Dynamical universality classes with kinematically constrained dynamics

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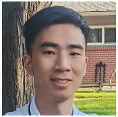

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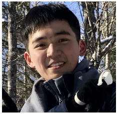

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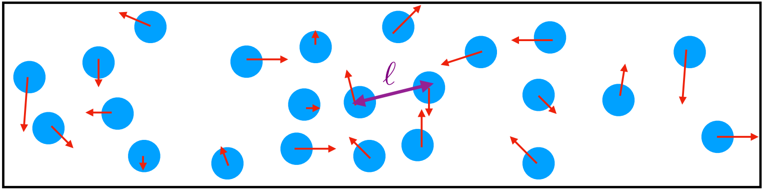
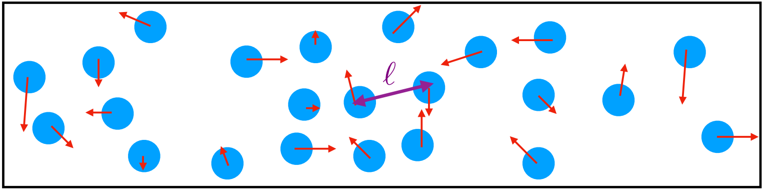
- Introduction to Hydrodynamics
- Dipole-Conserving Hydrodynamics
- Dipole and Momentum Conservation
- Summary

1





Solving classical equations for $(\mathbf{r}_1, \mathbf{p}_1, \ldots, \mathbf{r}_N, \mathbf{p}_N)$ we'd need to keep track of about $N = 10^{30}$ molecules ($6N$ degrees of freedom) to model air in this room. **Intractable!**

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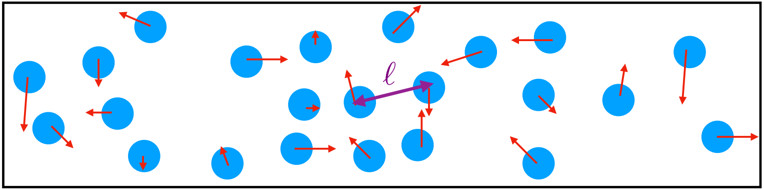

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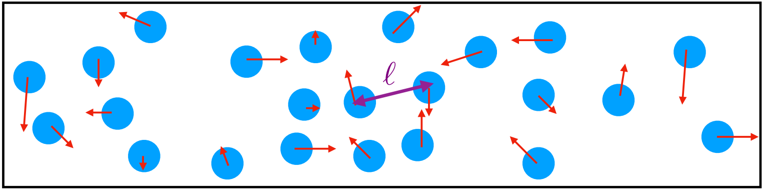

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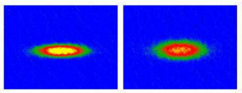
Hydrodynamics makes sense on length scales $L \gg \ell$, the "mean free path for collisions".

2

This universality is observed:

conventional gases:

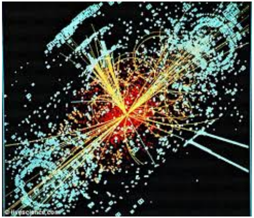
cold atoms:



electrons in solids:

conventional liquids:

quark-gluon plasma:



3 Sometimes, hydrodynamics is *unstable* to thermal fluctuations:

$$\partial_t \rho + \partial_t(\rho v_i) = 0.$$

$$\partial_t(\rho v_i) + \partial_j((a\rho + b\rho^2 + \cdots)\delta_{ij} + \rho v_i v_j) - \partial_j \eta_{ijkl} \partial_k v_l = \underbrace{\partial_j \tau_{kl}}_{\text{noise}}.$$

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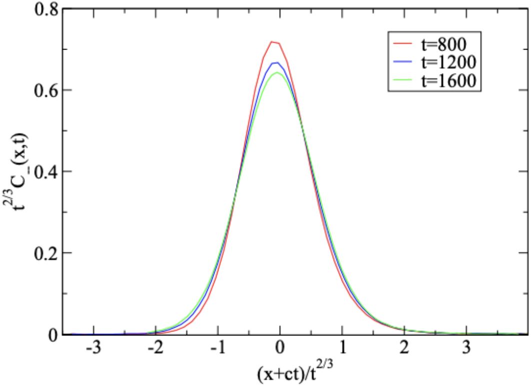
In $d = 1$, we miraculously know the endpoint of the hydrodynamic instability arising from these relevant nonlinearities. The hydrodynamic sound mode disperses like

$$\omega = \pm ck - \mathrm{i}\alpha k^{3/2} + \cdots$$

The resulting dynamical universality class is the Kardar-Parisi-Zhang fixed point.

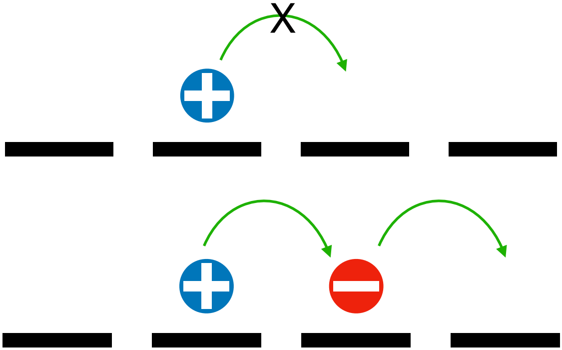
4 This fixed point can be readily seen in simulations of hydrodynamics (*classical* interacting particles) in $d = 1$.



[Das *et al*; 1404.7081]

5

What happens when the microscopic dynamics is subject to a non-trivial constraint, such as dipole conservation in addition to charge conservation?



We say that such theories have **fractons** – elementary excitations are immobile, and only move with other fractons.

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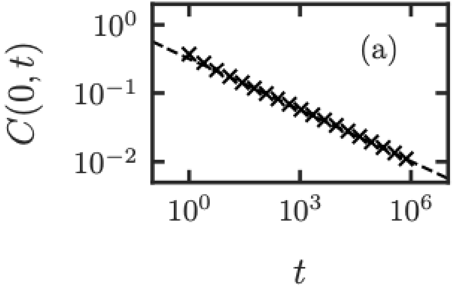
This result has been seen numerically:



$$C(0,t) = \langle \rho(0,0)\rho(0,t)\rangle$$

$$C(0,t) \sim t^{-1/4}$$

[Morningstar *et al*; 2004.00096], [Feldmeier *et al*; 2004.00635]

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Since dipole is conserved,

$$0 = \frac{\mathrm{d}}{\mathrm{d}t} \int \mathrm{d}x \rho \cdot x = - \int \mathrm{d}x \; x \partial_x J_x = \int \mathrm{d}x J_x$$

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If we try to write [Gromov, Lucas, Nandkishore; 2003.09429]

$$J_{xx} = a\rho + B\partial_x^2 \rho + \cdots ,$$

then since J_{xx} is time-reversal-odd while ρ is time-reversal-even, we need $a = 0$ (assuming microscopic time-reversal symmetry).

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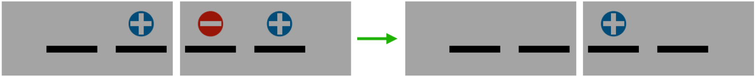

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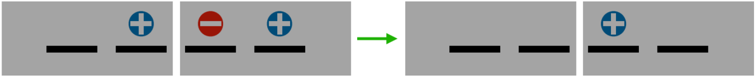

This is analogous to why there is no angular momentum density in the Navier-Stokes equations.

[Glorioso *et al*; 2007.13753]

9

The dipole-conserving subdiffusive universality class can arise due to *emergent constraints* on dynamics, such as in a tilted Fermi-Hubbard model:

$$H = -t \sum_{x,s} \left(c_{x,s}^{\dagger} c_{x+1,s} + c_{x+1,s}^{\dagger} c_{x,s} + F x c_{x,s}^{\dagger} c_{x,s} \right) + \sum_{x} U c_{x\uparrow}^{\dagger} c_{x\uparrow} c_{x\downarrow}^{\dagger} c_{x\downarrow}$$

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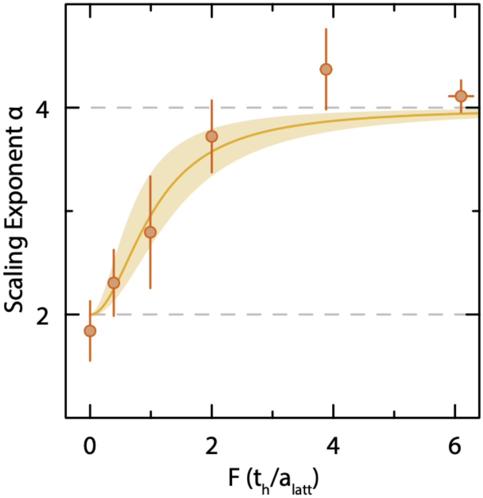
At long wavelengths, energy conservation is dominated by the tilt term, which is proportional to "dipole moment". One finds that charge and energy diffusion morph into:

$$\omega = \begin{cases} -\mathrm{i}AF^2 & \text{energy} \\ -\mathrm{i}\dfrac{Ck^4}{F^2} & \text{charge} \end{cases} .$$

as a consequence of energy conservation ultimately being dominated by dipole conservation.

10 This tilted Fermi-Hubbard model has been experimentally realized in a tilted optical lattice:



[Guardado-Sanchez *et al*; `1909.05848`]

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The conserved quantities are

$$P_i = \int \mathrm{d}^d x \ \pi_i,$$

$$Q = \int \mathrm{d}^d x \ \rho,$$

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The multipole algebra relating them is:

$$\{P_i, Q\} = \{D_i, Q\} = 0, \quad \{D_i, P_j\} = Q\delta_{ij}.$$

This last Poisson bracket/commutator will make hydrodynamics subtle!

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$$\pi_i \to \pi_i + c_i \rho$$

Or, if we write

$$\varphi_i = \frac{\pi_i}{\rho},$$

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φ_i's shift symmetry suggests it is a **Goldstone boson**...

13 The hydrodynamics is complicated – see paper for details. Schematically, the normal modes are:

$$\omega = \pm k^2 - \mathrm{i}k^4 \quad (\rho, \pi_{\parallel}),$$
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A scaling analysis reveals that in general there are relevant perturbations below $d = 4$.

We find "fractonic KPZ" dynamical universality class in $d = 1, 2, 3$!

14 Consider the Hamiltonian system

$$H = \sum_{i=1}^{N-1} \frac{(p_i - p_{i+1})^2}{2} + V(x_i - x_{i+1}), \quad V = \frac{1}{2}x^2 + \frac{g}{3}x^3 + \frac{g'}{4}x^4 + \cdots .$$

Just like momentum conservation implies symmetry $x_i \to x_i + c$, dipole conservation implies $p_i \to p_i + c$. [Glorioso *et al*; 2105.13365]

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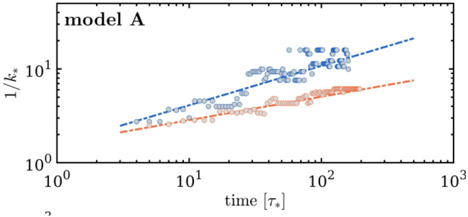
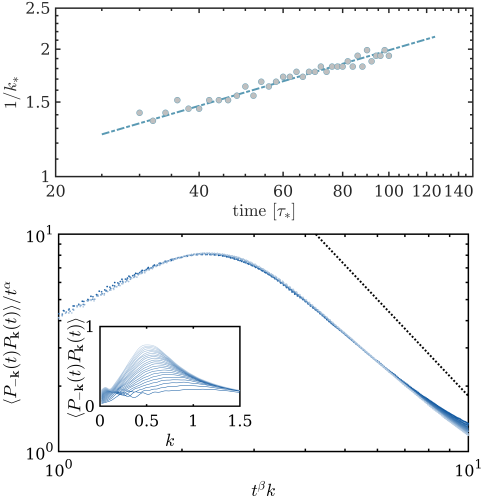
With random initial conditions, the typical wave number of fluctuations should obey

$$k_{\mathrm{typ}}(t) \sim t^{-1/z}.$$



In $d = 1$, when $g = 0$, $z \approx 4$; but when $g \neq 0$, $z \approx 2.5$!

15 However, these nonlinearities will stay relevant in $d = 2, 3$ as well. Using simulations with 6×10^5 degrees of freedom, we simulated $d = 2$ generalization and found that $z \approx 3$.



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Dipole symmetry is *spontaneously broken* in all dimensions. This avoids the Mermin-Wagner theorem because the symmetry is not compact (singlet states might not even exist in Hilbert space!)

17 Generalize to higher mutlipole conserving problems:

$$\left\{H, \sum_{j=1}^{N} x_i^q\right\} = \left\{H, \sum_{j=1}^{N} p_j\right\} = 0,$$

take (in $d = 1$) [Osborne, Lucas, 2111.09323]

$$H = \sum_{j=1}^{N-q-1} \det \begin{pmatrix} p_j & p_{j+1} & \cdots & p_{j+q} \\ 1 & 1 & \cdots & 1 \\ x_j & x_{j+1} & \cdots & x_{j+q} \\ \vdots & \vdots & & \vdots \\ x_j^{q-1} & x_{j+1}^{q-1} & \cdots & x_{j+q}^{q-1} \end{pmatrix}.$$

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Resulting hydrodynamics will break down for spatial dimension $d < 2 + 2q$: an infinite new family of dynamical universality classes!

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One can covariantly couple dipole-conserving fluids to classical background gravity. Suggests a route to placing more general fracton matter on curved space?