

Capture cross section measurements for ^{197}Au at GELINA

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Transmission

$$T = e^{-n \sigma_{\text{tot}}}$$

$$T_{\text{exp}} = \frac{C_{\text{in}}}{C_{\text{out}}}$$

- Incoming flux cancels
- Detection efficiency cancels

+ direct relation: $T \Leftrightarrow \sigma_{\text{tot}}$
 good geometry
 homogeneous sample

Reaction

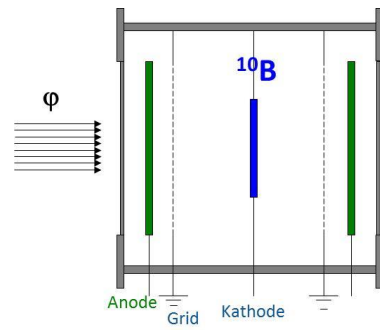
$$Y_r \cong (1 - e^{-n\sigma_{\text{tot}}}) \frac{\sigma_r}{\sigma_{\text{tot}}} + \dots$$

$$Y_{r,\text{exp}} = \frac{C_r}{\varepsilon_r \Omega_r P_r A_r \varphi}$$

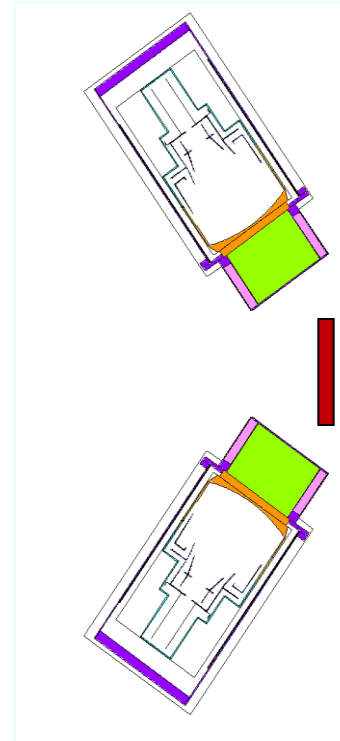
- φ Neutron flux
- ε_r Detection efficiency
- Ω_r solid angle (target-detector)
- P_r Escape probability
- A_r Effective area

+ complex relation : $Y \Leftrightarrow \sigma_r$
 $Y_r = f(\sigma_r, \sigma_{\text{tot}} \ \& \ \sigma_n)$
 only for $n\sigma_{\text{tot}} \ll 1$: $Y_r \cong n \sigma_r$

Flux measurement

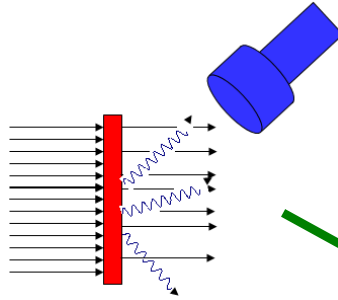


Capture detection system



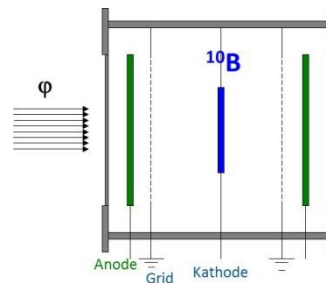
$$C_\gamma = \varepsilon_\gamma \Omega_\gamma P_\gamma Y_\gamma A_\gamma \varphi$$

- φ Neutron flux
- ε_γ Detection efficiency
- Ω_γ solid angle (target-detector)
- P_γ Escape probability
- A_γ Effective area



$$Y_{\gamma,exp} = \frac{\varepsilon_\varphi}{\varepsilon_\gamma} \frac{\Omega_\varphi}{\Omega_\gamma} \frac{P_\varphi}{P_\gamma} \frac{A_\varphi}{A_\gamma} \frac{C_\gamma}{C_\varphi} Y_\varphi$$

$$C_\varphi = \varepsilon_\varphi \Omega_\varphi P_\varphi Y_\varphi A_\varphi \varphi$$



$\Rightarrow Y_{\gamma,exp}$ is the ratio of results of 2 measurements

$\Rightarrow Y_\varphi$ is required,
i.e. standard reaction with known cross section

- Absolute measurements

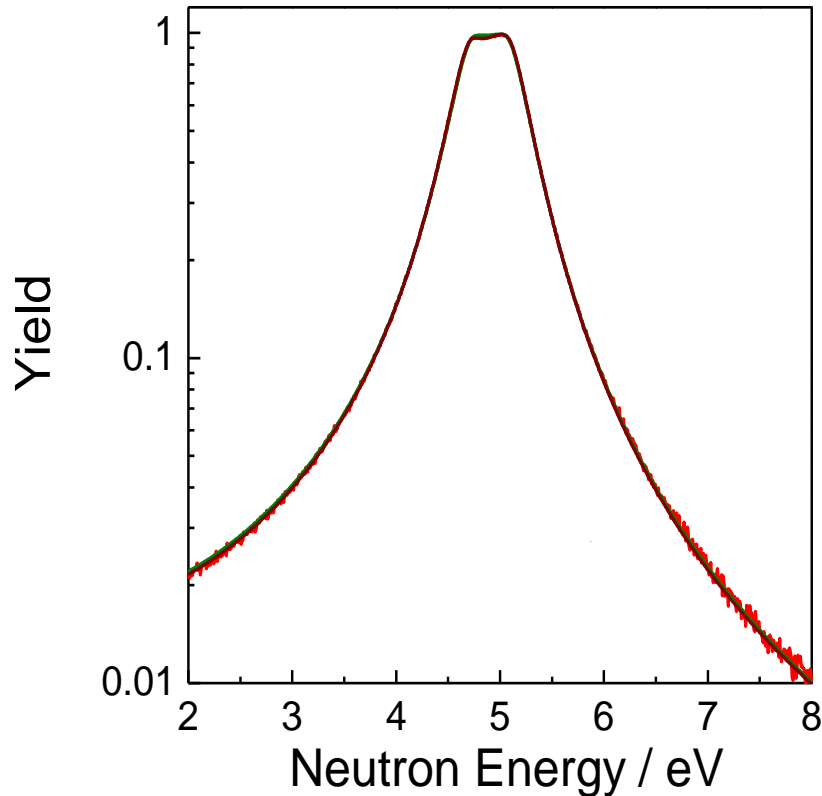
- All parameters (P , A , Ω , ε) have to be determined
- Y_φ has to be determined absolutely (absolute cross sections needed)

$$Y_{\gamma,\text{exp}} = \frac{\varepsilon_\varphi}{\varepsilon_\gamma} \frac{\Omega_\varphi}{\Omega_\gamma} \frac{P_\varphi}{P_\gamma} \frac{A_\varphi}{A_\gamma} \frac{C_\gamma}{C_\varphi} Y_\varphi$$

- Normalisation

- N accounts for all energy independent parameters & absolute value of neutron flux
- Y_φ only energy dependence is needed (shape of cross sections needed)
- N : determined at energy where Y_γ is known

$$Y_{\gamma,\text{exp}} = N \frac{C_\gamma}{C_\varphi} Y'_\varphi$$



$$Y_{\gamma} \cong \frac{\sigma_{\gamma}}{\sigma_{\text{tot}}} (1 - e^{-n\sigma_{\text{tot}}}) + \dots$$

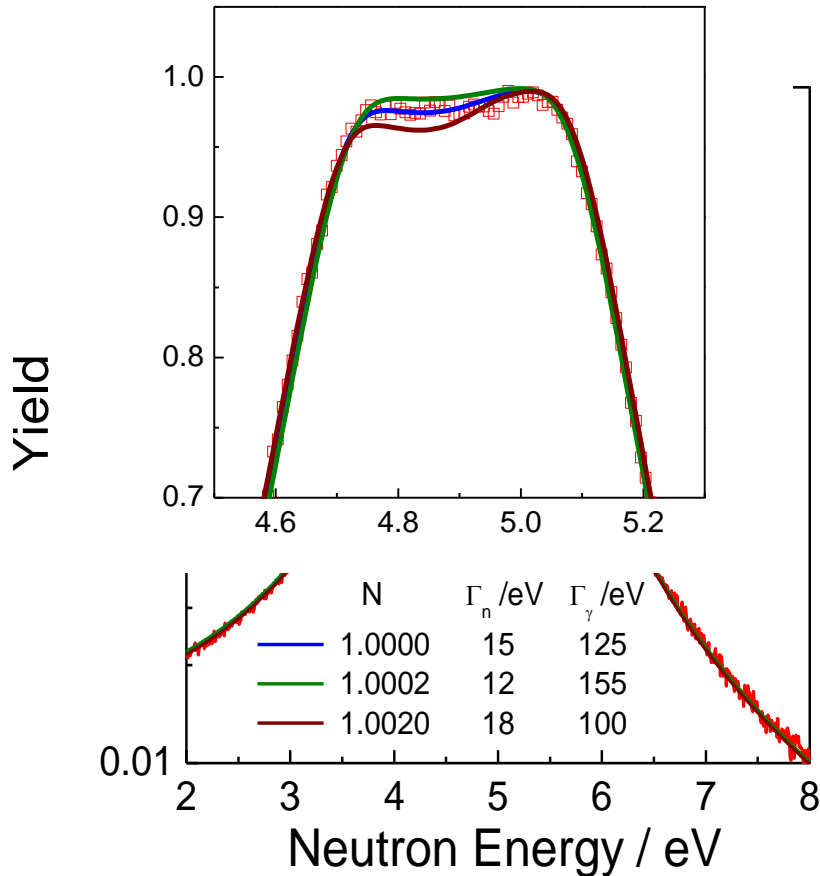
$$n\sigma_{\text{tot}} \gg 1 \text{ and } \sigma_{\gamma} \approx \sigma_{\text{tot}}$$

$$Y_{\gamma} \cong 1$$

$$\Rightarrow N \cong \frac{C_{\phi}}{C_{\gamma}} \frac{1}{Y_{\phi}}$$

σ_{ϕ} : only the relative energy dependence is required
 $\Rightarrow {}^{10}\text{B}(n,\alpha) \sim 1/v$

$$\frac{u_{Y_{\gamma,\text{exp}}}}{Y_{\text{exp},\gamma}} \leq 2\%$$



$$Y_\gamma \cong \frac{\sigma_\gamma}{\sigma_{\text{tot}}} (1 - e^{-n\sigma_{\text{tot}}}) + \dots$$

$$n\sigma_{\text{tot}} \gg 1 \text{ and } \sigma_\gamma \approx \sigma_{\text{tot}}$$

$$Y_\gamma \cong 1$$

$$\Rightarrow N \cong \frac{C_\phi}{C_\gamma} \frac{1}{Y_\phi}$$

N is independent of :

- sample thickness
- nuclear data

σ_ϕ : only the relative energy dependence is required
 $\Rightarrow {}^{10}\text{B}(n,\alpha) \sim 1/v$

$$\frac{u_{Y_{\gamma,\text{exp}}}}{Y_{\text{exp},\gamma}} \leq 2\%$$

- Neutron source

- moderated neutron beam
- 18° with normal of moderator face viewing FP4

- Filters :

- ^{10}B (0.005 at/b) overlap filter
- S and Na fixed black resonance filter

- Sample

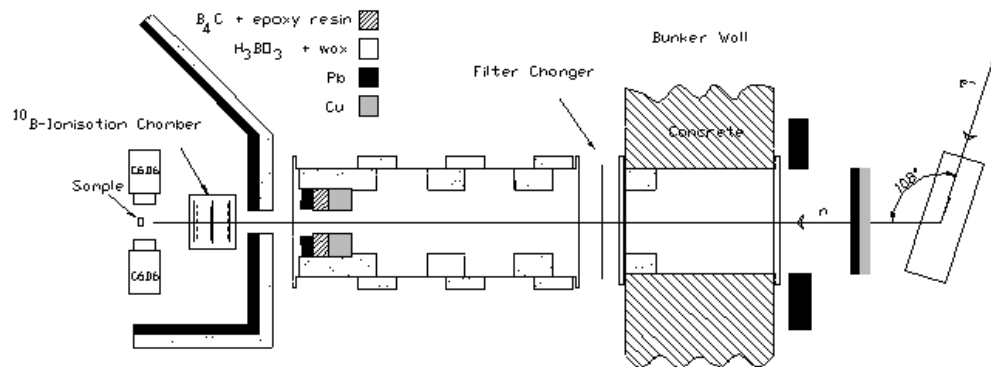
- Au-metal disc (80 mm diameter)
- $3.026 (0.001) 10^{-3}$ at/b & $5.596 (0.001) 10^{-3}$ at/b

- Neutron flux detector

- Frisch-gridded ionisation chamber
- $^{10}\text{B}(n,\alpha)$ reaction
- 2 back-to-back layers of ^{10}B (84 mm diameter)
- $2 \times 2.4 \cdot 10^{-6}$ at/b
- at 12.121 m from centre of neutron source

- Capture

- C_6D_6 -liquid NE230-scintillator
- 10 cm diameter
- 7.5 cm length
- at 12.938 m from centre of neutron source



- Measurement principles

- Total energy detection principle + Pulse Height Weighting Technique
- WF: Monte Carlo calculations
- Internal normalisation: 4.9 eV resonance

Total energy detection

- C_6D_6 liquid scintillators
 - 125°
 - Total energy detection principle + pulse height weighting technique
 - Weighting function: MC-simulations

$$C_w(T_n) = \int C_c(T_n, E_d) \text{WF}(E_d) dE_d$$

$$\varepsilon \propto E_\gamma \Rightarrow \varepsilon_c \propto S_n + E_n \frac{A}{1+A}$$

- Flux measurements (IC)
 - $^{10}\text{B}(n,\alpha)$



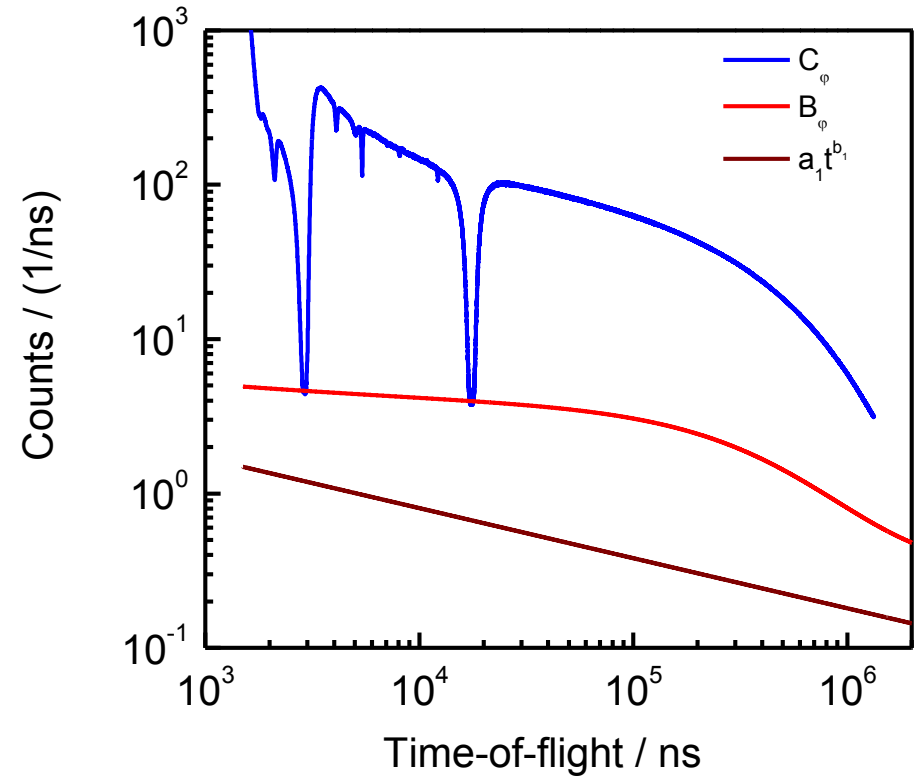
Background determination
 \Rightarrow black resonance technique



$$B_{\phi}(t) = a_0 + B_n(t) + B_{ov}(t)$$

- a_0 time independent ($< 10^{-1}$)
- $B_n(t)$ scattered neutrons
 $a_1 t^{b_1}$

Flux measurements



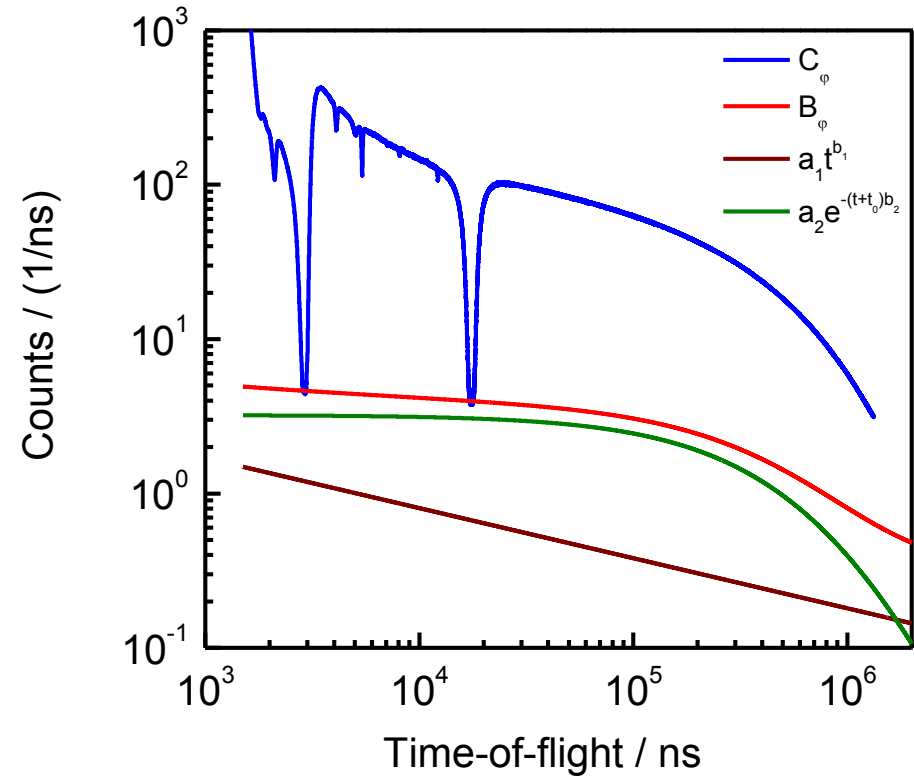
Background determination
 \Rightarrow black resonance technique



$$B_{\varphi}(t) = a_0 + B_n(t) + B_{ov}(t)$$

- a_0 time independent ($< 10^{-1}$)
- $B_n(t)$ scattered neutrons
 $a_1 t^{b_1}$
- $B_{ov}(t)$ overlap neutrons
 $a_2 e^{-b_2(t+t_0)}$

Flux measurements



$$B_{\varphi} = a_0 + a_1 t^{b_1} + a_2 e^{-b_2(t+t_0)}$$

Background determination
 \Rightarrow black resonance technique

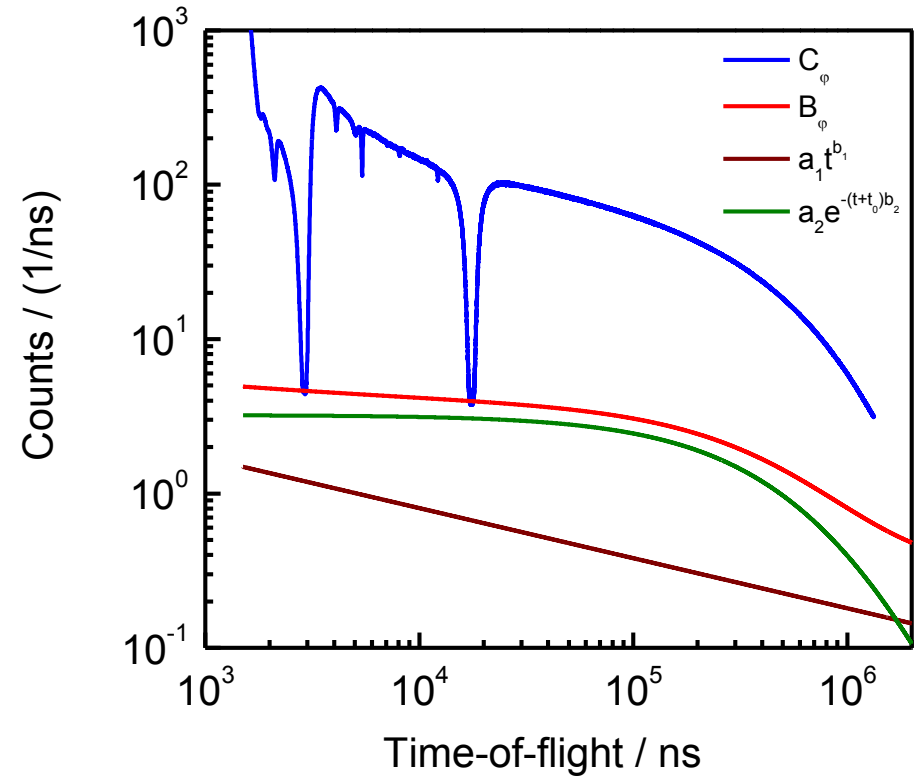


$$B_{\phi}(t) = a_0 + B_n(t) + B_{ov}(t)$$

- a_0 time independent ($< 10^{-1}$)
- $B_n(t)$ scattered neutrons
 $a_1 t^{b_1}$
- $B_{ov}(t)$ overlap neutrons
 $a_2 e^{-b_2(t+t_0)}$

Background influenced by sample
 \Rightarrow use of fixed background filters to adjust a_1 and a_2

Flux measurements



$$B_{\phi} = a_0 + a_1 t^{b_1} + a_2 e^{-b_2(t+t_0)}$$

Background determination
 \Rightarrow black resonance technique



$$B_{\varphi}(t) = a_0 + B_n(t) + B_{ov}(t)$$

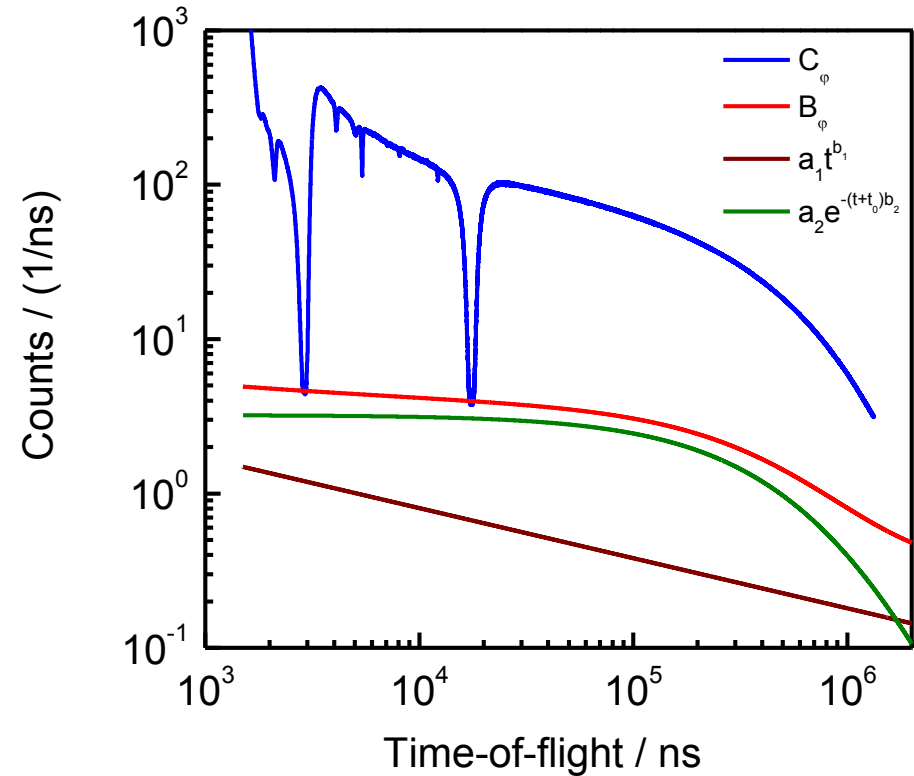
- a_0 time independent ($< 10^{-1}$)
- $B_n(t)$ scattered neutrons
 $a_1 t^{b_1}$
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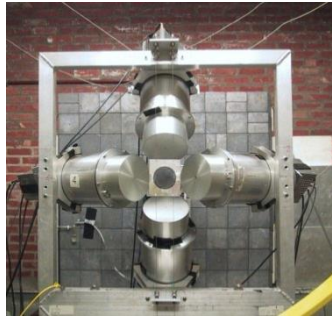
$$\frac{u_{B_{\varphi}}}{B_{\varphi}} \leq 3\% \Rightarrow \frac{u_{(C_{\varphi} - B_{\varphi})}}{C_{\varphi} - B_{\varphi}} \leq 0.3\%$$

Use of fixed BGR filters

$$B_{\varphi} = a_0 + a_1 t^{b_1} + a_2 e^{-b_2(t+t_0)}$$

Flux measurements

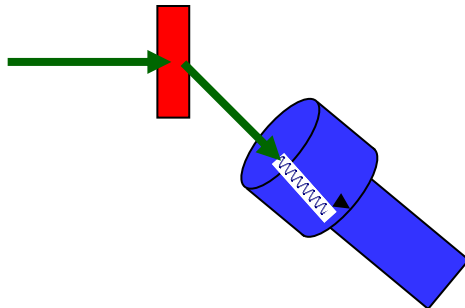




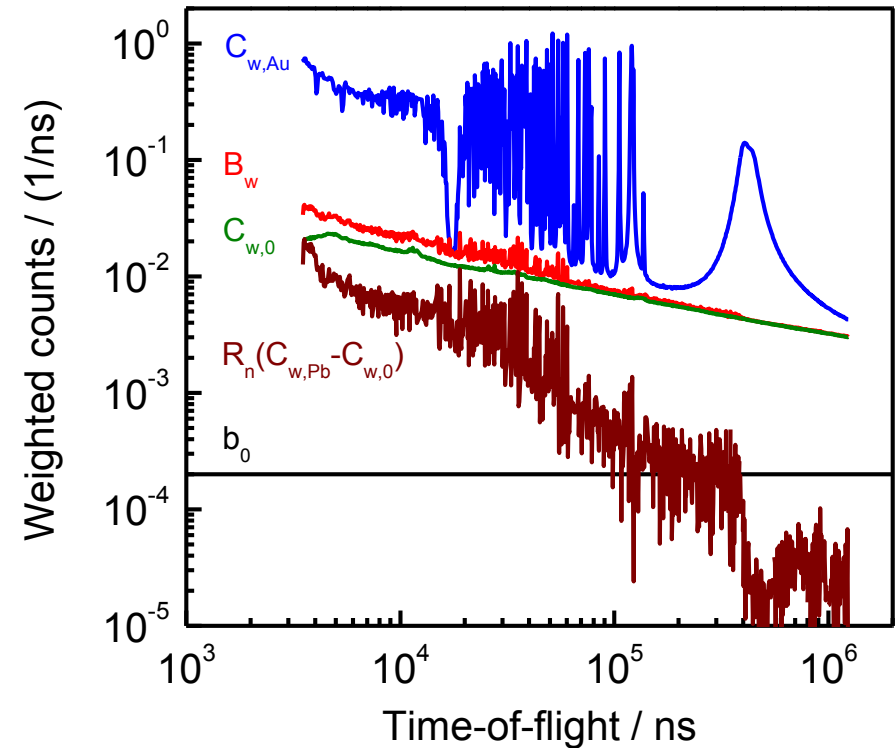
Background determination
 ⇒ additional measurements

$$B_w(t) = c_0 + C_{w,0}(t) + R_n(C_{w,Pb} - C_{w,0})(t)$$

- c_0 time independent background
- $C_{w,0}(t)$ neutrons scattered in environment + measurement without sample
- $C_{w,ns}(t)$ neutron sensitivity of detection system + measurements with ^{208}Pb metal disc



Capture measurements



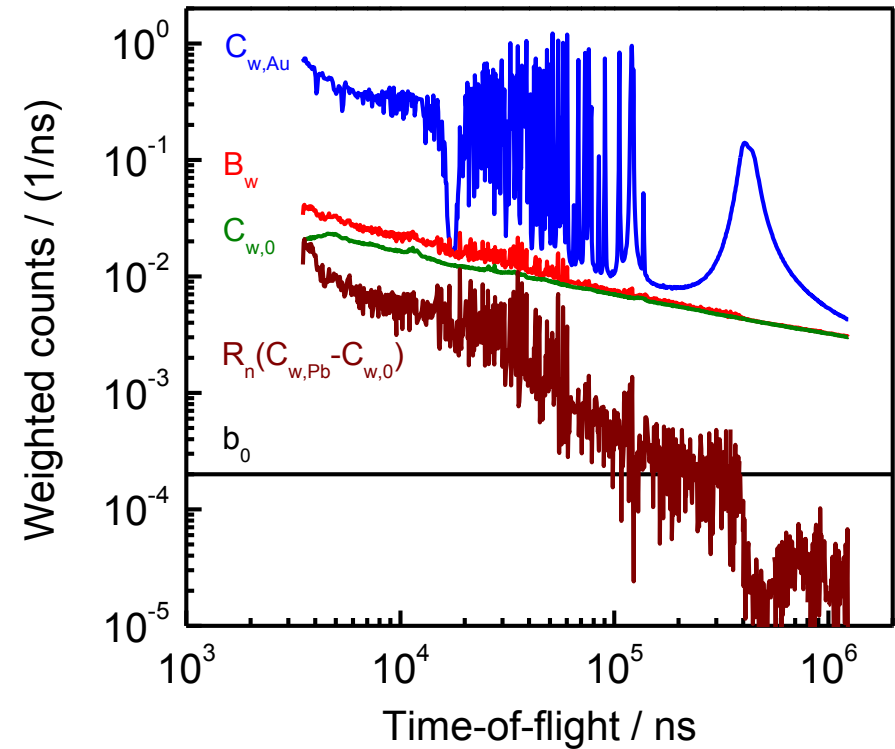


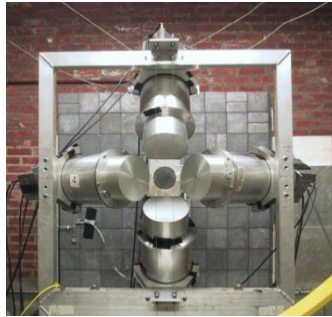
Background determination
 \Rightarrow additional measurements

$$B_w(t) = c_0 + C_{w,0}(t) + R_n(C_{w,Pb} - C_{w,0})(t)$$

- c_0 time independent background
 - $C_{w,0}(t)$ neutrons scattered in environment + measurement without sample
 - $C_{w,ns}(t)$ neutron sensitivity of detection system + measurement with ^{208}Pb sample
- $R_n(C_{w,Pb} - C_{w,0})$
 R_n is the ratio neutron yield $Y_{n,Au} / Y_{n,Pb}$

Capture measurements





Background determination
 \Rightarrow additional measurements

$$B_w(t) = c_0 + k_1 C_{w,0}(t) + k_2 R_n(C_{w,Pb} - C_{w,0})(t)$$

Uncertainties of systematic effects

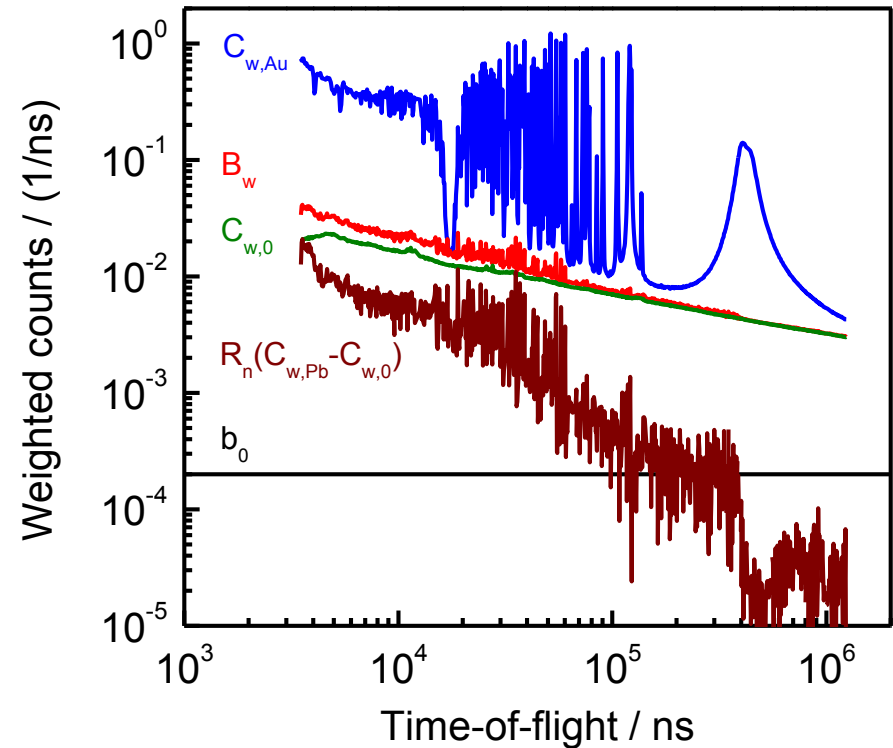
- $C_{w,0}(t)$ $k_1 = 1.00 \pm 0.03$

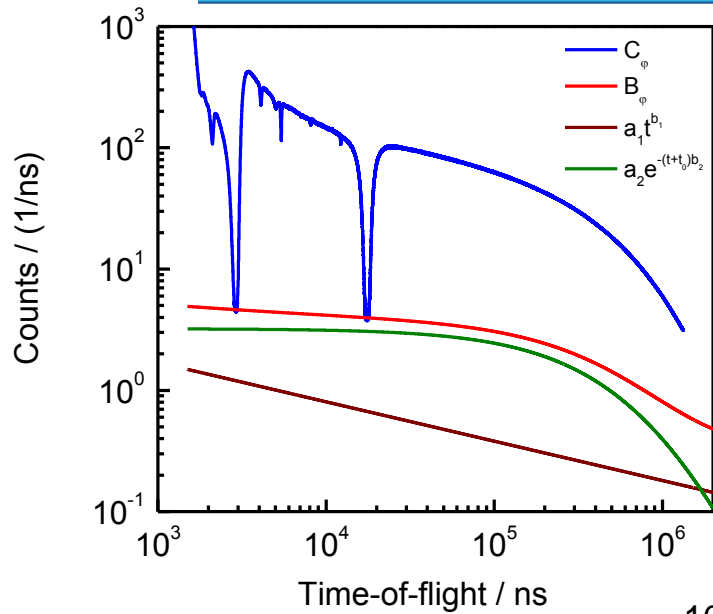
Use of fixed BGR filters

- $C_{w,ns}(t)$ $k_2 = 1.00 \pm 0.05$

$\Rightarrow (u_{k_1}, u_{k_2})$ correlated uncertainty components

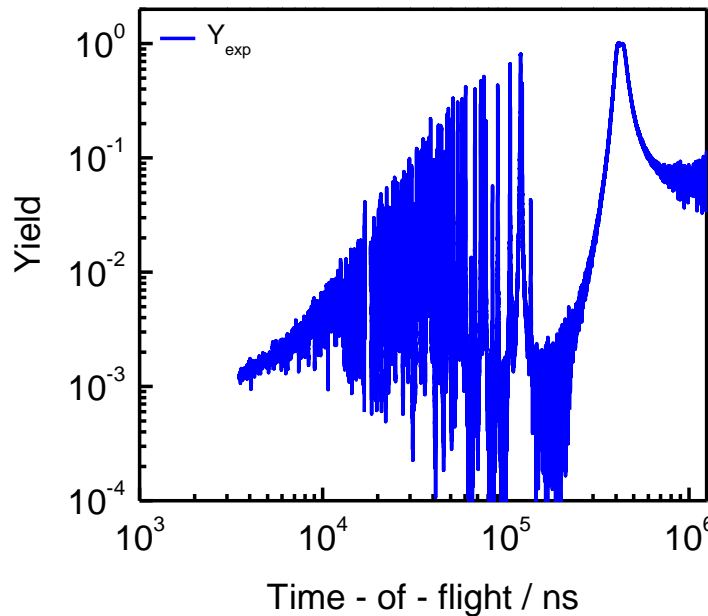
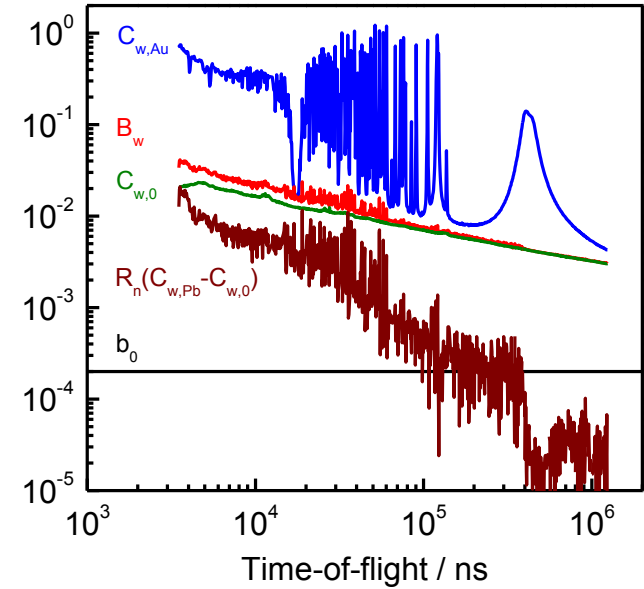
Capture measurements



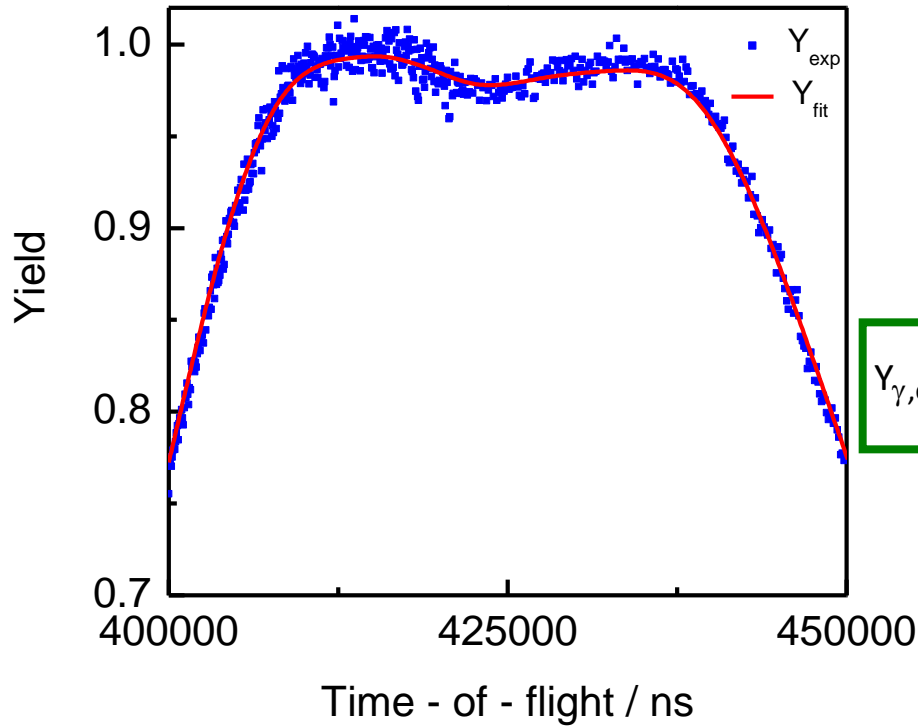


$$Y_{\gamma, \text{exp}} = N_C \frac{C_w - B_w}{C_\phi - B_\phi} Y_\phi$$

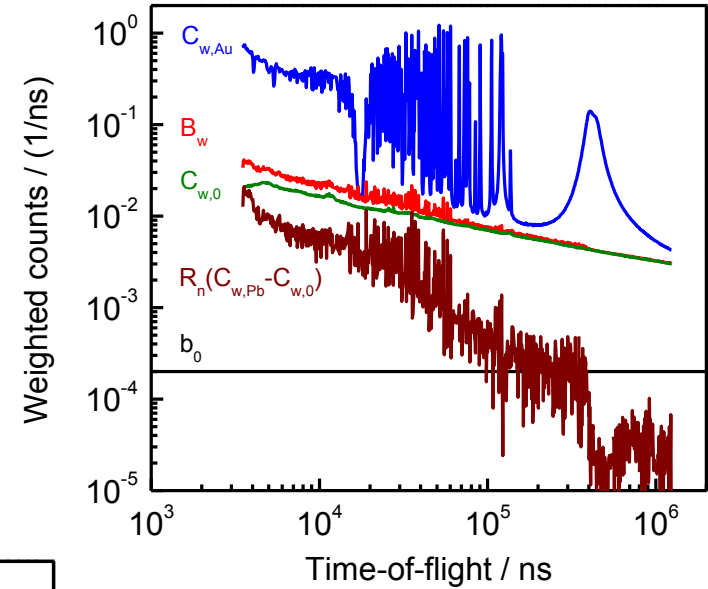
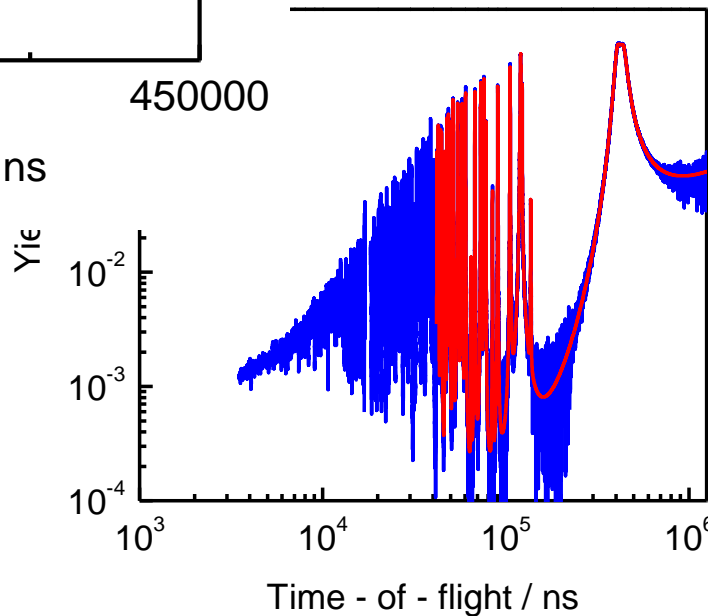
Weighted counts / (1/ns)



Use of fixed BGR filters :
reduces impact of systematic effects
due to background

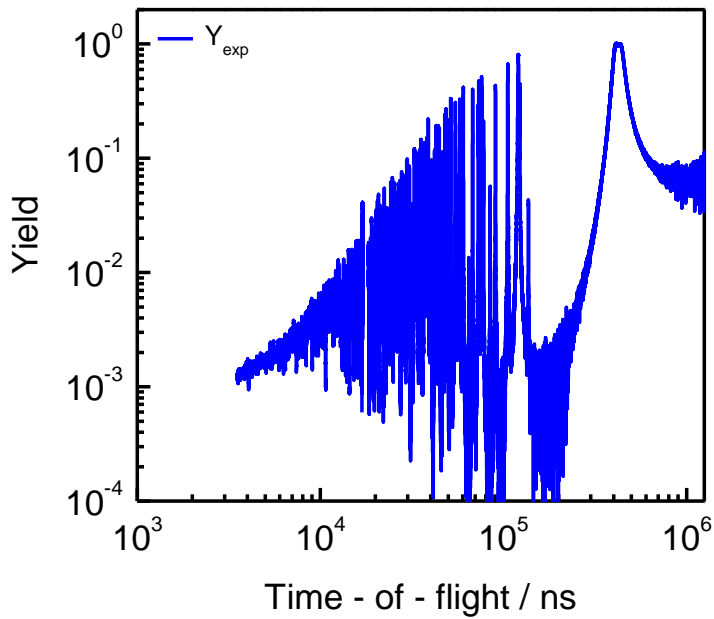


$$Y_{\gamma, \text{exp}} = N_C \frac{C_w - B_w}{C_\varphi - B_\varphi} Y_\varphi$$



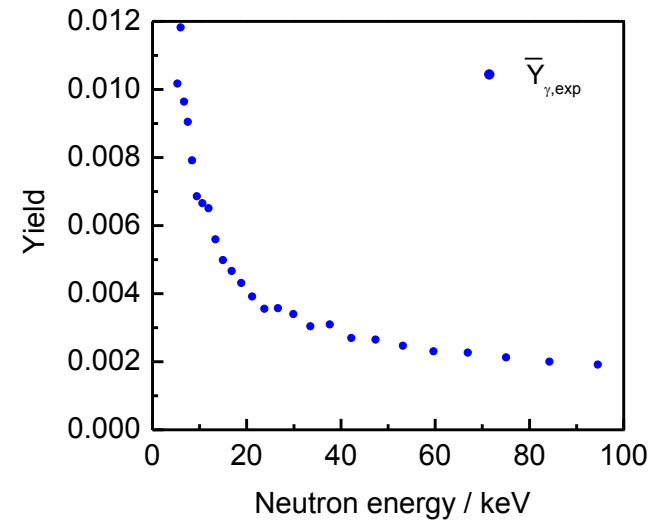
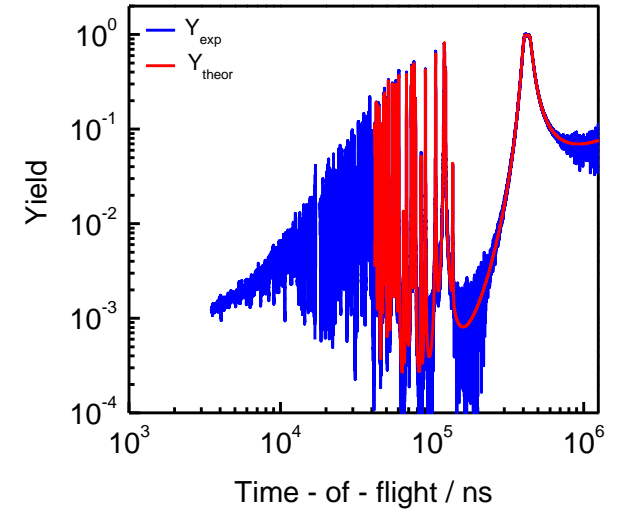
Saturated resonance at 4.9 eV
 with $\Gamma_n \ll \Gamma_\gamma$
 \Rightarrow no reference cross section
 except for shape of $^{10}\text{B}(n, \alpha)$

$$\frac{u_{N_C}}{N_C} \approx 1\%$$

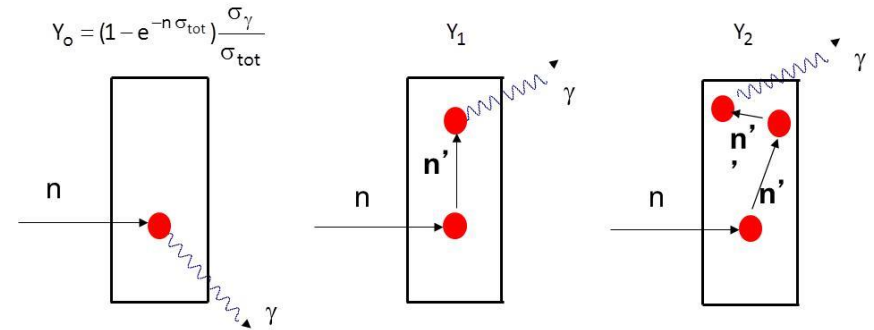
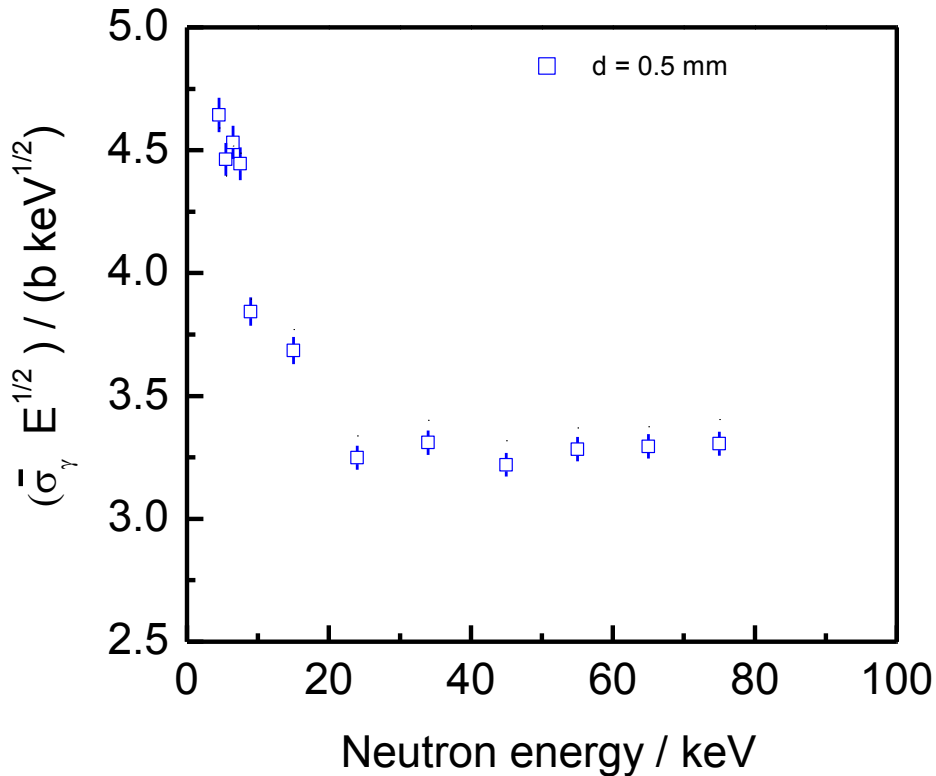


RRR
Resonance shape analysis

URR
Average parameters



$$\bar{\sigma}_{\gamma} = \frac{\bar{Y}_{\gamma, \text{exp}}}{n}$$

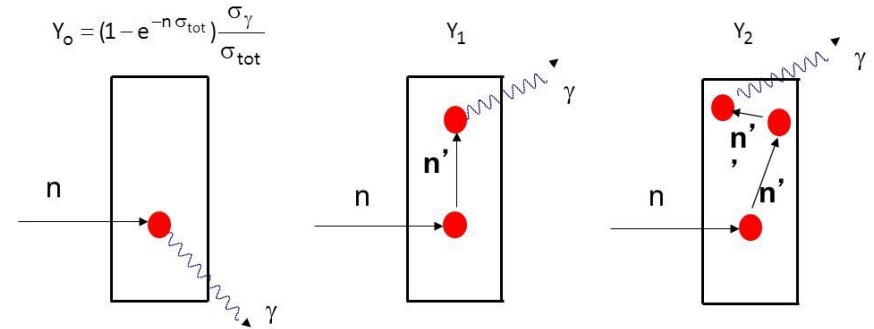
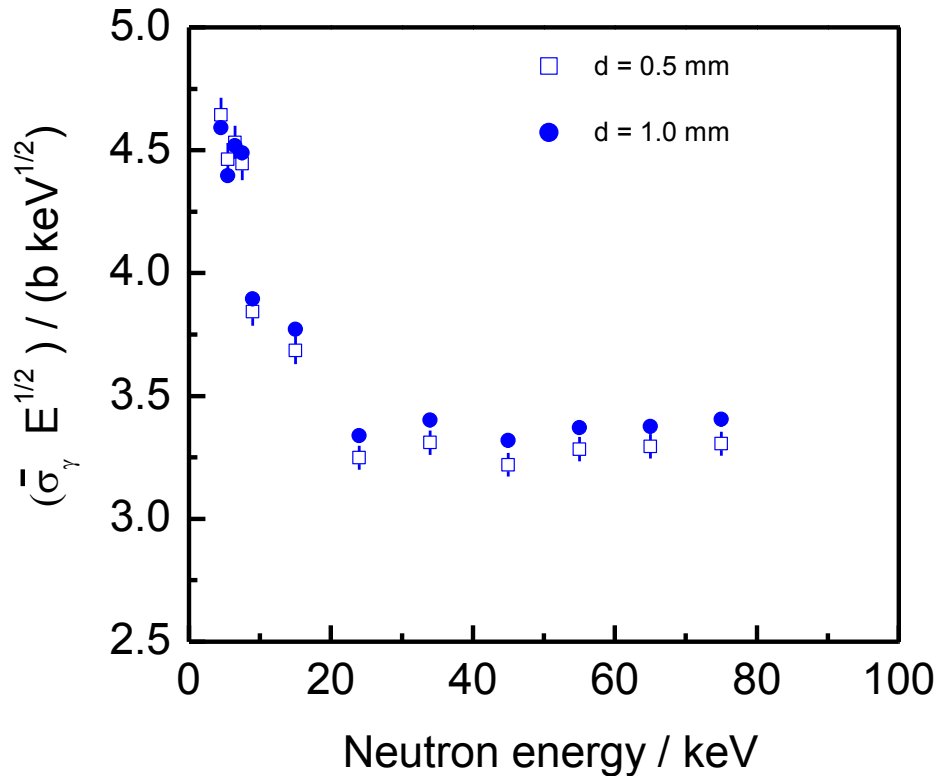


$$Y_0 = (1 - e^{-n\sigma_{\text{tot}}}) \frac{\sigma_{\gamma}}{\sigma_{\text{tot}}}$$

$$\bar{Y}_{\gamma, \text{exp}} = \bar{Y}_0 + \bar{Y}_1 + \bar{Y}_2 + \dots$$

$$\bar{Y}_{\gamma, \text{exp}} \neq n \bar{\sigma}_{\gamma}$$

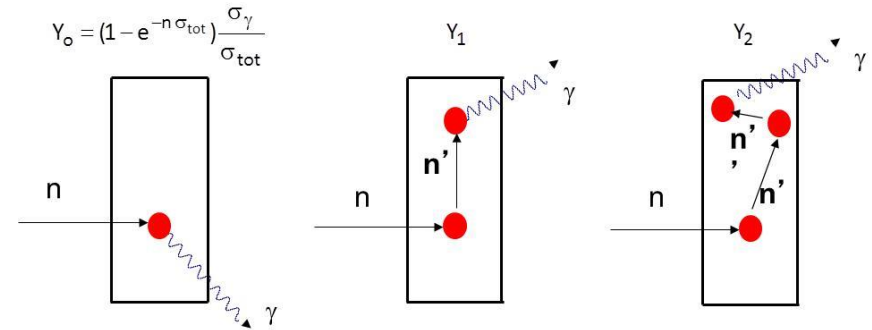
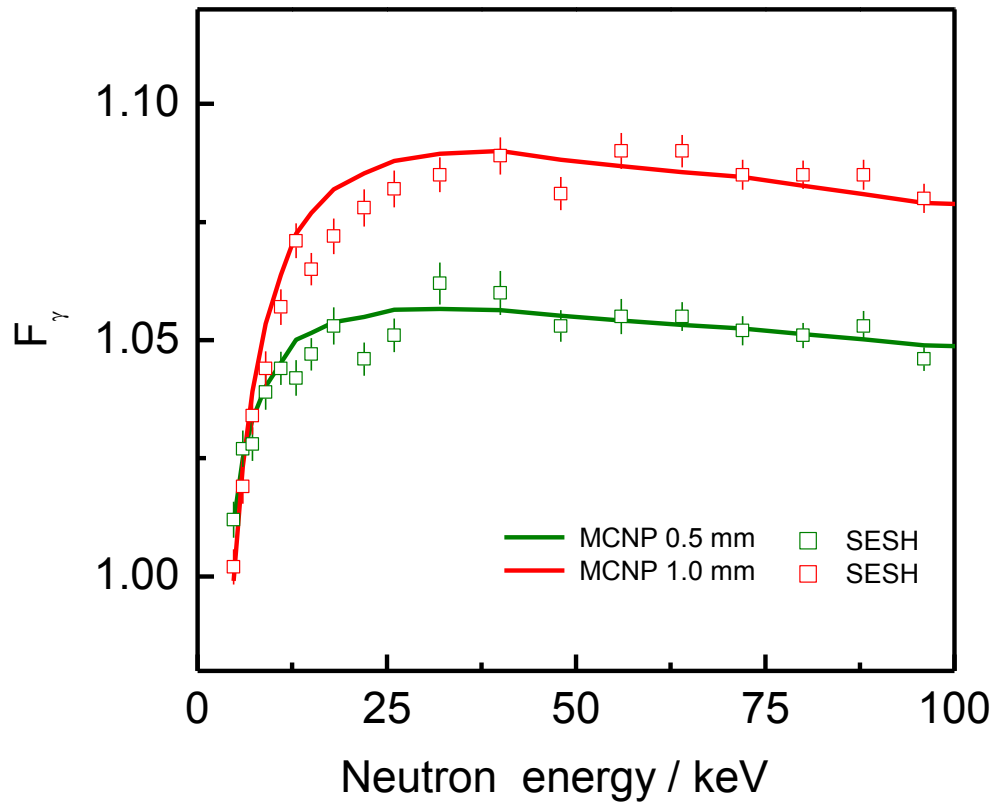
$$\bar{\sigma}_{\gamma} = \frac{\bar{Y}_{\gamma, \text{exp}}}{n}$$



$$\bar{Y}_{\gamma, \text{exp}} = \bar{Y}_0 + \bar{Y}_1 + \bar{Y}_2 + \dots$$

$$\bar{Y}_{\gamma, \text{exp}} \neq n \bar{\sigma}_{\gamma}$$

$$\bar{Y}_{\gamma, \text{exp}}(n) \cong n \bar{\sigma}_{\gamma} \quad \text{only for thin samples}$$



$$Y_0 = (1 - e^{-n\sigma_{\text{tot}}}) \frac{\sigma_{\gamma}}{\sigma_{\text{tot}}}$$

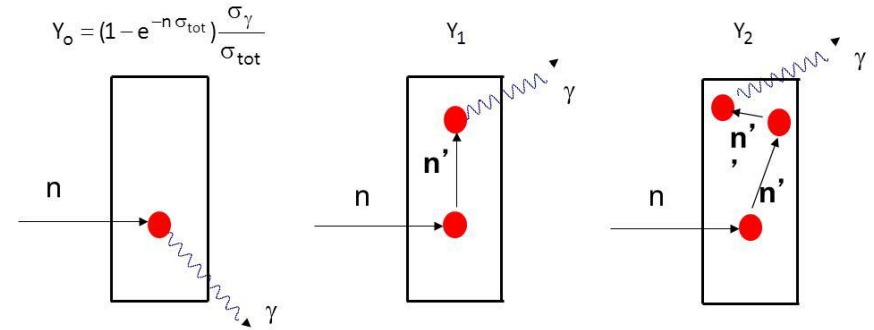
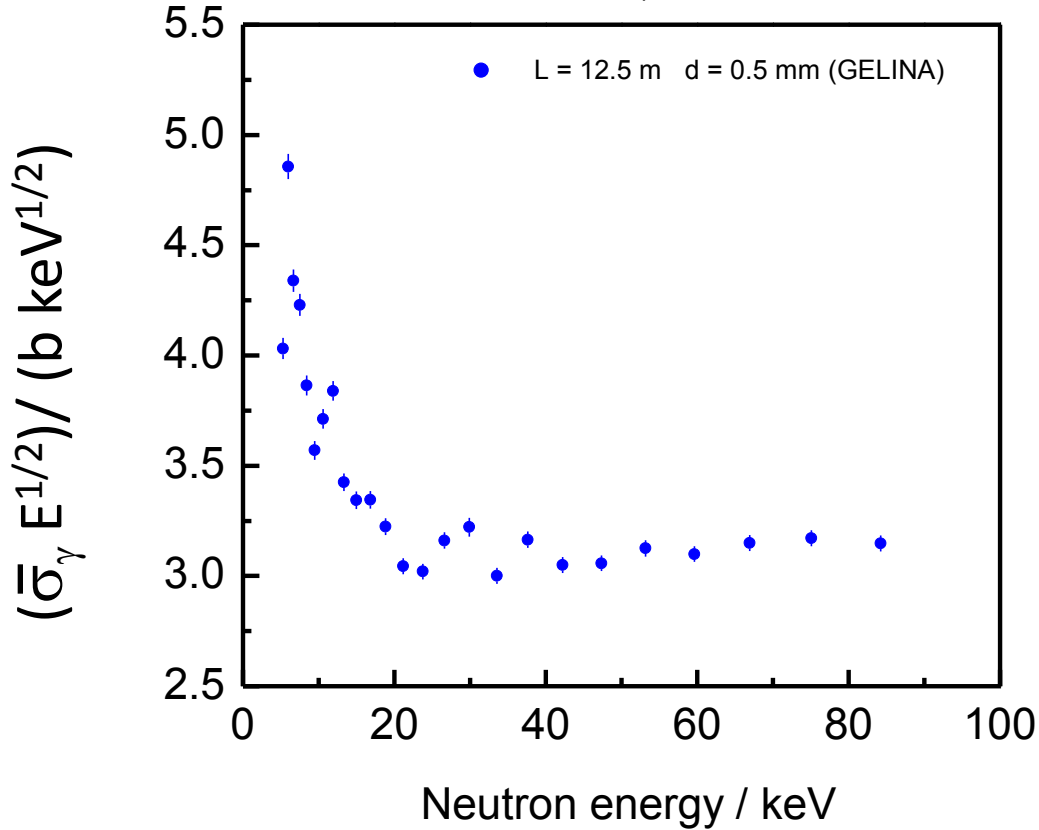
$$\bar{Y}_{\gamma, \text{exp}} = \bar{Y}_0 + \bar{Y}_1 + \bar{Y}_2 + \dots$$

$$\bar{Y}_{\gamma, \text{exp}} \neq n \bar{\sigma}_{\gamma}$$

$$\bar{Y}_{\gamma, \text{exp}}(n) = F_{\gamma} n \bar{\sigma}_{\gamma}$$

$$F_{\gamma} = \frac{\bar{Y}_{\gamma}(n)/n}{\bar{Y}_{\gamma}(n_{\text{thin}})/n_{\text{thin}}}$$

$$\bar{\sigma}_{\gamma} = \frac{\bar{Y}_{\gamma, \text{exp}}}{F_{\gamma} n}$$



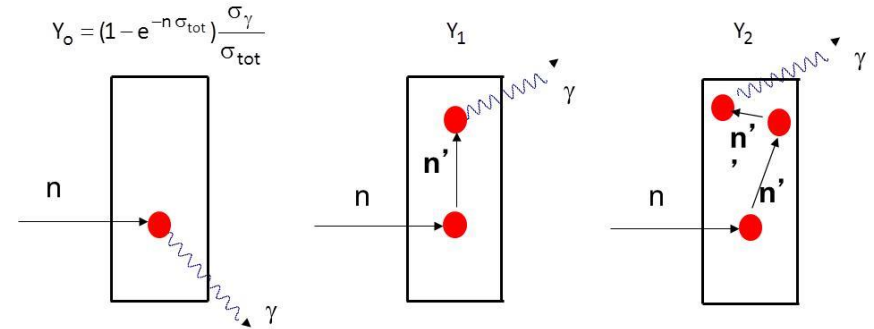
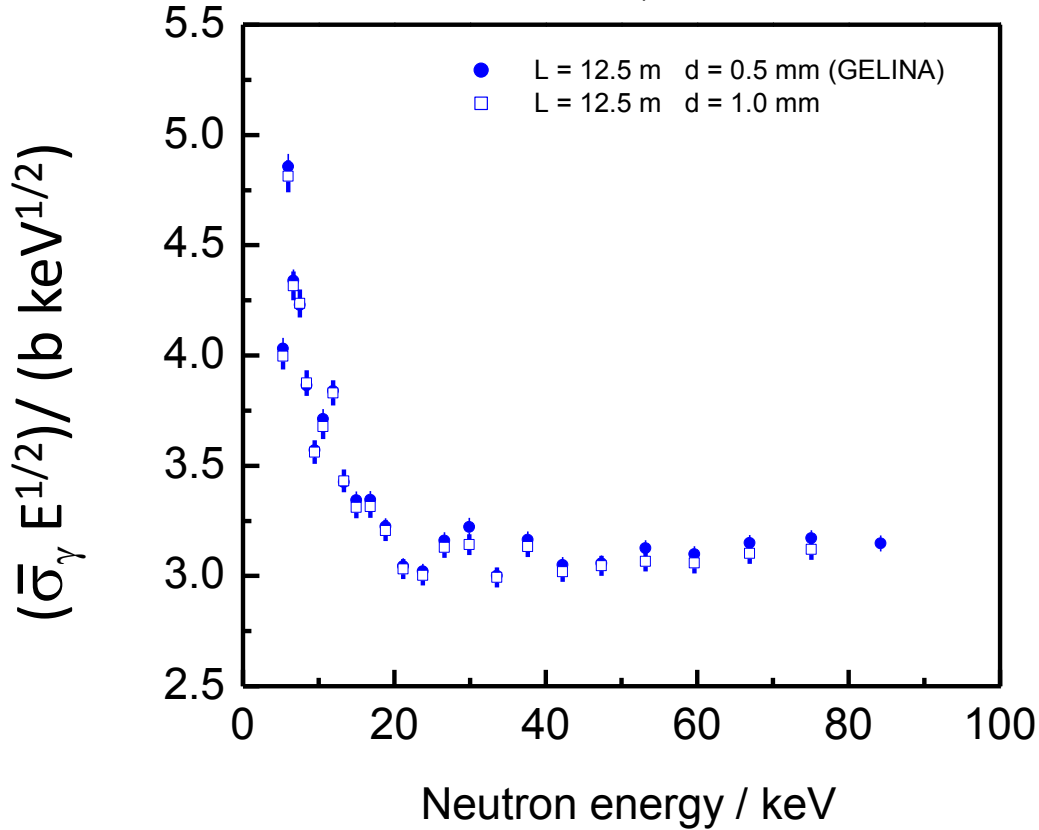
$$\bar{Y}_{\gamma, \text{exp}} = \bar{Y}_0 + \bar{Y}_1 + \bar{Y}_2 + \dots$$

$$\bar{Y}_{\gamma, \text{exp}} \neq n \bar{\sigma}_{\gamma}$$

$$\bar{Y}_{\gamma, \text{exp}}(n) = F_{\gamma} n \bar{\sigma}_{\gamma}$$

$$F_{\gamma} = \frac{\bar{Y}_{\gamma}(n)/n}{\bar{Y}_{\gamma}(n_{\text{thin}})/n_{\text{thin}}}$$

$$\bar{\sigma}_{\gamma} = \frac{\bar{Y}_{\gamma, \text{exp}}}{F_{\gamma} n}$$

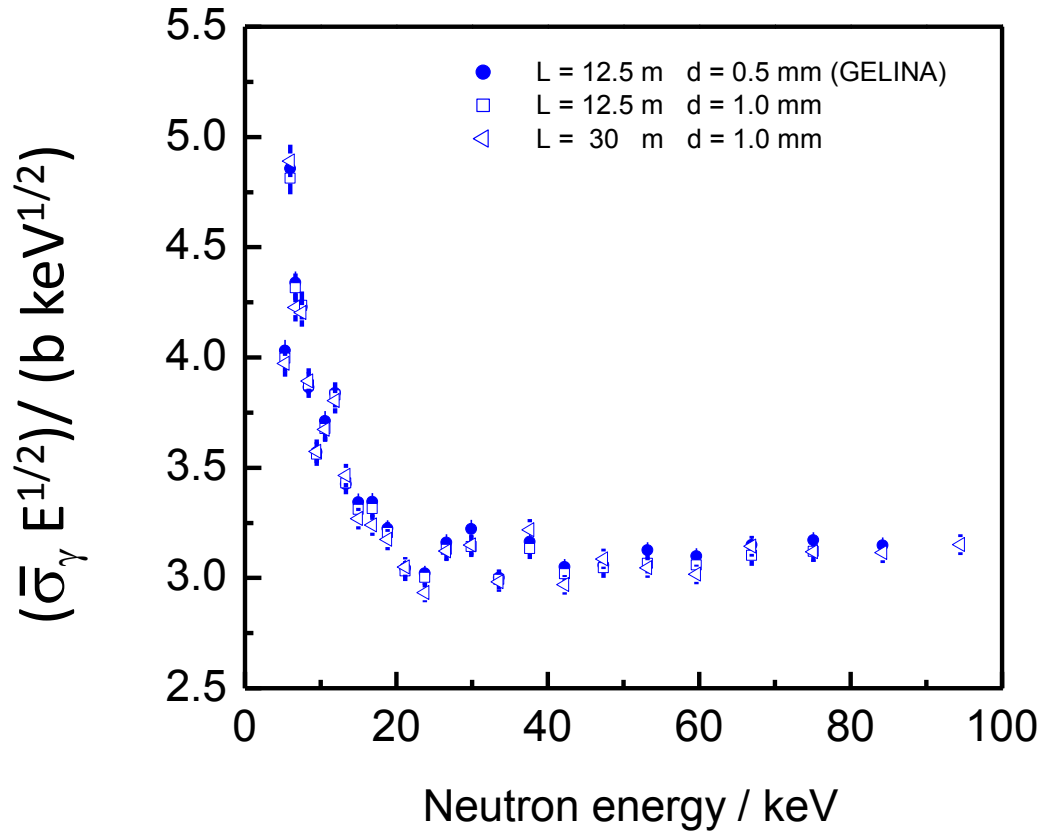


$$\bar{Y}_{\gamma, \text{exp}} = \bar{Y}_0 + \bar{Y}_1 + \bar{Y}_2 + \dots$$

$$\bar{Y}_{\gamma, \text{exp}} \neq n \bar{\sigma}_{\gamma}$$

$$\bar{Y}_{\gamma, \text{exp}}(n) = F_{\gamma} n \bar{\sigma}_{\gamma}$$

$$F_{\gamma} = \frac{\bar{Y}_{\gamma}(n) / n}{\bar{Y}_{\gamma}(n_{\text{thin}}) / n_{\text{thin}}}$$



$$Y_{\gamma, \text{exp}} = N_C \frac{C_w - B_w}{C_\varphi - B_\varphi} Y_\varphi$$

$$B_w(t) = c_0 + k_1 C_{w,0}(t) + k_2 R_n (C_{w,Pb} - C_{w,0})(t)$$

$$\frac{u_{N_C}}{N_C} \approx 1\% \quad \frac{u_{k_1}}{k_1} \approx 3\% \quad \frac{u_{k_2}}{k_2} \approx 5\%$$

$$\bar{\sigma}_\gamma = \frac{\bar{Y}_{\gamma, \text{exp}}}{F_\gamma n} \quad \frac{u_{\bar{\sigma}_\gamma}}{\bar{\sigma}_\gamma} \approx 1.2\%$$

Table 4. Average capture cross section ($\bar{\sigma}_\gamma$) and total uncertainty derived from the data obtained in this work. The information to derive the full covariance matrix based on the AGS concept (eq. (7)) is given: the diagonal elements of the uncorrelated components, $u_u = \sqrt{U_u}$ are in column 6, whereas columns 7–10 represent the matrix $S_{\eta=\{b_0, k_1, k_2, N_C\}}$. A high precision is given to ensure that the resulting covariance matrix can be inverted. The correction factor F_c for self-shielding multiple interaction is given in column 3.

E_l/eV	E_h/eV	F_c	$\bar{\sigma}_\gamma/\text{b}$	$u_{\bar{\sigma}_\gamma}/\text{b}$	AGS				
					u_u/b	S_{b_0}/b	S_{k_1}/b	S_{k_2}/b	S_{N_C}/b
3500	4000	0.9893	2.8696	0.0354	0.0084	-0.001731	-0.012957	-0.004330	0.031566
4000	4500	1.0022	2.2833	0.0284	0.0070	-0.001352	-0.010596	-0.003448	0.025116
4500	5000	1.0113	2.0888	0.0251	0.0058	-0.000981	-0.007942	-0.002375	0.022977
5000	5500	1.0180	1.5480	0.0190	0.0047	-0.000803	-0.006683	-0.001828	0.017028
5500	6000	1.0232	2.1886	0.0259	0.0057	-0.000734	-0.006767	-0.003384	0.024075
6000	6500	1.0273	1.7350	0.0207	0.0051	-0.000649	-0.006058	-0.001689	0.019085
6500	7000	1.0306	1.7219	0.0204	0.0049	-0.000567	-0.005428	-0.001737	0.018941
7000	8000	1.0345	1.5664	0.0184	0.0036	-0.000554	-0.005162	-0.001519	0.017230
8000	9000	1.0385	1.3120	0.0156	0.0034	-0.000494	-0.004555	-0.001419	0.014432
9000	10000	1.0414	1.1502	0.0137	0.0032	-0.000437	-0.004116	-0.001166	0.012652
10000	12000	1.0446	1.1625	0.0135	0.0023	-0.000374	-0.003588	-0.001109	0.012788
12000	14000	1.0475	0.9572	0.0113	0.0022	-0.000324	-0.003234	-0.000963	0.010529
14000	16000	1.0495	0.8569	0.0102	0.0022	-0.000283	-0.002963	-0.000830	0.009426
16000	18000	1.0509	0.8215	0.0097	0.0022	-0.000250	-0.002674	-0.000756	0.009037
18000	20000	1.0519	0.7329	0.0087	0.0021	-0.000225	-0.002411	-0.000705	0.008062
20000	24000	1.0529	0.6418	0.0076	0.0015	-0.000195	-0.002145	-0.000650	0.007060
24000	28000	1.0538	0.6165	0.0072	0.0015	-0.000168	-0.001929	-0.000703	0.006781
28000	32000	1.0542	0.5842	0.0076	0.0026	-0.000242	-0.002914	-0.000896	0.006426
32000	36000	1.0544	0.5160	0.0062	0.0016	-0.000144	-0.001835	-0.000669	0.005676
36000	40000	1.0544	0.5168	0.0061	0.0015	-0.000122	-0.001581	-0.000575	0.005685
40000	44000	1.0543	0.4709	0.0056	0.0014	-0.000103	-0.001343	-0.000487	0.005180
44000	52000	1.0539	0.4403	0.0051	0.0010	-0.000089	-0.001119	-0.000440	0.004843
52000	60000	1.0533	0.4192	0.0049	0.0011	-0.000088	-0.001074	-0.000610	0.004612
60000	68000	1.0526	0.3894	0.0045	0.0009	-0.000062	-0.000739	-0.000523	0.004284
68000	76000	1.0517	0.3771	0.0043	0.0009	-0.000054	-0.000632	-0.000500	0.004148
76000	84000	1.0508	0.3429	0.0039	0.0009	-0.000054	-0.000619	-0.000335	0.003772

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4500	5000	1.0113	2.0888	0.0251	0.0058	-0.000981	-0.007942	-0.002375	0.022977
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16000	18000	1.0509	0.8215	0.0097	0.0022	-0.000250	-0.002674	-0.000756	0.009037
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20000	24000	1.0529	0.6418	0.0076	0.0015	-0.000195	-0.002145	-0.000650	0.007060
24000	28000	1.0538	0.6165	0.0072	0.0015	-0.000168	-0.001929	-0.000703	0.006781
28000	32000	1.0542	0.5842	0.0076	0.0026	-0.000242	-0.002914	-0.000896	0.006426
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10000	12000	1.0446	1.1625	0.0135	0.0023	-0.000374	-0.003588	-0.001109	0.012788
12000	14000	1.0475	0.9572	0.0113	0.0022	-0.000324	-0.003234	-0.000963	0.010529
14000	16000	1.0495	0.8569	0.0102	0.0022	-0.000283	-0.002963	-0.000830	0.009426
16000	18000	1.0509	0.8215	0.0097	0.0022	-0.000250	-0.002674	-0.000756	0.009037
18000	20000	1.0519	0.7329	0.0087	0.0021	-0.000225	-0.002411	-0.000705	0.008062
20000	24000	1.0529	0.6418	0.0076	0.0015	-0.000195	-0.002145	-0.000650	0.007060
24000	28000	1.0538	0.6165	0.0072	0.0015	-0.000168	-0.001929	-0.000703	0.006781
28000	32000	1.0542	0.5842	0.0076	0.0026	-0.000242	-0.002914	-0.000896	0.006426
32000	36000	1.0544	0.5160	0.0062	0.0016	-0.000144	-0.001835	-0.000669	0.005676
36000	40000	1.0544	0.5168	0.0061	0.0015	-0.000122	-0.001581	-0.000575	0.005685
40000	44000	1.0543	0.4709	0.0056	0.0014	-0.000103	-0.001343	-0.000487	0.005180
44000	52000	1.0539	0.4403	0.0051	0.0010	-0.000089	-0.001119	-0.000440	0.004843
52000	60000	1.0533	0.4192	0.0049	0.0011	-0.000088	-0.001074	-0.000610	0.004612
60000	68000	1.0526	0.3894	0.0045	0.0009	-0.000062	-0.000739	-0.000523	0.004284
68000	76000	1.0517	0.3771	0.0043	0.0009	-0.000054	-0.000632	-0.000500	0.004148
76000	84000	1.0508	0.3429	0.0039	0.0009	-0.000054	-0.000619	-0.000335	0.003772

$$(C_w, C_{w,0}, C_{w,Pb}, C_\phi)$$

$$B_w(t) = b_0 + k_1 C_{w,0}(t) + k_2 R_n(C_{w,Pb} - C_{w,0})(t)$$

$$Y_{\gamma,exp} = N_C \frac{C_w - B_w}{C_\phi - B_\phi} Y_\phi$$

$$\bar{Y}_{exp}$$

$$\bar{\sigma}_\gamma = \frac{\bar{Y}_{\gamma,exp}}{F_\gamma n}$$

+ covariance data

u_u and $S_{(b_0, k_1, k_2, N_C)}$

$$V_{\vec{Z}} = D_{\vec{Z}} + S_{\vec{Z}} S_{\vec{Z}}^T$$

Table 4. Average capture cross section ($\bar{\sigma}_\gamma$) and total uncertainty derived from the data obtained in this work. The information to derive the full covariance matrix based on the AGS concept (eq. (7)) is given: the diagonal elements of the uncorrelated components, $u_u = \sqrt{U_u}$ are in column 6, whereas columns 7–10 represent the matrix $S_{\eta=\{b_0, k_1, k_2, N_C\}}$. A high precision is given to ensure that the resulting covariance matrix can be inverted. The correction factor F_c for self-shielding multiple interaction is given in column 3.

E_l/eV	E_h/eV	F_c	$\bar{\sigma}_\gamma/b$	$u_{\bar{\sigma}_\gamma}/b$	AGS				
					u_u/b	S_{b_0}/b	S_{k_1}/b	S_{k_2}/b	S_{N_C}/b
3500	4000	0.9893	2.8696	0.0354	0.0084	-0.001731	-0.012957	-0.004330	0.031566
4000	4500	1.0022	2.2833	0.0284	0.0070	-0.001352	-0.010596	-0.003448	0.025116
4500	5000	1.0113	2.0888	0.0251	0.0058	-0.000981	-0.007942	-0.002375	0.022977
5000	5500	1.0180	1.5480	0.0190	0.0047	-0.000803	-0.006683	-0.001828	0.017028
5500	6000	1.0232	2.1886	0.0259	0.0057	-0.000734	-0.006767	-0.003384	0.024075
6000	6500	1.0273	1.7350	0.0207	0.0051	-0.000649	-0.006058	-0.001689	0.019085
6500	7000	1.0306	1.7219	0.0204	0.0049	-0.000567	-0.005428	-0.001737	0.018941
7000	8000	1.0345	1.5664	0.0184	0.0036	-0.000554	-0.005162	-0.001519	0.017230
8000	9000	1.0385	1.3120	0.0156	0.0034	-0.000494	-0.004555	-0.001419	0.014432
9000	10000	1.0414	1.1502	0.0137	0.0032	-0.000437	-0.004116	-0.001166	0.012652
10000	12000	1.0446	1.1625	0.0135	0.0023	-0.000374	-0.003588	-0.001109	0.012788
12000	14000	1.0475	0.9572	0.0113	0.0022	-0.000324	-0.003234	-0.000963	0.010529
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16000	18000	1.0509	0.8215	0.0097	0.0022	-0.000250	-0.002674	-0.000756	0.009037
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40000	44000	1.0543	0.4709	0.0056	0.0014	-0.000103	-0.001343	-0.000487	0.005180
44000	52000	1.0539	0.4403	0.0051	0.0010	-0.000089	-0.001119	-0.000440	0.004843
52000	60000	1.0533	0.4192	0.0049	0.0011	-0.000088	-0.001074	-0.000610	0.004612
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76000	84000	1.0508	0.3429	0.0039	0.0009	-0.000054	-0.000619	-0.000335	0.003772

$$(C_w, C_{w,0}, C_{w,Pb}, C_\phi)$$

$$B_w(t) = b_0 + k_1 C_{w,0}(t) + k_2 R_n(C_{w,Pb} - C_{w,0})(t)$$

$$Y_{\gamma,exp} = N_C \frac{C_w - B_w}{C_\phi - B_\phi} Y_\phi$$

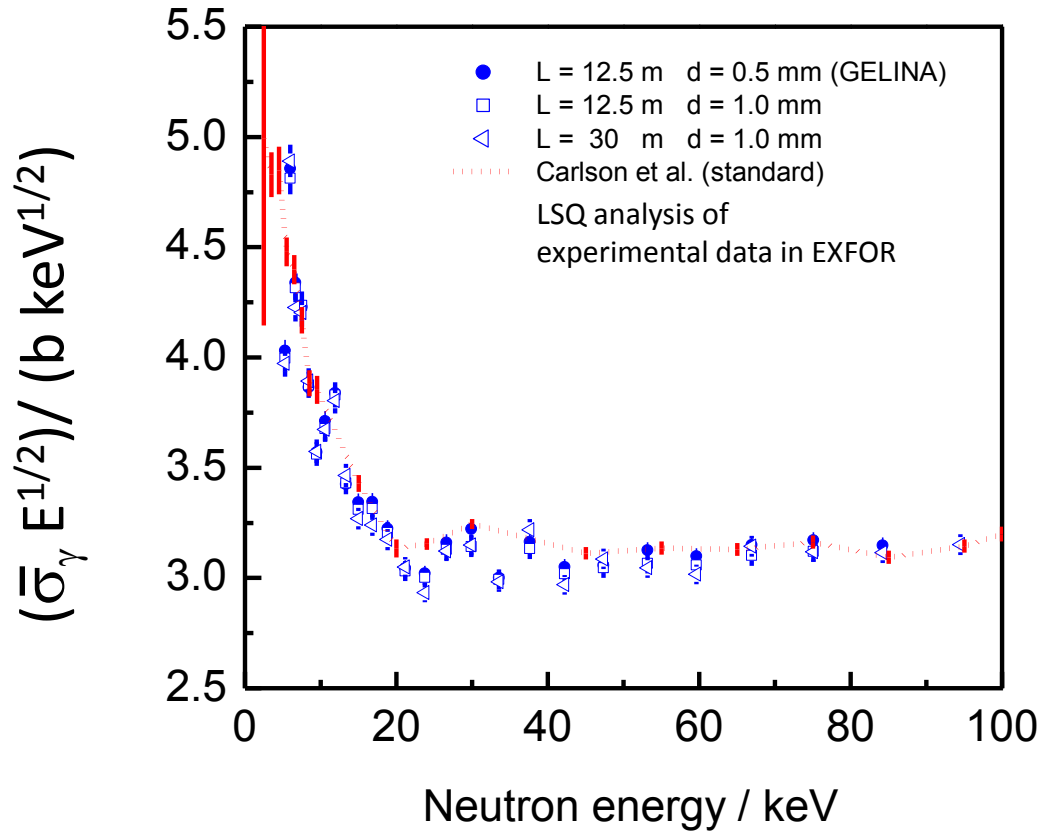
$$\bar{Y}_{exp}$$

$$\bar{\sigma}_\gamma = \frac{\bar{Y}_{\gamma,exp}}{F_\gamma n}$$

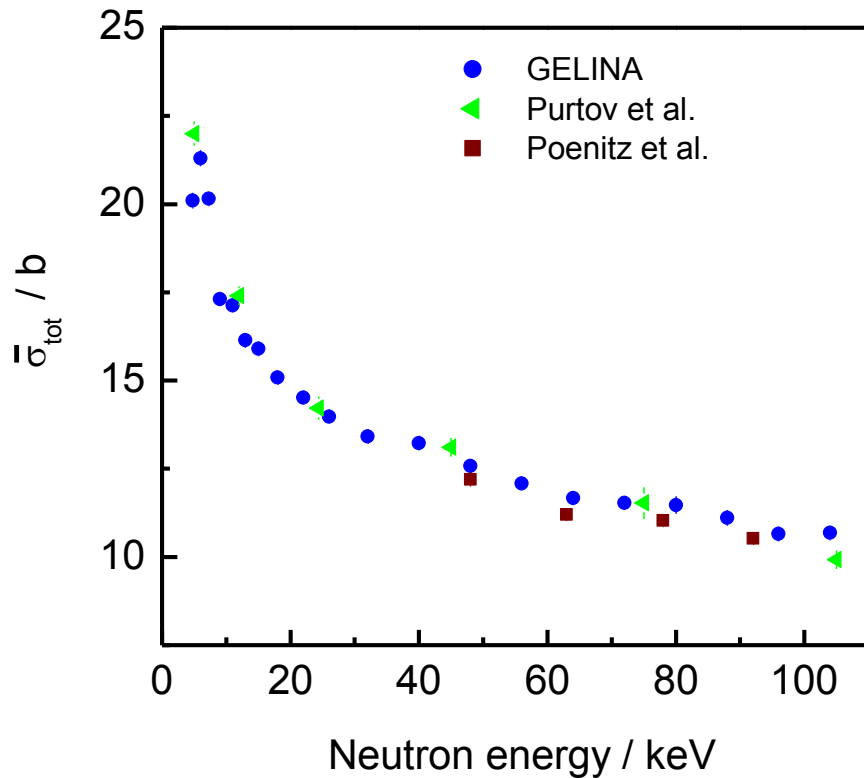
+ covariance data

u_u and $S_{(b_0, k_1, k_2, N_C)}$

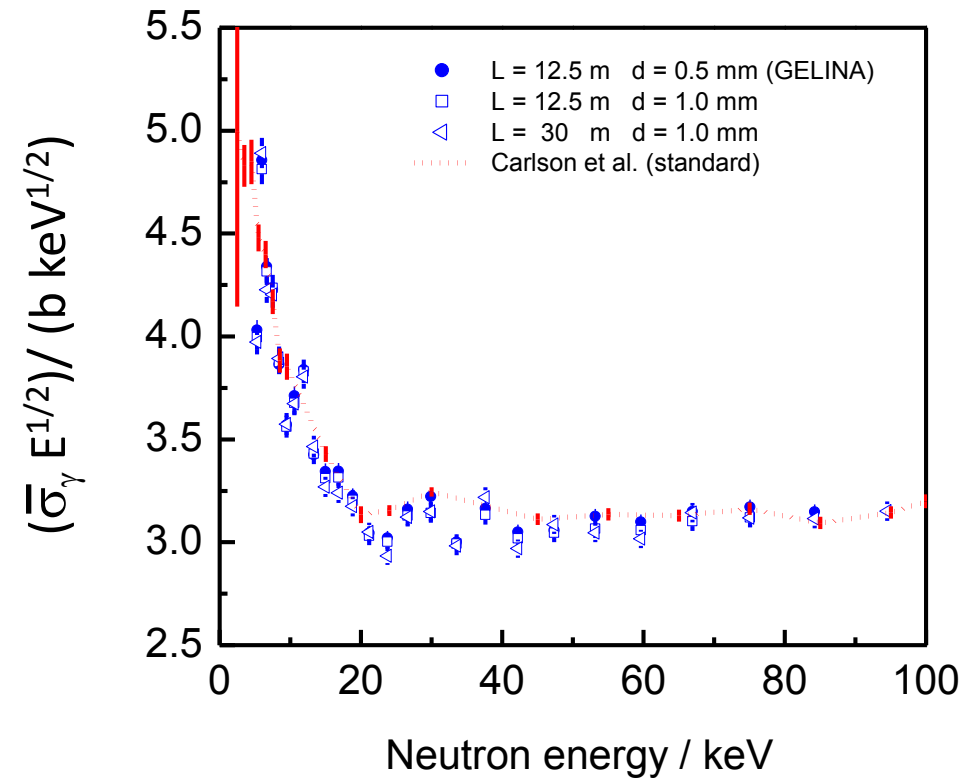
$$V_{\vec{Z}} = D_{\vec{Z}} + S_{\vec{Z}} S_{\vec{Z}}^T$$



$$\frac{u_{\sigma_\gamma}}{\sigma_\gamma} \approx 1.2\%$$



$$\frac{u_{\sigma_{\text{tot}}}^-}{\sigma_{\text{tot}}} \approx 1\% - 2\%$$



$$\frac{u_{\sigma_{\gamma}}^-}{\sigma_{\gamma}} \approx 1.2\%$$

