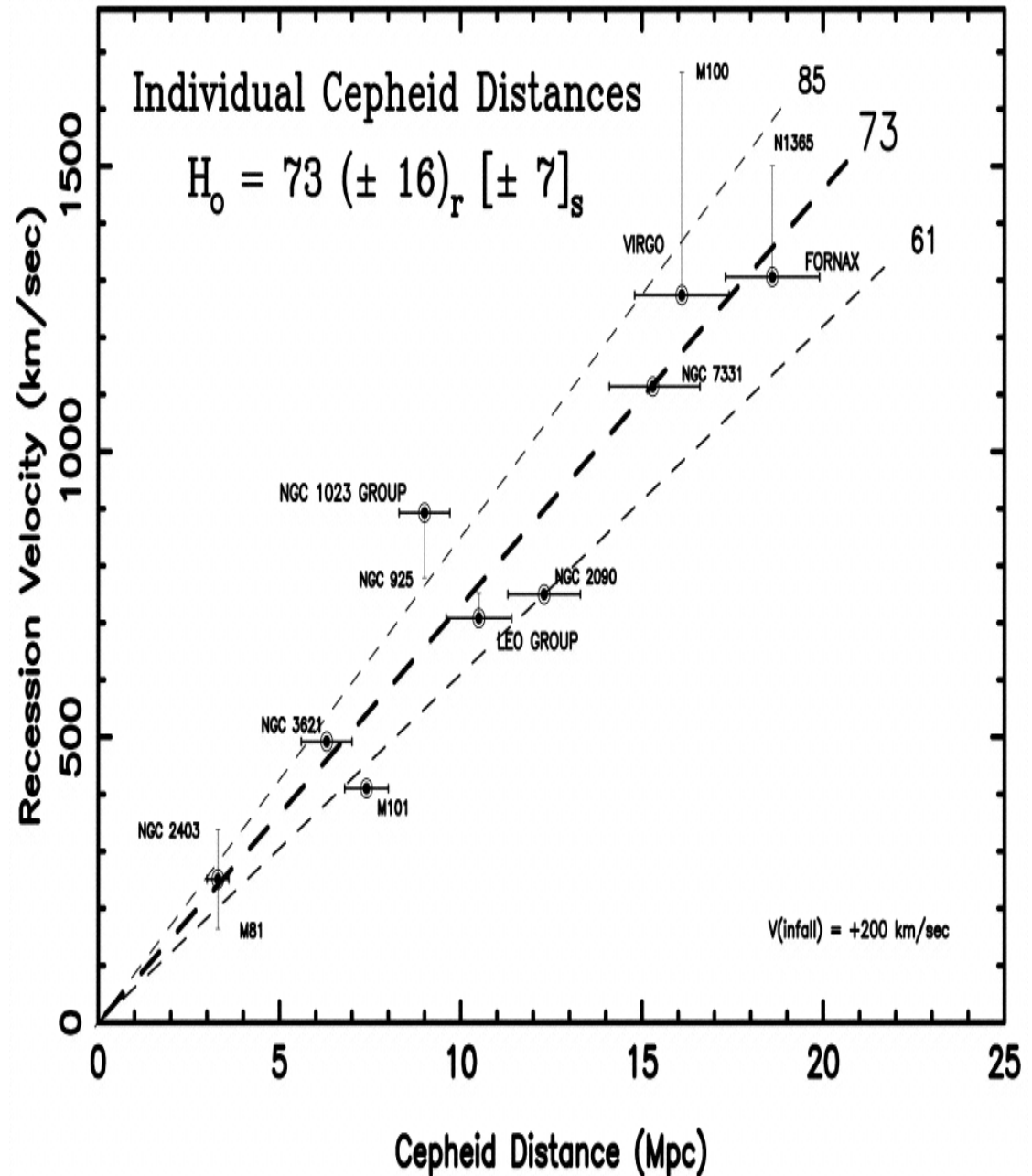


$$z = \frac{\lambda_{observed} - \lambda_{restframe}}{\lambda_{restframe}}$$

- Hubble's law from distant cepheid stars

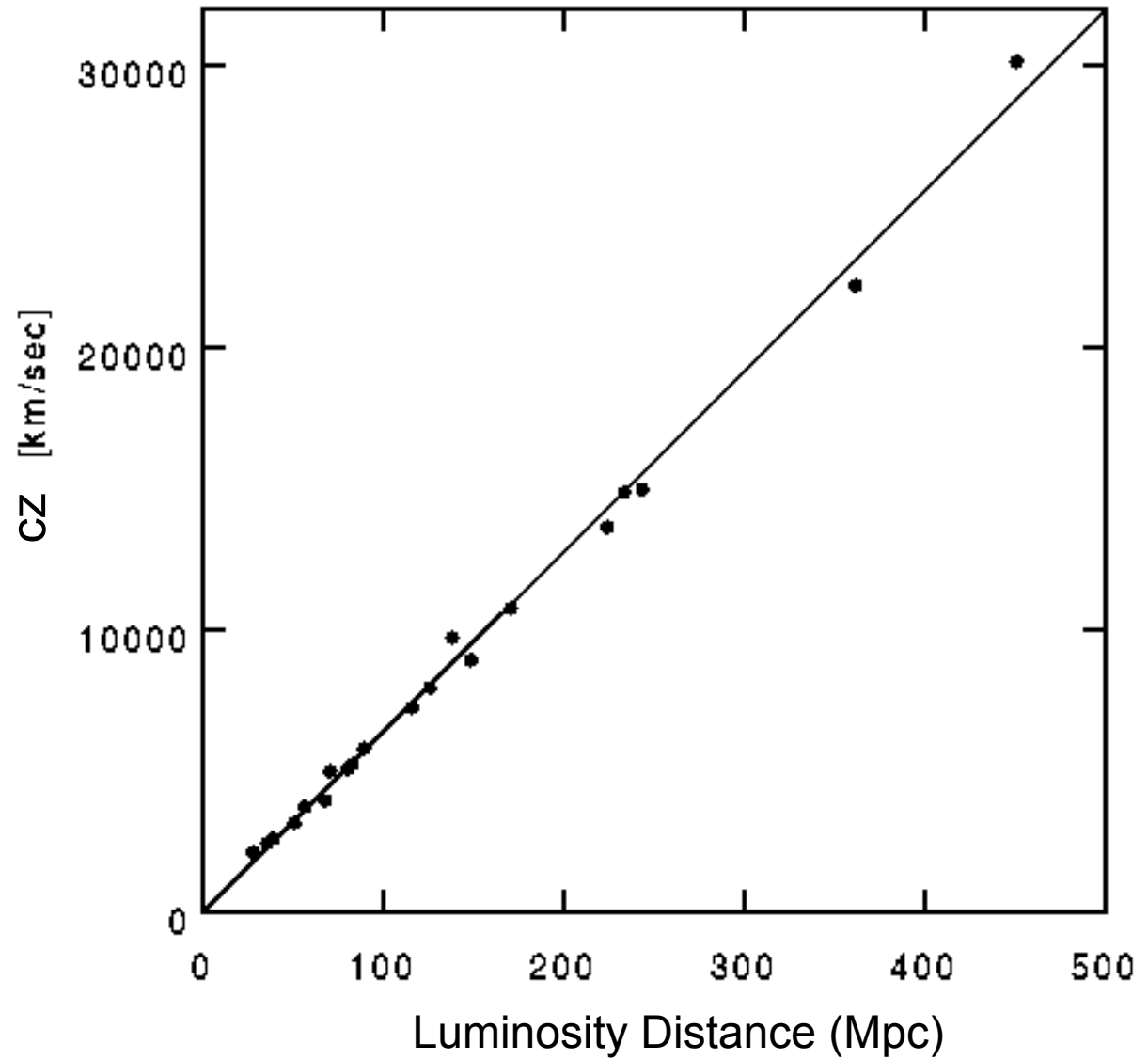
$$cz = H_0 D$$

- The redshift  $z$  is an empirical measure of the distance  $D$  at cosmological scales



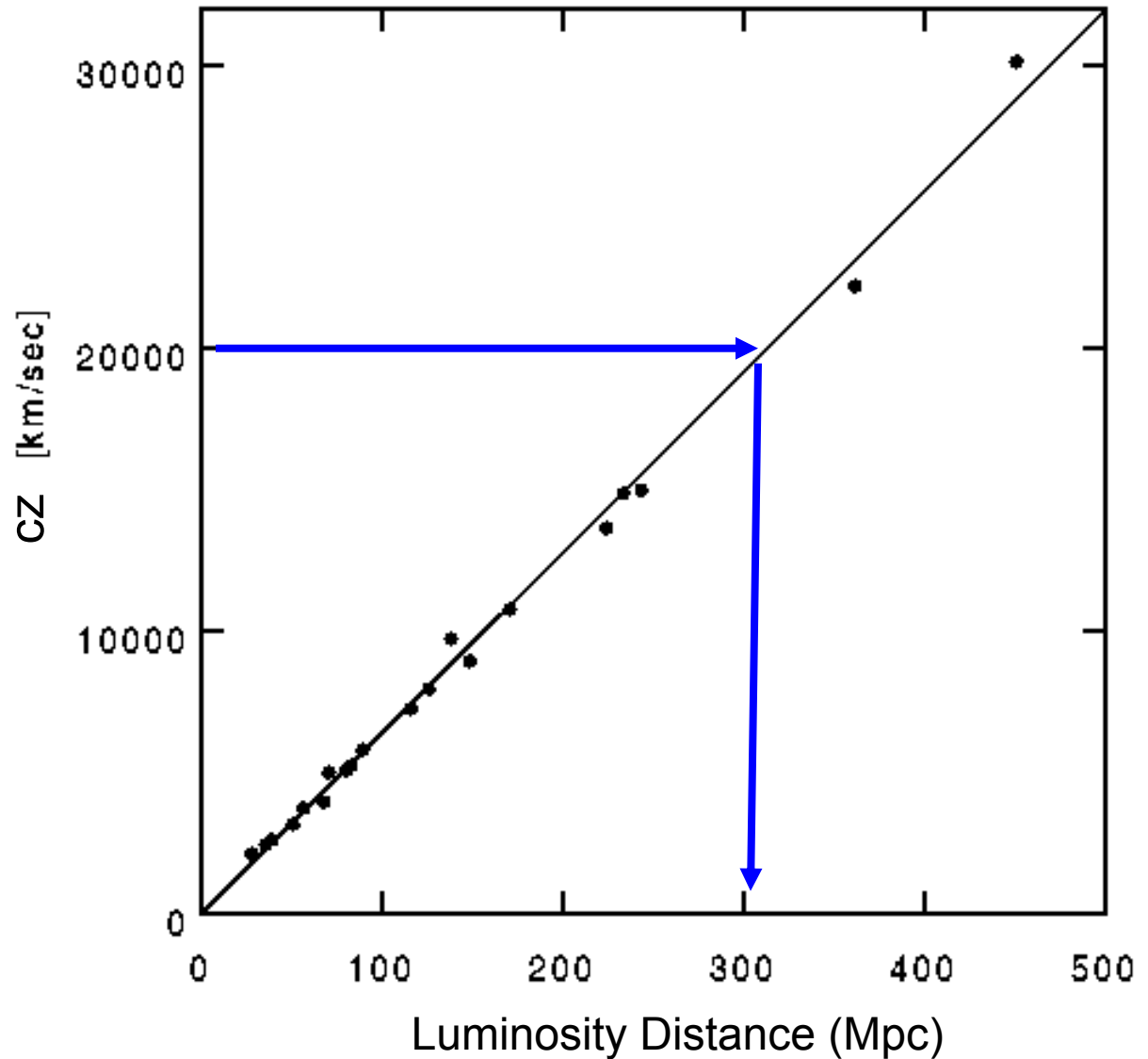


- Hubble's law with Supernovae 1a

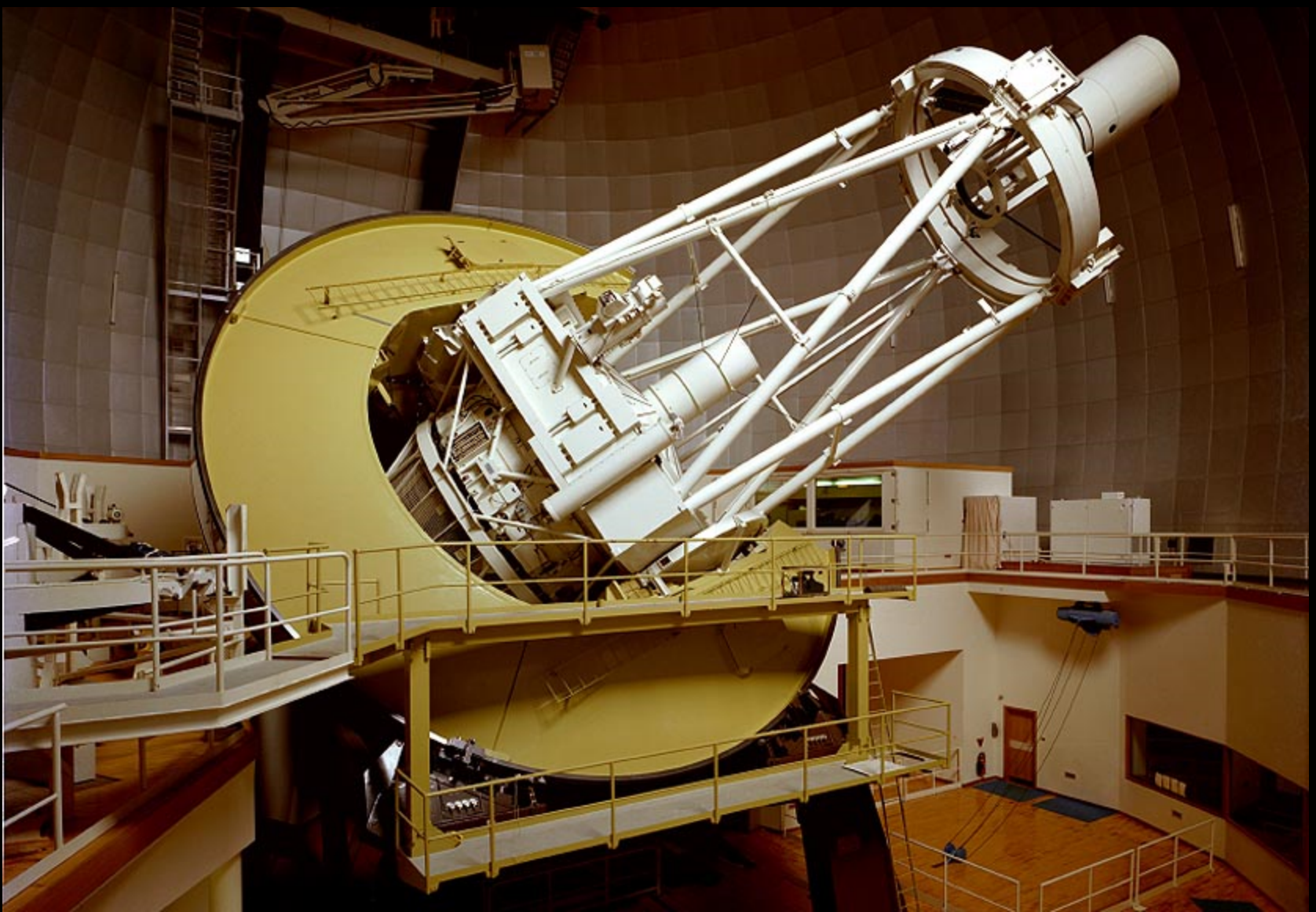


Riess et al. 1998

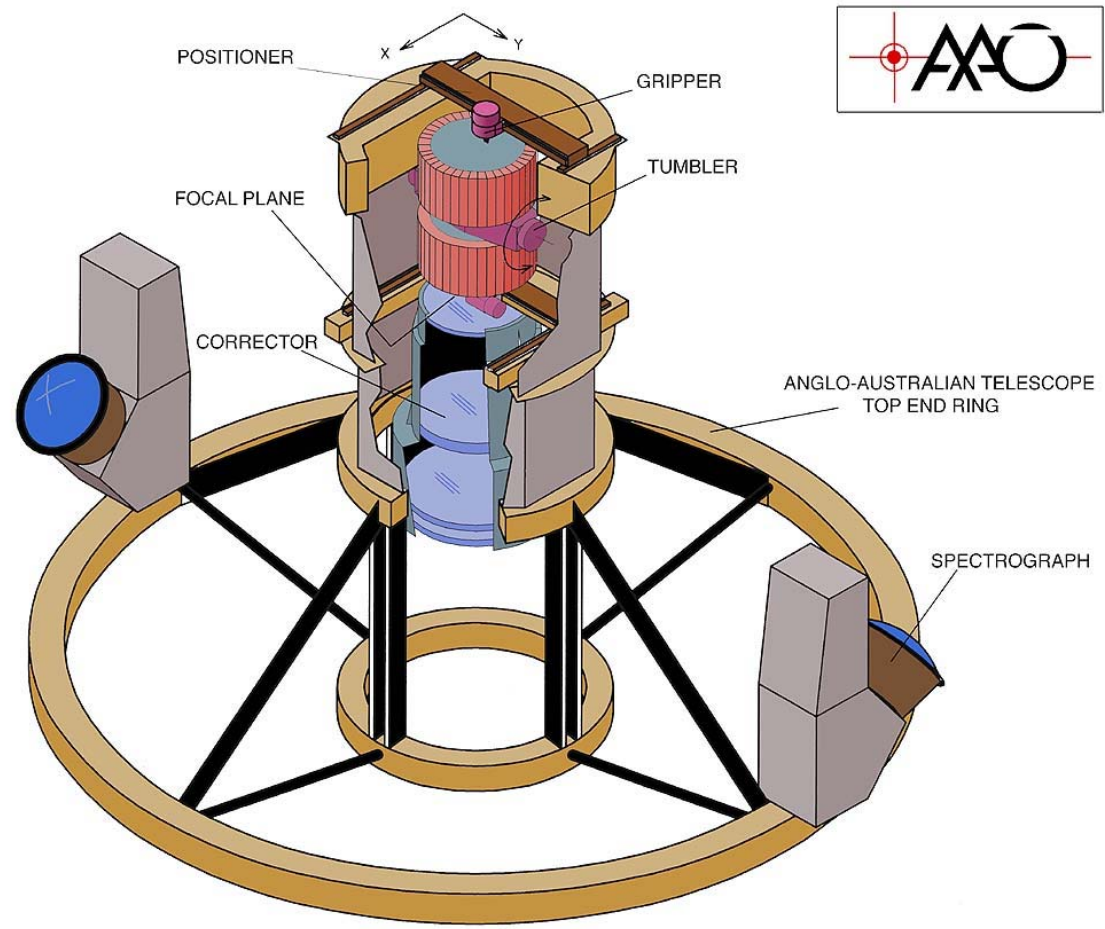
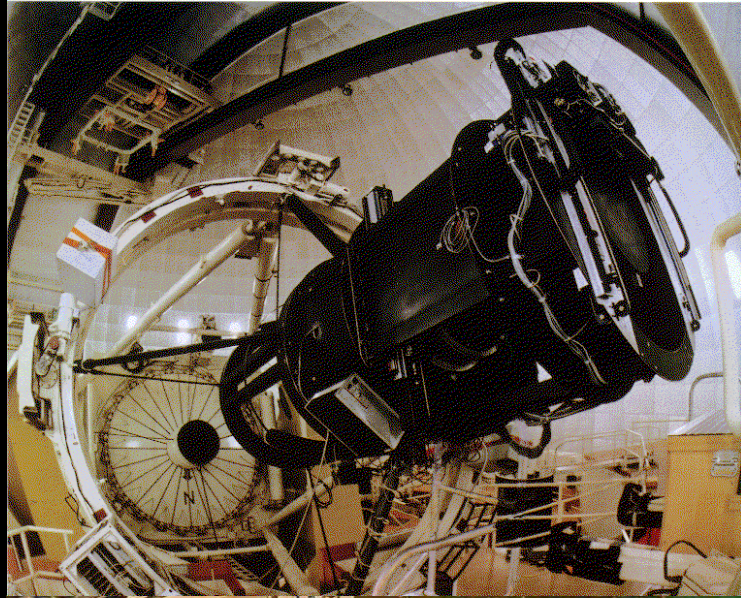
- Redshift as a distance indicator
- This can be done on very large samples of galaxies using redshift survey machines:



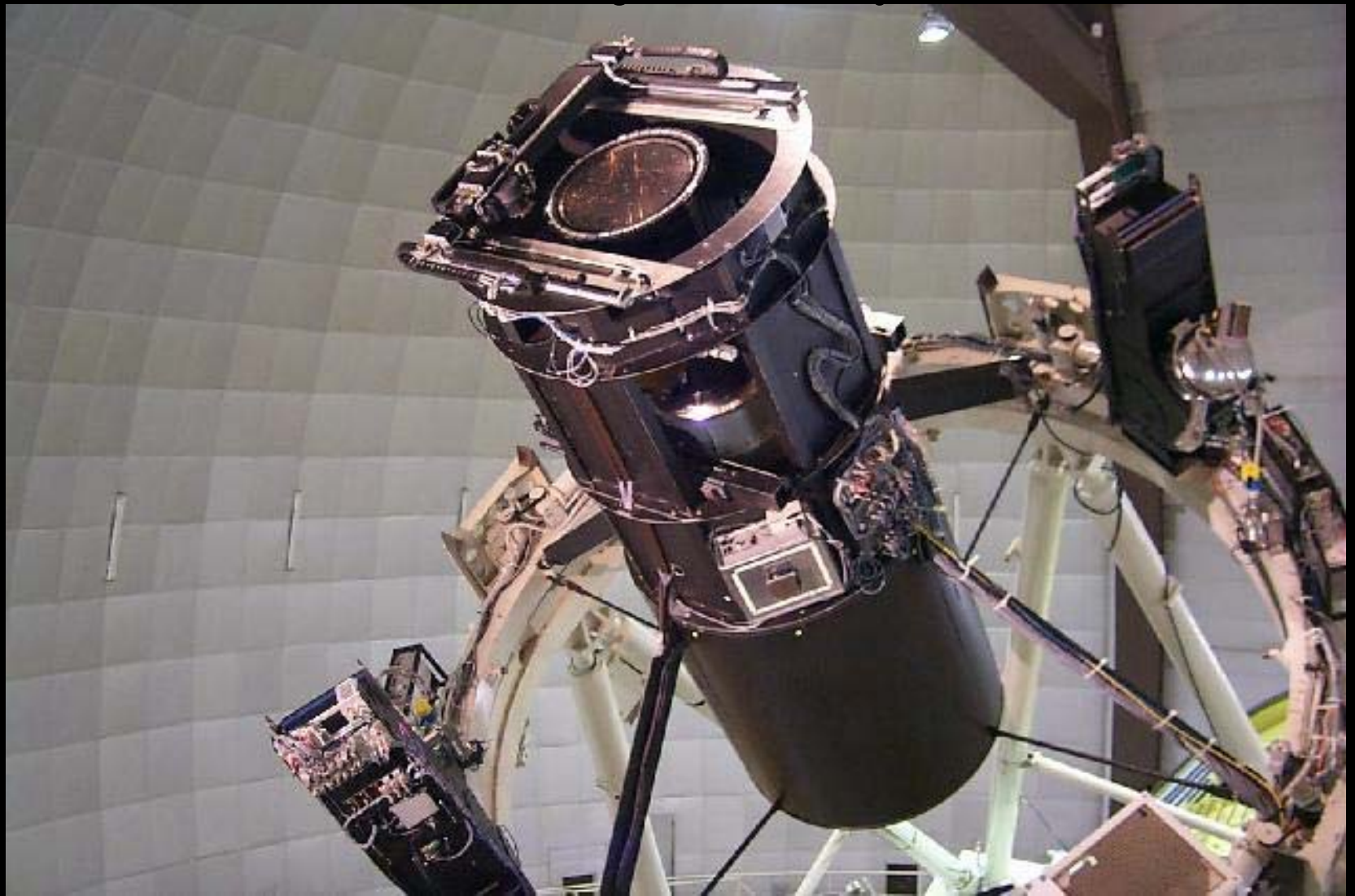
# Anglo-Australian Telescope (4 m)



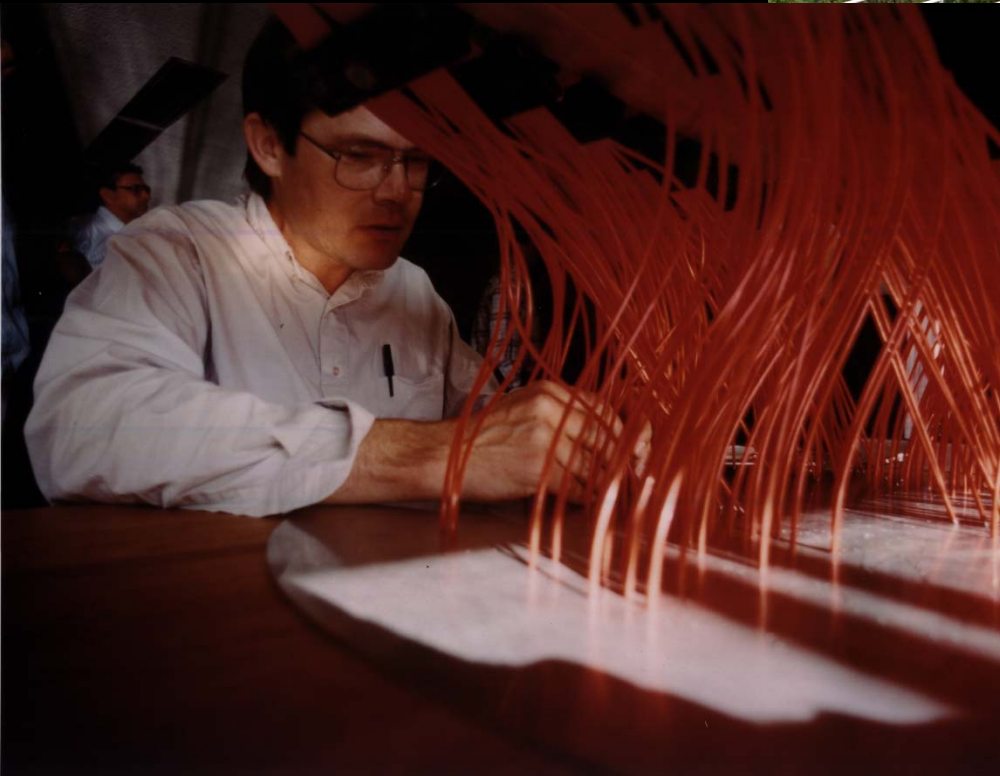
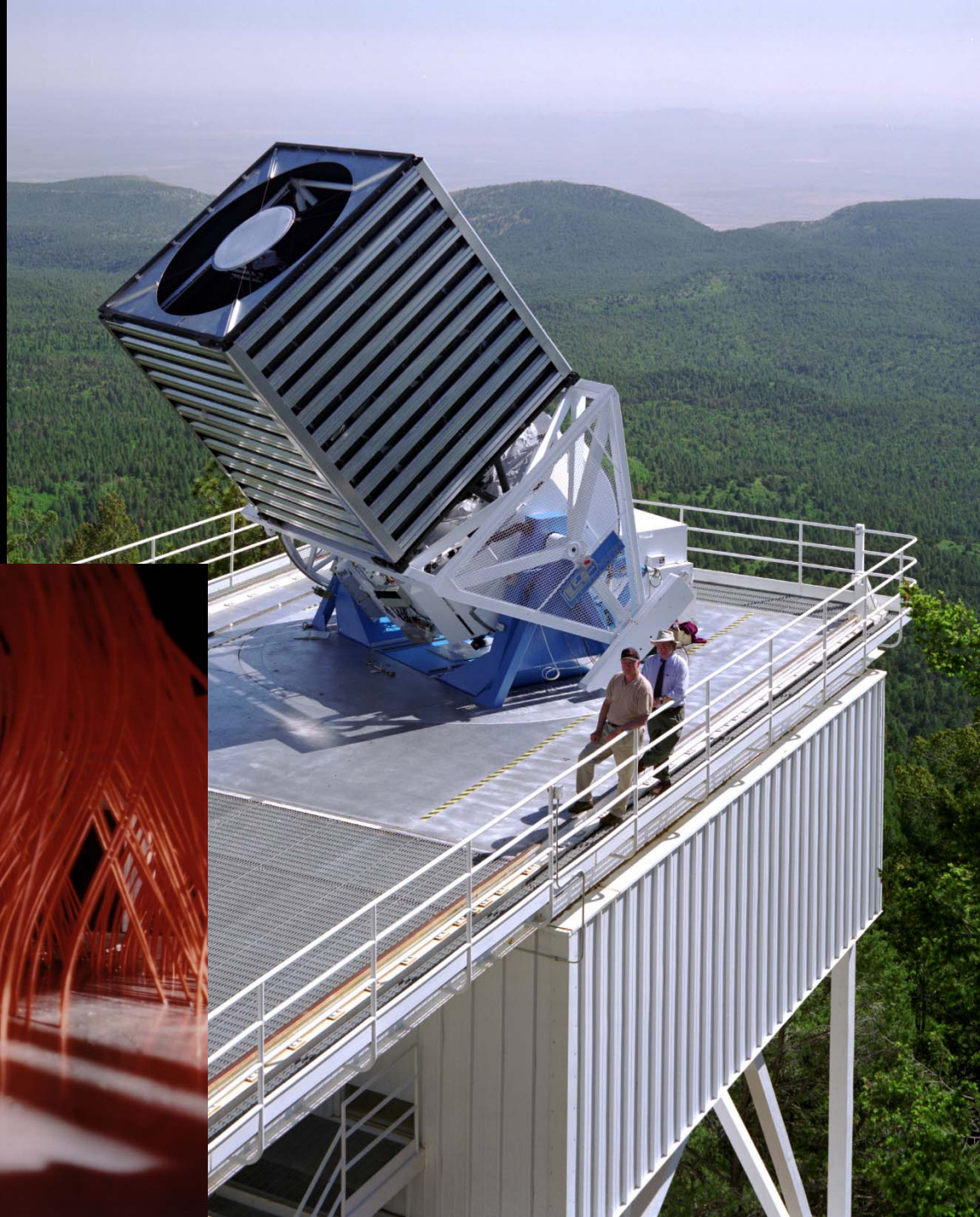
# TWO-DEGREE FIELD FACILITY

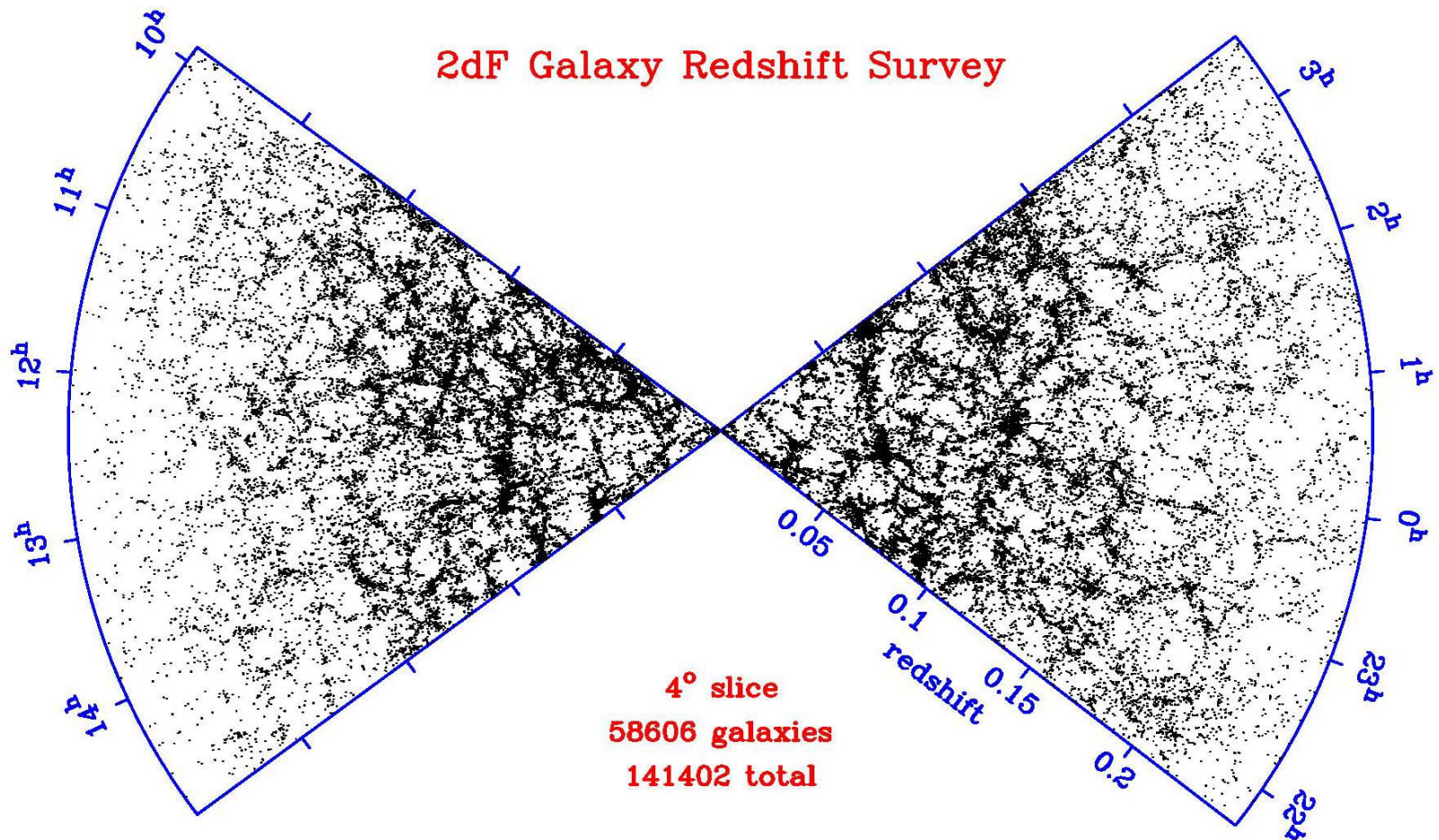


Robotized optical fiber spectrometer

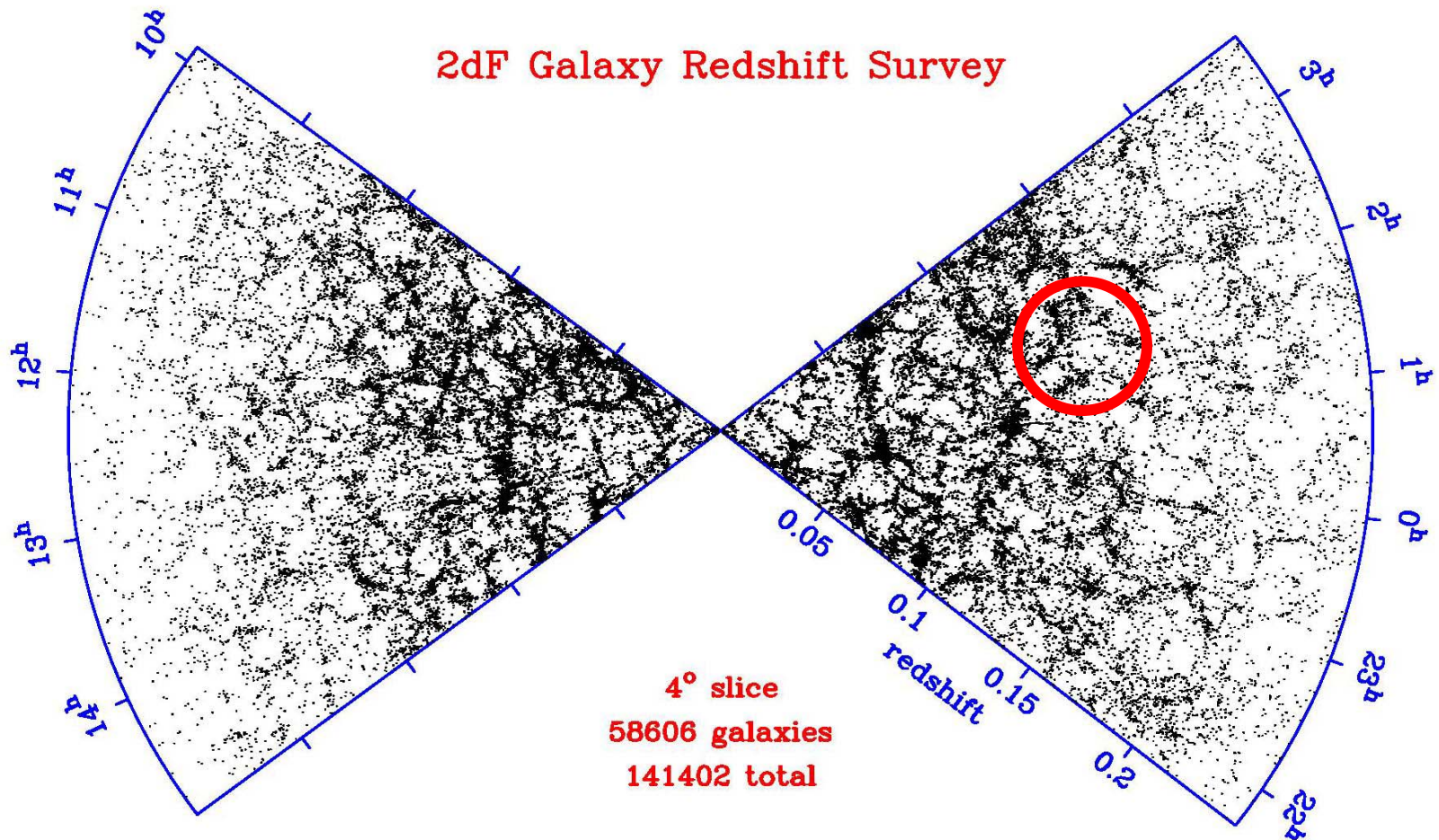


- Sloan Digital Sky Survey
- Dedicated telescope with optical fibers spectrometer
- Measured redshifts for more than  $10^6$  galaxies !



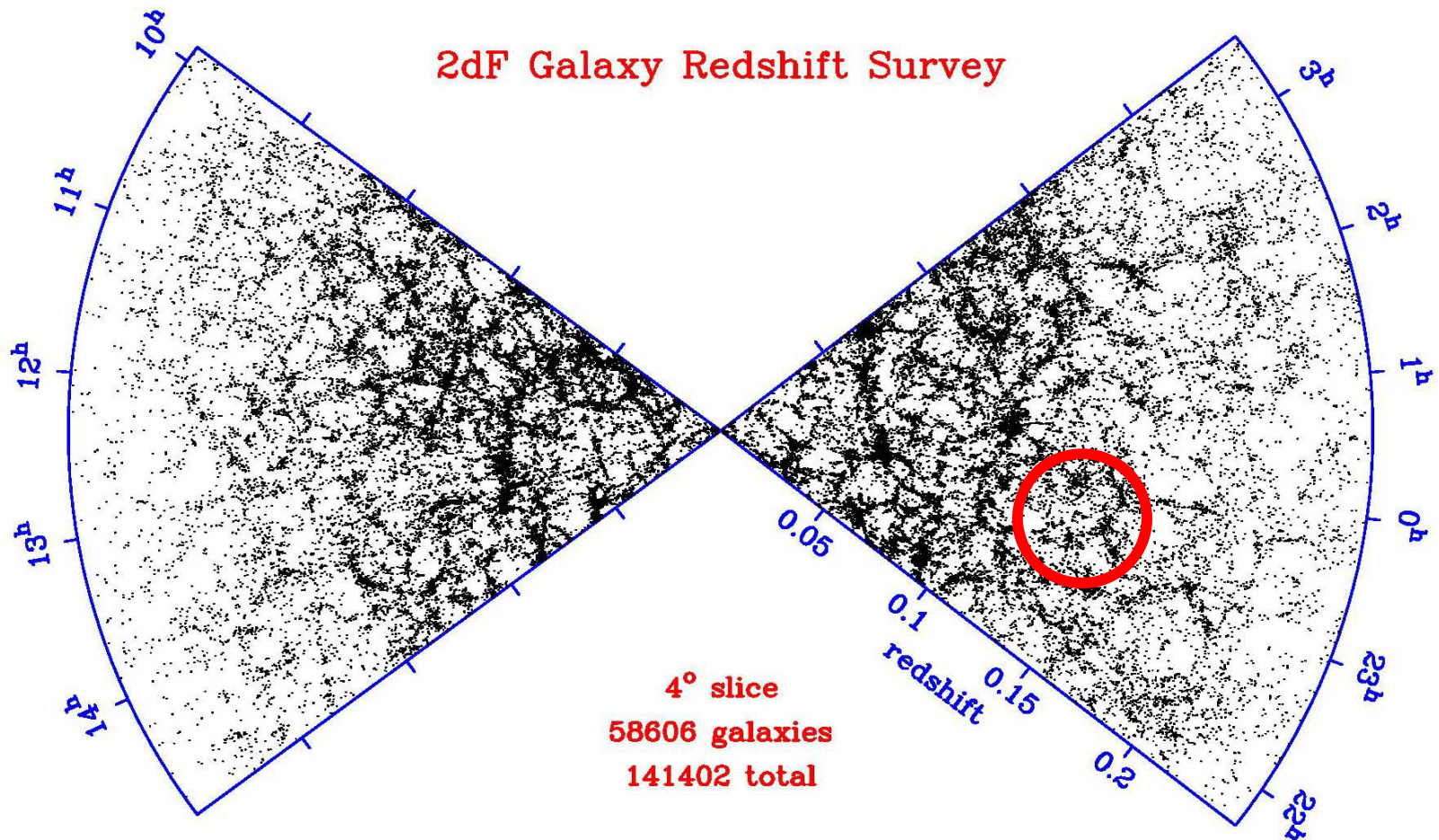


**FIGURE 2.** The distribution of galaxies in part of the 2dFGRS, drawn from a total of 141,402 galaxies: slices  $4^\circ$  thick, centred at declination  $-2.5^\circ$  in the NGP and  $-27.5^\circ$  in the SGP. Not all 2dF fields within the slice have been observed at this stage, hence there are weak variations of the density of sampling as a function of right ascension. To minimise such features, the slice thickness increases to  $7.5^\circ$  between right ascension  $13.1^h$  and  $13.4^h$ . This image reveals a wealth of detail, including linear supercluster features, often nearly perpendicular to the line of sight. The interesting question to settle statistically is whether such transverse features have been enhanced by infall velocities.

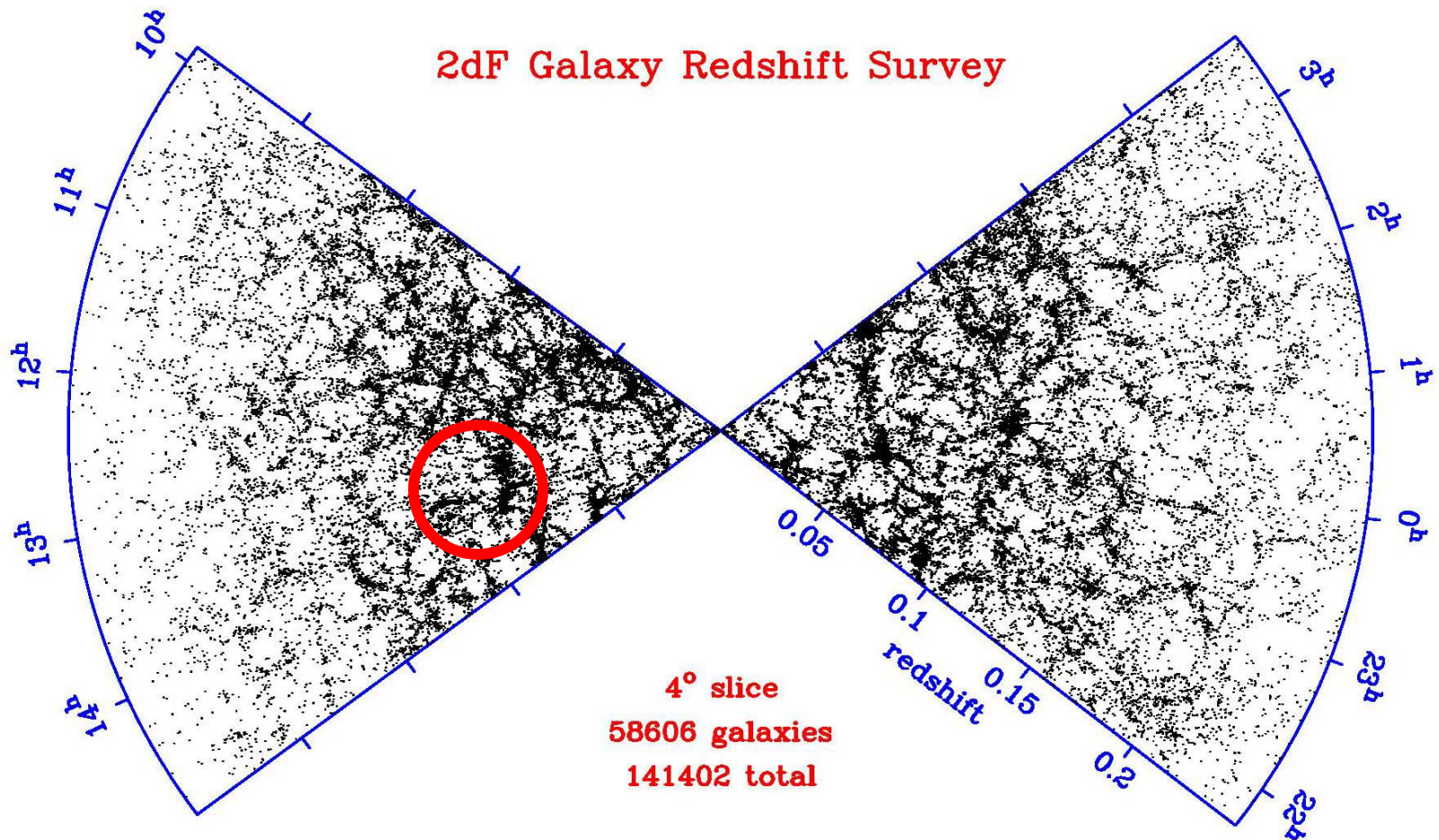


**FIGURE 2.** The distribution of galaxies in part of the 2dFGRS, drawn from a total of 141,402 galaxies: slices  $4^\circ$  thick, centred at declination  $-2.5^\circ$  in the NGP and  $-27.5^\circ$  in the SGP. Not all 2dF fields within the slice have been observed at this stage, hence there are weak variations of the density of sampling as a function of right ascension. To minimise such features, the slice thickness increases to  $7.5^\circ$  between right ascension  $13.1^h$  and  $13.4^h$ . This image reveals a wealth of detail, including linear supercluster features, often nearly perpendicular to the line of sight. The interesting question to settle statistically is whether such transverse features have been enhanced by infall velocities.



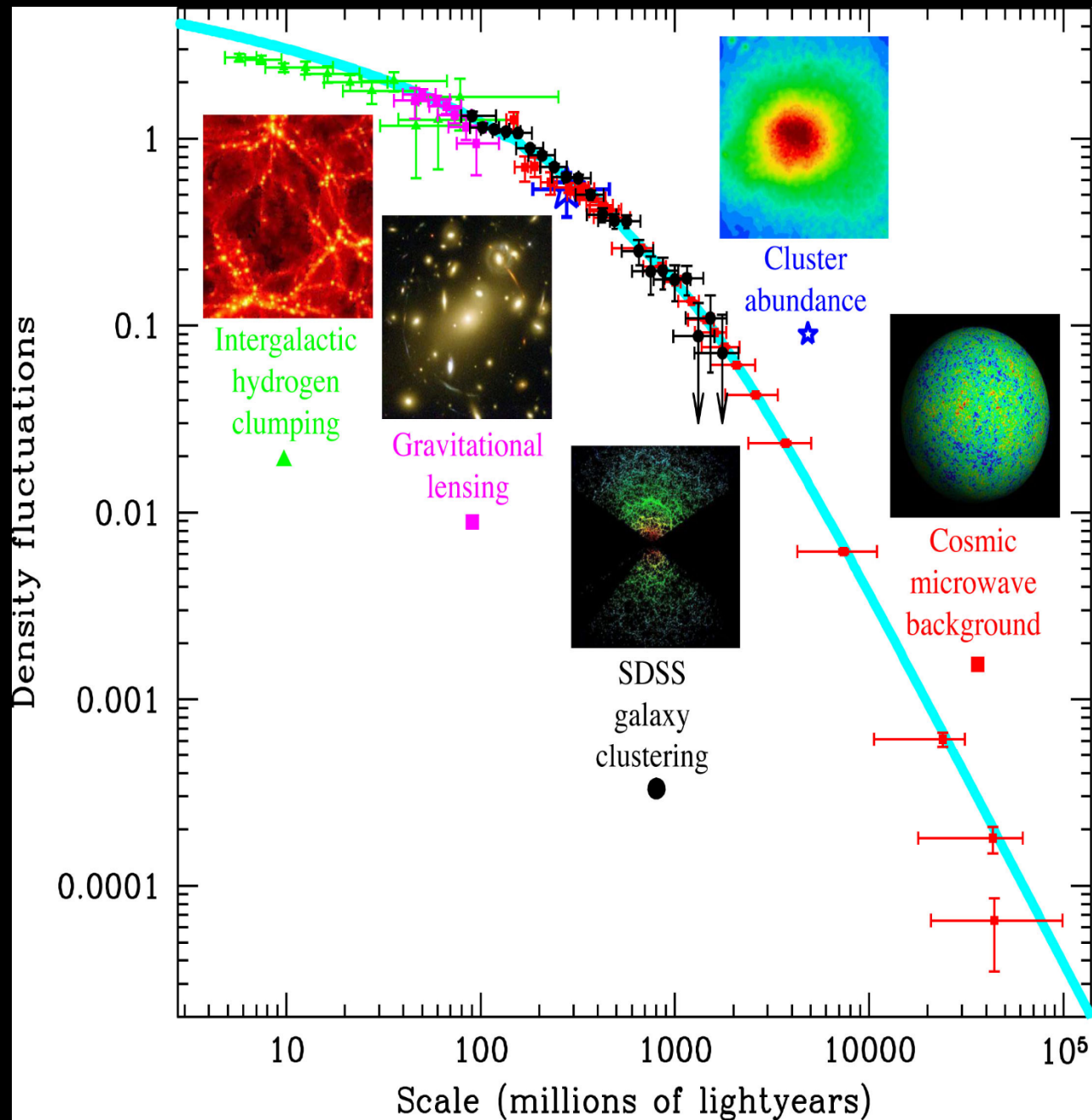


**FIGURE 2.** The distribution of galaxies in part of the 2dFGRS, drawn from a total of 141,402 galaxies: slices  $4^\circ$  thick, centred at declination  $-2.5^\circ$  in the NGP and  $-27.5^\circ$  in the SGP. Not all 2dF fields within the slice have been observed at this stage, hence there are weak variations of the density of sampling as a function of right ascension. To minimise such features, the slice thickness increases to  $7.5^\circ$  between right ascension  $13.1^h$  and  $13.4^h$ . This image reveals a wealth of detail, including linear supercluster features, often nearly perpendicular to the line of sight. The interesting question to settle statistically is whether such transverse features have been enhanced by infall velocities.



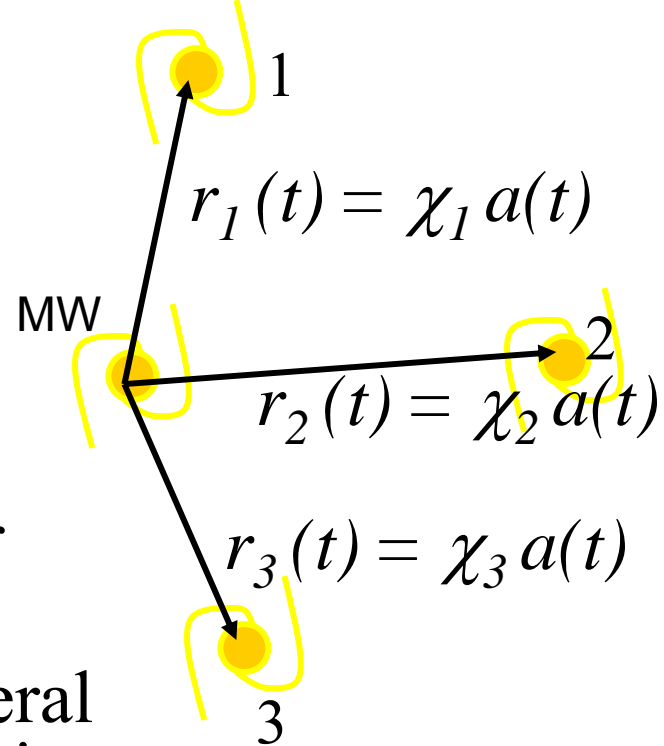
**FIGURE 2.** The distribution of galaxies in part of the 2dFGRS, drawn from a total of 141,402 galaxies: slices  $4^\circ$  thick, centred at declination  $-2.5^\circ$  in the NGP and  $-27.5^\circ$  in the SGP. Not all 2dF fields within the slice have been observed at this stage, hence there are weak variations of the density of sampling as a function of right ascension. To minimise such features, the slice thickness increases to  $7.5^\circ$  between right ascension  $13.1^h$  and  $13.4^h$ . This image reveals a wealth of detail, including linear supercluster features, often nearly perpendicular to the line of sight. The interesting question to settle statistically is whether such transverse features have been enhanced by infall velocities.

- The universe at large scales is statistically isotropic and homogeneous.
- Confirmed by the fact that if we increase the sampling volume, the rms galaxy density fluctuation decreases.



- The universe at large scales is statistically isotropic and homogeneous.
- However, the universe does not have to be static (and cannot, according to general relativity)
- An isotropic expansion or contraction preserves isotropy and homogeneity.

- Isotropic expansion or contraction :  
for every galaxy  $i$  :  
 $r_i$  = physical distance  
 $\chi_i$  = comoving distance  
 $a(t)$  = common scale factor  
conventionally  $a=1$  today
- FRW metric: the most general homogenous isotropic metric



$$(ds)^2 = c^2 dt^2 - a^2(t) \left[ \left( \frac{d\chi}{\sqrt{1 - k\chi^2}} \right)^2 - (\chi d\theta)^2 - (\chi \sin \theta d\varphi)^2 \right]$$

- $1/k$  = curvature of space
- In this metric, the redshift of distant sources is naturally predicted, if the universe is expanding.

# Cosmological Redshift

- In an expanding universe the wavelengths of photons expand in the same way as all other lengths ( $a(t)$ ).
- Consider a source at distance  $R(t)=a(t) \chi_1$  (comoving coordinate  $\chi_1$ )
- Photons emitted from the source propagate radially towards us along coordinate  $\chi$ , occupying sequentially all coordinates between  $\chi_1$  and 0.
- From the FRW metric, with radial propagation:

$$(ds)^2 = c^2 dt^2 - a(t)^2 \left[ \left( \frac{d\chi}{\sqrt{1-k\chi^2}} \right)^2 + (\cancel{\chi d\theta})^2 + (\cancel{\chi \sin\theta d\phi})^2 \right]$$

- Assuming  $ds=0$  for photons ( $v=c$ ),  $cdt = a(t) \frac{d\chi}{\sqrt{1-k\chi^2}}$



# Cosmological Redshift

- Consider a first crest of the EM wave emitted at time  $t_1$  and received at time  $t_o$ ; the next crest is emitted at  $t_1 + \lambda_1/c$  and received at  $t_o + \lambda_o/c$ . Since  $\chi_1$  is constant, we have that

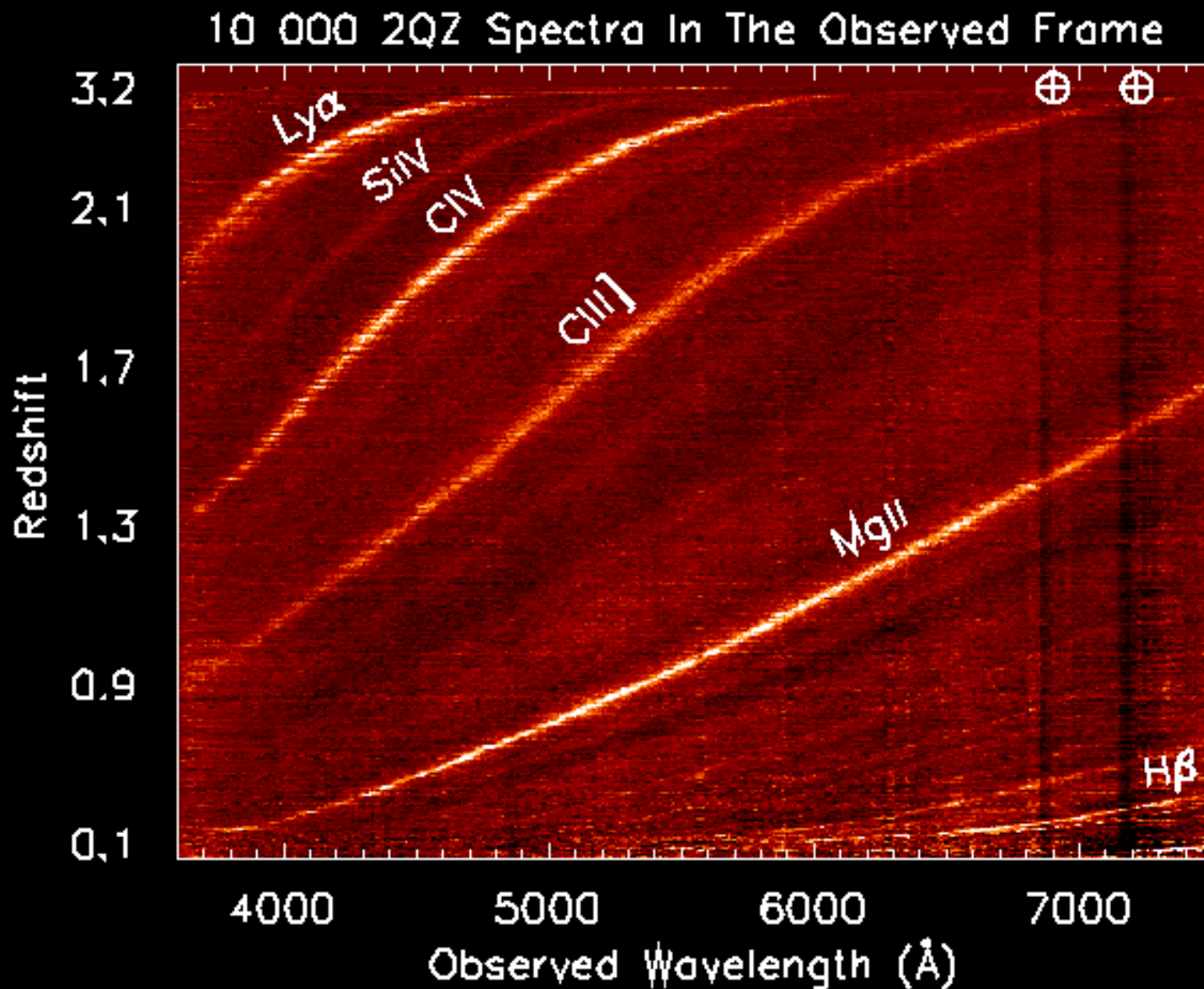
$$\int_0^{\chi_1} \frac{d\chi}{\sqrt{1 - k\chi^2}} = \int_{t_1}^{t_o} \frac{cdt}{a(t)} = \int_{t_1 + \lambda_1/c}^{t_o + \lambda_o/c} \frac{cdt}{a(t)} \Rightarrow \int_{t_1}^{t_1 + \lambda_1/c} \frac{cdt}{a(t)} = \int_{t_o}^{t_o + \lambda_o/c} \frac{cdt}{a(t)}$$

- However, the times  $\lambda_o/c$  e  $\lambda_1/c$  are both  $\ll H_o^{-1}$ , the typical timescale for of  $a(t)$ . So we can safely consider  $a(t)$  as constant in the integrals, and we get

$$\frac{c}{a(t_1)} \frac{\lambda_1}{c} = \frac{c}{a(t_o)} \frac{\lambda_o}{c} \Rightarrow \frac{a(t_o)}{a(t_1)} = \frac{1}{a(t_1)} = \frac{\lambda_o}{\lambda_1} \stackrel{\text{Redshift of the source}}{\downarrow} = (1 + z_1)$$

- The wavelengths of photons elongate in the same way as all other cosmological distances, following the same scale factor  $a(t)$ . In particular, **redshift implies expansion.**

10000 optical spectra of QSOs (with important emission lines) as measured from the 2dF survey, plotted in vs the observed wavelengths. The redshift scale is not linear.





$$v = c$$

Observing far = observing the past

$$\lambda_{obs}/\lambda_{em} = 1 + z = 1/a_{em}$$

In an expanding universe  $a_{em} < 1$

Observing far = measuring long  $\lambda$ s