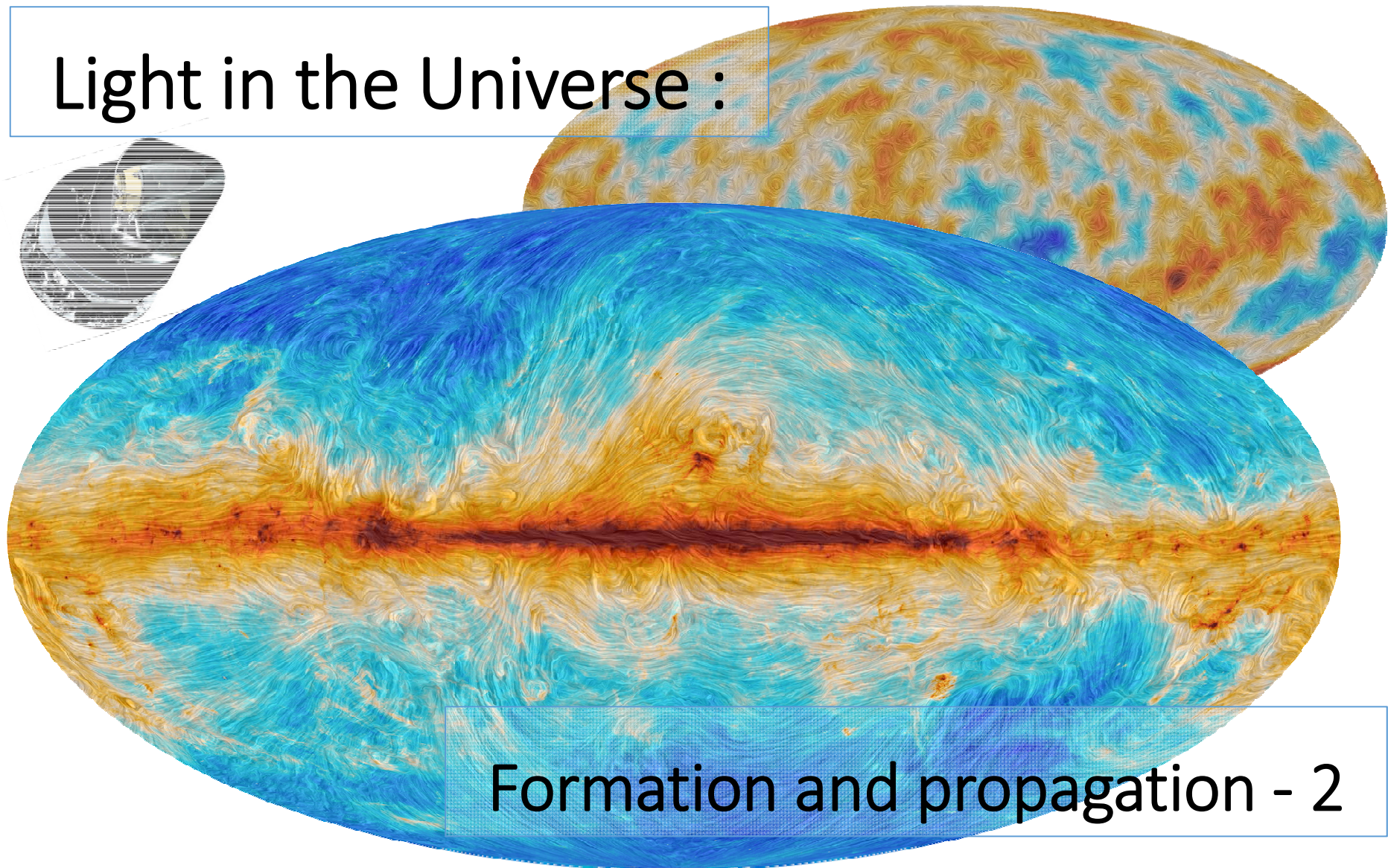
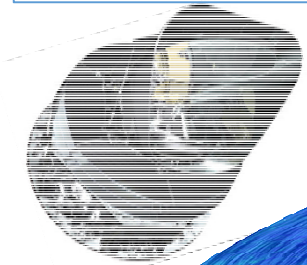


Light in the Universe :



Formation and propagation - 2

Paolo de Bernardis

Dipartimento di Fisica, Sapienza Università di Roma

Winter College on Optics 2015 - Trieste, 9-20 February

$$v = c$$

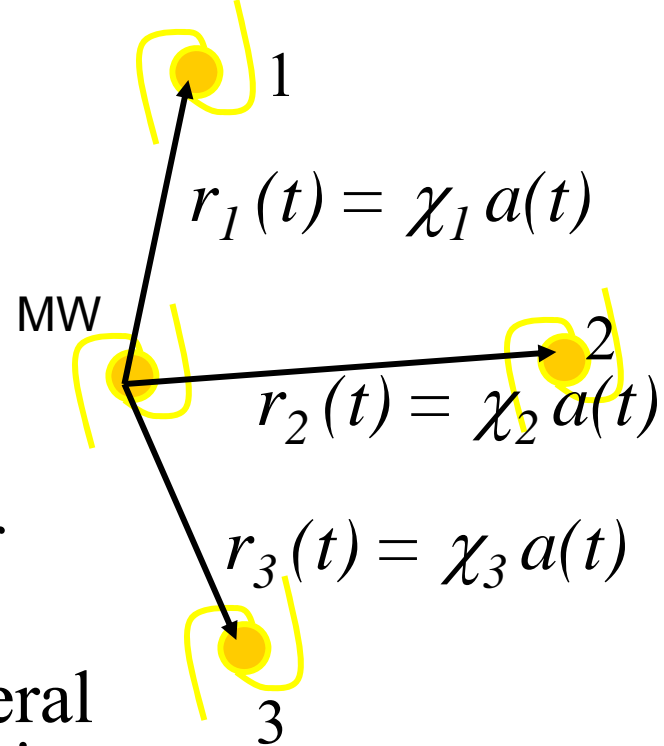
Observing far = observing the past

$$\lambda_{obs}/\lambda_{em} = 1 + z = 1/a_{em}$$

In an expanding universe $a_{em} < 1$

Observing far = measuring long λ s

- Isotropic expansion or contraction :
for every galaxy i :
 r_i = physical distance
 χ_i = comoving distance
 $a(t)$ = common scale factor
conventionally $a=1$ today
- FRW metric: the most general homogenous isotropic metric



$$(ds)^2 = c^2 dt^2 - a^2(t) \left[\left(\frac{d\chi}{\sqrt{1 - k\chi^2}} \right)^2 - (\chi d\theta)^2 - (\chi \sin \theta d\varphi)^2 \right]$$

- $1/k$ = curvature of space
- In this metric, the redshift of distant sources is naturally predicted, if the universe is expanding.

Energy density

- Matter : $\rho_m = \rho_{m_0} a^{-3}$

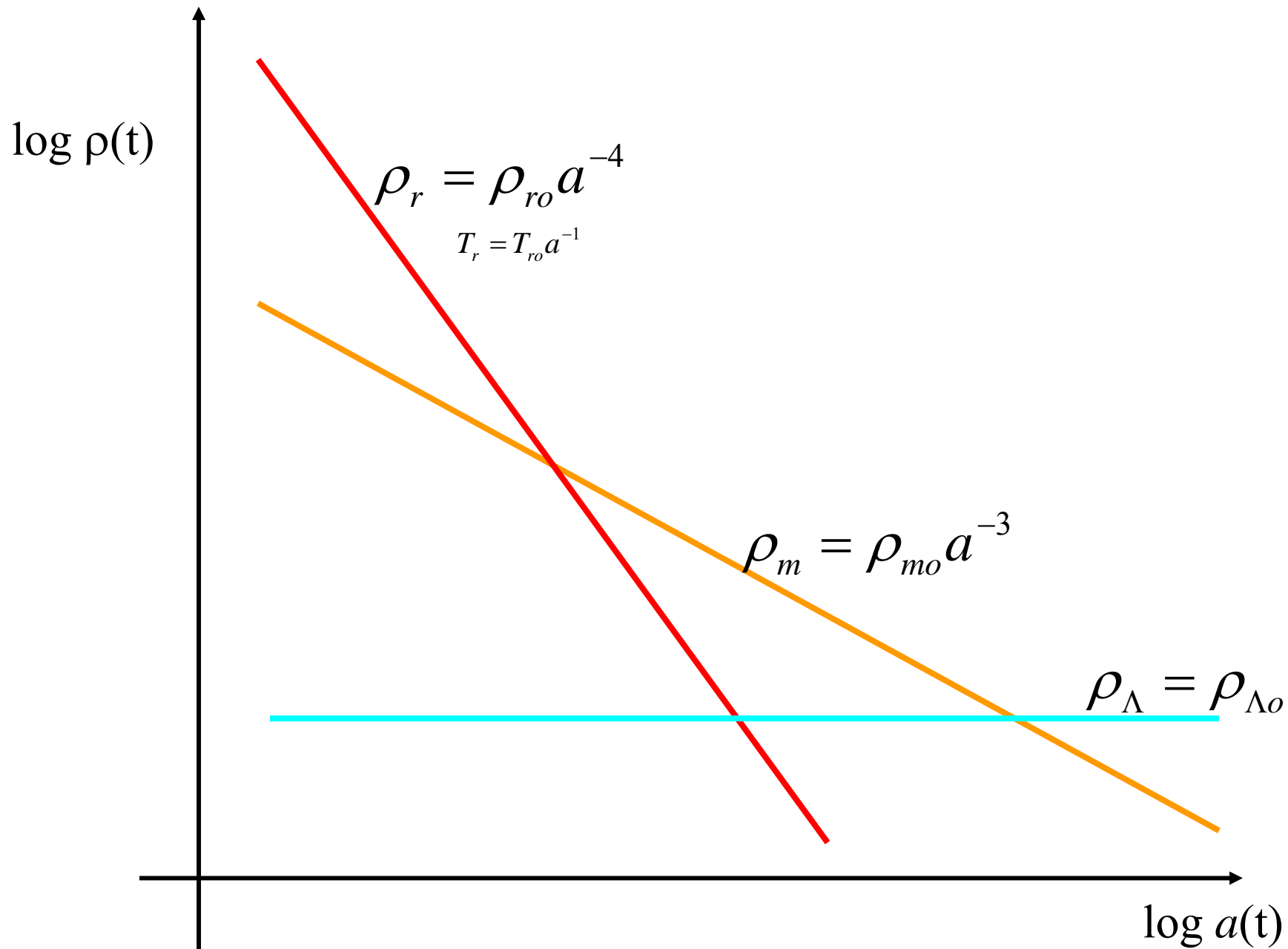
- Radiation : $\rho_r = n_r h\nu = n_r \frac{hc}{\lambda} = n_{r_0} a^{-3} \frac{hc}{\lambda_0 a} = \rho_{r_0} a^{-4}$

Note that, for a blackbody,

$$\rho_{rBB} = \sigma T_r^4 \quad \rightarrow \quad T_r = T_{r_0} a^{-1}$$

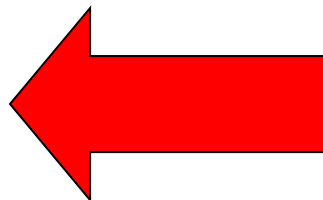
- Cosmological Constant :

$$\rho_\Lambda = \rho_{\Lambda_0}$$



$\log \rho(t)$

$$\rho_r = \rho_{r0} a^{-4}$$
$$T_r = T_{r0} a^{-1}$$

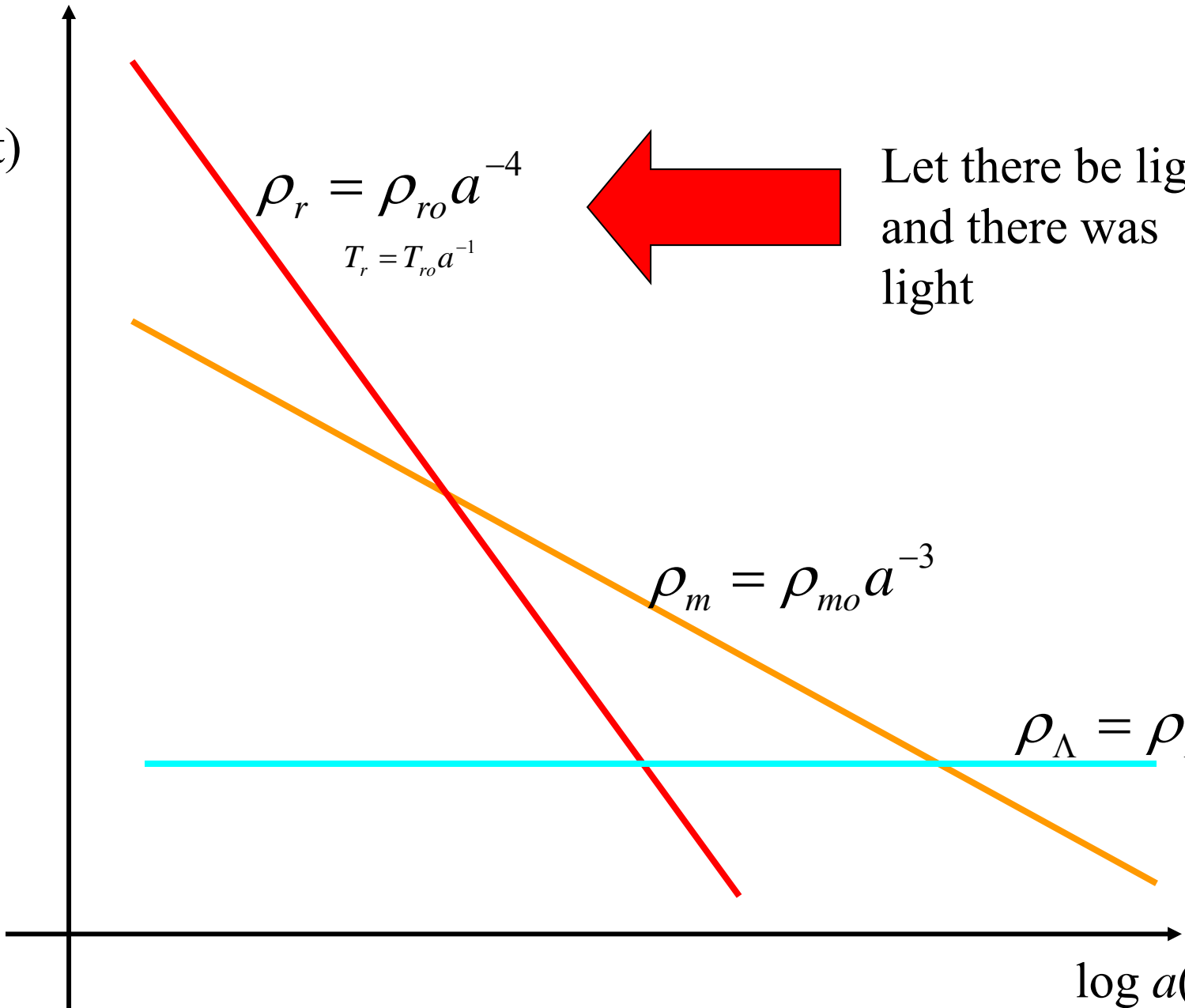


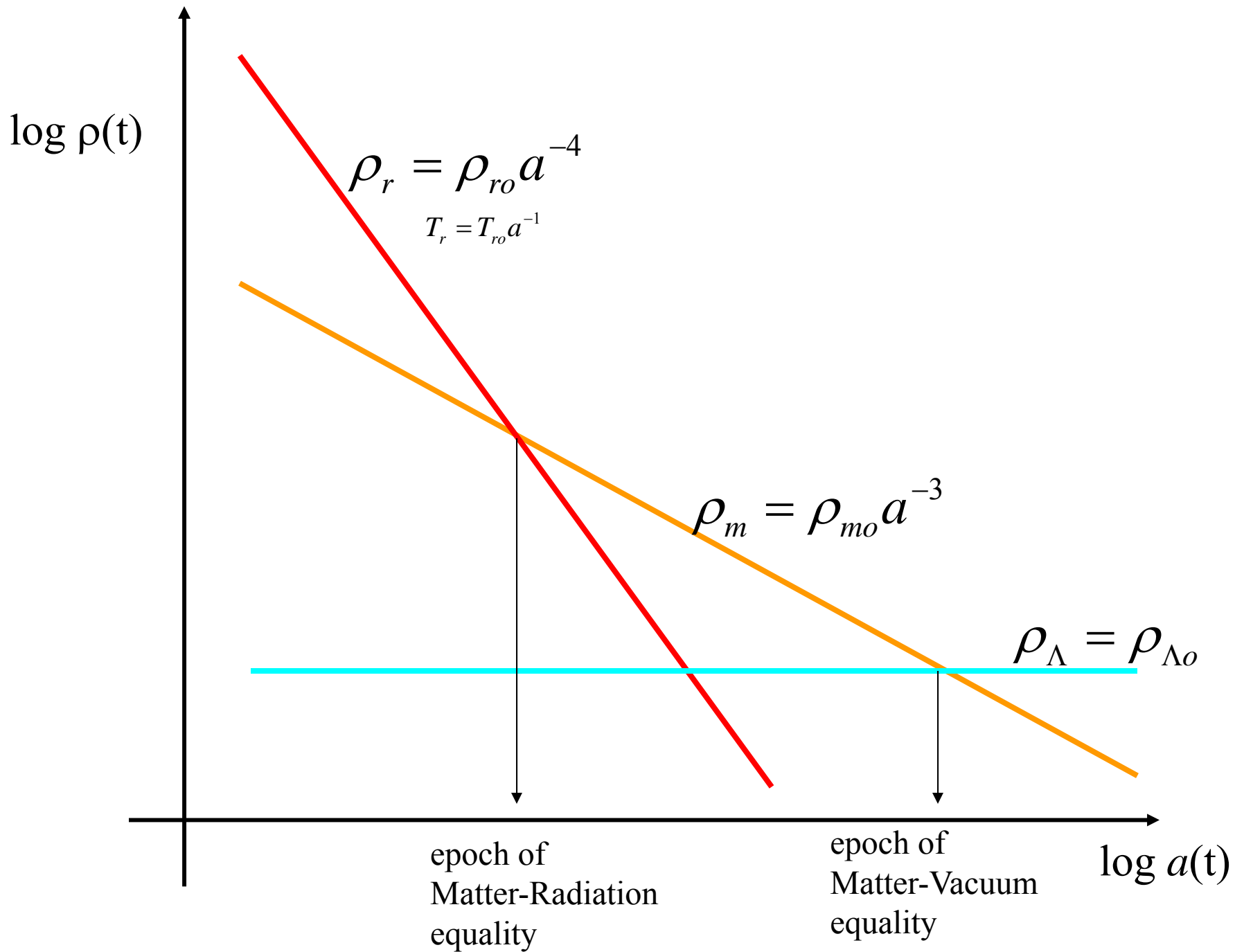
Let there be light..
and there was
light

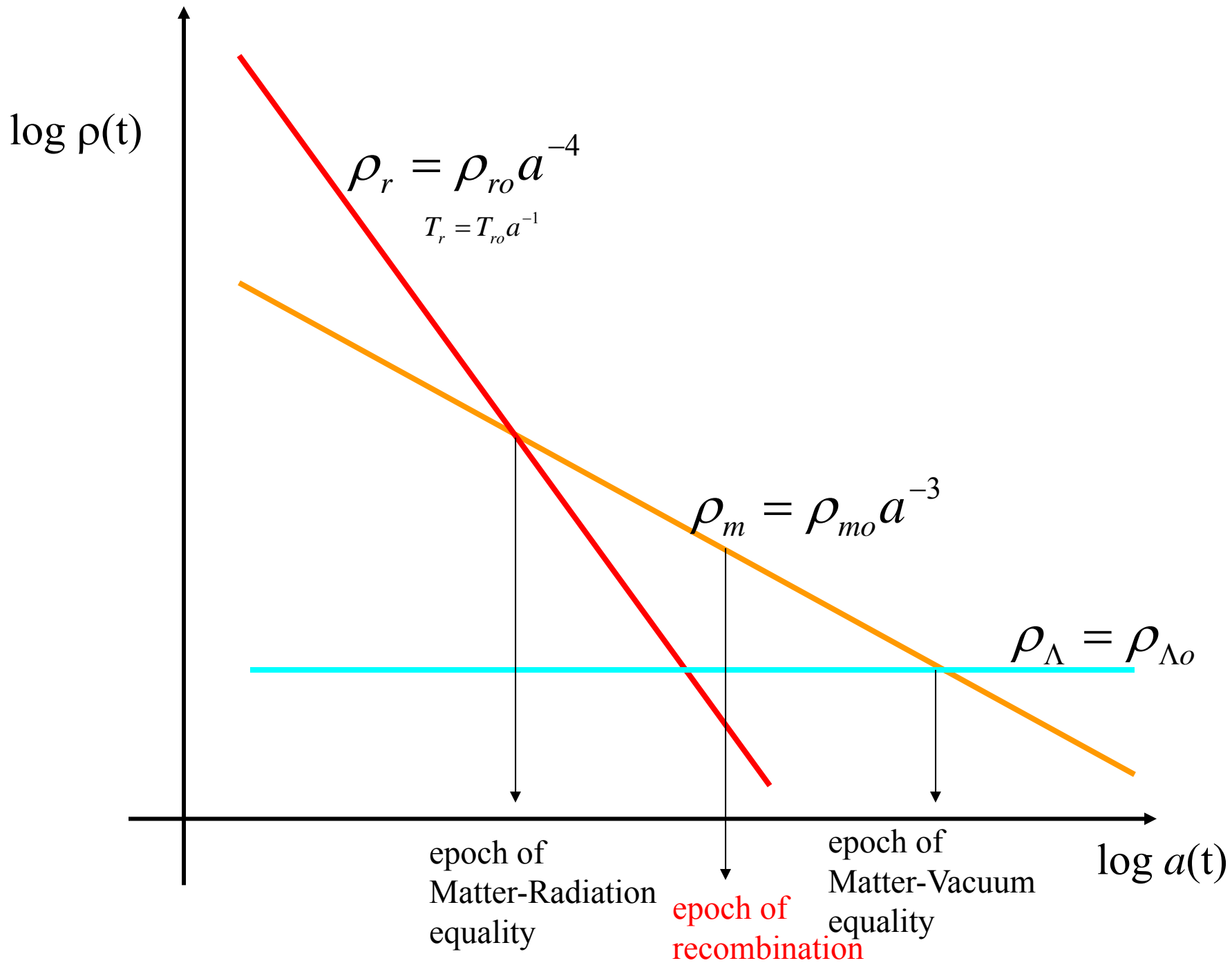
$$\rho_m = \rho_{m0} a^{-3}$$

$$\rho_\Lambda = \rho_{\Lambda0}$$

$\log a(t)$







Primeval Fireball

- If the Universe is expanding, it was denser and hotter in the past.
- Both matter and radiation were hotter in the past, when the scale factor was smaller.
- In a sufficiently early phase of the Universe, the temperature of matter was so high that matter (H) was ionized. As a consequence, Thomson scatterings were so frequent to keep matter and radiation in thermal equilibrium.
- This early phase is called the **Primeval Fireball**
- The universe was full of ionized matter and radiation in thermal equilibrium (blackbody radiation).
- This primeval fireball phase ended when the universe cooled down enough to allow the combination of electrons and protons in hydrogen atoms (3000K, 380000 yrs after the big bang): this is the so called **recombination** epoch.

The cosmic Blackbody

- Blackbody radiation present in the primeval fireball should still be present in the universe.
- And should still have a blackbody spectrum, with a temperature reduced by the ratio of the scale factor now and the scale factor when radiation was released, i.e. at recombination.
- In fact :

Evolution of a blackbody spectrum in an expanding universe.

- Specific energy density of a blackbody:

$$u_\nu = \frac{8\pi\nu^2}{c^3} h\nu \frac{1}{e^{h\nu/kT} - 1} = \frac{8\pi h \nu^3}{c^3 (e^{h\nu/kT} - 1)}$$

$$u_\lambda = \frac{dE}{d\lambda} = \frac{dE}{d\nu} \frac{\nu}{\lambda} = u_\nu \frac{\nu}{\lambda} = \frac{8\pi h \nu^4}{\lambda c^3 (e^{h\nu/kT} - 1)} = \frac{8\pi h c}{\lambda^5 (e^{hc/\lambda kT} - 1)}$$

- So the energy density is $u_\lambda d\lambda = \frac{8\pi h c}{\lambda^5 (e^{hc/\lambda kT} - 1)} d\lambda$

- Due to expansion and redshift: $\frac{\lambda}{\lambda_o} = \frac{a}{a_o} \quad ; \quad \frac{u_\lambda d\lambda}{u_{\lambda_o} d\lambda_o} = \left[\frac{a}{a_o} \right]^4$

Evolution of a blackbody spectrum in an expanding universe

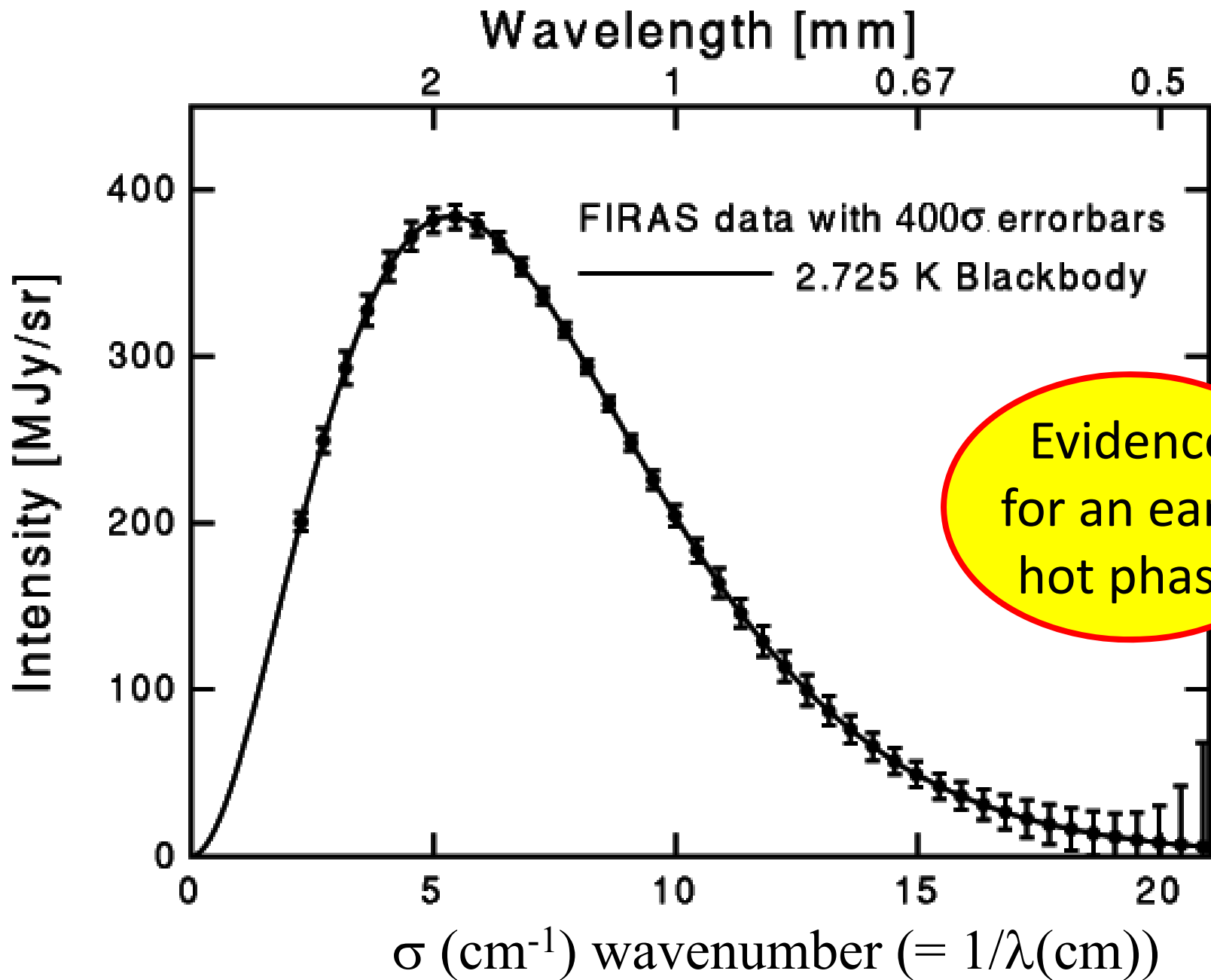
$$u_{\lambda_o} d\lambda_o = \left[\frac{a}{a_o} \right]^4 u_{\lambda} d\lambda = \left[\frac{a}{a_o} \right]^4 \frac{8\pi hc}{\lambda_o^5 \left[\frac{a}{a_o} \right]^5 \left(e^{hc/\lambda_o \left[\frac{a}{a_o} \right] kT} - 1 \right)} d\lambda_o \left[\frac{a}{a_o} \right]$$

$$\Rightarrow u_{\lambda_o} d\lambda_o = \frac{8\pi hc}{\lambda_o^5 \left(e^{hc/\lambda_o \left[\frac{a}{a_o} \right] kT} - 1 \right)} d\lambda_o$$

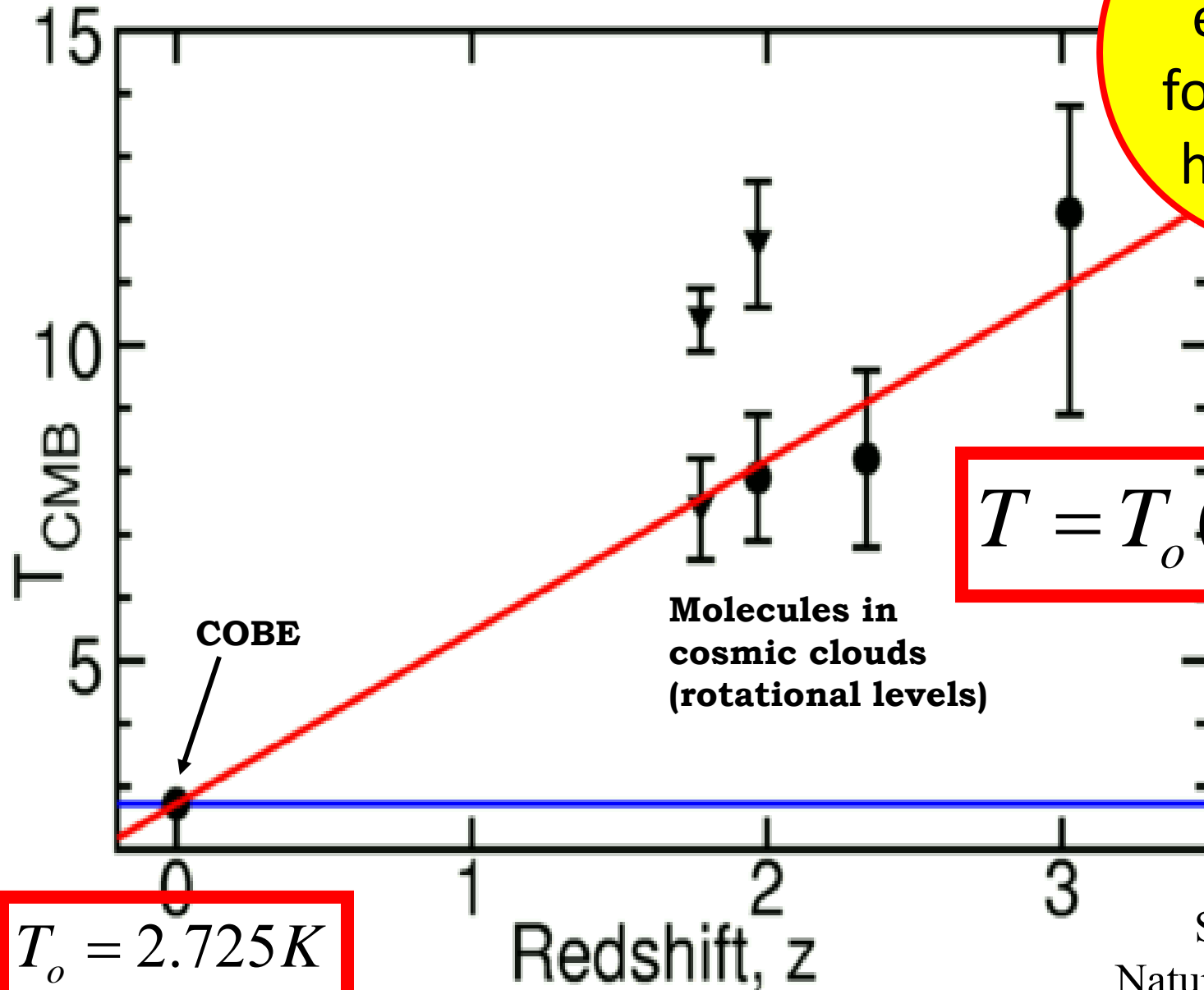
$$u_{\lambda_o} d\lambda_o = \frac{8\pi hc}{\lambda_o^5 \left(e^{hc/\lambda_o kT_o} - 1 \right)} d\lambda_o$$

$$T = T_o \frac{a_o}{a}$$

- A blackbody spectrum remains a blackbody, but its temperature scales as the inverse of the scale-factor.
- The photons of the primeval fireball should still be around, as a low-temperature blackbody.



Primeval Fireball



Additional evidence for an early hot phase

$$T = T_0 (1 + z)$$

Molecules in cosmic clouds (rotational levels)

COBE

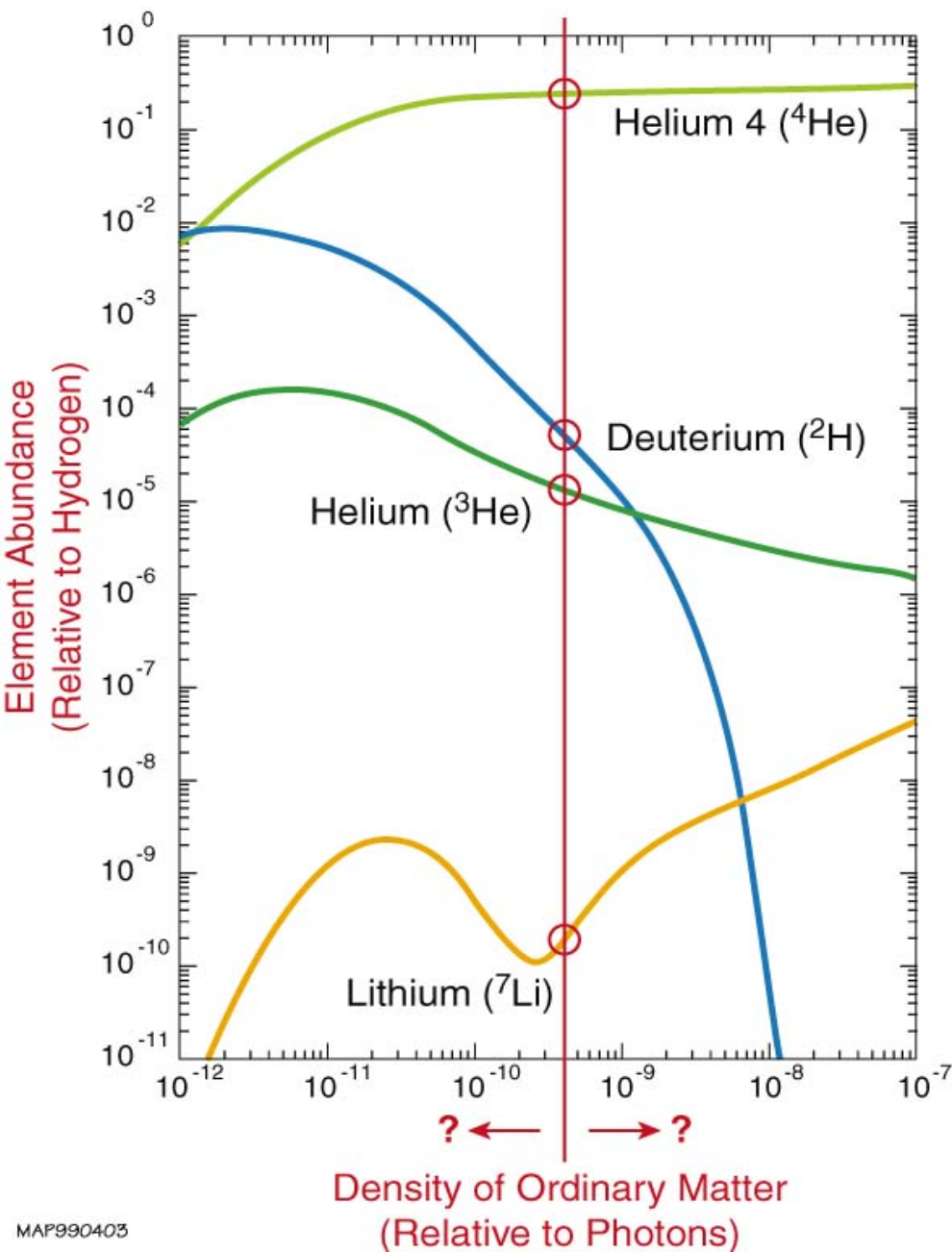
$$T_0 = 2.725\text{K}$$

CMB and cosmology

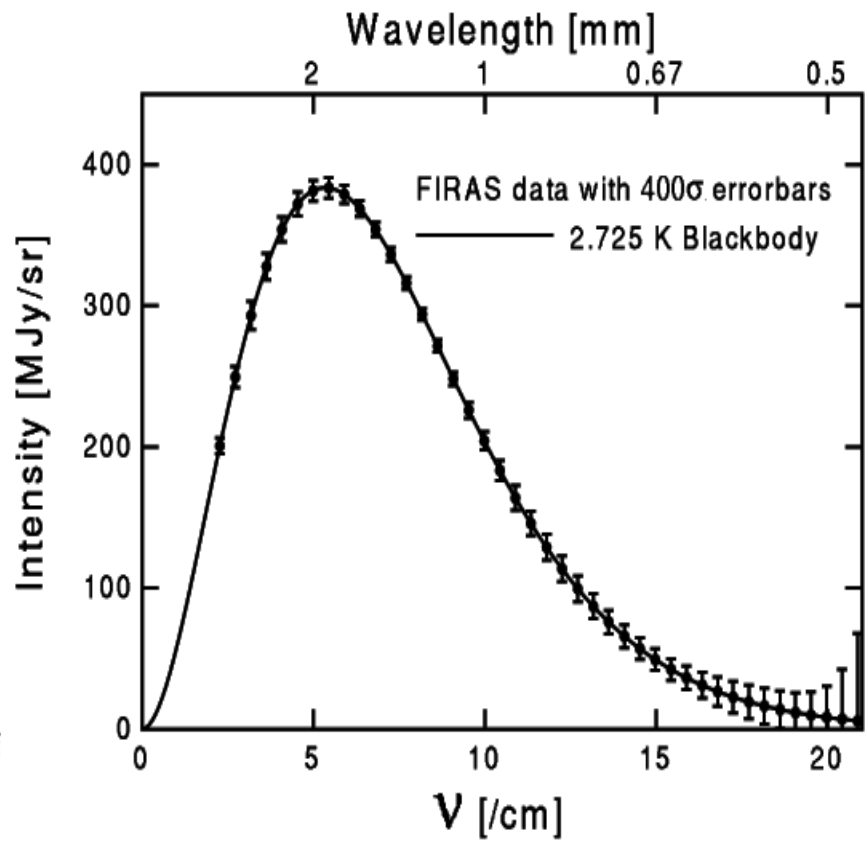
- We will see later how these measurements have been obtained.
- Now let's stress how important for cosmology is the presence of the Cosmic Microwave Background, and how this radiation formed, according to modern cosmology.
- A blackbody with $T=2.725\text{K}$ consists of 400 g/cm^3 . This is a density way higher than the average density of matter in the universe. **About 10^9 times higher.** CMB photons are the most abundant particles in the universe.
- The BB nature of the CMB is a **direct confirmation that the universe underwent a hot early phase.**
- There is no other way to produce such a perfect blackbody spectrum, filling the sky with an almost perfect isotropy.
- We need to investigate when was formed, and when was released.

Nucleosynthesis and the CMB

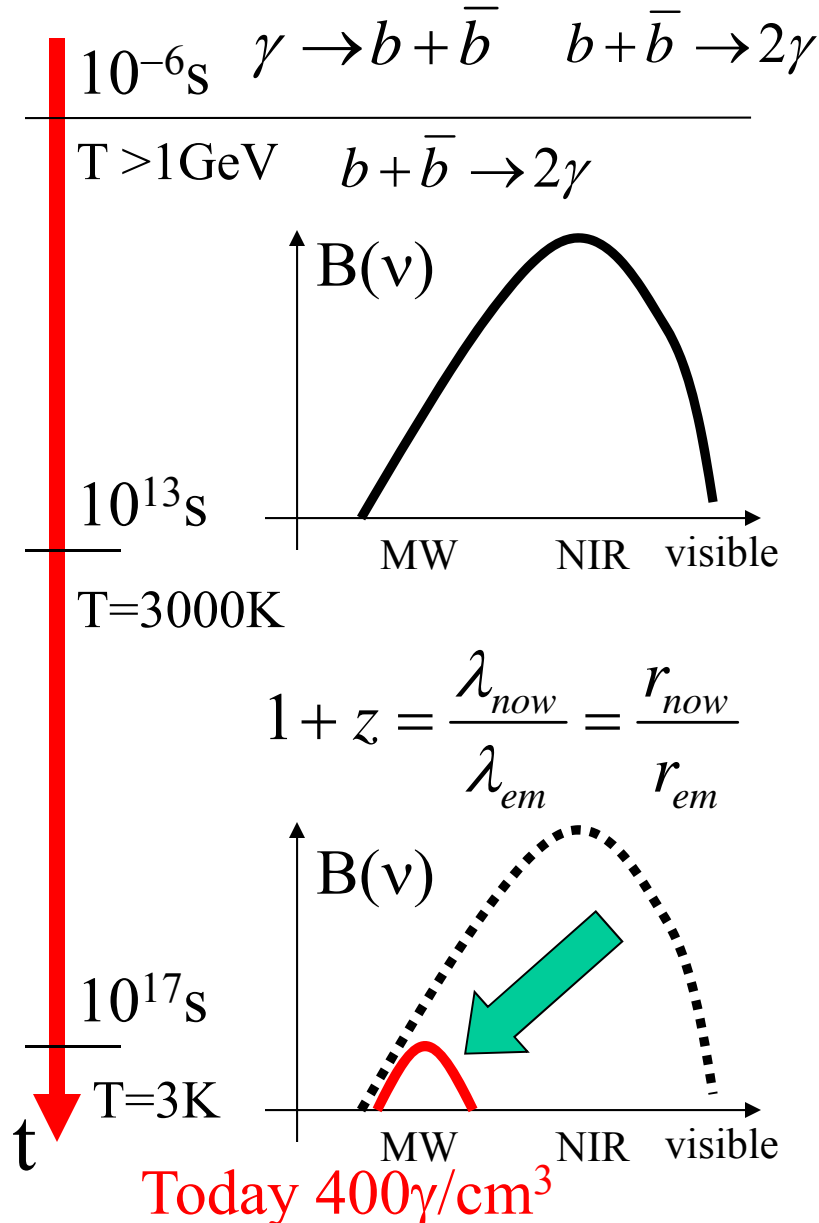
- At an even earlier phase, the temperature was high enough that nuclear reactions produced light elements starting from a plasma of simple particles (nucleosynthesis).
- The observed primordial abundance of light elements can be produced only if an abundant background of photons is present ($10^9 \gamma/\text{baryon}$). (G.Gamow).
- This background is observed today as the Cosmic Microwave Background.



Synergic Evidence for an early hot phase



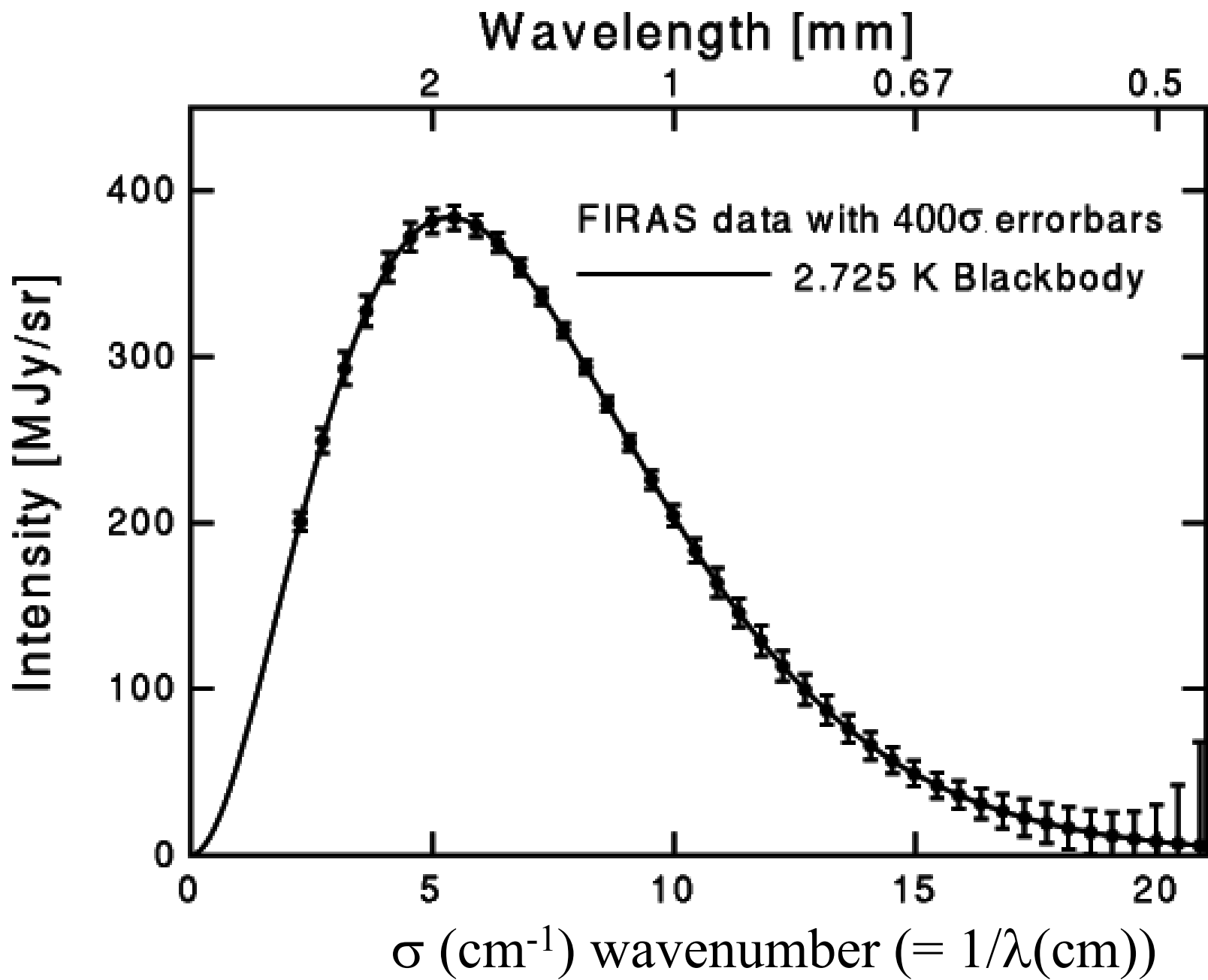
What is the CMB



According to modern cosmology:

An abundant background of photons filling the Universe.

- **Generated** in the very early universe, less than 4 ms after the Big Bang ($10^9\gamma$ for each baryon) from a small $b - \bar{b}$ asymmetry
- **Thermalized** in the primeval fireball (in the first 380000 years after the big bang) by repeated scattering against free electrons
- **Redshifted** to microwave frequencies ($z_{\text{CMB}}=1100$) **and diluted** in the subsequent 14 Gyrs of expansion of the Universe

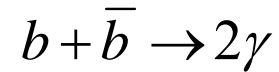
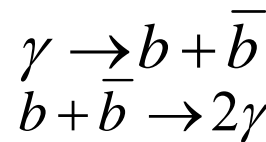


When were CMB photons generated ?

in the very early universe

(a few μs after the big bang, at

baryogenesis)



When and where did they last scatter
against matter ?

in the early universe

(380000 yrs after the big bang, at

recombination)

Recombination epoch

- Here the primeval plasma cools down enough to allow the formation of the first hydrogen atoms.
- electrons and protons can combine if the energy of the photons of the primeval fireball is less than the binding energy, which, for the fundamental state of H is $B=13.6\text{eV}$.
- So one would naively say that the temperature of matter and radiation has to be less than $T=B/k=156000\text{K}$
- However, in a blackbody distribution, there is a tail of photons with energy much higher than the average energy. Since the number of photons is so high with respect to the number of particles, the temperature has to be much less than 156000K .
- The equilibrium between electrons, protons, H atoms is described by Saha's equation, which can be found from the equilibrium distributions:

$$n_i \cong g_i \left(\frac{m_i k T}{2\pi h^2} \right)^{3/2} e^{(\mu_i - m_i c^2)/kT}$$

- where $i=e,p,H$. Equilibrium maintained by $p + e \leftrightarrow H + \gamma$

Recombination epoch

$$n_i \cong g_i \left(\frac{m_i kT}{2\pi h^2} \right)^{3/2} e^{(\mu_i - m_i c^2)/kT} \quad p + e \leftrightarrow H + \gamma$$

- To balance the chemical potentials we have $\mu_e + \mu_p = \mu_H$.
- Expressing μ_i with n_i , and defining the hydrogen binding energy as $B = (m_p + m_e - m_H)c^2$ we find Saha's equation:

$$\frac{n_H}{n_p n_e} \cong \left(\frac{h^2}{2\pi m_e kT} \right)^{3/2} e^{B/kT}$$

- We define η as the ratio of baryons to photons $\eta = n_b/n_\gamma \sim 5 \times 10^{-10}$, and x as the ionized fraction $x = n_e/n_b = n_p/n_b$ so we can write $n_p = n_e = x n_b = x \eta n_\gamma$ and $n_H = (1-x)n_b = (1-x)\eta n_\gamma$

Recombination epoch

- Inserting the definitions above into Saha's equation we find

$$\frac{1-x}{x^2} \cong n_\gamma \eta \left(\frac{h^2}{2\pi m_e kT} \right)^{3/2} e^{B/kT}$$

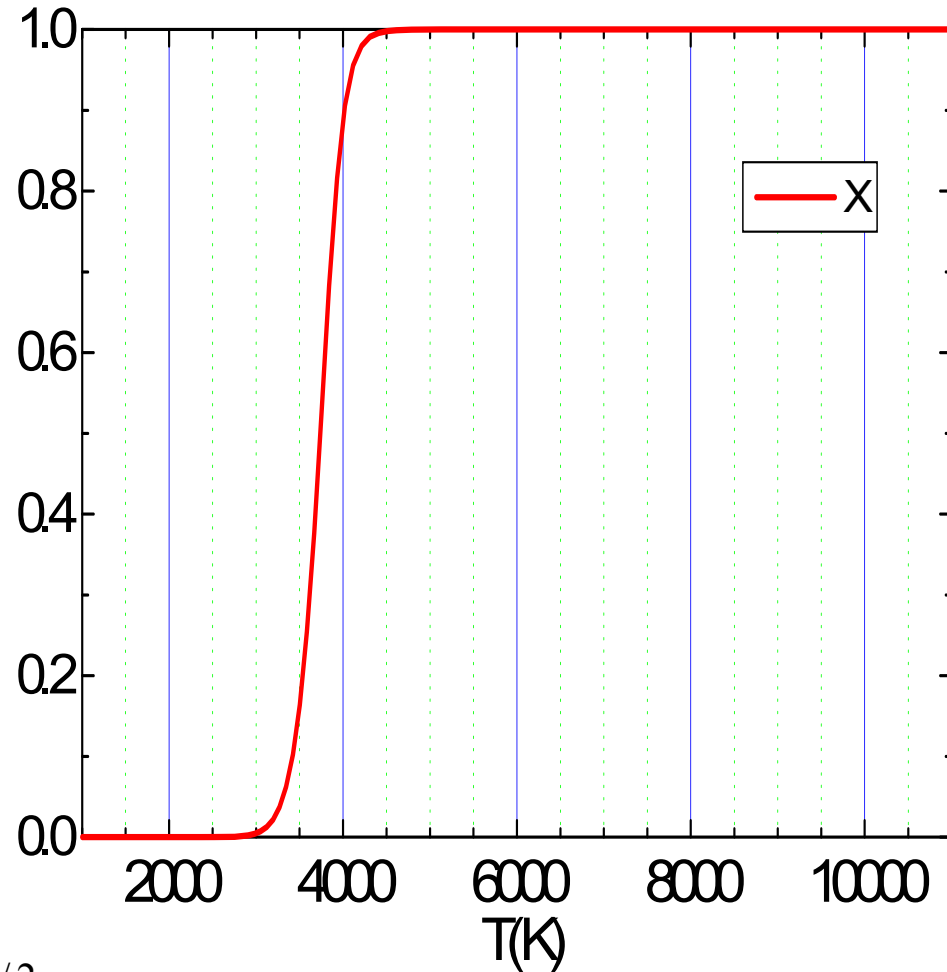
but $n_\gamma = n_{\gamma_0} (T/T_0)^3$ so

$$\frac{1-x}{x^2} \cong n_{\gamma_0} \eta \left(\frac{h^2 (T/T_0)}{2\pi m_e kT_0} \right)^{3/2} e^{B/kT} \quad \times$$

whose solution is

$$x = \frac{\sqrt{1+4f(T)} - 1}{2f(T)}$$

with $f(T) = n_{\gamma_0} \eta \left(\frac{h^2 (T/T_0)}{2\pi m_e kT_0} \right)^{3/2} e^{B/kT}$

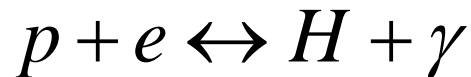


Recombination epoch

- Since η is very small, the actual temperature of recombination is $T_{\text{rec}} \sim 3000\text{K}$, not 156000K . So

$$(1+z_{\text{rec}}) = a_0/a_{\text{rec}} = T_{\text{rec}}/T_0 \sim 3000/2.725 \sim 1100$$

- **Recombination happened when all lengths were 1100 times smaller than today.**
- To convert this into a time, we need to find $a(t)$.
- We need to notice, also, that Saha's equation is valid only in thermodynamic equilibrium, i.e. as long as the reaction



is active.

- This will not be active anymore once most of the electrons and protons have combined into H atoms: when their density will be low enough, the reaction rate will be too low compared to the expansion, which separates particles preventing reactions. So we expect a small residual ionization to survive (residual x is around 10^{-4})

Recombination epoch

- Since η is very small, the actual temperature of recombination is $T_{\text{rec}} \sim 3000\text{K}$, not 156000K . So $(1+z_{\text{rec}}) = T_{\text{rec}}/T_0 \sim 1100$.
- **Recombination happened when all lengths were 1100 times smaller than today.**
- When most protons and electrons combine, interactions of photons and matter become so rare that the universe becomes transparent. (The cross-section of a Hydrogen atom is 1800 times smaller than the Thomson cross-section. Basically, most CMB photons do not interact with matter anymore, before reaching our CMB telescopes.
- **Note: We can make astronomical observations using electromagnetic radiation only within a sphere whose radius is as large as the look-back distance to recombination.**
- We cannot measure anything more distant than that, since the Universe was not transparent before recombination. It was completely opaque, like the interior of the sun.
- So we really need to find $a(t)$

Radiation Phase

- GR recipe to find $a(t)$: given the metric and the composition in terms of mass-energy, write down Einstein's equations. In our case the result is the Friedmann equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = H_o^2 \left\{ \Omega_{Ro} a^{-4} + \Omega_{Mo} a^{-3} + \Omega_{Ko} a^{-2} + \Omega_{\Lambda} \right\} \approx H_o^2 \Omega_{Ro} a^{-4}$$

- Note that the expansion rate \dot{a}/a tends to infinity at the beginning (near the Big Bang, radiation dominated), and then decreases with time.
- In this phase the solution $a(t)$ can be found analytically:

$$\frac{a^2}{2} = H_o \sqrt{\Omega_{Ro}} t \Rightarrow a(t) = \left\{ 2\sqrt{\Omega_{Ro}} \right\}^{1/2} (H_o t)^{1/2}$$

- We know H_o from Hubble's law. Ω_R has contributions from CMB photons, but also from all other relativistic particles present at early epochs. So the extrapolation using only the energy density of the CMB would not be precise.

log $\rho(t)$

$$\rho_r = \rho_{r0} a^{-4}$$

The radiation phase continues until the energy density of radiation becomes comparable to the energy density of non relativistic matter.

$$\rho_{R0} a_{eq}^{-4} = \rho_{M0} a_{eq}^{-3} \longrightarrow$$

$$a_{eq} = \frac{\rho_{R0}}{\rho_{M0}} \approx \frac{\sigma T_{CMB}^4 / c^3}{\Omega_{mo} \rho_{co}}$$

$$a_{eq} \approx 4 \times 10^{-5}$$

$$t_{eq} \approx 2000 \text{ y}$$

very rough

$$\rho_m = \rho_{m0} a^{-3}$$

log $a_{eq}(t)$

log $a(t)$

Note: at the end of the radiation phase the temperature $T = T_{CMB}/a$ was still $> 10^5$ K
The universe was still ionized and opaque.

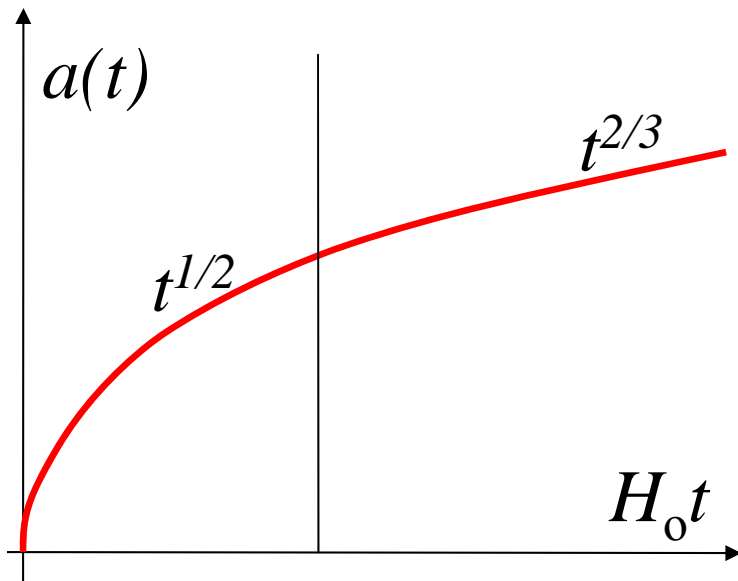
Matter Phase

- When the energy of non-relativistic matter becomes dominant

$$\left(\frac{\dot{a}}{a}\right)^2 = H_o^2 \left\{ \Omega_{Ro} a^{-4} + \Omega_{Mo} a^{-3} + \Omega_{Ko} a^{-2} + \Omega_{\Lambda} \right\} \approx H_o^2 \Omega_{Mo} a^{-4}$$

- In this phase the solution $a(t)$ can be found analytically:

$$\frac{2}{3} a^{3/2} \Big|_{a_o}^a = H_o \sqrt{\Omega_{mo}} (t - t_o) \Rightarrow a(t) = \left(\frac{3}{2} \sqrt{\Omega_{Mo}} H_o (t - t_o) + a_o^{3/2} \right)^{2/3}$$



From this equation we can estimate how long it took to go from $a=10^{-5}$ (end of radiation phase) to $a=10^{-3}$ (recombination).

The result is 380000 years.
This number is important for the following.

Vacuum Phase

- When the energy of non-relativistic matter becomes negligible wrt the energy of the vacuum-like term

$$\left(\frac{\dot{a}}{a}\right)^2 = H_o^2 \left\{ \Omega_{Ro} a^{-4} + \Omega_{Mo} a^{-3} + \Omega_{Ko} a^{-2} + \Omega_{\Lambda} \right\} \approx H_o^2 \Omega_{\Lambda}$$

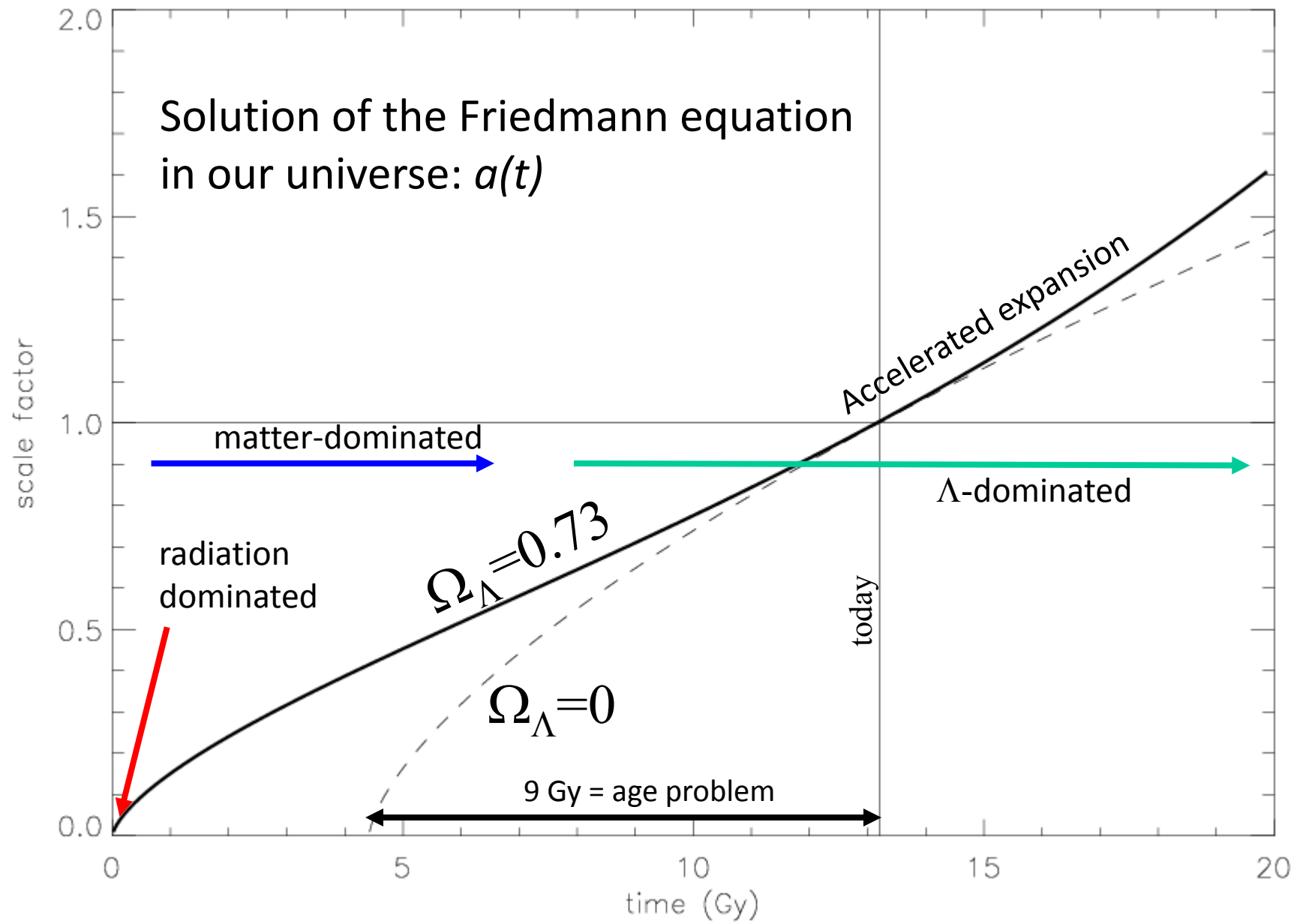
- In this phase the solution $a(t)$ can be found analytically:

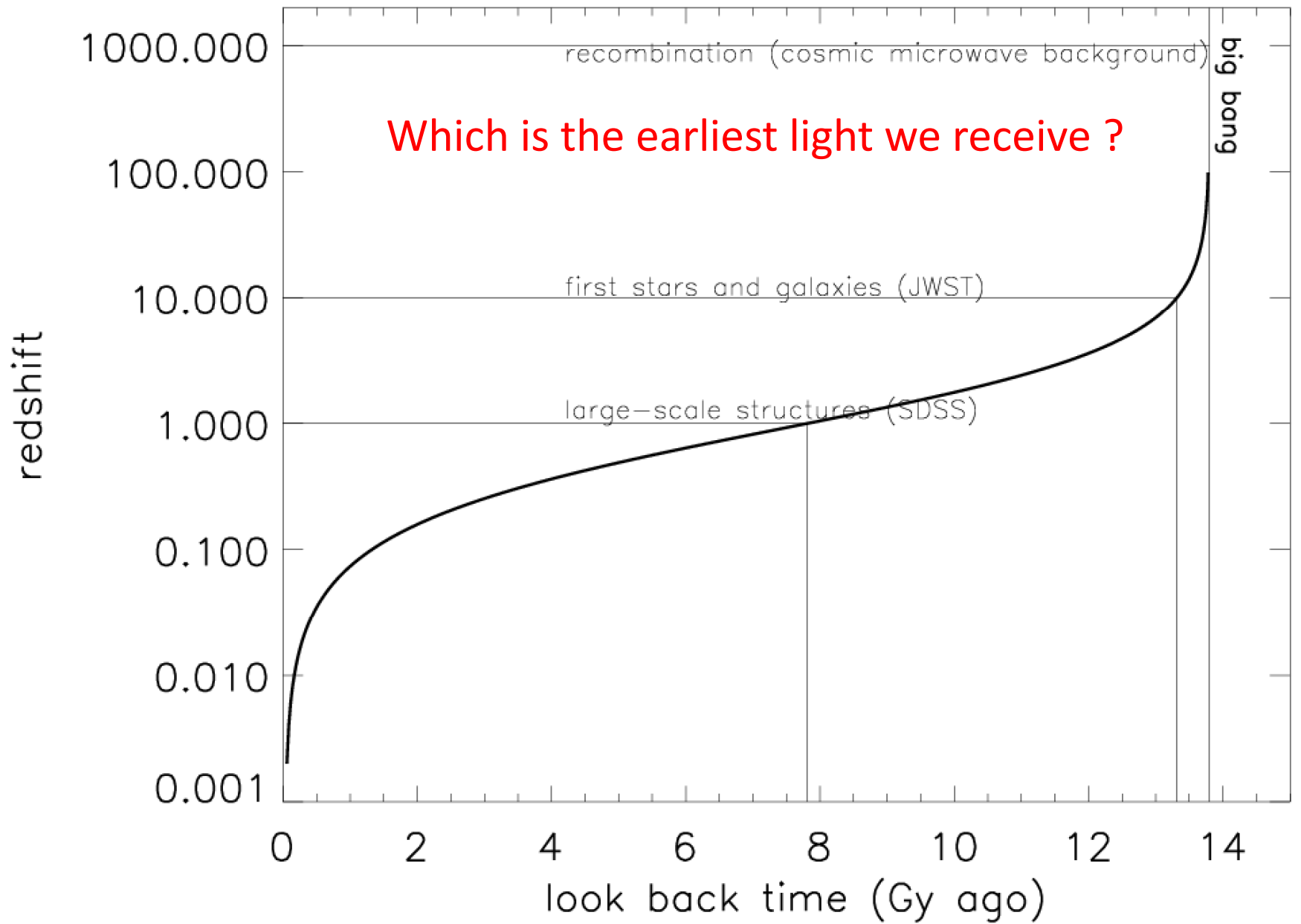
$$\ln a \Big|_{a_o}^a = H_o \sqrt{\Omega_{\Lambda}} (t - t_o) \Rightarrow a(t) = a_o e^{\sqrt{\Omega_{\Lambda}} H_o (t - t_o)}$$

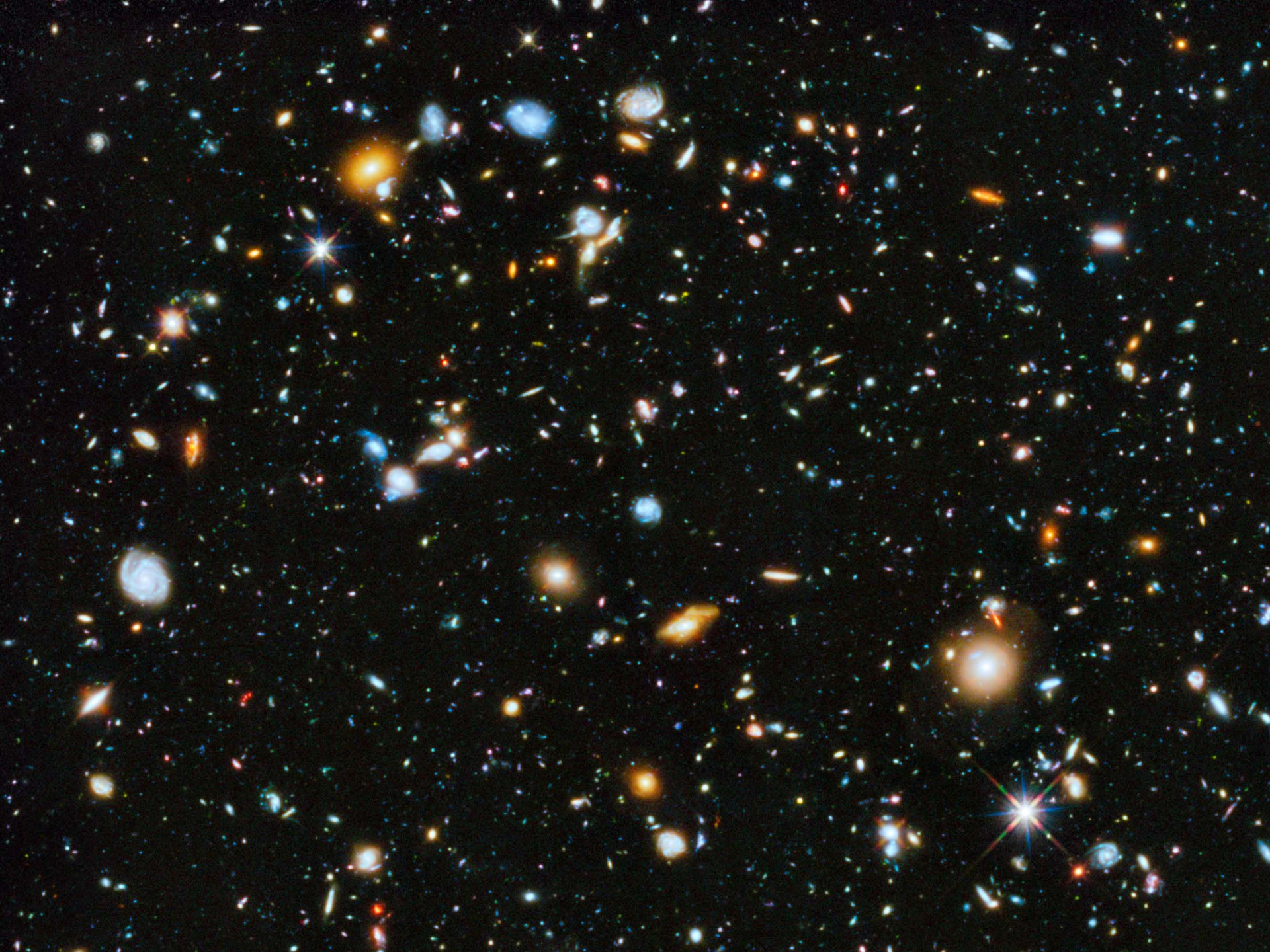
Exponential, accelerated phase, started a few billion years ago. $\Omega_{mo}=0.3$, $\Omega_{\Lambda}=0.7$ from observations.

- So, putting all together :

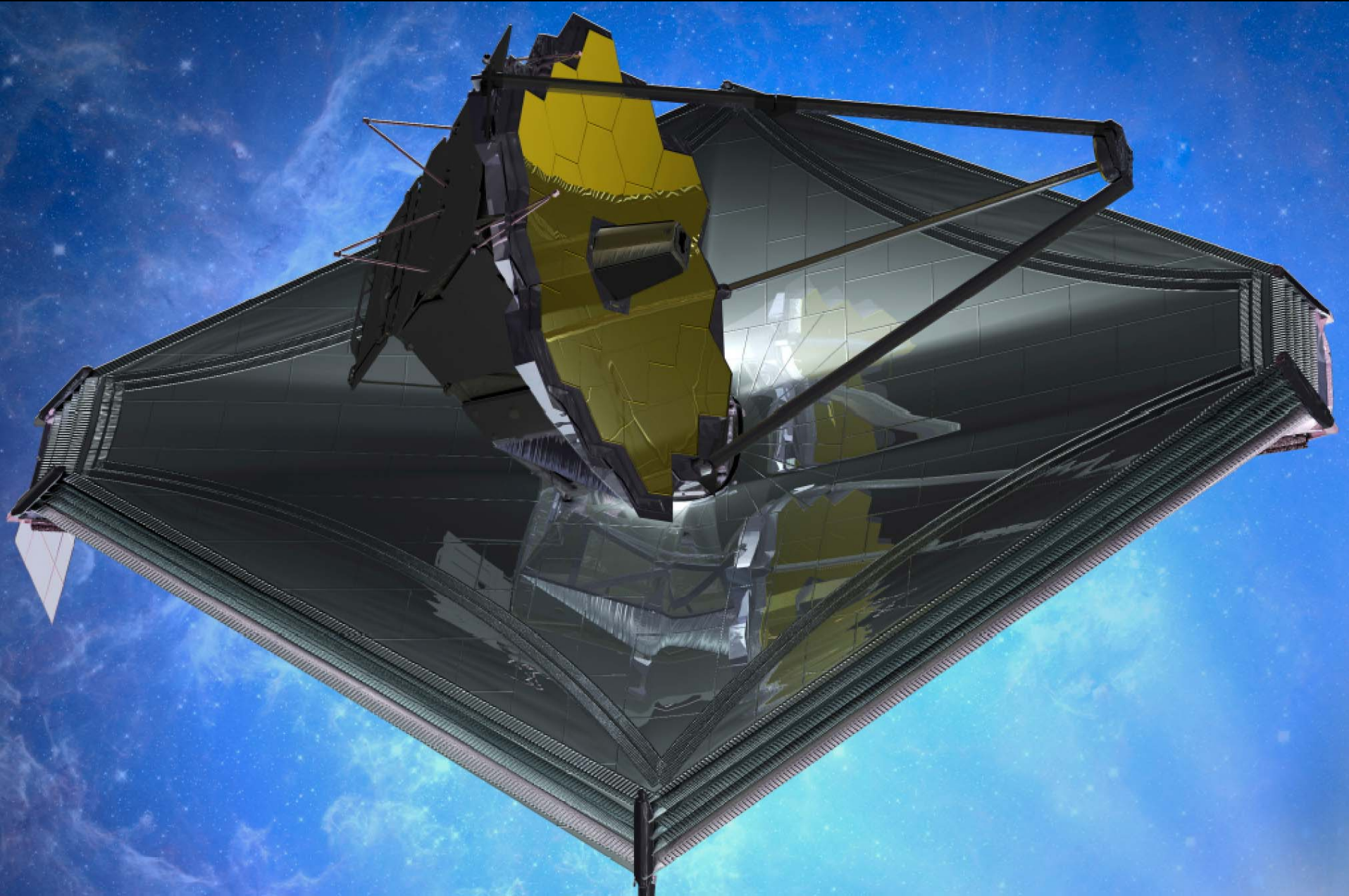
Solution of the Friedmann equation in our universe: $a(t)$

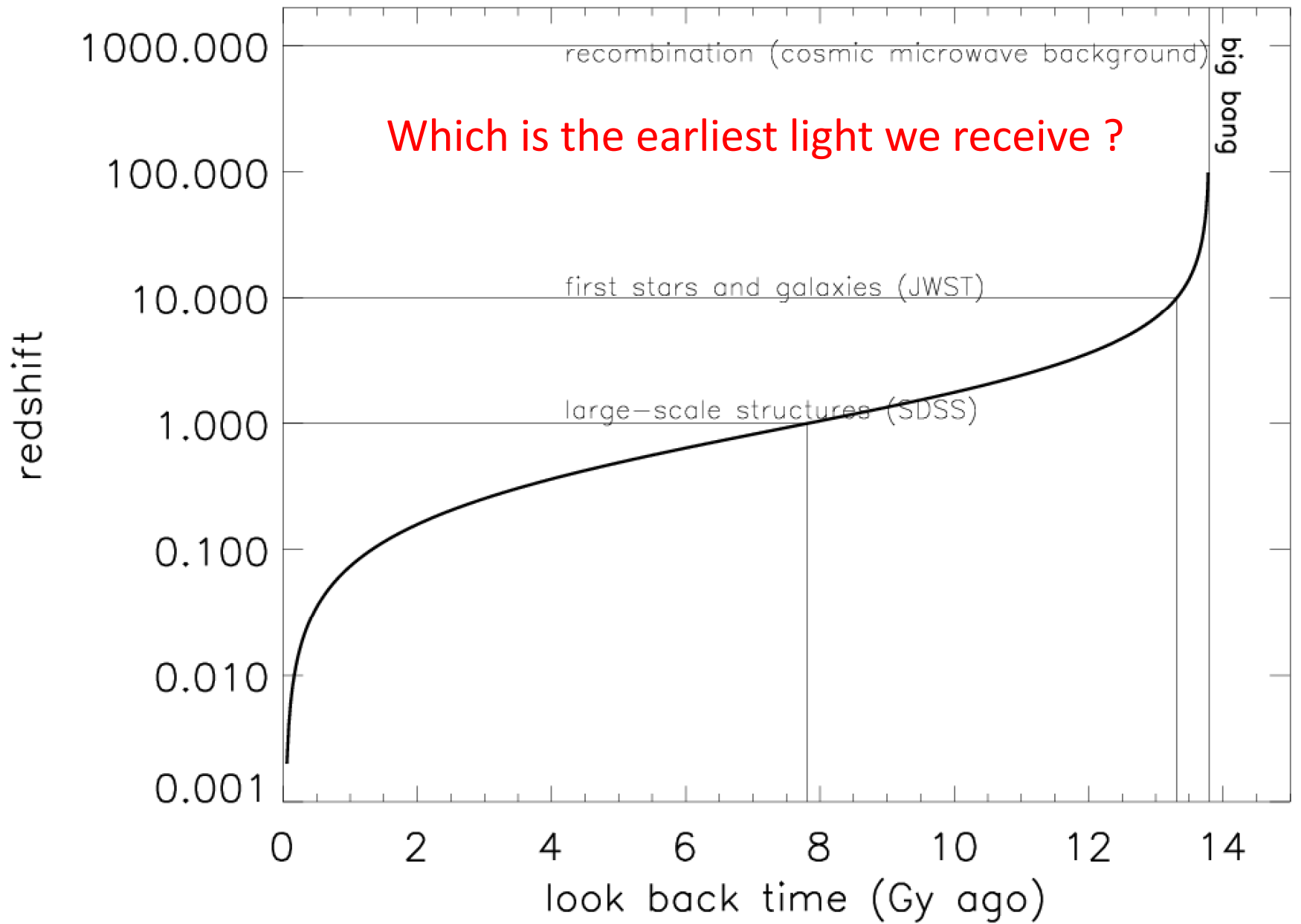






JWST





Isotropy of the CMB ?

Several physical phenomena happening at recombination and before suggest that the CMB should be (slightly) anisotropic.

Horizons – Inflation – seeds of structures

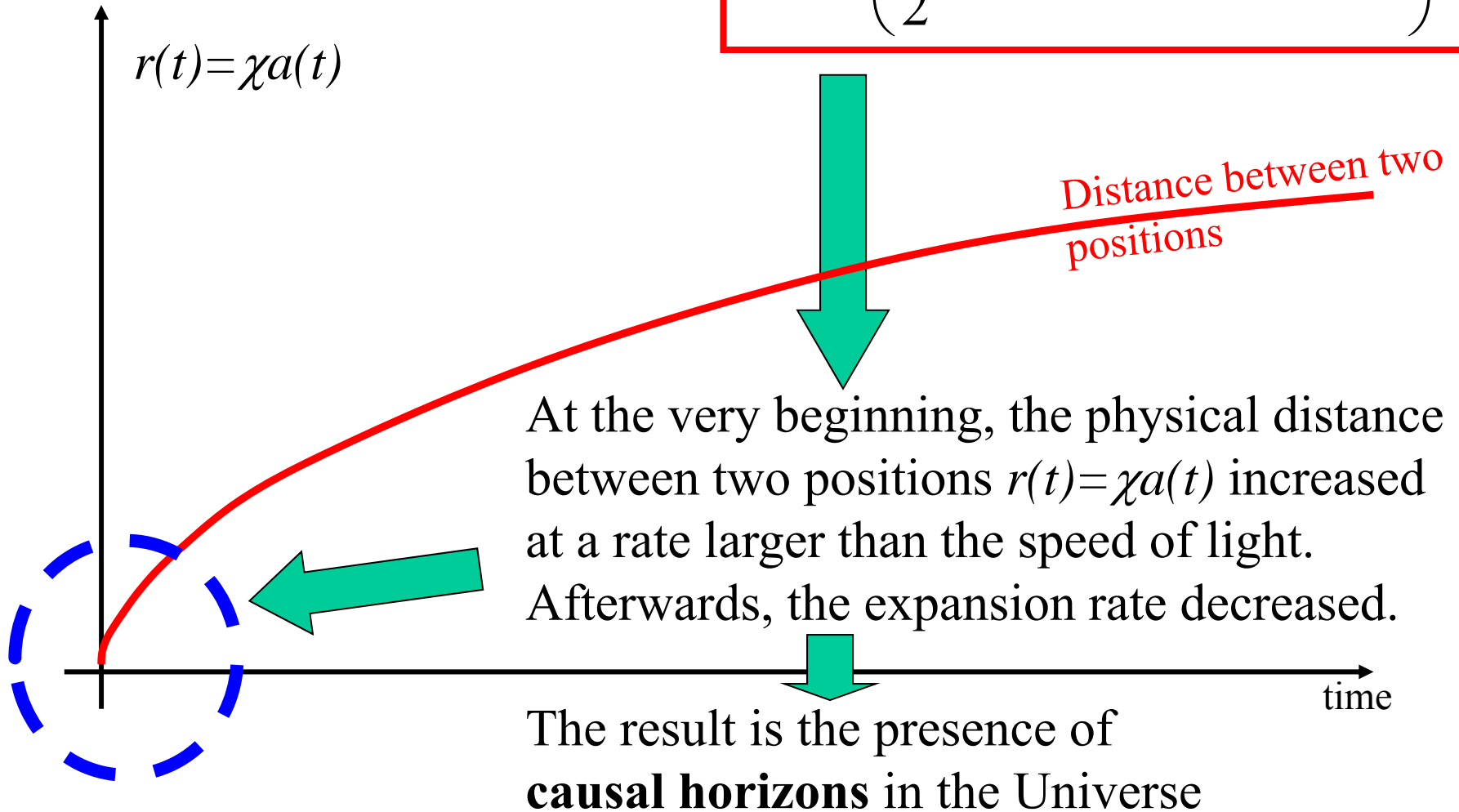
Oscillations in the primeval fireball

$$\left(\frac{\dot{a}}{a}\right)^2 \approx H_o^2 \Omega_{Ro} a^{-4}$$

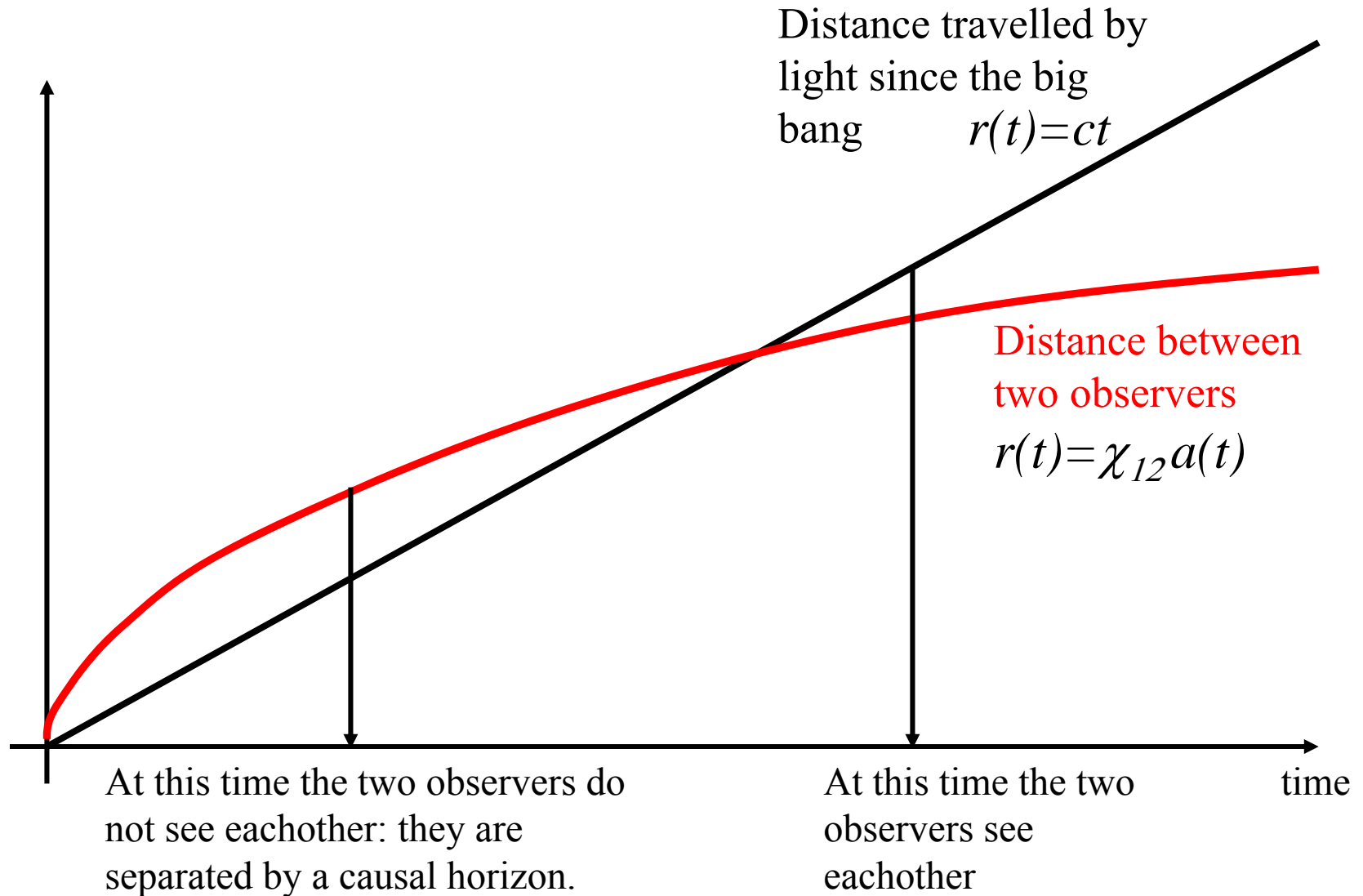
$$a(t) = \left\{ 2\sqrt{\Omega_{Ro}} \right\}^{1/2} (H_o t)^{1/2}$$

$$\left(\frac{\dot{a}}{a}\right)^2 \approx H_o^2 \Omega_{Mo} a^{-4}$$

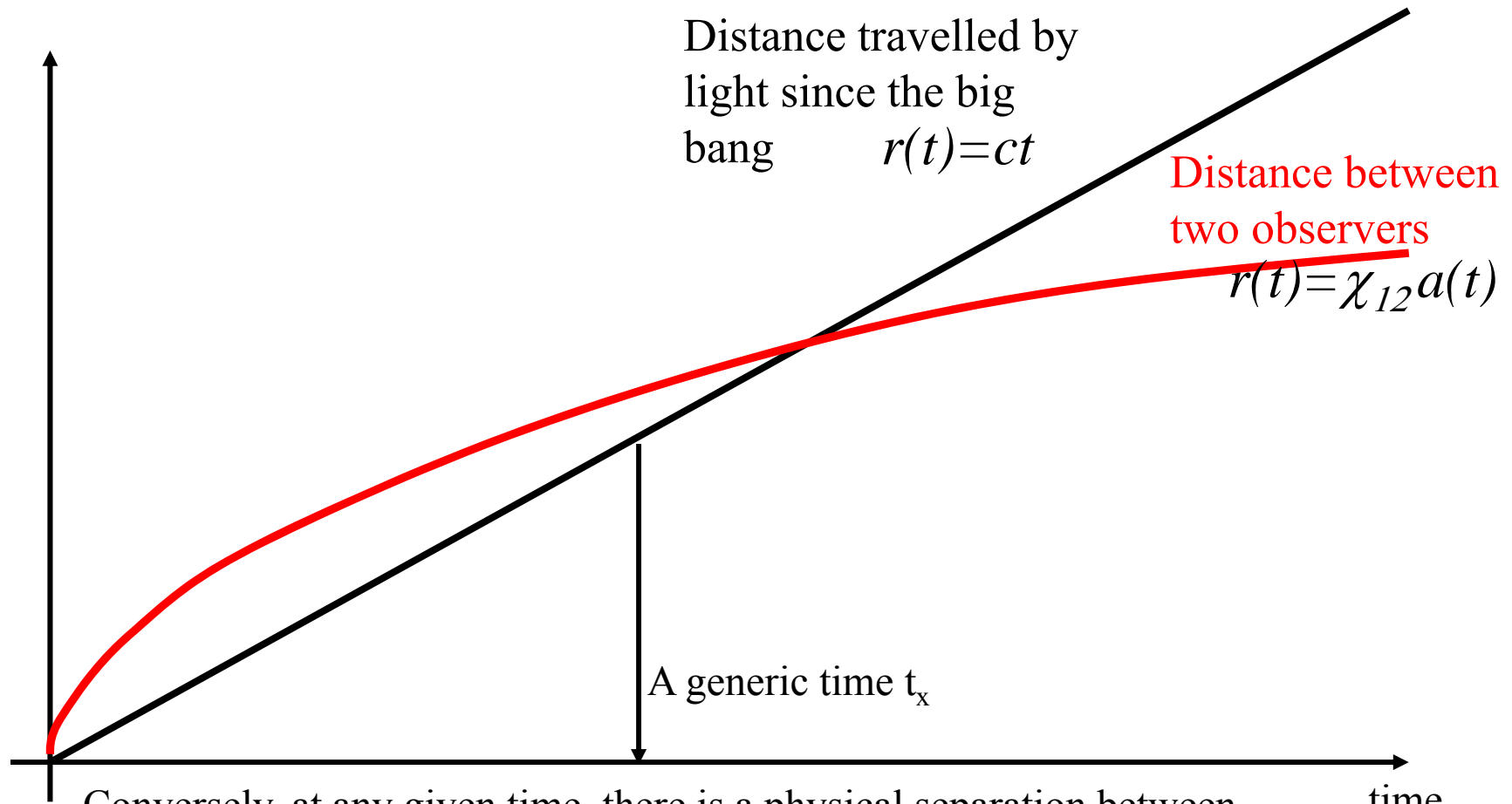
$$a(t) = \left(\frac{3}{2} \sqrt{\Omega_{Mo}} H_o (t - t_o) + a_o^{3/2} \right)^{2/3}$$



Expansion vs Horizon

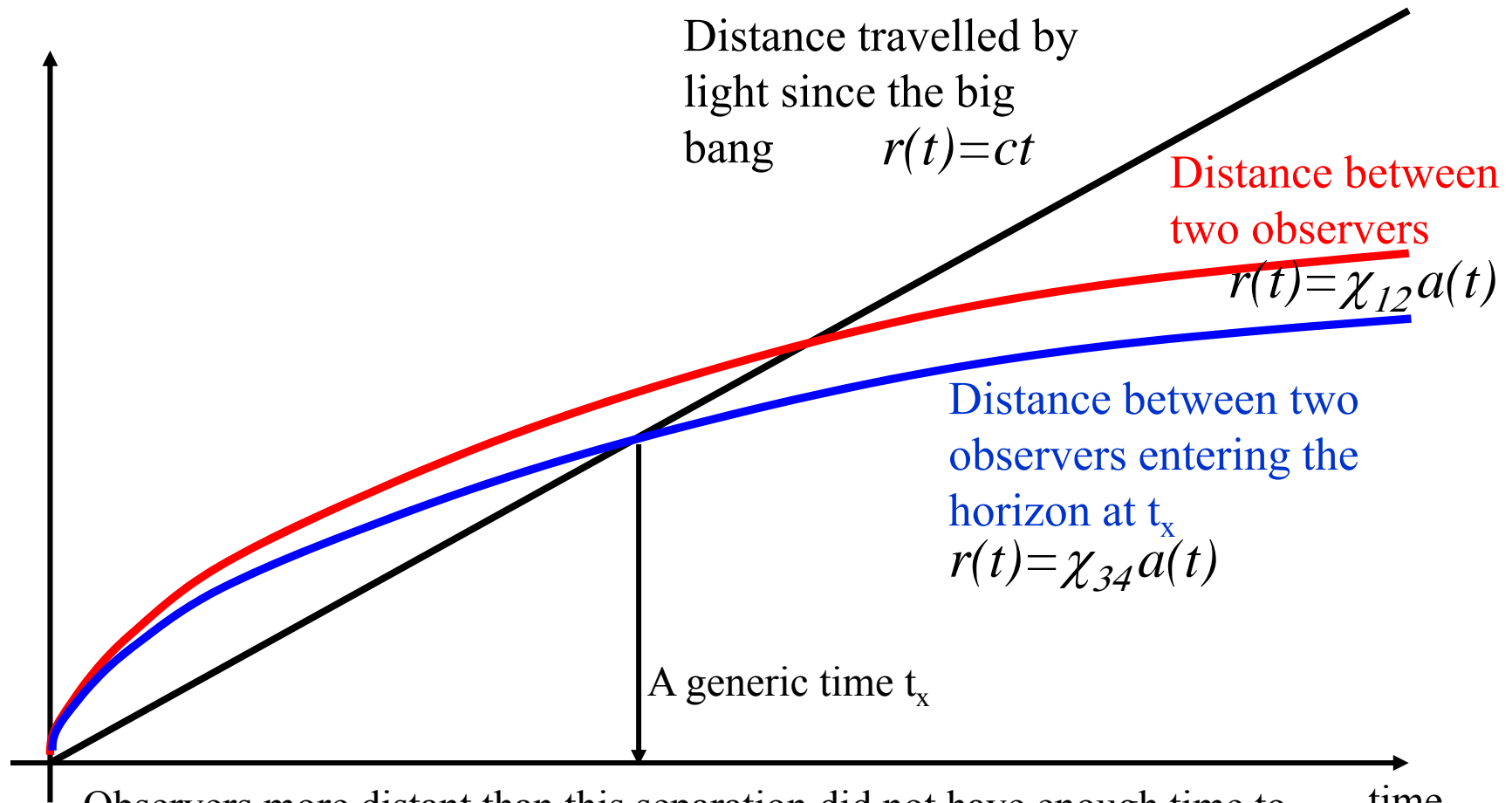


Horizons at recombination



Conversely, at any given time, there is a physical separation between observers which marks the causal horizon: observers more distant than this separation did not have enough time to exchange light signals. They are causally disconnected.

Horizons at recombination



Observers more distant than this separation did not have enough time to exchange light signals. They are causally disconnected.

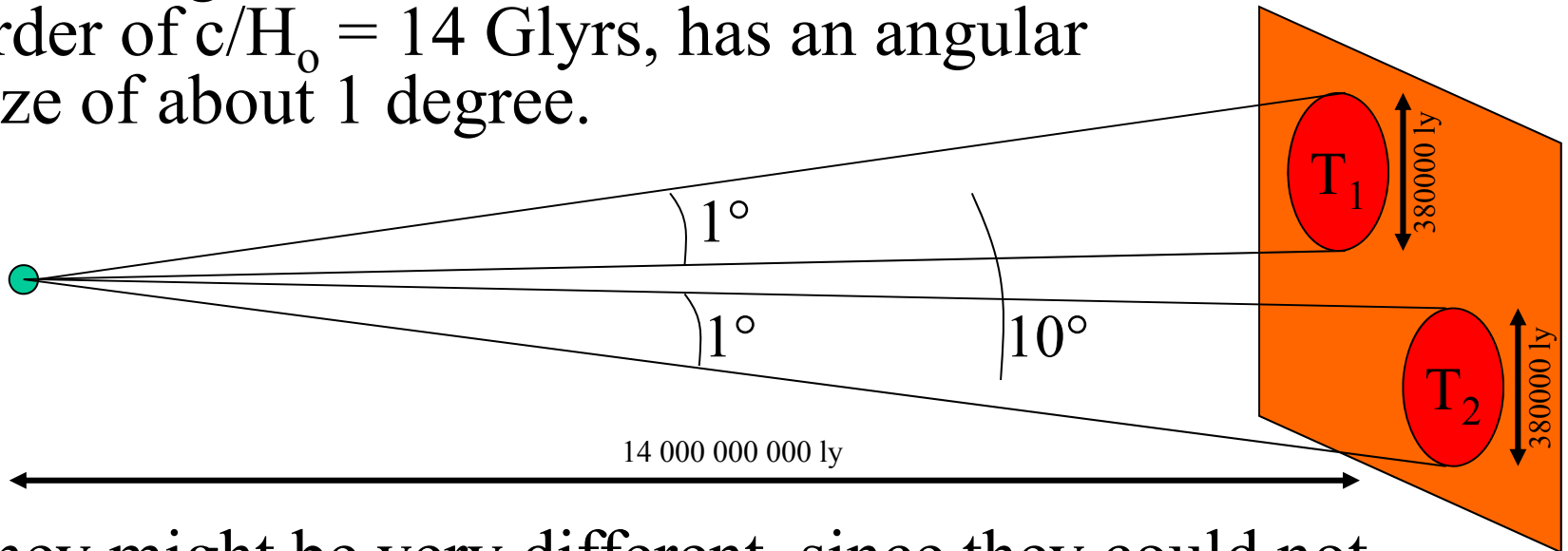
Observers closer than this separation have been in causal contact.

Horizons

- The physical phenomena happening within the causal horizon are different from the phenomena with scales larger than the horizon.
- Forces are transmitted at most at the speed of light, so phenomena outside the causal horizon are frozen until they enter the horizon.
- We should be able to *see* the effects of causal horizons, impressed in the *image* of the CMB.

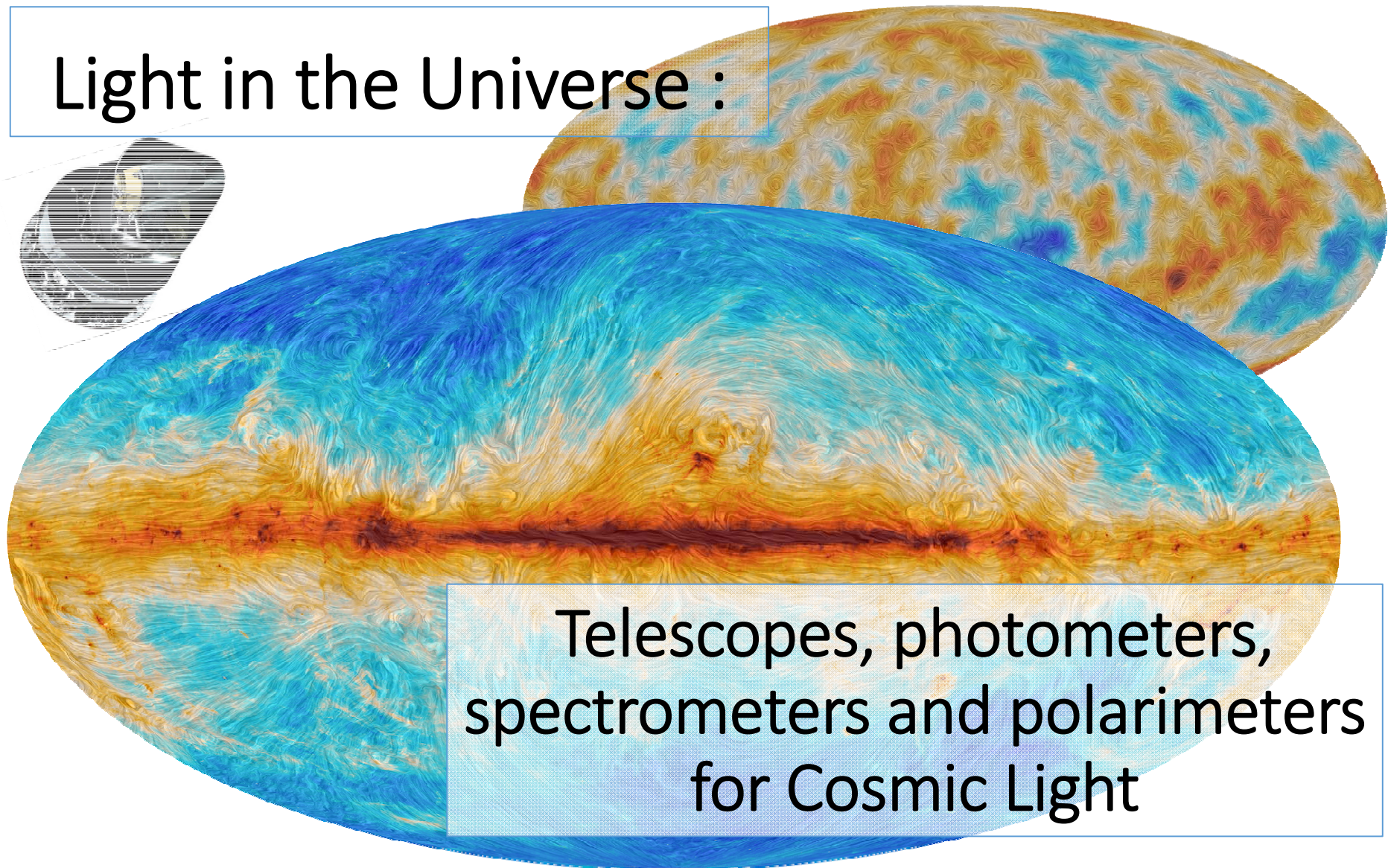
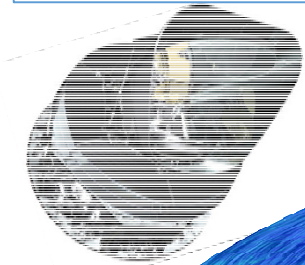
Horizons

- At recombination ($t=380000$ years), only regions of the Universe closer than 380000 light years have had the possibility (enough time) to interact.
- That length, as seen from a distance of the order of $c/H_0 = 14$ Glyrs, has an angular size of about 1 degree.



- They might be very different, since they could not interact during all the previous history of the Universe, from the Big Bang to recombination !
- However, measurements show that this is not the case.
- CMB anisotropy measurements.

Light in the Universe :

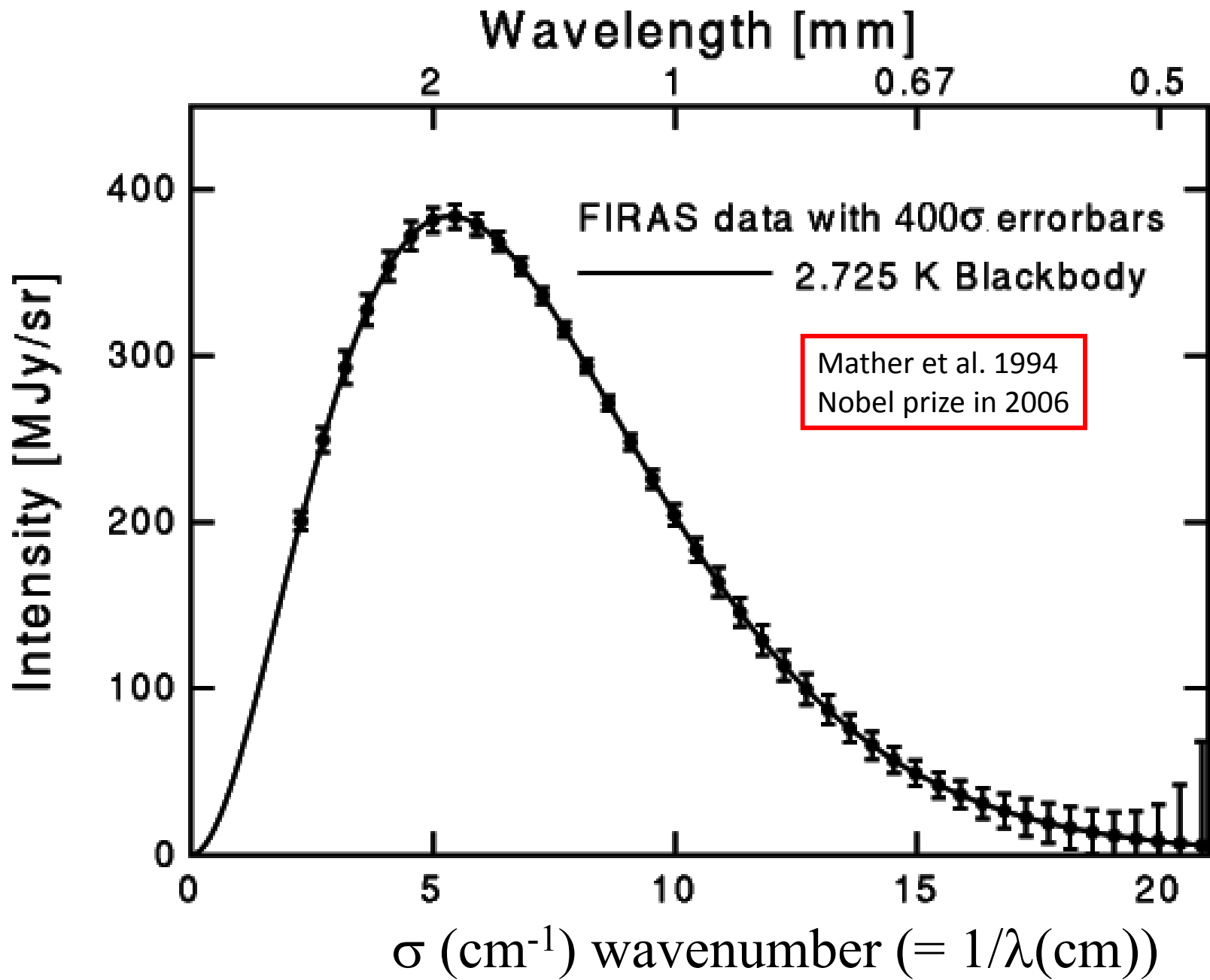


Telescopes, photometers,
spectrometers and polarimeters
for Cosmic Light

Paolo de Bernardis

Dipartimento di Fisica, Sapienza Università di Roma

Winter College on Optics 2015 - Trieste, 9-20 February



- The spectrum

$$B(\nu, T) = \frac{2h}{c^2} \frac{\nu^3}{e^x - 1}$$

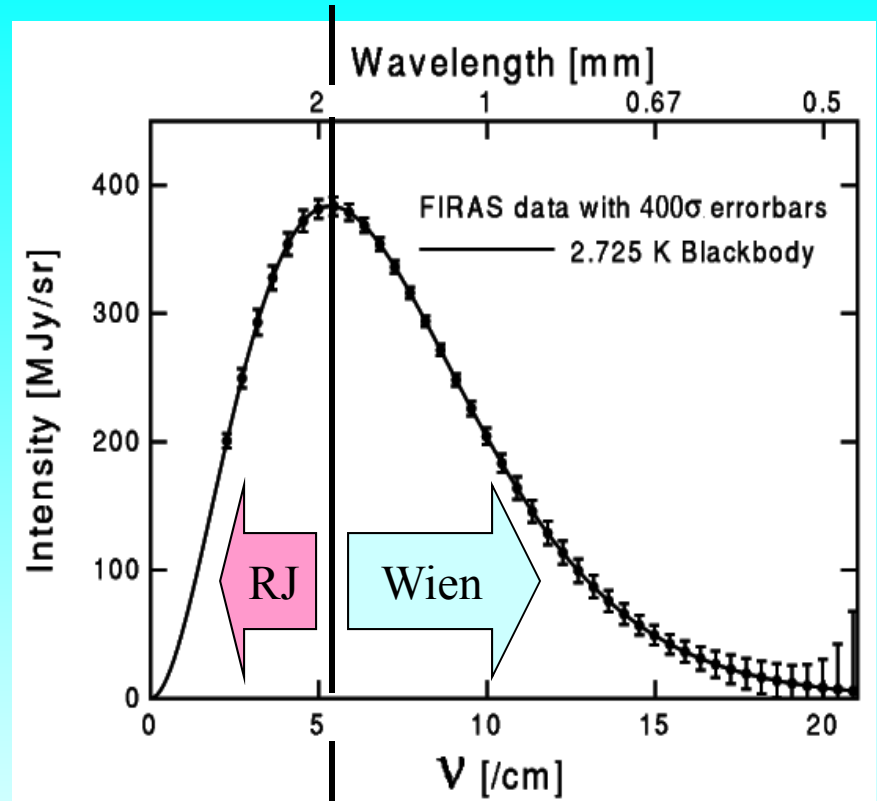
$$T_{CMB} = 2.725 K$$

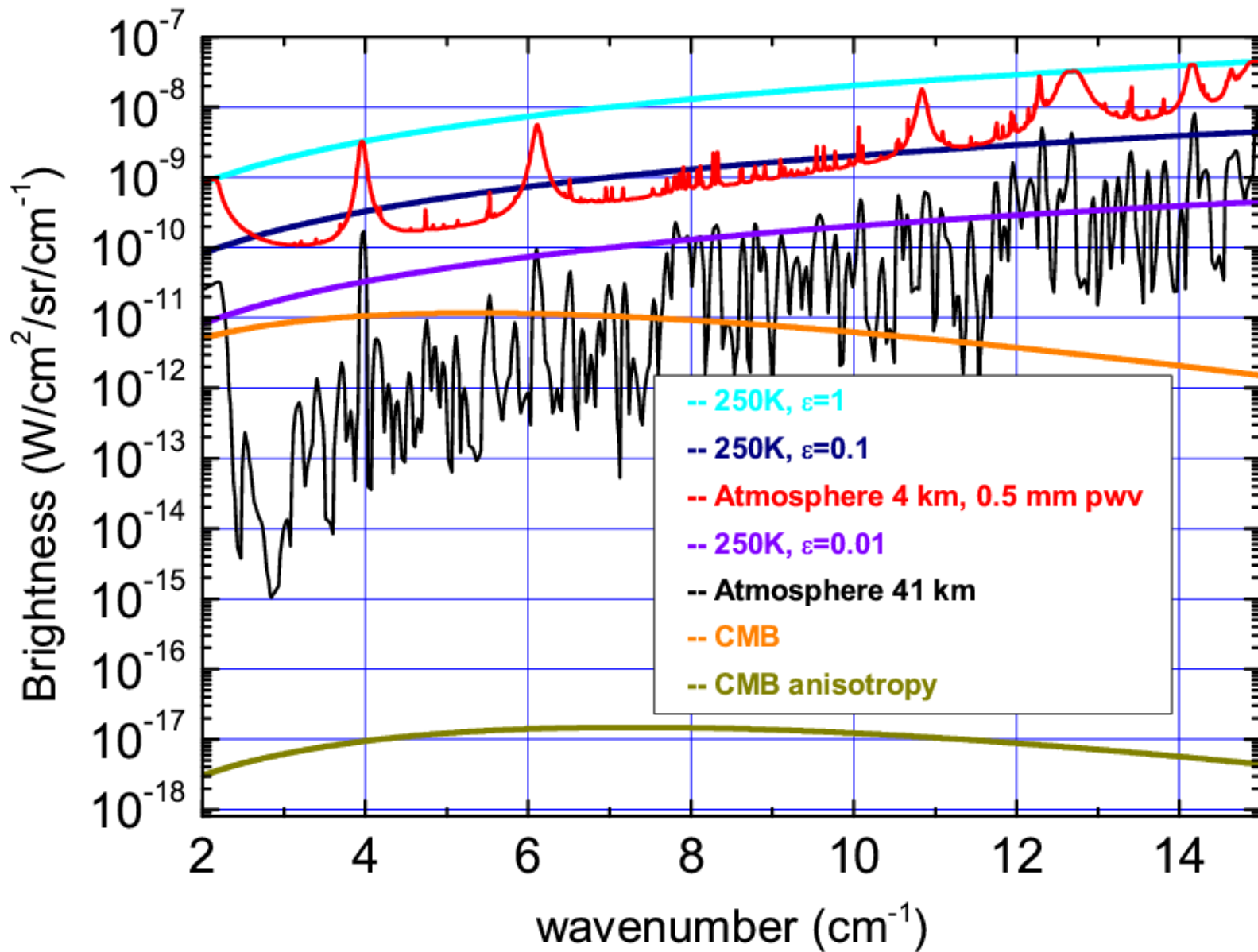
$$x_{CMB} = \frac{h\nu}{kT_{CMB}} \approx \frac{\nu}{56 \text{ GHz}}$$

$$1 - e^{-x_{\max}} = \frac{x_{\max}}{3} \Rightarrow x_{\max} = 2.82 \Rightarrow$$

$$\nu_{\max} = 159 \text{ GHz} \quad (\sigma_{\max} = 5.31 \text{ cm}^{-1})$$

$$B(\lambda, T) = \frac{\nu}{\lambda} B(\nu, T) \Rightarrow \lambda_{\max} = 1.06 \text{ mm}$$





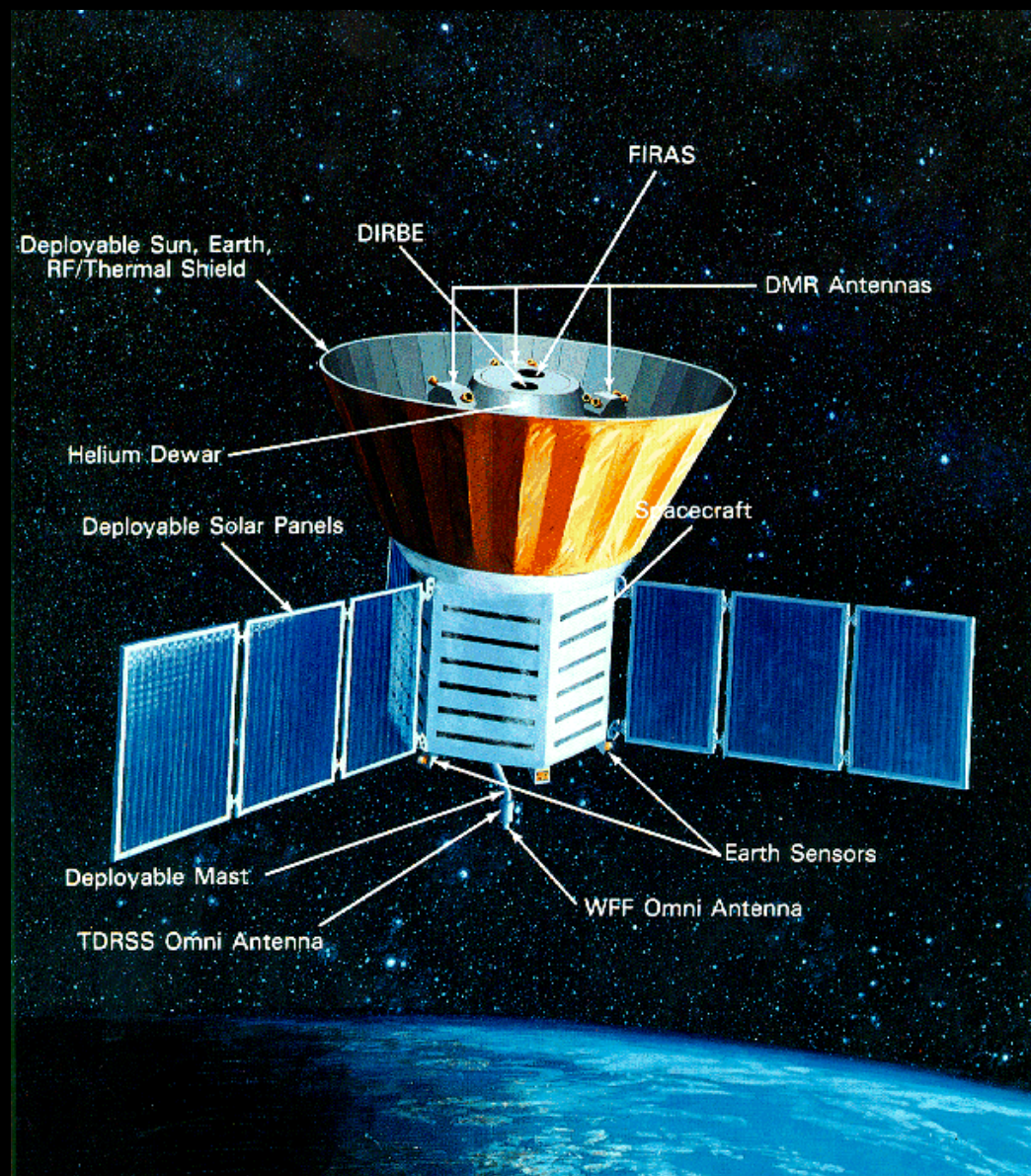
Environment !

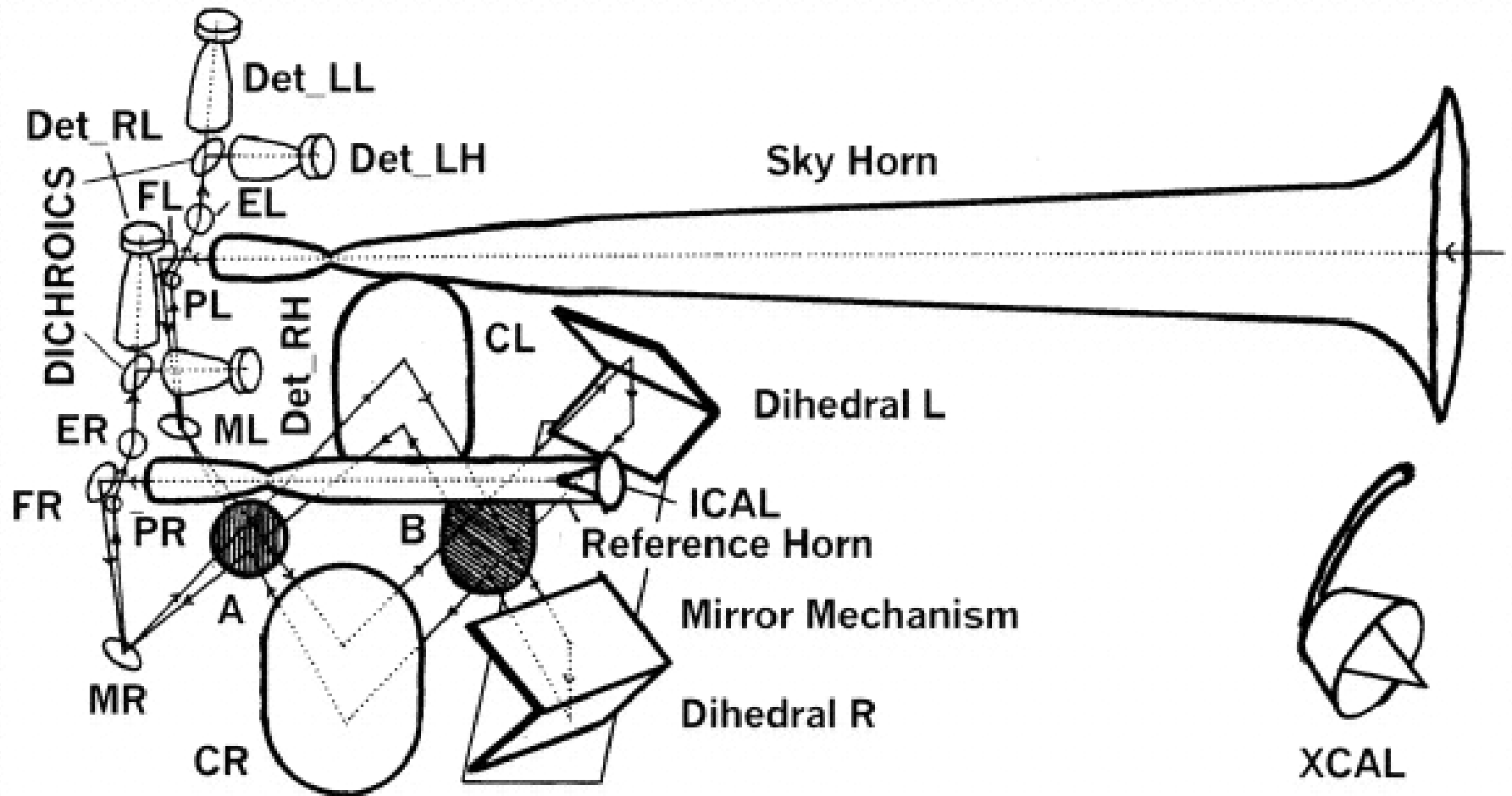


Space : to remove atmospheric emission
 Cryogenics : to limit instrumental emission

CMB spectrum measurements: COBE-FIRAS

- COBE-FIRAS was a cryogenic Martin-Puplett Fourier-Transform Spectrometer with composite bolometers.
- It was placed in a 400 km orbit, above the earth atmosphere.
- A *null* instrument comparing the specific sky brightness to the brightness of a cryogenic Blackbody





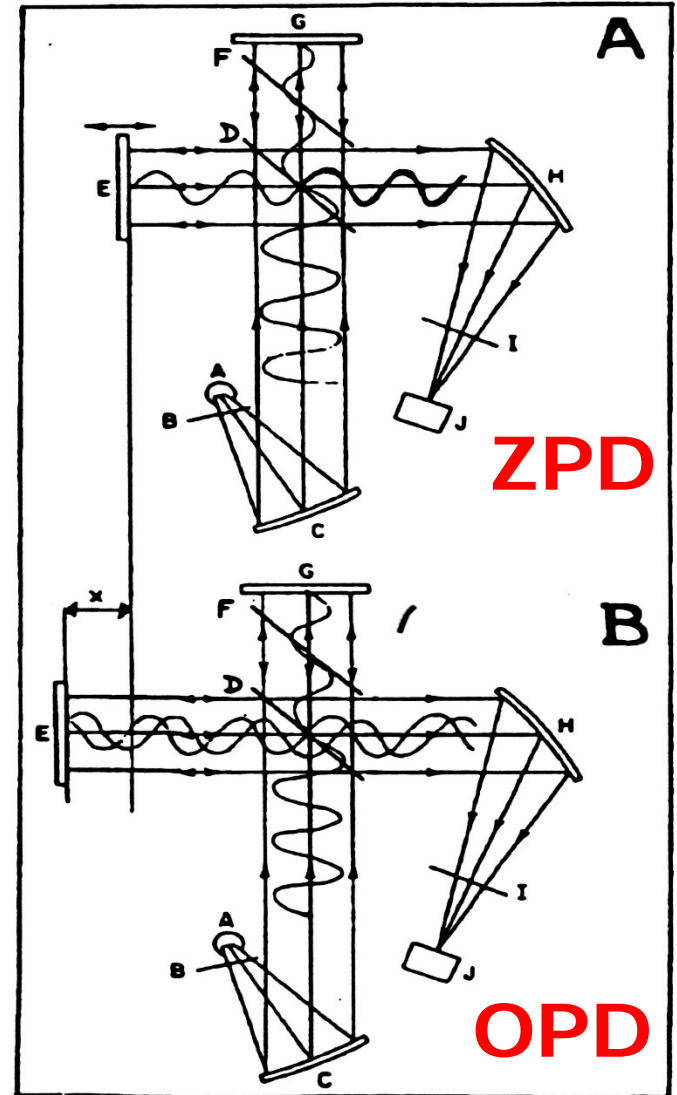
All this is cooled at 2K (-271°C)

Fourier Transform Spectrometers (FTS)

- To measure spectra, you use interference (prism, grating, FP ...)
- In the case of the FTS only 2 light beams interfere: this is the simplest experimental configuration, but results in a complex encoding of the spectrum.

Recipe for a FTS

- Take the beam to be analyzed (A), transform it into a quasi-parallel beam (C), and split it in two beams (D).
- Delay one of the two beams (E), driving it along a longer optical path (x).
- Recombine the beams on the detector (H and J), and record the detected power vs. the optical path difference (this is called the interferogram).
- The interferogram is the Fourier transform of the spectrum of the incoming radiation (as shown below).

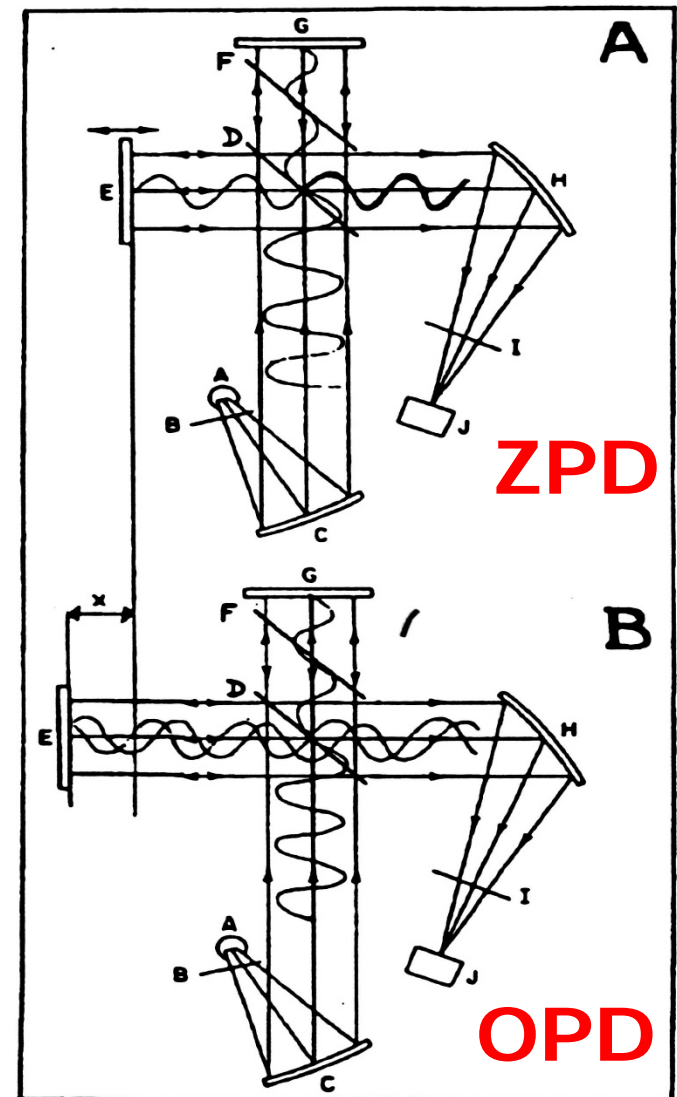


Elementary theory of the FTS

- The OPD (optical path difference) is $2x$.
- For a perfectly monochromatic radiation with wavenumber σ ($=1/\lambda$) the resulting field on the detector will be

$$E(t) = E_o(\sigma)RT(\sigma)\cos(2\pi\sigma t) + E_o(\sigma)RT(\sigma)\cos(2\pi\sigma t + 4\pi\sigma x)$$

- Here RT is the efficiency of the beamsplitter (frequency dependent, in general)



Elementary theory of the FTS

$$E(t) = E_o(\sigma)RT(\sigma)\cos(2\pi\sigma t) + E_o(\sigma)RT(\sigma)\cos(2\pi\sigma t + 4\pi\sigma x)$$

- The power on the detector $I(x)$ will be proportional to the mean square electrical field:

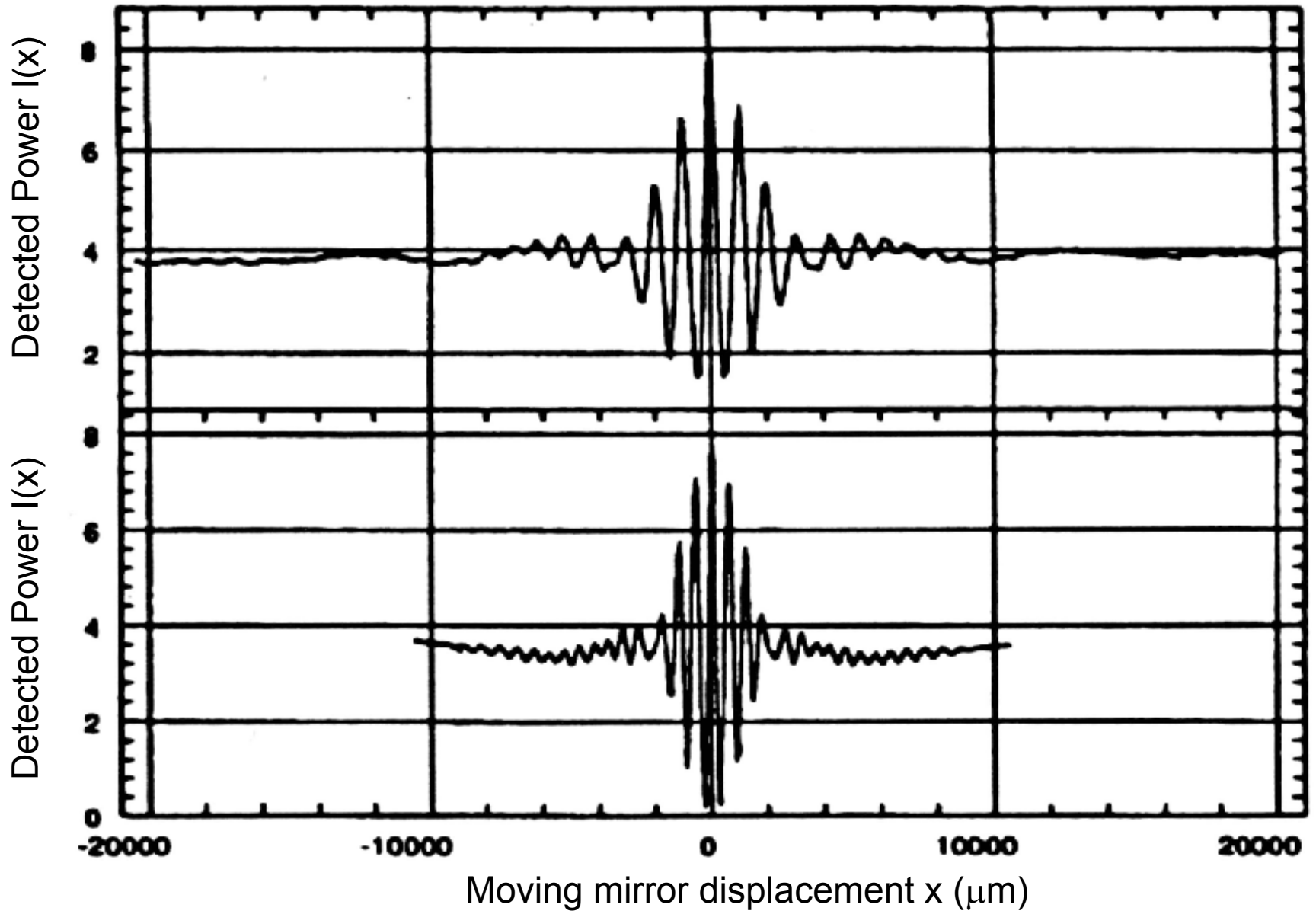
$$\begin{aligned} I(x) &\propto \langle E(t)^2 \rangle = \langle E(t)E^*(t) \rangle = \\ &= E_o^2(\sigma)rt(\sigma) \left[e^{i2\pi\sigma t} + e^{i2\pi\sigma t} e^{i4\pi\sigma x} \right] \left[e^{-i2\pi\sigma t} + e^{-i2\pi\sigma t} e^{-i4\pi\sigma x} \right] = \\ &= E_o^2(\sigma)rt(\sigma) \left[1 + e^{-i4\pi\sigma x} + e^{i4\pi\sigma x} + 1 \right] = E_o^2(\sigma)rt(\sigma)2(1 + \cos 4\pi\sigma x) \end{aligned}$$

- So the interferogram is $I(x) - \langle I \rangle = rt(\sigma)\cos(4\pi\sigma x)$
- If the input radiation is not monochromatic, and each wavenumber has amplitude $S(\sigma)$:

$$I(x) - \langle I \rangle = \int_0^{\infty} S(\sigma)rt(\sigma)\cos(4\pi\sigma x)d\sigma$$

The specific Brightness and the interferogram are related by a Fourier Transform

$$I(x) - \langle I \rangle = \int_0^{\infty} S(\sigma) \text{rt}(\sigma) \cos(4\pi\sigma x) d\sigma$$



Elementary theory of the FTS

$$S(\sigma)rt(\sigma) = \int_{-\infty}^{\infty} (I(x) - \langle I \rangle) \cos(4\pi\sigma x) dx$$

- For obvious reasons we cannot extend x to infinity ! If the maximum displacement of the moving mirror is x_{\max} , all we can do is to compute

$$S'(\sigma)rt(\sigma) = \int_{-x_{\max}}^{x_{\max}} (I(x) - \langle I \rangle) \cos(4\pi\sigma x) dx$$

- S' is an approximation of the real spectrum S
- The main difference is in the effective spectral resolution of the spectrometer, which for S' is limited to approx. $1/(2x_{\max})$.

Spectral Resolution

- Consider a monochromatic line with wavenumber σ_o : the interferogram is

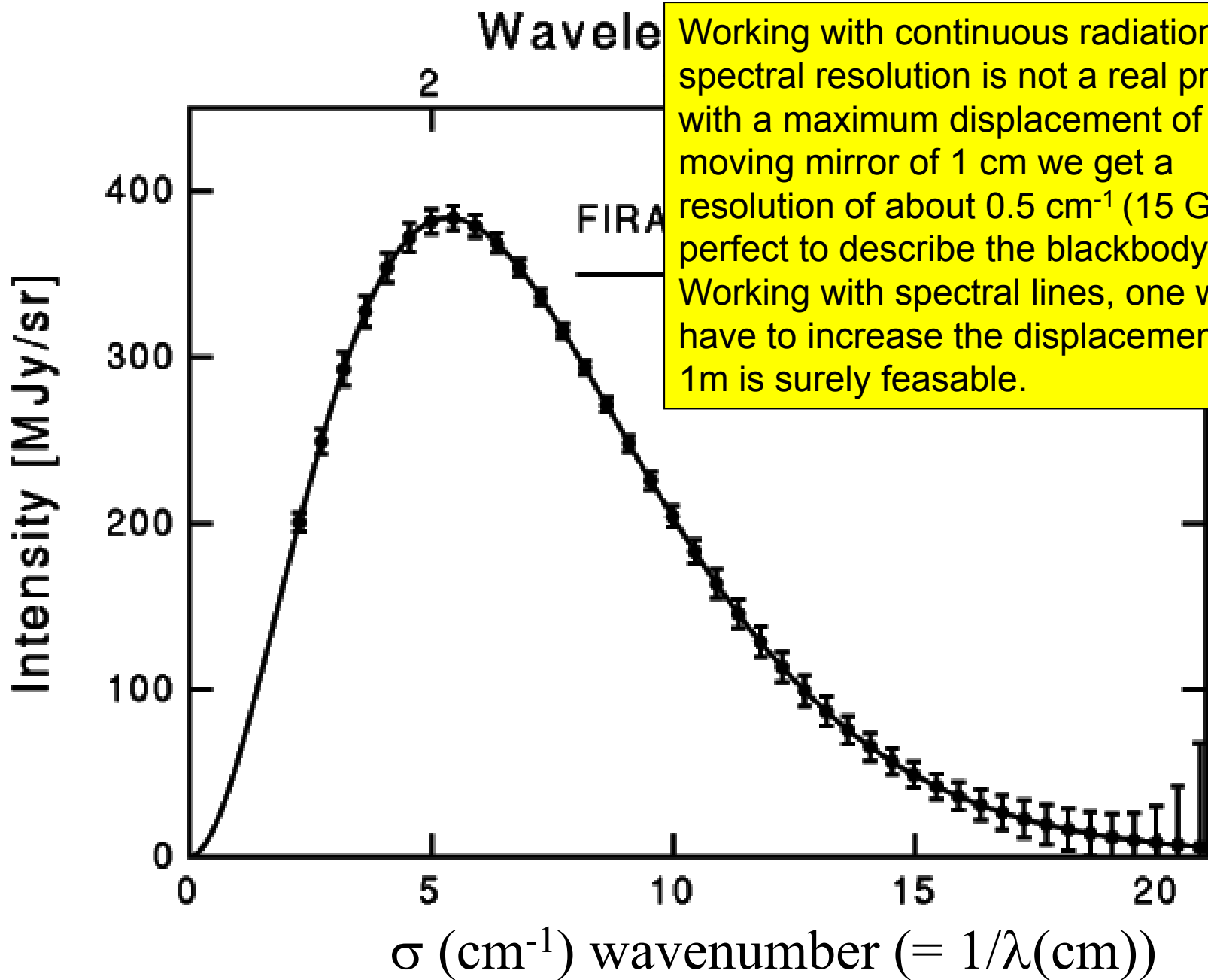
$$I(x) - \langle I \rangle = I_o \cos(4\pi\sigma_o x)$$

- The Fourier integral, limited to $\pm x_{\max}$, is:

$$S'(\sigma) = I_o \int_{-x_{\max}}^{x_{\max}} \cos(4\pi\sigma_o x) \cos(4\pi\sigma x) dx \quad \Rightarrow$$

$$S'(\sigma) = I_o x_{\max} \frac{\sin 4\pi(\sigma - \sigma_o)x_{\max}}{4\pi(\sigma - \sigma_o)x_{\max}}$$

- This is an approximation of the real $S(\sigma)$ which would be a delta function centered in σ_o : in place of a delta, we get a sinc, with a half-width approx. $1.23/(2x_{\max})$.



Working with continuous radiation, low spectral resolution is not a real problem: with a maximum displacement of the moving mirror of 1 cm we get a resolution of about 0.5 cm⁻¹ (15 GHz), perfect to describe the blackbody curve. Working with spectral lines, one would have to increase the displacement. 1m is surely feasible.

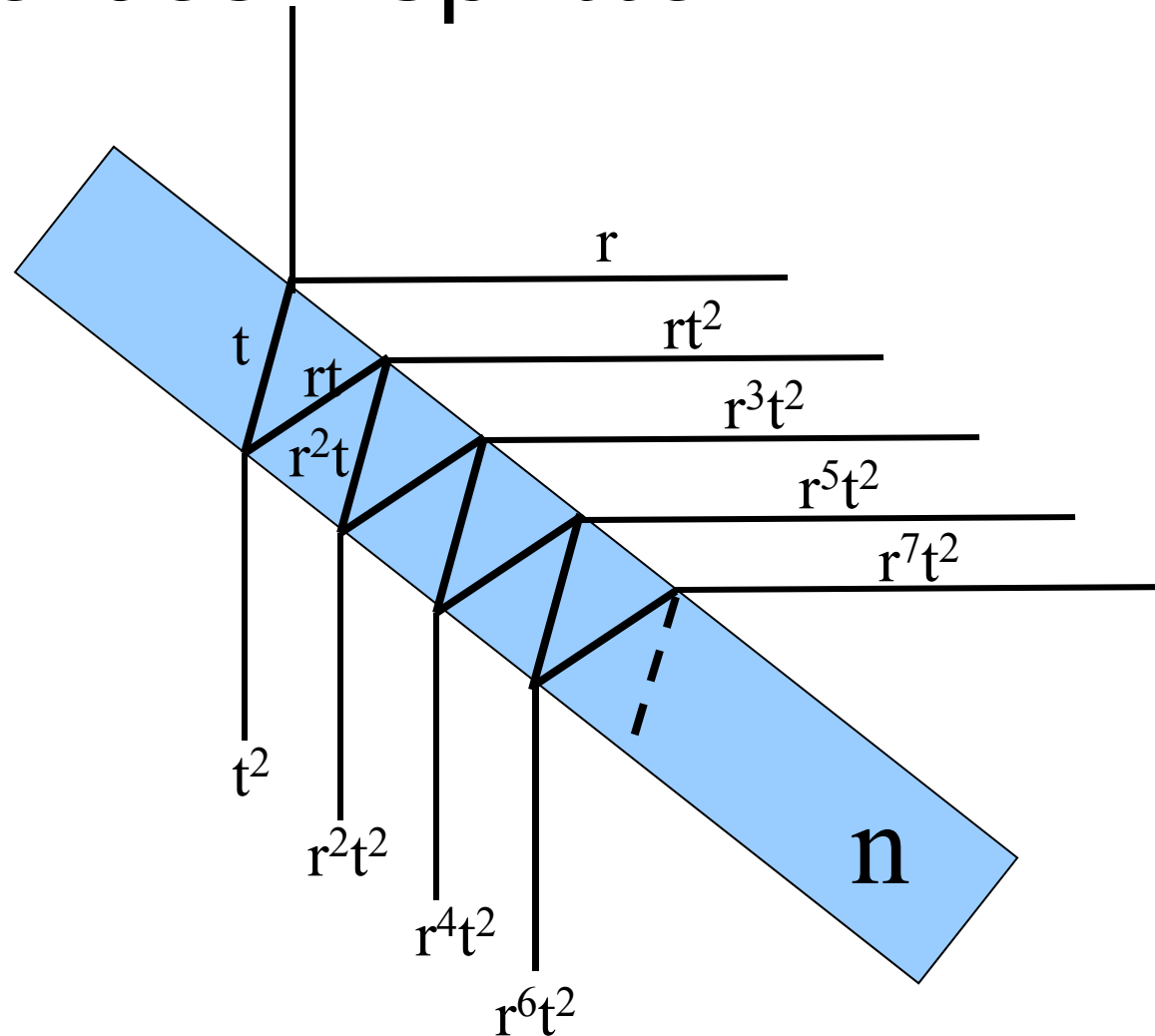
Beamsplitter problems

$$S'(\sigma) \text{rt}(\sigma) = \int_{-x_{\max}}^{x_{\max}} (I(x) - \langle I \rangle) \cos(4\pi\sigma x) dx$$

- What we get is the input spectrum times the efficiency of the beamsplitter.
- If the latter goes to zero, we cannot retrieve the spectrum.
- So we need good beamplitters, ideally with $rt=0.25$, independent on frequency.

the beamsplitter

- The simplest beamsplitter is a dielectric slab, with refraction index n and thickness t .
- Due to multiple reflections inside the slab, the transmitted and reflected fields can be computed as the sum of an infinite number of components with decreasing amplitude (a converging series) and increasing phase delay.



$$\delta = 4\pi n d \cos \theta' \sigma$$

$$E = E_o (-r \cos 2\pi\sigma ct + rt^2 \cos(2\pi\sigma ct + \delta) + r^3t^2 \cos(2\pi\sigma ct + 2\delta) + r^5t^2 \cos(2\pi\sigma ct + 3\delta) + \dots)$$

From this, the efficiency $rt(\sigma)$ is computed

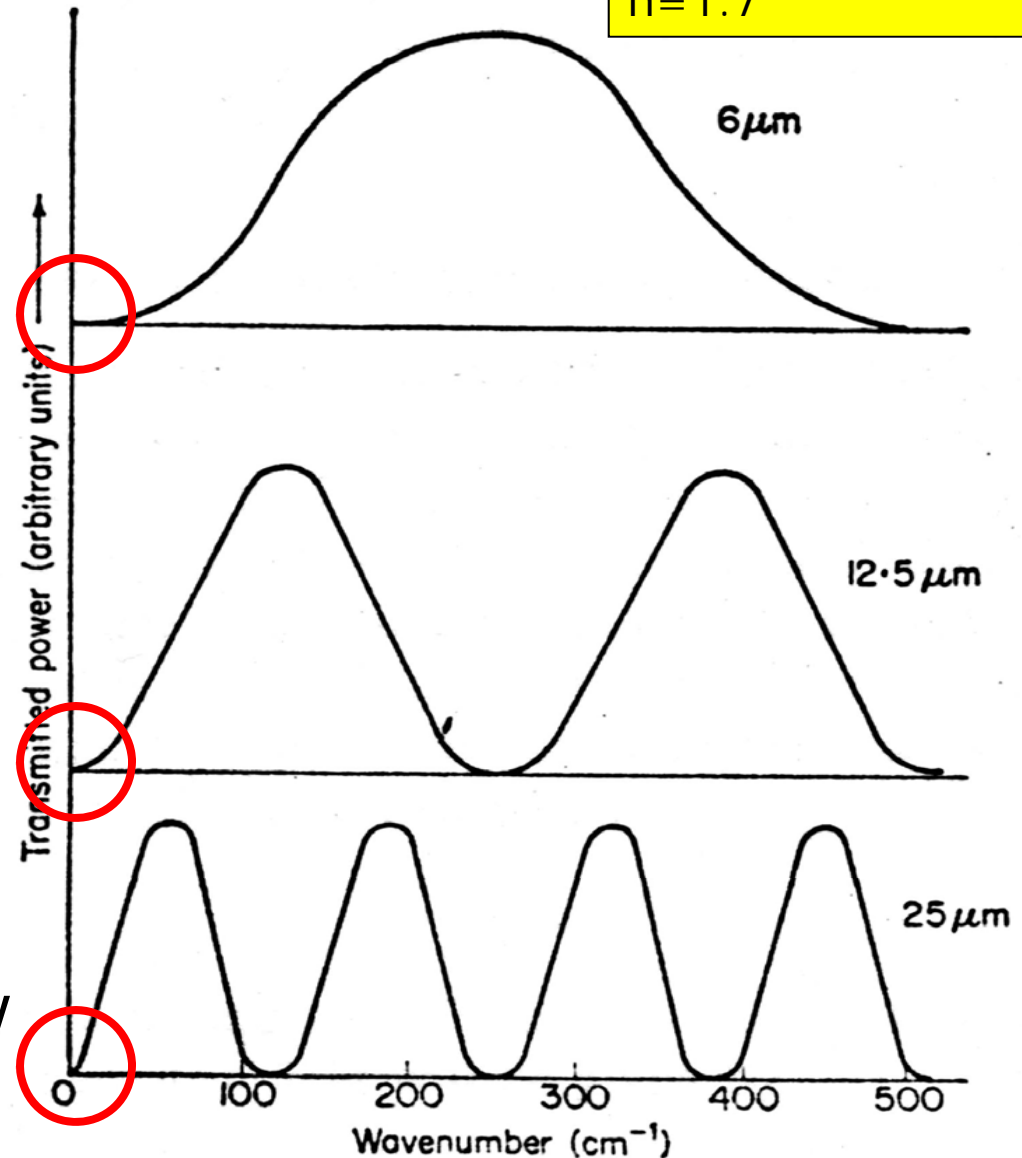
the beamsplitter

Polyethylene
Terephthalate
(mylar or melinex)
 $n=1.7$

- The efficiency is a periodic function with zeros at wavenumbers

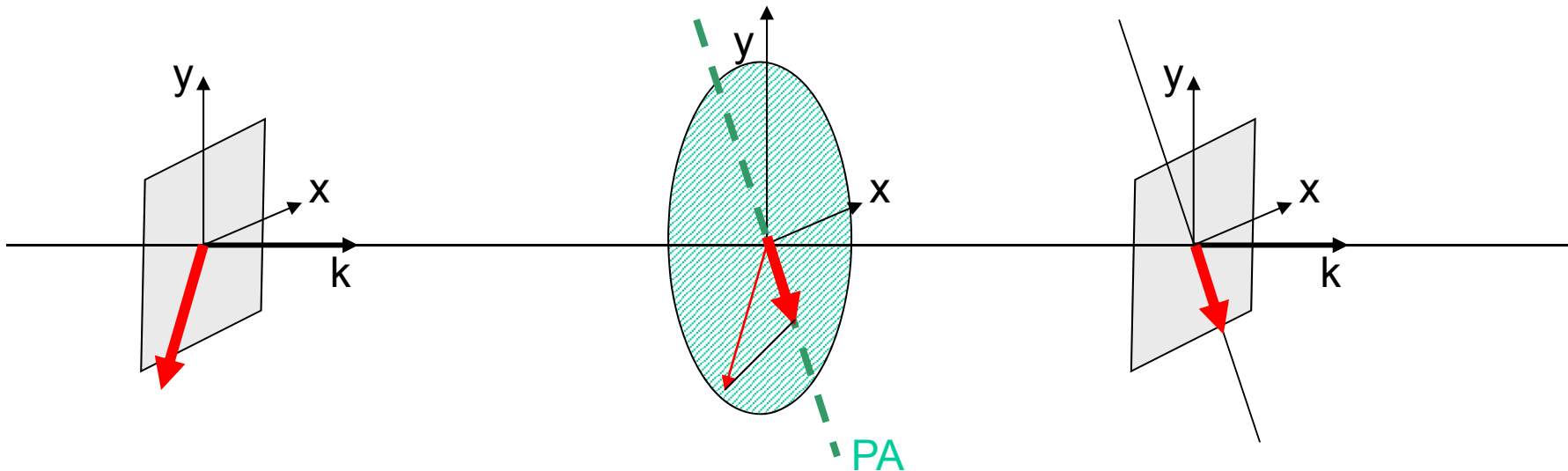
$$\sigma_m = \frac{m}{nd \cos \theta'}$$

- Whatever thickness and refraction index you select, this is not efficient at low frequencies.



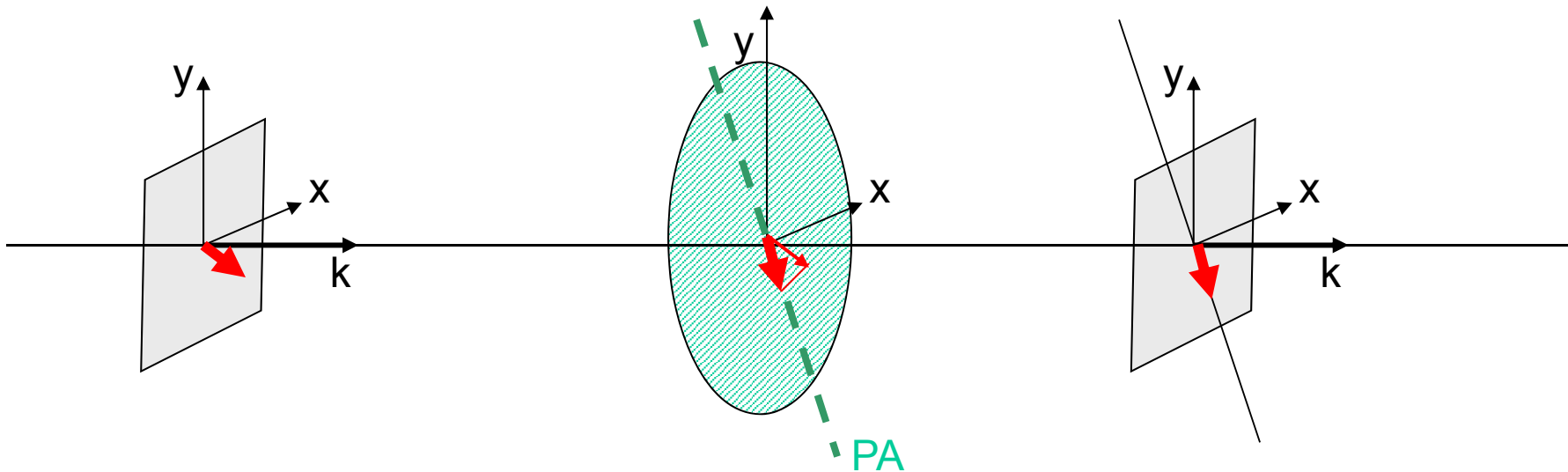
Linear Polarizers

- Linear polarizers can be used as high efficiency achromatic beamsplitters at long wavelengths.
- A linear polarizer is an optical device transmitting only the projection of the E field of the EM wave parallel to a given direction, which is called the principal axis of the polarizer.
- Unpolarized radiation (where the E field direction in the wavefront is random) is transformed into linearly polarized radiation (where the E field direction is constant) when crossing a polarizer.



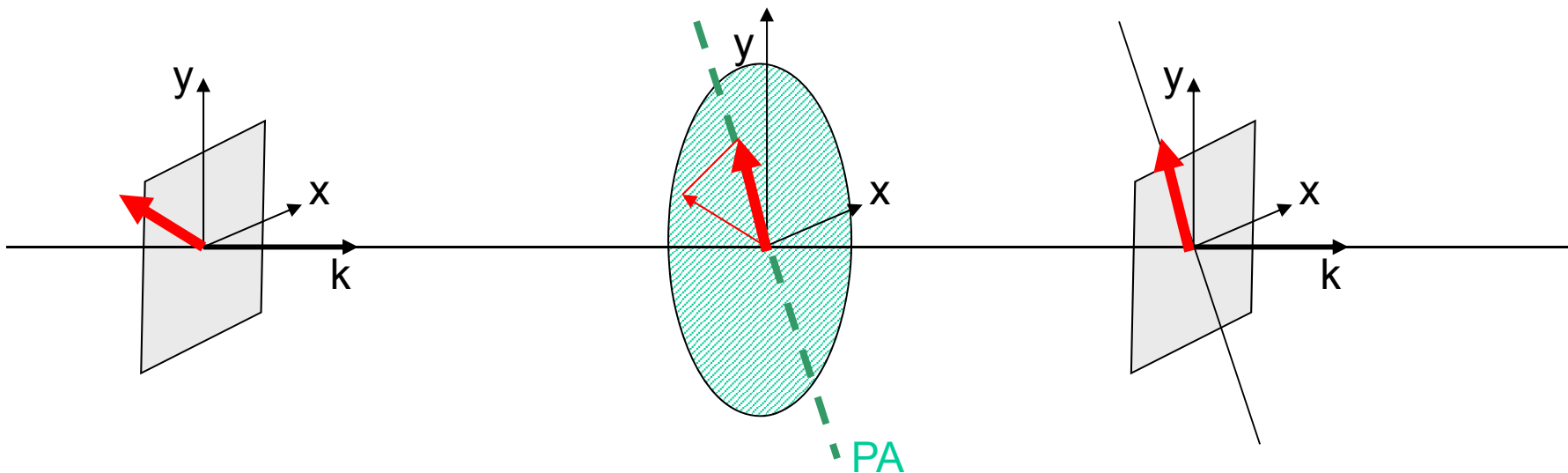
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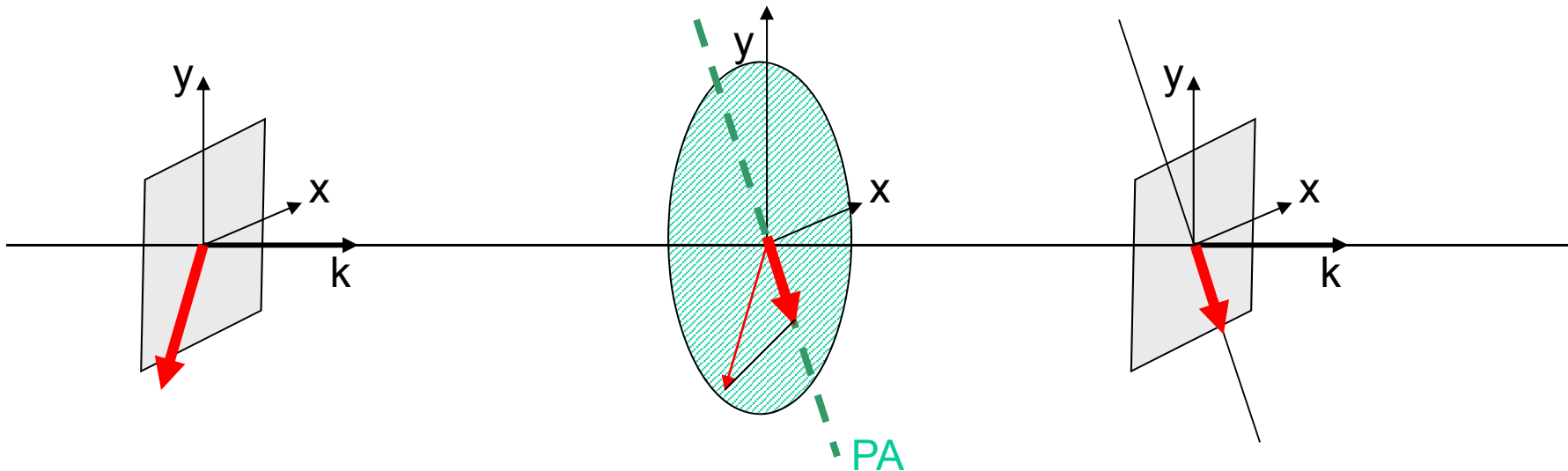
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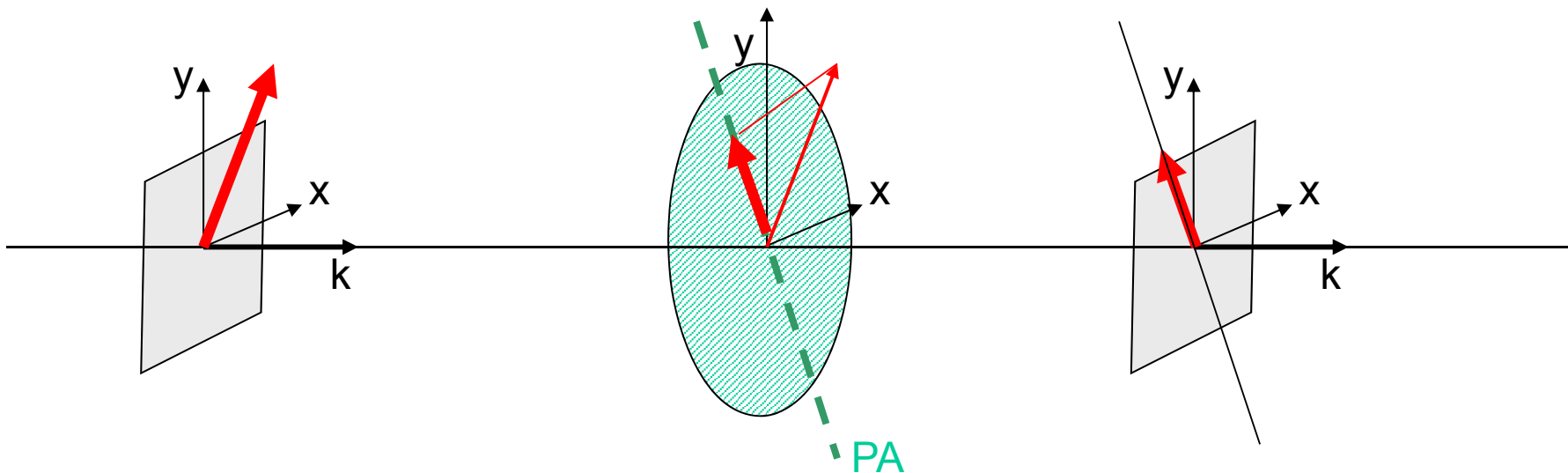
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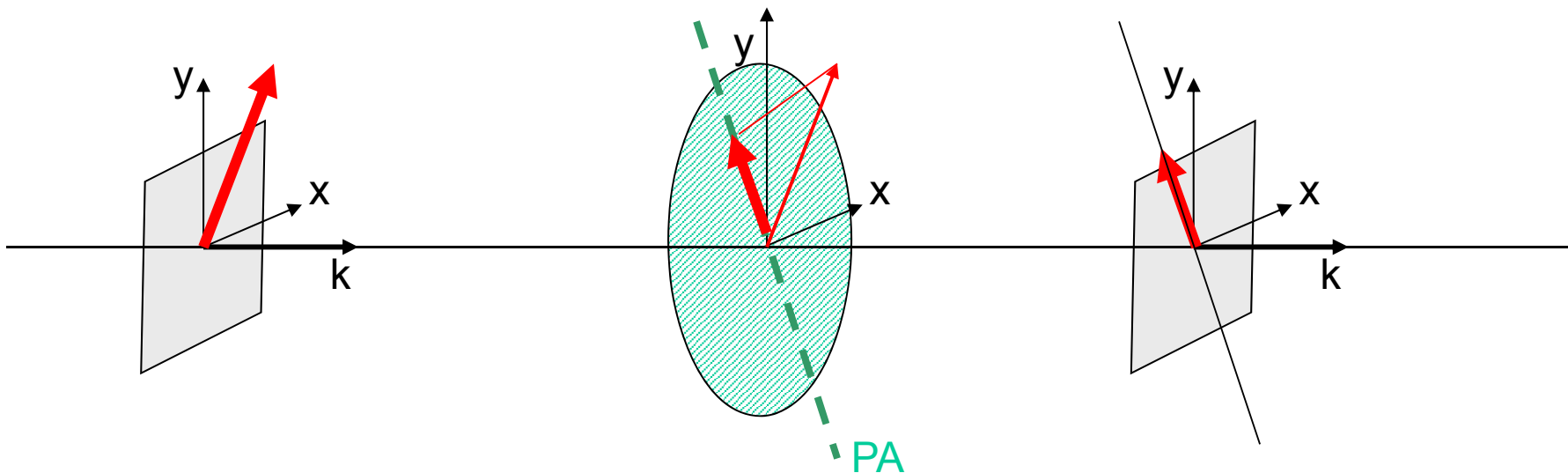
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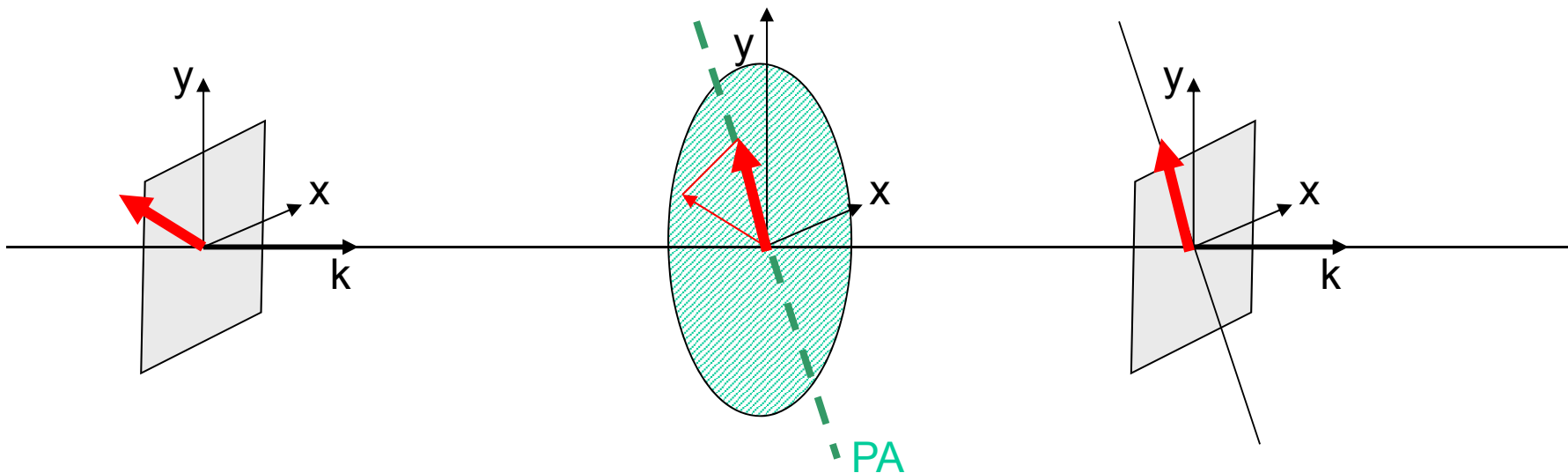
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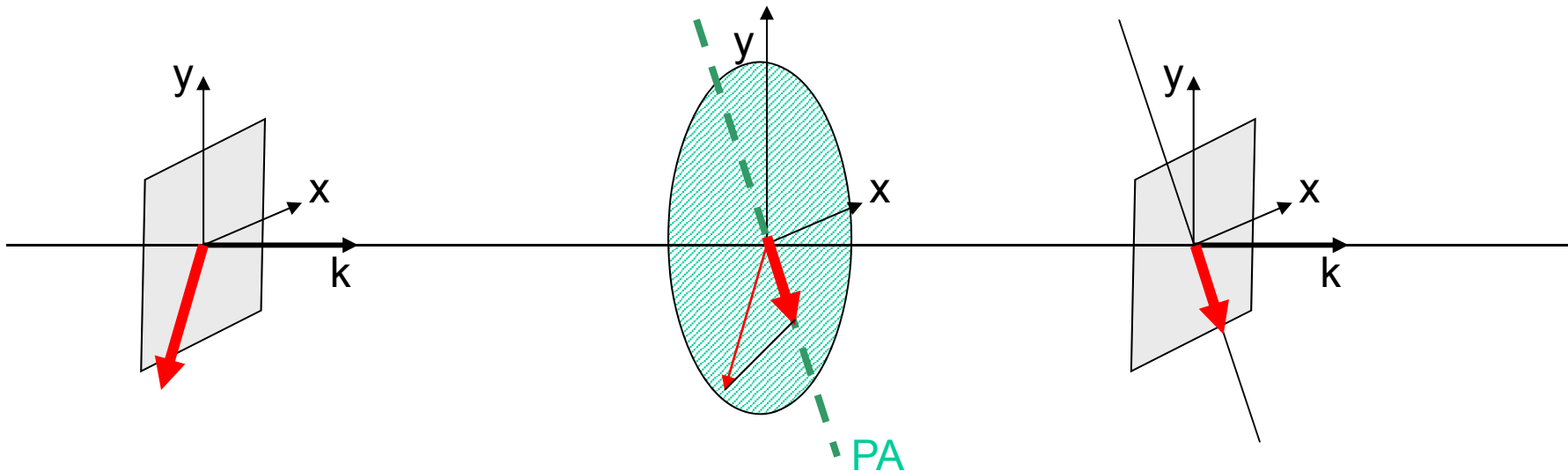
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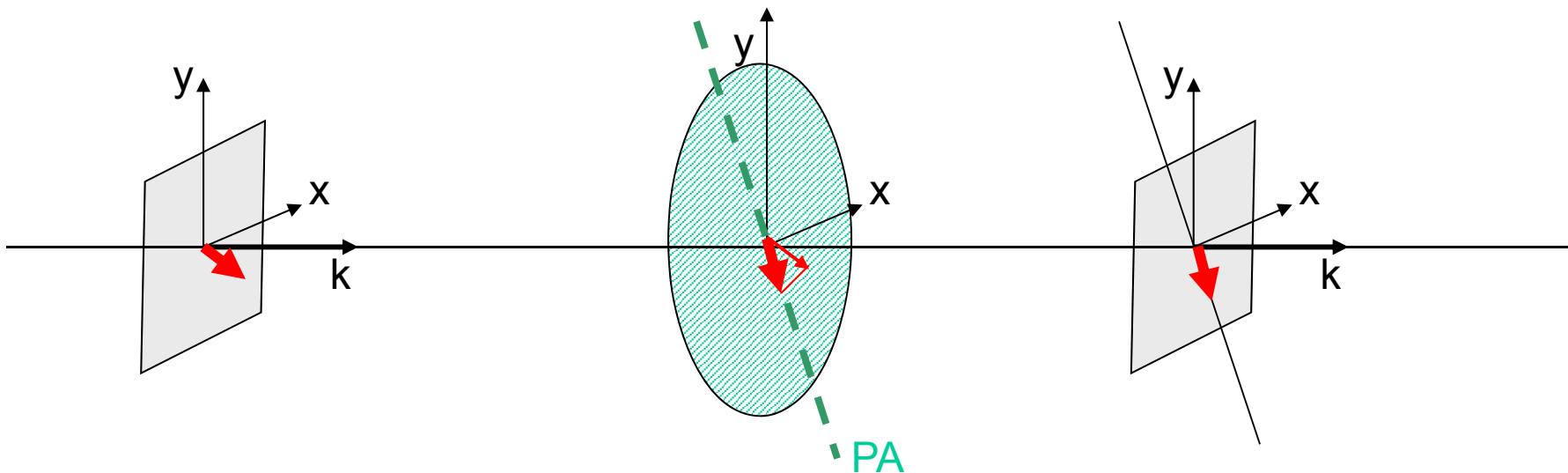
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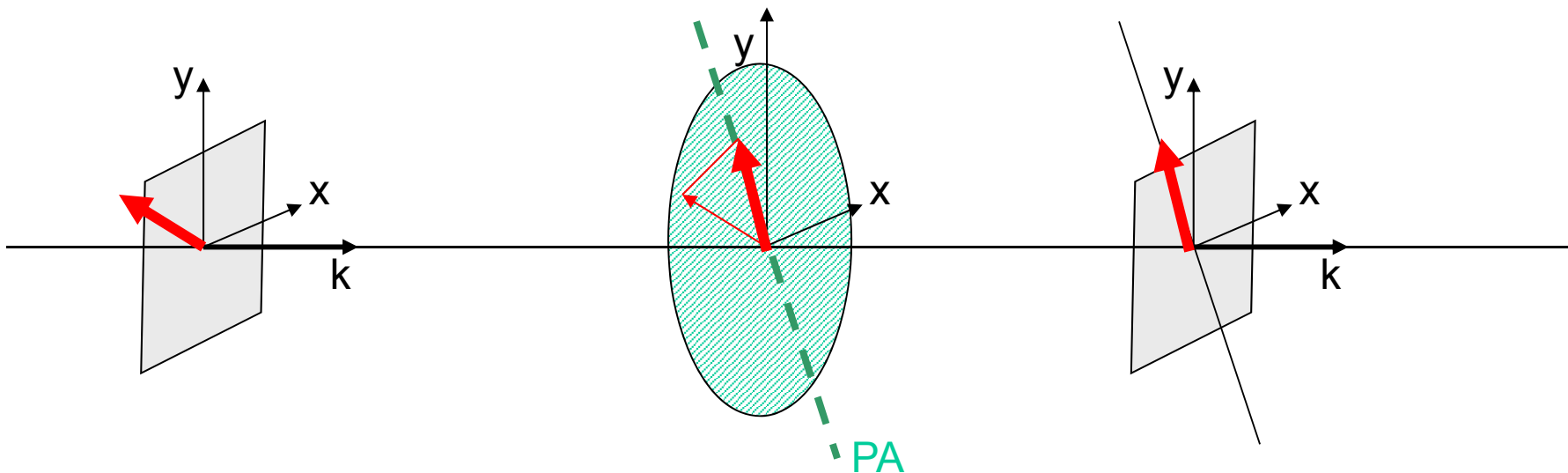
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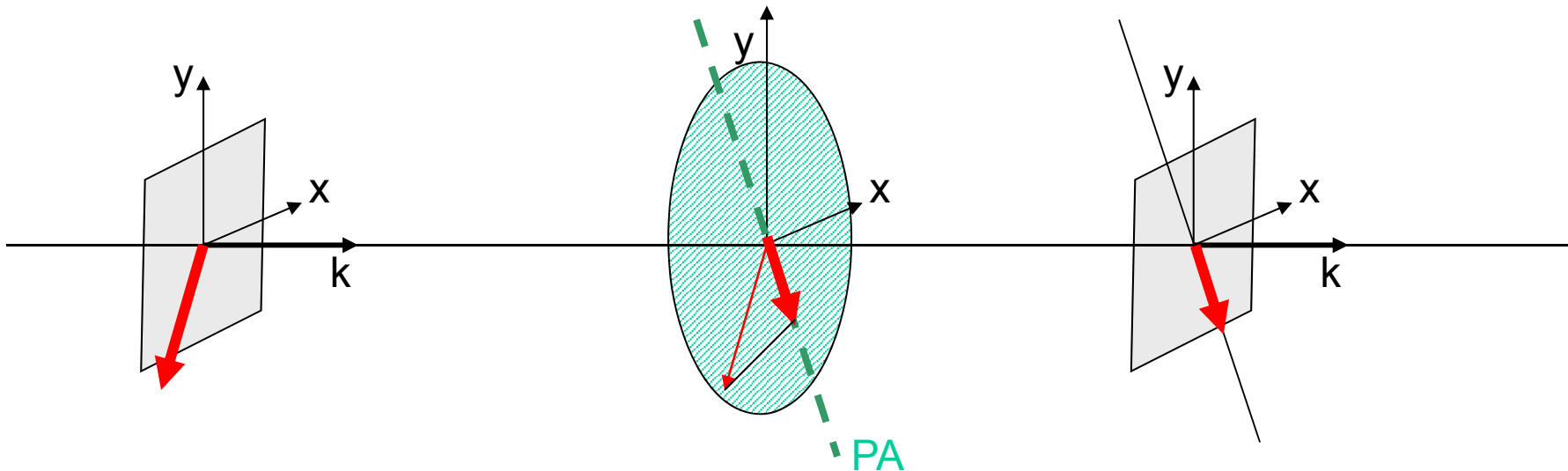
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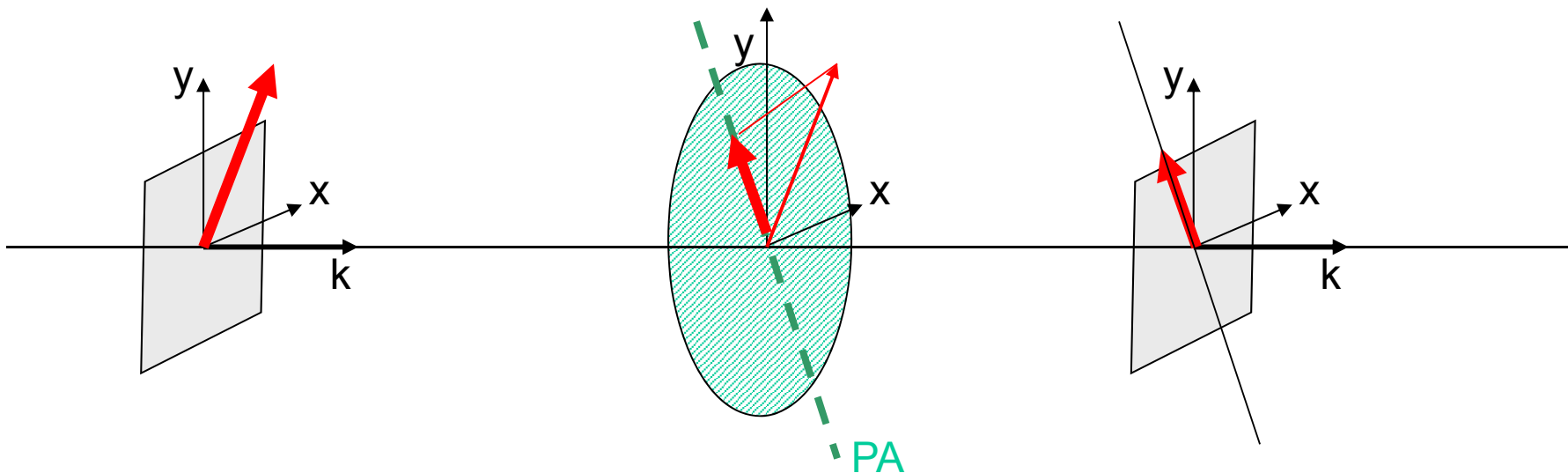
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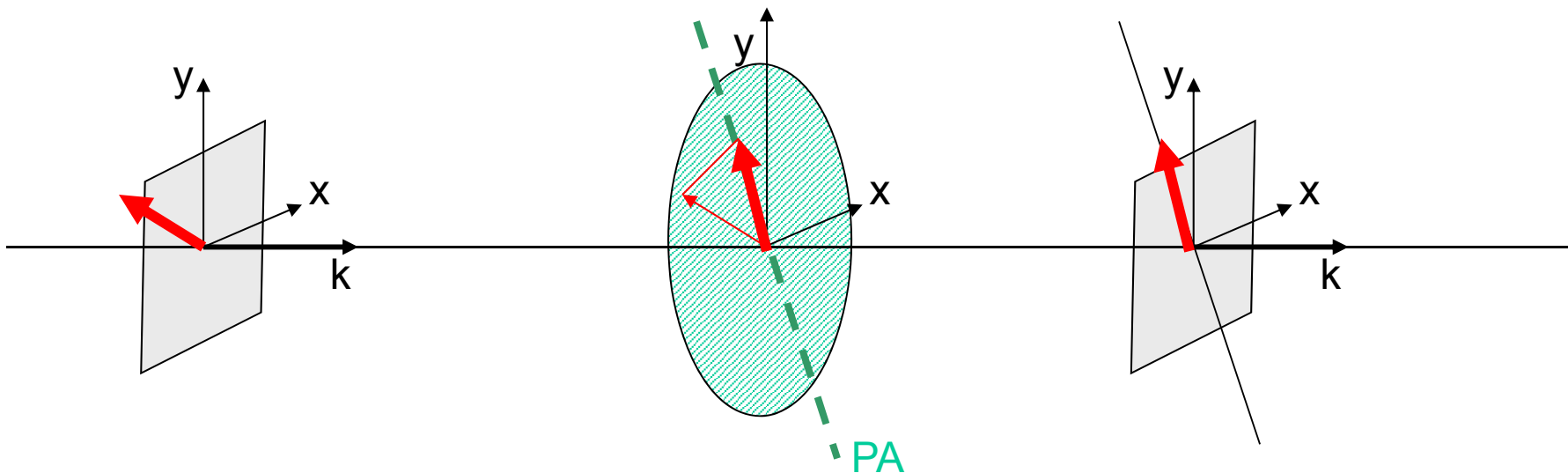
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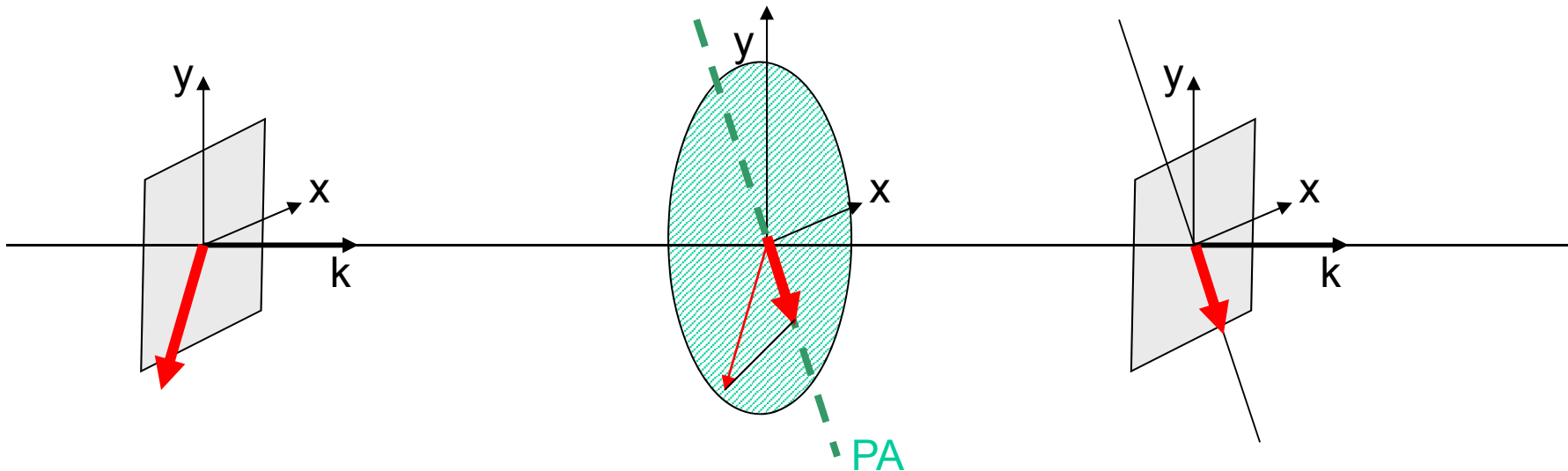
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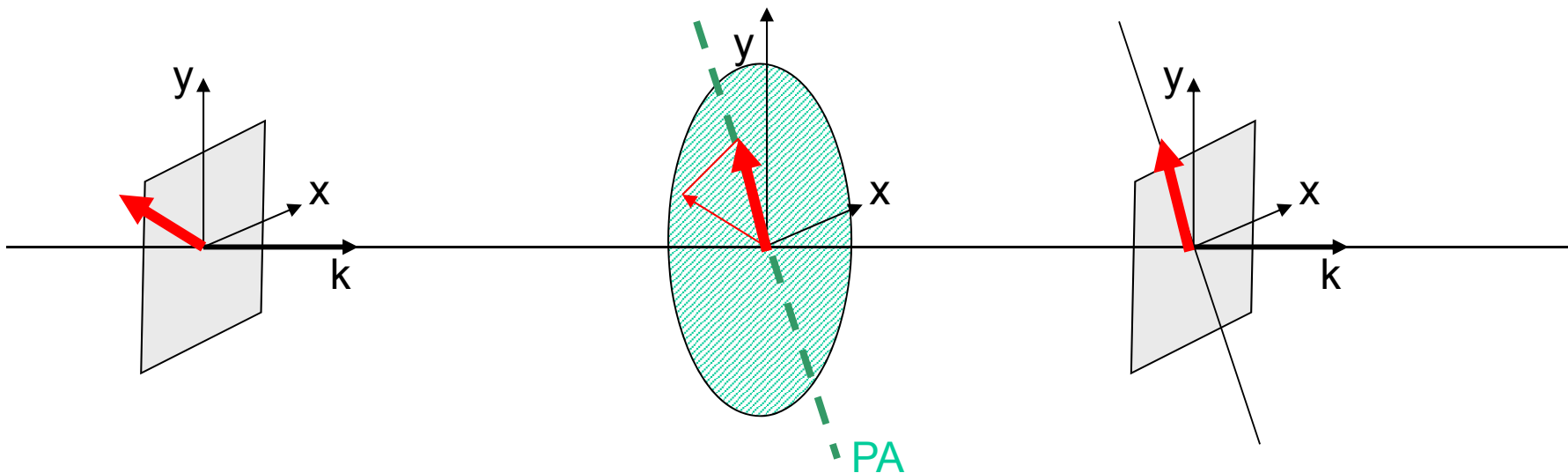
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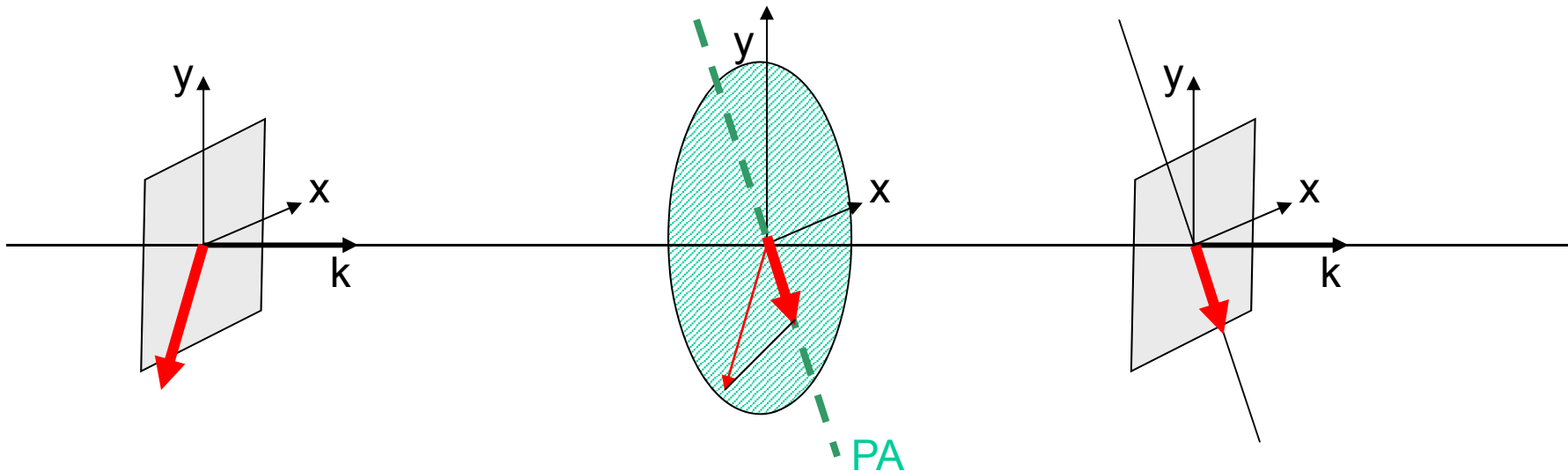
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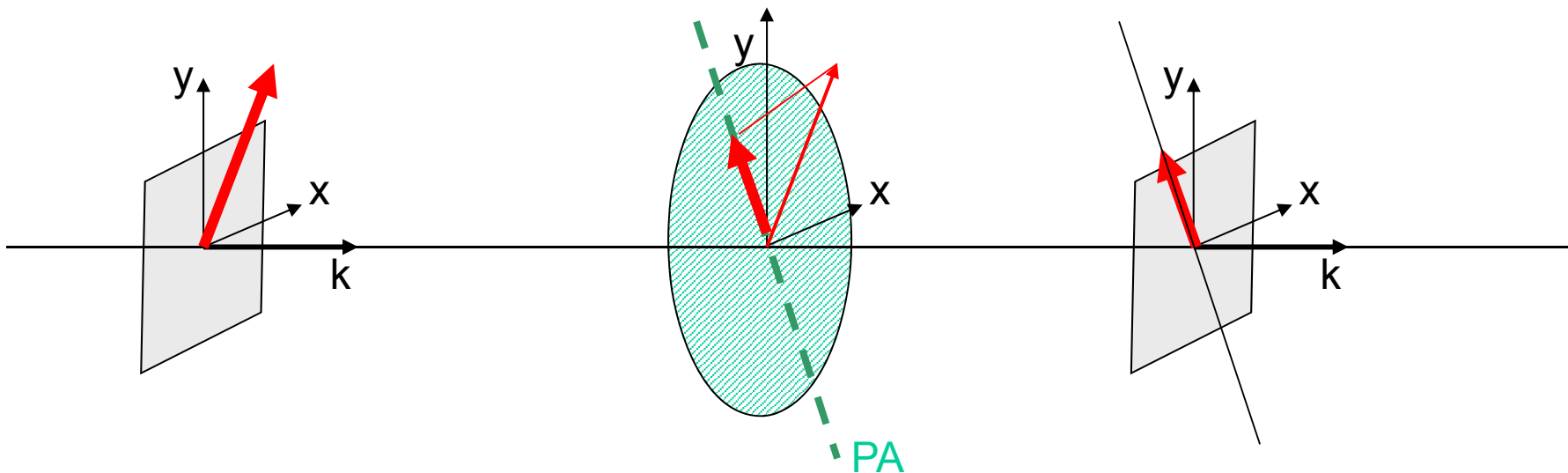
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Linear Polarizers

- The transmitted field E' can be computed projecting the incoming field E along the Principal Axis:

$$E_{par} = E_x \cos \phi + E_y \sin \phi$$

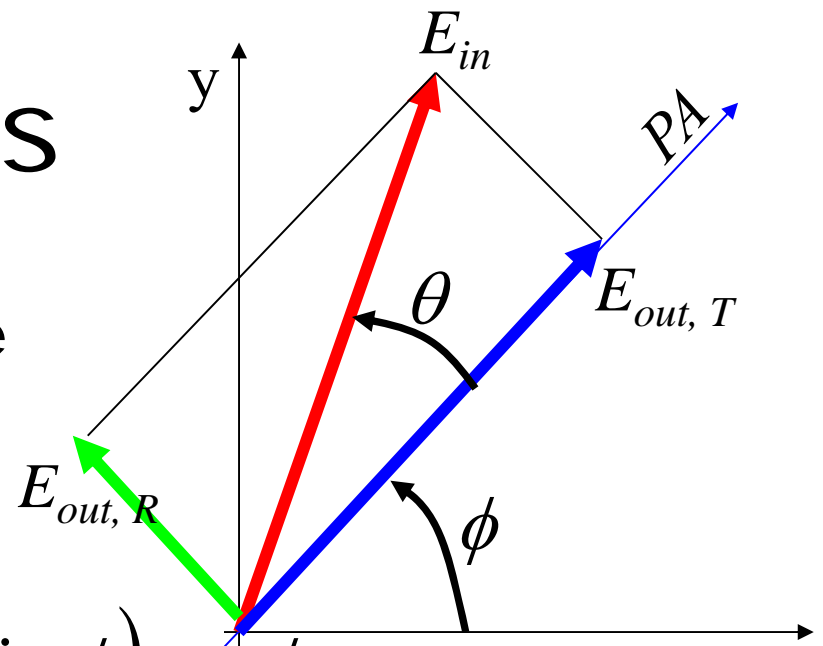
$$\begin{cases} E'_x = E_{par,x} = (E_x \cos \phi + E_y \sin \phi) \cos \phi \\ E'_y = E_{par,y} = (E_x \cos \phi + E_y \sin \phi) \sin \phi \end{cases}$$

- The component of the incoming field orthogonal to the PA is either absorbed or reflected, depending on the particular polarizer used.

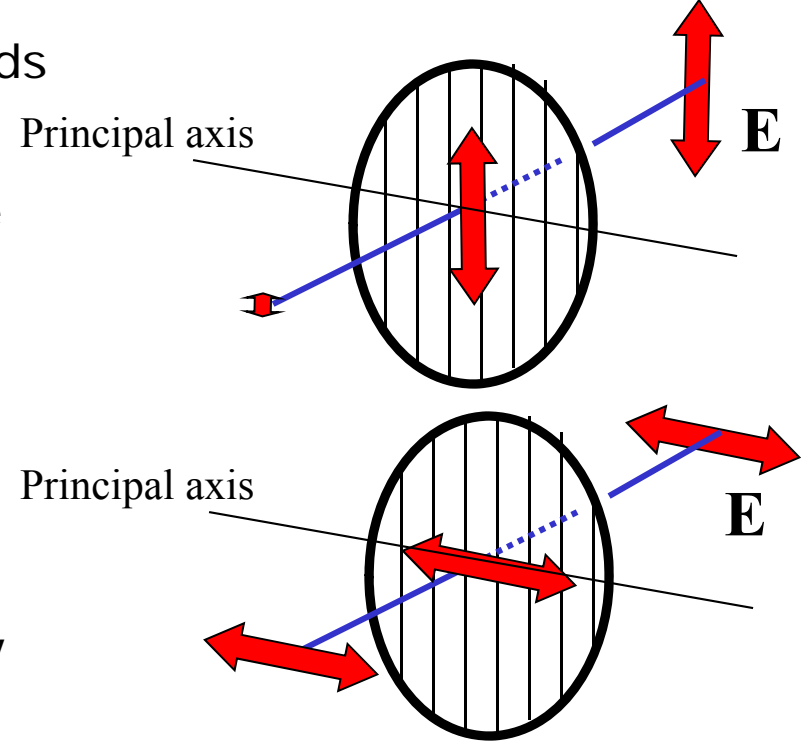
- Note that

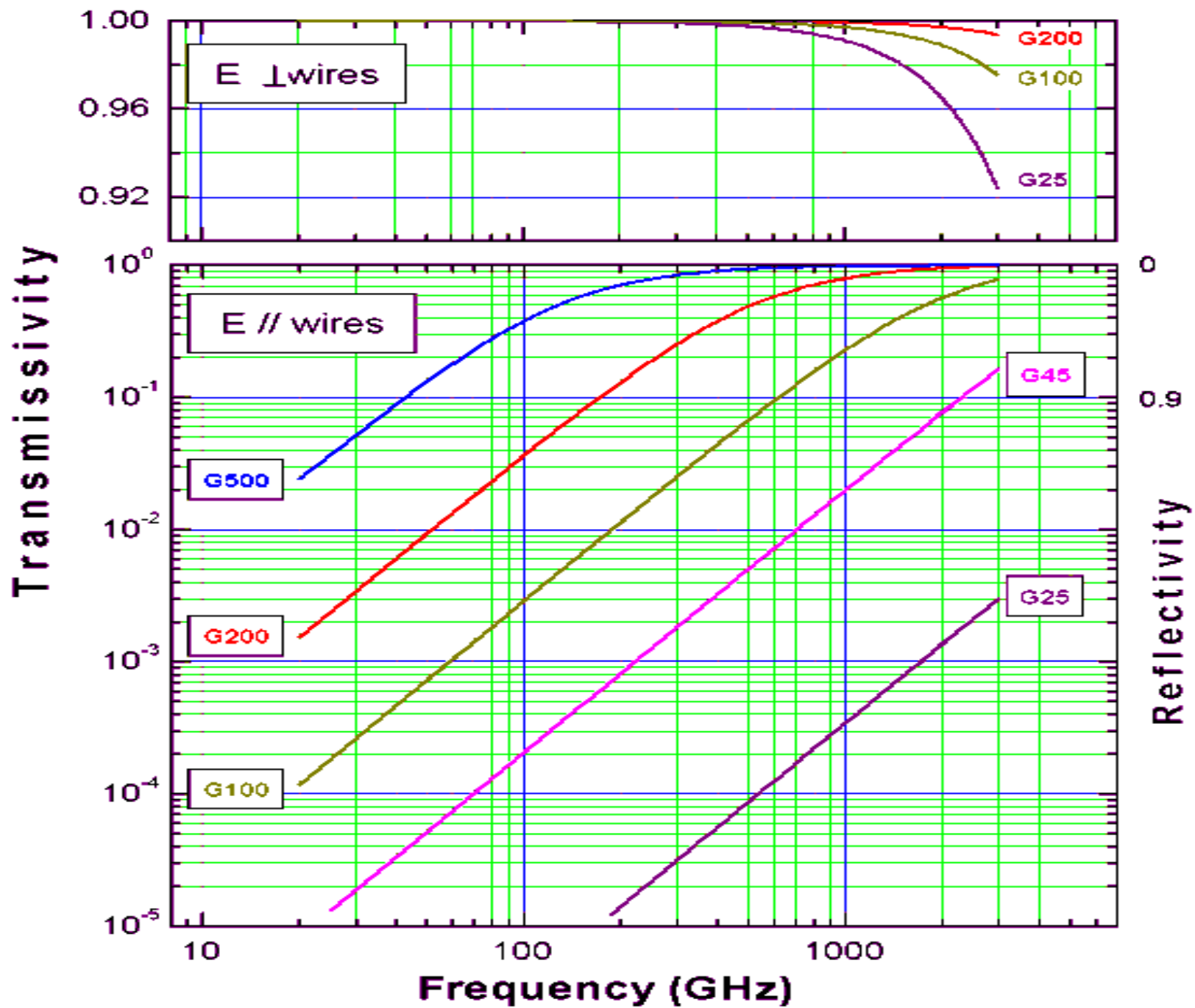
$$I' = |E'|^2 = |E_{par}|^2 = |E|^2 \cos^2 \theta = I \cos^2 \theta \quad (\text{Malus law})$$

- So, for $\theta = 45^\circ$ incidence, half of the intensity is transmitted (and half is reflected or absorbed).



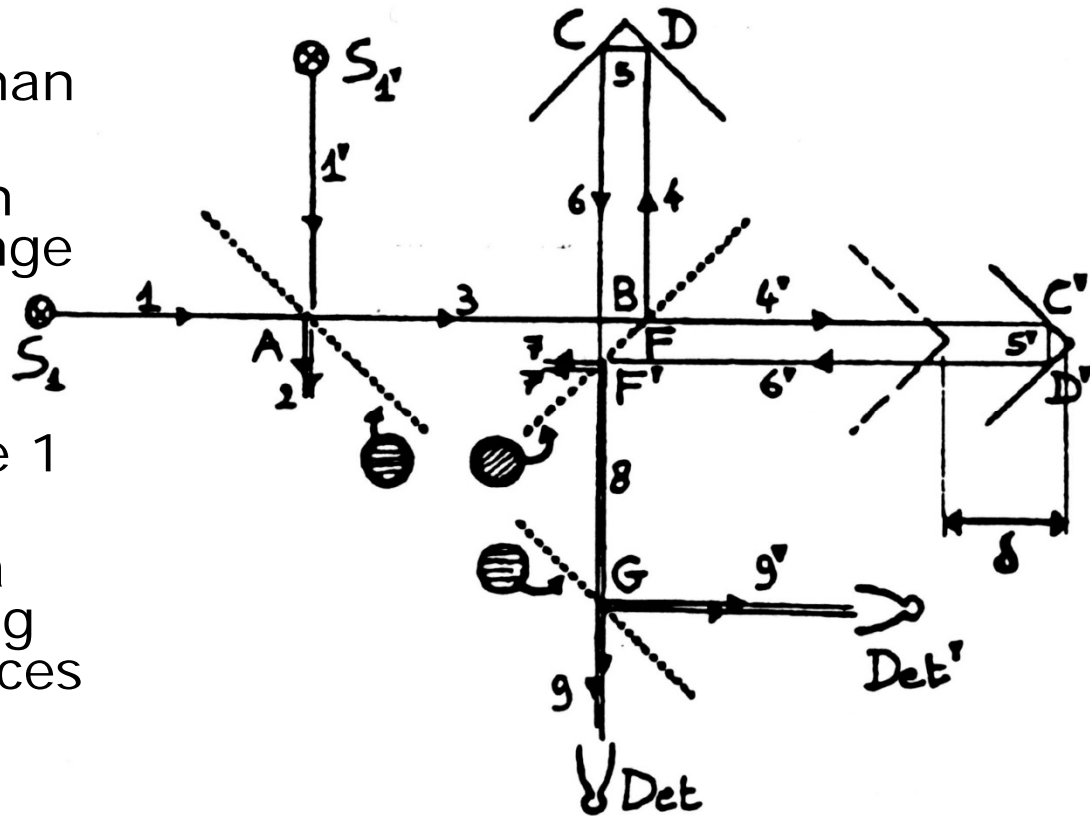
- At long wavelengths metallic wire grids act as ideal polarizers:
- Radiation with E parallel to the wires induces a current in the wires, so the polarizer acts as a metallic mirror: radiation is fully reflected and is not transmitted.
- Radiation with E orthogonal to the wires cannot induce a current in the wires, so it is transmitted.
- Radiation at a generic angle from the wires is partially transmitted (orthogonal component) and partially reflected (parallel component).
- If the spacing of the wires a and their diameter d are much less than the wavelength, the wire grid is very close to an ideal polarizer, with its principal axis orthogonal to the wires.
- Wire grid polarizers can be used as **ideal beamsplitters** for radiation at 45° wrt the wires: half of the intensity is transmitted, and half is reflected, without any wavelength dependence.
- Wire grids can be machined easily using a lathe and tungsten wire, which is available in long coils.





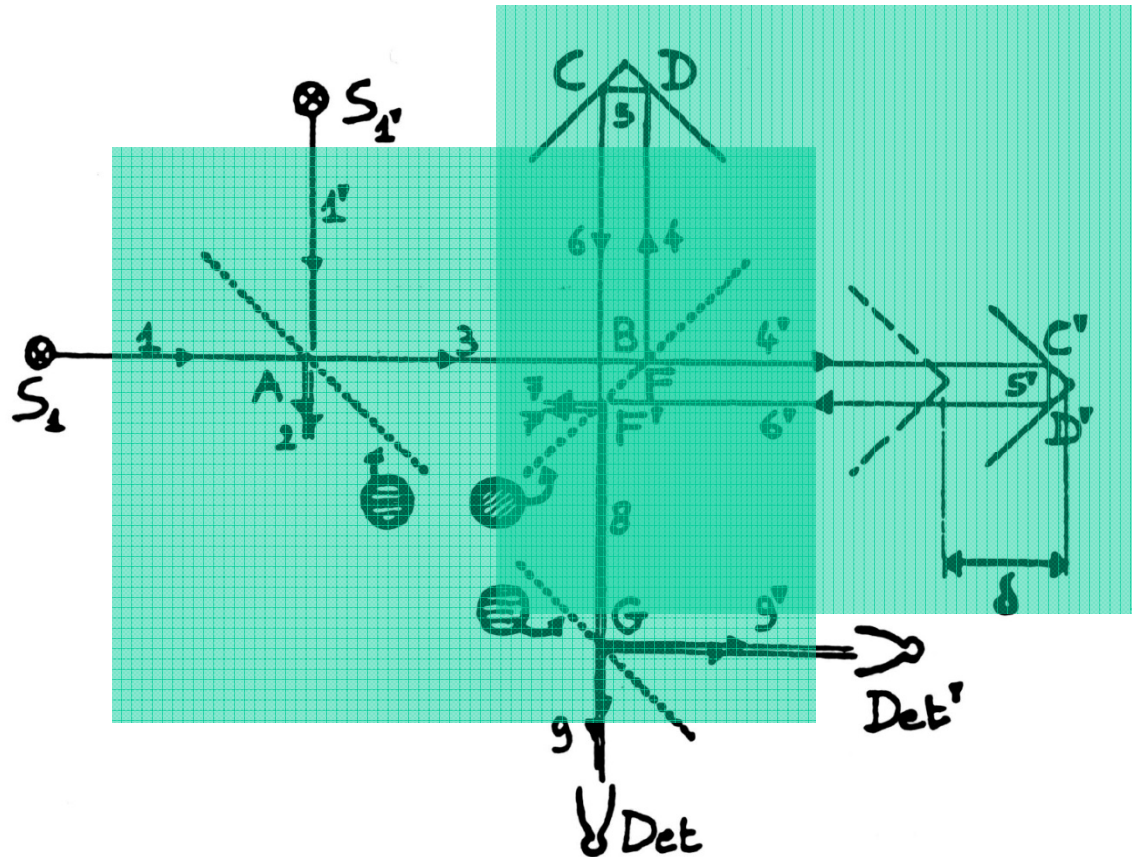
Martin Puplett Interferometer

- In the Martin-Puplett configuration, radiation is prepared by the first polarizer, then is split by the second, and is recombined by the third.
- There are two sources. The beam from source 1' is reflected one time more than the beam from source 1.
- For each metallic reflection there is a 180° phase change of the electric field, so the detector will measure the difference in spectral brightness between source 1 and source 1'.
- The instrument becomes a zero instrument, comparing the brightness of two sources (see later).
- In the case of FIRAS one source was the sky, the other one was an internal blackbody.



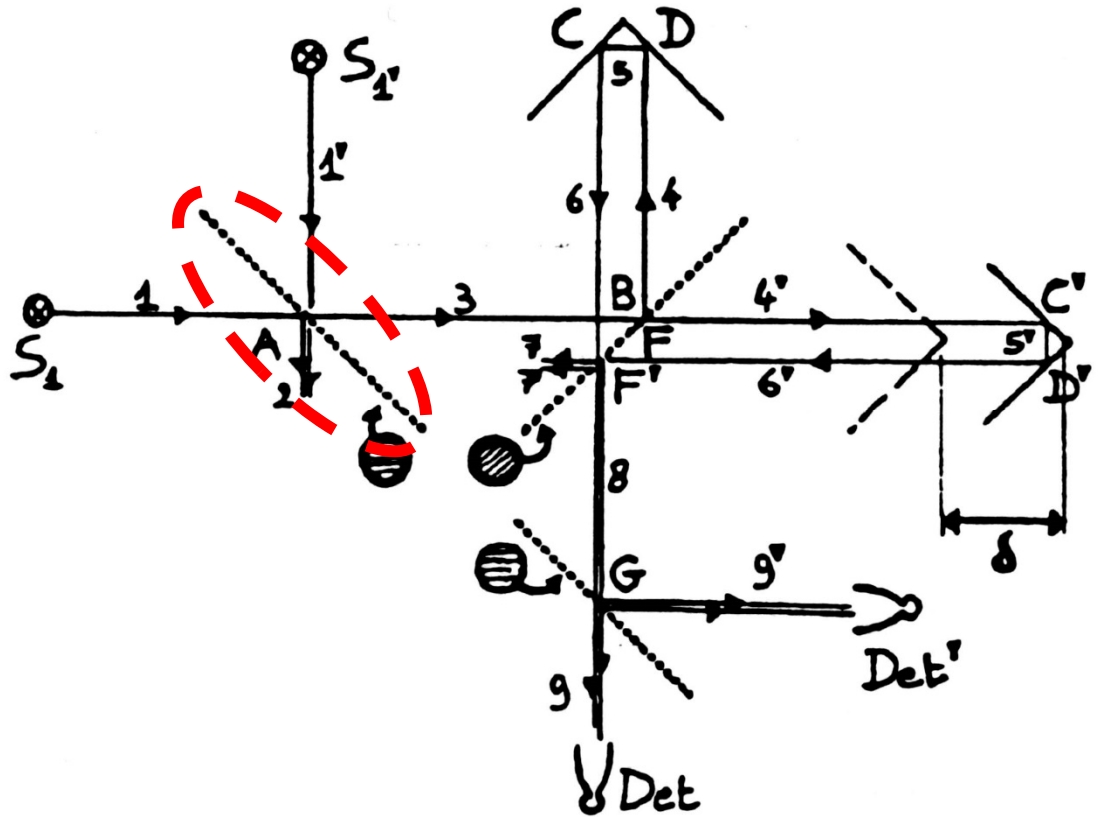
Martin Puplett Interferometer

- Two input ports and two output ports.
- Uses two unpolarized sources, and two detectors sensitive to the power.
- Let's study the operation following the beams.



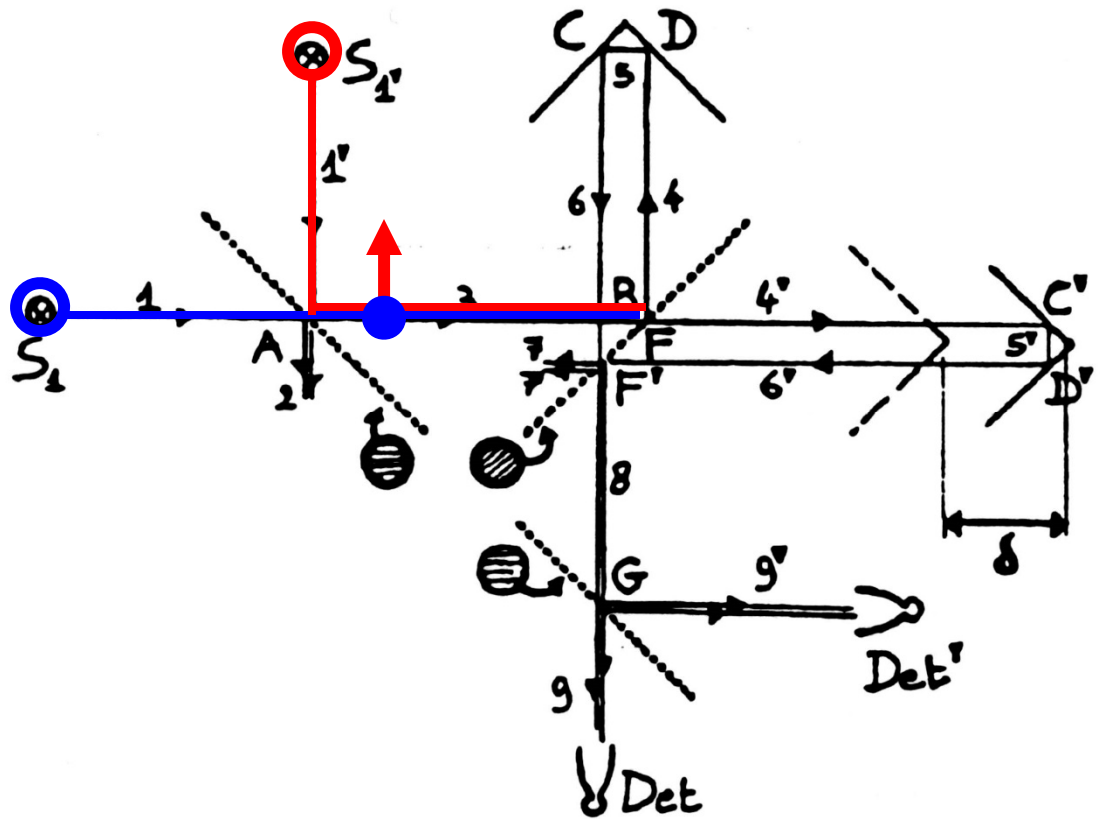
Martin Puplett Interferometer

- The input polarizer A reflects radiation from source S_1 , and transmits radiation from S_1
- Assume that the polarizer wires are horizontal (main axis of the polarizer vertical)
- The beam at position 3 has a vertical component from S_1 and a horizontal component from S_1 ,



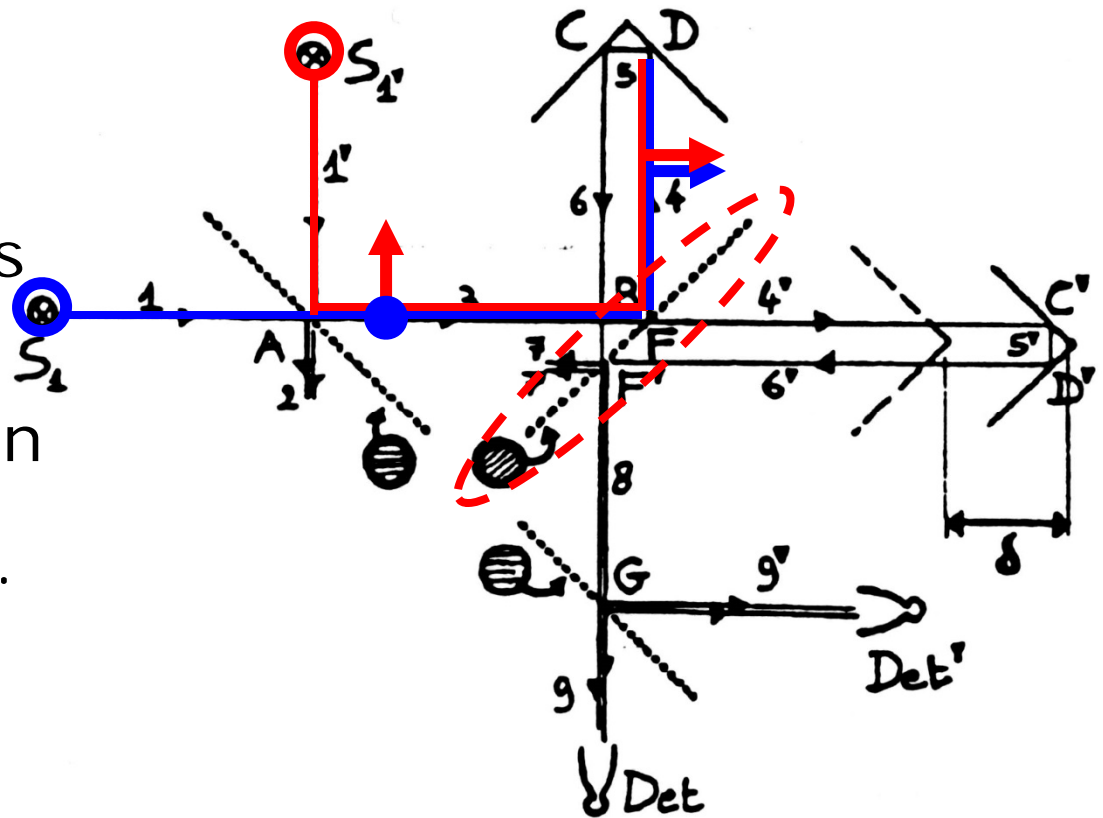
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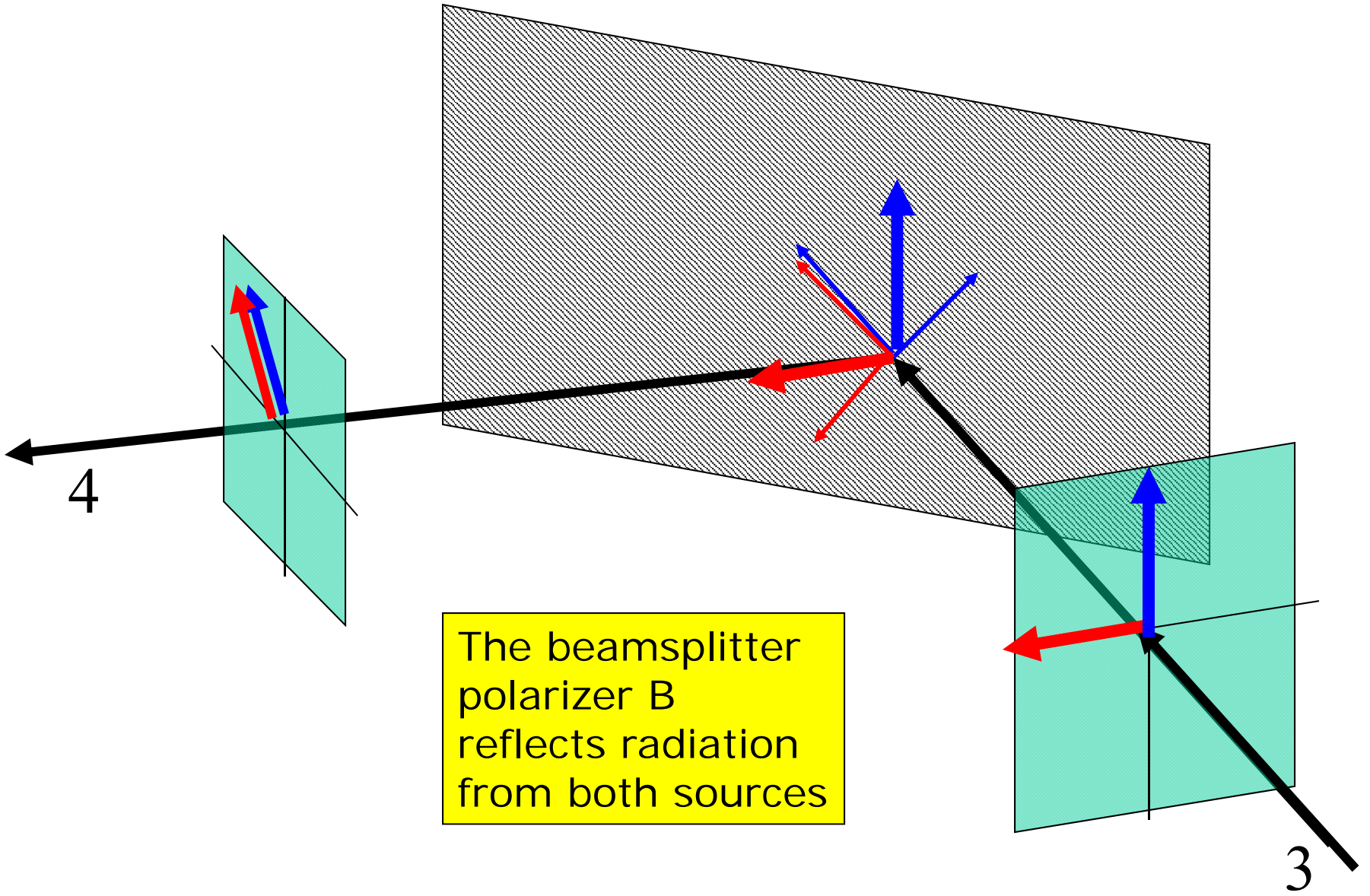
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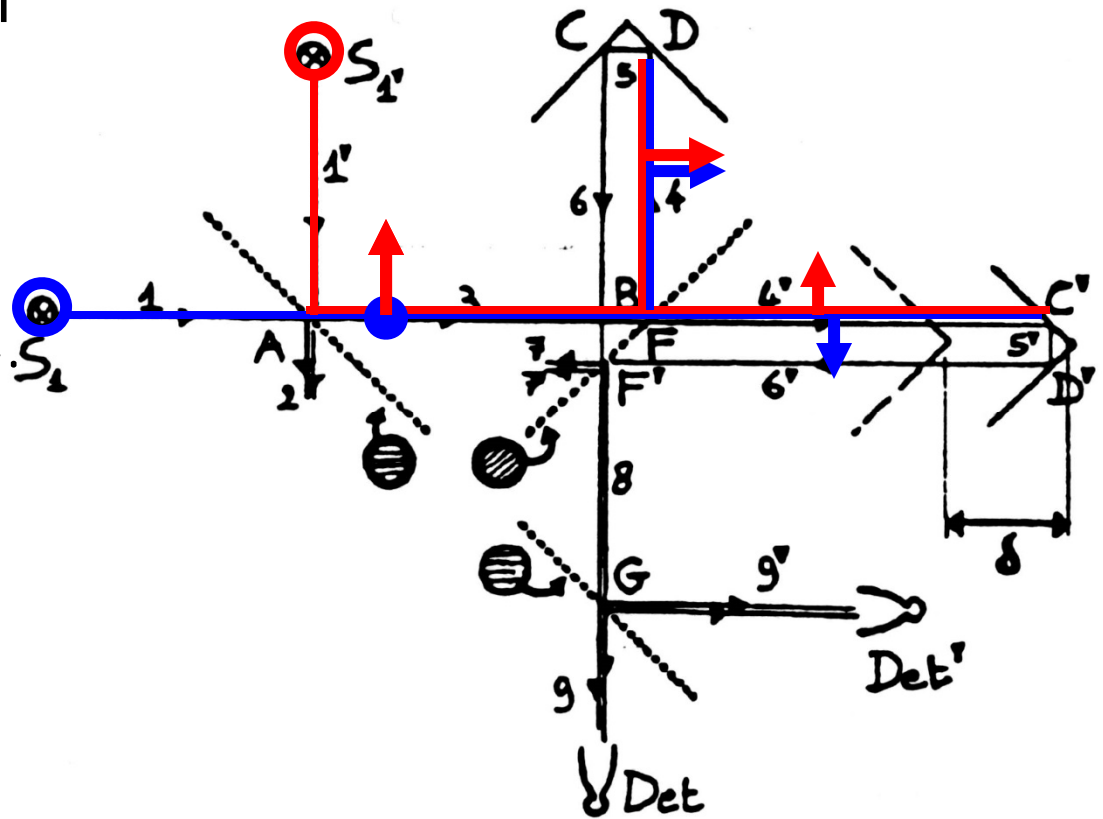
- The wires of the beamsplitter polarizer B are oriented to be seen by incoming radiation at 45° from the drawing plane.
- In this way, B reflects a fraction of the vertical component from \mathbf{S}_1 and a fraction of the horizontal component from \mathbf{S}_1' .
- So beam 4 will be polarized at 45° and will consist of equal contributions from both \mathbf{S}_1 e da \mathbf{S}_1' .

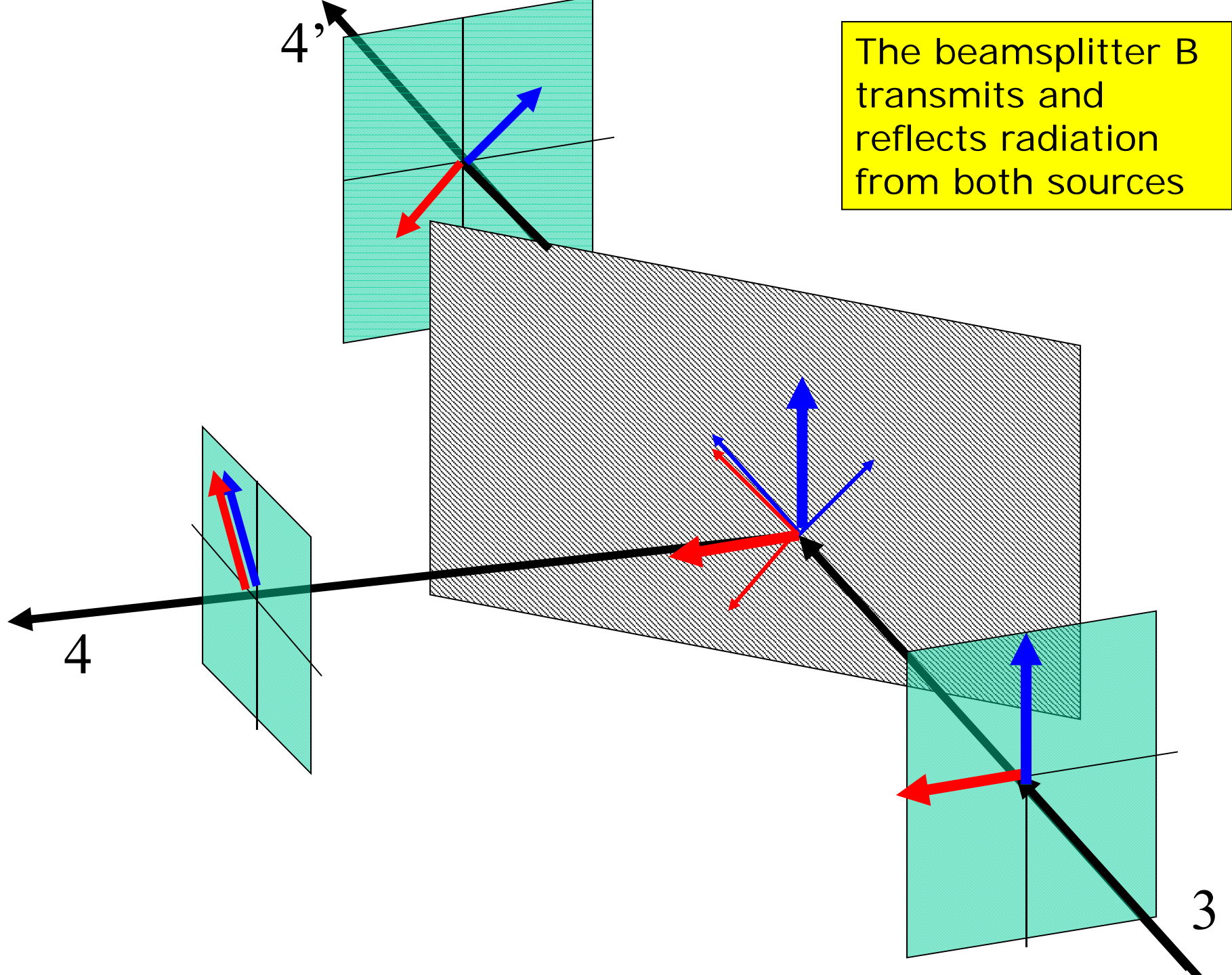




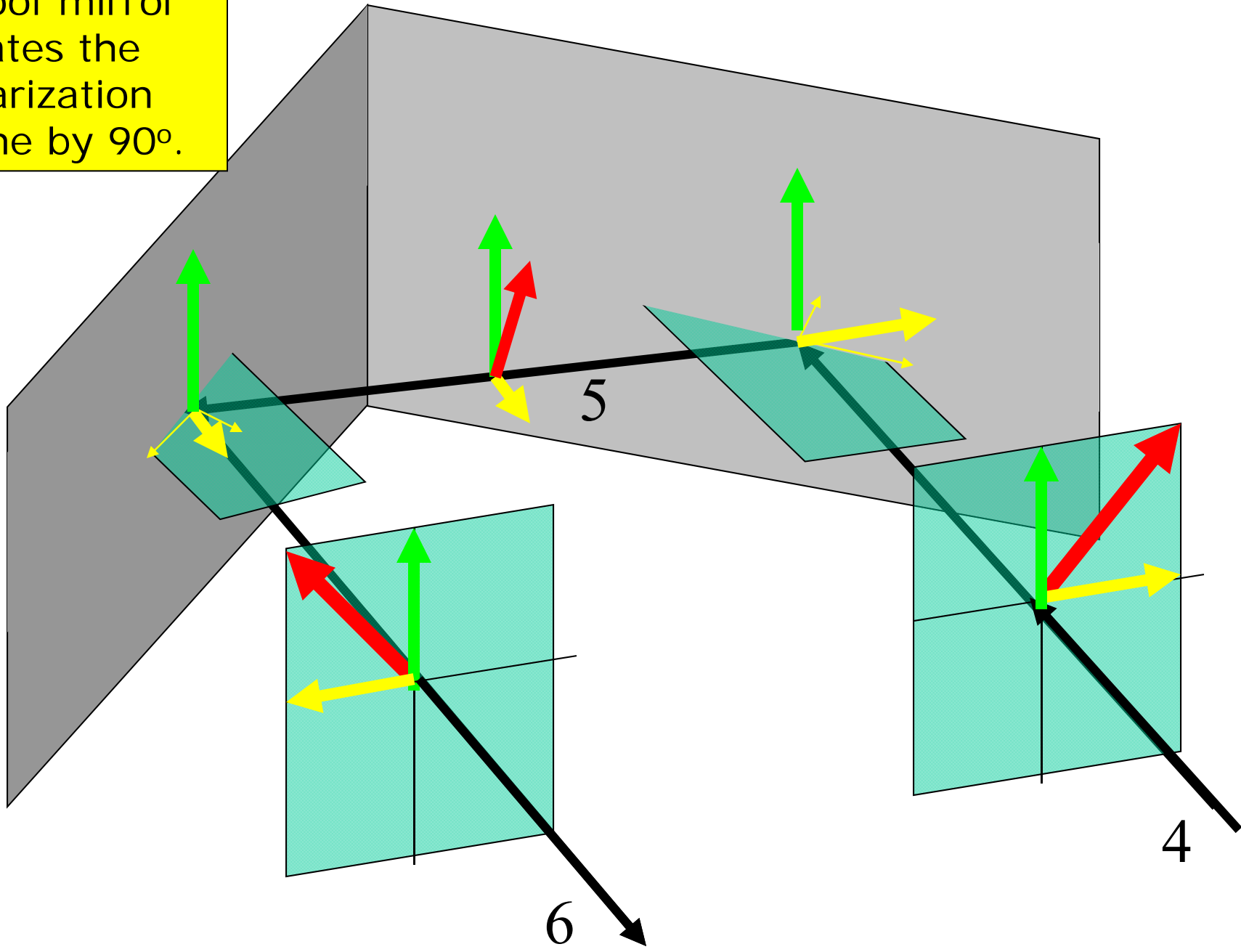
Martin Puplett Interferometer

- In addition, B **transmits** a fraction of the vertical component from S_1 and a fraction of the horizontal polarization from S_1' .
- So beam 4' is polarized at 45° and consists of equal contributions from both S_1 and S_1' .



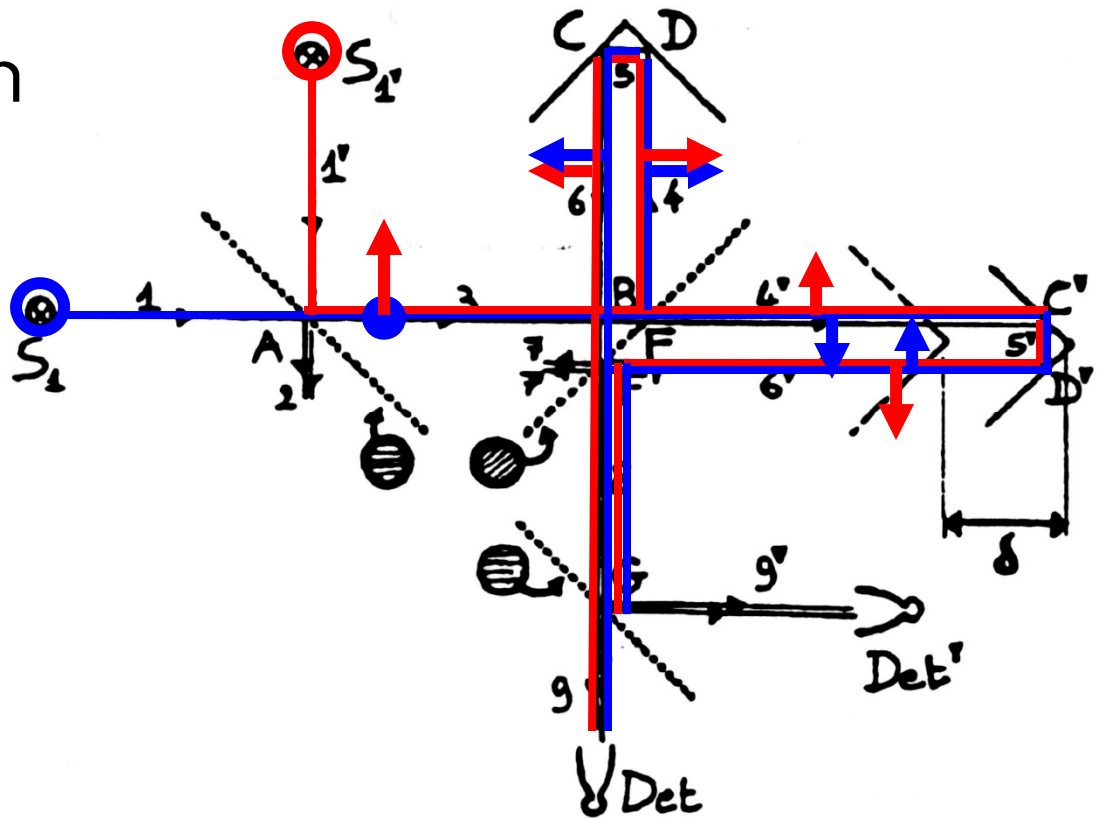


A roof mirror rotates the polarization plane by 90°.



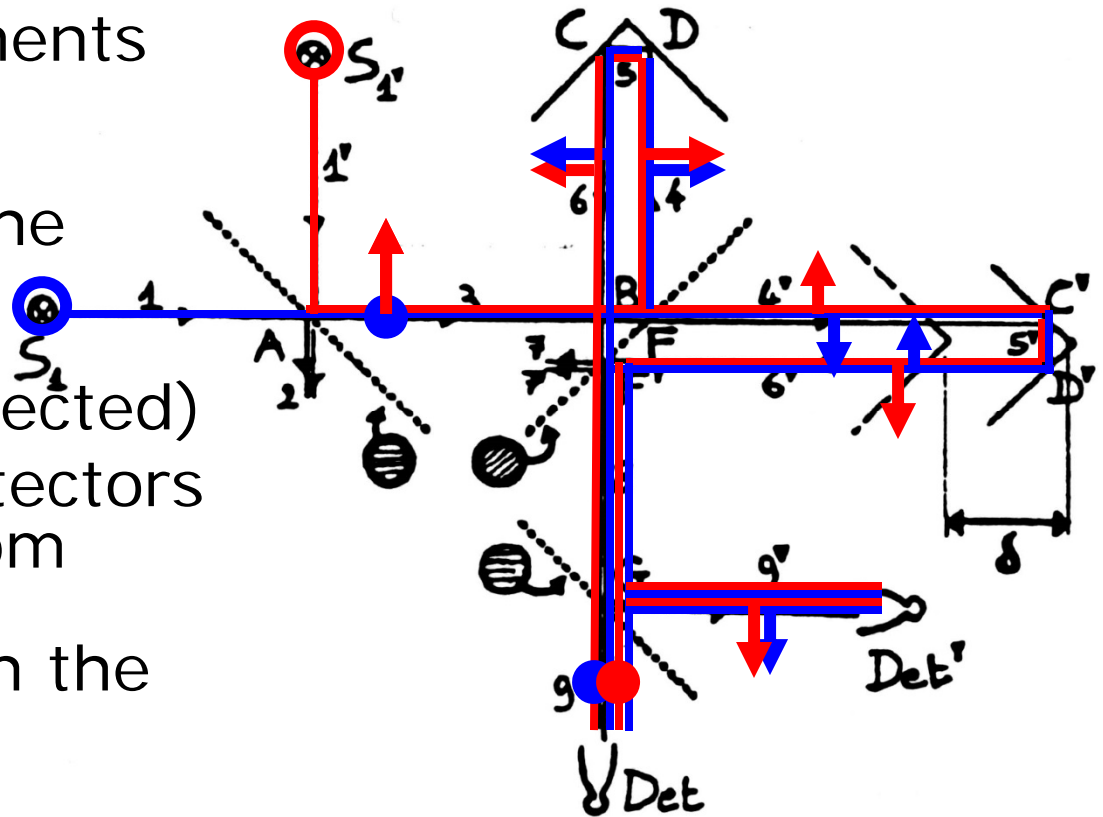
Martin Puplett Interferometer

- In the same way the roof mirror $C'D'$ reflects beam $4'$ back towards the beamsplitter (as $6'$)
- The polarization plane is rotated by 90° , so that beam $6'$, which had been transmitted (as $4'$) now is reflected towards the detectors.



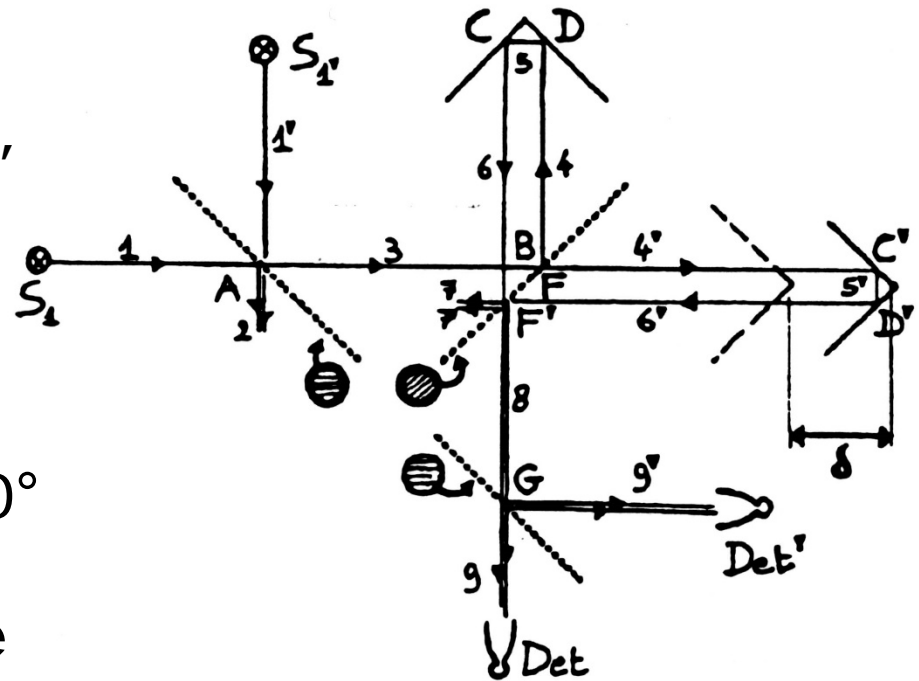
Martin Puplett Interferometer

- The output polarizer G has wires parallel to the drawing plane.
- The rays coming from the beamsplitter, which are at 45° , have both horizontal and vertical components (coming from both sources) so they contribute to both the beams towards the detectors (both transmitted and reflected)
- In this way both detectors receive radiation from both sources, which passed through both the arms of the interferometer.



Martin Puplett Interferometer

- The fundamental difference is that radiation from source S_1 , underwent 4 reflections, while radiation from S_1 underwent only 3 reflections. Since each reflection produces a 180° phase shift, the instrument measures the **diffence** between interferograms produced by S_1 , and S_1



Quantitative treatment: uses Jones Calculus

- Jones matrices are used to describe linearly polarized radiation (Jones 1941)
- The interaction of the E field of the EM wave with an optical component is described by a 2x2 matrix:

$$\begin{pmatrix} E_{x,OUT} \\ E_{y,OUT} \end{pmatrix} = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} \begin{pmatrix} E_{x,IN} \\ E_{y,IN} \end{pmatrix}$$

- They work only for fully polarized radiation. For partially polarized radiation one can use Muller calculus (losing any phase information).

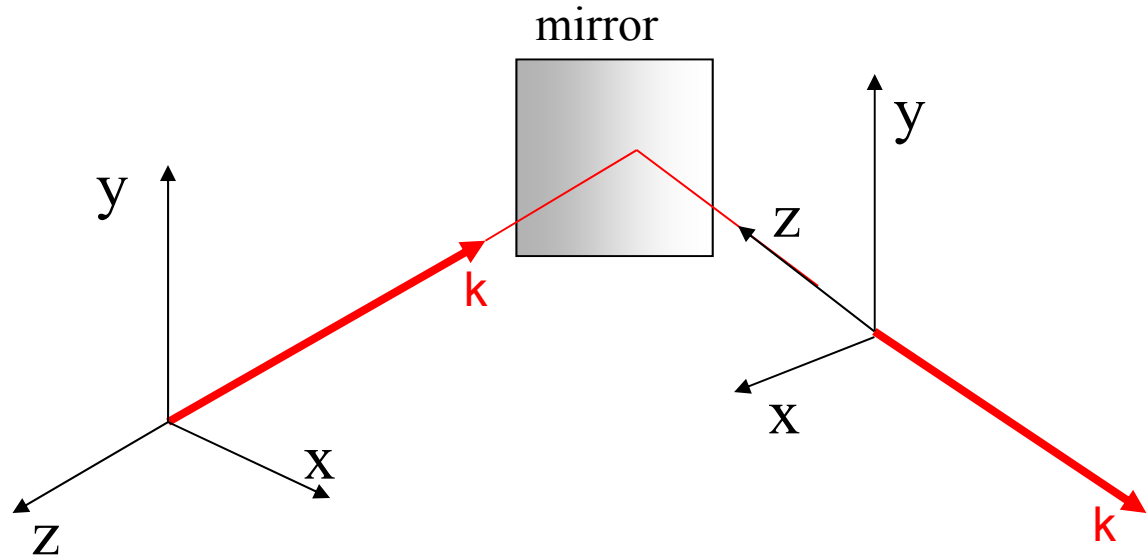
E field

If E is the amplitude of the electric field of the EMW, it is represented as in the following examples:

- Linear polarization aligned along x axis $E \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
- Linear polarization aligned along y axis $E \begin{pmatrix} 0 \\ 1 \end{pmatrix}$
- 45° from x axis $\frac{E}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$
- -45° from x axis $\frac{E}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$
- Circular polarization (right) $\frac{E}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}$
- Circular polarization (left) $\frac{E}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}$

Reference system

- Comoving with the light beam :



Mirrors

- Ideal single mirror, orthogonal to xz plane:
- Ideal roof mirror, orthogonal to xz plane:

$$M = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$RM = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Linear polarizers

- Transmission :
Polarizer with horizontal
principal axis: $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

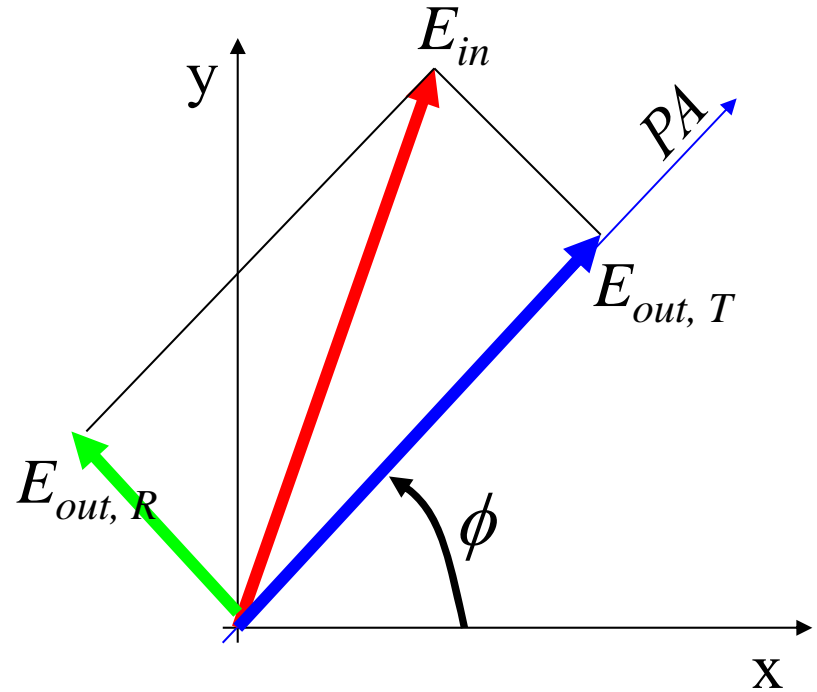
- Transmission :
Polarizer with vertical
principal axis: $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

- Transmission :
Polarizer with principal
axis at angle ϕ from x
axis

$$P_t(\phi) = \begin{pmatrix} \cos^2 \phi & \cos \phi \sin \phi \\ \cos \phi \sin \phi & \sin^2 \phi \end{pmatrix}$$

- Reflection : Polarizer
with principal axis at
angle ϕ from x axis

$$P_r(\phi) = \begin{pmatrix} \sin^2 \phi & -\cos \phi \sin \phi \\ \cos \phi \sin \phi & -\cos^2 \phi \end{pmatrix}$$



Delay

- Introduced by an optical path difference $\delta=4\pi\sigma x$: this is common for both polarizations, so

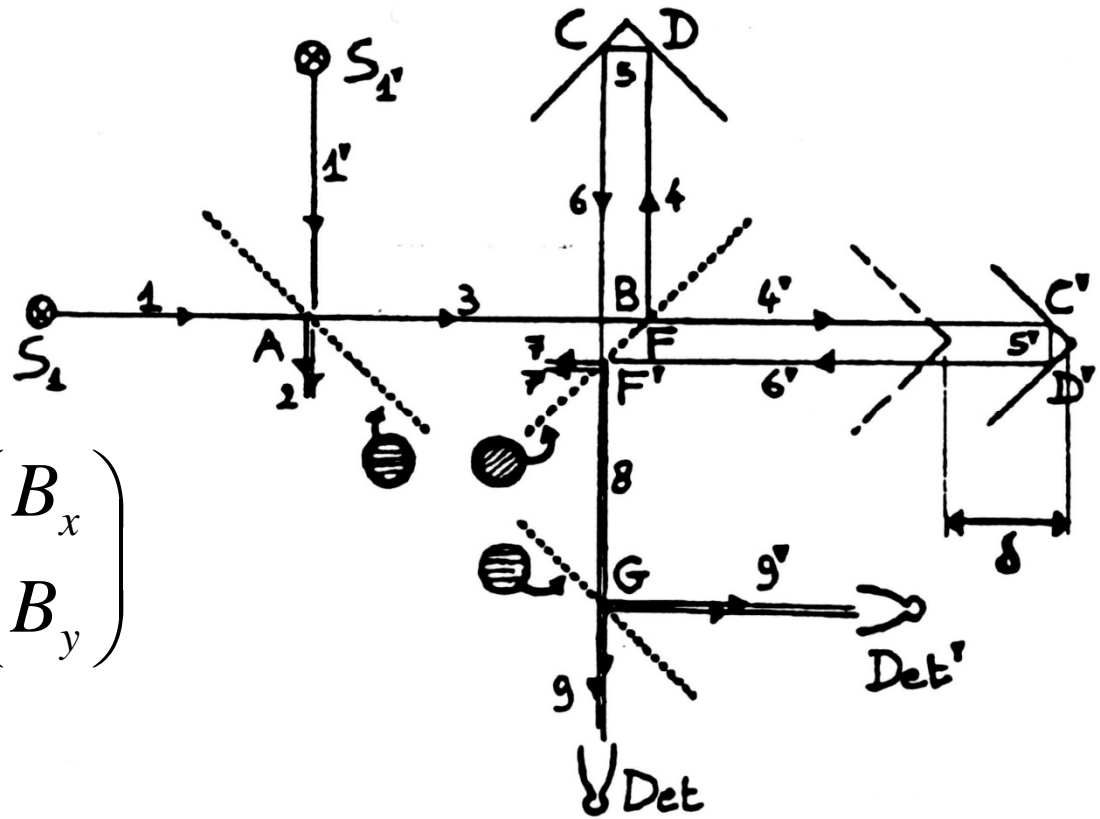
$$D(\delta) = \begin{pmatrix} e^{i\delta} & 0 \\ 0 & e^{i\delta} \end{pmatrix}$$

- The two sources S_1 and S'_1 , are described by Jones vectors

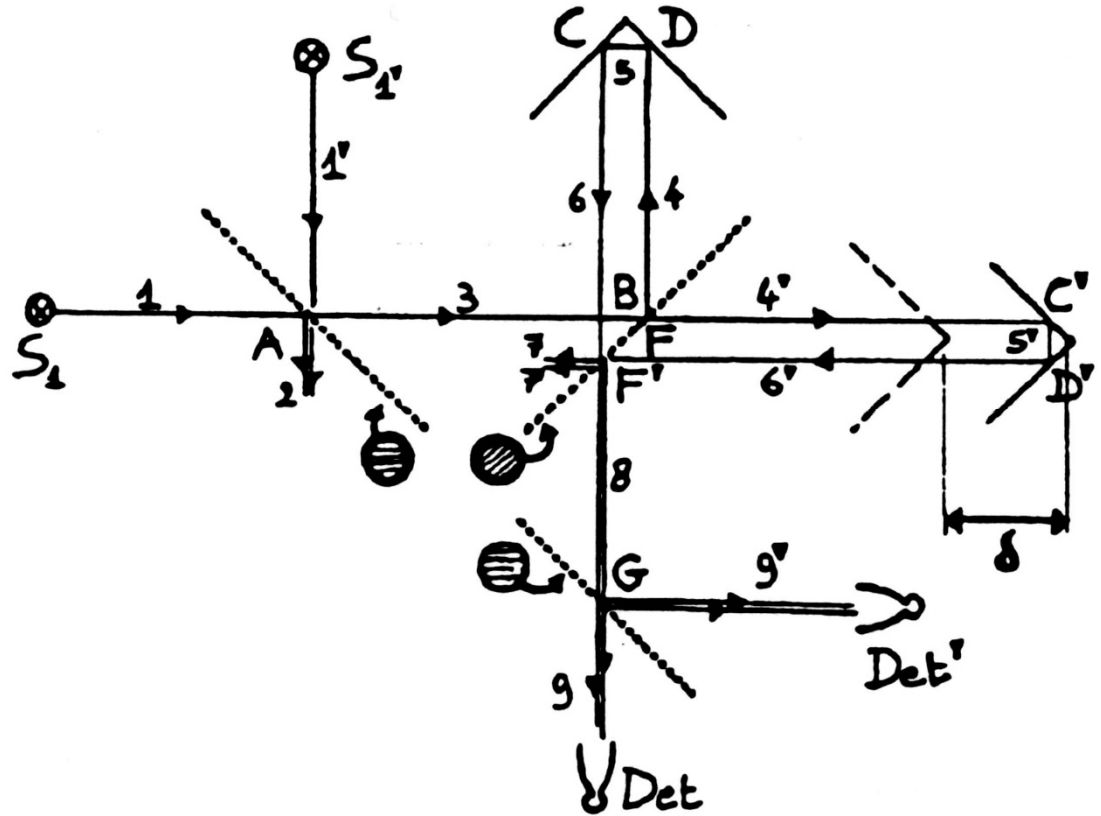
$$S_1 = \begin{pmatrix} A_x \\ A_y \end{pmatrix} \quad S'_1 = \begin{pmatrix} B_x \\ B_y \end{pmatrix}$$

- beam 3, after the input polarizer (with horizontal principal axis), is

$$S_3 = P_t(0)S_1 + P_r(0)S'_1 = \begin{pmatrix} A_x \\ -B_y \end{pmatrix}$$



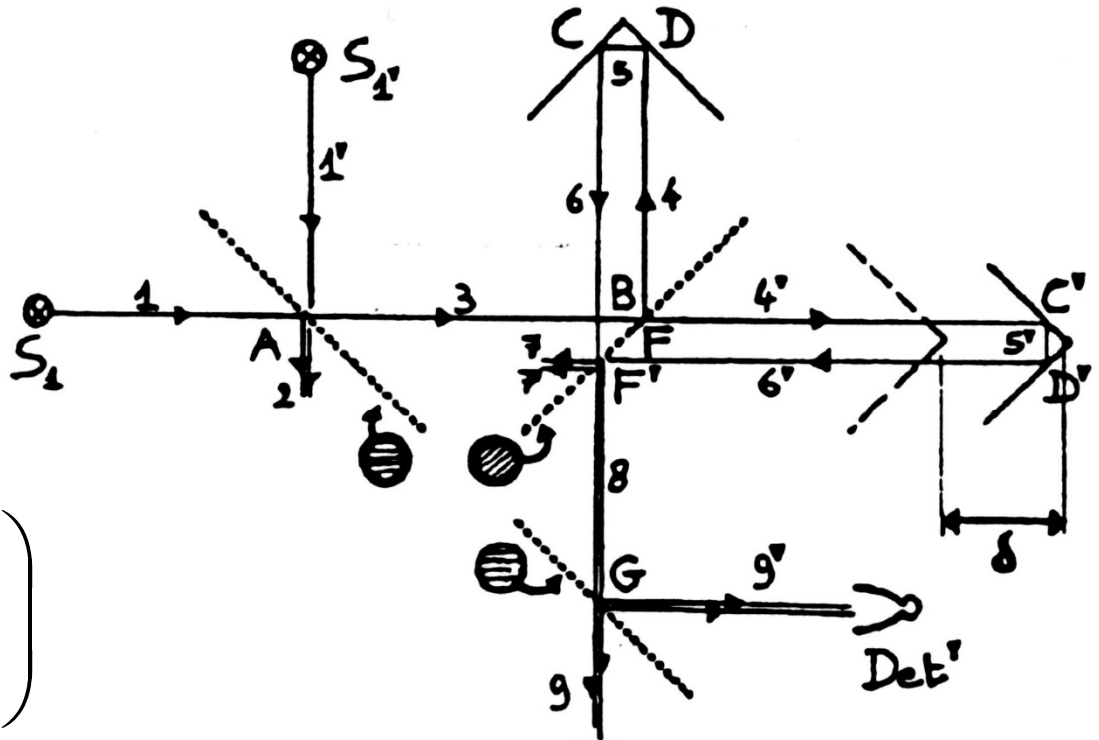
- beam 4 (reflected by the beamsplitter) and beam 4' (transmitted by the beamsplitter) will be:



$$S_4 = P_r(\pi/4)S_3 = \frac{1}{2} \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} A_x \\ -B_y \end{pmatrix} = \frac{1}{2} \begin{pmatrix} A_x + B_y \\ A_x + B_y \end{pmatrix}$$

$$S_4' = P_t(\pi/4)S_3 = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} A_x \\ -B_y \end{pmatrix} = \frac{1}{2} \begin{pmatrix} A_x - B_y \\ A_x - B_y \end{pmatrix}$$

- Since roof mirrors are represented by unity matrix, we have also



$$S_6 = S_4 = \frac{1}{2} \begin{pmatrix} A_x + B_y \\ A_x + B_y \end{pmatrix}$$

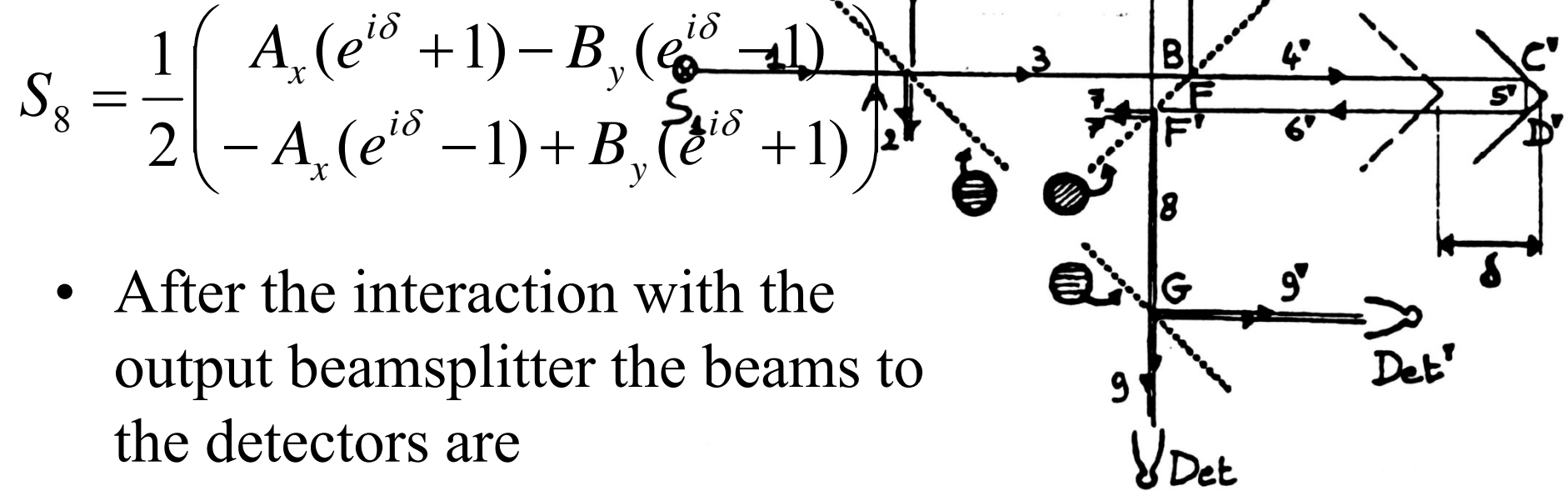
$$S'_6 = DS'_4 = D \frac{1}{2} \begin{pmatrix} A_x - B_y \\ A_x - B_y \end{pmatrix} = \frac{1}{2} \begin{pmatrix} (A_x - B_y) e^{i\delta} \\ (A_x - B_y) e^{i\delta} \end{pmatrix}$$

- S_6 is transmitted by the beamsplitter, while S'_6 is reflected, so

$$S_8 = P_t(\pi/4)S_6 + P_r(3\pi/4)S'_6 = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} S_6 + \frac{1}{2} \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix} S'_6$$

$$S_8 = P_t(\pi/4)S_6 + P_r(3\pi/4)S'_6 = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} S_6 + \frac{1}{2} \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix} S'_6$$

• So



• After the interaction with the output beamsplitter the beams to the detectors are

$$S_9 = P_t(0)S_8 = \frac{1}{2} \begin{pmatrix} A_x(e^{i\delta} + 1) - B_y(e^{i\delta} - 1) \\ 0 \end{pmatrix}$$

$$S'_9 = P_r(0)S_8 = \frac{1}{2} \begin{pmatrix} 0 \\ A_x(e^{i\delta} - 1) - B_y(e^{i\delta} + 1) \end{pmatrix}$$

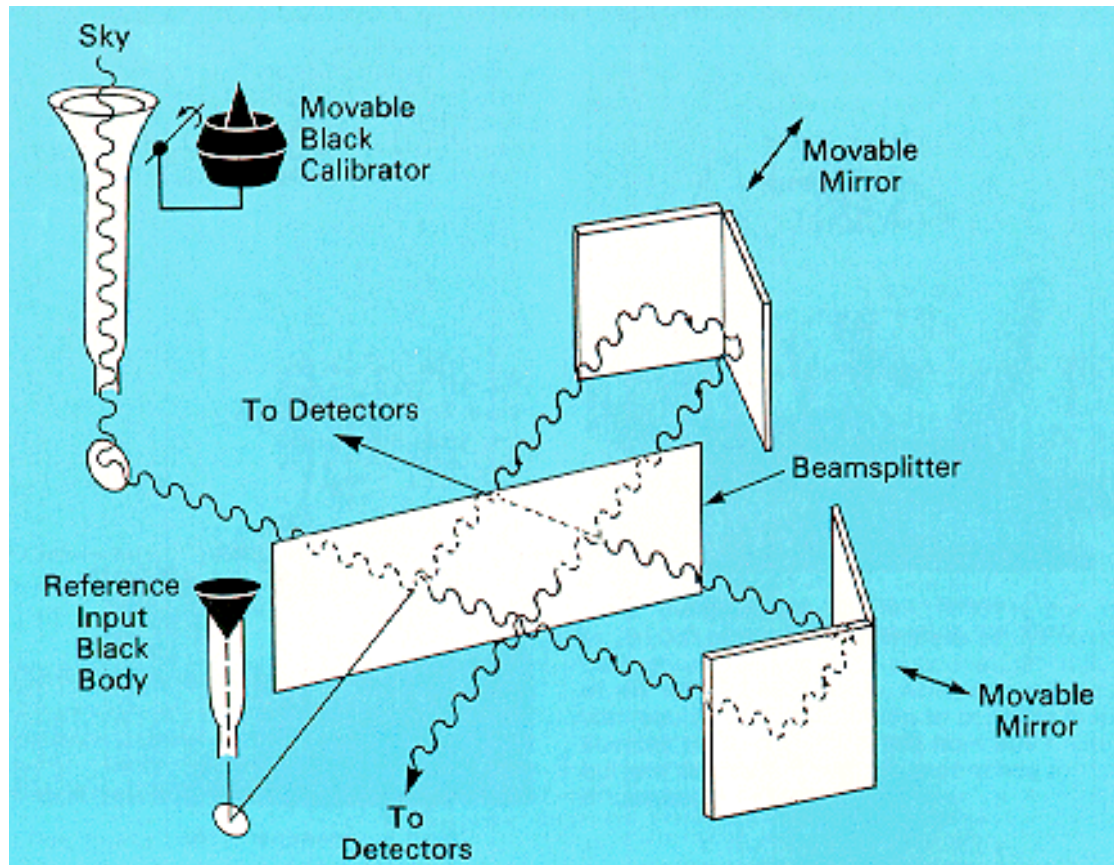
- Now we can compute the power on the detectors:

$$I = E_x E_x^* + E_y E_y^* \quad \rightarrow$$

$$I_9 = \frac{1}{2} [A_x^2 (1 + \cos \delta) + B_y^2 (1 - \cos \delta)] = \frac{A_x^2 + B_y^2}{2} + \frac{A_x^2 - B_y^2}{2} \cos \delta$$

$$I'_9 = \frac{1}{2} [A_x^2 (1 - \cos \delta) + B_y^2 (1 + \cos \delta)] = \frac{A_x^2 + B_y^2}{2} - \frac{A_x^2 - B_y^2}{2} \cos \delta$$

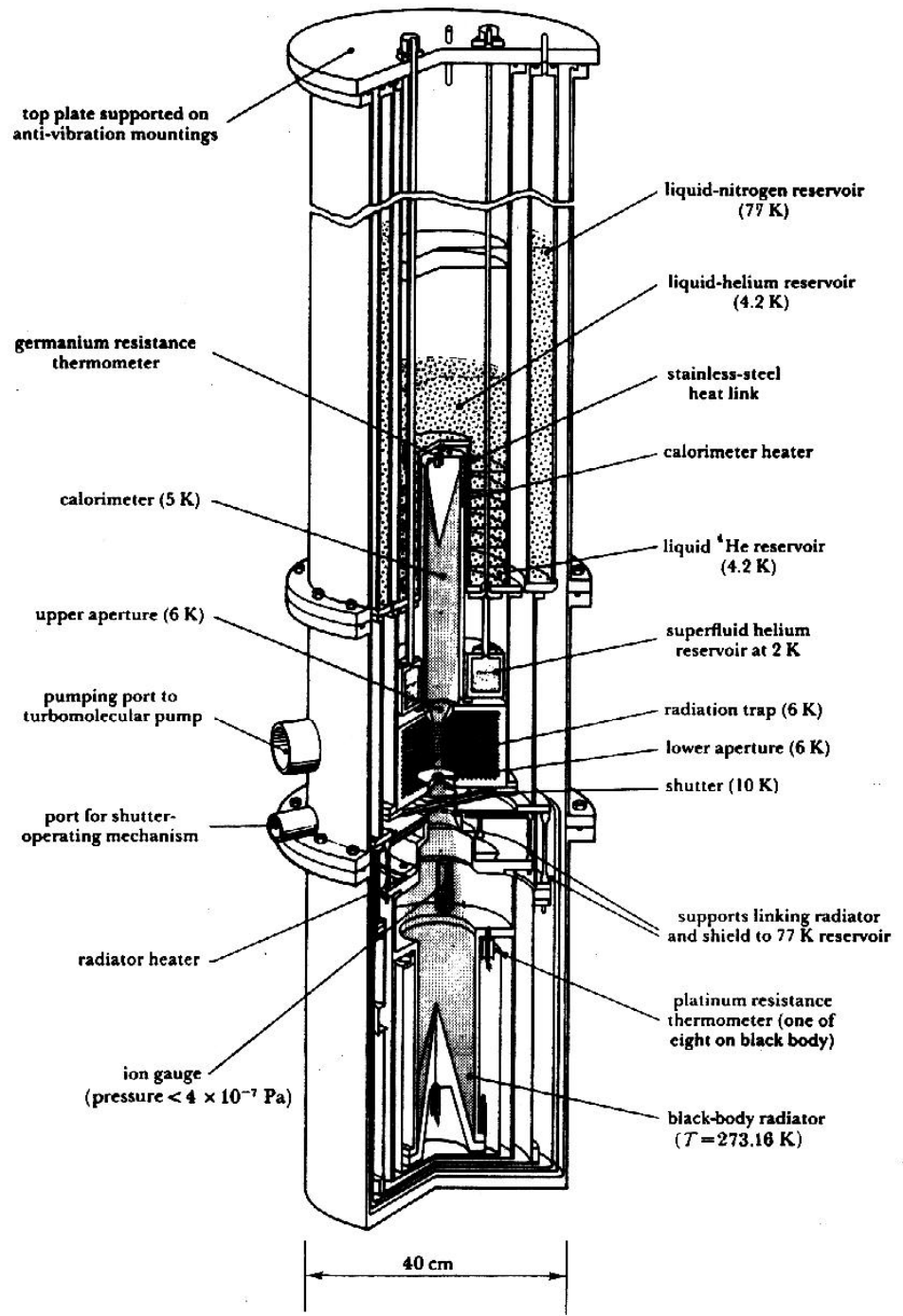
- Both detectors measure a **constant intensity** (equal to half of the sum of the intensities from the two sources), **plus a modulated intensity** (modulated by the optical path difference), whose amplitude is the **difference** of the spectra from the two sources.
- **The interferogram is zero if the two sources have the same spectrum.**



$$I_{SKY}(x) = C \int_0^{\infty} [S_{SKY}(\sigma) - S_{REF}(\sigma)] rt(\sigma) \{1 + \cos[4\pi\sigma x]\} d\sigma$$

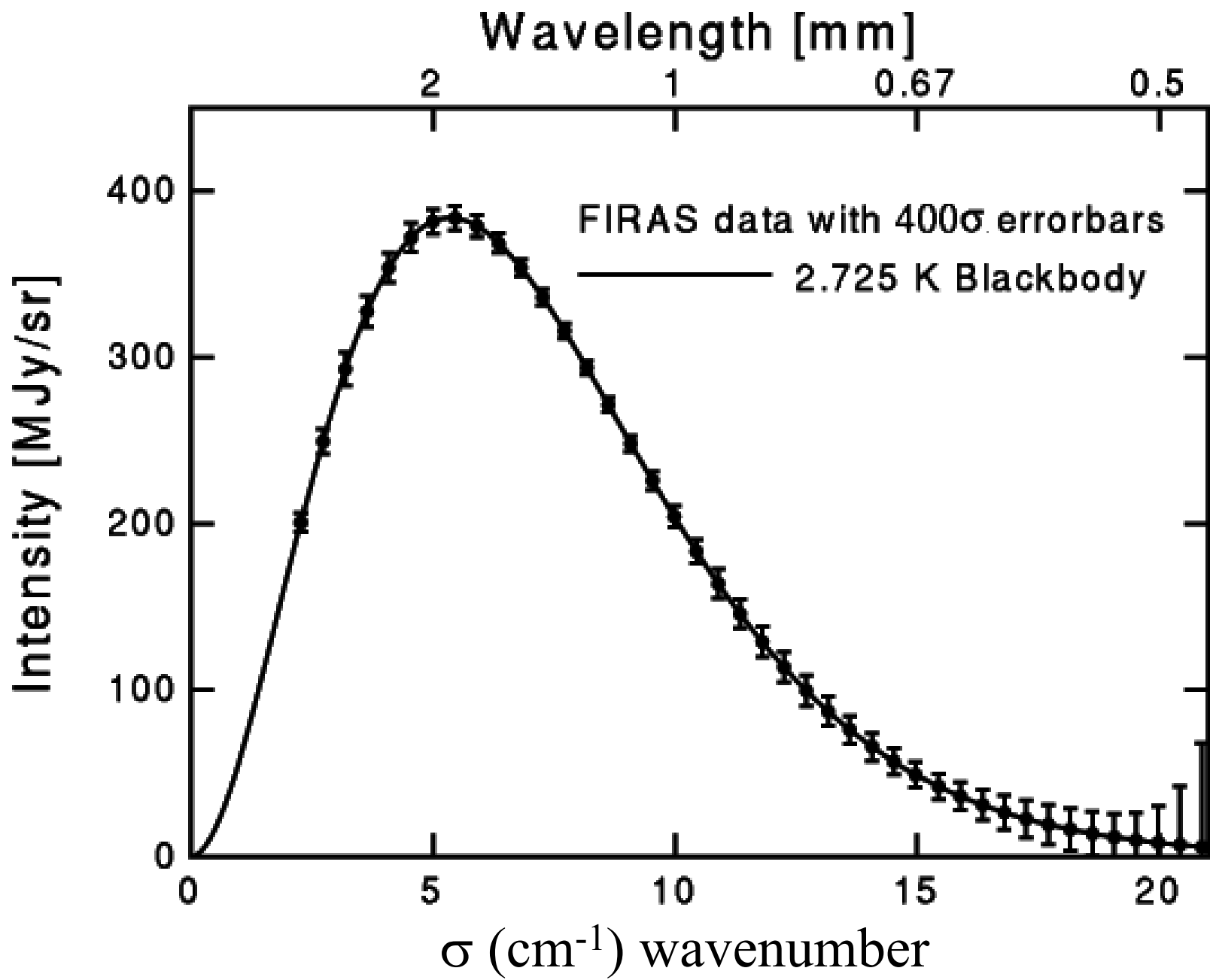
$$I_{CAL}(x) = C \int_0^{\infty} [S_{CAL}(\sigma) - S_{REF}(\sigma)] rt(\sigma) \{1 + \cos[4\pi\sigma x]\} d\sigma$$

- To measure a few K blackbody, you need a cryogenic reference blackbody, with variable temperature. Otherwise you do not null the signal.
- Practical design of a Blackbody cavity: see e.g. Quinn T.J., Martin J.E., (1985), *Phil. Trans. R. Soc. Lond.*, A316, 85: who made a radiometric measurement of the Boltzmann constant (precise to 5 significant digits !)

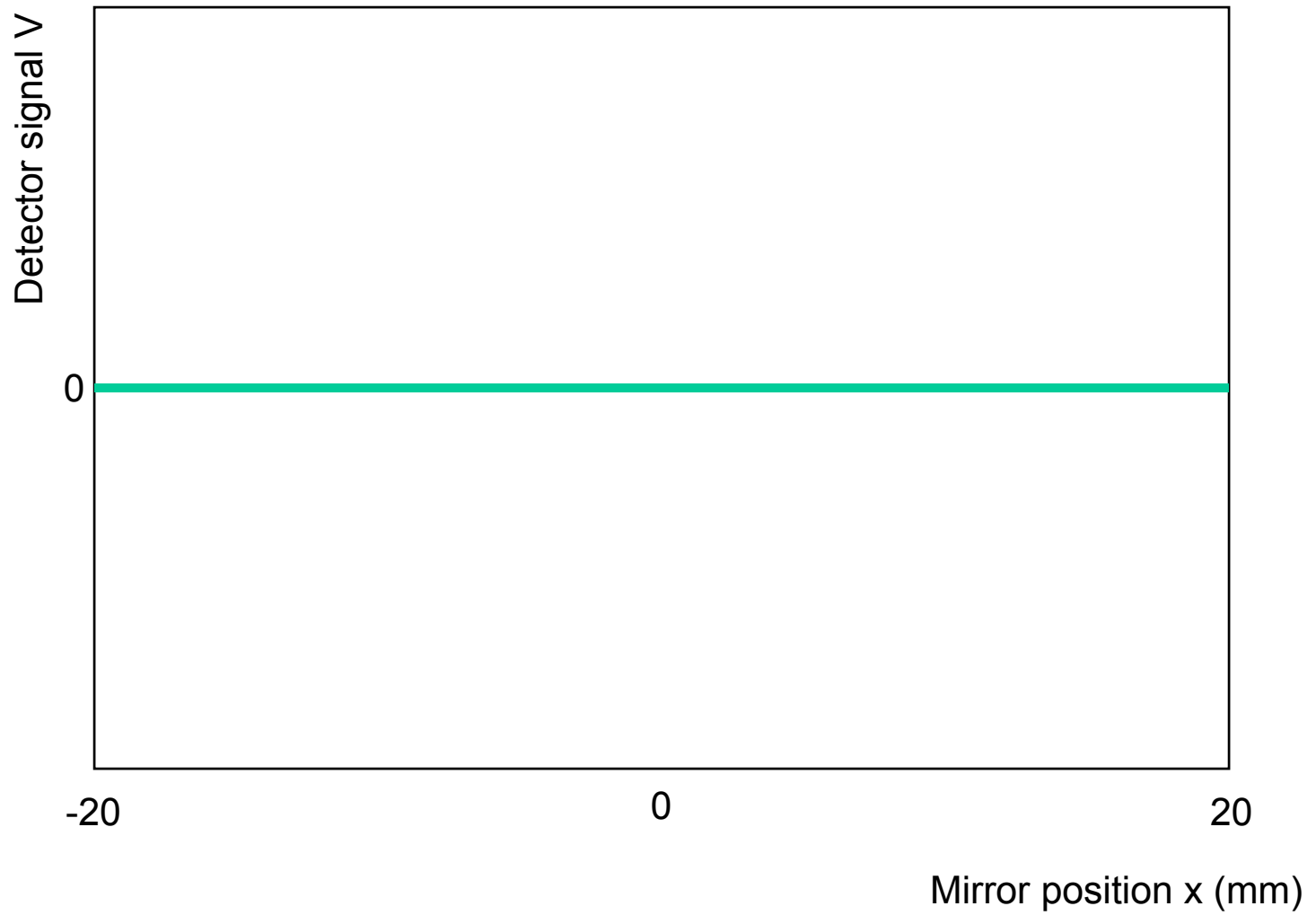


FIRAS

- The FIRAS guys were able to change the temperature of the internal blackbody until the interferograms were flat zero.
- This is a null measurement, which is much more sensitive than an absolute one: (one can boost the gain without saturating !).
- This means that the difference between the spectrum of the sky and the spectrum of a blackbody is zero, i.e. **the spectrum of the sky is a blackbody with the same temperature as the internal reference blackbody.**
- This also means that the internal blackbody is a real blackbody: it is unlikely that the sky can have the same deviation from the Planck law as the source built in the lab.

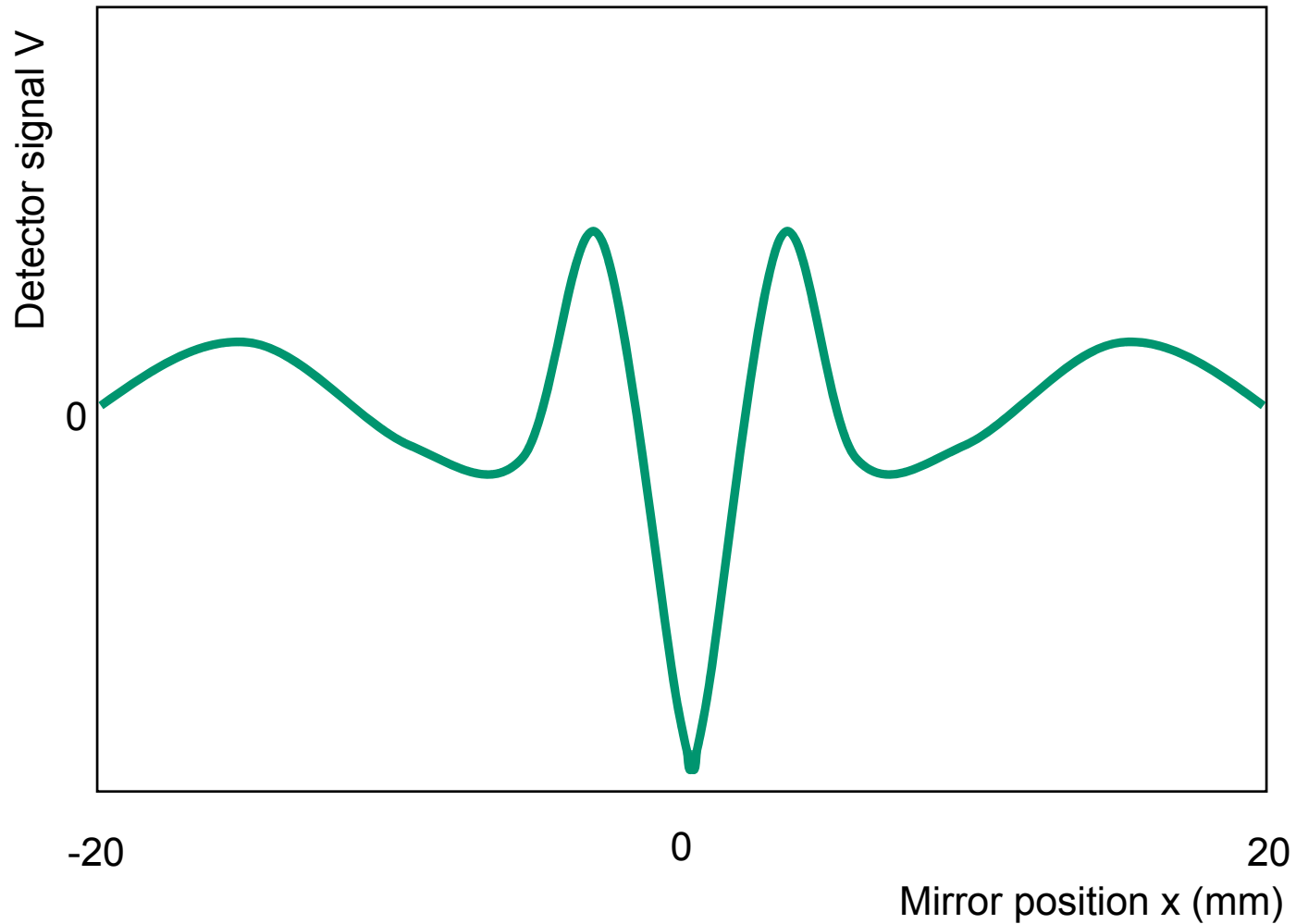


What was actually measured :



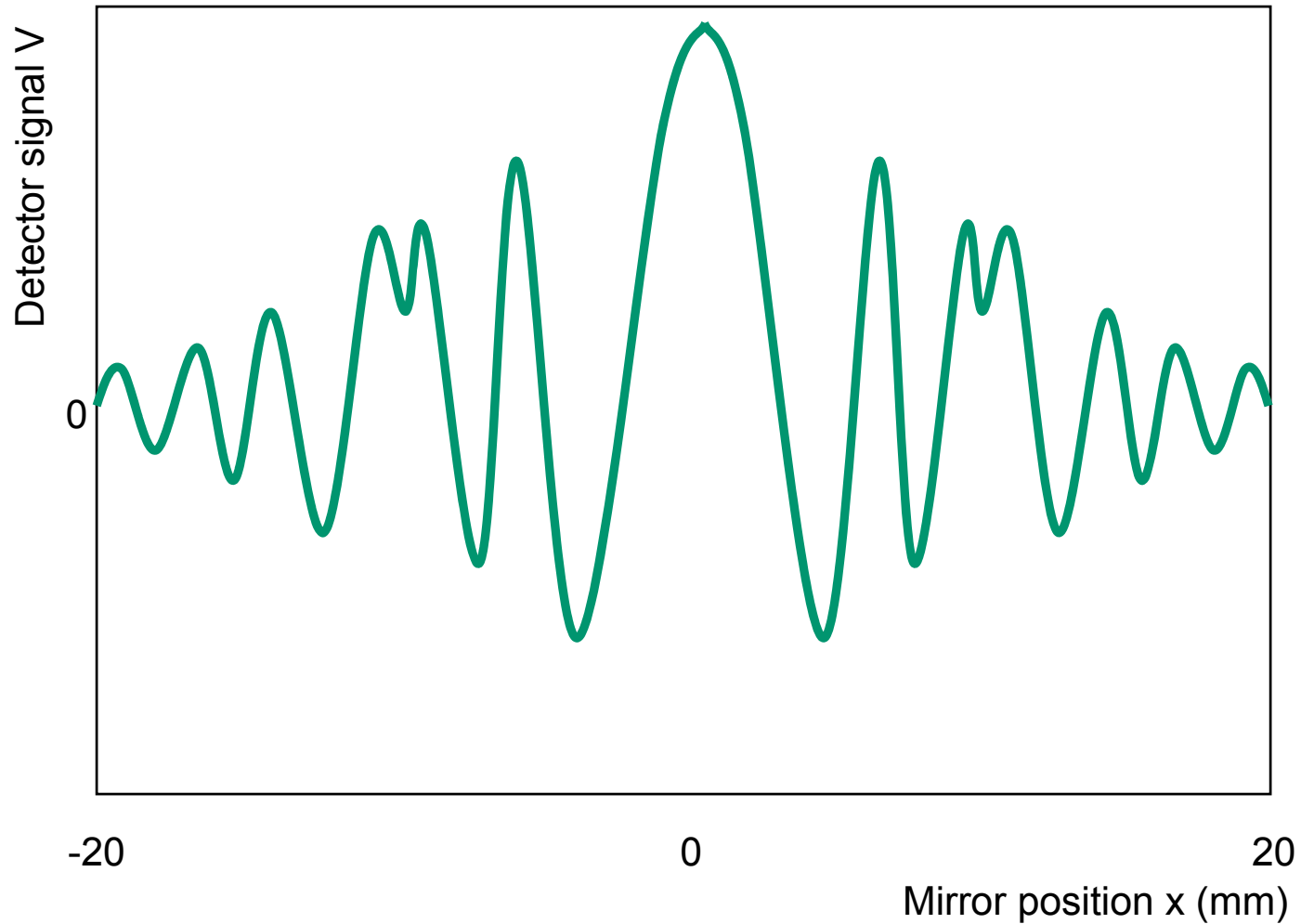
What was NOT measured :

i.e. what you expect if, whatever they are, the internal reference and the sky brightness are not the same.

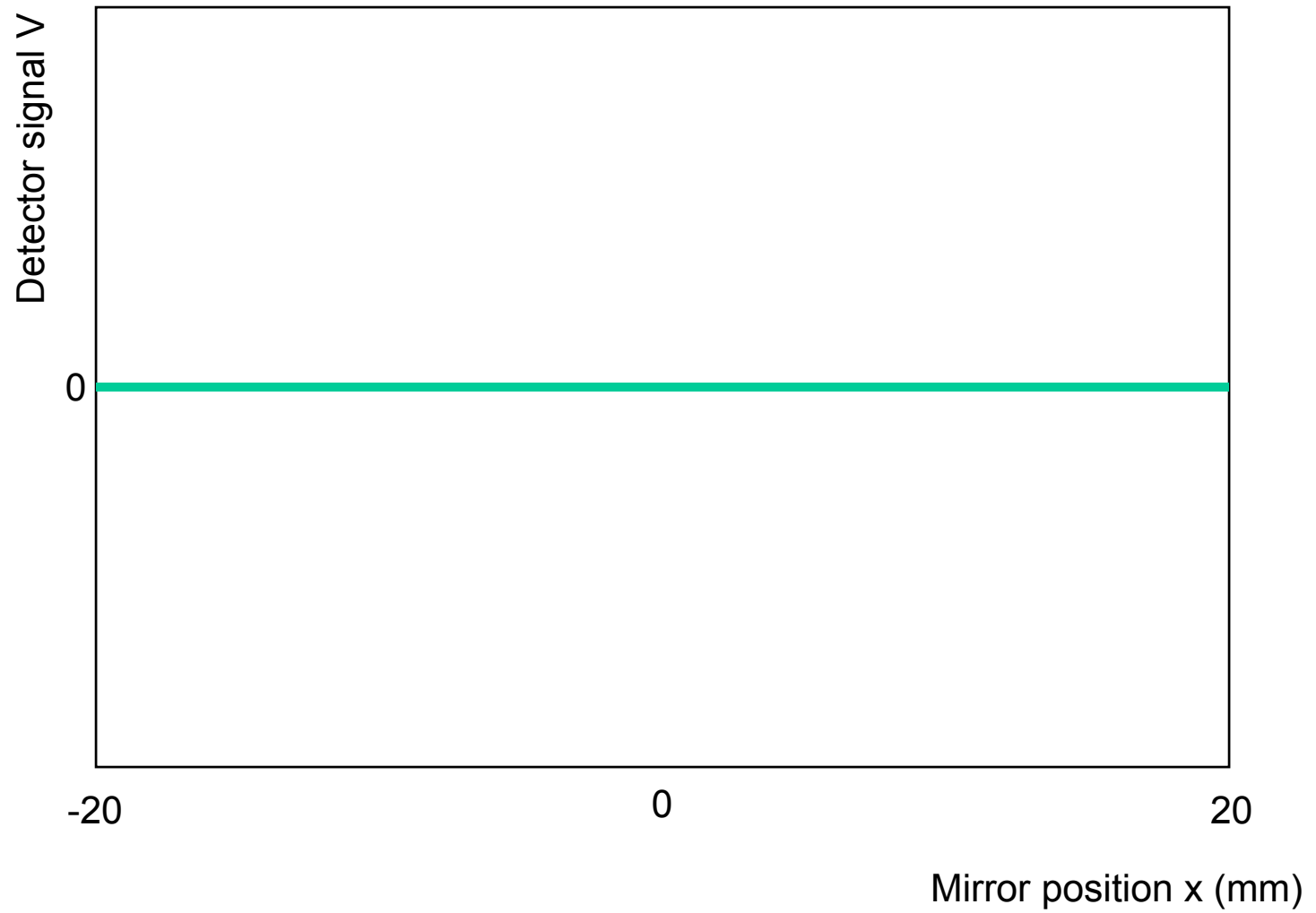


What was NOT measured :

i.e. what you expect if, whatever they are, the internal reference and the sky brightness are not the same.



What was actually measured :

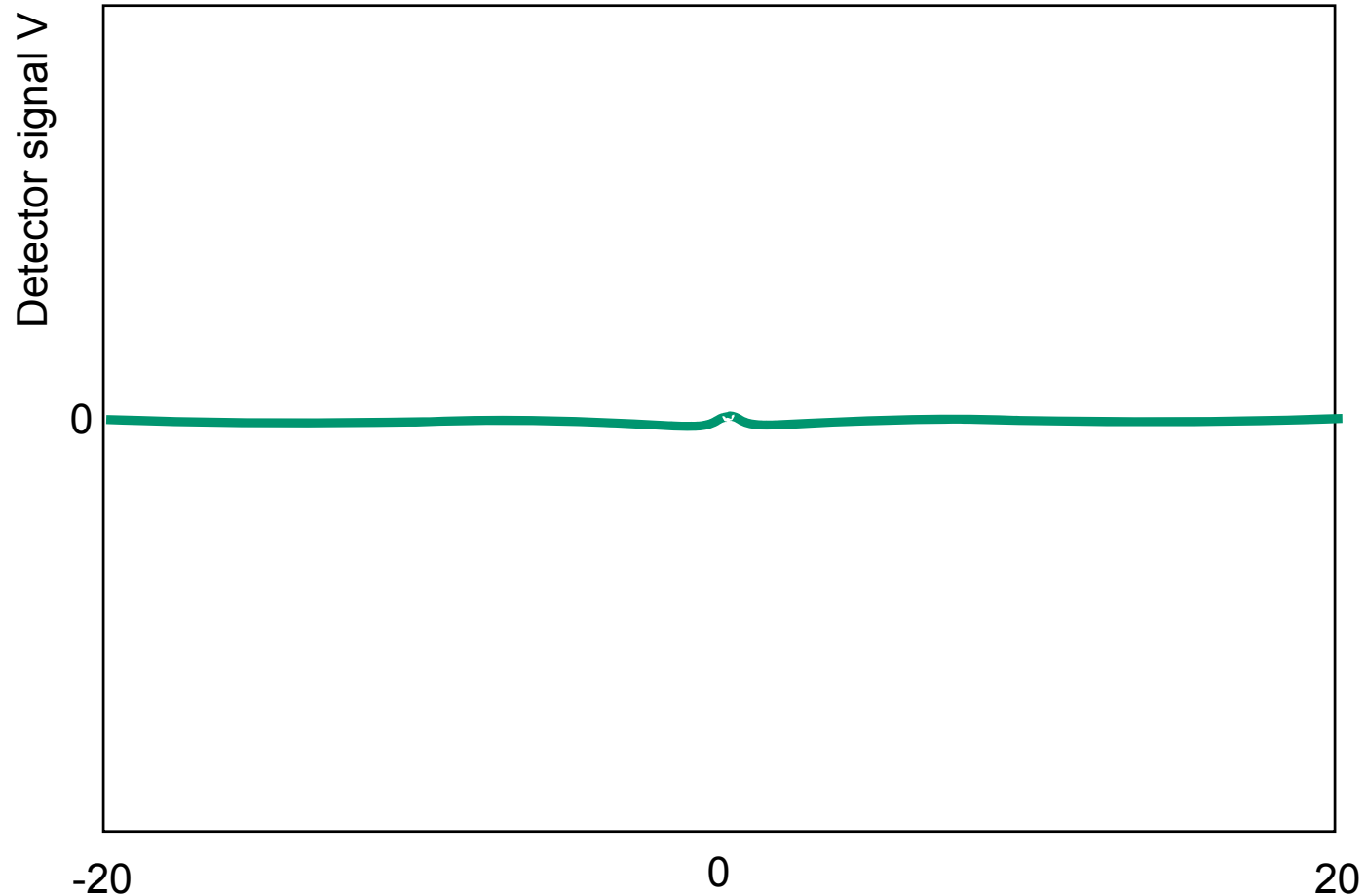


Note that :

As soon as you start to detect any small deviation from zero ..

- 1) You need accurate calibration of C and $rt(\sigma)$ to convert it into a $S_{\text{sky}} - S_{\text{ref}}$ signal
- 2) You start asking yourself if this deviation is in the sky or is in your reference blackbody

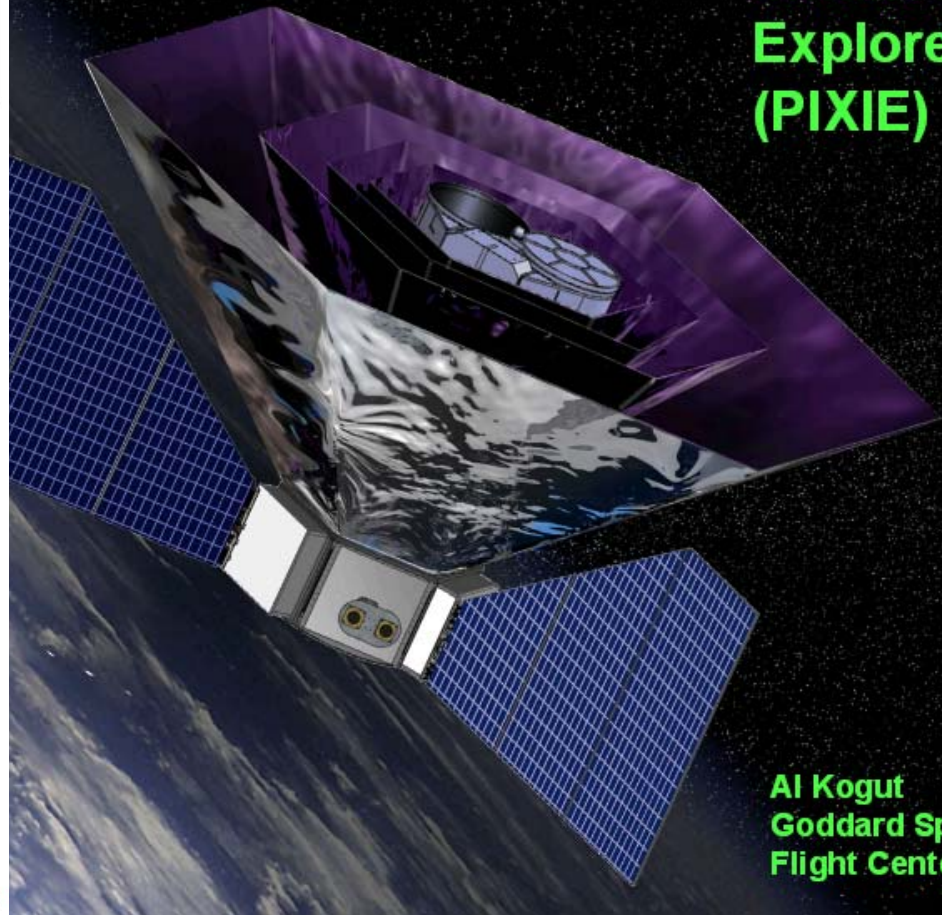
Being a differential instrument, your FTS cannot tell you the answer.



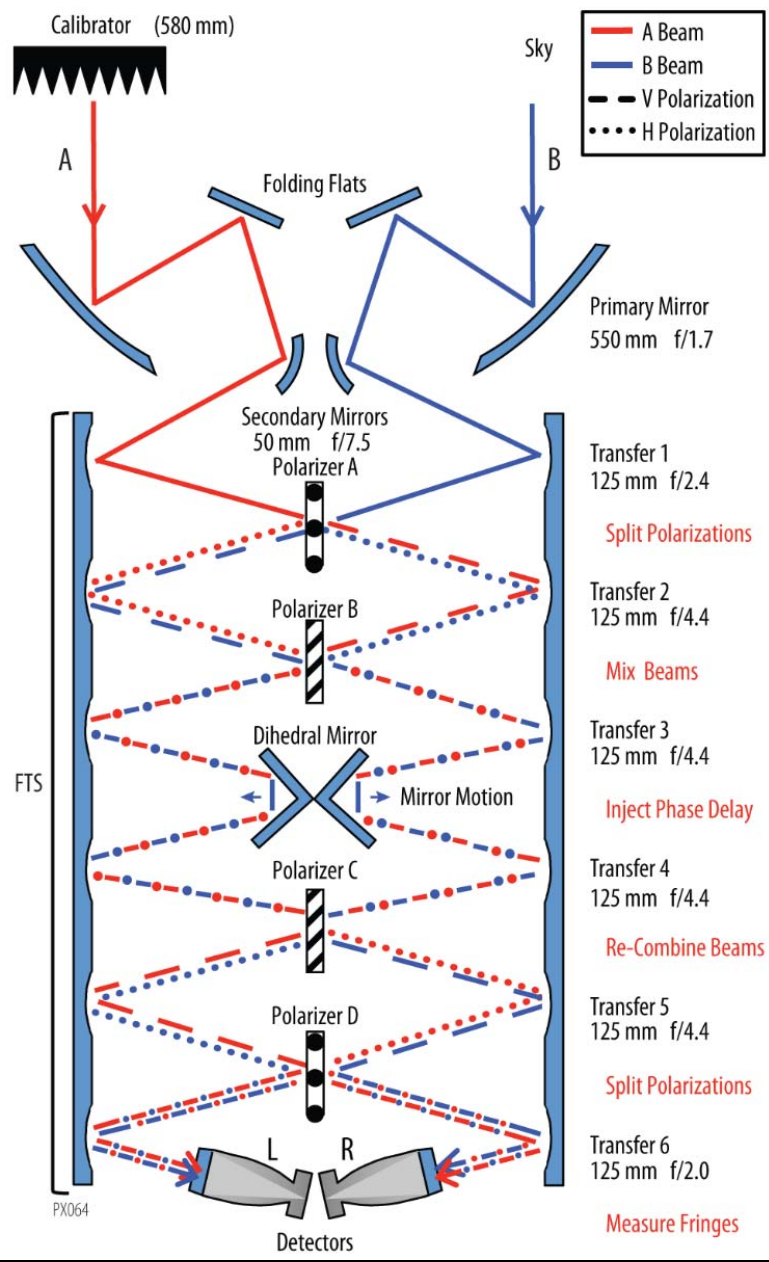
In COBE-FIRAS the deviations were less than the noise, i.e. less than 10^{-4} of the peak brightness of the CMB.

Mirror position x (mm)

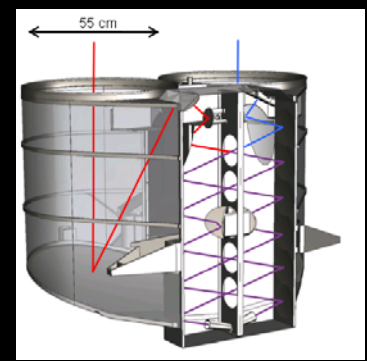
Primordial Inflation Explorer (PIXIE)



Al Kogut
Goddard Space
Flight Center



Looking at CMB deviations from Planck's law of the order of $<10^{-7}$!

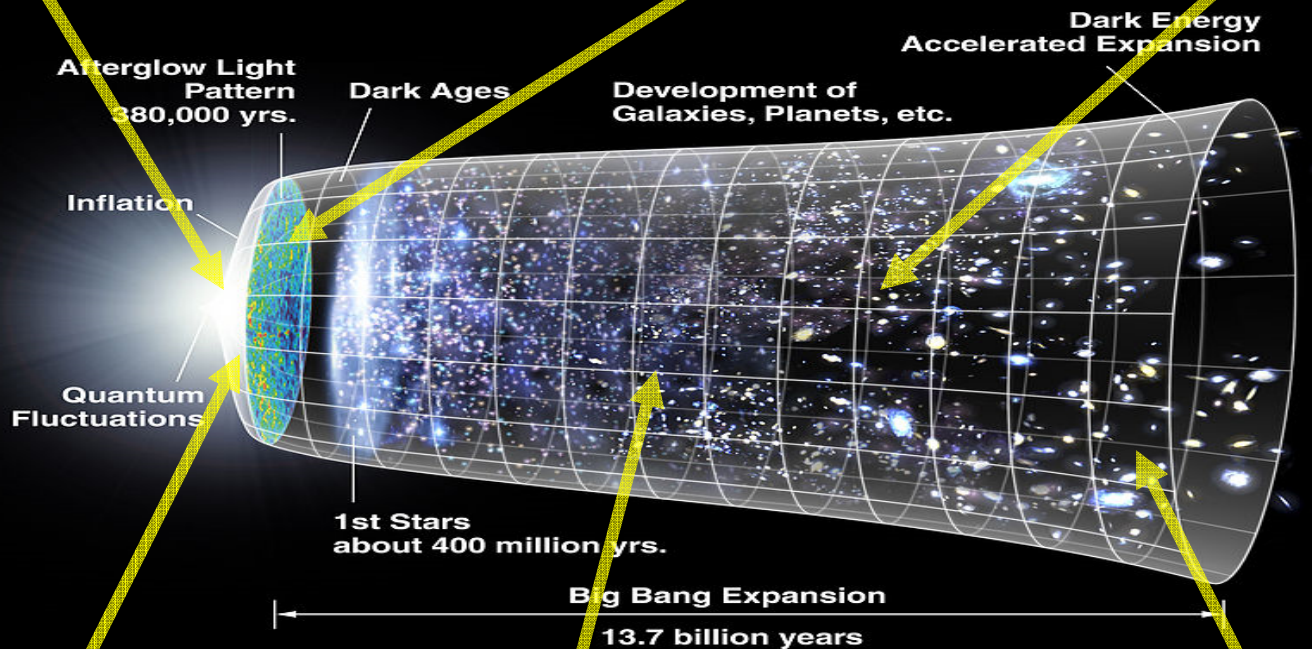


theory tells us that CMB measurements can probe all phases of the evolution of the Universe, but need to study the tiniest details: anisotropy and polarization

measurement of CMB polarization, Gaussianity, Search for tensor perturbations produced during inflation. Initial quantum fluctuations and inflation

Physics of the primeval fireball (acoustic oscillations of the primeval plasma). Physics of recombination

LSS (galaxy clusters, filaments) via Sunyaev-Zeldovich effect (SZ):



Probe epochs before recombination and new physics using CMB spectral distortion measurements

Map the gravitational potential all the way to $z=1100$ through CMB lensing

Dipole: our *absolute* motion in the universe