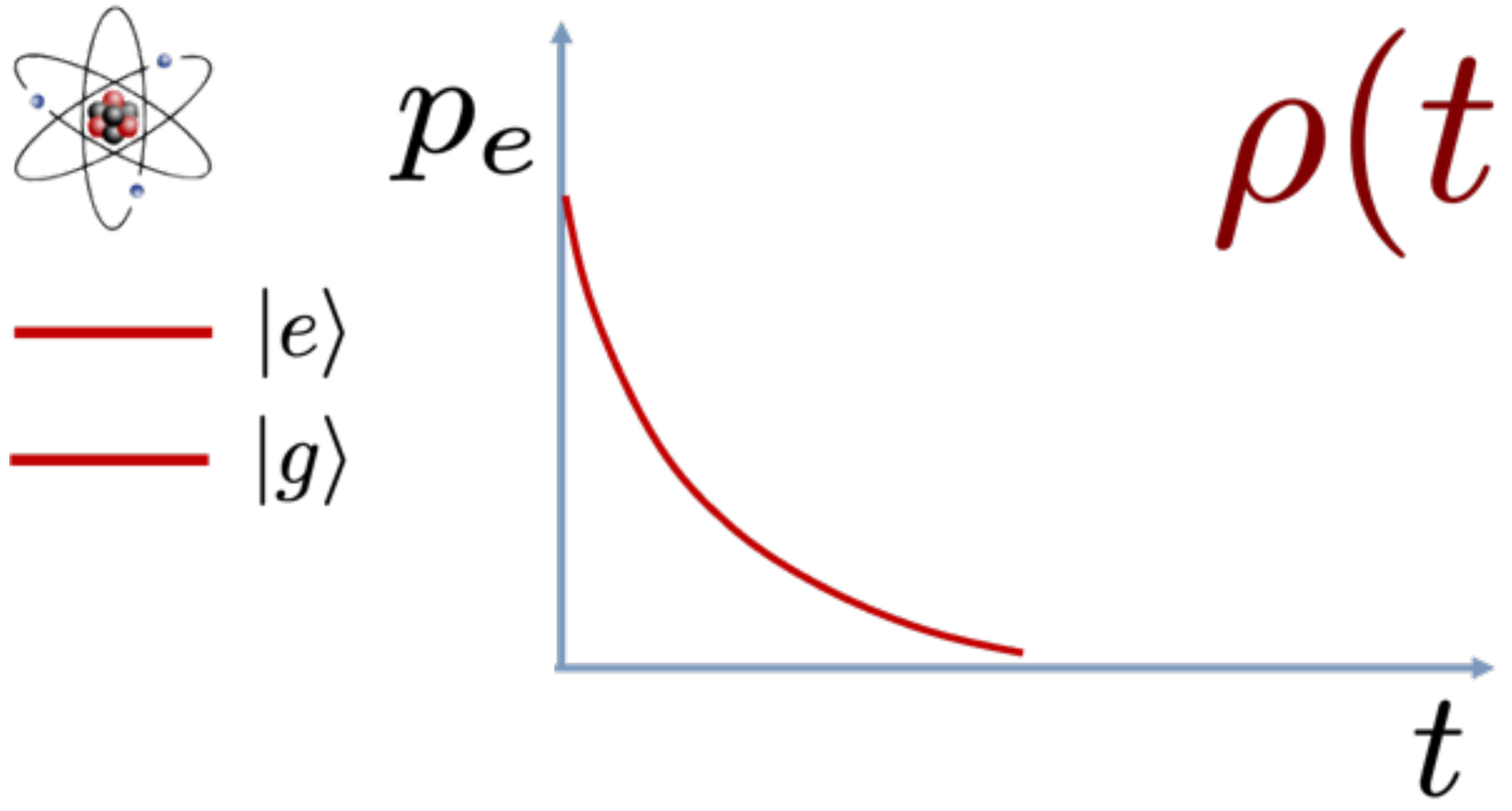
Measurement Induced Transitions

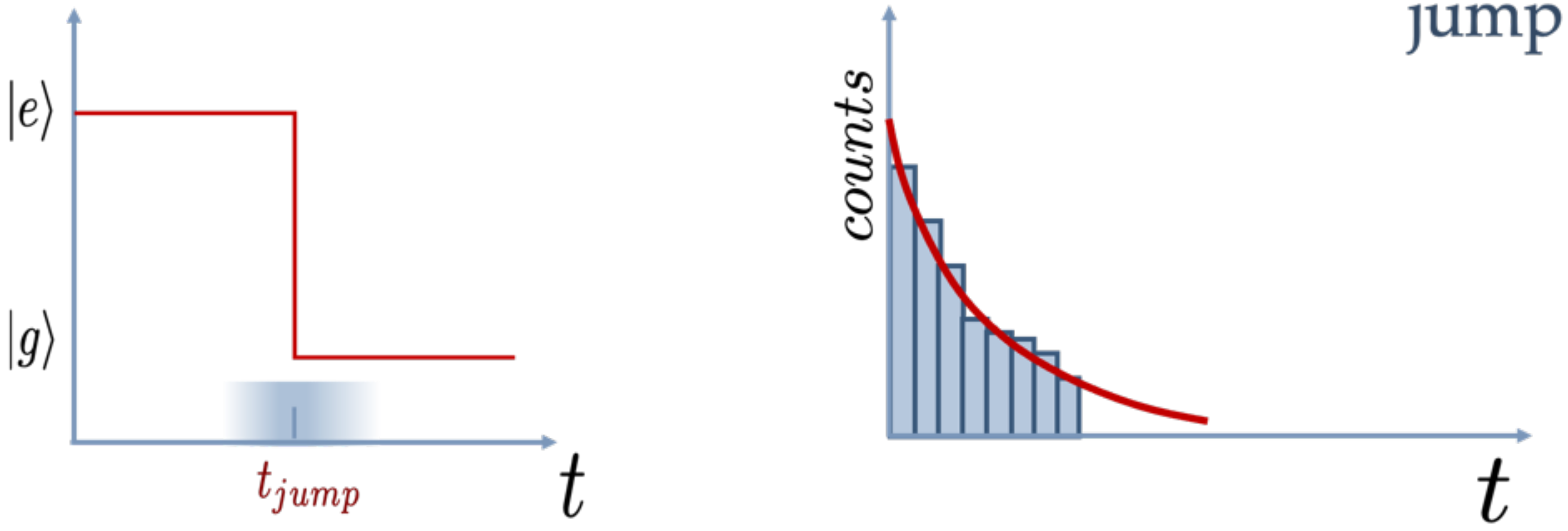
Rosario Fazio

The *Abdus Salam* International Centre for Theoretical Physics, Trieste
&
Università di Napoli *"Federico II"*



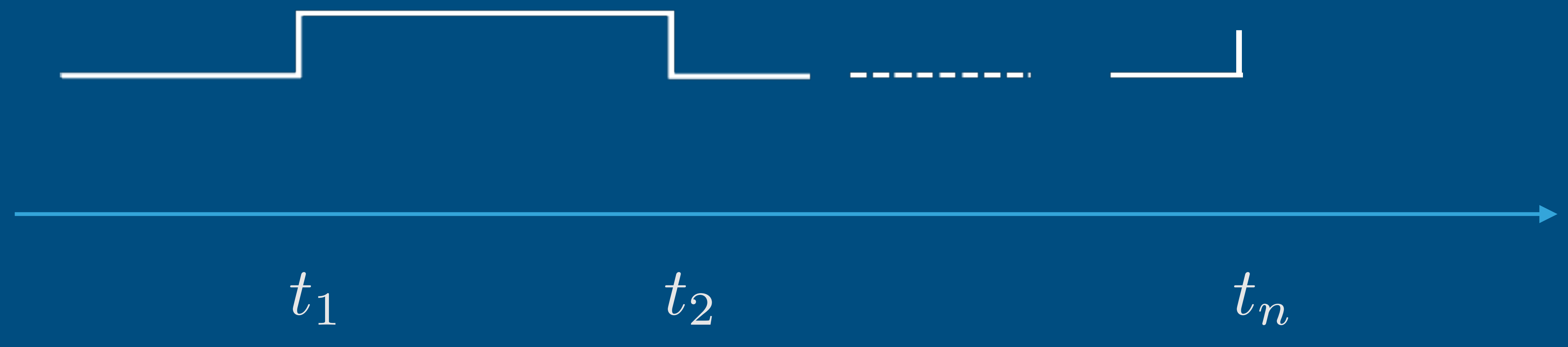
$\rho(t)$

Continuously monitoring: $|e\rangle$ decays with an abrupt jump

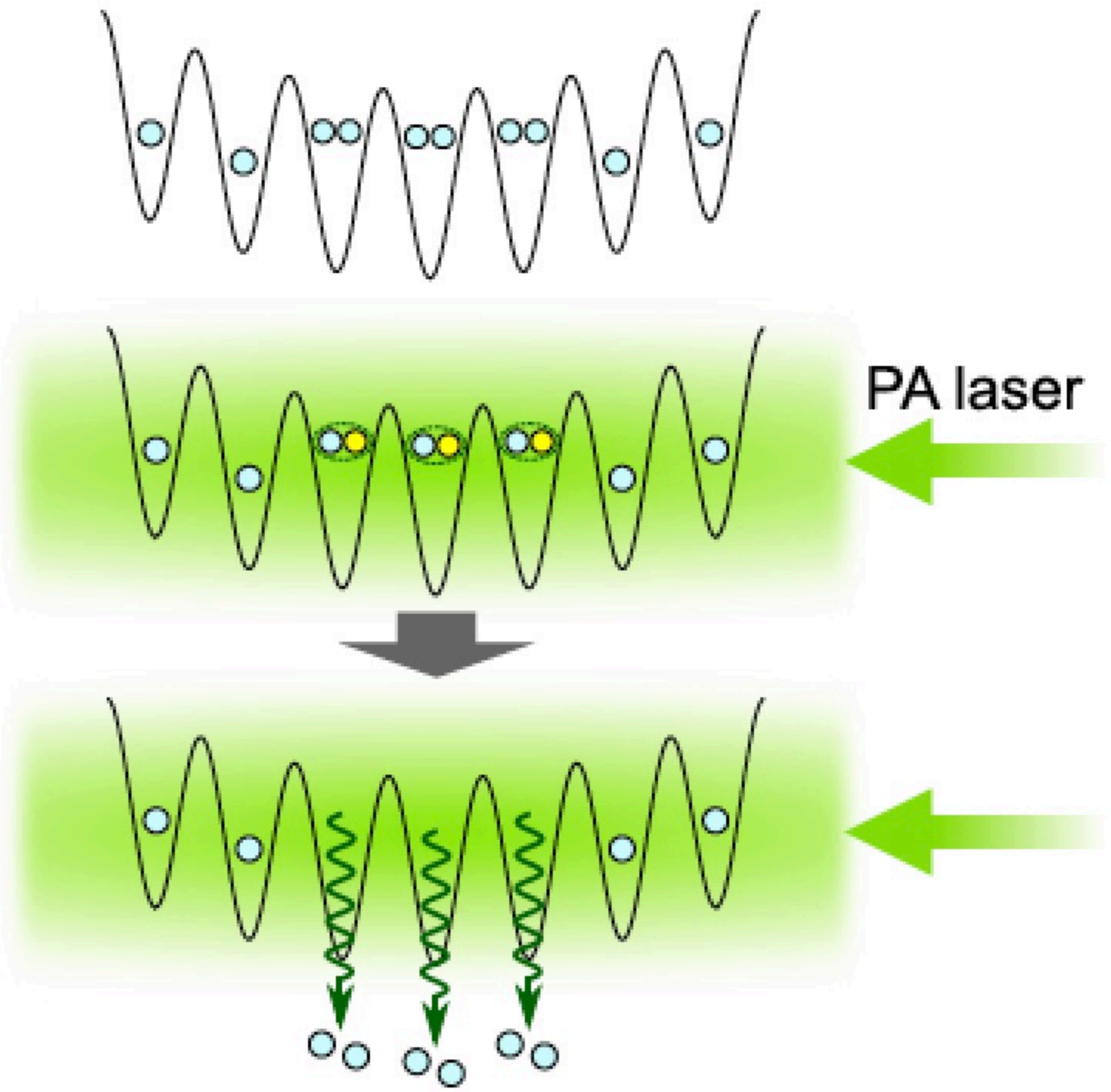


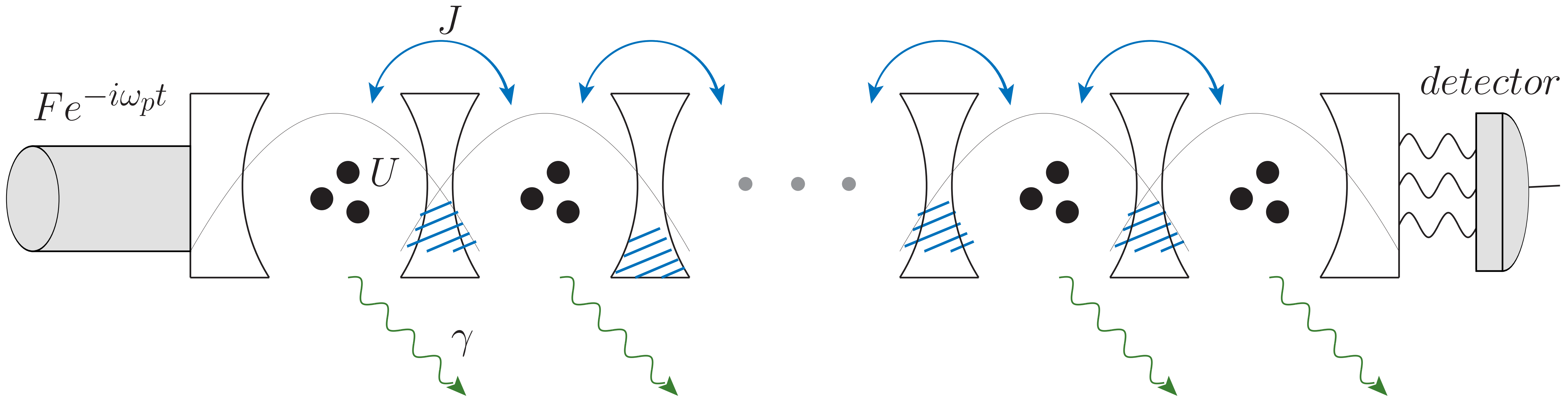
C.W. Gardiner and P. Zoller, *Quantum Noise*
H. Wiseman and G. Milburn, *Quantum measurement and Control*
...

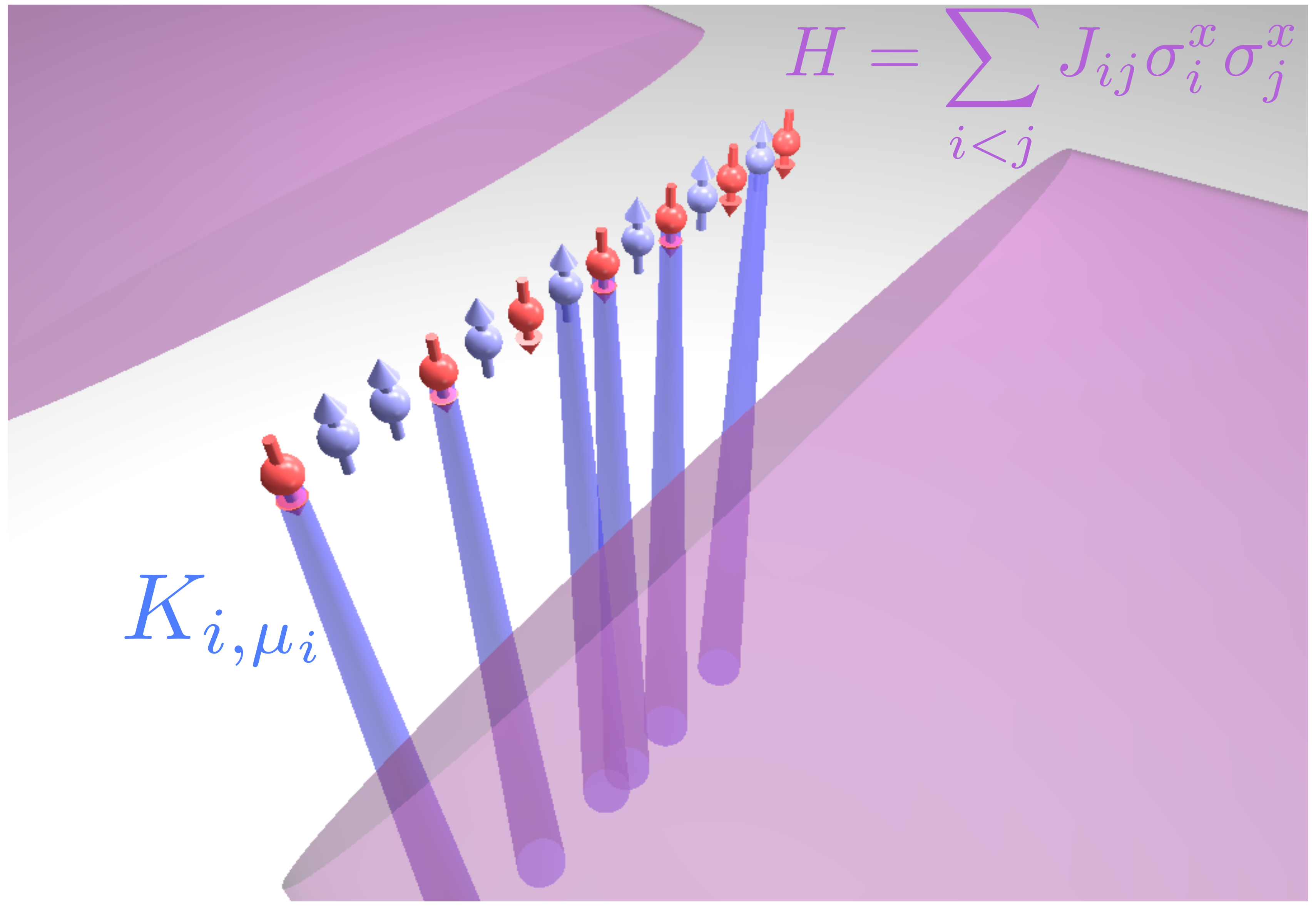
$|\psi(t), \mathcal{R}(t)\rangle$



Open many-body systems







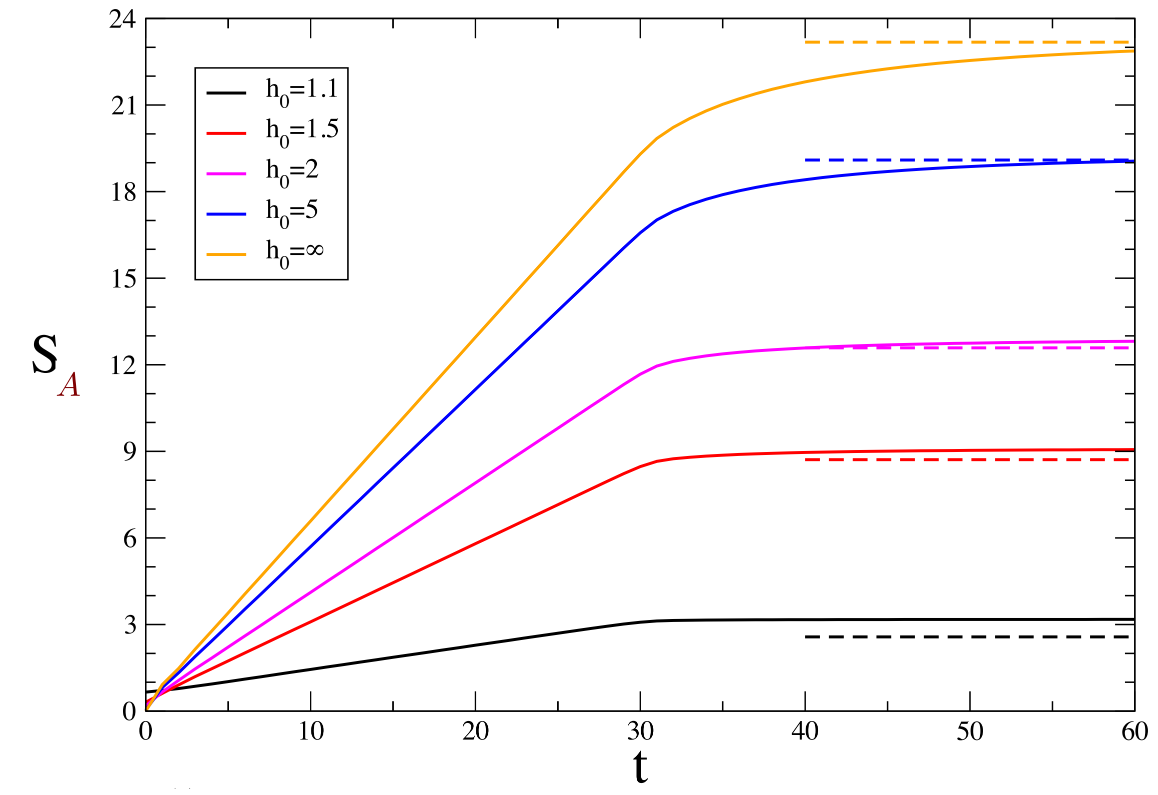
Entanglement growth

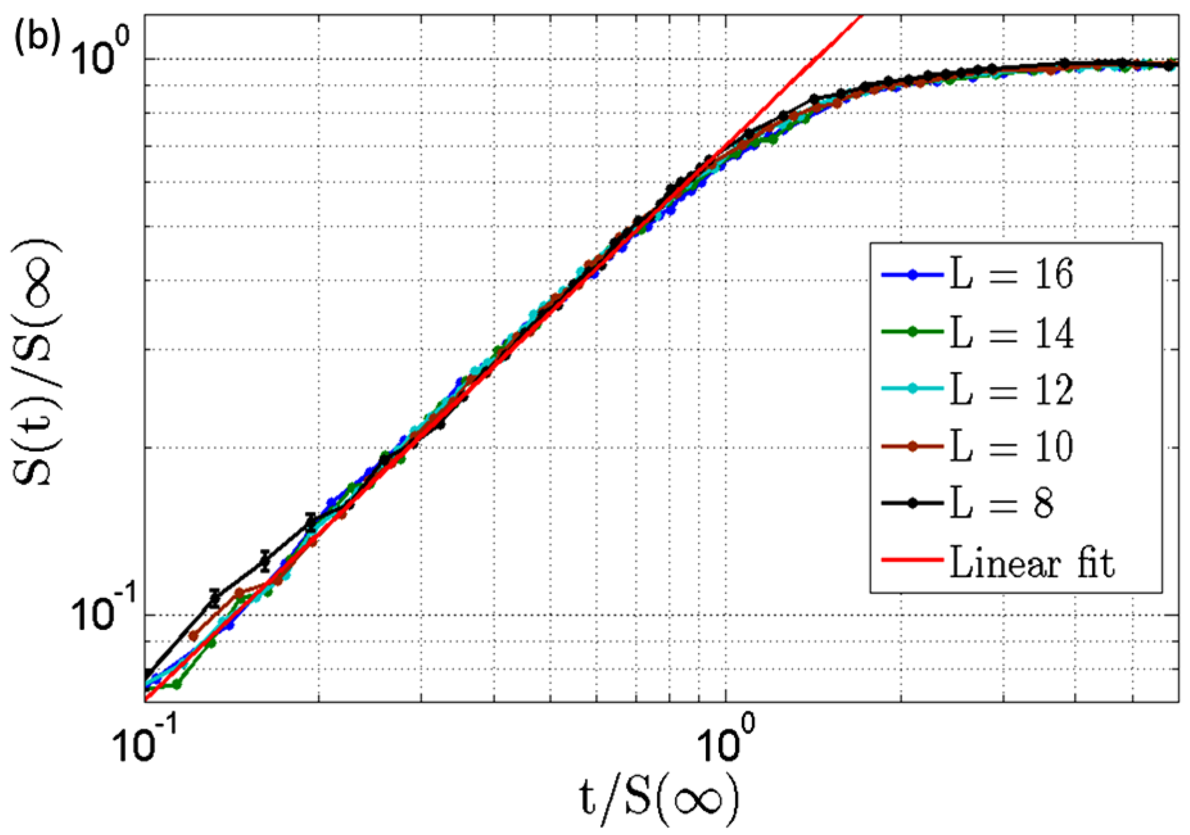
Unitary evolution in a quantum many-body system leads to a growth of entanglement (as quantified by the entanglement entropy).

$|\psi(t)\rangle$ Entanglement entropy

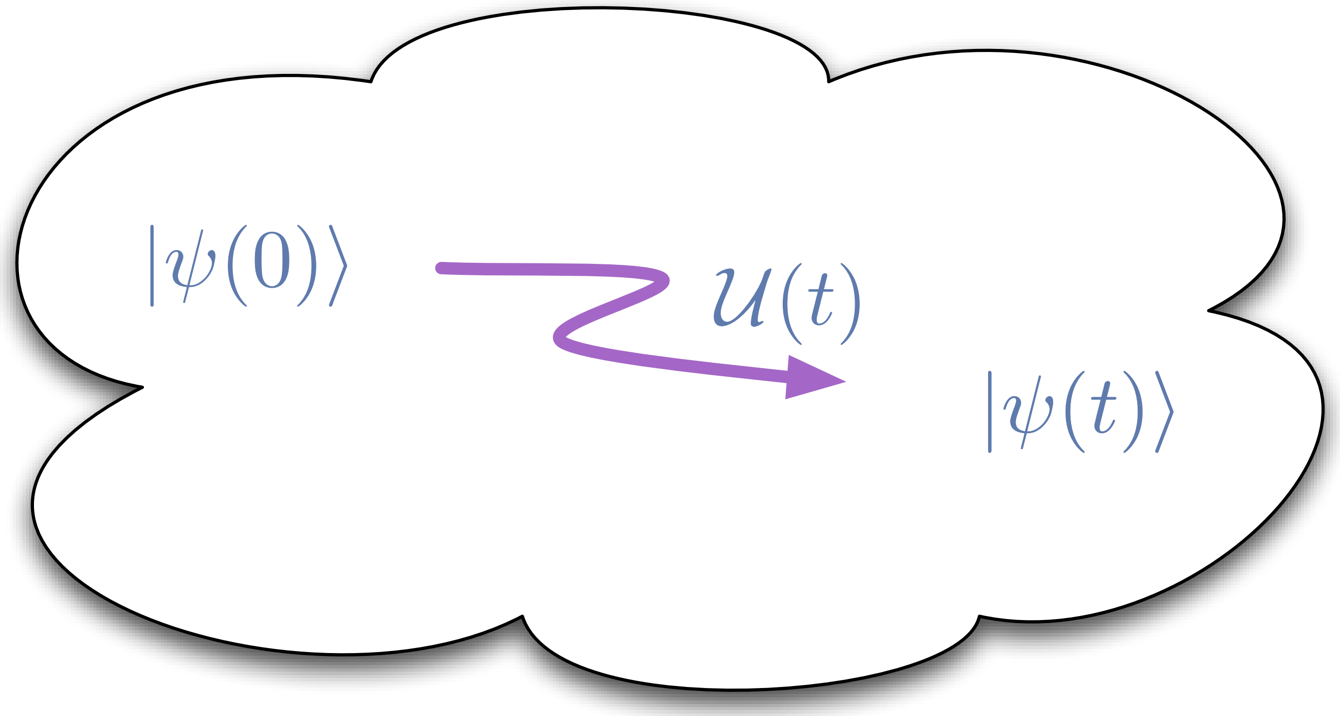
A

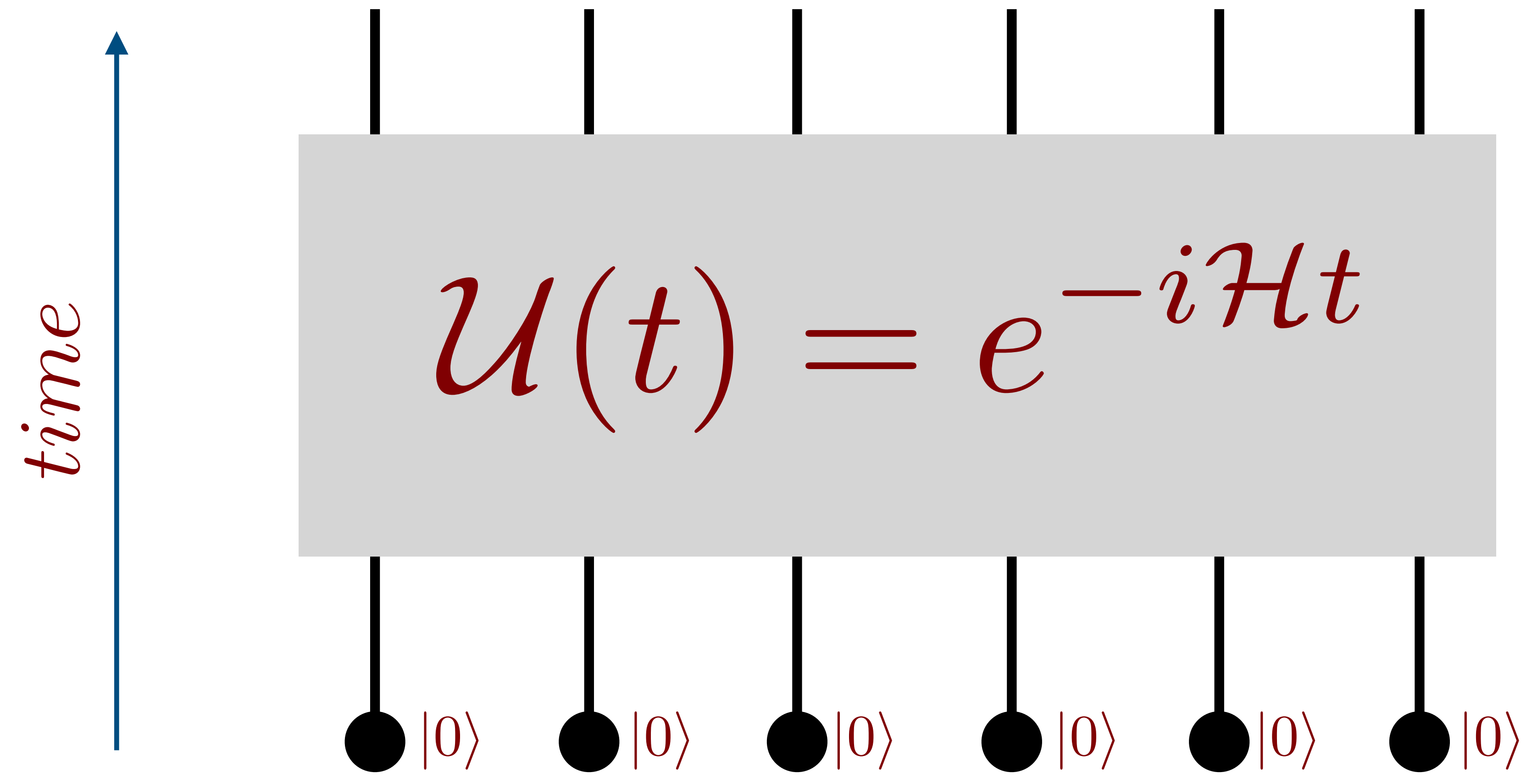
$\rho_A = \mathrm{Tr}_B |\psi(t)\rangle\langle\psi(t)| \qquad S_A = -\mathrm{Tr}[\rho_A \ln \rho_A]$

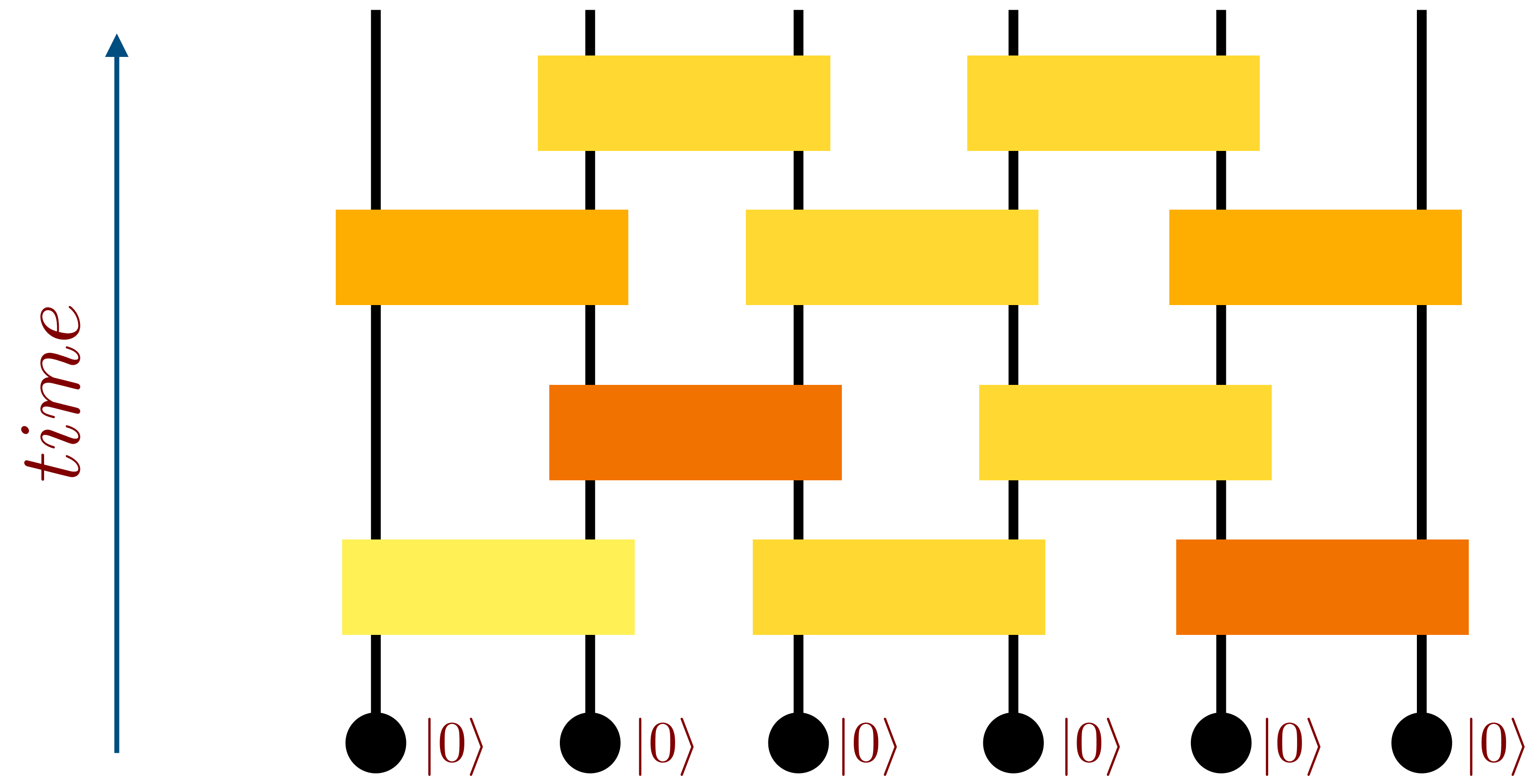




see e.g.
P. Calabrese and J. Cardy, J. Stat. Mech. P04010 (2005)
H. Kim and D.A. Huse, Phys. Rev. Lett. 111, 127205 (2013)







Entanglement growth

Ⓧ Example: Projective measurement

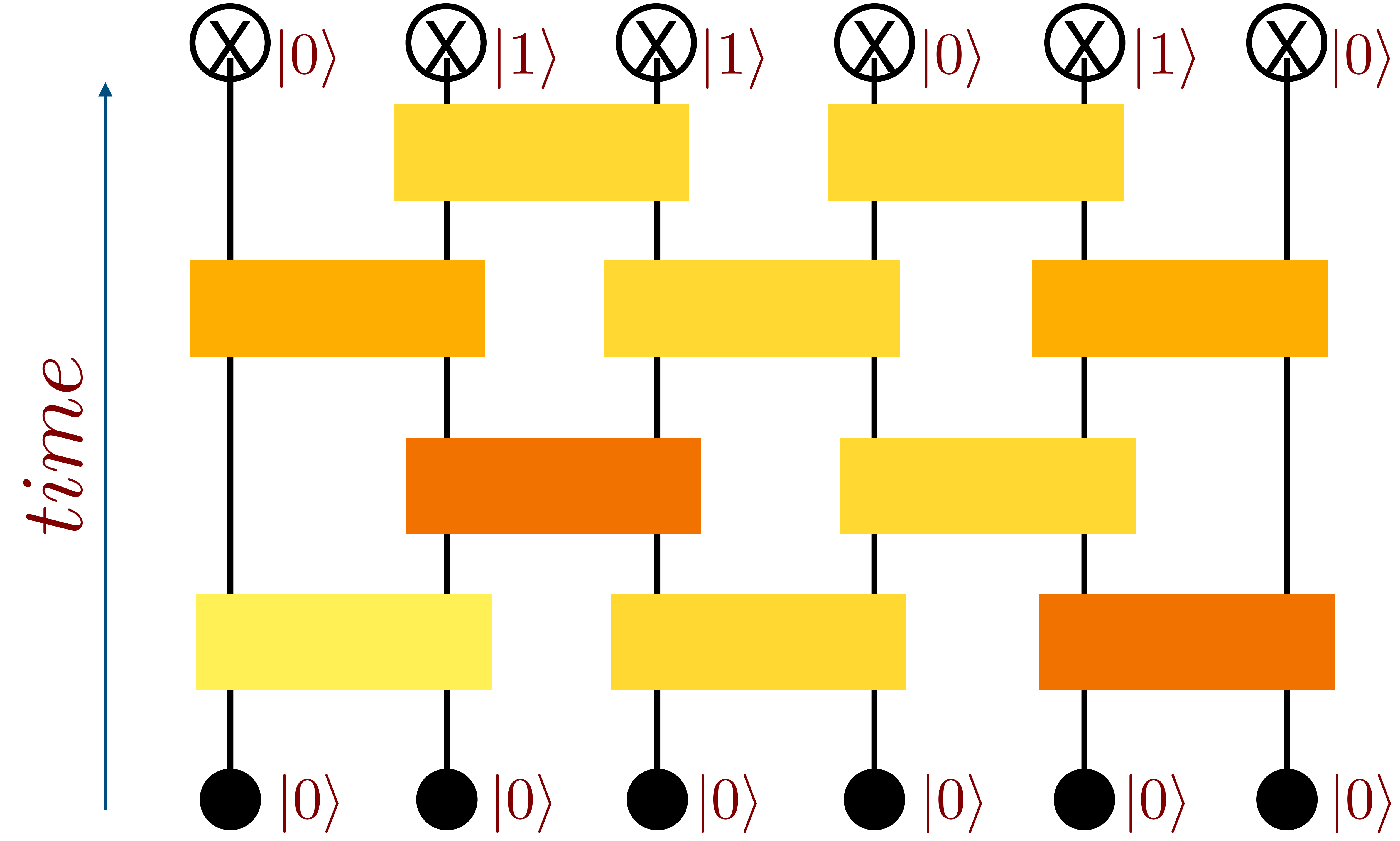
The growth of the entanglement can be contrasted by performing local measurement during the evolution (the environment can "measure" the system)

What are the consequences of the competition between unitary evolution and local (generalized) measurement ?

⇩

Dynamics of monitored many-body systems

… back to separable state



Entanglement/(measurement induced) transitions

Y. Li, X. Chen, and M. Fisher, Phys. Rev. B **98**, 205136 (2018)
B. Skinner, J. Ruhman, and A. Nahum, Phys. Rev. X **9**, 031009 (2019)
X. Cao, A. Tilloy, and A. D. Luca, SciPost Phys. **7**, 24 (2019)
M. J. Gullans, and D. A. Huse, Phys. Rev. X **10**, 041020 (2020)
Y. Bao, S. Choi, and E. Altman, Phys. Rev. B **101**, 104301 (2020)
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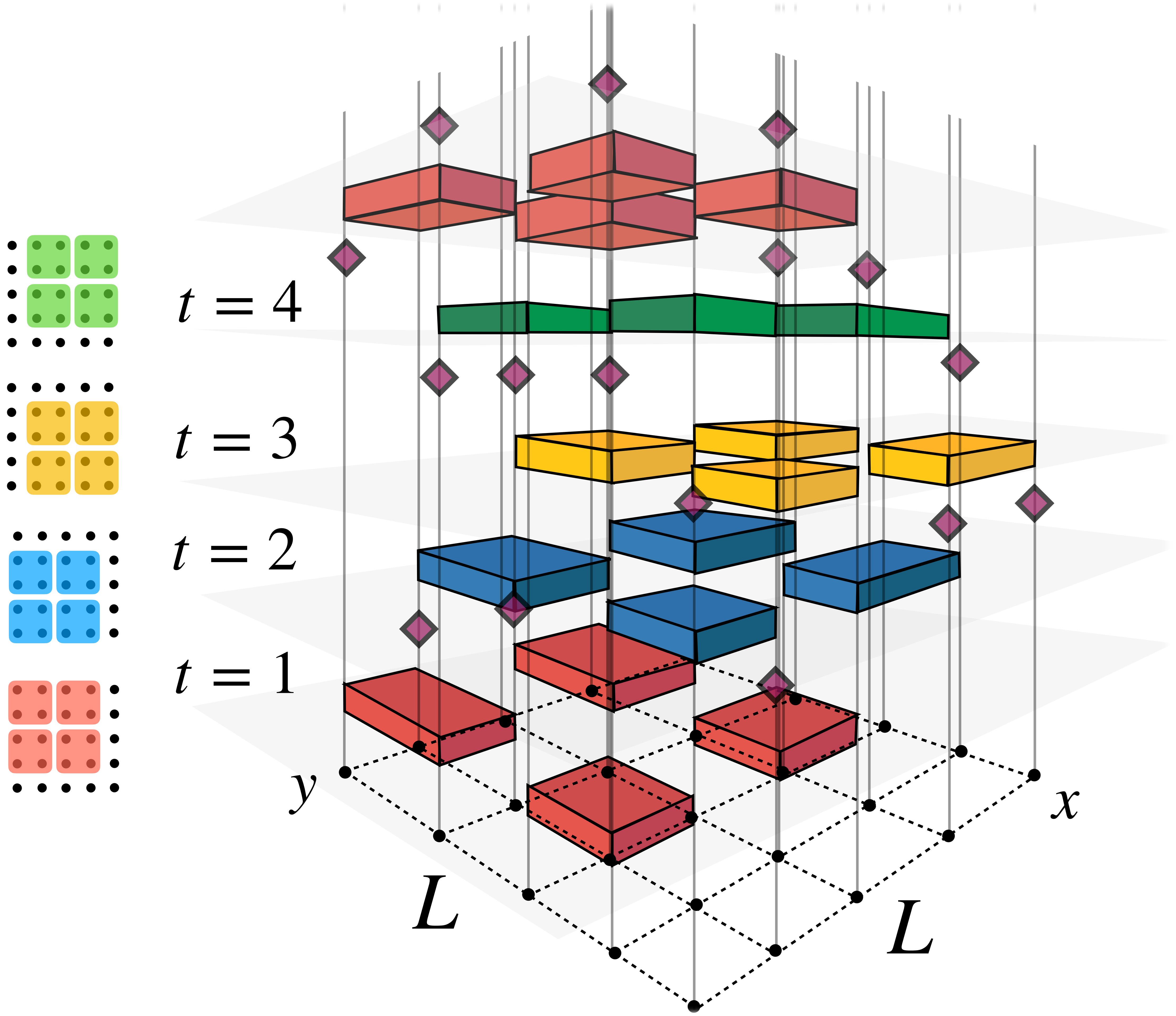


Competition between unitary dynamics and "non-unitary maps" may lead to transitions in the behaviour of entanglement in the steady state

Entanglement Transitions

In collaboration with

Giuliano Chiriacò
Marcello Dalmonte
Shraddha Sharma
Emanuele Tirrito
Mikheil Tsitsishvili

Marco Schirò
Xhek Turkeshi

Piotr Sierant

Federica Surace

Guido Pagano

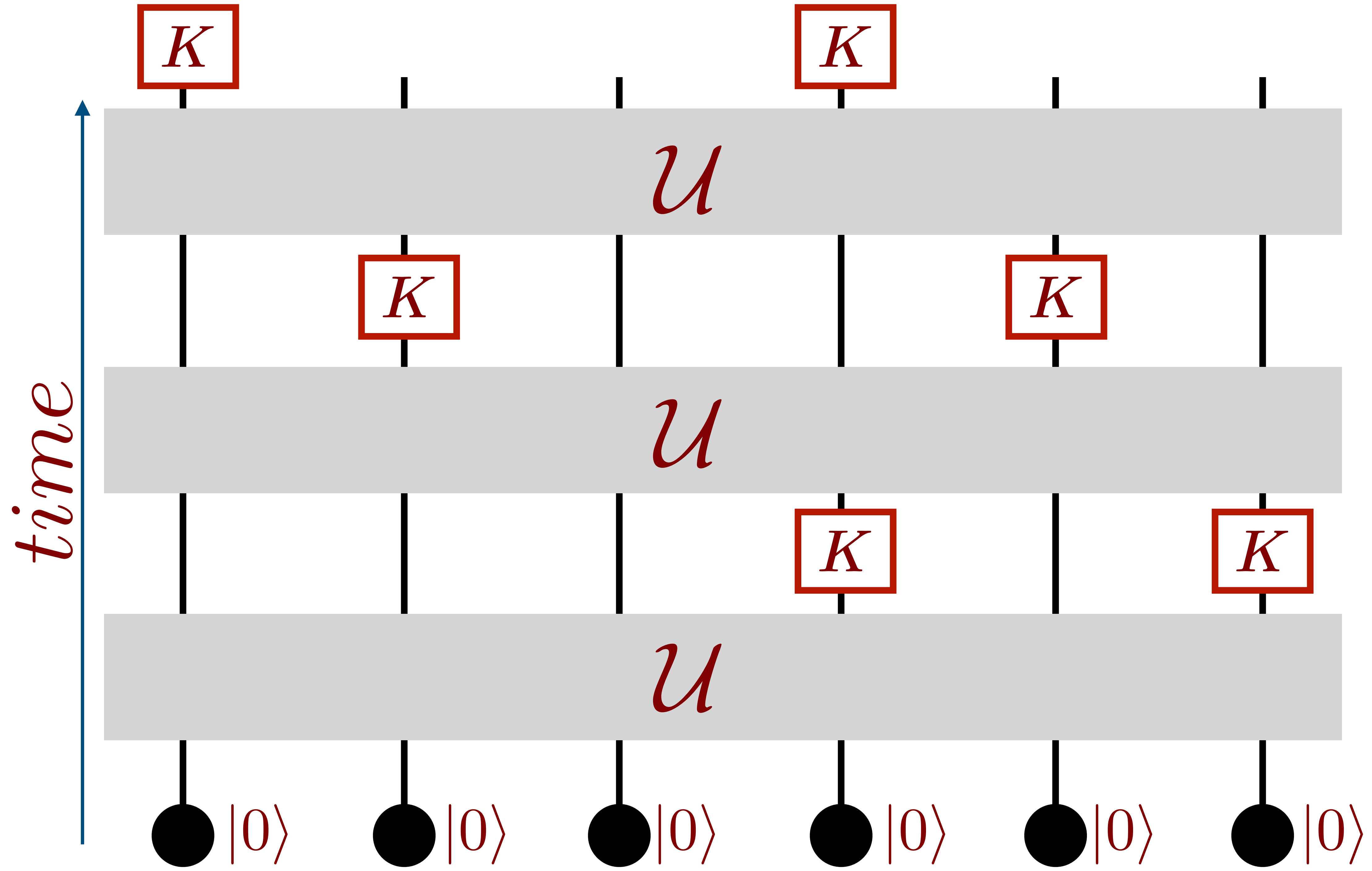
Alberto Biella

Dario Poletti

Mario Collura
Alessandro Santini

X. Turkeshi, R. Fazio, and M. Dalmonte, Phys. Rev. B **98**, 102, 014315 (2020)
X. Turkeshi, A. Biella, R. Fazio, M. Dalmonte, and M. Schirò, Phys. Rev. B **103**, 224210 (2021)
P. Sierant, G. Chiriacò, F.M. Surace, S. Sharma, X.Turkeshi, M. Dalmonte, R. Fazio, and G. Pagano, Quantum **6**, 638 (2022)
S. Sharma, X. Turkeshi, R. Fazio, and M. Dalmonte, SciPost Phys. Core 023 (2022)
X. Turkeshi, R. Fazio, M. Dalmonte, and M. Schirò, Phys. Rev. B **105**, L241114 (2022)
E. Tirrito, A. Santini, R. Fazio, and M. Collura, arXiv:2212.09405 (2022)
G. Chiriacò, M. Tsitsishvili, D. Poletti, R. Fazio, and M. Dalmonte, arXiv:2302.10563 (2023)

Entanglement/(measurement induced) transitions



$$|\psi'\rangle = \mathcal{U}|\psi\rangle$$

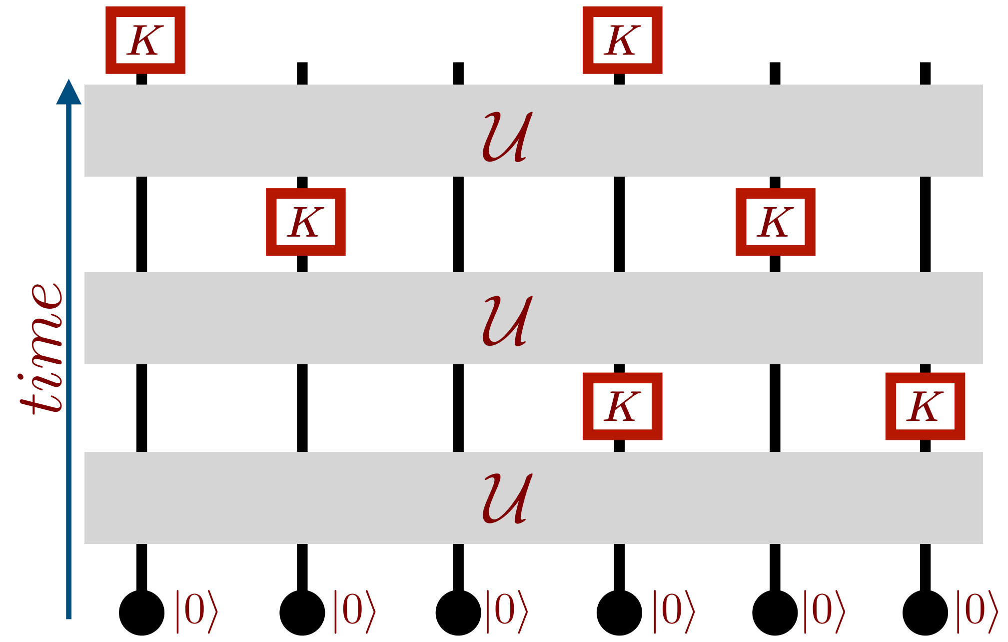
$$|\psi'\rangle = \frac{K_\mu|\psi\rangle}{\sqrt{\langle\psi|K_\mu^\dagger K_\mu|\psi\rangle}}.$$

with probability given by the Born rule

$$\mathcal{P}(\mu) = \langle\psi|K_\mu^\dagger K_\mu|\psi\rangle$$

Kraus operators $\boxed{K}$

Two cases



1)

$$K_0 = \sqrt{p}|0\rangle\langle 0| \qquad K_1 = \sqrt{p}|1\rangle\langle 1| \qquad K_2 = \sqrt{1-p}\,\mathbb{I}$$

2)

$$K_0 = \sqrt{p}|0\rangle\langle 0| \qquad K_1 = \sqrt{p}|0\rangle\langle 1| \qquad K_2 = \sqrt{1-p}\,\mathbb{I}$$

Kraus map

$$\rho' = \sum_\alpha K_\alpha \rho K_\alpha^\dagger$$

From Kraus to Lindblad equation

$$K_0 = 1 - dt\left(\mathcal{H} - \frac{1}{2}L^\dagger L\right)$$

$$\dot{\rho} = -i\left[\mathcal{H}, \rho\right] + L\rho L^\dagger - \frac{1}{2}\left\{L^\dagger L, \rho\right\}$$

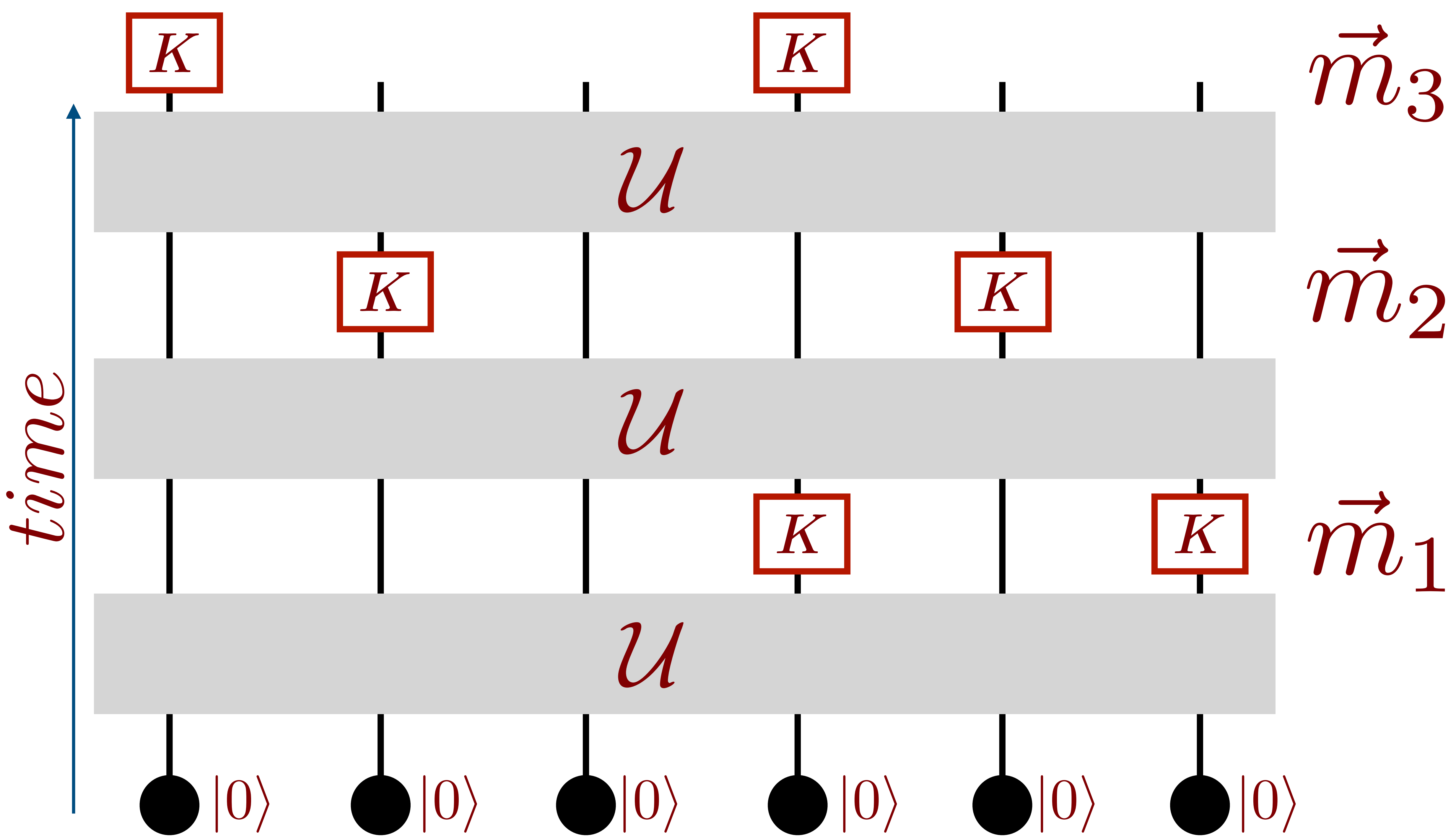
$$K_1 = \sqrt{dt}L$$

Open systems - quantum trajectories

Monitored systems

Consider for the moment projective measurements. At each stage one record the result of the measurement *m*. This data set forms a trajectory

$$\mathcal{R} \equiv \vec{m}_1, \vec{m}_2, \cdots$$



After n-steps of the evolution the wave-function will depend on the measurement record $|\psi(t)\rangle_{\mathcal{R}}$

"Two types" of averages

$\langle O \rangle_{\mathcal{R}}$ and $\langle O \rangle = \sum_{\mathcal{R}} \langle O \rangle_{\mathcal{R}} = \mathrm{Tr}\rho O$

Averaging the state over all trajectories is equivalent to evaluate the average over the density matrix. This is because it is a linear quantity in the state.

If one consider non-linear (in the state) quantities, performing the averages in different order will make a difference.

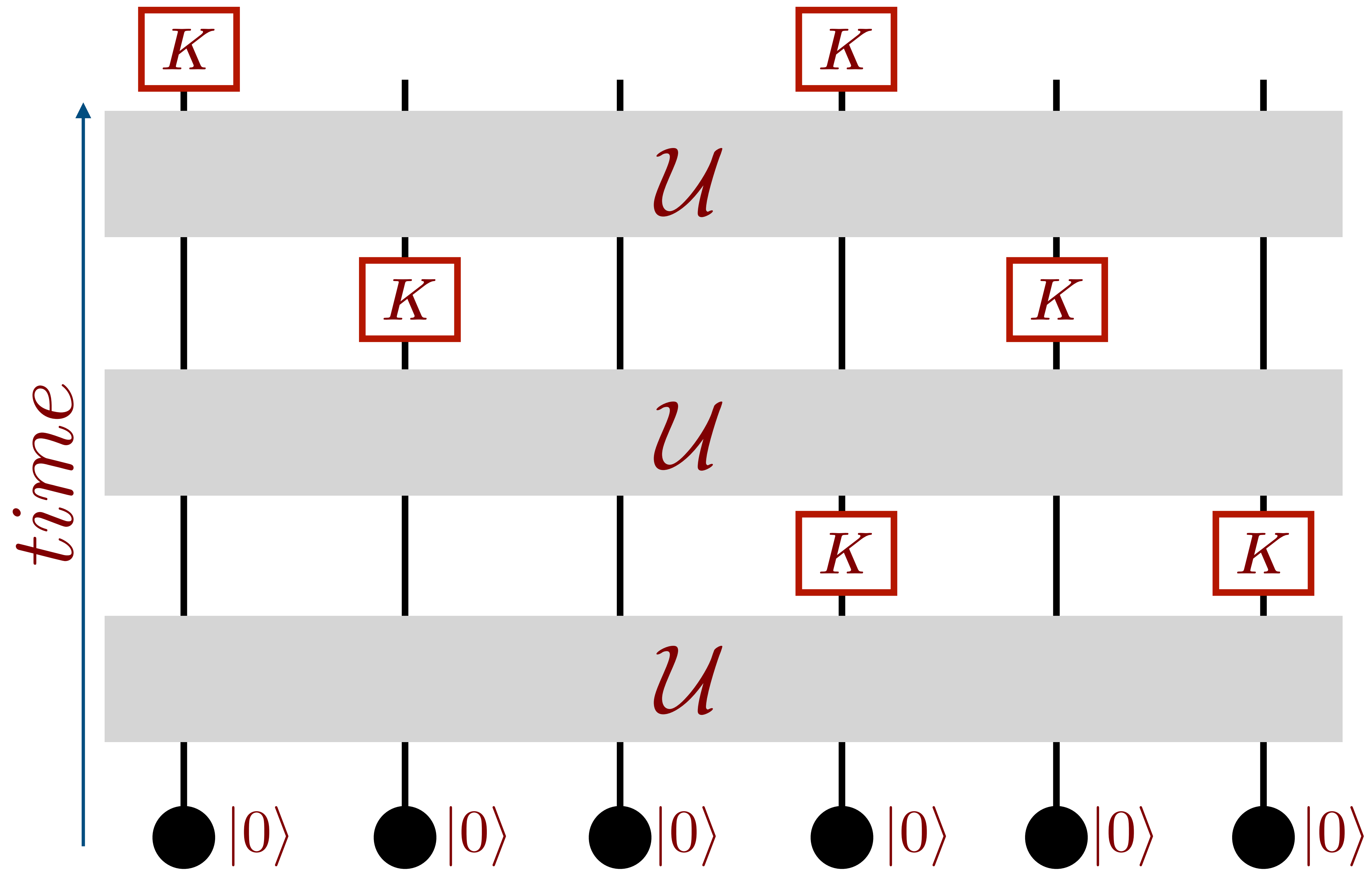
Example of such a quantity is a block entropy

$$S_2 = -\sum_{\mathcal{R}} \log \mathrm{Tr}_A \rho^2_{A,\mathcal{R}}$$

Open systems - quantum trajectories

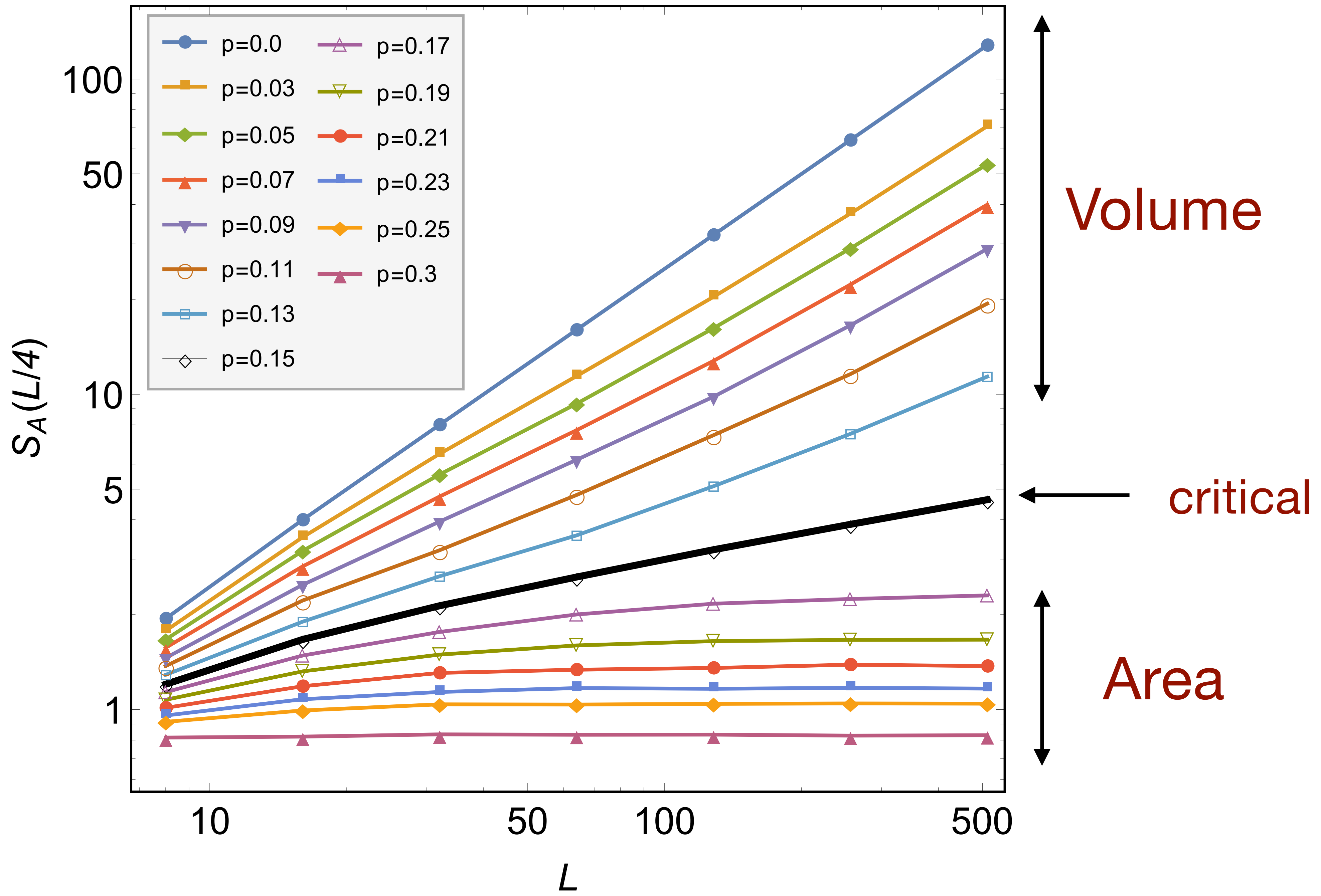
The K-operations are projective measurements along the z-direction

The average density matrix in the long-time limit approaches the identity. All averages of observables are trivial.



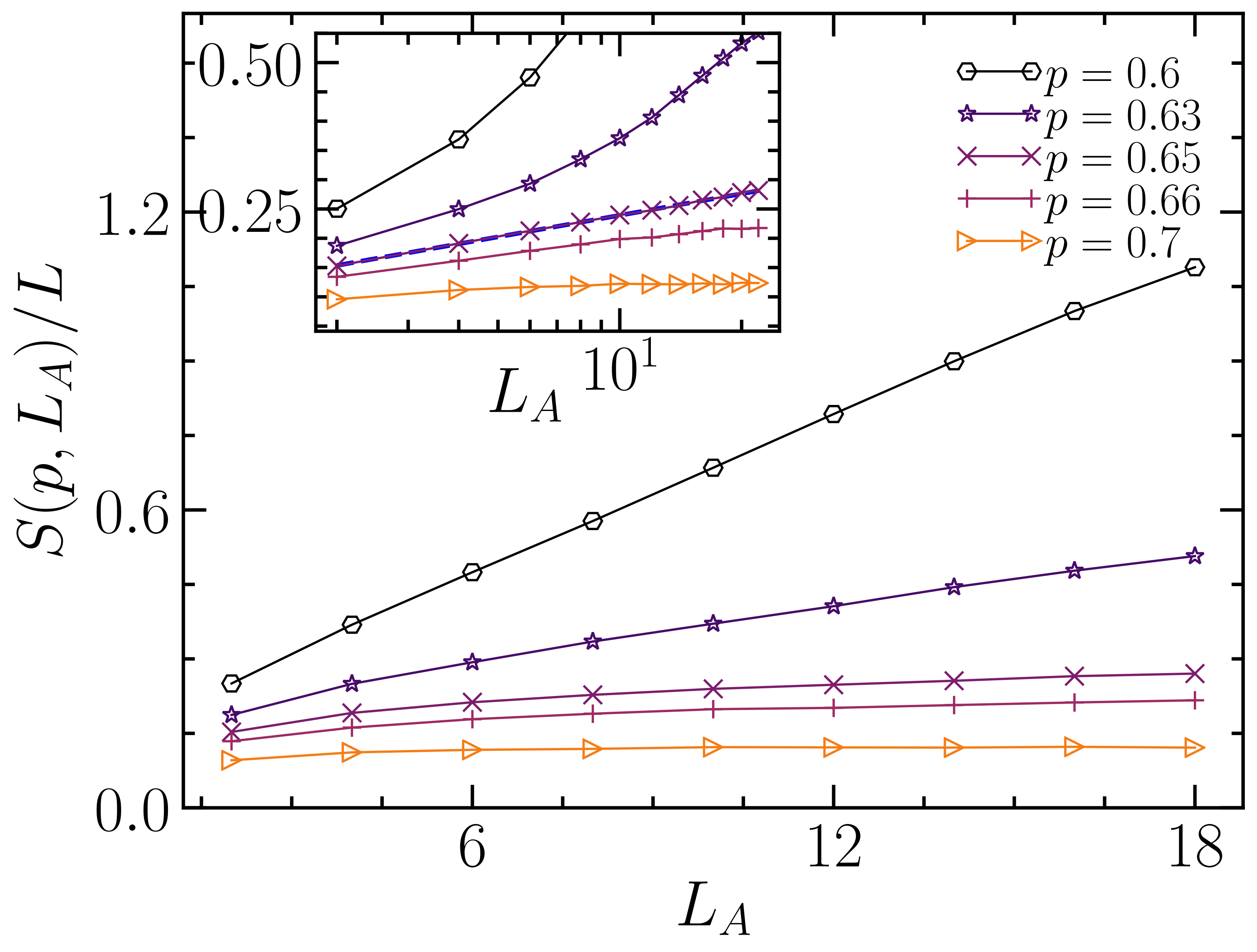
Unitary evolution leads to volume-law in the entanglement

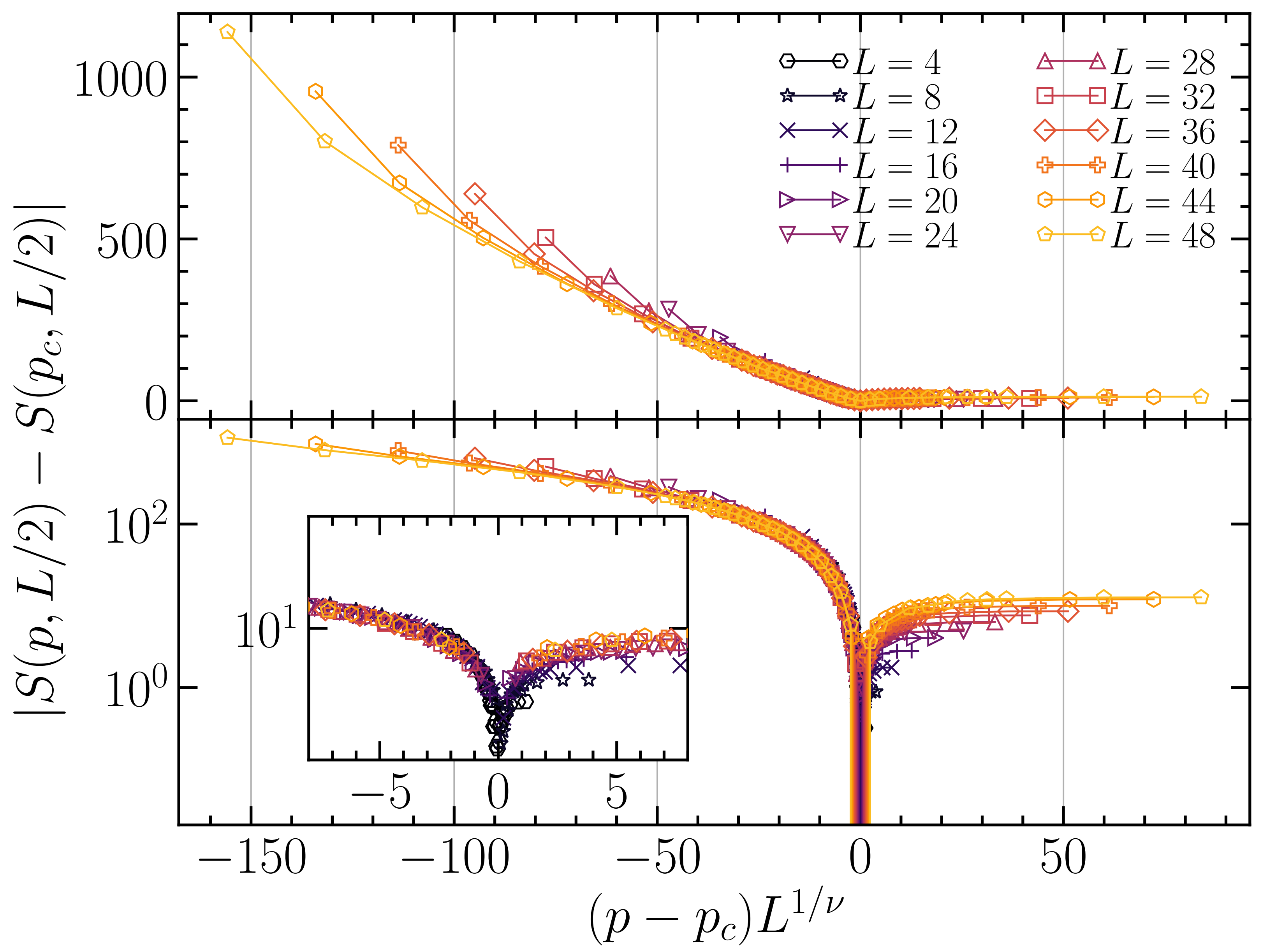
Non-unitary local operations favour a separable state (area - law)

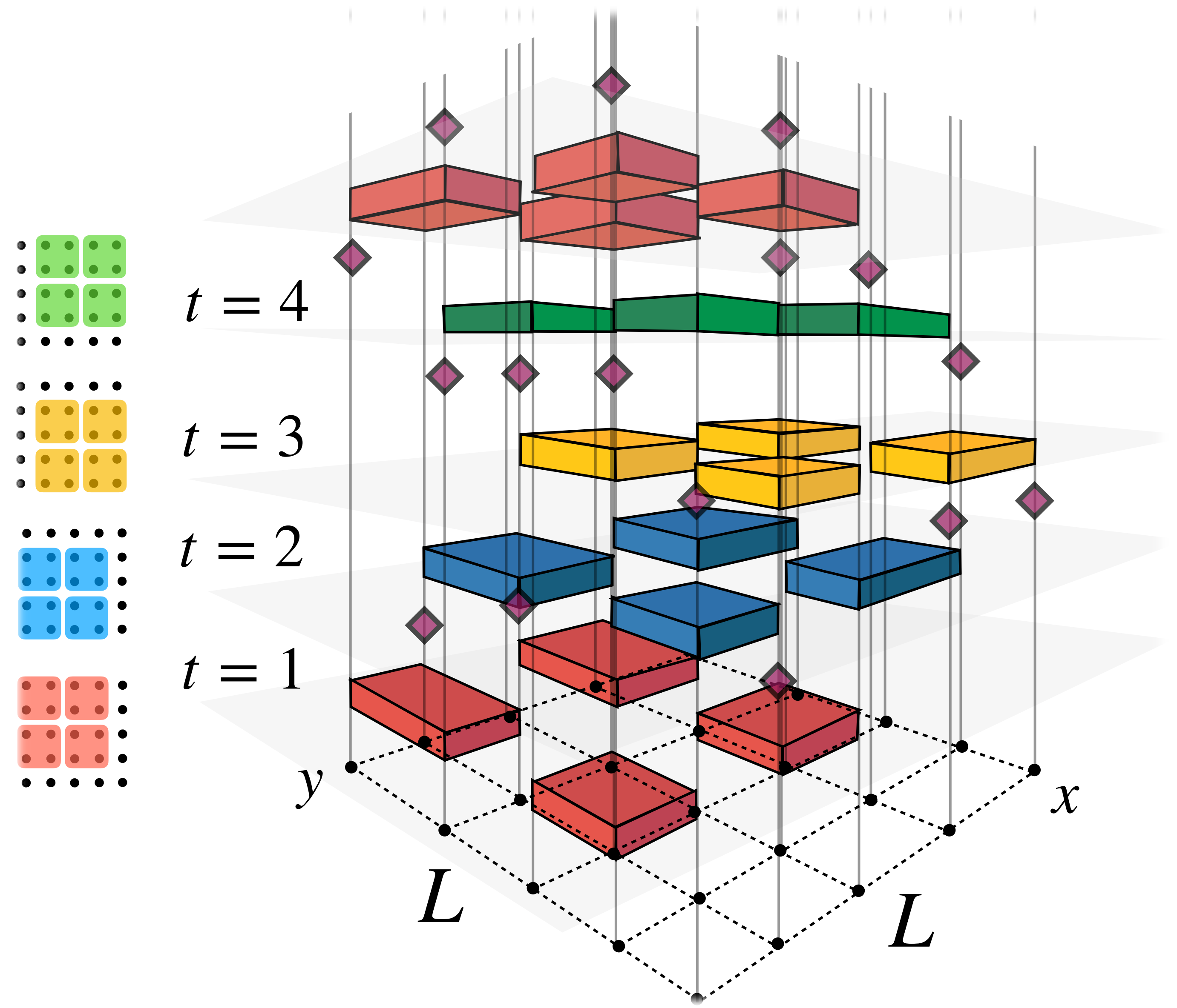


Y. Li, X. Chen, and M. Fisher, Phys. Rev. B **98**, 205136 (2018)

Measurement-induced criticality in (2+1)-d hybrid quantum circuits







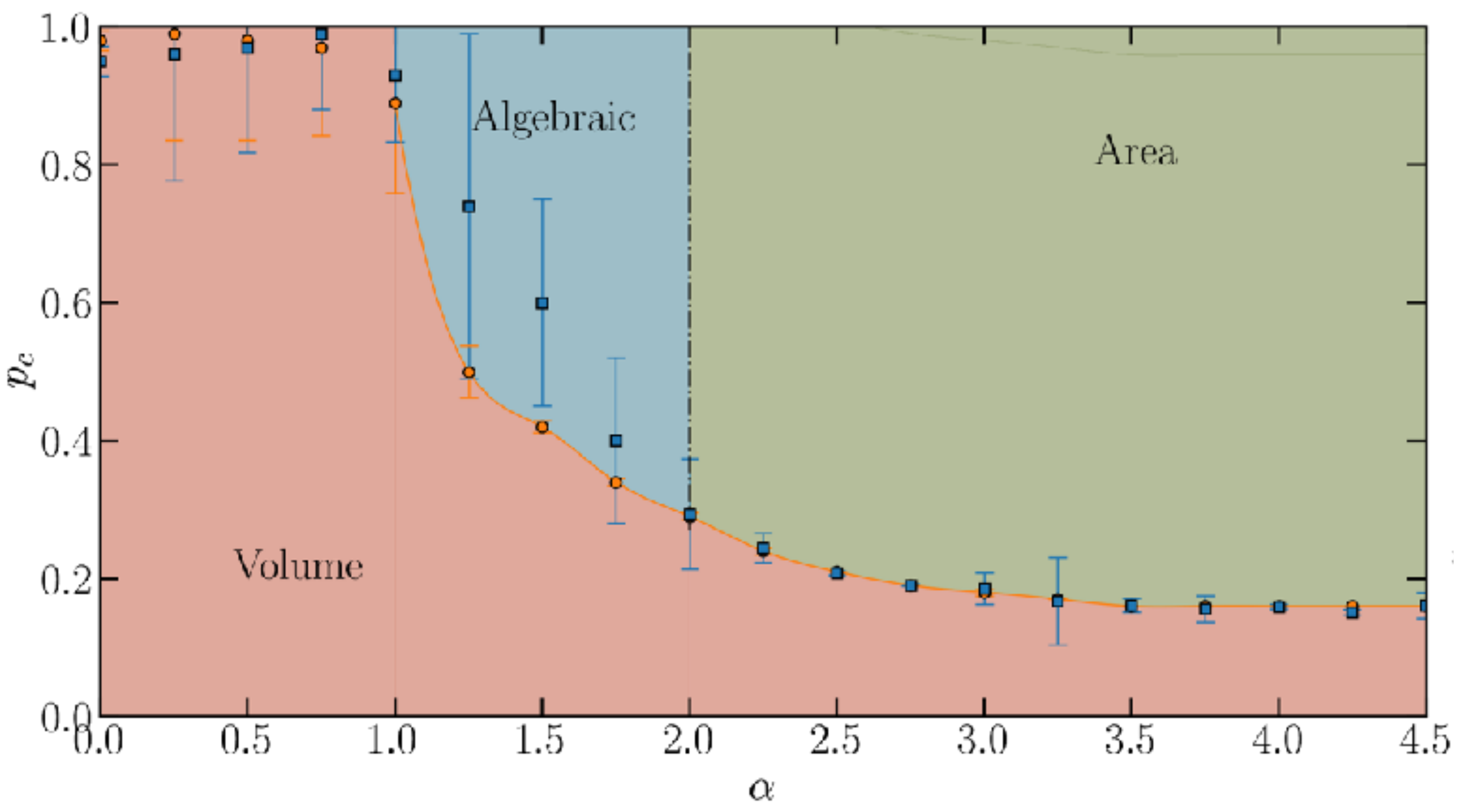
There is a universal critical behavior for 2+1D hy- brid quantum circuits

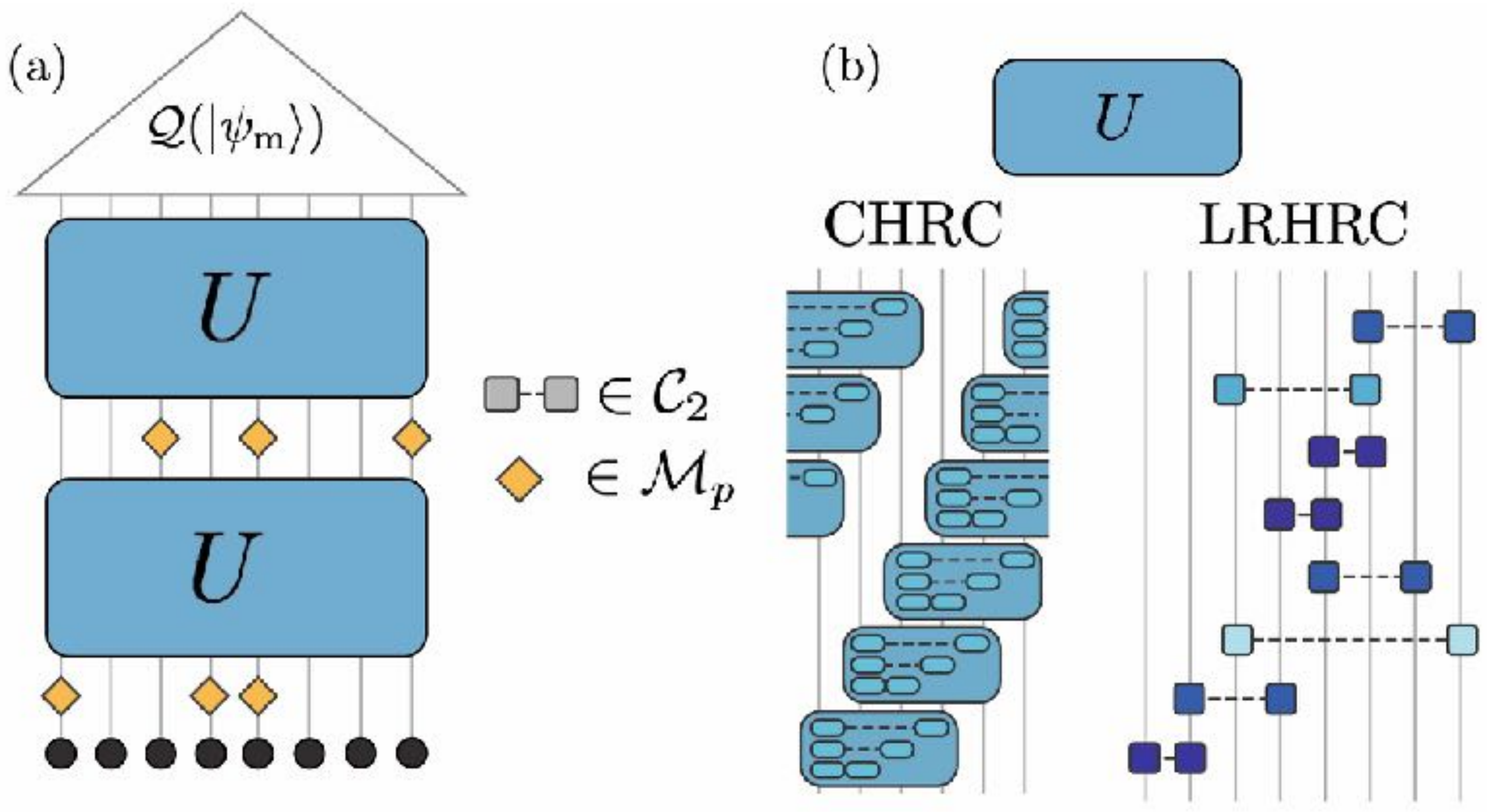
The critical point shows a multiplicative logarithmic correction to the area law entanglement entropy

The universality class is different from that of 3D percolation

X. Turkeshi, R. Fazio, and M. Dalmonte, Phys. Rev. B **102**, 014315 (2020)

Measurement-induced criticality in long-range hybrid quantum circuits





In the case of power-law distributed gates the universality class of the phase transition changes continuously with the parameter controlling the range of interactions.

For intermediate values of the control parameter, we find a non-conformal critical line which separates a phase with volume-law scaling of the entanglement entropy from one with sub-extensive scaling. Within this region, the entanglement entropy and the logarithmic negativity present a cross-over from a phase with algebraic growth of entanglement with system size, and an area-law phase

S. Sharma, X. Turkeshi, R. Fazio, and M. Dalmonte, SciPost Phys. Core 023 (2022)

Open systems - Steady state

$$\rho_s = \rho(t \to \infty)$$

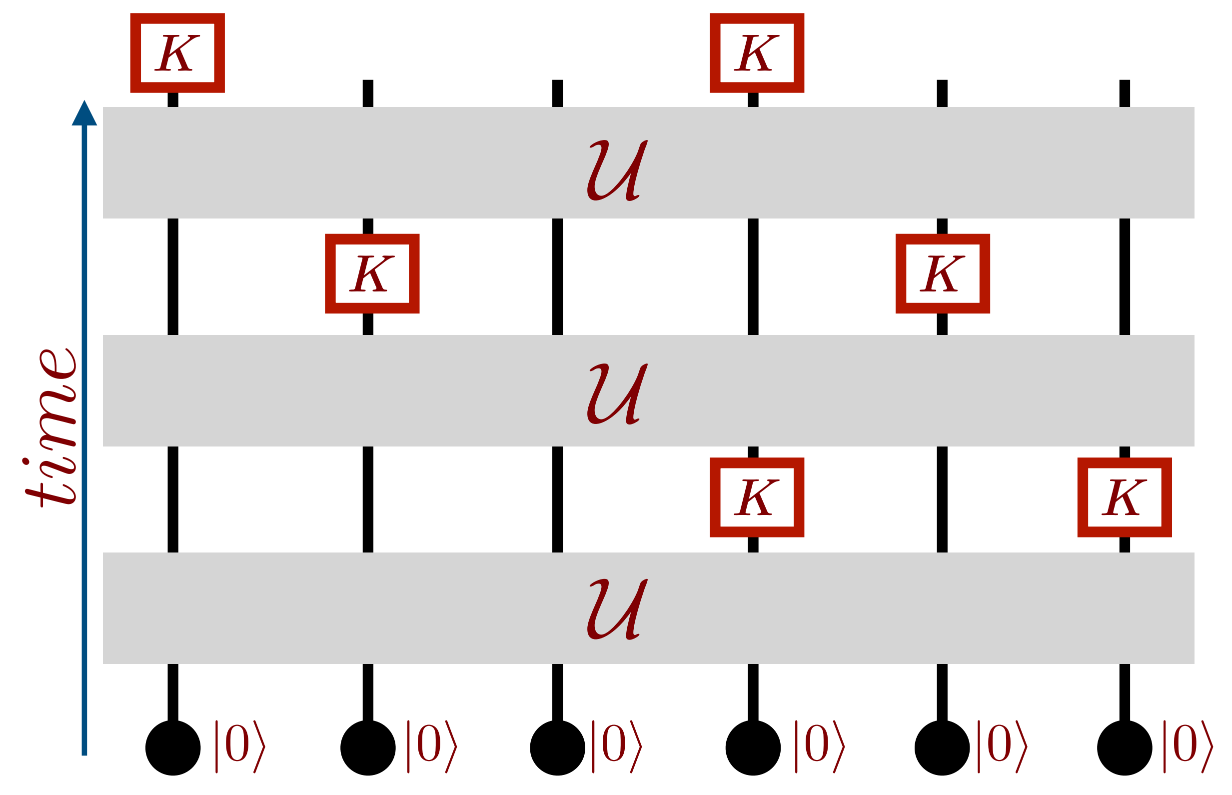
$$\sum_k p_k |\chi_k\rangle\langle\chi_k|$$

Symmetry breaking
(dissipative phase transitions)

mixed
non-equilibrium

correlations vs quantum correlations (entanglement)

Dissipative vs Entanglement transitions



$$\mathcal{H} = \sum_{ij} J_{ij}\sigma_i^x\sigma_j^x - h\sum_i \sigma_i^z \qquad J_{ij} = \frac{J}{|i-j|^\alpha}$$

A new framework to simulaneously investigate dissipative and measurement-induced phase transitions in experimentally realizable in trapped ions. A kicked Ising model with resetting

Both a DPT and a MIPT emerge as a result of the interplay between the unitary dynamics and the ran- dom quantum measurements.

$$K_0 = \sqrt{p}|0\rangle\langle 0|$$

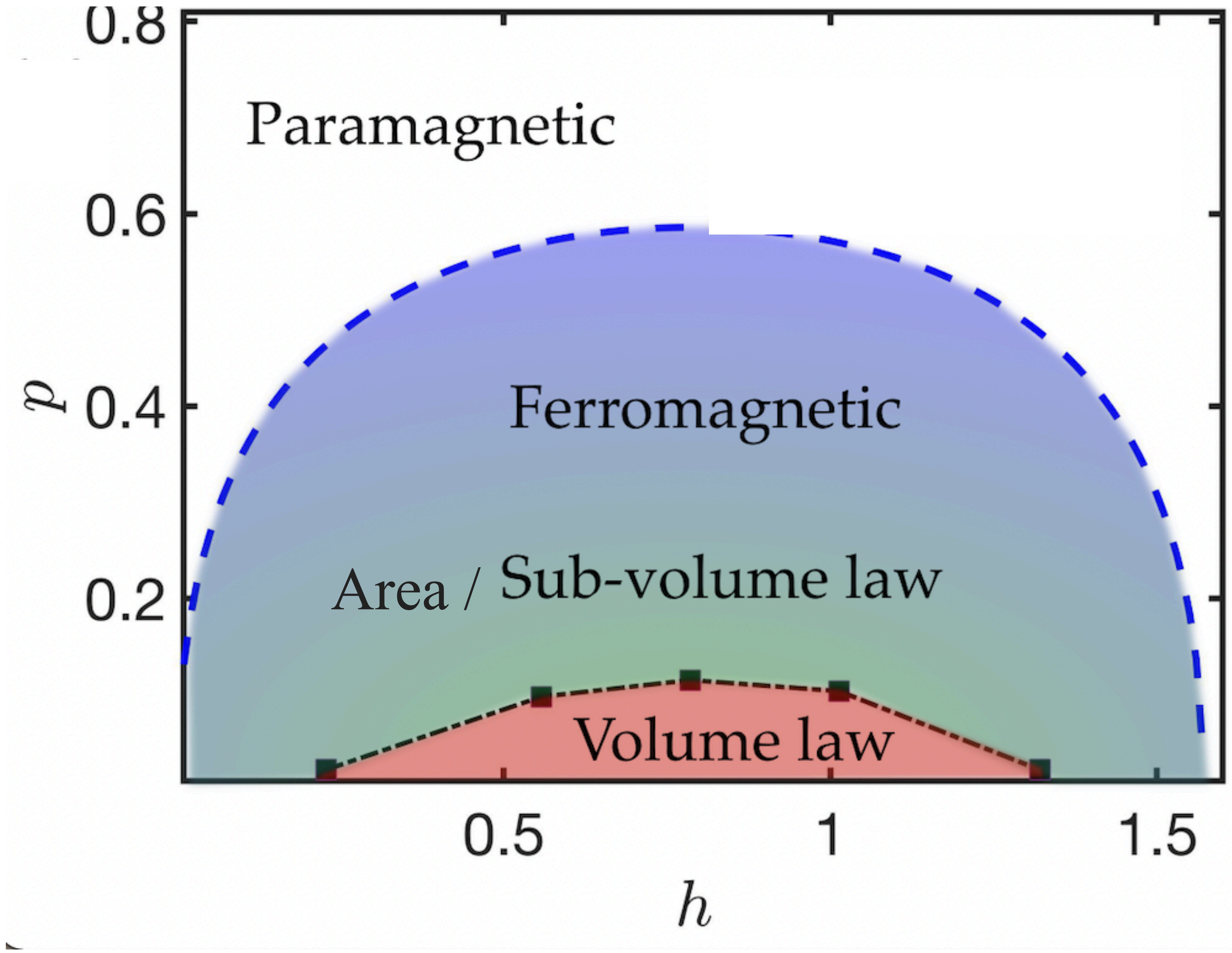
$$K_1 = \sqrt{p}|0\rangle\langle 1|$$

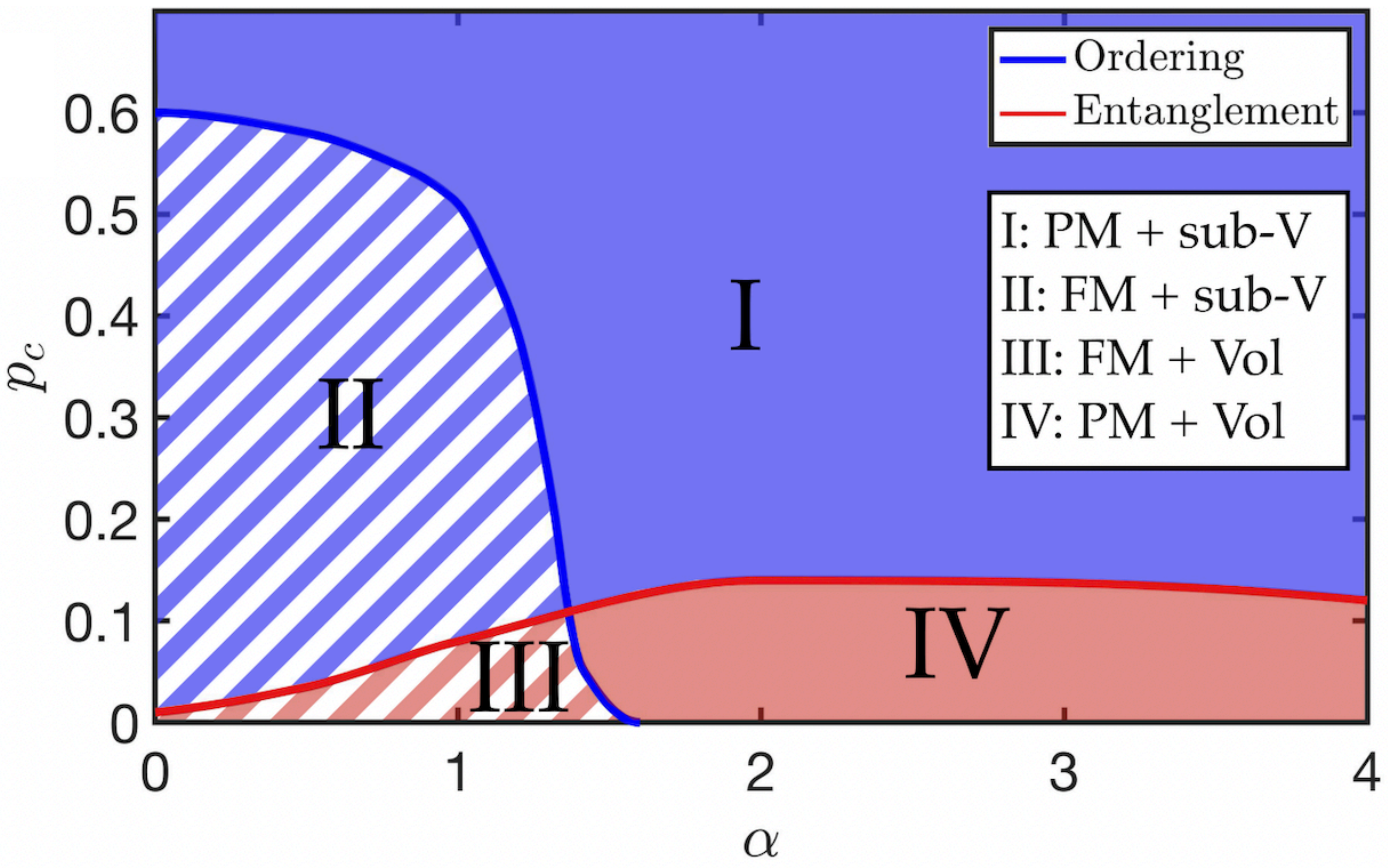
$$K_2 = \sqrt{1-p}\,\mathbb{I}$$

The non-unitary consists in a local resetting where spins are independently reset to the down state with probability p

P. Sierant, G. Chiriacò, F.M. Surace, S. Sharma, X.Turkeshi, M. Dalmonte, R. Fazio, and G. Pagano, Quantum **6**, 638 (2022)

Phase Diagram





A region of coexistence of the ordered phase and the area law phase appears for intermediate values of *p*.

Qualitative diagram of the interplay between ordering and entanglement transition as a function of the range of interactions α and the resetting probability p

P. Sierant, G. Chiriacò, F.M. Surace, S. Sharma, X.Turkeshi, M. Dalmonte, R. Fazio, and G. Pagano, Quantum **6**, 638 (2022)

Thank you!