

Adaptation: Universal Feature common to biological systems

- Two facets in adaptation

(1) adaptation --- 'essential variables' return to the original values (or within a range around them()) independent of environmental conditions

Cannon's Homeostasis

(eg. Body temperatures remains within a certain range) -- ('Wisdom of the body') → Wiener's feedback,

Keep 'micro-macro' consistency—Le Chatelier

(2) Change to a fitter state (higher survivability, growth) (here focus on the scale <<evolution)

- (1)(2) seemingly contradictory,,, but,,, somehow both are achieved
- For different time scales
- For different variables

Actually the two are studied rather independently

Dynamical systems view:

- (1) Some variables respond and come back to the original
- (2) Some variables change (switch to a different attractor, or by bifurcation) so that the 'fitness' is increased

Model for adaptation

(cf) perfect vs partial adaptation

Koshland, Oosawa ; Asakura-Honda model

'Homeostasis' after external change, most variables return to the original. Just few absorb the external change

Minimum model (1-degree of freedom)

$du/dt = f(u, v; S)$, $dv/dt = g(u, v; S)$ if $u^* = \text{indep't of } S$

$f = S - h(u, v)$ $g = (uv - v) / \tau$: (eg. $h = uv + u$) $\rightarrow u^* = 1$

$f = S - (u + v)$, $g = (S - v) / \tau$ $u^* = 1$

$f = S(1 - u) - uv$, $g = (S - v) / \tau$ $u^* = 1/2$

u shows perfect adaptation

$\rightarrow$ (more realistic models with gene expression, epigenetic modification, metabolic reaction)

Difference in time scale: fast response, slow relaxation to come back to the original

A simple example with catalytic reaction

PRL 107, 048301 (2011)

PHYSICAL REVIEW LETTERS

22 JULY 2011

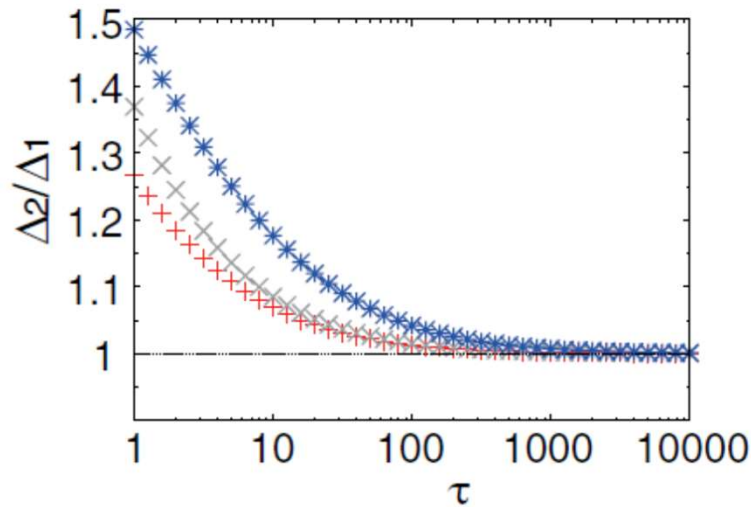
Weber's Law for Biological Responses in Autocatalytic Networks of Chemical Reactions

Masayo Inoue¹ and Kunihiko Kaneko²

introduced in [5]: $X_0 + X_1 \rightarrow 2X_1$; the model also includes (a) the synthesis of X_0 from an external resource chemical S as $S \rightarrow X_0$ and (b) the degradation of X_0 and X_1 . By representing the concentrations of the two chemicals as x_0 and x_1 and suitably scaling the time and concentration variables, we get the rate equation as

$$dx_0/dt = S - x_0x_1 - x_0, dx_1/dt = (x_0x_1 - x_1)/\tau. \quad (1)$$

on the steady state $x_0^* = 1$, $x_1^* = S - 1$ [12]. According to a linear stability analysis, this state is stable when $S > 1$, i.e., as long as $x_1^* > 0$. It should be noted that x_0^* is independent of S . The chemical concentration responds to the



Fast variable changes initially while slow variable is insensitive. Later slow variable changes, and fast variable returns to the original

FIG. 1 (color online). Plot of ratio of maximum response amplitude Δ_2/Δ_1 (ordinate axis) versus τ (abscissa axis). $\Delta_i = x_0^{\text{peak}i} - x_0^*$ with the change $S_0^i \rightarrow pS_0^i$. $\Delta_2/\Delta_1 = 1$ implies the independence of the response amplitude from the S_0^i value, or

$(x_0^{\text{peak}1} - x_0^*)/(x_0^{\text{peak}2} - x_0^*)$ obtained by multiplying S p -fold from S_0^1 or S_0^2 , respectively, as a function of τ . The ratio is close to unity; it is independent of the initial S_0 as long as $\tau/t^{\text{peak}} \sim \tau S_0 \gtrsim 100$. In a previous study, we

Weber Law: change in Log --basic

Change S from S_0 to pS_0 , peak change in x_0

We assume $\tau \gg 1$, which is required to ensure a fast response and slow adaptation. Under the adiabatic limit, x_1 changes more slowly than x_0 does. During the fast response of x_0 to the change in S , x_1 can be assumed to remain at the steady-state value under the condition of $S = S_0$. Then, the peak value of x_0 is obtained from $(dx_0/dt)_{x_0=x_0^{\text{peak}}} = 0$ by fixing the value of x_1 to $S_0 - 1$.

A straightforward calculation gives us $x_0^{\text{peak}} = p$. In a

**If timescale τ is
Separated the
response is
independent of the
absolute value S**

similar way, the peak time t^{peak} is estimated from $(dx_0/dt) \sim S - S_0 x_0$ and t^{peak} is given by $\sim 1/S_0$ [13].

The direct solution of $(dx_0/dt) = S - S_0 x_0$ is monotonic, but with inclusion of higher-order term in $1/(\tau S_0)$, x_0 shows a peak at t^{peak} .

Fast change is absorbed by slow change (LeChatelier)

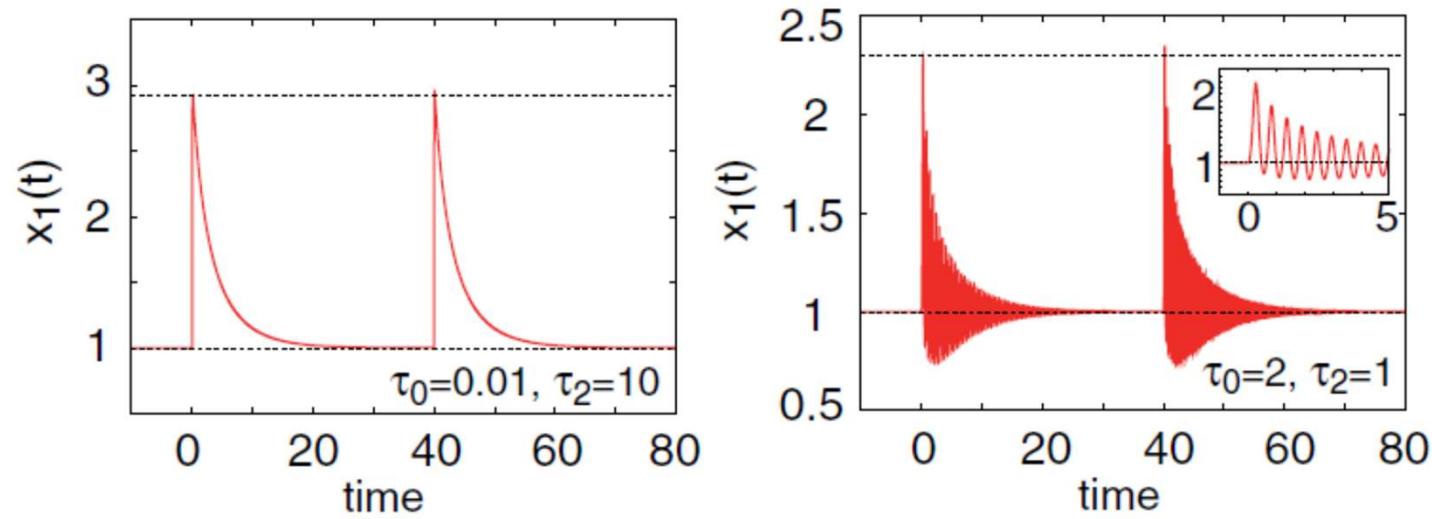
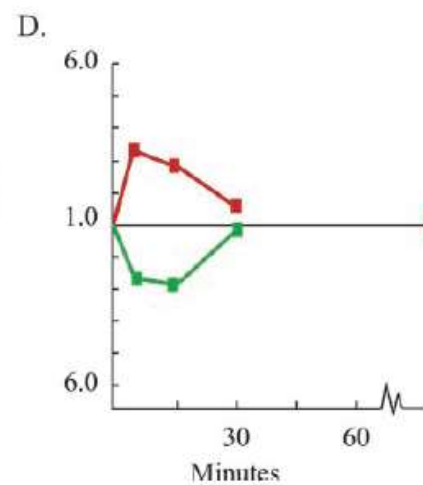
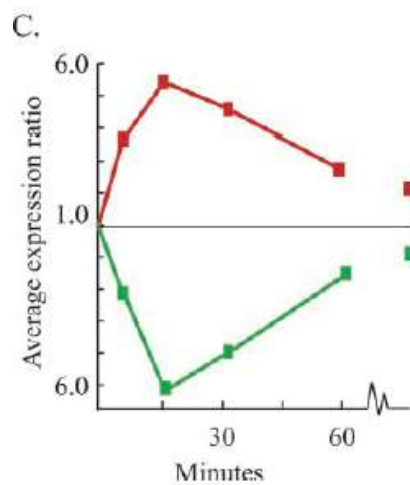
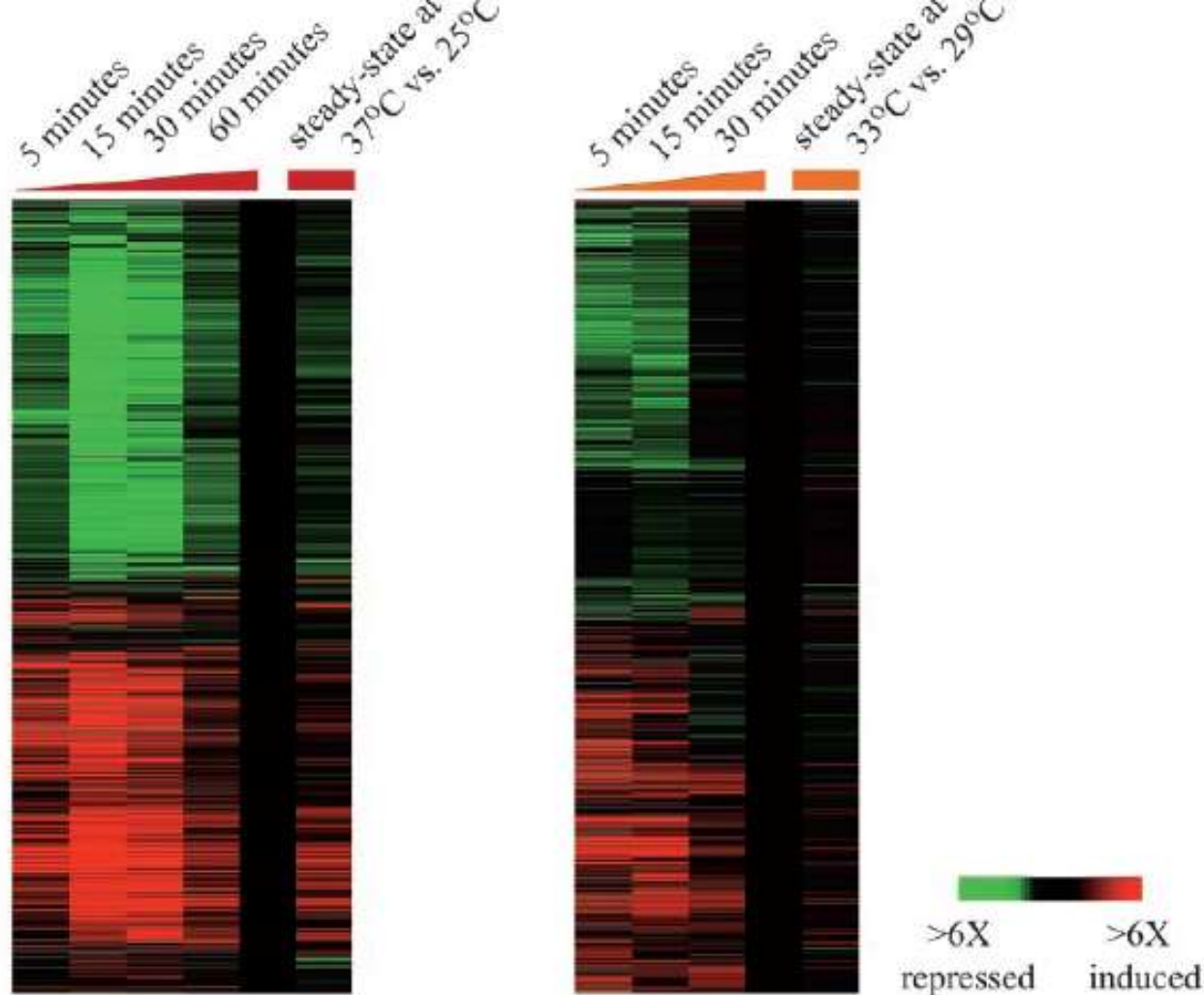


FIG. 2 (color online). Behaviors of an adaptive variable (x_1) in the $N = 3$ case. Responses corresponding to $S = 100 \rightarrow 200$ at $t = 0$ and $S = 200 \rightarrow 400$ at $t = 40$ are shown, respectively. (Left) $\tau_0 = 0.01$, $\tau_1 = 1$, $\tau_2 = 10$ and (Right) $\tau_0 = 2$, $\tau_1 = 1$, $\tau_2 = 1$, zoomed in the inset.

- Exp: microarray analysis (gene expression dynamics) by measuring temporal changes after change in environmental condition
- Many expressions show perfect or partial adaptation; timescales to return are distributed
- ‘conservative tendency in a biological system’: to keep many components at the original level (probably ‘good’ state for survival is rare, so that the life system tries to keep it)
- If some variables regarded as parameters, the parameters have tendency for adaptation (cf Ashby’s ultrastability)
- ‘excitable system in a high-dimension?’



**Gesch et al, --Yeast,
after stress, many
expression levels have
tendency to come back
towards the original**

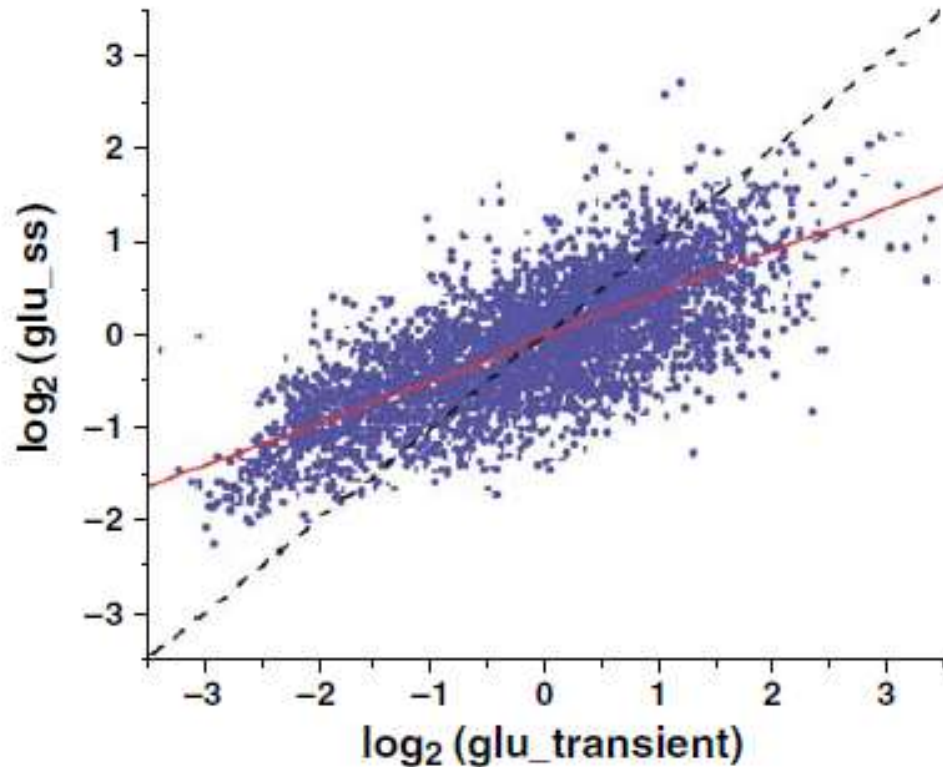
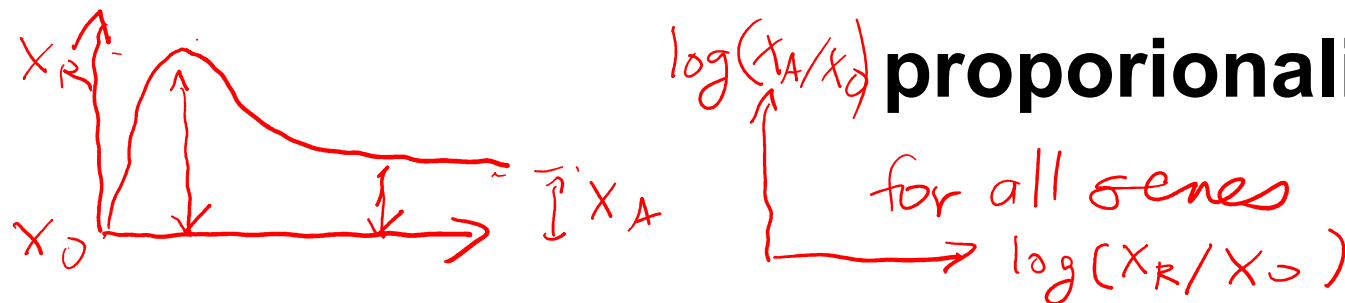


Figure 7 Steady-state transcriptional pattern is well predicted from the transient response. The steady-state expression level (average of the two measurement points in glucose steady state) versus the transient expression level (4th time point in Figure 1B) for the experiment with 40mM 3AT. The Pearson correlation between the transient response and steady state is 0.76. The slope deviation from the reference dashed black line represents the overshoot or undershoot in transient expression levels compared to the steady state.

**Stern,,.Braun
(MSB2007)
Yeast. Global trend in
response/adaptation**

**Thousands of gene
expression levels
show partial
adaptation**

**Further there is
common
proportionality**

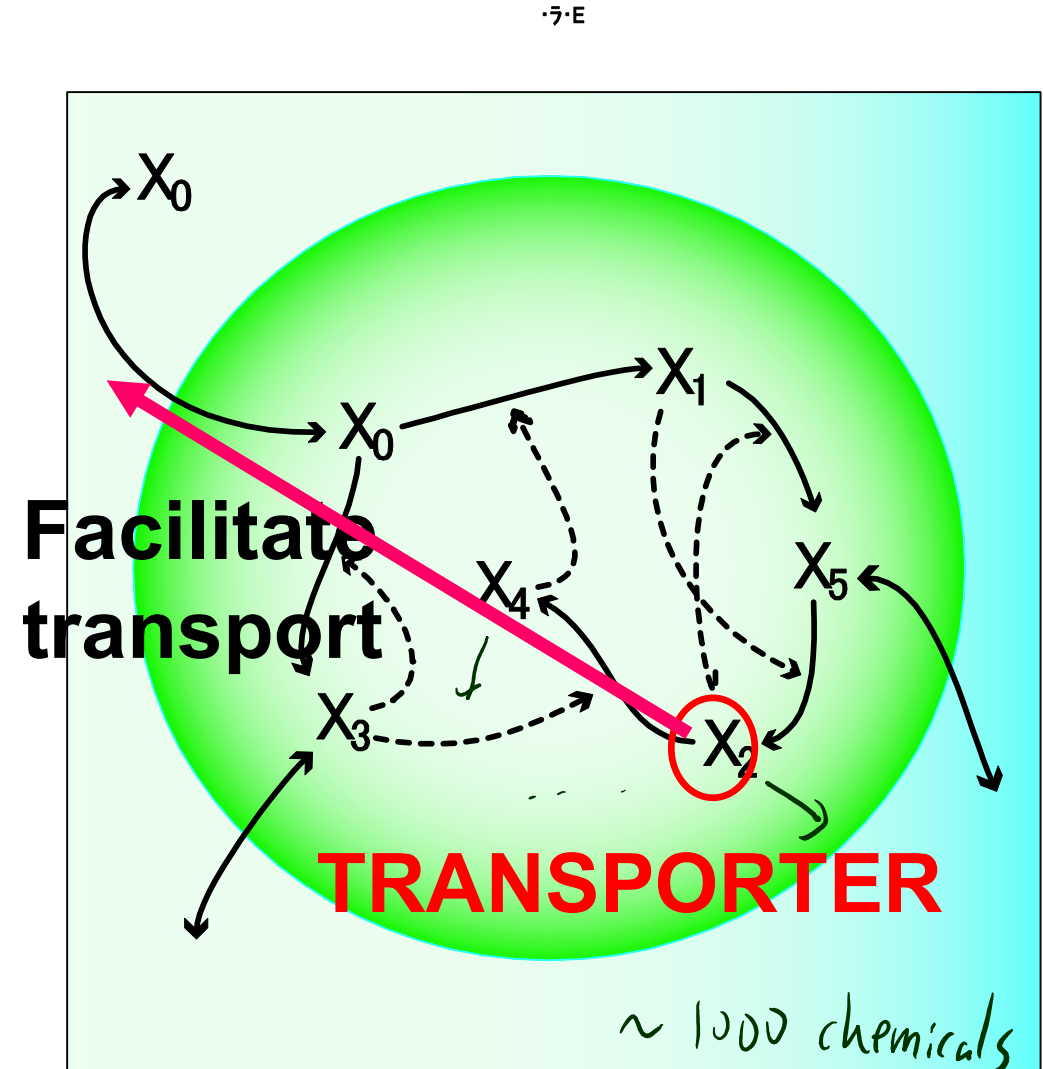


Microscopic Model → **Adaptation to Criticality**

Transport of resources is usually facilitated by

'transporter' molecule
(**active transport**) instead of passive diffusion
→ **self-tune the balance of concentrations of nutrient and catalytic chemicals**

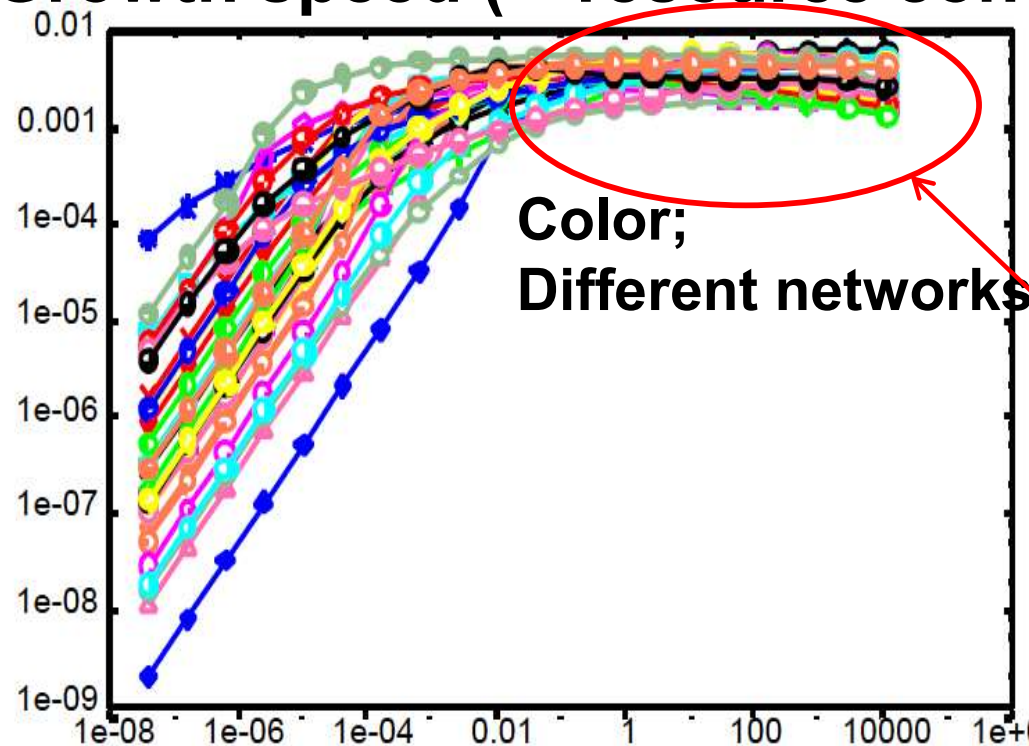
adaptive to environment
(Furusawa, KK, PRL, 2012)



Mutual Dependency leads to maintenance of reproduc

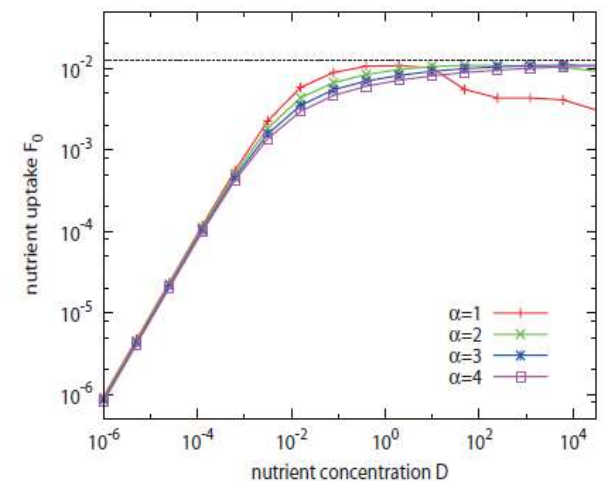
As long as external resource concentration is sufficient, cellular system adapts to a 'critical' state

Growth speed ($\propto$ resource conc.)

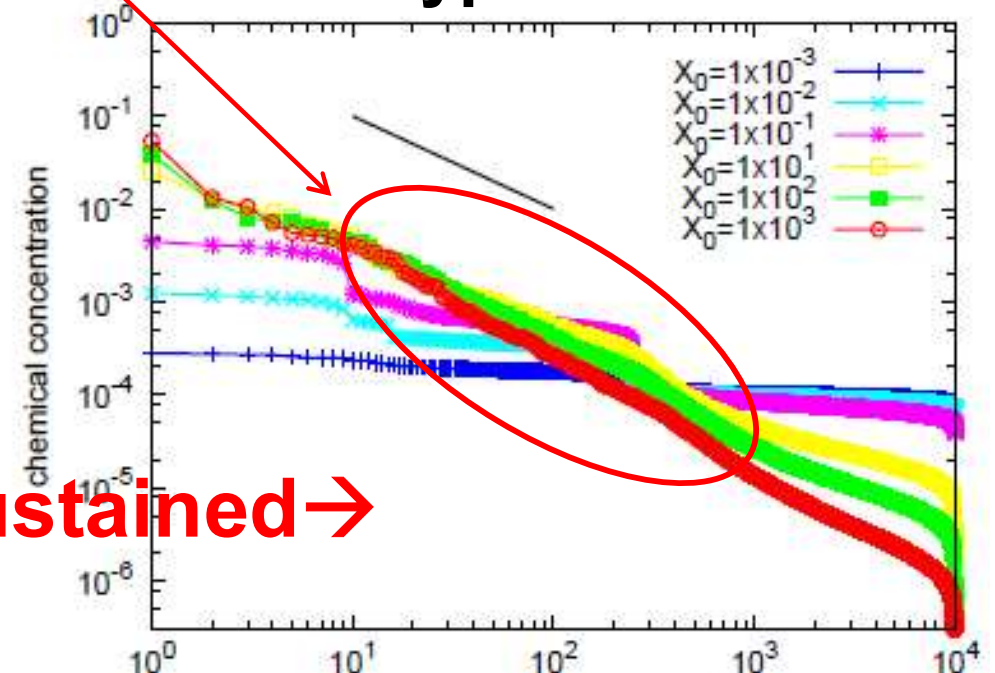


concentration of external resource

Power law abundances is sustained $\rightarrow$



Dependence on order of (for $\alpha=1,2,3,4$) catalysis oscillatory dynamics can appear due to delay in triggering resource - mean-field-type calculations



Layer 0 (resource) $\rightarrow$ L1 $\rightarrow$ L2 $\rightarrow$... $\rightarrow$ Lk
catalyst : mean field

$$dm_0/dt = Sm_k^\alpha - (M/N)(1 - m_0)m_0 - Sm_k^\alpha m_0$$

$$dm_j/dt = (M/N)((1 - m_0)m_{j-1} - (1 - m_0)m_j) - Sm_k^\alpha m_j,$$

$j = 1, \dots, k$, and $M = \rho N$ indicates the mean number of reaction paths

By setting $dm_0/dt = 0$, we obtain $F_0 = Sm_k^\alpha = \rho m_0$, **Growth rate**

$dm_j/dt = 0$, we get $m_j = m_{j-1}(1 - m_0)$. Thus, we get $m_k = m_0(1 - m_0)^k$.

S is large, $m_k \propto (1 - m_0)^k$ follows;

$$F_0 \sim \rho, \text{ if}$$

at k -th layer obeys $m_k = m_0(1 - m_0)^k$. On the other hand, at each k -th layer, there are $\sim (\rho N)^k$ chemical species. Hence, the ranking of the chemical at k -th layer, denoted by r_k , in the order of abundances increases as $r_k \sim (\rho N)^k$ when ρN is enough large, and thus $k = \log(r_k)/\log(\rho N)$. From these equations, we obtain $m(r_k) = m_0(1 - m_0)^{\log(r_k)/\log(\rho N)}$, where $m(r_k)$ represents the chemical concentration of r_k -th ranked chemical. Thus,

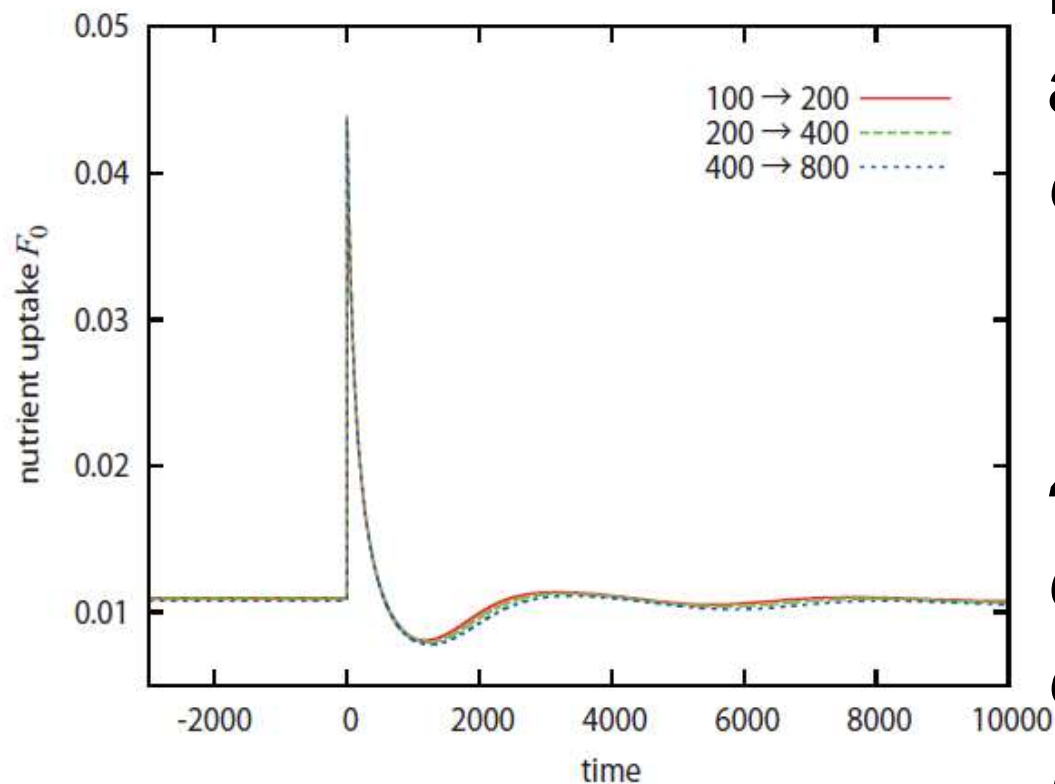
$$\log m_k = \log m_0 - \alpha \log(r_k) \tag{S1}$$

Adaptation dynamics (Fold Change Detection)

Change in environment (Resource)

Adaptive dynamics (growth speed first changes and returns to the original **Fold-change detection:**

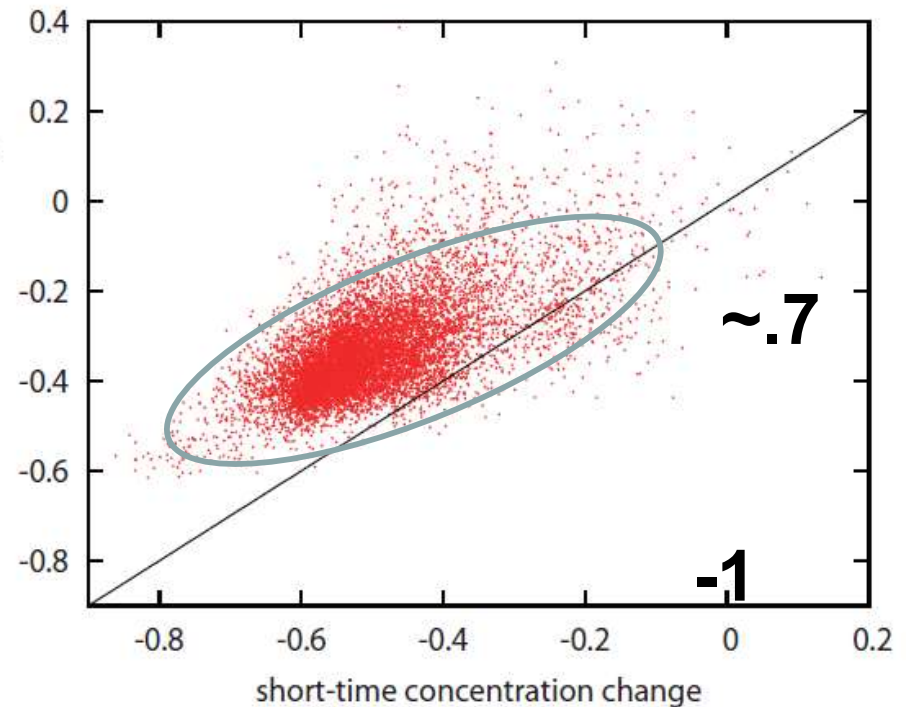
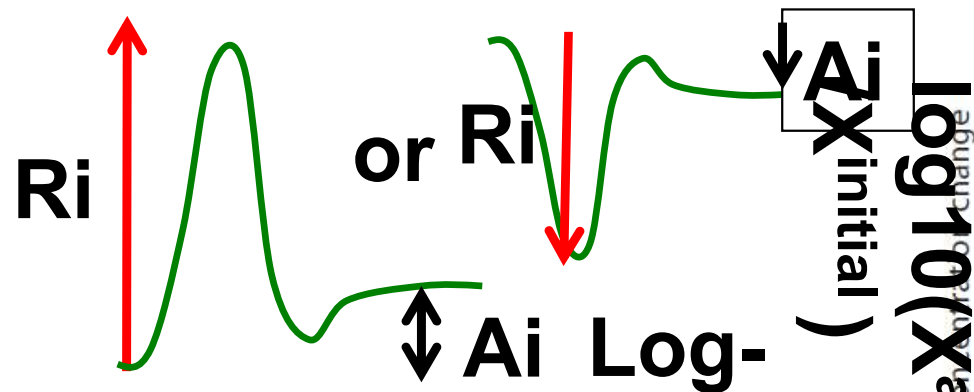
The adaptive dynamics depends only on the ratio of resources before and after. *e.g., after change of external resources*
100 → 200, 200 → 400, 400 → 800, identical dynamics common in responses of present cells



(Goentoro-Kirschner, Alon et al)

Common High-dimensional Adaptation dynamics

all chemicals show 'partial' adaptation



All of 10000 chemicals show imperfect adaptation.

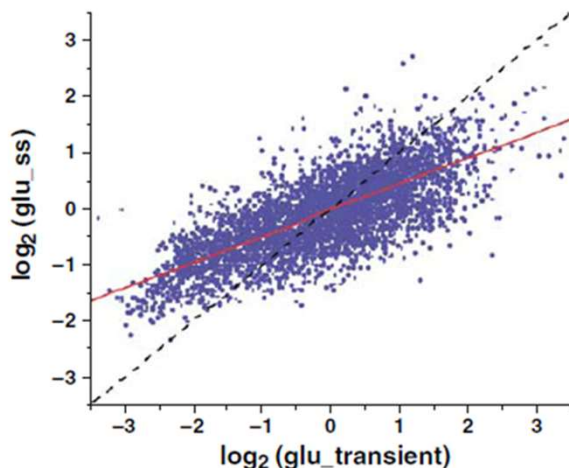
Response/Adapt

$R_i = C * A_i$ with $1 > C > 0$

$\log_{10}(X^{\text{response}} / X^{\text{initial}})$

Yeast, change in expression level of each gene by

Genome-wide transcriptional plasticity underlies cellular adaptation to novel challenge



- Ideal-Cell-Model of this version

(1)Optimal growth is achieved

(2)Power Law in abundances (Zipf's law)

(3)Adaptation dynamics of growth rate with FCD

(4)General trend of partial adaptation

just by catalytic reaction network +feedback from transporter(=enzyme).

Interestingly, (1)-(4) agree well with the observations in the present cells

(1)-(3) is explained by layer-mean-field theory

Change in enzyme abundances → change in reaction rate → autonomous regulation in time scale? (→Next)

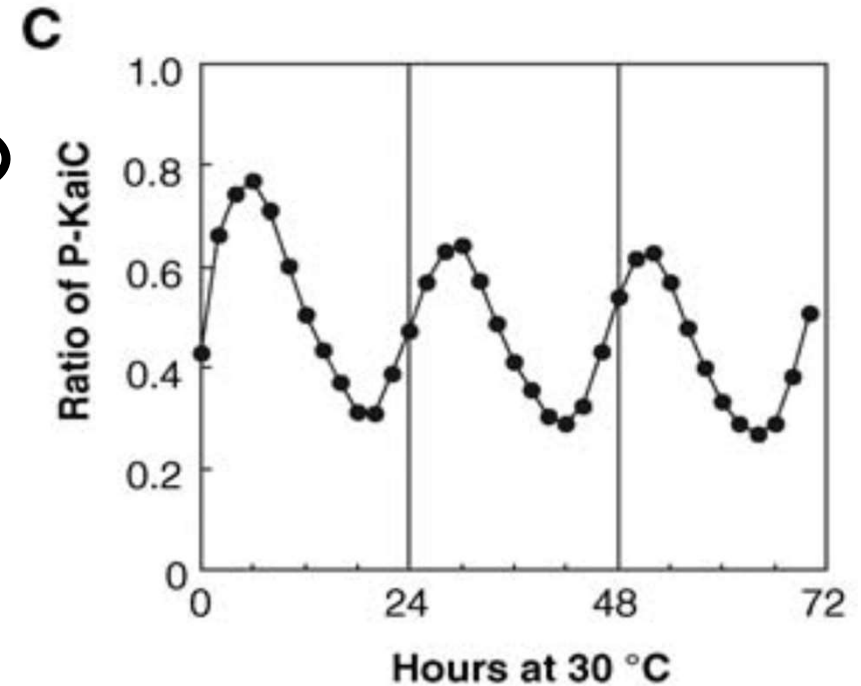
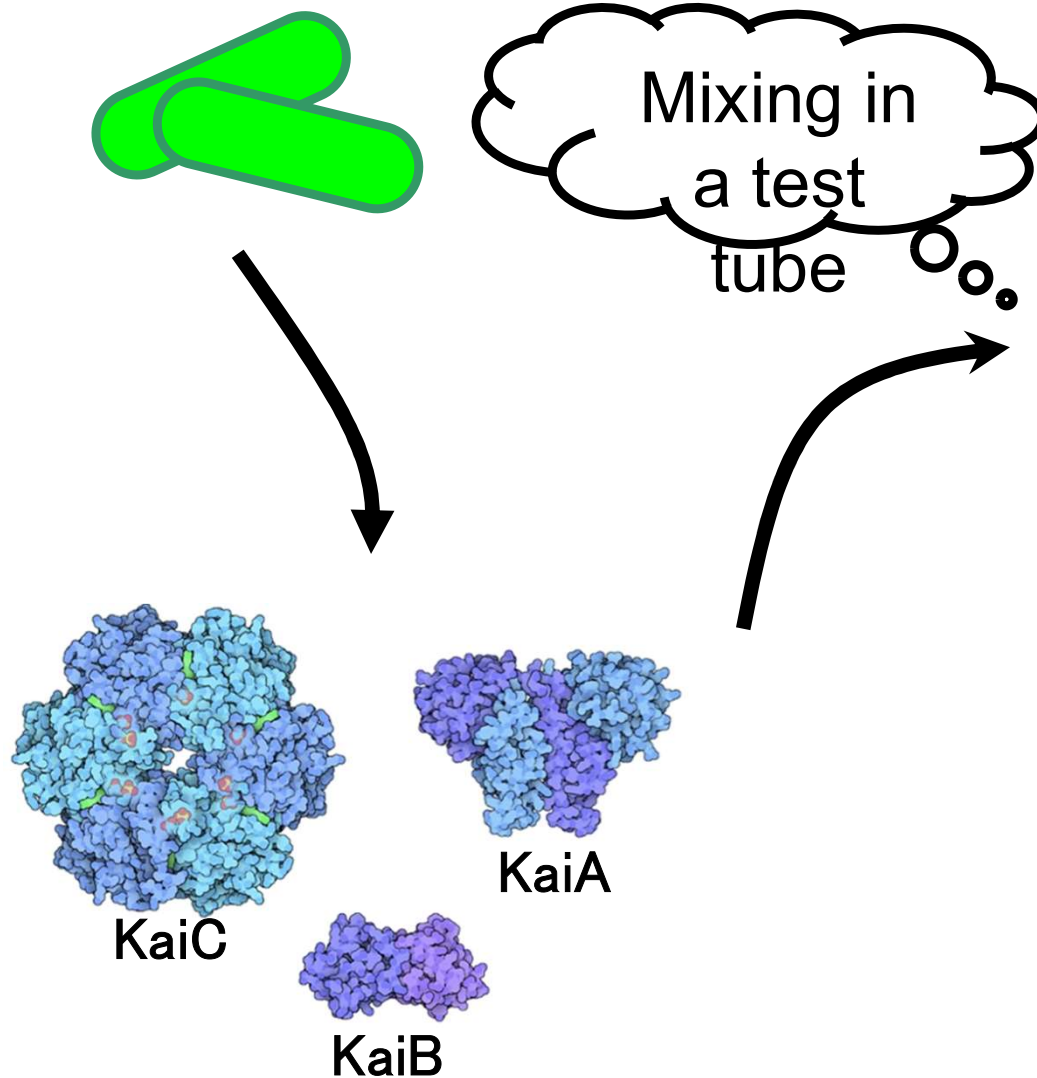
- Robustness of Circadian rhythm (period)
(*T.S.Hatakeyama and KK, PNAS, 2012*)

Circadian rhythm: generated in vitro by just a few proteins (*KAI ABC experiments by Kondo*)

1. ~24 hour rhythm (Slow from protein timescales)
2. Period is insensitive to temperature change
3. But, synchronize with external periodic change (eg. 24-hr temperature change)

Response to external change + homeostasis =
Basic problem in biology, common to adaptation

In-vitro reconstruction of circadian rhythm (Takao Kondo's group)



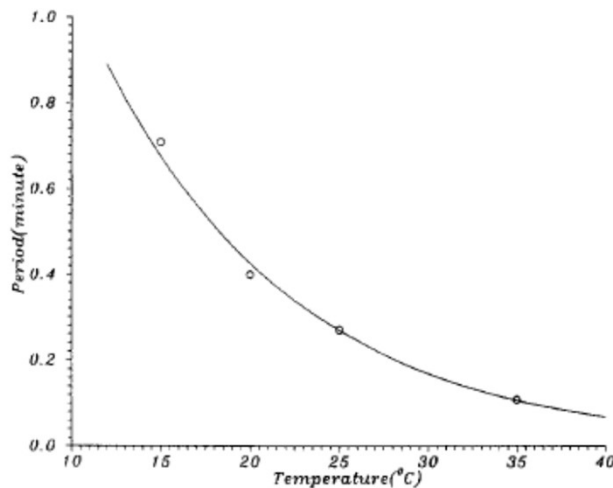
(Nakajima et al., Science, 2005)

(Illustrated by David Goodsell)

Temperature Compensation

In vitro Kai-protein oscillator

B-Z reaction



25°C

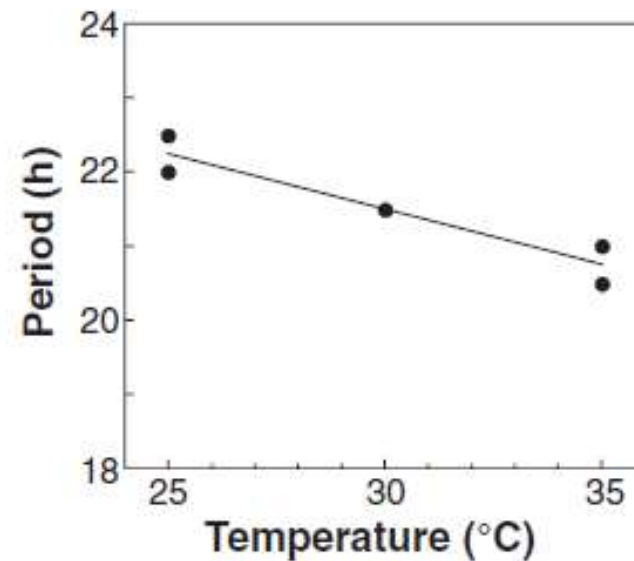
0.3 minute

35°C

0.15 minute

50%

Extracted from cyanobacteria)



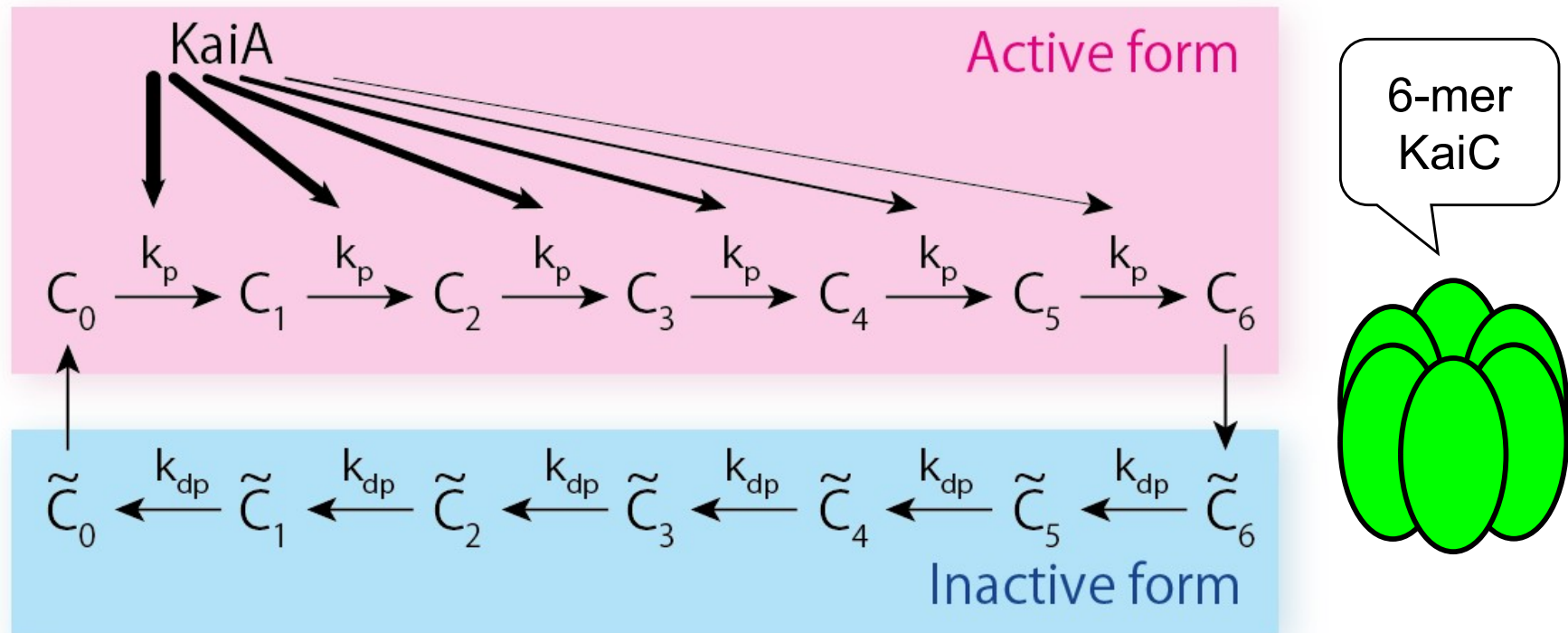
21–22 hours

95%

- Question: reaction rate typically changes with $\exp(-E/kT)$. How can the period be insensitive to temperature?
- Temperature affect to amplitude (entrainment)
→ $E \neq 0$
- Since the period is 'long', large E would be expected for some reactions
- → Slowness is not explained by smallness of $\exp(-E/kT)$
- System-level compensation?

- Core Idea – enzyme limited competition
- The reaction rate by enzymatic reaction is given by $r = A \exp(-E/kT)$ A: concentration of free enzyme
- (i) rhythm consists of several reaction steps forming a circuit where abundances circulate
- (ii) Same enzyme is used by substrates in the circuit
- (iii) These enzymatic reactions rate-limit the cycle
- (iv) increasing T, substrates that bind enzyme increase with $\exp(-E/kT)$, so that
available free enzyme A decreases with $\exp(E/kT)$
- Total reaction rate r is independent of temperature

Reaction process (phosphorylation/dephosphorylation)



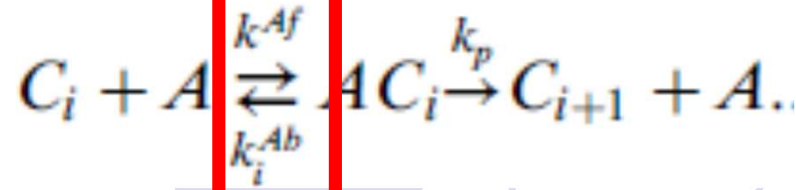
$$k_p = A_p \exp(-\beta E_p)$$

$$k_{dp} = A_{dp} \exp(-\beta E_{dp})$$

$$(E_p > E_{dp})$$

Adapted from (van Zon Lubensky, ten Wolde., PNAS 2007)

Model equation



Fast — — equilibrium



$$\frac{d[C_i]}{dt} = \frac{k_p[A]}{K_{i-1} + [A]}[C_{i-1}] - \frac{k_p[A]}{K_i + [A]}[C_i] + \delta_{i,0}b_i[\tilde{C}_i] - \delta_{i,N}f_i[C_i]$$

$$\frac{d[\tilde{C}_i]}{dt} = k_{dp}([\tilde{C}_{i+1}] - [\tilde{C}_i]) - \delta_{i,0}b_i[\tilde{C}_i] + \delta_{i,N}f_i[C_i]$$

$$K_i (= k_i^{Ab} / k_i^{Af})$$

$$K_i = K_0 \alpha^i (\alpha > 1.0).$$

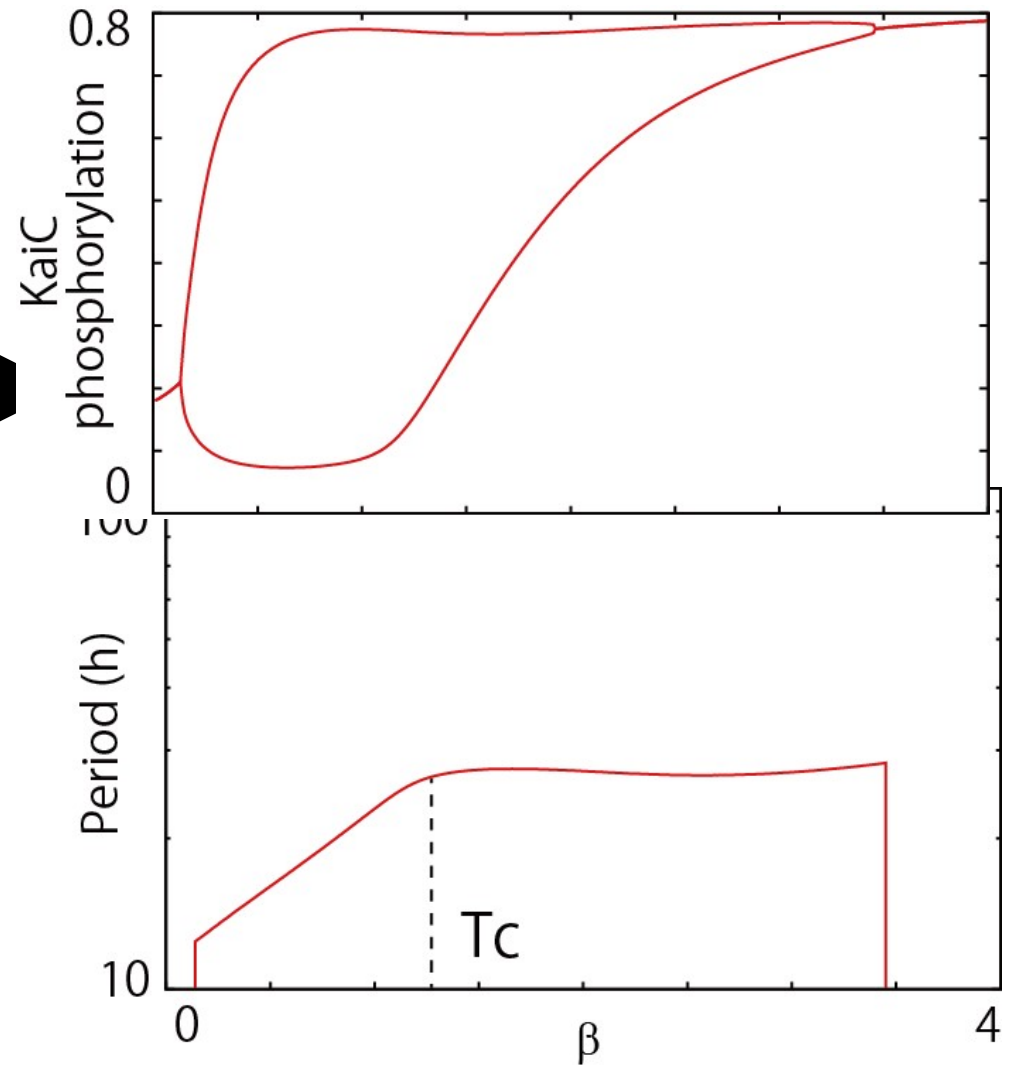
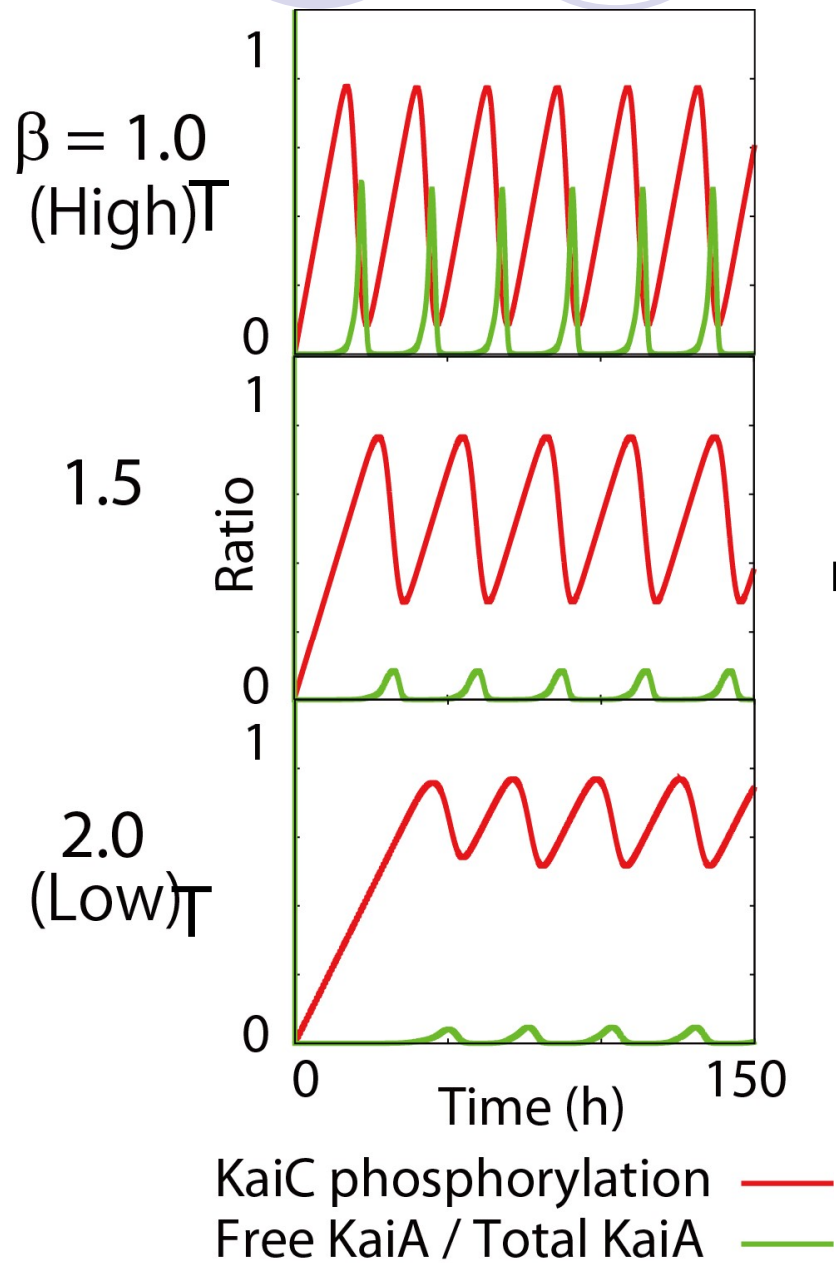
$$[A]_T = [A] + \sum_{i=0}^{N-1} \frac{[A][C_i]}{K_i + [A]}$$

Total enzyme (const)
= Free+ Bound

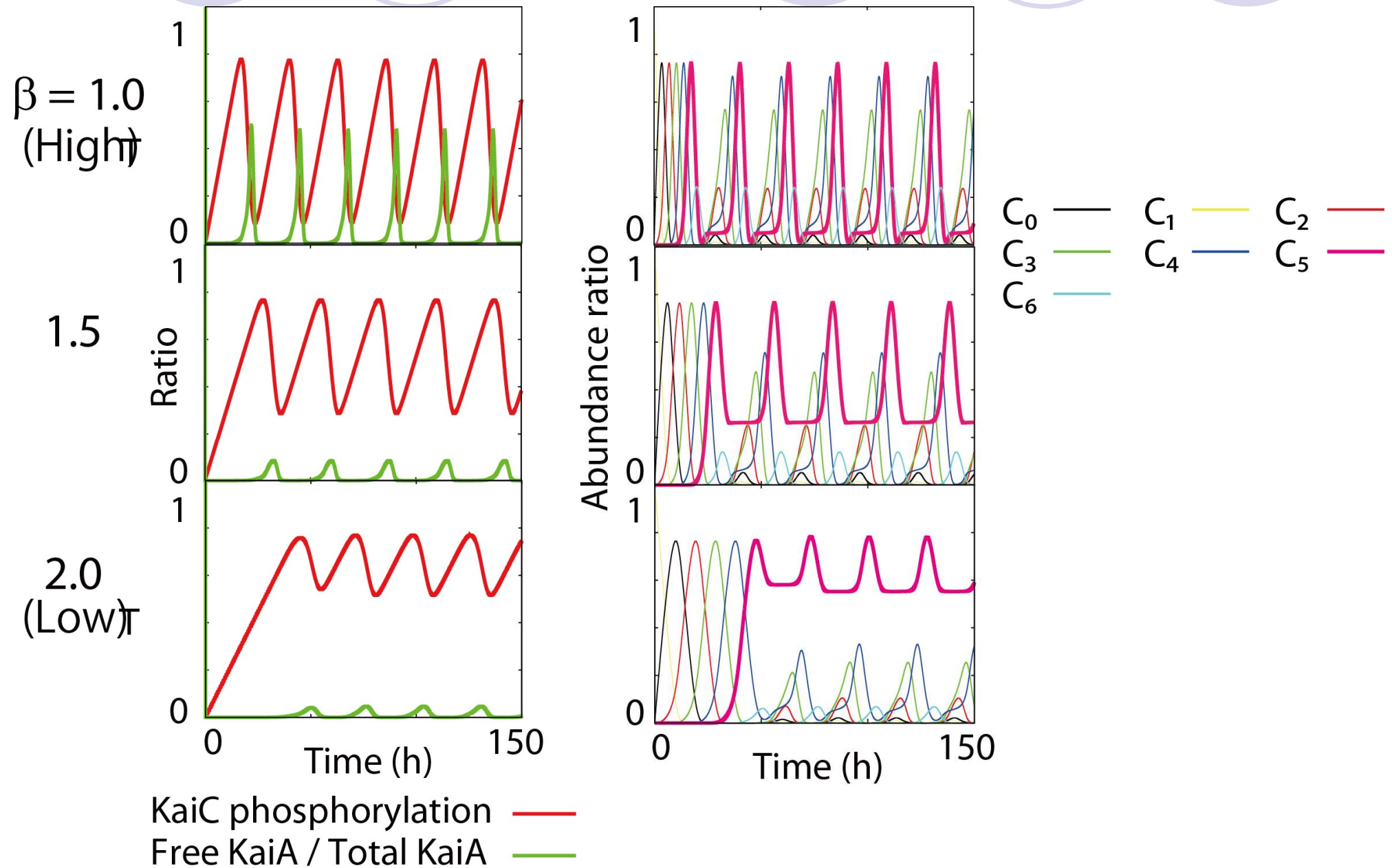
$$k_p = A_p \exp(-\beta E_p)$$

$$k_{dp} = A_{dp} \exp(-\beta E_{dp})$$

Below certain T_c , the period is insensitive to temperature, but the amplitude is lowered with lowering temperature

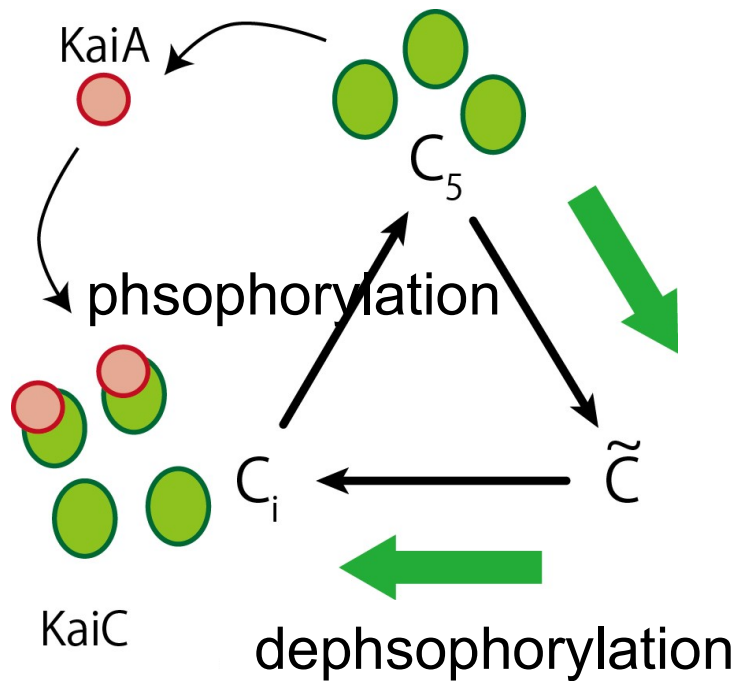


Below T_c , C_5 is accumulated, which leads to shortage of free Kai A enzyme



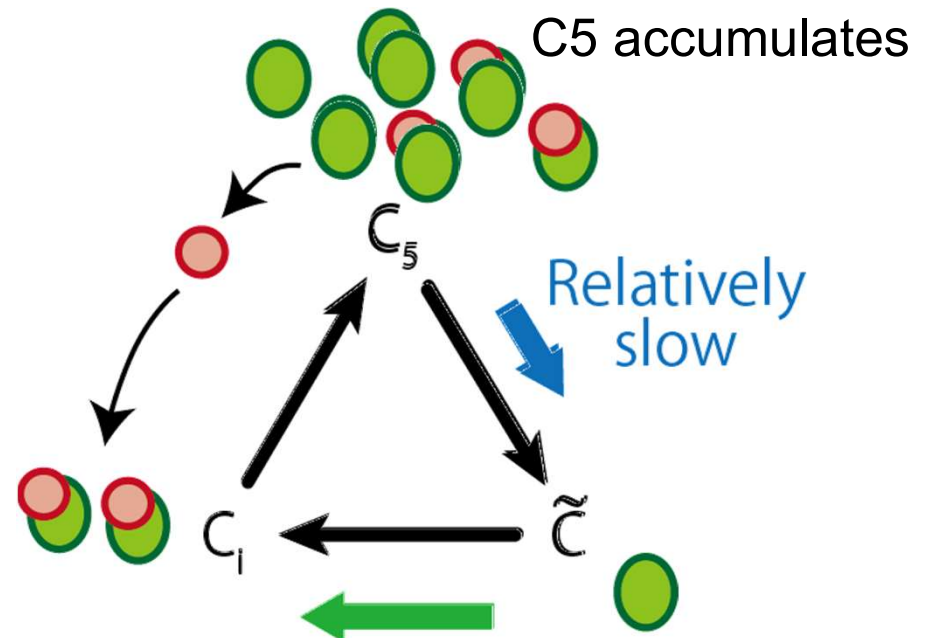
At the temperature-compensation region, phosphorylation is rate-limited $\leftarrow E_p > E_d$

At the critical temperature (T_c)



High temperature

Below T_c

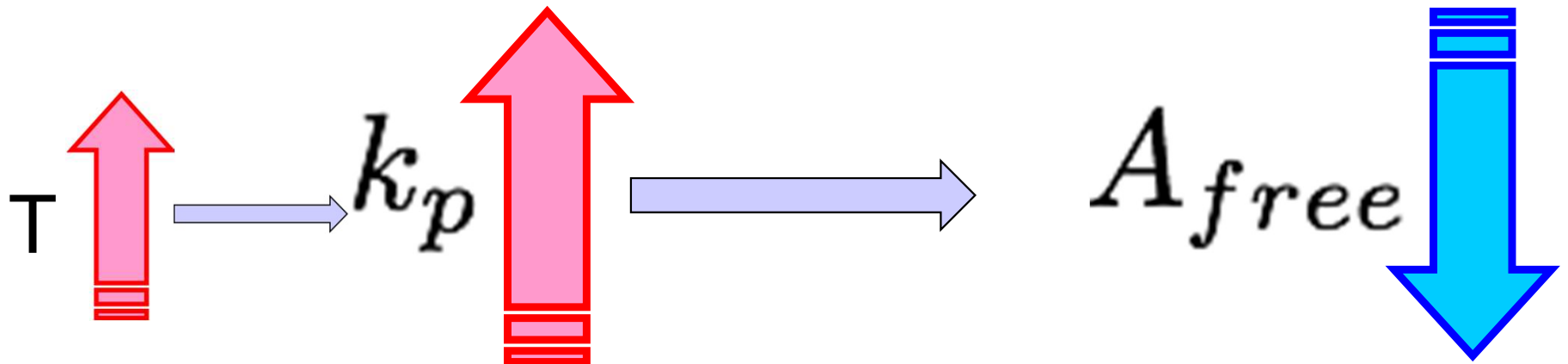


Enzyme-limited competition (ELC)

By lowering (**increasing**) temperature →

Abundances of reacting substrates at the phosphorylation circuit decrease (**increase**) →

Abundances of free enzyme (that is not bound with substrates) increase (**decrease**)



(details) Enzyme-limited competition (ELC)

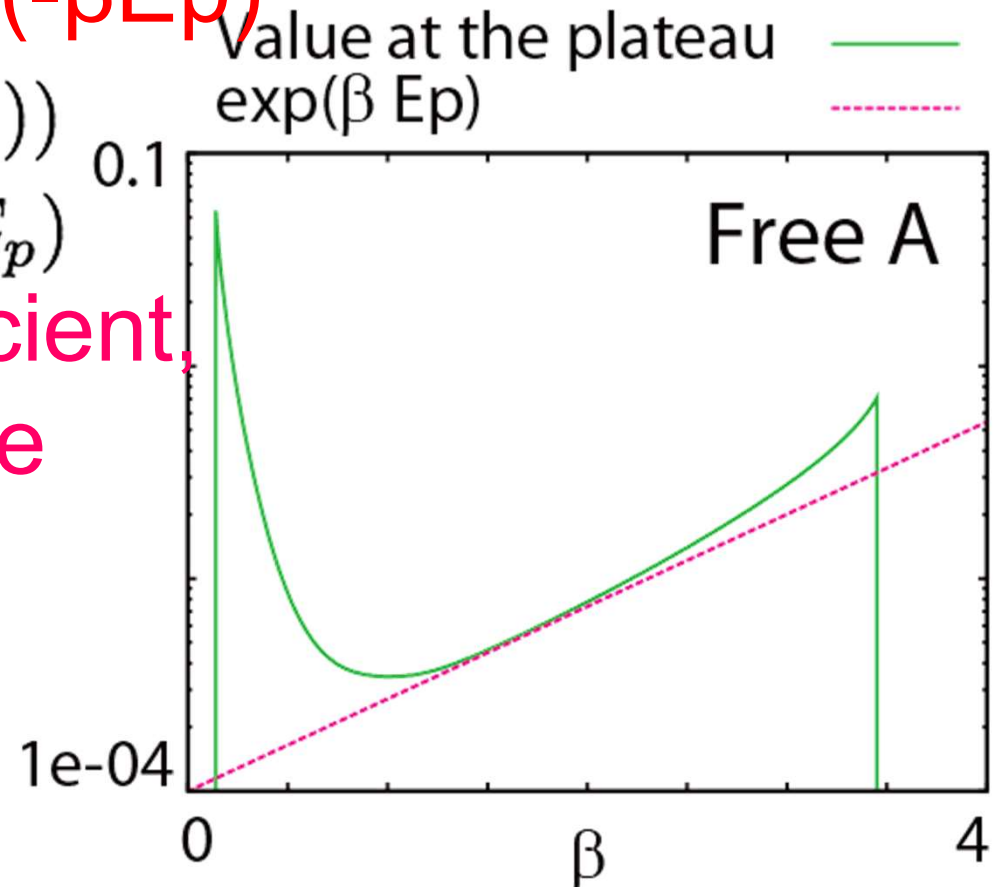
(1) Flow at phosphorylation process change with $\exp(-\beta E_p)$

$$\Sigma \tilde{C} \propto \exp(-\beta(E_p - E_{dp}))$$

$$C_i \sim k_{dp} \Sigma \tilde{C} \propto \exp(-\beta E_p)$$

(2) When A_{total} is not sufficient, due to ELC, available free enzyme decreases with $\exp(\beta E_p)$ against T

Phosphorylation process slows down at phosphorylation level $m \sim 2$, while C5 is accumulated



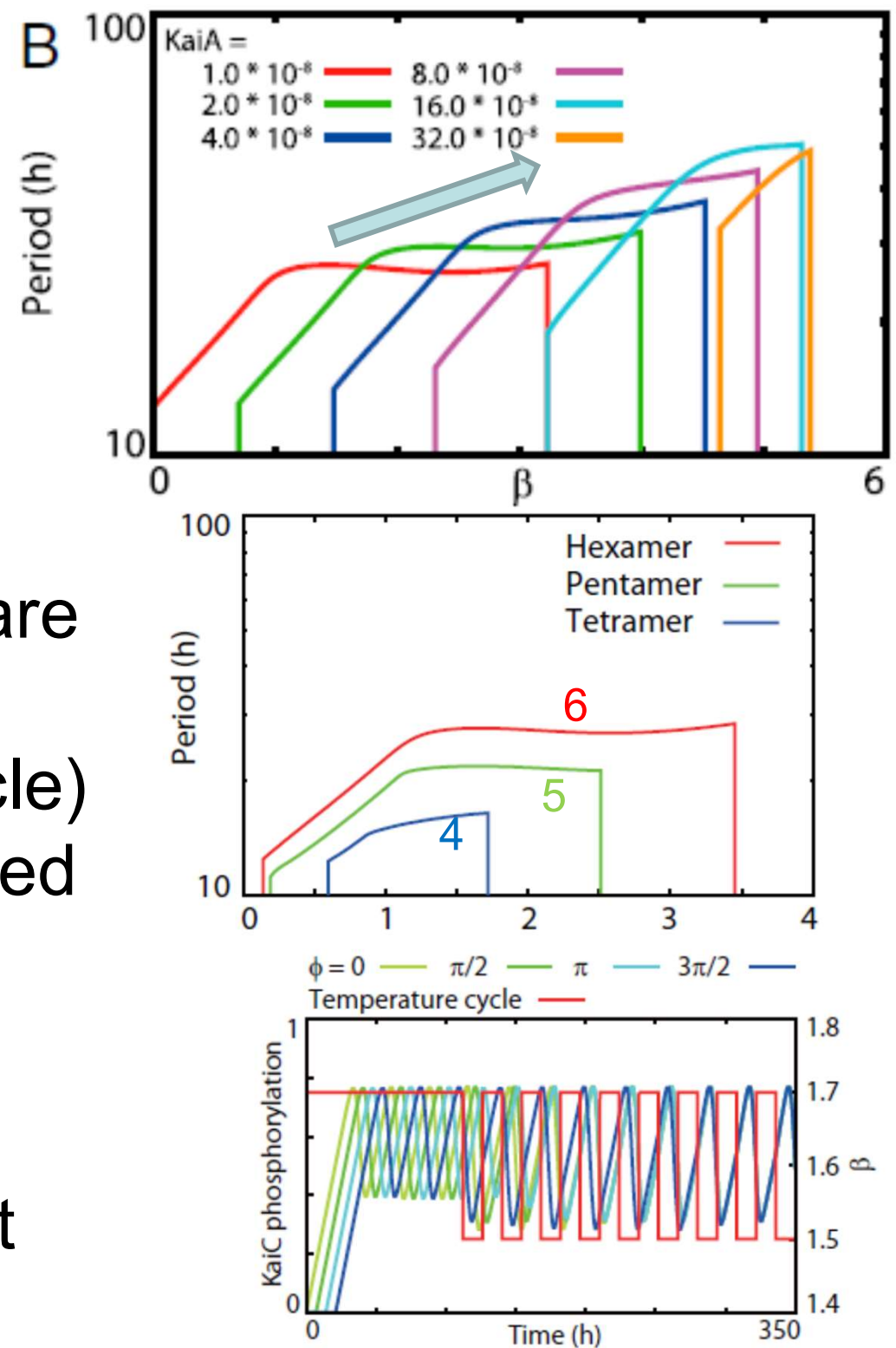
$$A \simeq \frac{A_{\text{total}}}{1 + C_m/K_m} \simeq \frac{A_{\text{total}} K_m}{C_m} \propto \exp(\beta E_p).$$

$$A_{\text{free}} \sim \exp(\beta E_p)$$

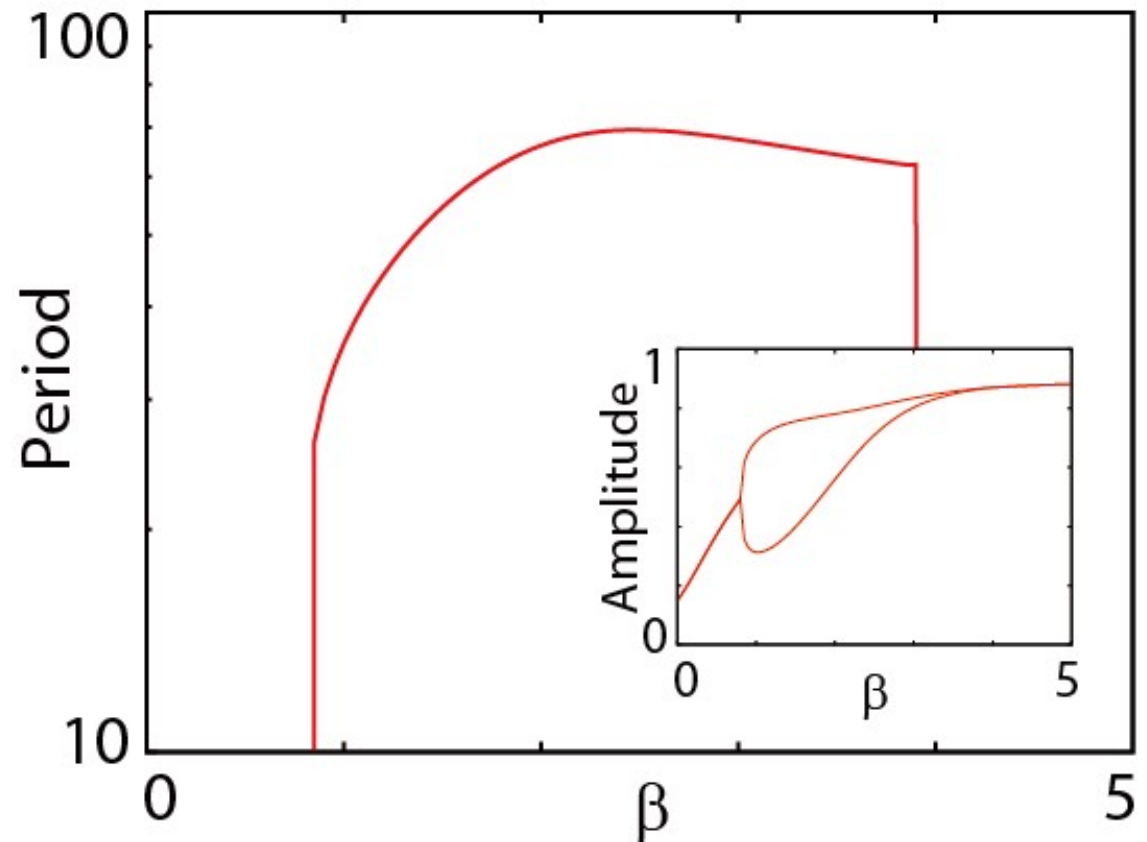
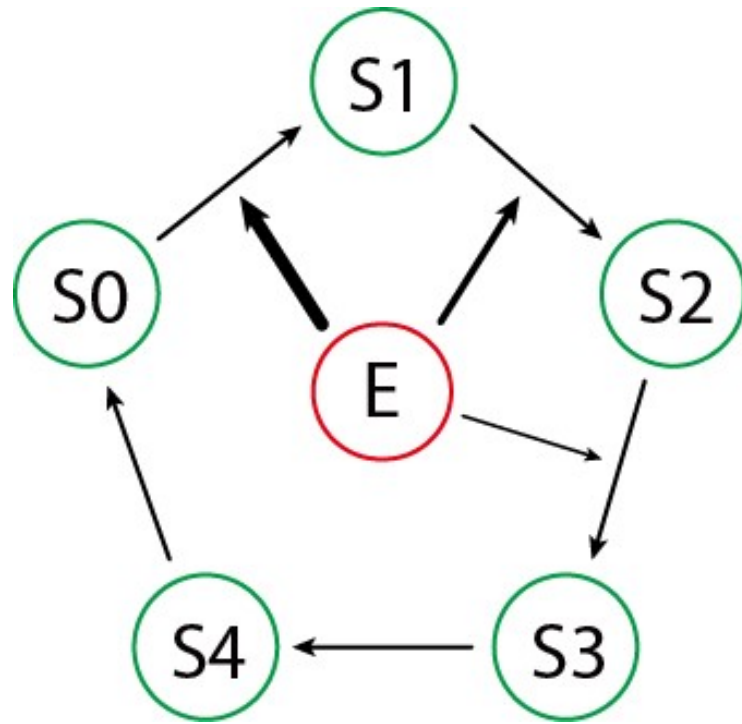
$$k_p A_{\text{free}} \sim \exp(-\beta E_p) \exp(\beta E_p) \sim 1$$

● A bit more...

- (i) As the total Kai A increases, the temperature compensation region decreases
- (ii) If phosphorylation sites are less, temperature compensation (and limit cycle) region decreases, >3 required
- (iii) Against cyclic change in temperature, the entrainment occurs (as in experiments)



Generality: The ELC mechanism is confirmed for much simpler cyclic reaction to lead to temperature compensation



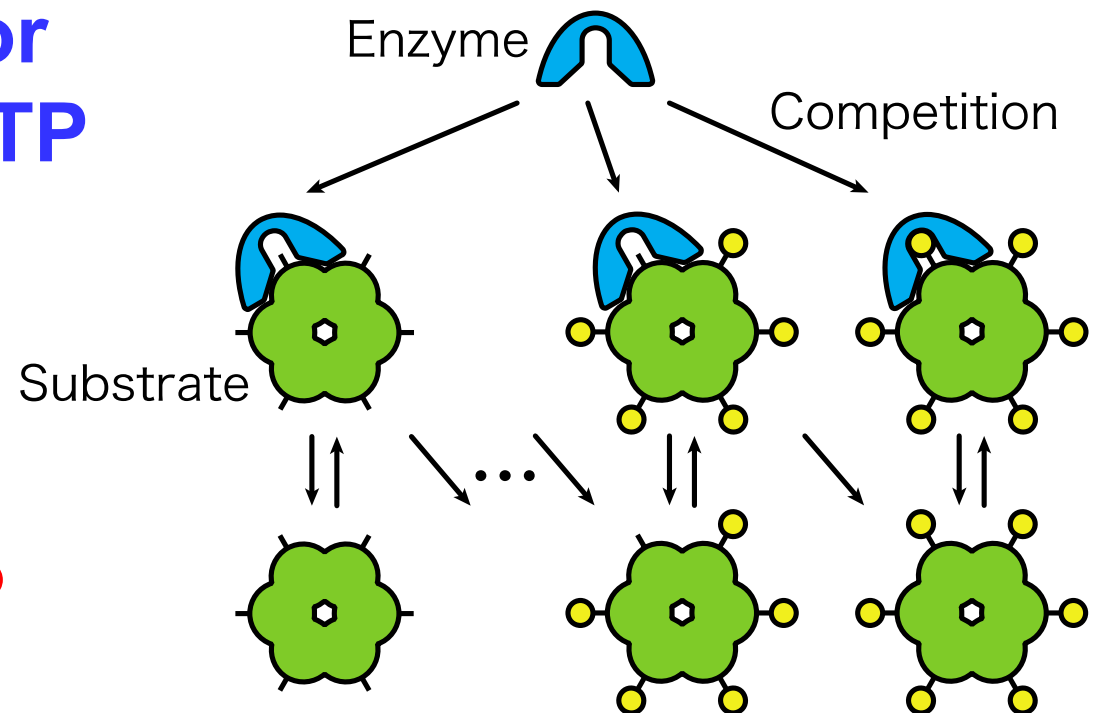
Summary of this part

- Temperature compensation is achieved by autonomous change in free enzyme
- → No need for balance mechanism by fine tuning (through evolution)

Enzyme-limited competition (ELC)

→ Other robust behavior
(e.g., to the change in ATP
concentration)

homeostasis in
biological system?



Robustness vs plasticity in biological clocks

PRL 115, 218101 (2015)

PHYSICAL REVIEW LETTERS

week ending
20 NOVEMBER 2015

Reciprocity Between Robustness of Period and Plasticity of Phase in Biological Clocks

Tetsuhiro S. Hatakeyama* and Kunihiro Kaneko

- Robustness
 - Temperature compensation
 - Nutrient compensation
- Plasticity
 - Temperature-entrainability
 - Food-entrainability
 - Light/dark-entrainability



Physics ▾



Synopsis: Robust Yet Flexible Clocks

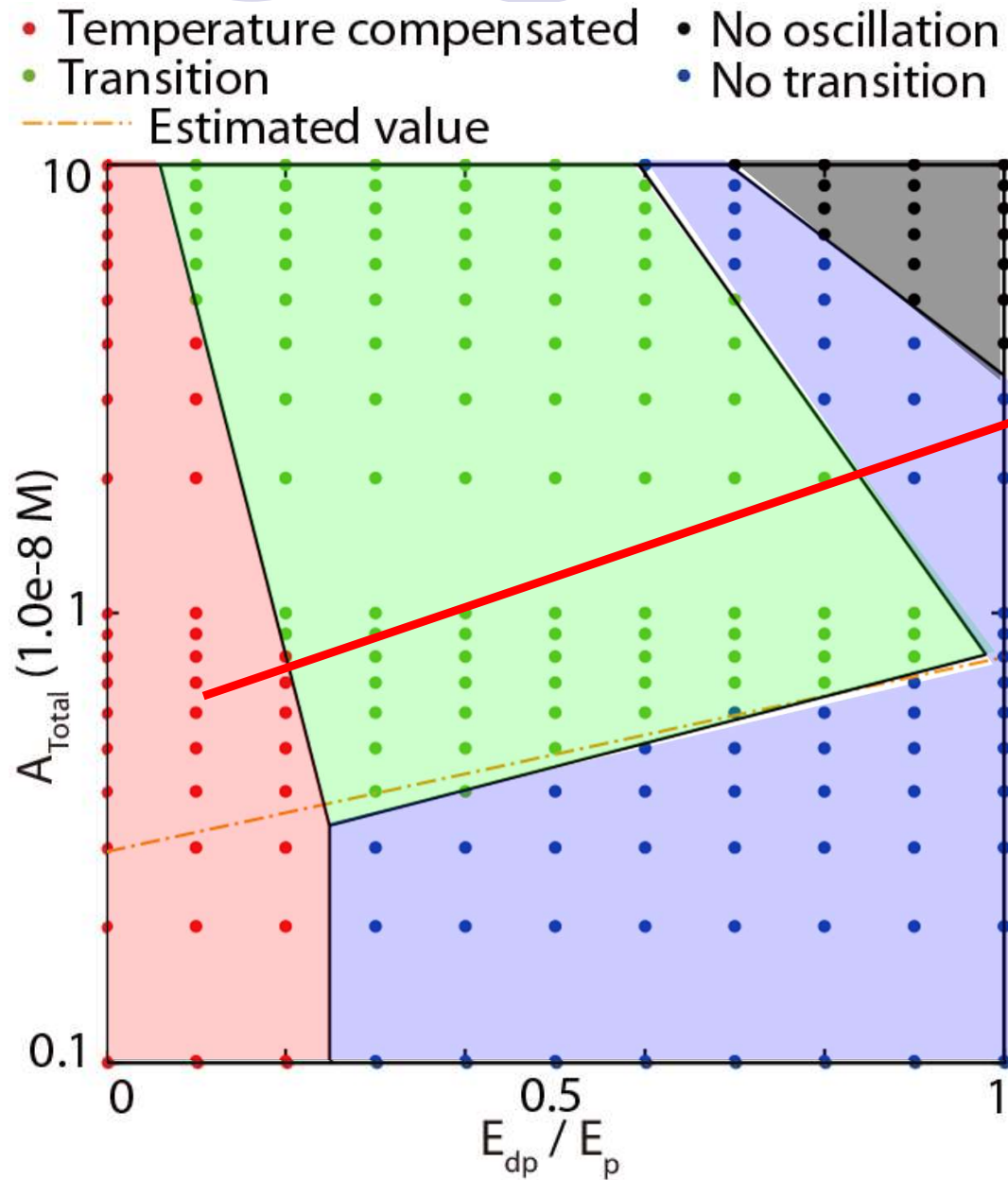
November 19, 2015

A theoretical analysis explains why circadian clocks can be robust but also able to adapt to environmental changes.



!Stockphoto.com/MarkSwallow

Phase Diagram



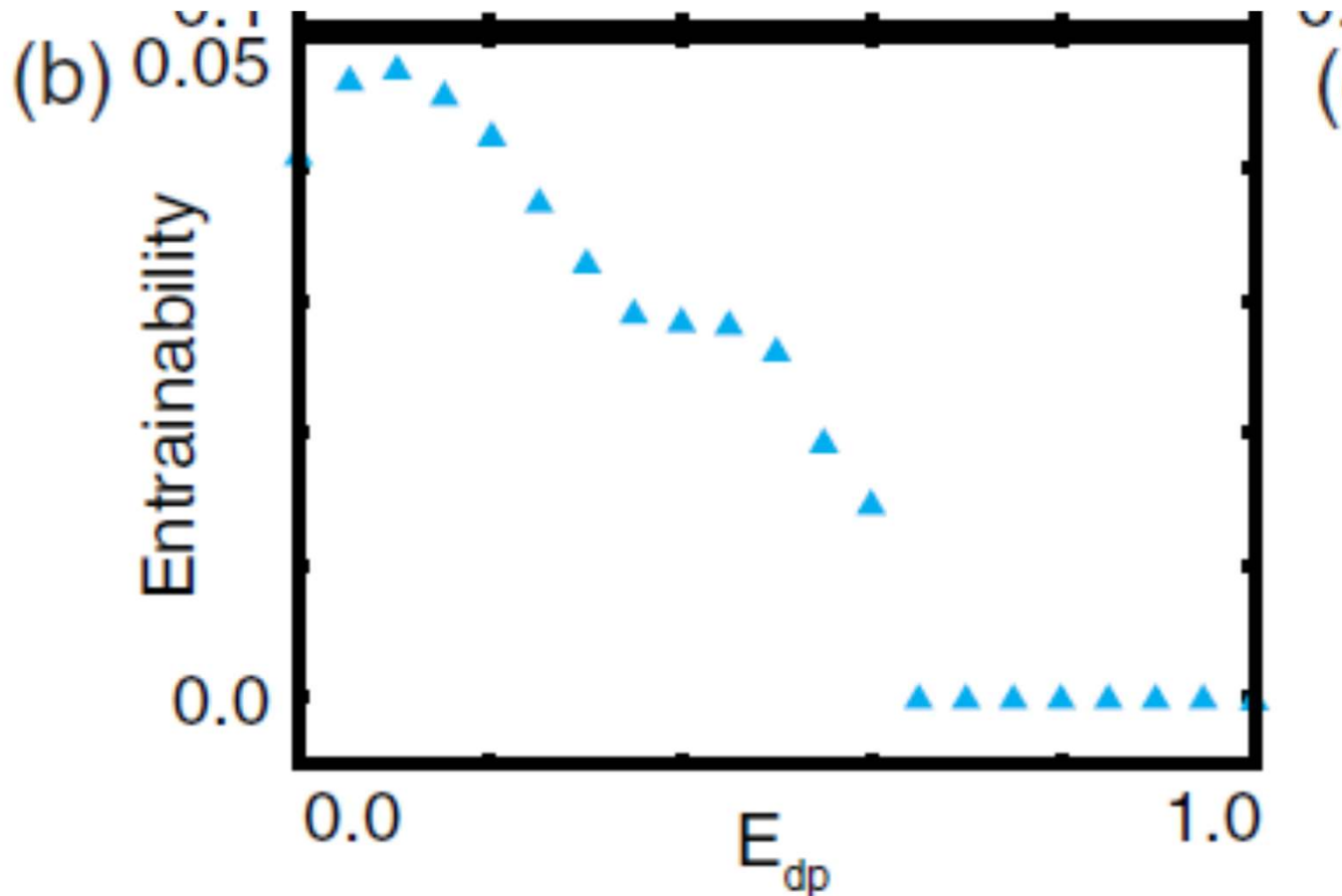
Condition for the present
Temperature Compensation

1 $E_{\text{dp}} / E_{\text{p}}$ small
($E_{\text{p}} \gg E_{\text{dp}}$)

2 A_{total} small

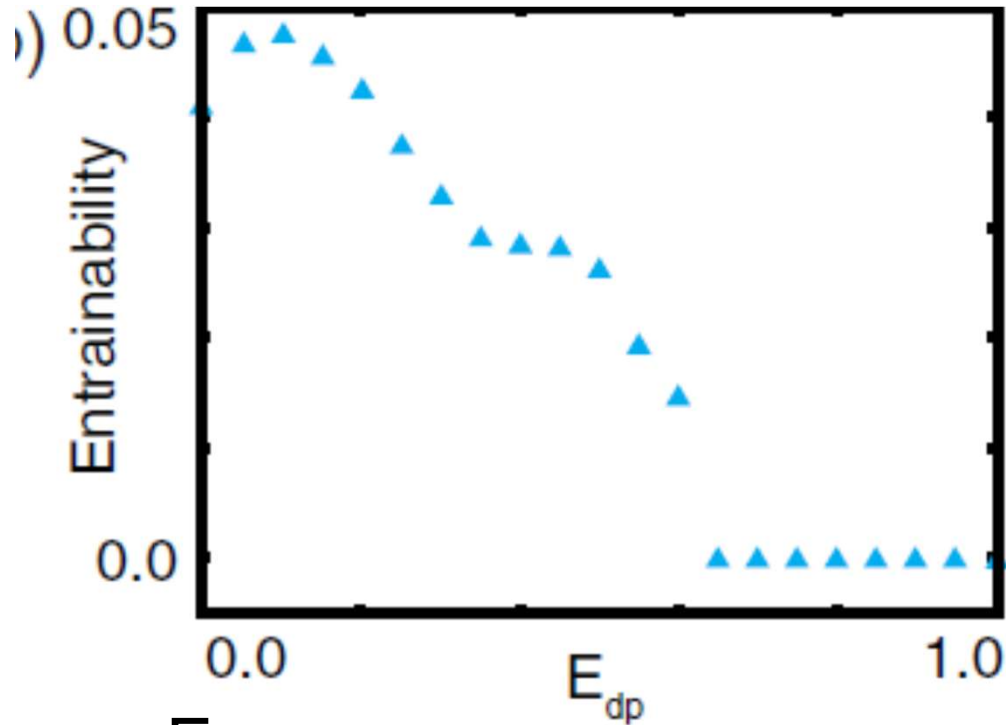
As the condition is relaxed, period robustness gets worse

Entrainability=inverse of time needed
to entrain



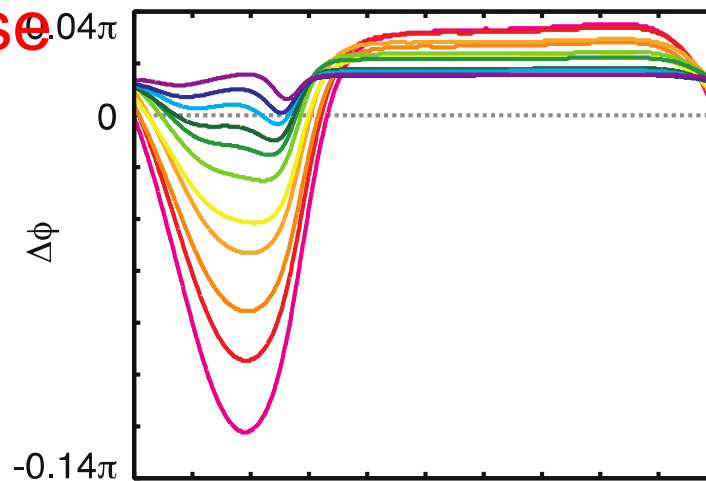
**Well temperature-compensated clock shows
high temperature-entrainability**

Entrainability=inverse of
time needed to entrain

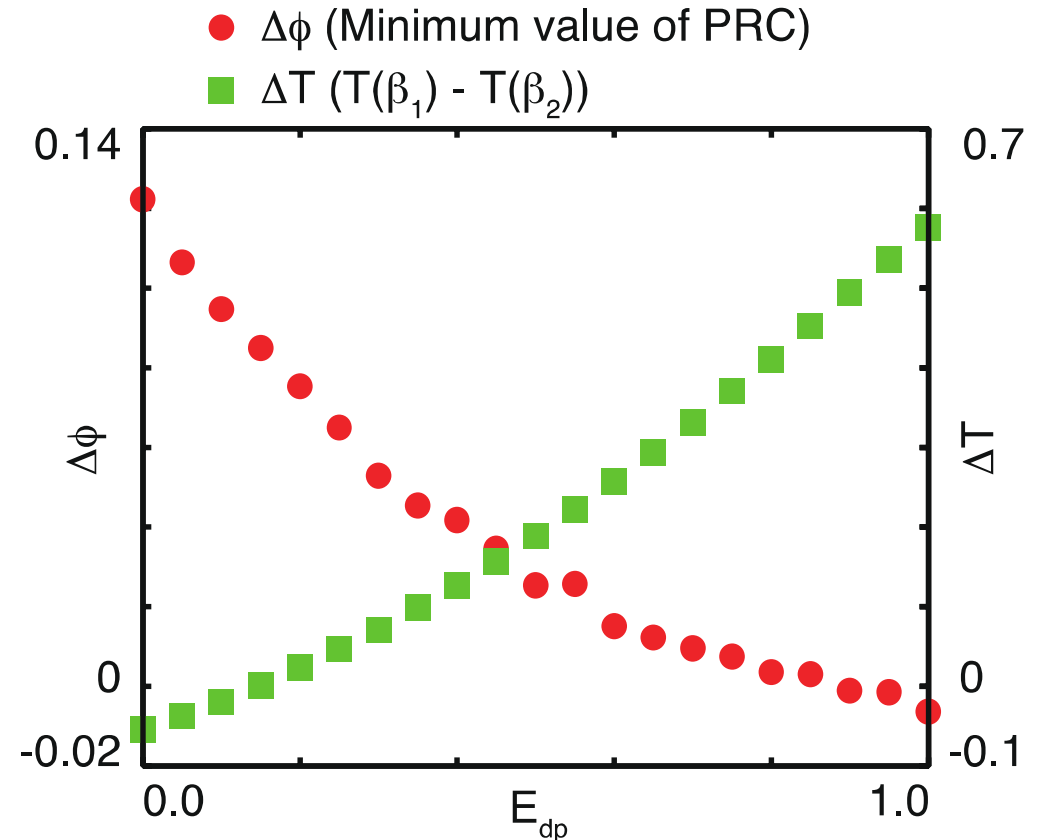


E_{dp} : dephosphorylation energy,
smaller $\rightarrow$ temp compensation

Phase Response
curve $\rightarrow$ degree
of plasticity in
phase $\Delta\phi$

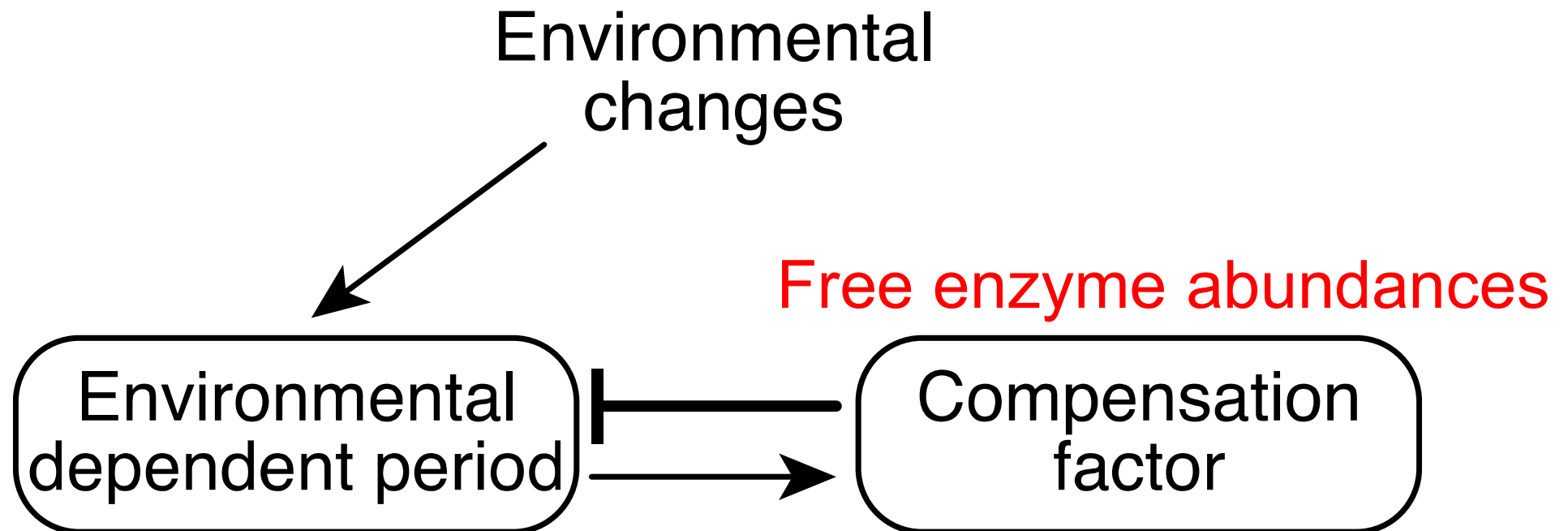


Reciprocity relationship



Reciprocity relationship
between $\Delta\phi$ and ΔT

Reciprocity relationship – general. – compensation needs time



compensation needs time (delay),
during which phase is shifted,

Strong compensation

→ Low environmental dependence
and large phase shift :

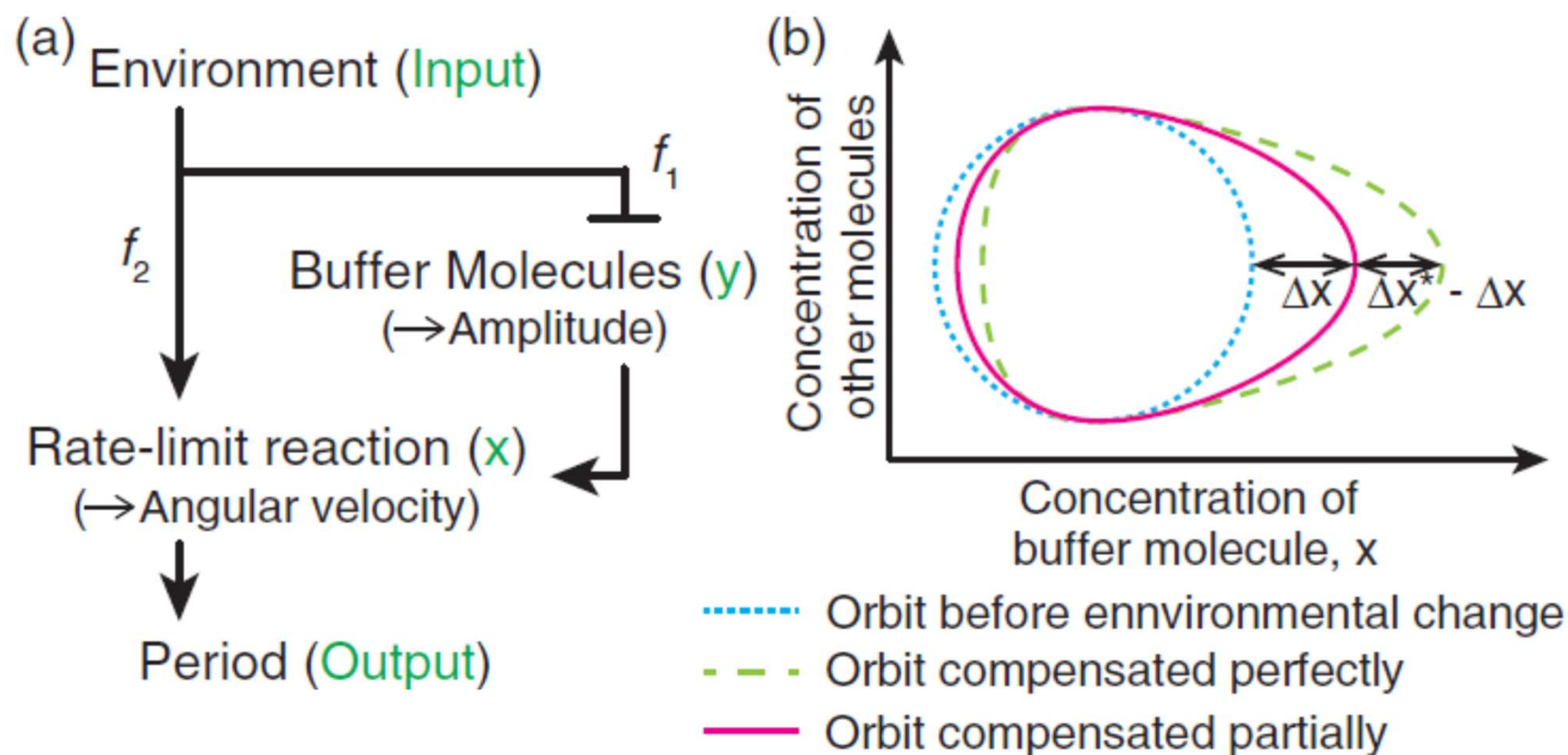


FIG. 3 (color online). Schemes of the reciprocity between the robustness of period and plasticity of phase. (a) Schematic networks of a generic (bio)chemical oscillator exhibiting homeostasis of period. Pointed and flat arrowheads indicate positive and negative regulation, respectively. Correspondences with a simple feedforward adaptation motif are represented by green characters in parentheses. (b) Scheme of limit-cycle orbits with compensation of the period against environmental change. Blue dotted line

Summary of this part

- Robustness & Plasticity: Two important features in biological systems
- But they seem to be in opposite direction (consider a process deepening the potential valley)
- Possible answer:
 - 1) Conjugate variable (eg phase and period), robustness to one variable $\sim$ plasticity to the other (reciprocity)
 - 2) Buffer process for adaptation (robustness) provides the basis for reciprocity

→ Generalized?

Q

Consider a model that shows the adaptation in the sense of robustness (homeostasis), in which most variables have tendency to return to the original. Then, examine if there is a 'plastic' variable. Discuss, if possible, relationship between robustness and plasticity