Recall

- Def R.
- **Thm** Hyp
- $\{\psi^n_\omega\}$ $\cancel{E^n_\omega}$ $\mathcal{C}_n$, $\mathcal{C} = \cap \cup \mathcal{C}_n$
- **Thm** (de Carvalho, Lyubich, M)
  (Similar Gambaudo, v. Strien Tresser)

a) $\mathcal{C}_F$ is Cantor Set.
b) $\forall n\ \exists$ periodic orbit P_n of period 2^n.
c) $\forall x \notin \cup W^s(P_n)$: $\omega(x) = \mathcal{C}$.

$$\boxed{\text{d)}\quad \begin{array}{ccc} h: \mathcal{C}_F & \longrightarrow & \mathcal{C}_{f_*} \\ \circlearrowleft & & \circlearrowleft \\ F & & f_* \end{array} \quad \text{homeo}}\qquad ?$$

Thm: $\exists!\ \mu$ on $\mathcal{C}_F$

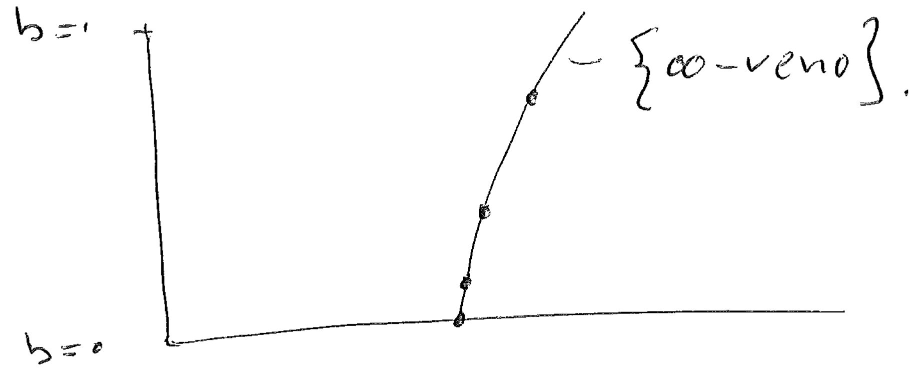
$$\chi_0 = 0, \quad \chi_- = \ln b_F$$

where

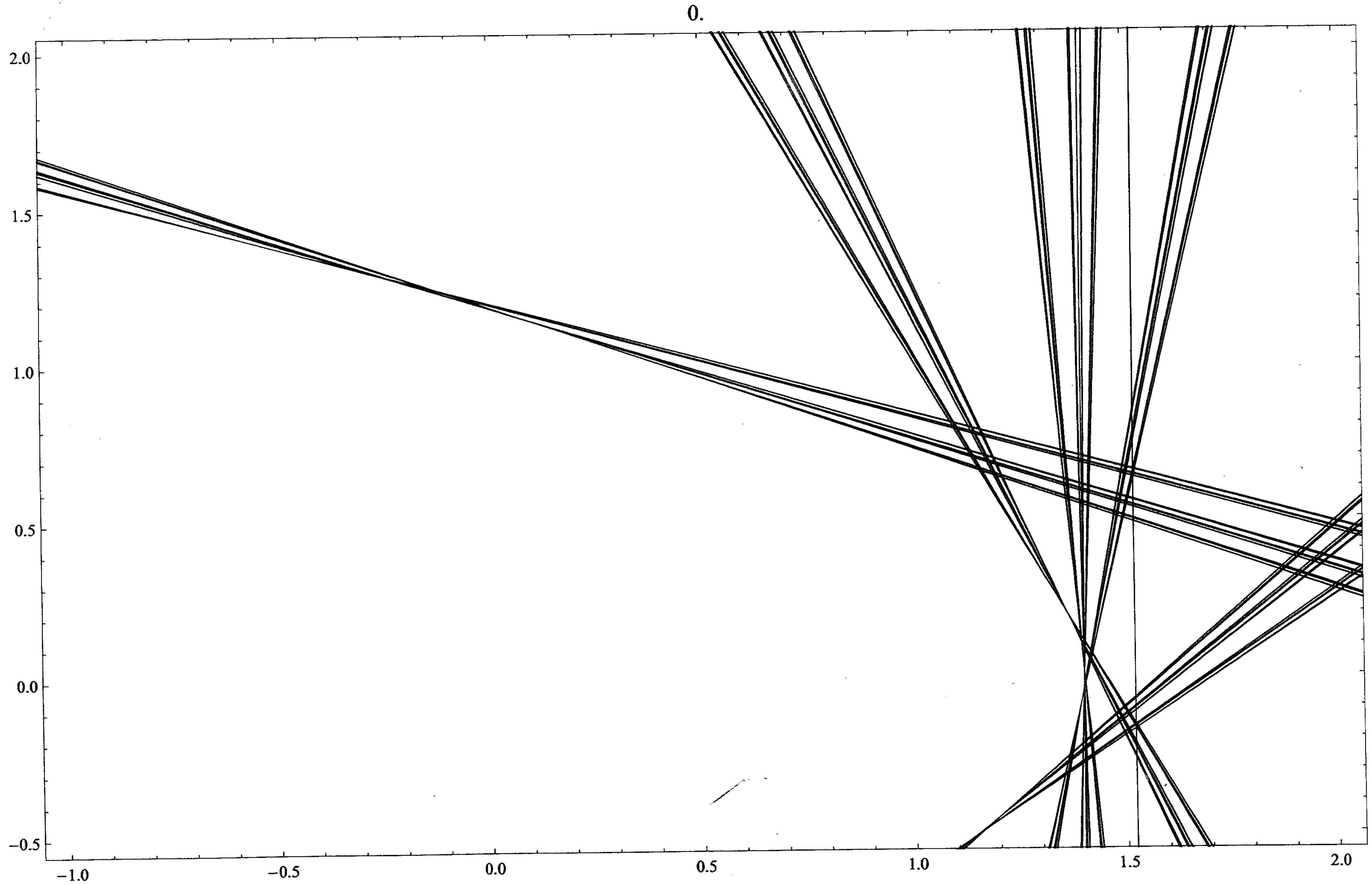
b_F is the Avarage Jacobian

$$\boxed{b_F = e^{\int \mathrm{Jac}(F)\, d\mu}}$$

The invariant line field for $x_0 = 0$ are the "Tangent lines" to the Cantor set $\mathcal{C}_F$.

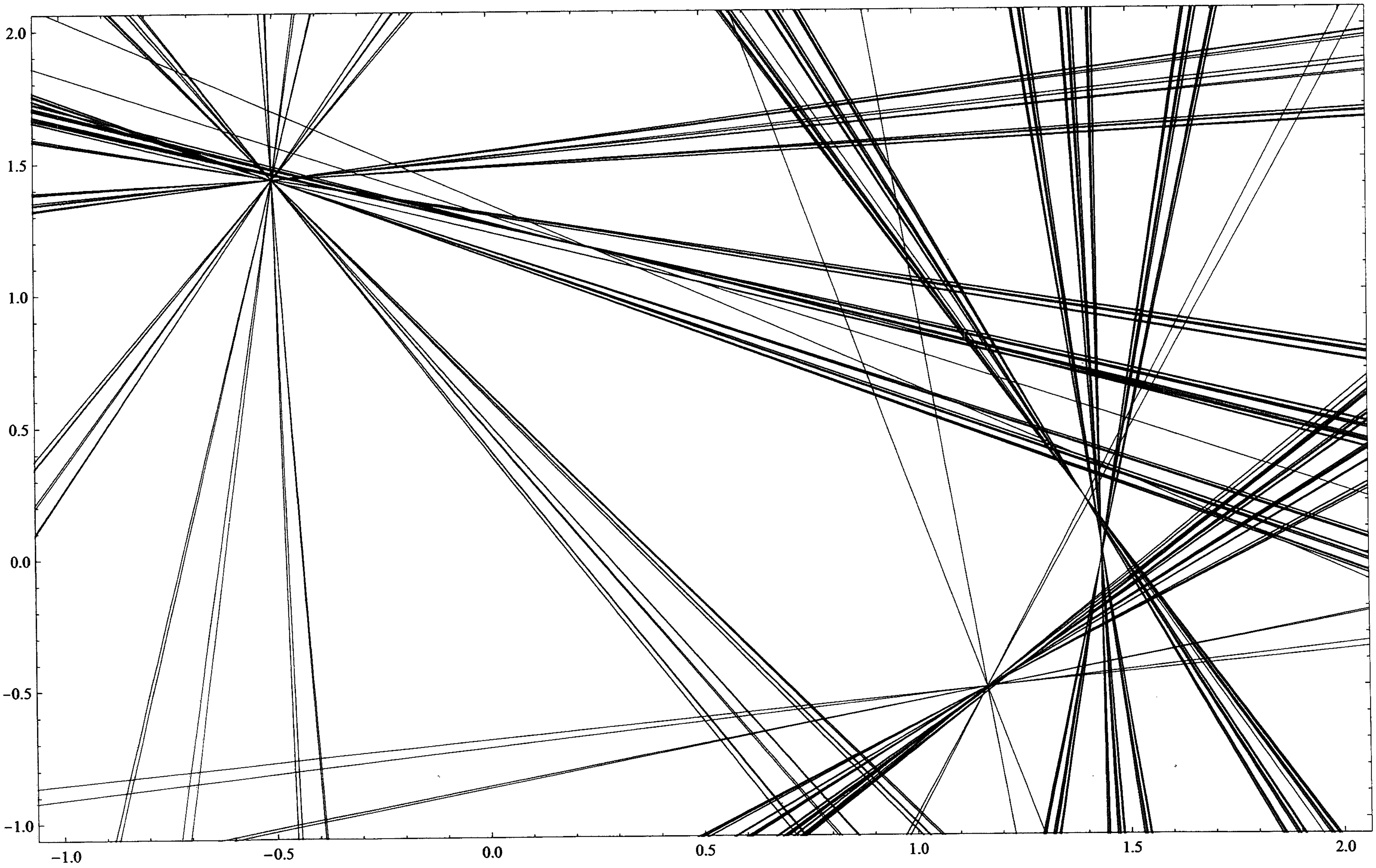


Pics

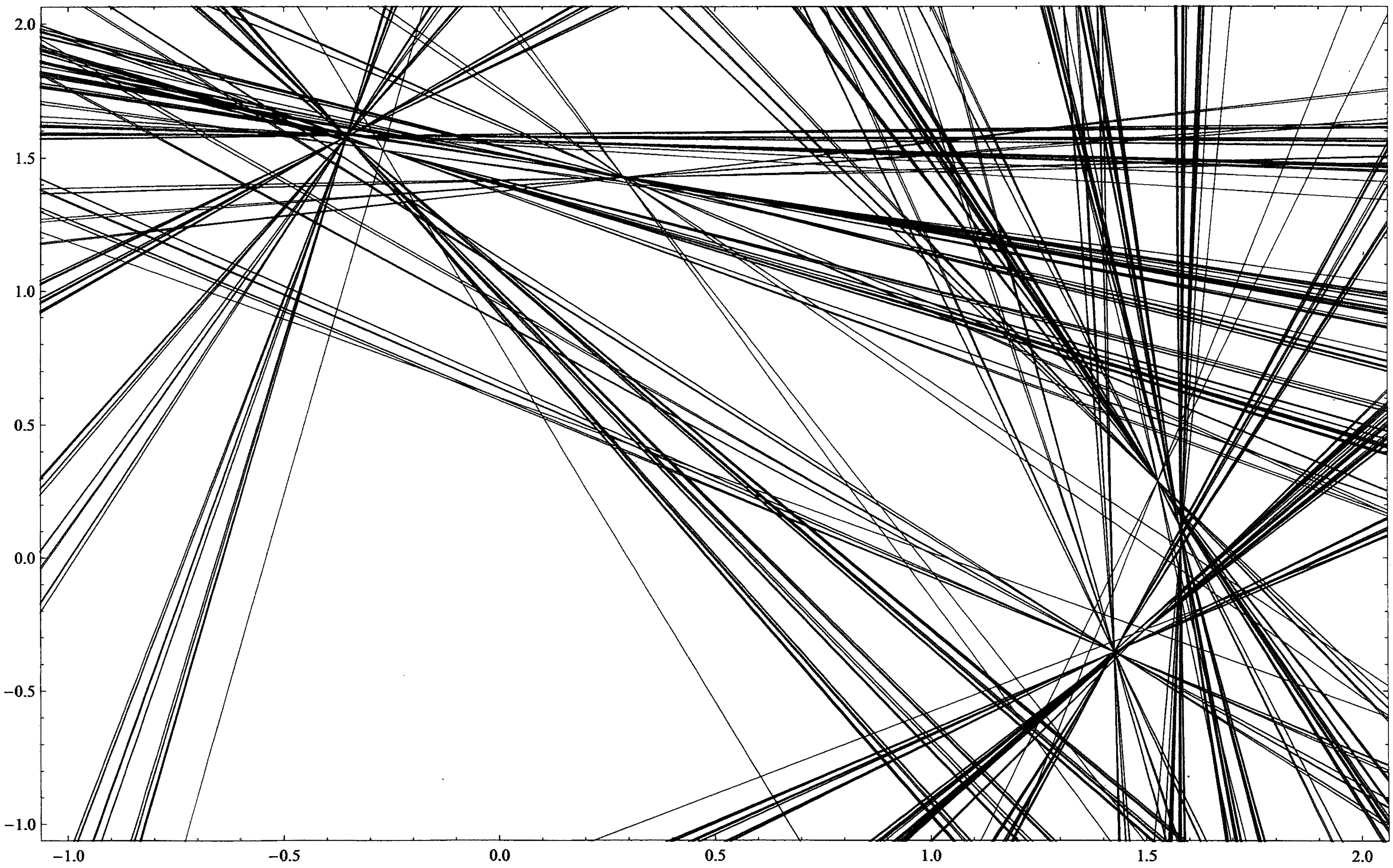


0.
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0

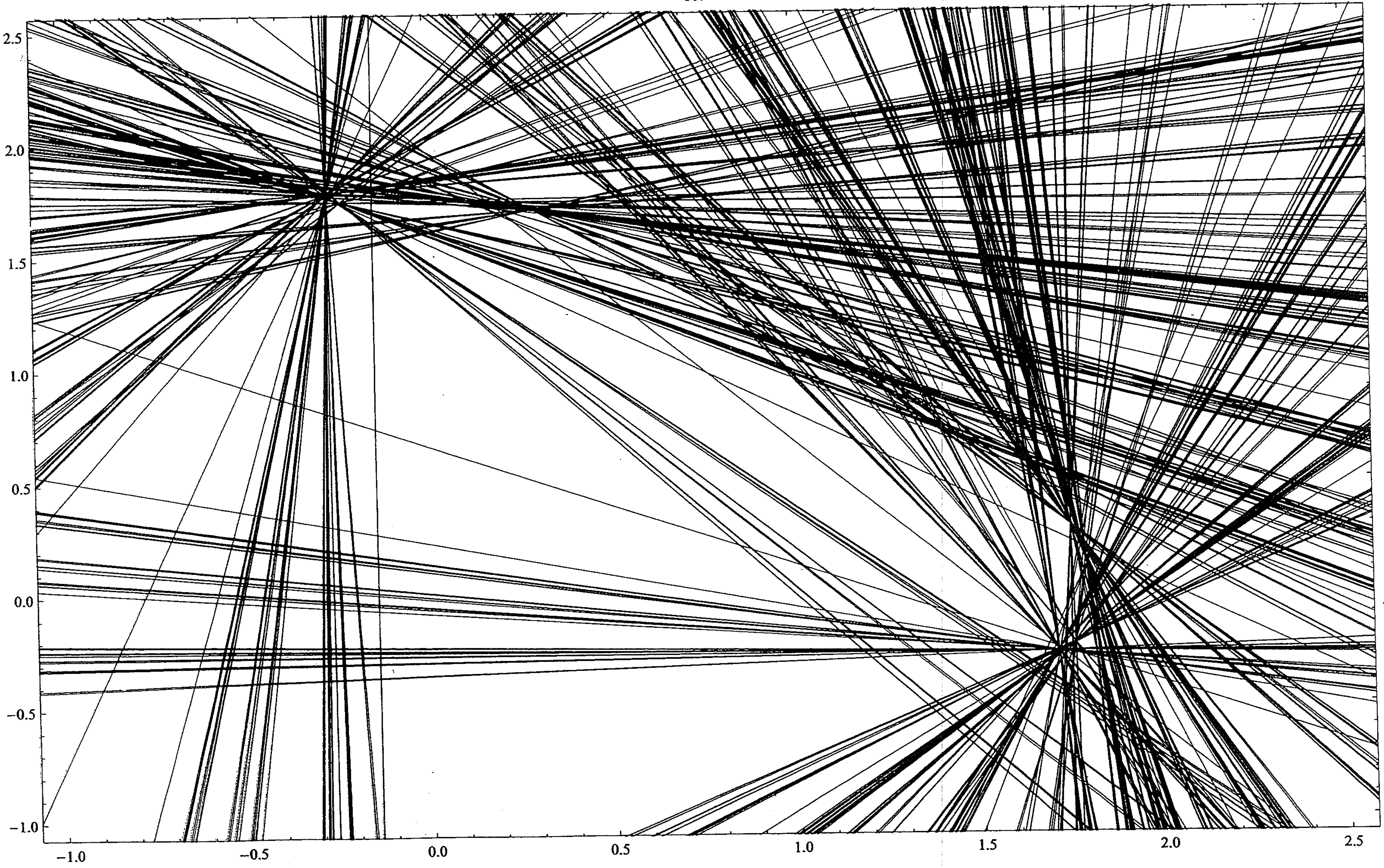
0.1



0.4



0.7



Thm: (CLM)

There is no cont. invariant line field on $\mathcal{C}_F$ when $b_F > 0$

Cor: $\mathcal{C}_F$ is not contained in a smooth curve.

↳ 2D Dynamics.

Thm (LM)

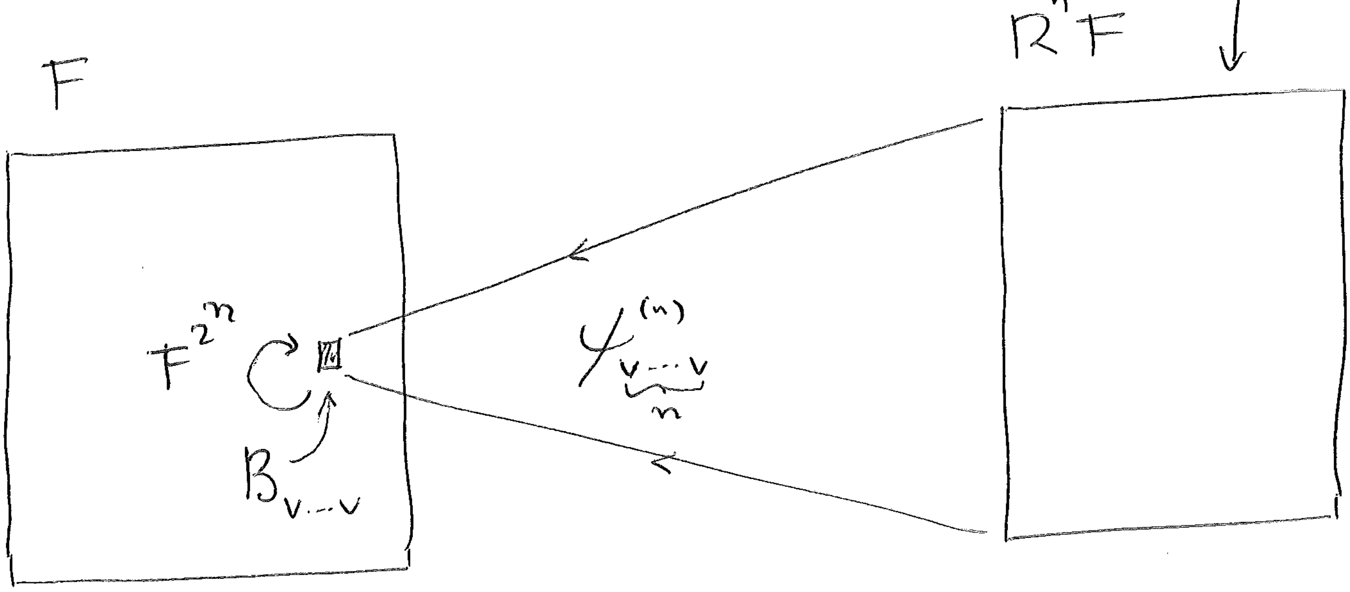
$\mathcal{C}_F$ is contained in a rectifiable curve.

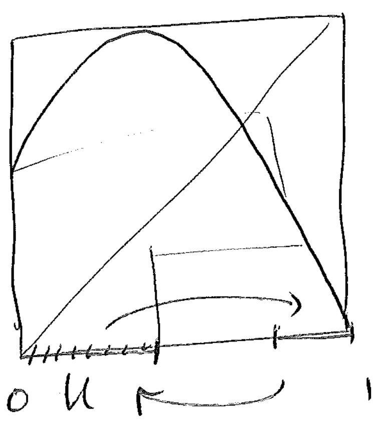
<u>Thm</u> $\exists \rho < 1$ (Universality).

$$R^n F = \begin{pmatrix} f_n(x) - b_F^{2^n} a(x) y (1 + O(\rho^n)) \\ x \end{pmatrix}$$

where $f_n \xrightarrow[\exp]{} f_* \in \mathcal{U}$

and $a(x)$ analytic universal.





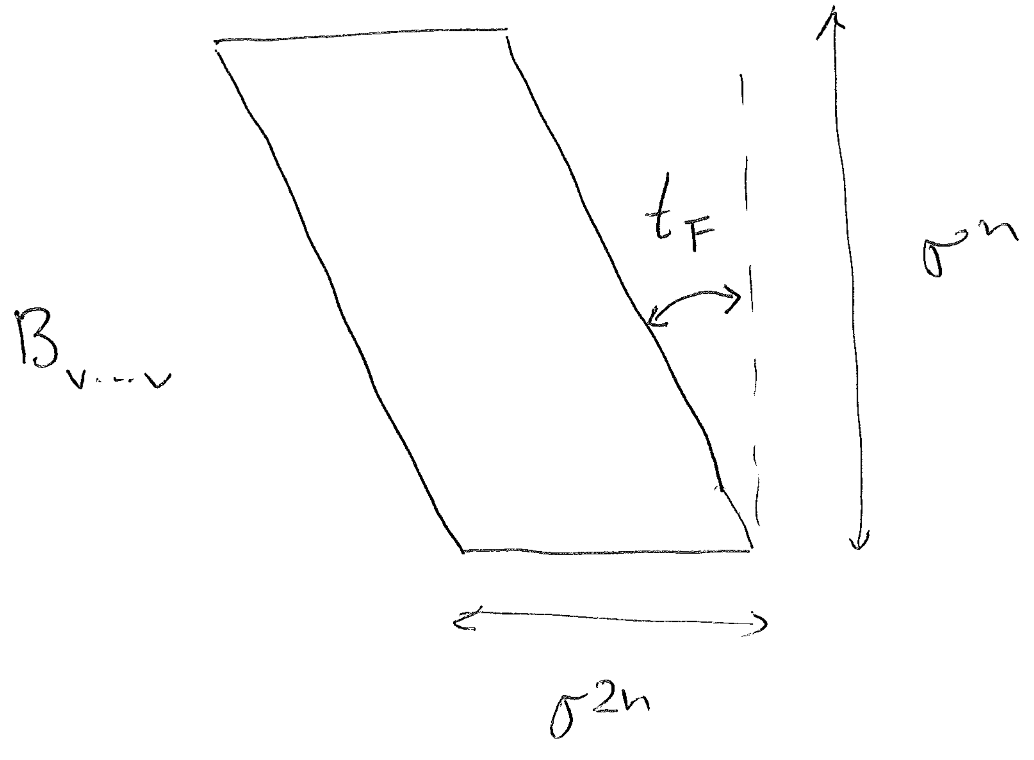
$$|u| = \sigma \approx \frac{1}{2.6\ldots}$$

Thm (Scaling Universality)

$$\psi^{(n)}_{v\cdots v} = \begin{pmatrix} 1 & t_F \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \sigma^{2n} & 0 \\ 0 & (-\sigma)^n \end{pmatrix} \begin{pmatrix} S_n(x,y) \\ y \end{pmatrix}$$

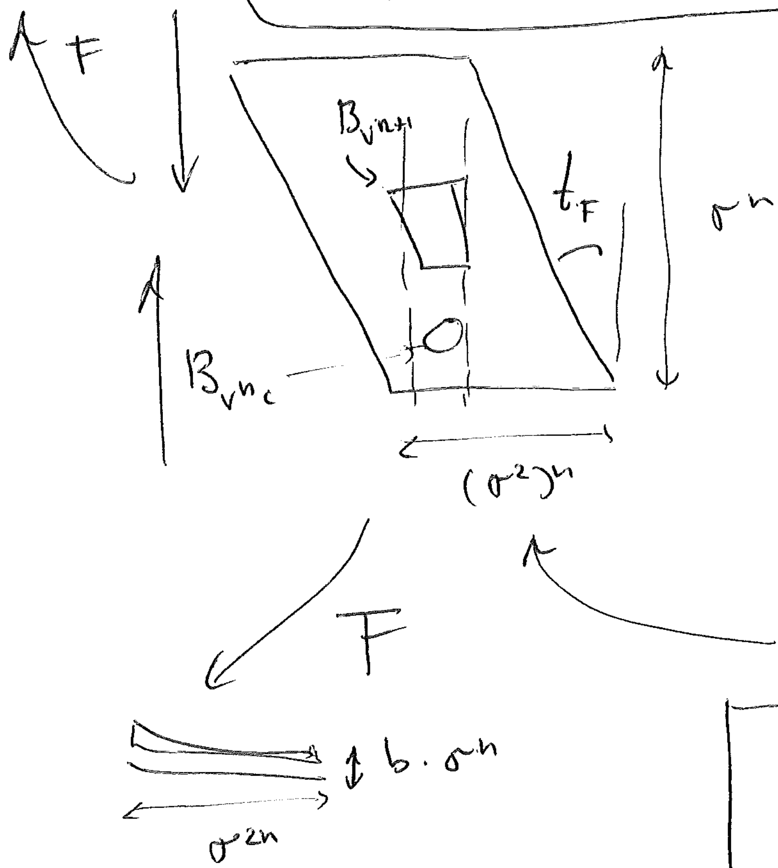
- $t_F \asymp b_F$
- $S_n(x,y) = v(x) + a_F y^2 + O(\rho^n)$

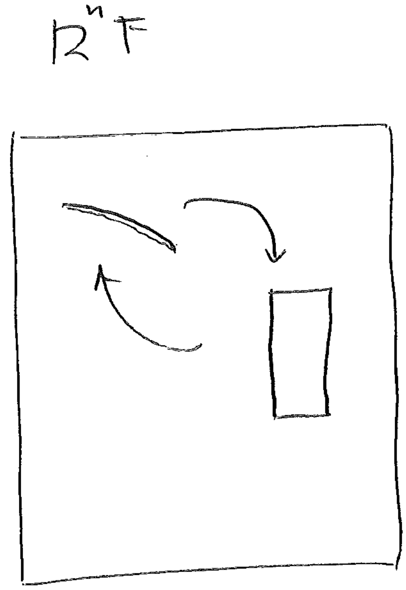
$\uparrow$ analytic universal.



Key Observation

F has strong vertical contraction $\simeq b$.





Condition

$$\boxed{b\cdot\sigma^n \asymp \sigma^{2n}}$$

$$\Updownarrow$$

$B_{v^{n+1}}$ and $B_{v^n_c}$ are above each other

Thm (CLM) (Non Rigidity).

$$h : \mathcal{C}_{F_1} \longrightarrow \mathcal{C}_{F_2}$$

$$b_{F_1} > b_{F_2}$$

h is at most $C^{\frac{1}{2}\left[1+\frac{\ln b_{F_1}}{\ln b_{F_2}}\right]}$

Thm (Hazard, Lyubich, M)

$\exists\, A \subset [0,1]$, $|A| = 1$.

If $b_F \in A$ **then**

$\mathcal{C}_F$ has no A priori Bounds

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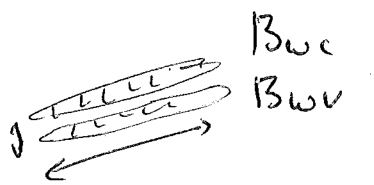
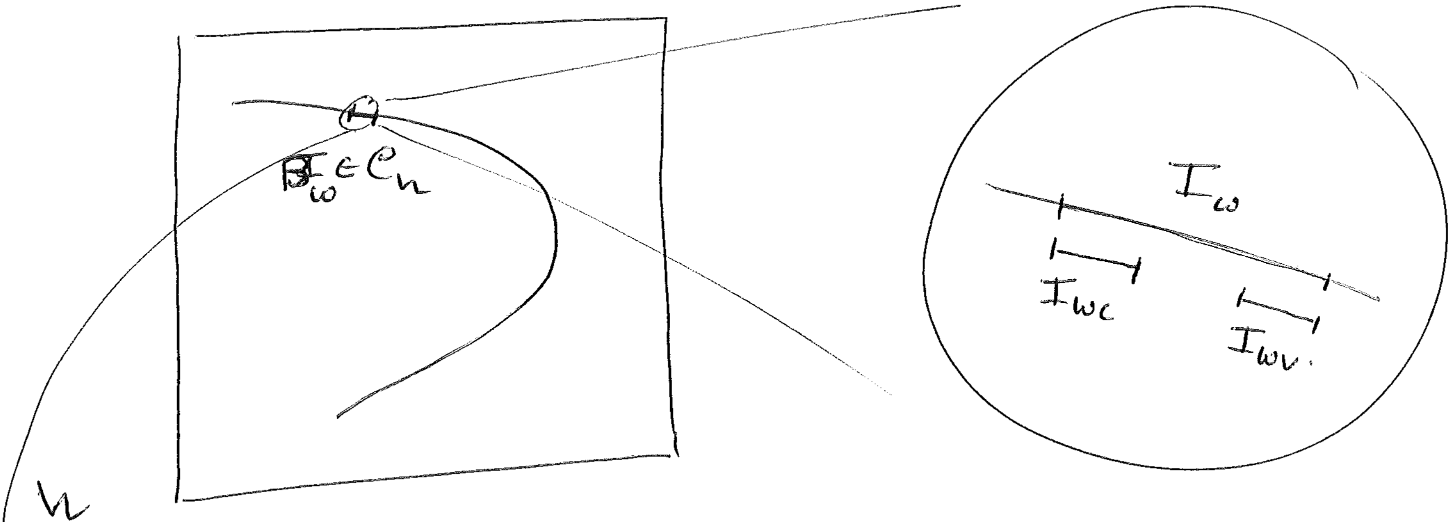


"

$$A = \left\{ b \;\middle|\; b^{2^n} \asymp \sigma^k \quad \text{for } \infty\text{-many } n \text{ and } k \right\}.$$

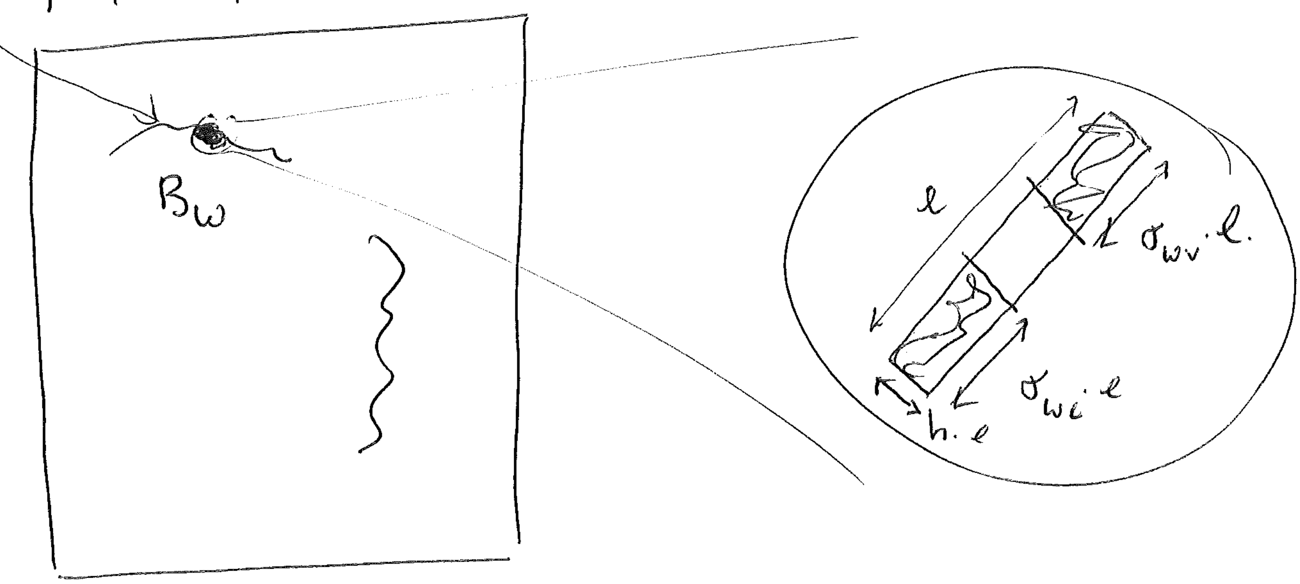
Scaling Structure

$F_* = \begin{pmatrix} f_*(x) \\ x \end{pmatrix}$



Scaling Ratio : $\sigma_{wc} = \dfrac{|I_{wc}|}{|I_w|} \in \Sigma_{f_*}$

$F : b_F > 0$



B_ω is ε- Standard

- $|h| < \varepsilon$
- $|\sigma_\omega^F - \sigma_\omega^{f_*}| < \varepsilon$.

$$\mathcal{S}_n(a) = \{ B \in \mathcal{C}_n \mid \varepsilon\text{-standard} \}.$$

<u>Thm</u> (Lyubich, M) (Probabilistic Universality).

$\exists\, \theta < 1$

$$\mu(\mathcal{S}_n(\theta^n)) > 1 - \theta^n.$$

<u>Meaning</u>: $\mathcal{C}_n$ has 2^n pieces and only $\theta^n \cdot 2^n$ of them do <u>not</u> look like their 1D counterpart in ~~$\mathcal{C}_{f_*}$~~ $\mathcal{C}_{f_*}$

Almost all are exp close to 1D.

Thm (LM) (Probabilistic Rigidity)

$\exists\, \beta > 0$ (universal).

$\forall F$ (∞-reno)

$$X_1^F \subset X_2^F \subset \cdots \subset \mathcal{C}_F$$

- $\mu(X_n^F) \longrightarrow 1$.
- X_n^F is contained in a $C^{1+\beta}$-curve.
- $h : X_n^F \longrightarrow h(X_n^F) \subset \mathcal{C}_{f_*}$ is $C^{1+\beta}$

Rmk: X_n^F has a topological definition.

If $b_{F_1} = b_{F_2}$ then $h : \mathcal{C}_{F_1} \to \mathcal{C}_{F_2}$

$$h : X_n^{F_1} \longrightarrow X_n^{F_2} \quad C^{1+\beta}$$

Thm (LM)

b_F is a top. inv.

$b_{F_1} \neq b_{F_2}$: no Rigidity.

But also $F_1 \not\sim_{top} F_2$

Maybe the paradigm

$$\boxed{\text{Top} \Longrightarrow \text{Geo}}$$

Still Holds:

Question: $b_{F_1} = b_{F_2}$

$\Longrightarrow$ $h : C_{F_1} \to C_{F_2}$ is $C^{1+\beta}$.