PROBLEM LIST 2. EXAMPLES.

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(1) Given $1 \leq \ell_c < d$, start with $A \in \mathrm{SL}(d,\mathbb{Z})$ with all eigenvalues of different modulus and construct by deformation on one fixed point a partially hyperbolic diffeomorphism with a central bundle E^c of dimension ℓ_c which cannot be splitted into more subbundles and which is neither uniformly contracted nor uniformly expanded.

(2) Characterize the pairs $A \in \mathrm{SL}(d,\mathbb{Z})$, $v \in \mathbb{R}$ such that $x \mapsto Ax + v$ in $\mathbb{T}^d$ is transitive (i.e. it has a dense forward orbit).
(*Hint:* f is transitive if and only if for every $U, V \subset \mathbb{T}^d$ non-empty open sets, there exists $n \geq 1$ such that $f^n(U) \cap V \neq \emptyset$.)

(3) Show that left translation by $\begin{pmatrix} e^t & 0 \\ 0 & e^{-t} \end{pmatrix}$ in $PSL(2,\mathbb{R})/_\Gamma$ is partially hyperbolic where Γ is isomorphic to the fundamental group of a closed surface and $\Gamma \subset \mathrm{PSL}(2,\mathbb{R})$ is discrete.