Design and operation of accelerator chain and storage rings

S. Di Mitri

Elettra Sincrotrone Trieste

Univ. of Trieste, Dept. of Physics

simone.dimitri@elettra.eu

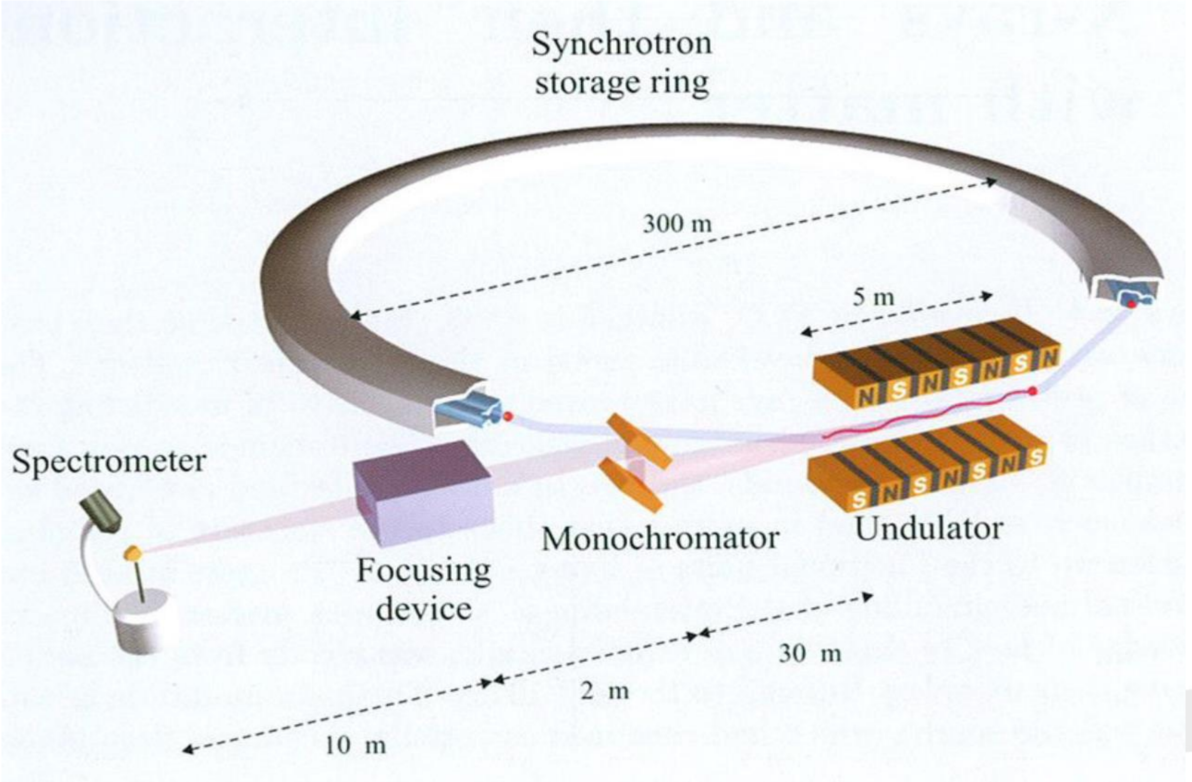
Outline

1. How does a synchrotron light source look like ?
2. What are the main physical processes ?
3. How does it work ?

- ❑ Synchrotron radiation – recap
- ❑ Single particle linear motion
  - Longitudinal – phase stability, synchrotron oscillations
  - Transverse – dispersion, betatron tunes, emittance
- ❑ Perturbations to linear dynamics
  - Equilibrium distribution
  - Chromaticity, resonances, dynamic aperture
  - Beam lifetime
- ❑ Operation
  - Beam injection and storage.
  - Brilliance, diffraction limit.

Synchrotron Light Sources

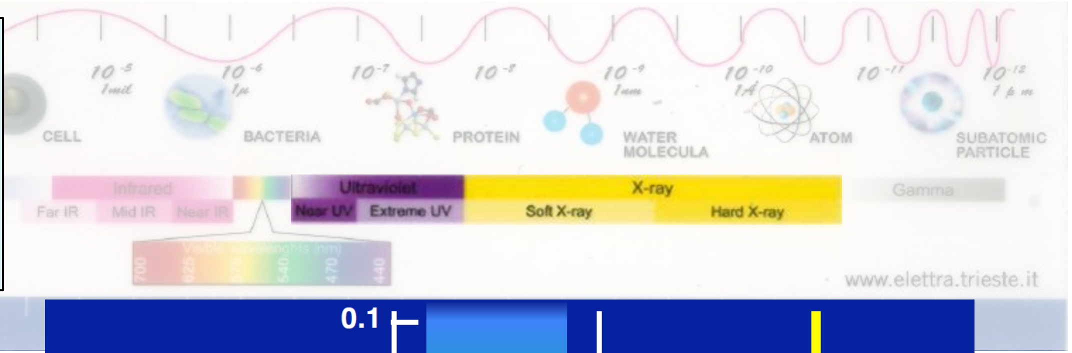
Serving as a Microscope



Why X-rays ?

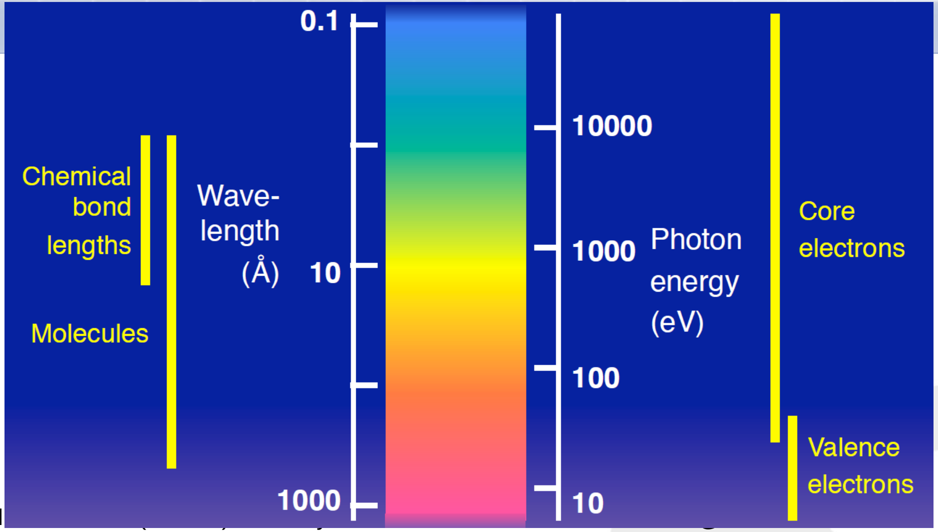
$$\begin{cases} \Delta x \cdot \Delta p \geq \frac{1}{2} \\ \Delta p \sim \frac{1}{\lambda} \end{cases} \Rightarrow \lambda \leq \Delta x$$

Spatial resolution



X-rays are ideal probes of chemical bonds, where most of science is rooted.

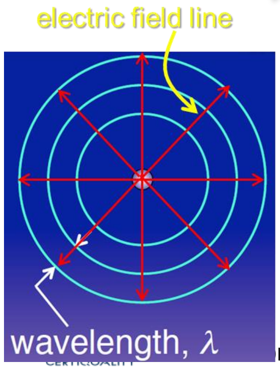
They can be used to visualize proteins structure, molecular dynamics, atomic levels and orbitals…

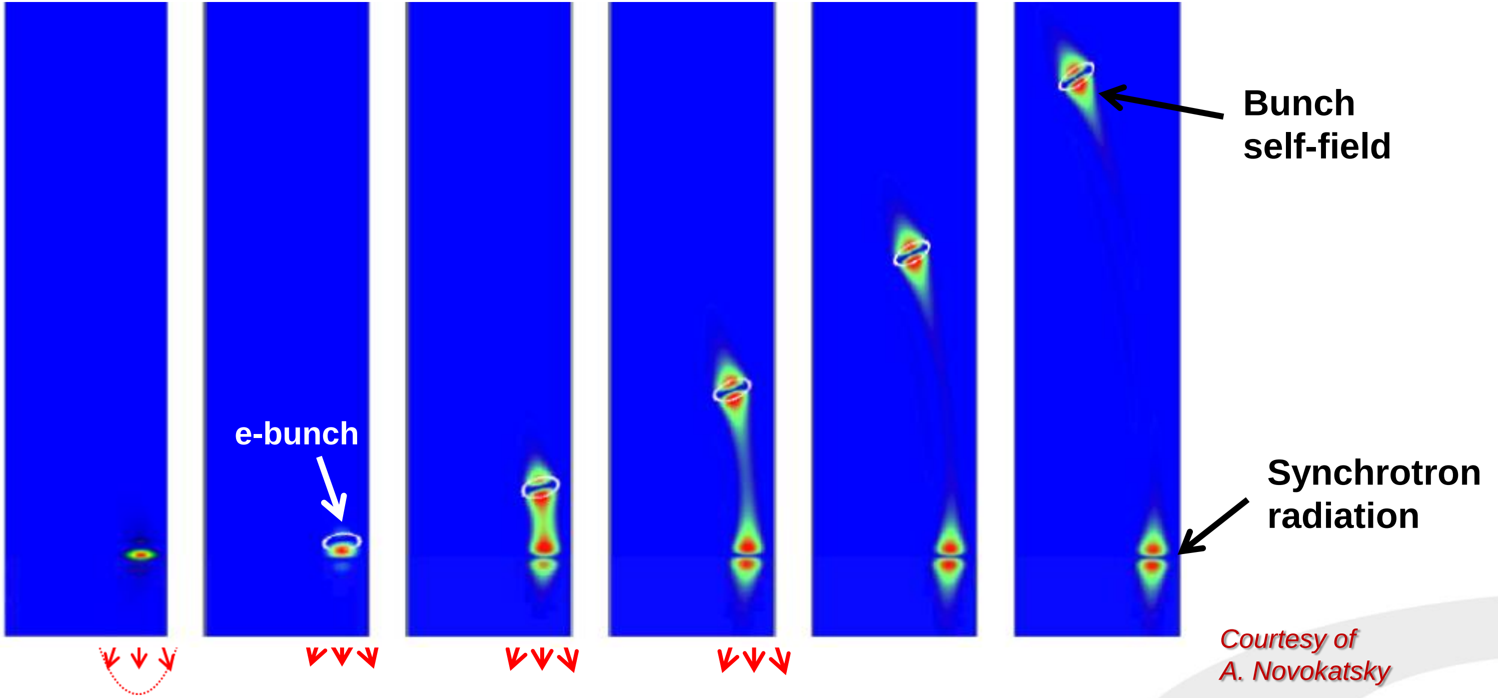


Synchrotron Radiation

Synchrotron radiation is e.m. energy de-coupled from a charge by centripetal acceleration. For example, an ultra-relativistic electron in a magnetic dipole field.

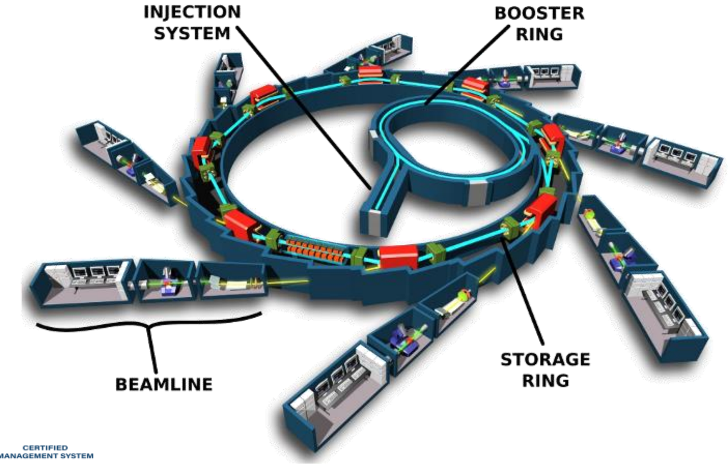
$$\vec{E}_r \sim \frac{q}{r^3}\hat{r}$$

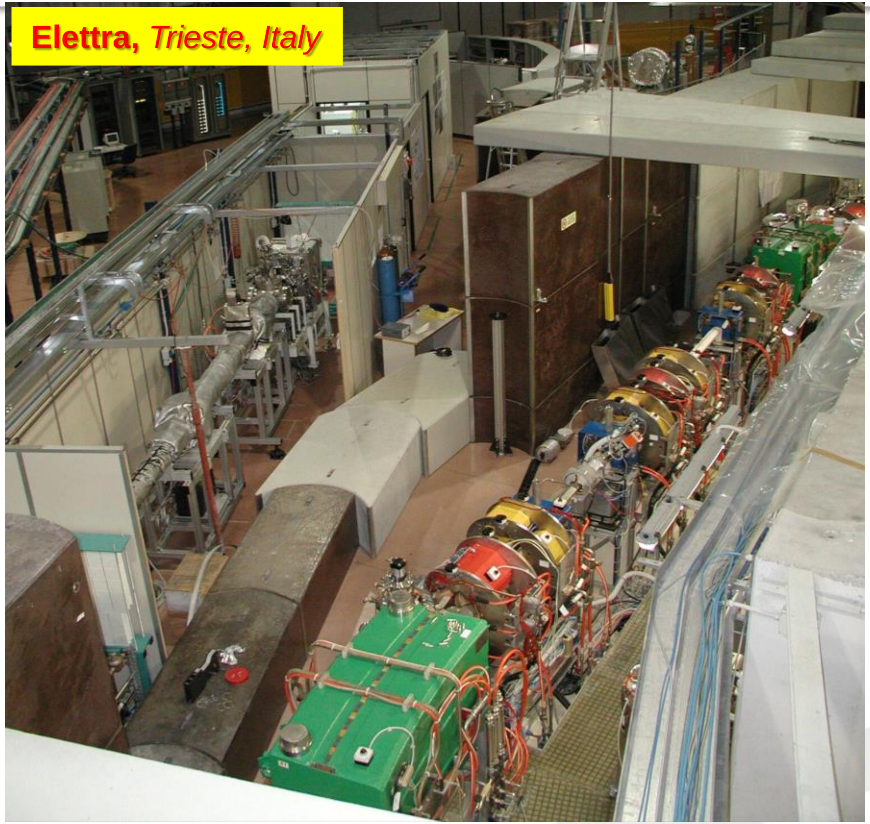




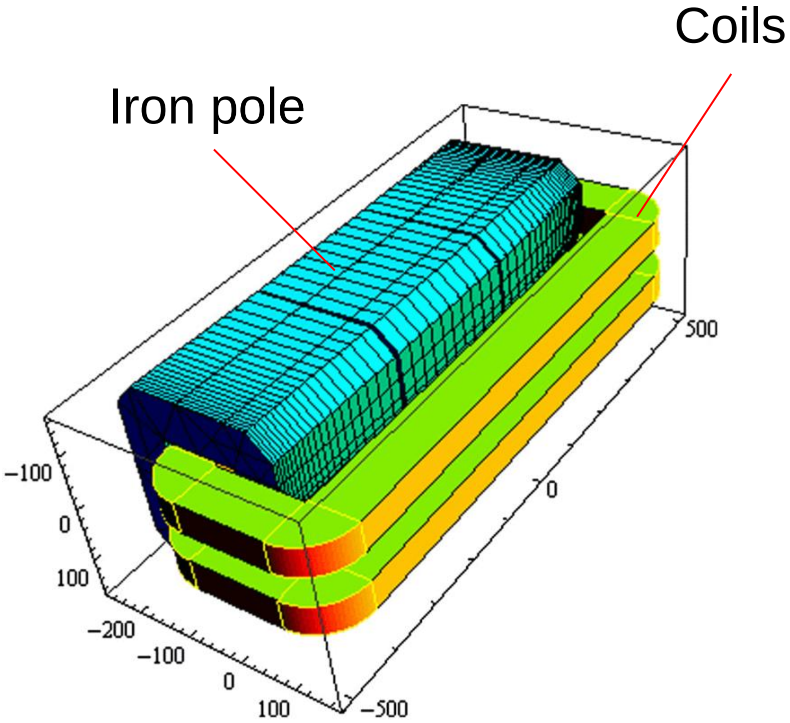
Courtesy of A. Novokatsky

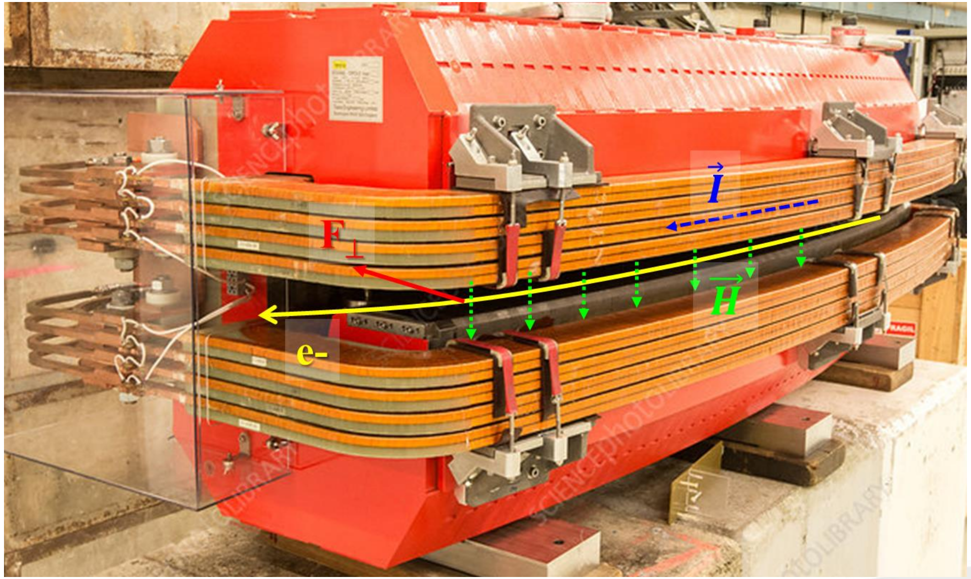
Bird-Eye View





Dipole Magnet





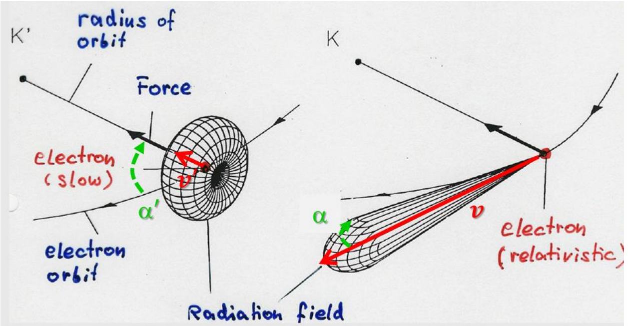
Synchrotron Radiation – Recap

$$P_{SR} \propto \frac{\gamma^4}{R^2} = B^2 \frac{E^4}{m_0^4}$$

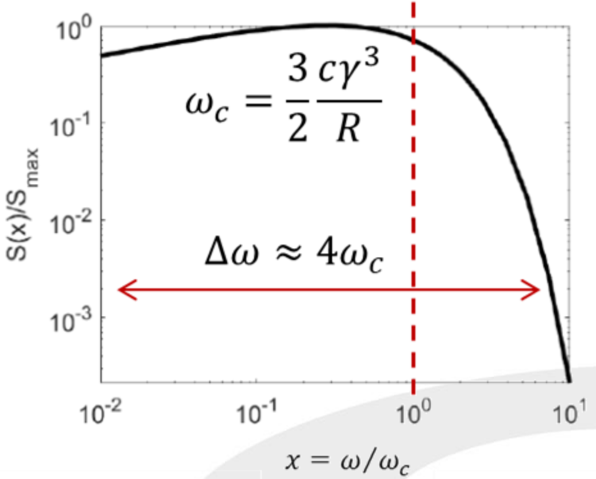
Stronger field stimulates larger emission

Strong increase with particle's ***total energy***

Leptons radiate more than hadrons



Cutoff frequency
$\propto \boldsymbol{\gamma^3}$ (at R=const.)



Forward collimated emission, α **≈ 1/γ**

simone.dimitri@elettra.eu

Development of SRLS

$$\omega_c \approx \frac{c\gamma^3}{R}$$

- **First observation**:

  1947, General Electric, 70 MeV synchrotron

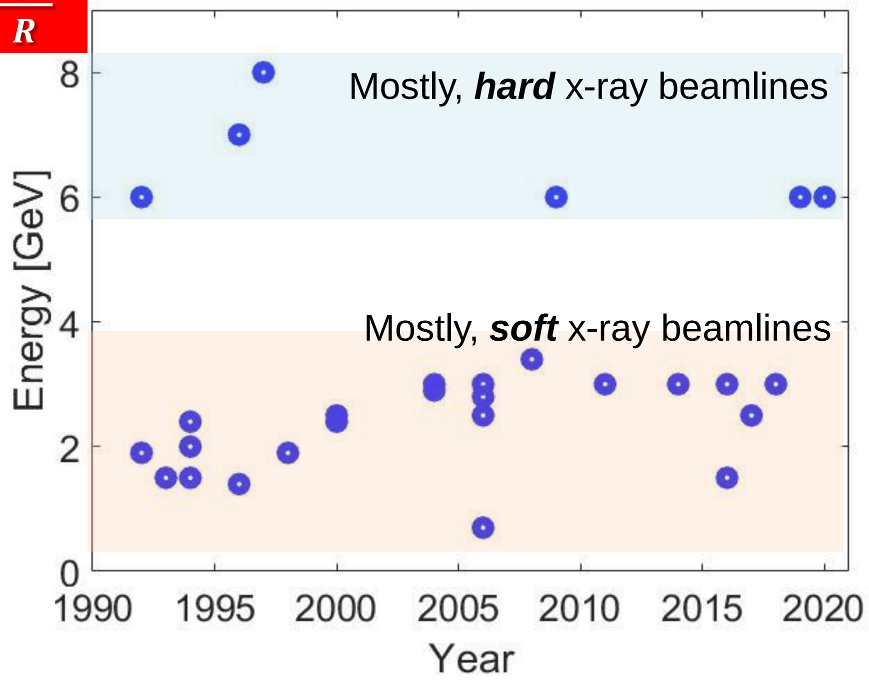
- **First user experiments**:

  1956, Cornell, 320 MeV synchrotron

- **1st generation light sources**: machine built for High Energy Physics or other purposes used parasitically for synchrotron radiation

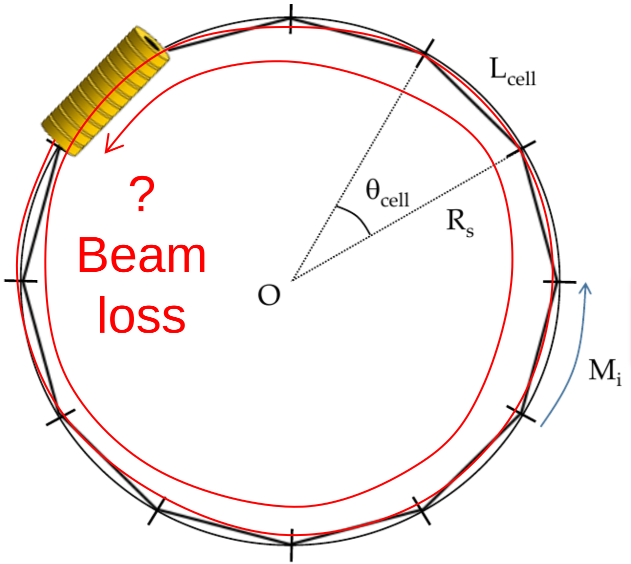
- **2nd generation light sources**: purpose built synchrotron light sources, SRS at Daresbury was the first dedicated machine (1981 – 2008)

- **3rd generation light sources**: optimised for high brilliance with low emittance and Insertion Devices; ESRF, Diamond,



Longitudinal Beam Dynamics

Accelerating RF Cavity

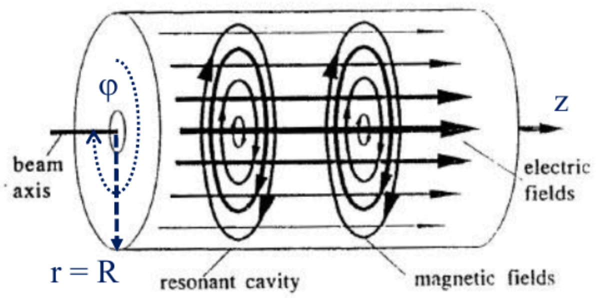


$$\frac{mv_z^2}{R} = F_{L,x} = ev_zB_y \Rightarrow p_z = eB_yR$$

RF cavities replenish the beam by the energy lost every turn $\Rightarrow$ beam energy is constant *on average* in a turn

Longitudinal electric field:

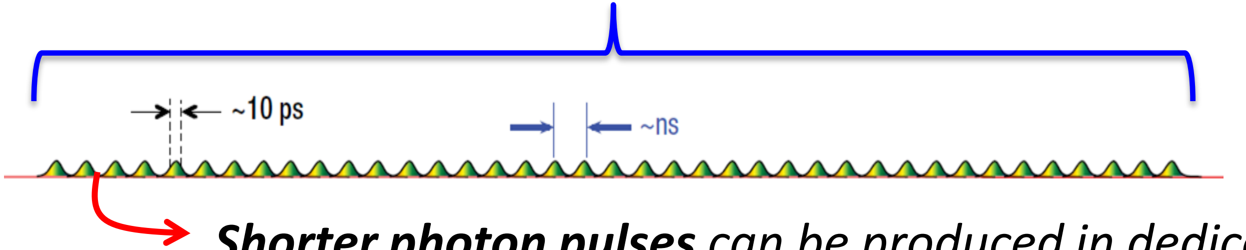
$$E_z \approx E_{z,0}\cos(\omega t + \phi_0)$$



Synchronization of particle's arrival time and RF field: $\omega = h\omega_{riv}, \quad h \in \dot{\mathbb{N}}(\gg 1)$

Filling Pattern

$T_{riv} \sim 1 - \text{few } \mu s$

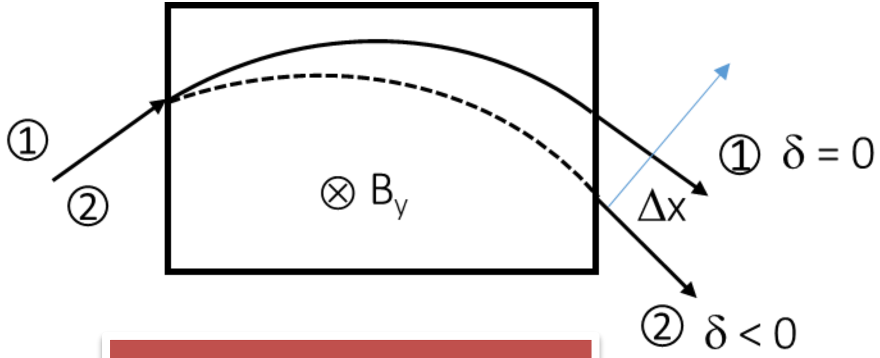


***Shorter photon pulses** can be produced in dedicated few bunches schemes or with more advanced e-beam manipulations.*

- **1.5 - 8 GeV, 200 – 500 mA, 100 – 1000 bunches per turn**
- **10 – 50 photon beamlines operating simultaneously**
- **> 5000 hours per year (24h, 7/7), ~1000 hours reserved for machine physics**
- **> 1000 users / year**

Energy–Dispersion Function

High energy electrons on a circular path



$$E \longrightarrow E - E_{sr} + \ldots$$

$$\delta := \Delta E / E$$

$$p_z = eB_yR$$

Dispersion function:

$$D_x(s) := \frac{x(s)}{\delta}$$

Momentum Compaction, Slip Factor

Orbit difference:

$$dR = \frac{C_2 - C_1}{\theta_b} = \frac{1}{\theta_b}\left(\oint ds_2 - \oint ds_1\right) = \frac{1}{\theta_b}\oint d\theta\,[(R_1 + x) - R_1] = \frac{1}{\theta_b}\oint x d\theta = \langle x \rangle_\theta$$

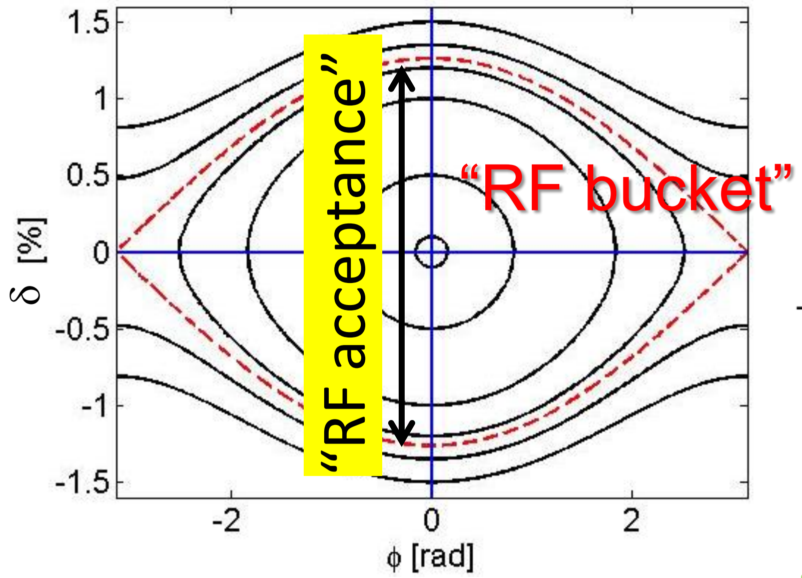
Orbit difference per unit of energy deviation ("momentum compaction"):

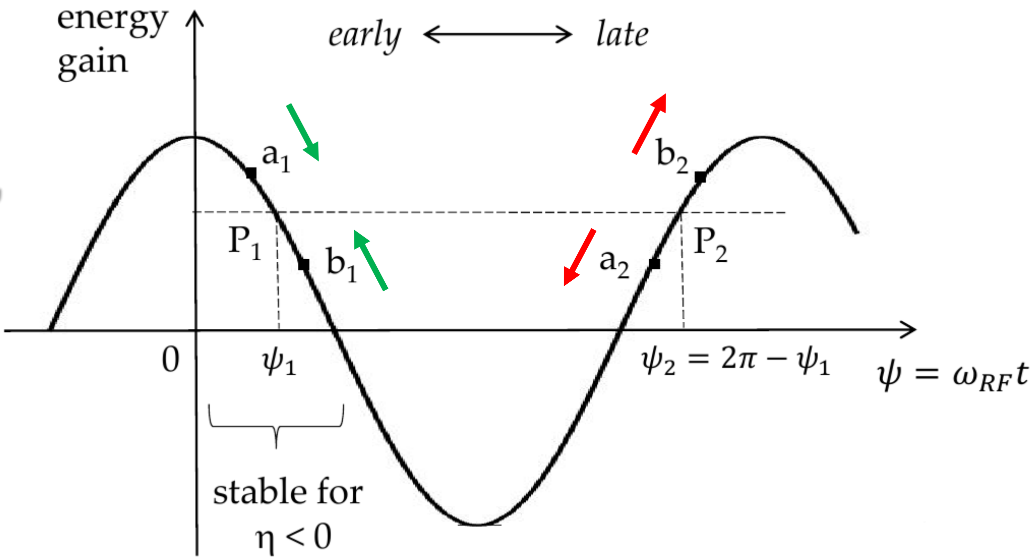
$$\alpha_c = \frac{dR/R}{\delta} = \frac{1}{R}\frac{\langle x \rangle_\theta}{\delta} = \frac{\langle D_x \rangle_\theta}{R} = \frac{1}{R\theta_b}\int d\theta D_x = \frac{1}{C}\int ds \frac{D_x(s)}{R(s)}$$

Revolution frequency difference per unit of energy deviation ("slip factor"):

$$\eta := \frac{d\omega/\omega_s}{dp_z/p_{z,s}} = \frac{1}{\gamma^2} - \alpha_c \xrightarrow{\text{GeV energies}} -\alpha_c$$

Phase Stability





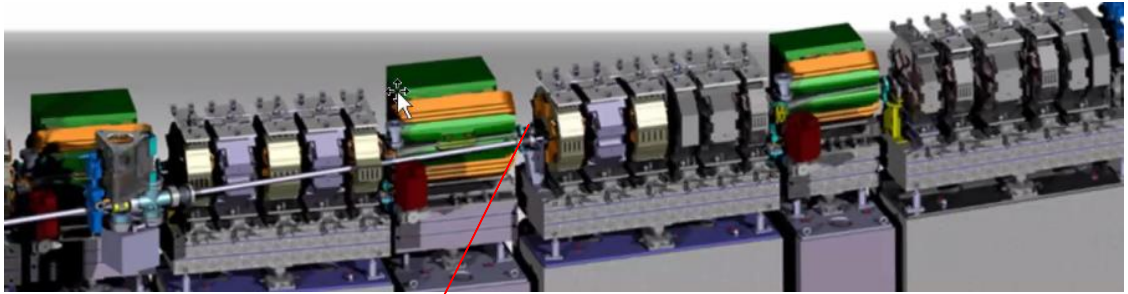
Synchrotron oscillations:

$$\Omega_s(t) := \frac{2\pi}{T_s} = \sqrt{-\frac{qV_0\eta\omega_{RF}\sin\psi_s}{Cp_s}}$$

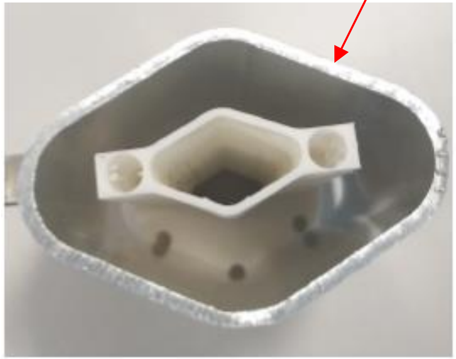
Transverse Beam Dynamics

simone.dimitri@elettra.eu

Magnetic Lattice



Vacuum chamber (Al, Cu, Steel) at ultra-low pressure (< 10^{-9} mbar), to avoid gas-scattering



Particle beam must be kept in!

---> ***external focusing***

$$\frac{|\vec{F}_e|}{|\vec{F}_m|} = \frac{q|\vec{E}|}{q|\vec{v}\wedge\vec{B}|} = \frac{E}{vB} \equiv 1 \quad \Rightarrow \quad \frac{|\vec{E}|}{|\vec{B}|} = \beta c$$

300 MV/m !

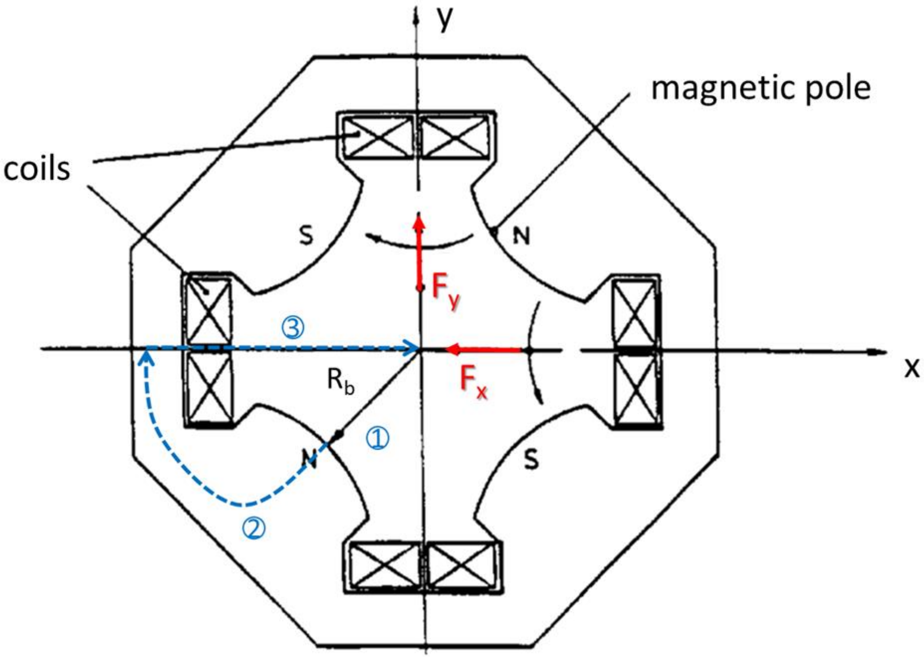
1 Tesla …

Quadrupole Magnet

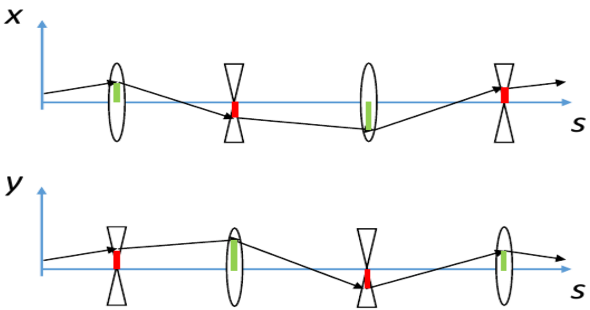
Normalized quadrupole strength:

$$k[m^{-2}] = 0.2998 \frac{g[T/m]}{p_z[GeV/c]}$$

$$f = \frac{1}{k l_q}$$ focal length



Betatron Motion



Betatron oscillations

$$x(s) = x_\beta(s) + x_\epsilon(s) =$$
$$= x_\beta(s) + D_x(s)\delta$$

Alternated gradient strengths (*Hill's eqs. assume* *linear motion* *&* *no frictional forces*):

$$\ddot{y}(s) - k(s)(1-\delta)y(s) = 0$$

$$\ddot{x}(s) + \left[k(s)(1-\delta) + \frac{1}{R(s)^2}\right] x(s) = 0$$

"Strong" focusing

Relative energy deviation

"Weak" focusing

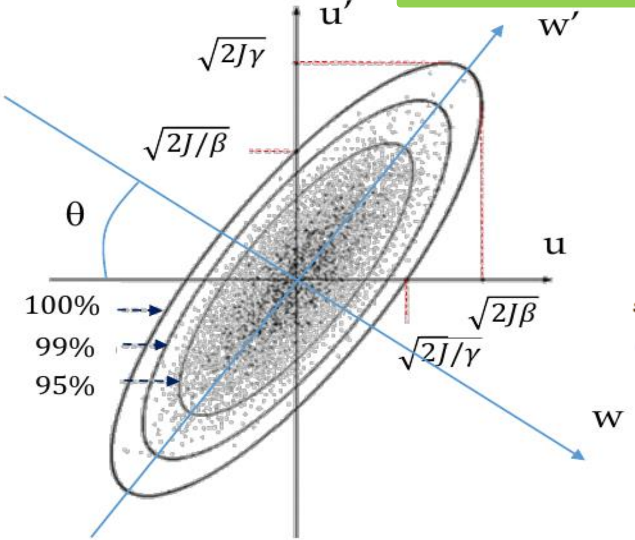
Phase Space

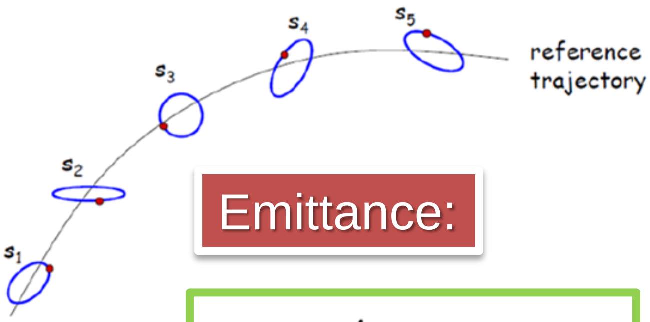
$$\begin{cases} u(s) = \sqrt{2J_u\beta_u}\cos\Delta\mu_u \\ u'(s) = -\sqrt{\frac{2J_u}{\beta_u}}\left(\alpha_u\cos\Delta\mu_u + \sin\Delta\mu_u\right) \end{cases}$$

Quasi-harmonic oscillator in (x,x’) and (y,y’)
----> the oscillation amplitude depends on s: $\beta_u(s), \alpha_u(s)$

Betatron tune:

$$Q_u = \frac{\Delta\mu_u}{2\pi} = \oint \frac{ds}{\beta_u(s)} \equiv \frac{2\pi R_s}{\overline{\beta_u}}$$





Emittance:

$$\epsilon_u = \frac{Area}{\pi} = \sigma_u \sigma_{u'}$$

Charge Distribution at Equilibrium

Liouville's Theorem

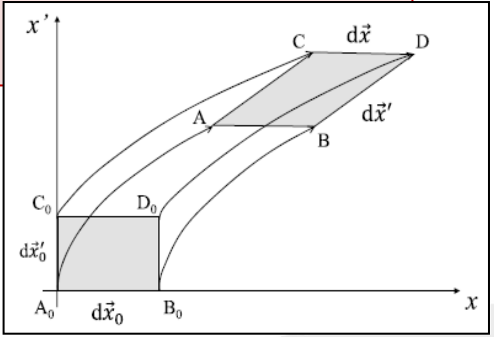
The dynamics of a non-dissipative system obeys ***Hamilton's equations:*** $\dot{q} = \frac{\partial H}{\partial p}$ $\dot{p} = -\frac{\partial H}{\partial q}$

The ***phase space area*** (hyper-volume) in proximity of an orbit is a ***constant of motion***.

A storage ring is ***not*** a Hamiltonian system because of radiation emission and acceleration.

However, it behaves ***as if it were*** a Hamiltonian system (see more later).

⇒ *The "phase space" beam emittance is a constant of motion*



A storage ring is ***not*** a ***linear*** system because of high order magnetic field components.

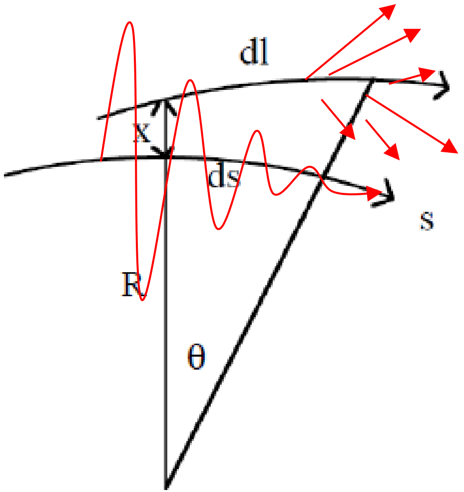
However, it behaves as a ***linearized system***.

⇒ *The "statistical" beam emittance is a constant of motion*

Radiation Damping, Quantum Excitation

Oscillations:

$$\begin{cases} \epsilon(t) = A_\epsilon(t)\cos(\Omega_s \frac{\Delta z}{c} + \phi_0) \equiv A_\epsilon(t)\cos\phi \\ \tau(t) = -\left(\frac{\alpha_c}{E_0\Omega_s}\right) A_\epsilon(t)\sin\phi \end{cases} \Rightarrow \begin{cases} A_\epsilon^2 = \epsilon^2 + \tau^2\left(\frac{E_0\Omega_s}{\alpha_c}\right)^2 \\ \langle\epsilon^2(t)\rangle_\phi = \frac{A_\epsilon^2(t)}{2} \end{cases}$$



Emission of synchrotron radiation:

$$\langle\delta A_\epsilon^2\rangle_\phi = \langle\delta\epsilon^2\rangle + \langle\delta\tau^2\rangle\left(\frac{E_0\Omega_s}{\alpha_c}\right)^2 = 2\langle\epsilon\delta\epsilon\rangle + \frac{1}{2}\langle(2\delta\epsilon)\delta\epsilon\rangle = -2\langle\epsilon u\rangle + \langle u^2\rangle \approx$$

$$\approx -2\langle\epsilon\frac{du}{d\epsilon}\epsilon\rangle + \langle u^2\rangle = -A_\epsilon^2\frac{du}{d\epsilon} + \langle u^2\rangle$$

"damping" "excitation"

Characteristic **damping time** to reach the **equilibrium** Gaussian distribution, in all planes:

Equilibrium:

$$\langle\frac{d}{dt}\langle\delta A_\epsilon^2\rangle_\phi\rangle_R \cong \frac{d\langle A_\epsilon^2\rangle_R}{dt} = -\langle A_\epsilon^2\rangle_R\langle\frac{d}{dt}\frac{du}{d\epsilon}\rangle_R + \langle\frac{d}{dt}\langle u^2\rangle_\phi\rangle_R \equiv 0$$

$$\tau \approx T_0\frac{E_0}{U_0}$$

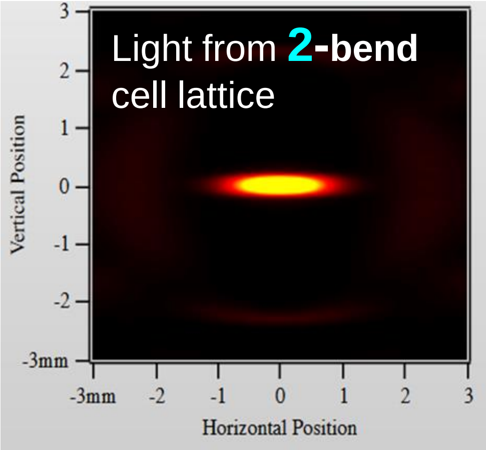
Beam Size and Emittance

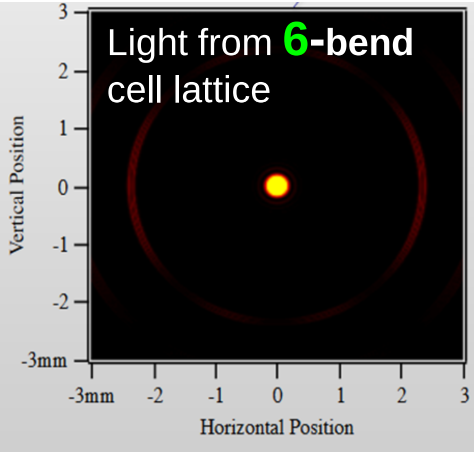
$$u(s) = \sqrt{2J_u\beta_u}\cos\Delta\mu_u$$

$$\sigma_x = \sqrt{\varepsilon_x\beta_x}$$

depends on quads strength only; varies through the lattice

constant through the lattice ("equilibrium")





$$\epsilon_{x,eq} = C_e \frac{\gamma^2}{J_x} \frac{\langle H_x \rangle_R}{R}$$

$$\frac{\langle H_x \rangle_R}{R} \approx \frac{1}{R}\left(\frac{1}{\beta_x}\langle D_x^2\rangle + \beta_x\langle D_x'^2\rangle\right) \propto \frac{\theta_b}{l_b}\left[\frac{l_b^2\theta_b^2}{4\beta_x} + \beta_x\theta_b^2\right] \propto \theta_b^3\left(\frac{l_b}{\beta_x} + \frac{\beta_x}{l_b}\right) \propto \left(\frac{2\pi}{N_b}\right)^3$$

$$\Rightarrow \epsilon_{x,eq} = F\frac{C_e}{J_x}\frac{\gamma^2}{N_b^3}$$

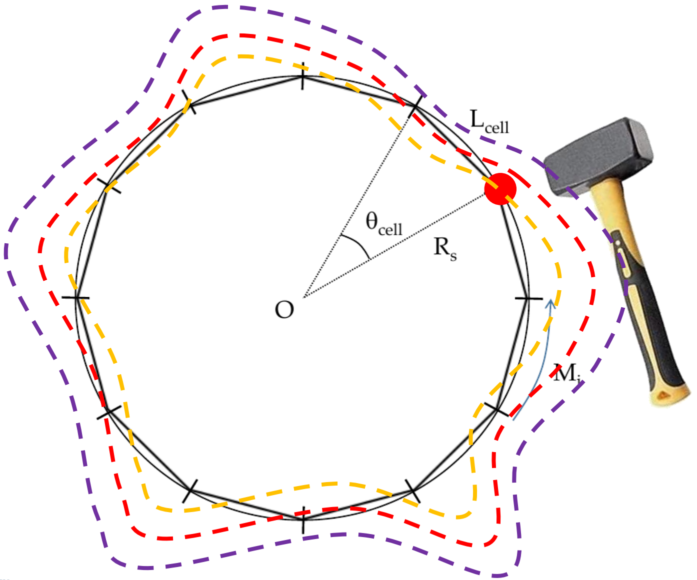
This is driving world-wide upgrades to multi-bend lattices (4th generation). Radiation is far more collimated and more intense – higher "brilliance"!

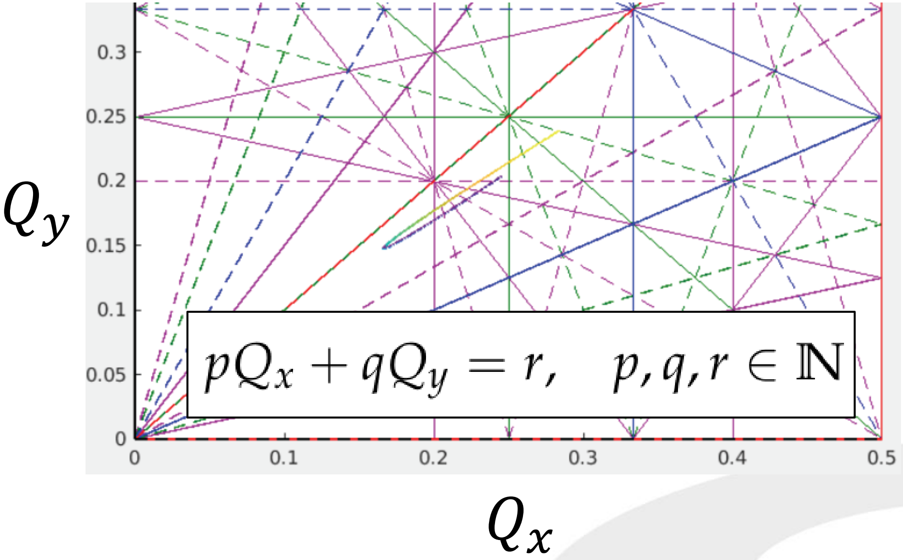
Nonlinear Dynamics

Resonances

$$Q_u = \frac{\Delta\mu_u}{2\pi} = \oint \frac{ds}{\beta_u(s)} \equiv \frac{2\pi R_s}{\overline{\beta_u}}$$

The error sum coherently if the particles goes back to it with same amplitude and phase (position and angle) every r-turns



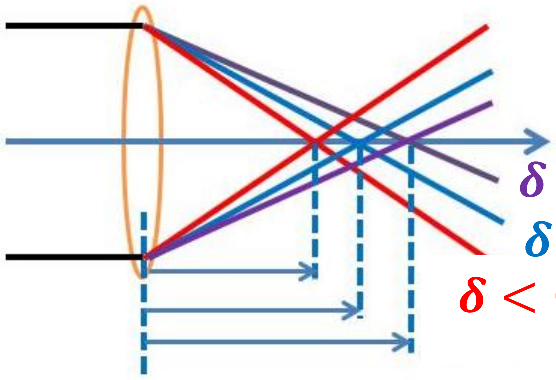


$$pQ_x + qQ_y = r, \quad p,q,r \in \mathbb{N}$$

simone.dimitri@elettra.eu

Chromaticity

Particles at (slightly) different energies are focused differently:



1. Phase advance

$$\tan(\Delta\mu_u) \approx -\beta_u \frac{u'}{u}$$

2. Small phase variation by error kick:

$$\mathrm{d}(\tan(\Delta\mu_u)) \approx d(\Delta\mu_u) \approx -\beta_u \frac{du'}{u}$$

3. Quad error kick: $\Delta u' \approx \boldsymbol{k\delta} \cdot ds \cdot u$

4. Local tune change:

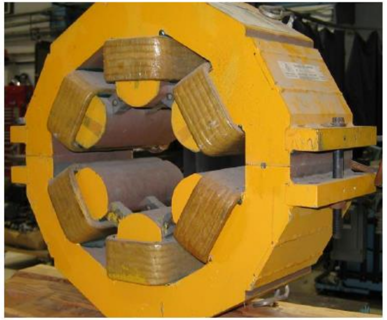
$$\mathrm{d}Q_u = \frac{d(\Delta\mu_u)}{2\pi} \approx -\frac{1}{2\pi}\beta_u\, k\delta ds$$

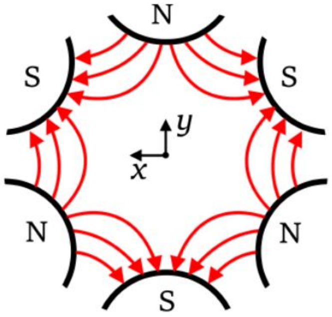
5. Global tune change (chromaticity):

$$\xi_u^{nat} := \frac{\Delta Q_u}{\delta} = -\frac{1}{4\pi}\oint ds\beta_u(s)k(s)$$

Sextupole Magnet

$$\begin{cases} \xi_x^{cor} = \frac{\Delta Q_x}{\delta} = -\frac{1}{4\pi}\oint \beta_x(s)\left[k(s) + m(s)\eta_x(s)\right] ds \\ \xi_y^{cor} = \frac{\Delta Q_y}{\delta} = -\frac{1}{4\pi}\oint \beta_y(s)\left[-k(s) + m(s)\eta_x(s)\right] ds \end{cases}$$

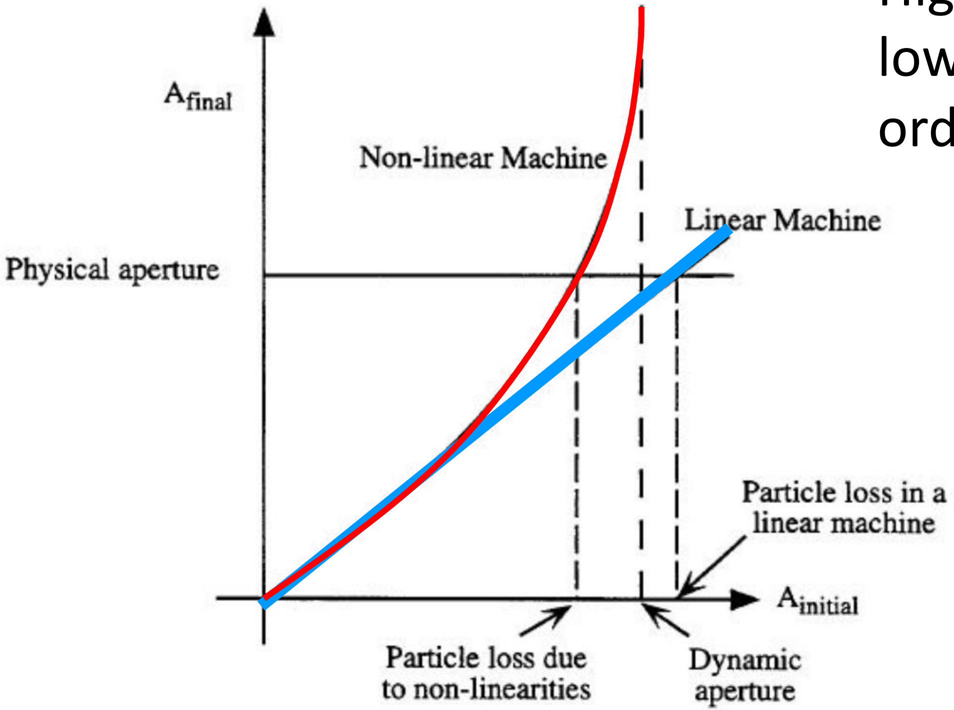




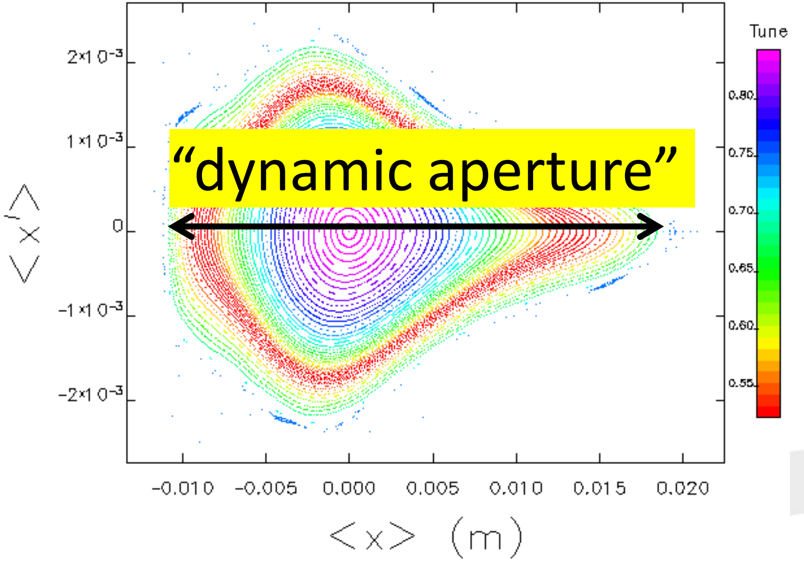
N.B.: $$m_{sext} \propto \frac{1}{\eta_x} \propto \frac{1}{\theta_b} = \frac{N_b}{2\pi}$$

Sextupoles acts as a quadrupole of normalized gradient proportional to the dispersion function.

Dynamic Aperture



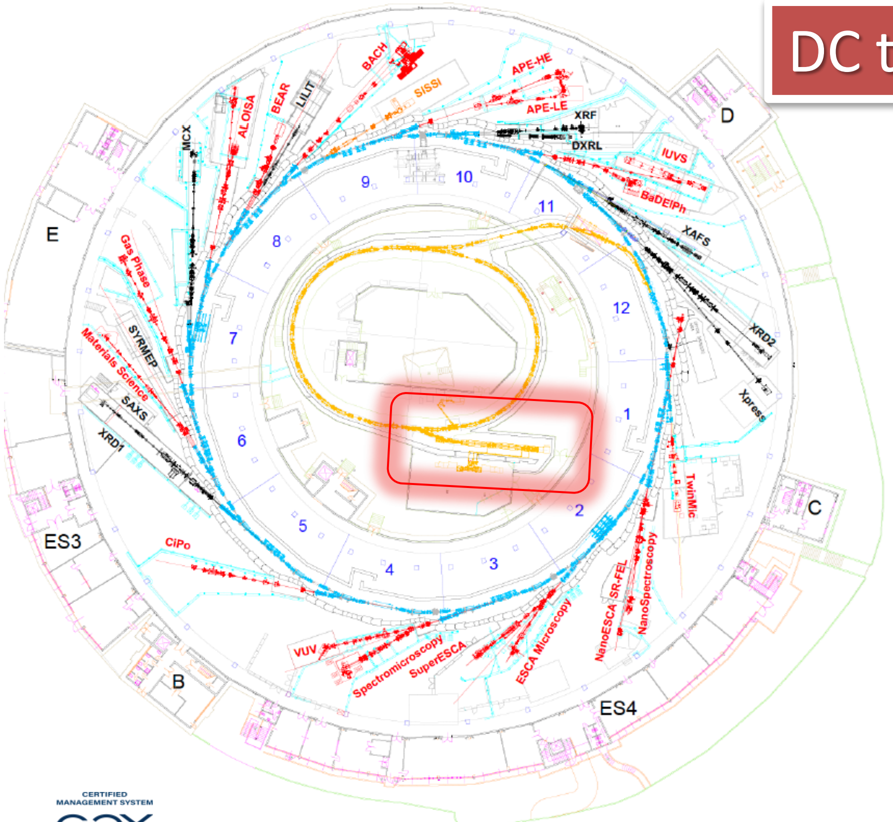
Higher order multi-pole magnets correct lower order errors, but they add higher order perturbations (nonlinear motion).

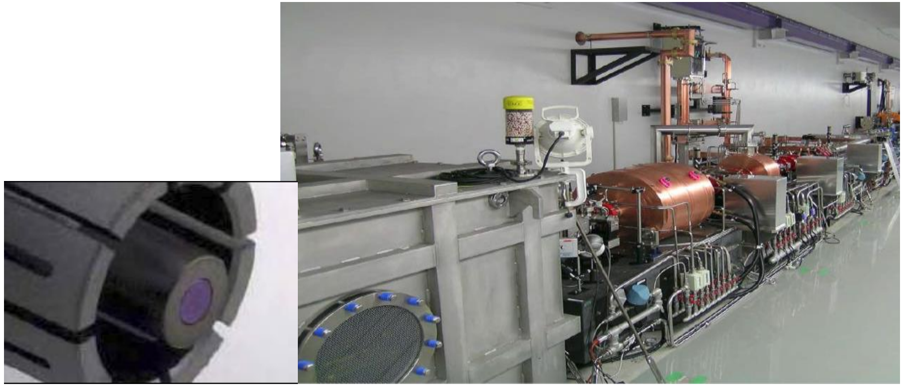


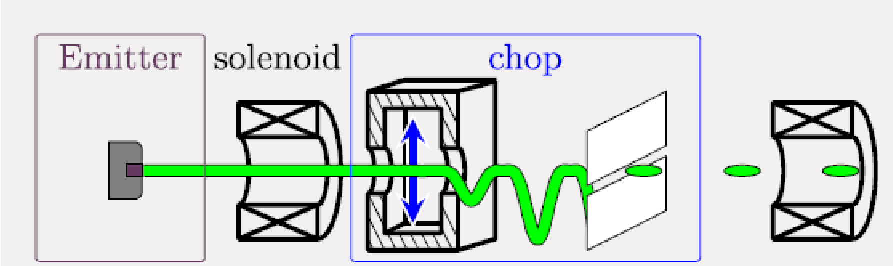
Beam Storage

Injection Chain - LINAC

DC thermo-ionic Gun + “buncher” + RF





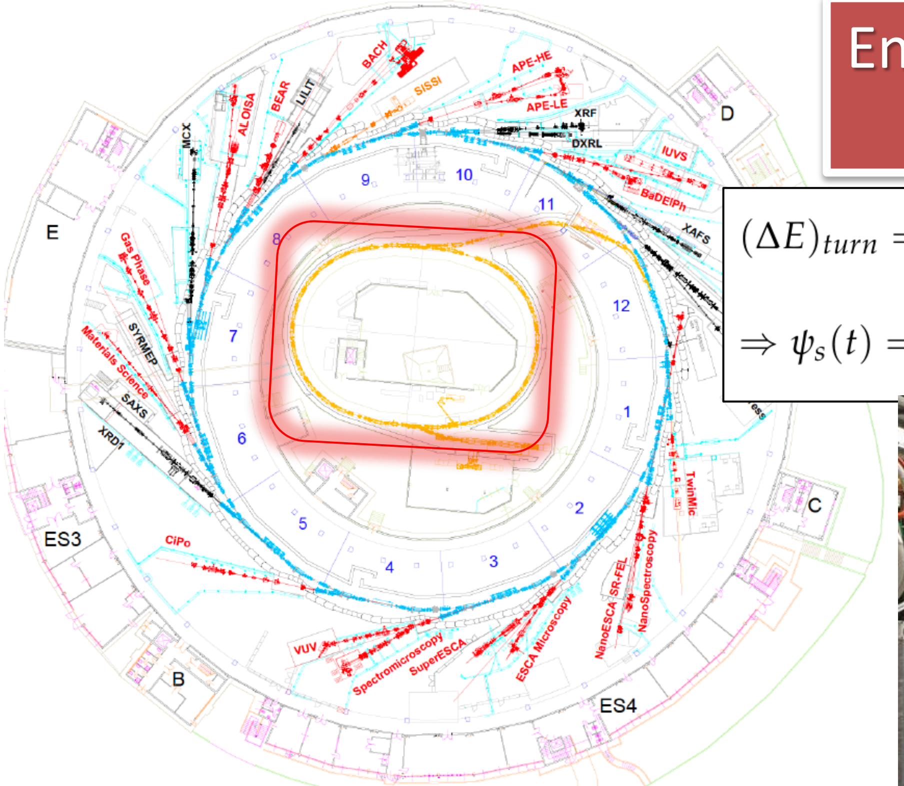


Injection chain - BOOSTER

Energy ramp ---> magnetic field ramp, frequency shift

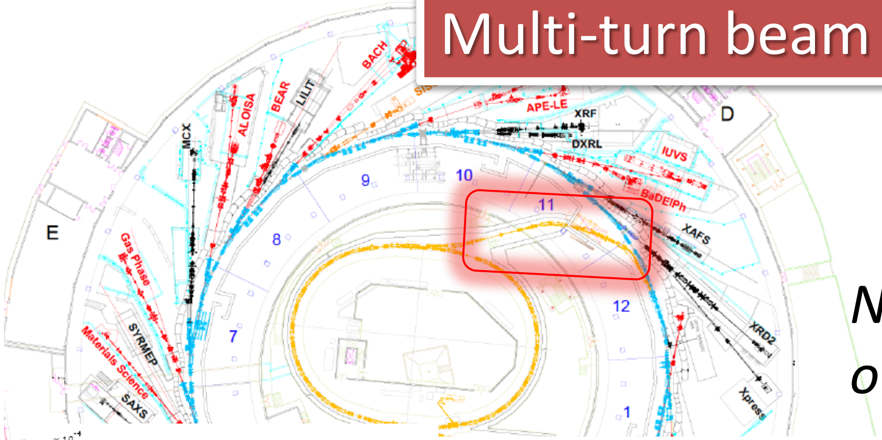
$$(\Delta E)_{turn} = (\Delta p_z)_{turn}\beta c = e\dot{B}_y r T_0 \beta c = 2\pi R_s r q \dot{B}_y \equiv q V_0 cos(\psi_s - \psi_0)$$

$$\Rightarrow \psi_s(t) = \psi_0 + \arccos\left(2\pi R_s r \frac{\dot{B}_y}{V_0}\right)$$



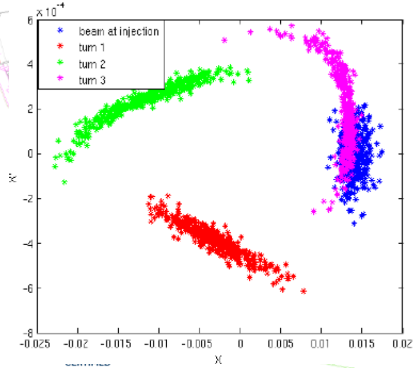
Injection chain - BUMP

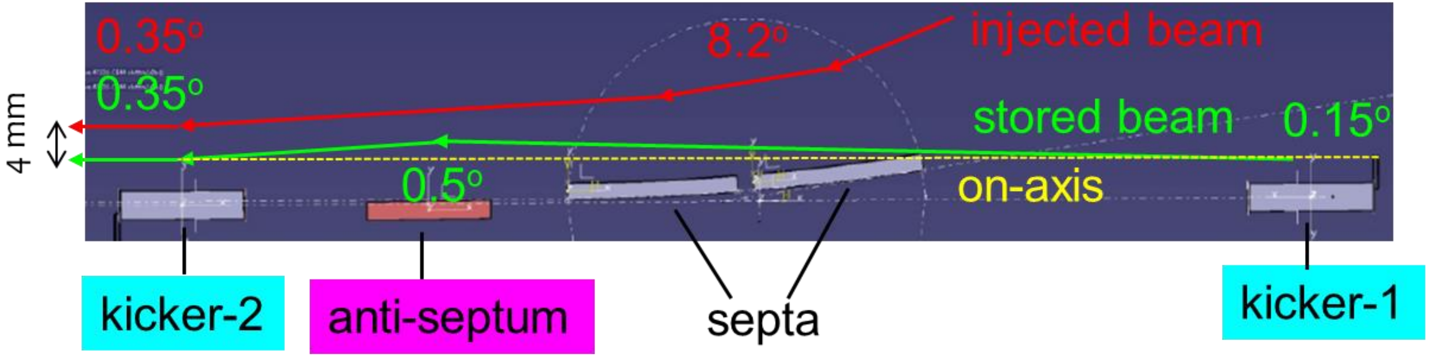
Multi-turn beam accumulation (e.g., off-axis injection)



- ➢ Injection efficiency.
- ➢ Transparency to stored beam (users)

New schemes: single-kicker, on-axis (swap-out), longitudinal injection, linac,…

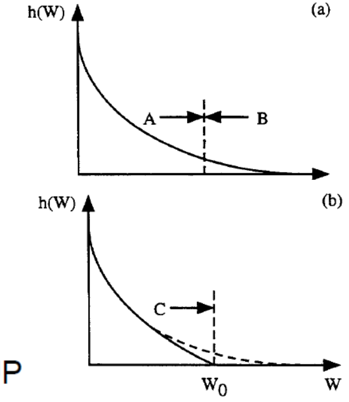




Beam Lifetime

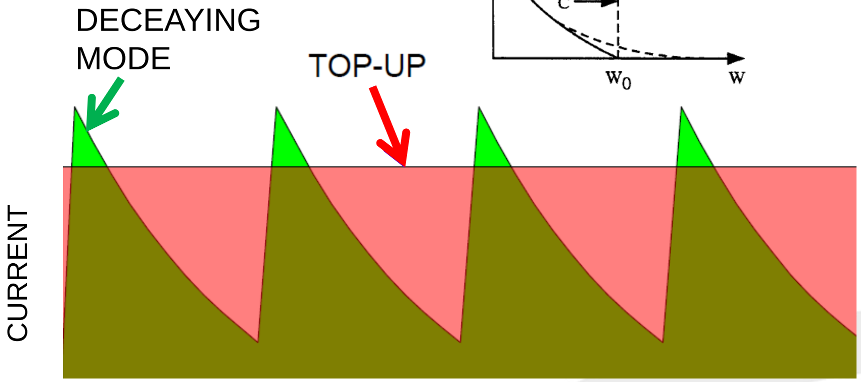
Quantum lifetime:

$$\left(\frac{dN}{dt}\right)_{W_c} = \left(\frac{dN}{dW}\frac{dW}{dt}\right)_{W_c}, where \begin{cases} dN(W) = N\rho dW \\ W(t) = \hat{W}e^{-\frac{2t}{\tau}} \end{cases} \Rightarrow \begin{cases} \frac{dN}{dW} = \frac{N}{\langle W\rangle}e^{-\frac{W}{\langle W\rangle}} \\ \frac{dW}{dt} = -\frac{2}{\tau} \end{cases}$$



$$\left(\frac{dN}{dt}\right)_{W_c} = -\frac{2N}{\tau}\frac{W_c}{\langle W\rangle}e^{-\frac{W_c}{\langle W\rangle}} \Rightarrow \begin{cases} N(t) = N_0 e^{-\frac{t}{\tau_q}} \\ \tau_q = \frac{\tau}{2}\frac{\langle W\rangle}{W_c}e^{\frac{W_c}{\langle W\rangle}} = \frac{\tau}{2}\frac{e^{\xi}}{\xi} \end{cases}$$

Due to physical or dynamic boundaries, the beam current decreases exponentially.



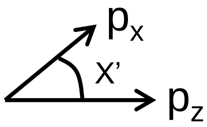
Particle Scattering

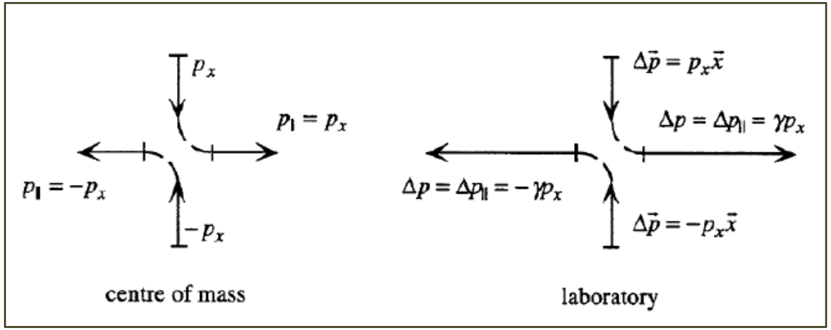
Touschek scattering:

single, large angle event

If two particles collide in the c.m. frame transferring their (transverse) momentum $\vec{p}_i' = (p_x', 0)$ into (longitudinal) momentum $\vec{p}_f' = (0, p_z') = (0, p_x')$,

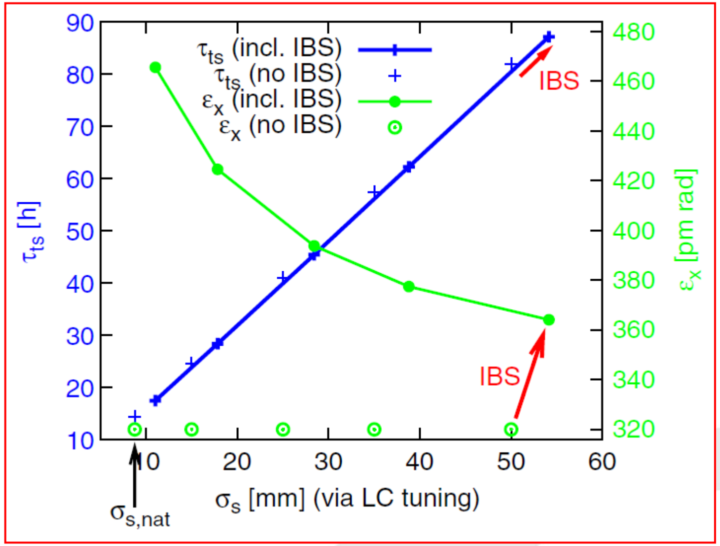
$$\Rightarrow \frac{\Delta p_z}{p_z} \approx \gamma \sqrt{\frac{\epsilon_u}{\beta_u}}, \quad u = x, y$$ must be << long. acceptance





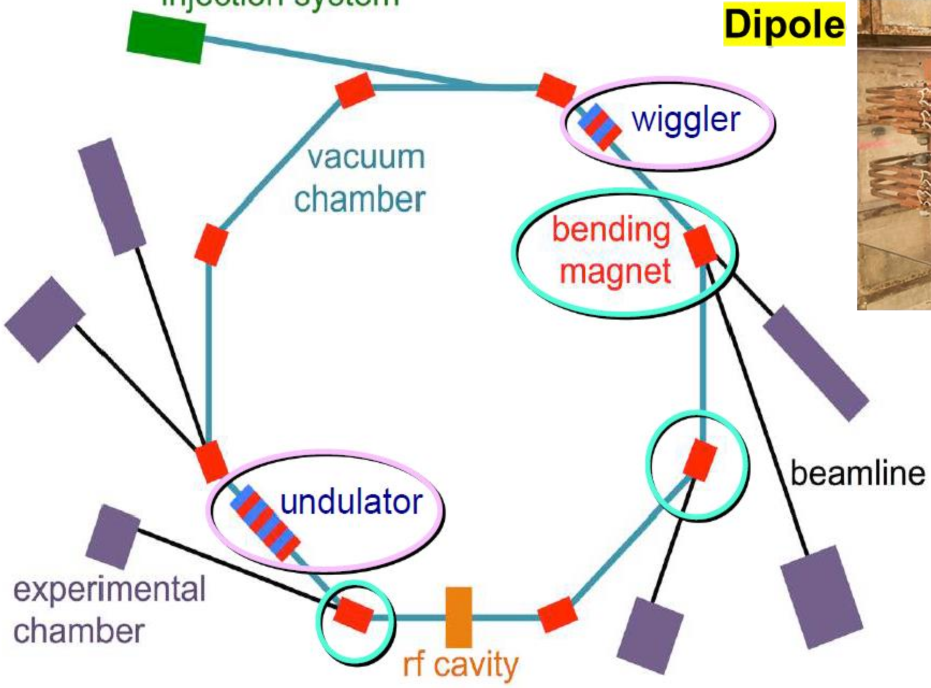
Intrabeam scattering:

multiple small angle events (diffusion)

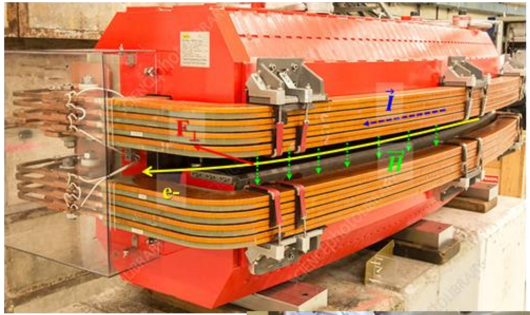


Diffraction Limited Light Sources

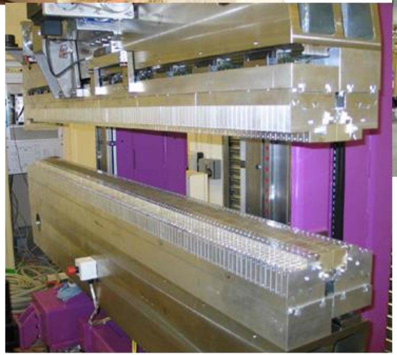
Insertion Devices



Dipole



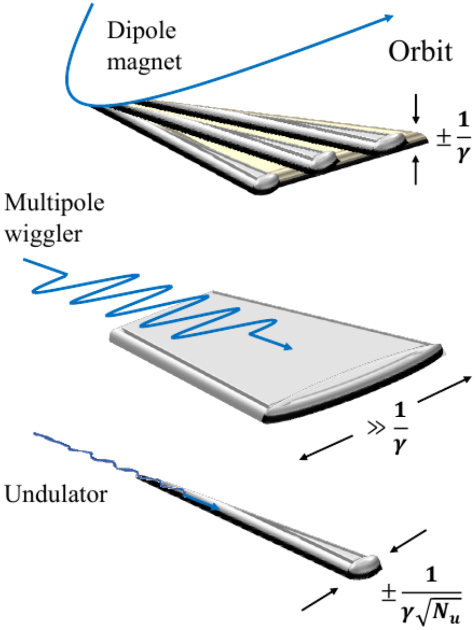
Superconducting wiggler

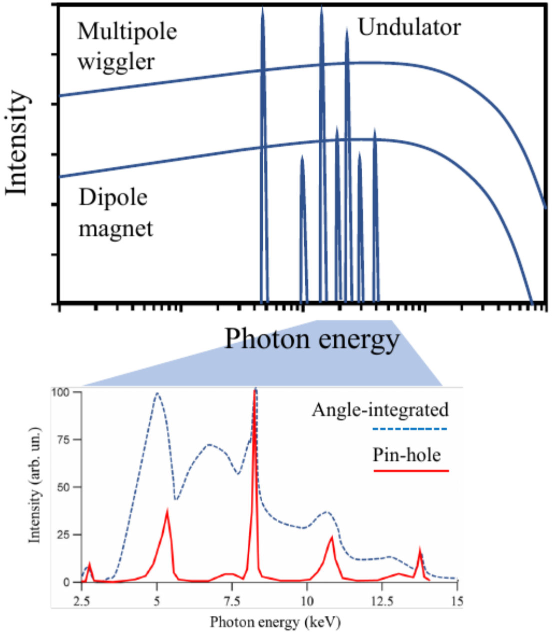


Variable gap undulator

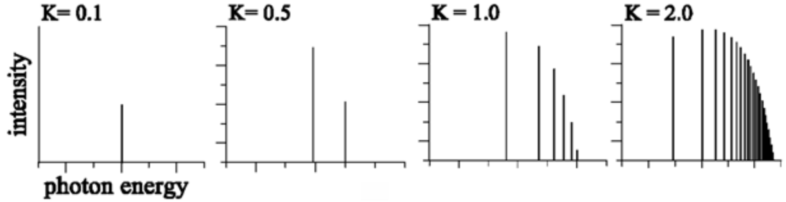
Light Shaping

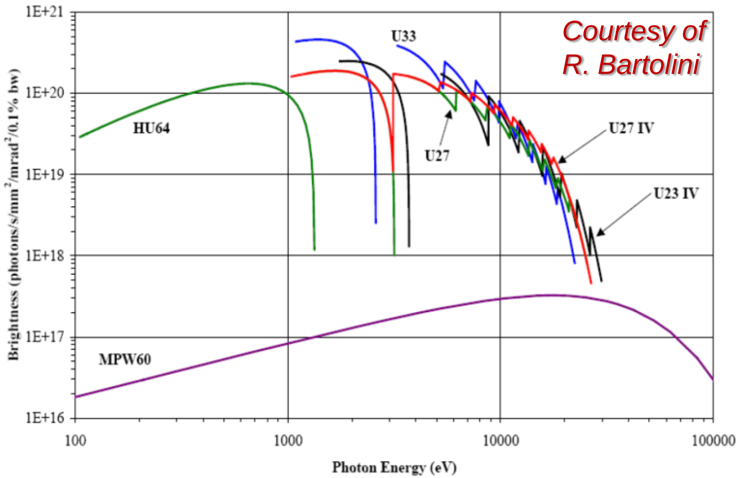
dipole, wiggler, undulator





wiggler vs. undulator





Courtesy of R. Bartolini

Maximizing the Brilliance

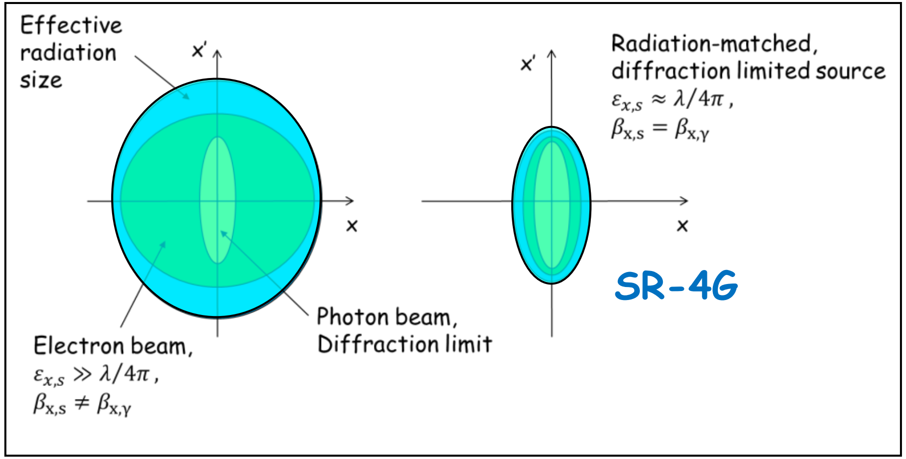
Brilliance = 6-D photon density

$$B_\gamma = \frac{dN_\gamma/dt}{4\pi^2\Sigma_x\Sigma_{x'}\Sigma_y\Sigma_{y'}\Delta\omega/\omega}$$

e-beam–radiation matching

e-beam at the diffraction limit

$$B_\gamma = \frac{dN_\gamma/dt}{\Delta\omega/\omega}\frac{1}{(\lambda^2/2)(\kappa+1)}$$



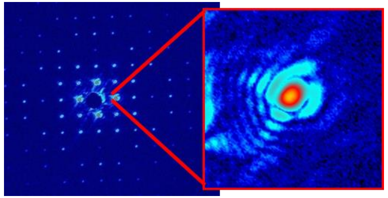
$$\varepsilon_{x,s} = \varepsilon_R = \frac{\lambda}{4\pi}$$

$$\kappa = \frac{\varepsilon_{y,s}}{\varepsilon_{x,s}} \leq 1$$ **Coupling coefficient**

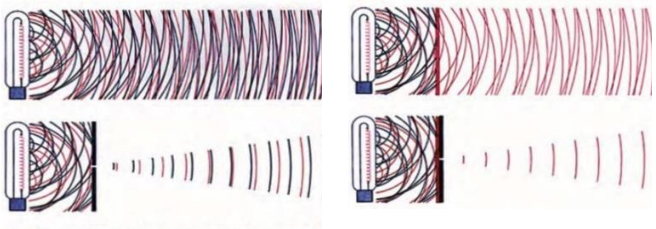
This ensures a high degree of ***transverse coherence*** at wavelengths $\lambda \geq 4\pi\varepsilon_{x,s}$

Transverse Coherence

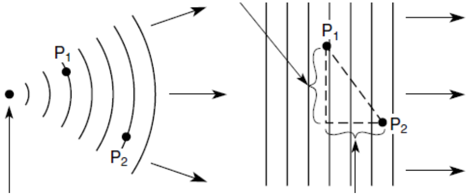
Interference fringes



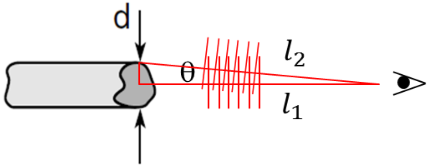
Collimated, monochromatic light



Phase-correlated field

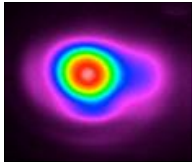


Classical model: path length over which two waves become **out of phase**



Uncertainty Principle: the smallest **phase space area** occupied by the light pulse

$$\Delta x \Delta p_x \geq \frac{\hbar}{2} \quad and\ \theta = \frac{\Delta p_x}{p_z} \cong \frac{\Delta p_x}{(h/\lambda)} \quad \Rightarrow \frac{d}{2}\theta_c = \frac{\lambda}{4\pi}$$

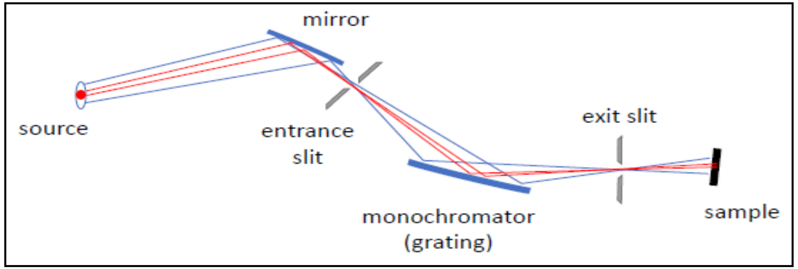


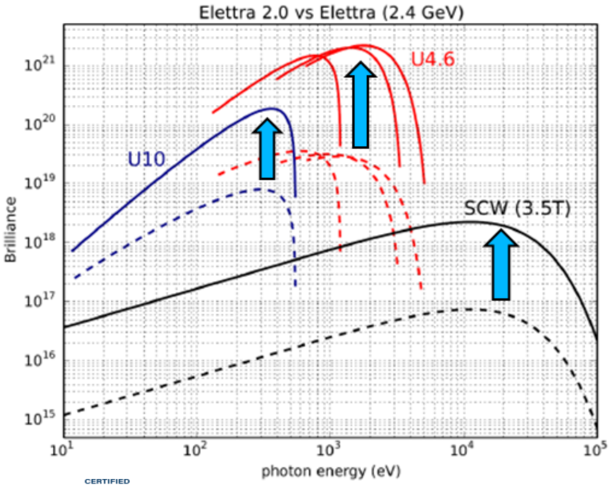
TEM00

Minimum transverse phase space area (**"emittance"**) of a transversally coherent light pulse

Brilliance for Beamlines

- $\frac{dN_\gamma/dt}{\Sigma_x\Sigma_{x'}\Sigma_y\Sigma_{y'}}$ is a **conserved quantity** in a *perfect* optical system. However, a ***real beamline*** includes slits, mirrors, gratings, etc. for manipulation of the light pulse. They show geometrical and surface **imperfections**, which are stronger for larger spatial and angular **footprint of the light on the optical elements**.





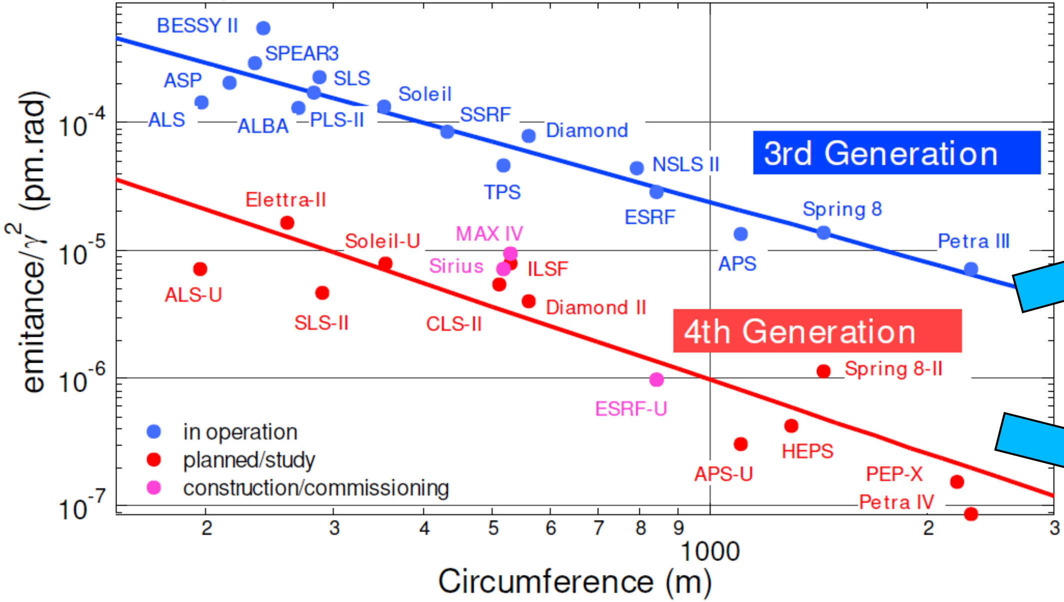
Machine design at the state-of-the-art is a good investment because **the higher the brilliance at the *source*, the higher the brilliance at the *sample!***

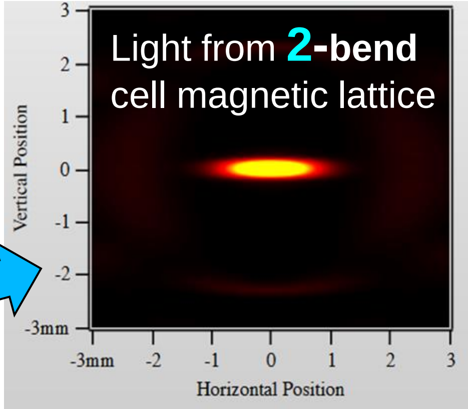
Brilliance for Experiments

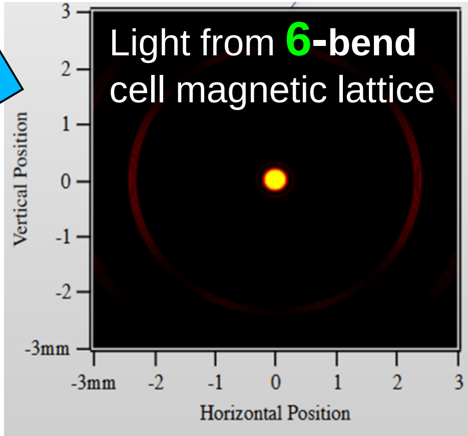
- Reduction in the source emittance, thus increase in **brilliance,** will lead to:
  - significant ***gain in the emitted or transmitted signals*** from the samples;
  - ***reduced acquisition time*** for all types of spectroscopies and x-ray scattering techniques;
  - implementation of ***photon-hungry techniques*** such as: high pressure experiments with anvil cells and dilute samples, and spin-resolved ARPES;
  - improvement of the ***lateral resolution*** with focusing optics down to a few-nm scale (e.g. nano-PES, nano-ARPES)
- Higher degree of transverse **coherence** will open unique opportunities for:
  - ***Coherent Diffraction Imaging*** (CDI) with chemical specificity
  - ***Ptychography***
  - ***X-ray photon correlation spectroscopy*** (XPCS)

Diffraction-Limited SRLS

Courtesy of R. Bartolini







- ❑ **Smaller gap – shorter period – stronger field undulators**
- ❑ **Transverse coherence at higher photon energies**

Conclusions

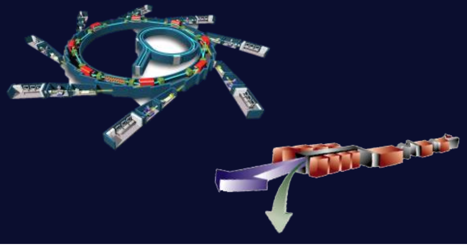
Strong Points of SRLS

- Synchrotrons provide light up to **tens of beamlines simultaneously**, each beamline receiving light from a dedicated insertion device.
- **Large flexibility** in tuning or selecting radiation wavelength and intensity. Spectrum from IR to hard x-rays.
- High **average radiation power** at the expense of low peak power (incoherent emission) and long pulses (several 10s ps).
- Extremely **stable**.
- **Transverse coherence** in X-rays.

The "LEAPS" EU Initiative

A new consortium of excellence in Europe devising a transformative level of coordination and integration

13 European Synchrotron Radiation and **6** FEL Facilities are joining forces to master the challenges of the next decades.



simone.dimitri@elettra.eu

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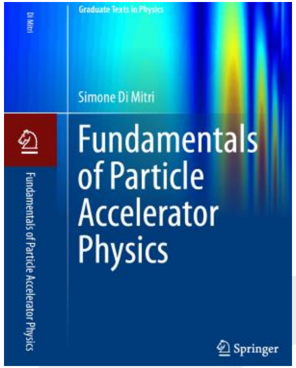


Elettra Sincrotrone Trieste

Back up slides

Radiated Power

v << c: $P'_{SR} \propto |\boldsymbol{S}'| \propto |E'_x|^2 \propto \left(\frac{F'_x}{m_e}\right)^2 \propto a'^2_x$

Stronger field **stimulates larger emission**

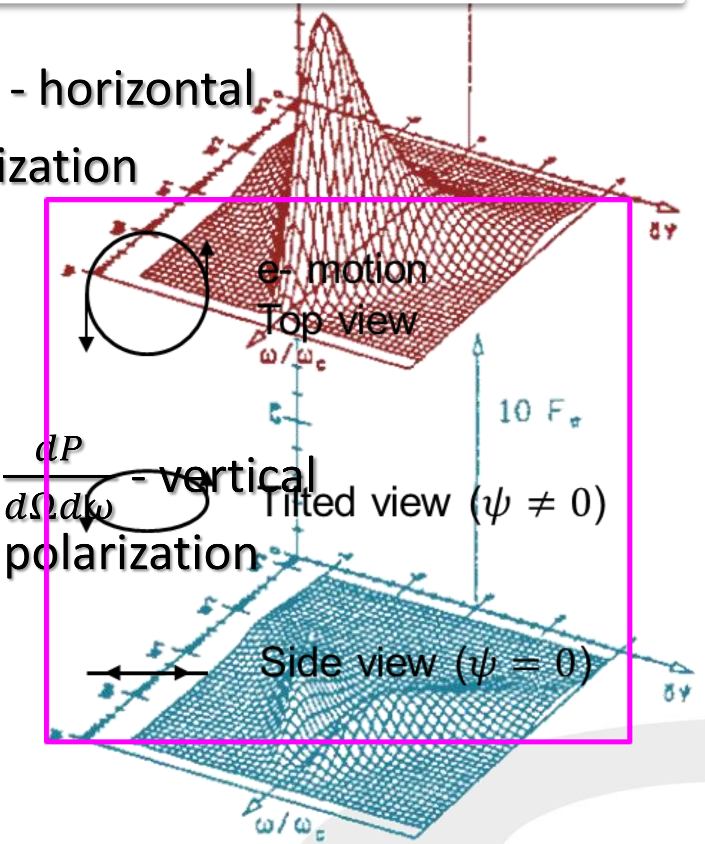
Lorentz-transform to Lab frame:

$$\begin{cases} t' \to t/\gamma \\ x' \to x \end{cases} \Rightarrow a'^2_x \to \gamma^4 \, a^2_x$$

$$P_{SR} \propto \gamma^4 = \frac{E^4}{m_0^4}$$

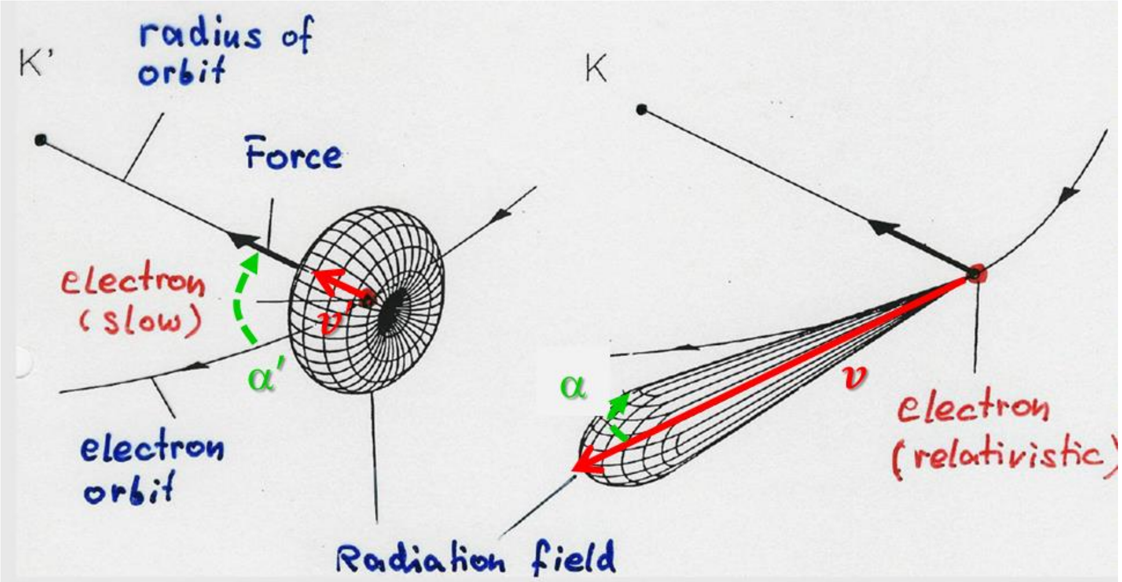
Strong increase with particle *total energy*

"Light" particles **(e.g., leptons) radiate *more than "heavy"* ones (e.g., hadrons)**



Angular Distribution

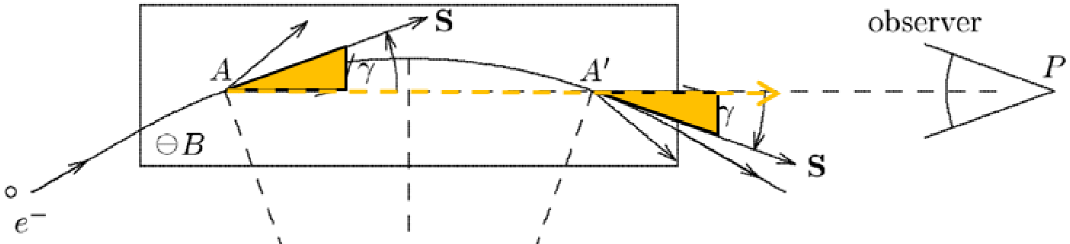
Electron rest frame:
electric dipole emission, max. intensity at $\alpha' = \pi/2$.



Lab reference frame:
forward collimated emission, $\alpha \approx 1/\gamma$.

$$\tan\alpha = \left.\frac{\sin\alpha'}{\gamma(\beta + \cos\alpha')}\right|_{\alpha'=\frac{\pi}{2}} = \frac{1}{\beta\gamma} \cong \frac{1}{\gamma} << 1$$

Spectrum

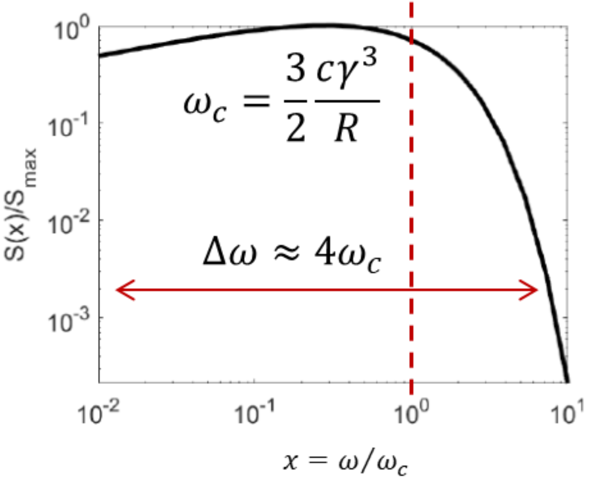


Dipole emission is a ***short*** light flash of duration Δt_{SR}.

The spectral bandwidth is ***broad:*** $\Delta\omega/\omega_c \approx 1$.

$$\Rightarrow \omega_c \approx \Delta\omega \approx \frac{1}{\Delta t_{SR}} \approx \frac{c\gamma^3}{R}$$

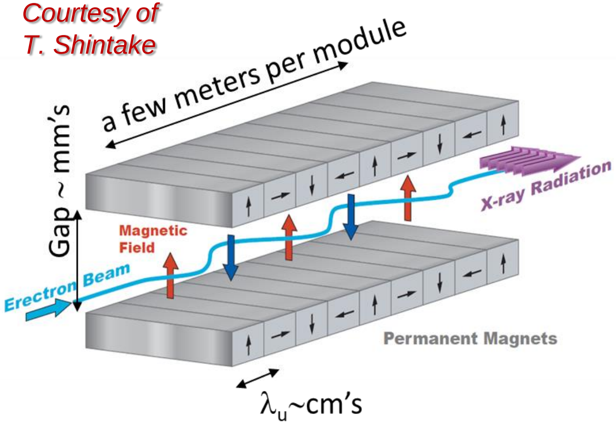
$$\Delta t_{SR} = t_A - t_{A'} = \frac{2R}{c}\left(\frac{1}{\beta\gamma} - \sin\frac{1}{\gamma}\right) \cong \frac{R}{c\gamma^3}$$

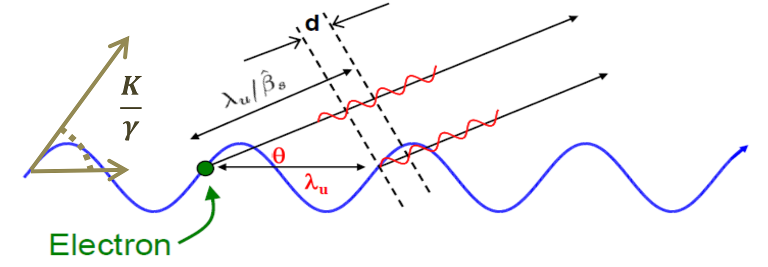


Undulator Spontaneus Emission

Elettra Sincrotrone Trieste

Courtesy of T. Shintake





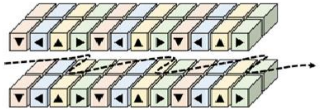
Undulator parameter

$$K = \frac{e B_y \lambda_u}{2\pi m_0 c}$$

Constructive interference: $\frac{\lambda_u}{v_z} - \frac{\lambda_u \cos\theta}{c} = \frac{\lambda}{c} \Rightarrow \lambda = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right)$

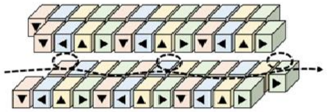
central wavelength

red-shift off-axis



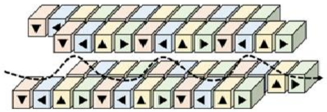
Shift = 0

Horizontal polarization



Shift = $\lambda_u/4$

Circular (right) polarization



Shift = $\lambda_u/2$

Vertical polarization

The electrons' orbit determines the light polarization, tuned via variable gap/phase of arrays

Brilliance

Brilliance = 6-D photon density:

$$B_\gamma = \frac{dN_\gamma/dt}{4\pi^2 \Sigma_x \Sigma_{x'} \Sigma_y \Sigma_{y'} \Delta\omega/\omega}$$

Effective radiation size (at the source)

$$\Sigma_u = \sqrt{\sigma_{u,s}^2 + \sigma_{u,R}^2} \cong \sqrt{(\beta\varepsilon)_{u,s} + (\beta\varepsilon)_{u,R}}$$

$$\Sigma_{u'} = \sqrt{\sigma_{u',s}^2 + \sigma_{u',R}^2} \cong \sqrt{(\varepsilon/\beta)_{u,s} + (\varepsilon/\beta)_{u,R}}$$

- It is maximized by **source-radiation matching:** $\beta_{u,s} = \beta_{u,R}$

$$B_\gamma = \frac{dN_\gamma/dt}{4\pi^2 \Delta\omega/\omega} \frac{1}{(\varepsilon_{x,s} + \varepsilon_R)(\varepsilon_{y,s} + \varepsilon_R)}$$

- and by a **diffraction limited source:**

$$B_\gamma = \frac{dN_\gamma/dt}{\Delta\omega/\omega} \frac{1}{(\lambda^2/2)(\kappa + 1)}$$

$$\varepsilon_{x,s} = \varepsilon_R = \frac{\lambda}{4\pi}$$

$$\kappa = \frac{\varepsilon_{y,s}}{\varepsilon_{x,s}} \leq 1$$ **Coupling coefficient**