Zariski finiteness theorem and some properties of ring of invariants.

Abstract: The main aim of the talk is to discuss two theorems which belong to the interface of commutative algebra, algebraic geometry and invariant theory:

The first is a new proof of a special case of the Zariski finiteness theorem, namely : Let T be an affine factorial domain $\mathbb{C}$. Let S be an inert subring of T such that the transcendental degree of S over $\mathbb{C}$ is 2. Then S is finitely generated algebra over $\mathbb{C}$.

The other is a criterion for the quotient $\mathbb{A}^3_C//\mathbb{G}_a$, to be a Zariski locally trivial $\mathbb{A}^2$-bundle over C, where C is a smooth complex affine curve. More precisely we show that if R is a regular complex affine domain of dimension 1 and suppose the additive group $\mathbb{G}_a$ acts by R-automorphisms on $R[X, Y, Z]$ such that the singular fibers of the morphism Spec $R[X, Y, Z]^{\mathbb{G}_a} \to$ Spec R are normal, then Spec $R[X, Y, Z]^{\mathbb{G}_a}$ is Zariski locally an $\mathbb{A}^2$-bundle over Spec R.

The novelty feature in both proof is the use of standard algebraic topology. All these results are obtained in joint work with R.V. Gurjar and B. Hajra and published in Transformation Groups journal.

Link: https://link.springer.com/article/10.1007/s00031-020-09594-0