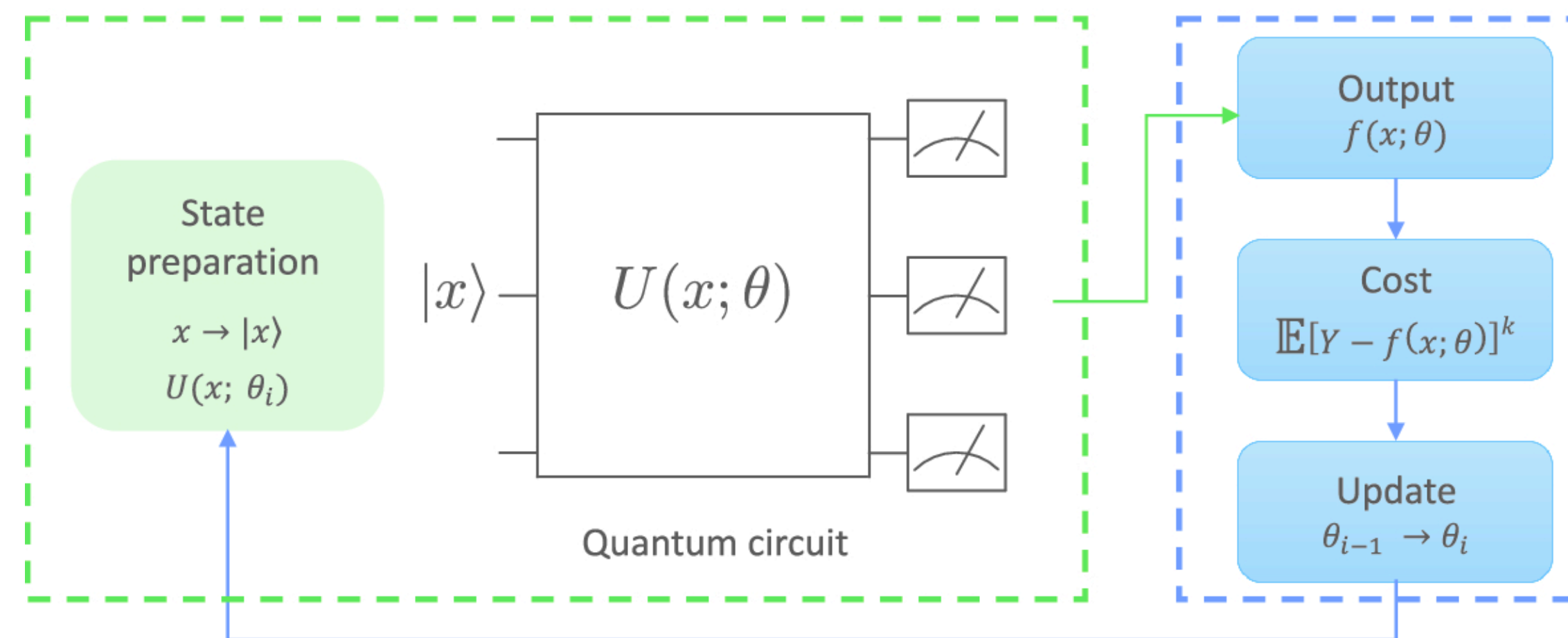
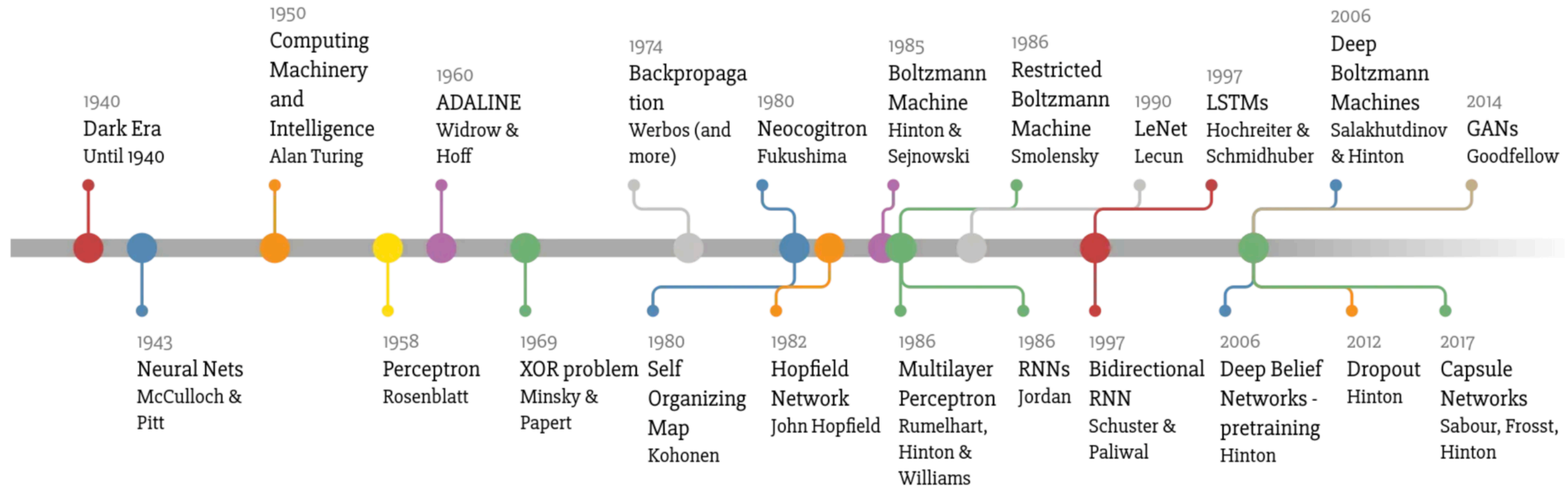


Training quantum models at scale

Amira Abbas,
University of KwaZulu-Natal

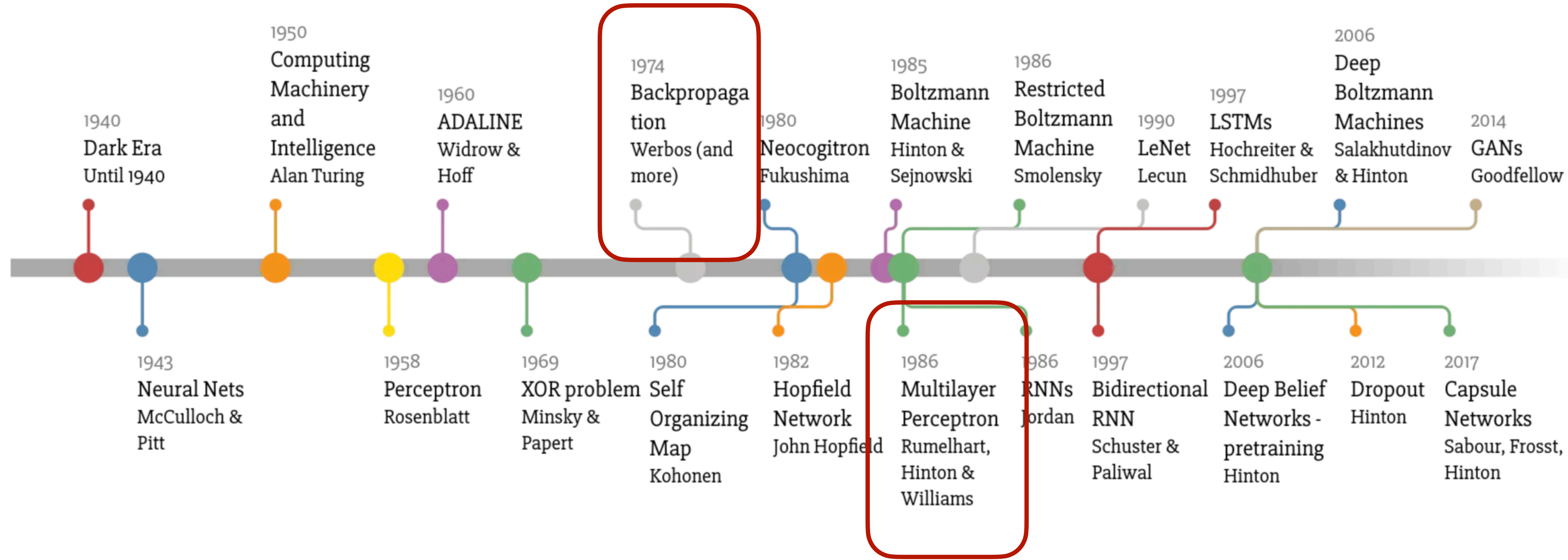


Deep Learning Timeline



Made by Favio Vázquez

Deep Learning Timeline



Made by Favio Vázquez

~175 billion parameters

A

Tell me in 2 sentences why backpropagation is so important.



Backpropagation is important because it allows artificial neural networks to learn and improve their performance over time through the adjustment of their weights. Without backpropagation, training complex neural networks with many layers and connections would be impractical, if not impossible.

chatGPT

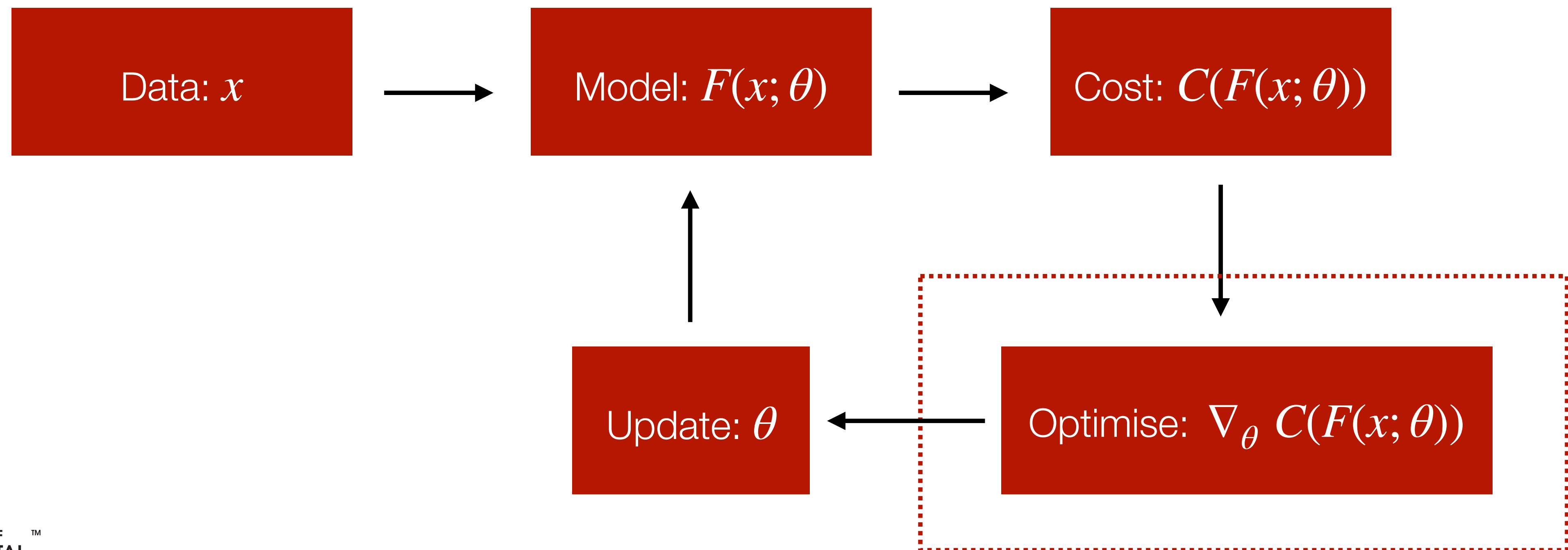
How does backpropagation work?

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- A recipe to compute gradients

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- The first computationally efficient method update parameters of a neural network

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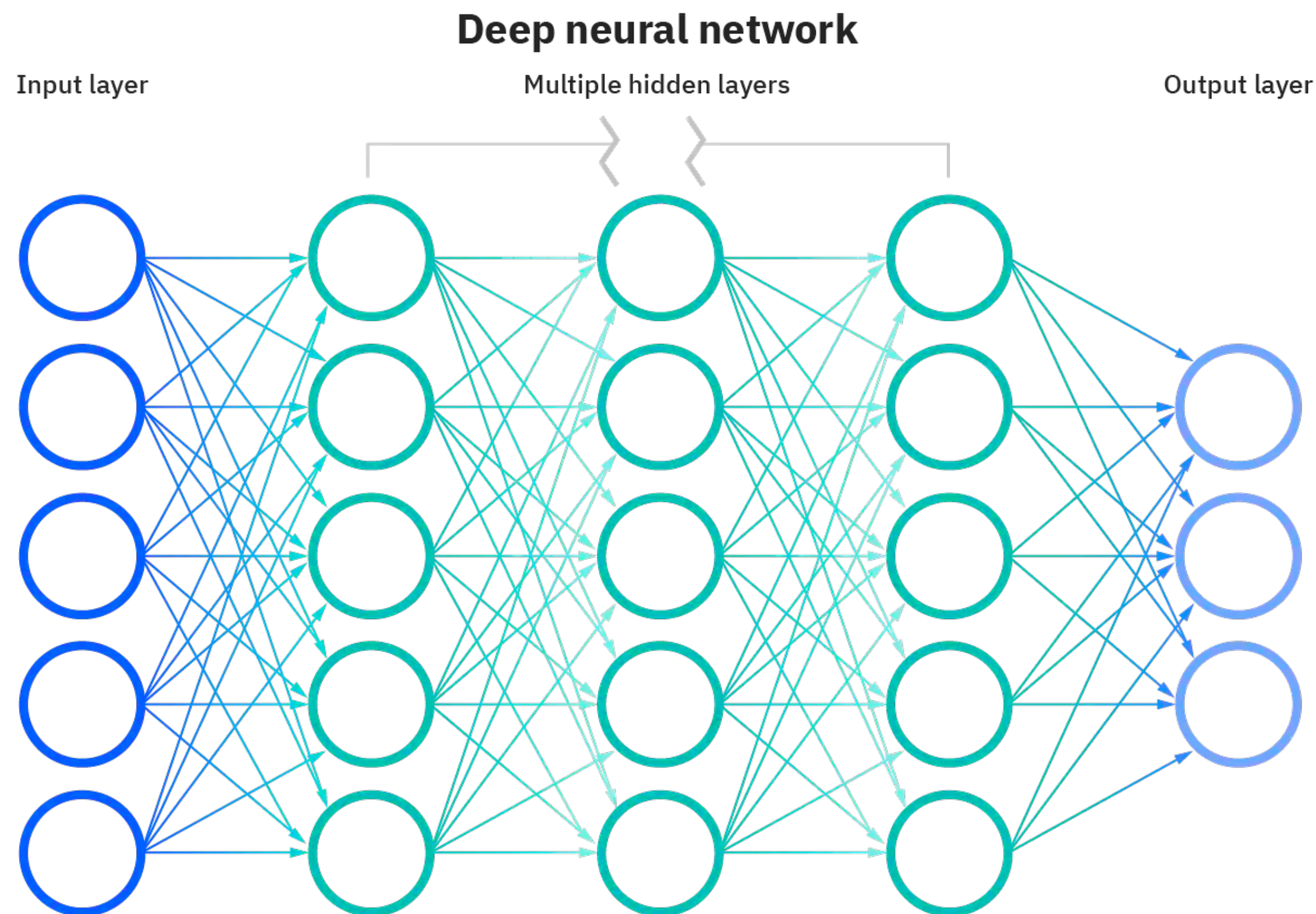
- A recipe to compute gradients
- The first computationally efficient method update parameters of a neural network

$$f(X; \vec{\theta}) = \sigma(\theta_L(\sigma(\theta_{L-1} \dots \theta_1(X))))$$

- Often solely attributed to the chain rule

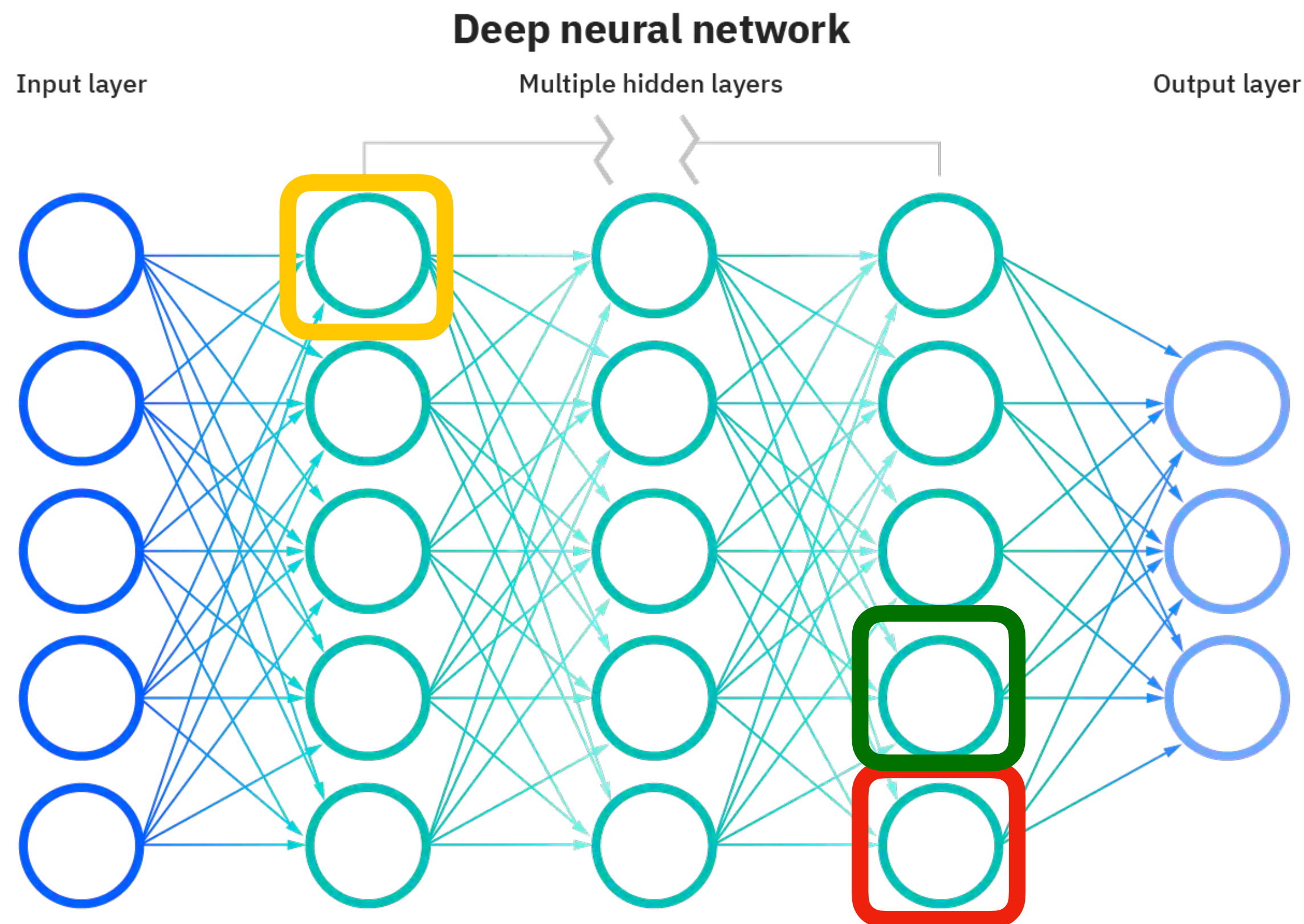
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$$\frac{\partial f(X; \vec{\theta})}{\partial \theta_1}$$

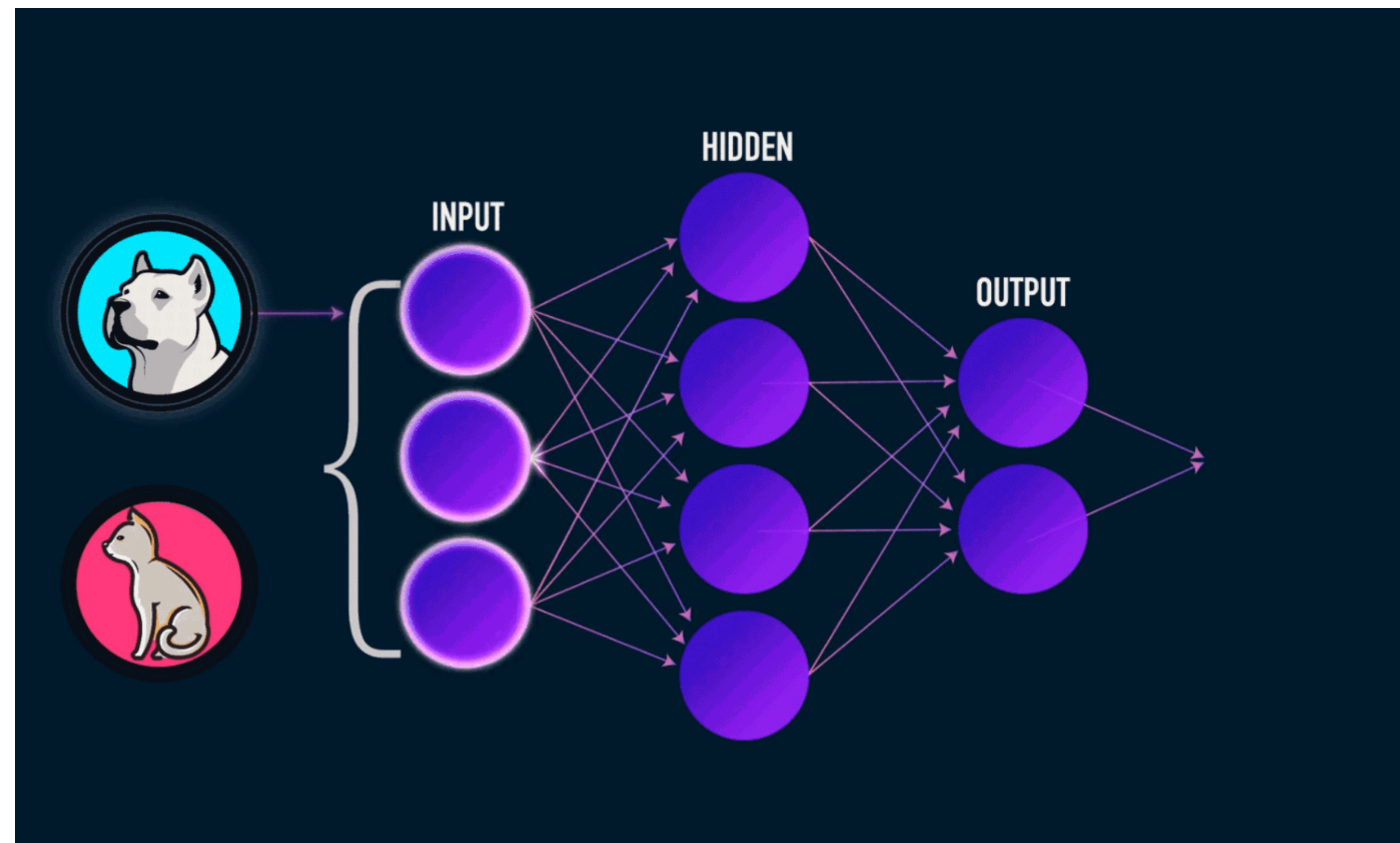
⋮

$$\frac{\partial f(X; \vec{\theta})}{\partial \theta_{L-1}}$$

$$\frac{\partial f(X; \vec{\theta})}{\partial \theta_L}$$

How does backpropagation work?

- As a neural network function is being computed, intermediate information is cleverly stored and reused for gradient computation - dynamic programming



Memory and time

Memory and time

- Neural network with M parameters

$$F(\theta), \theta \in \mathbb{R}^M$$

Memory and time

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- Cost to compute the function in time:
- Cost to compute the function in memory:

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$$\text{TIME}(F(\theta))$$

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$$\omega_1, \omega_2 \in [2, 4]$$

An example: naive gradient computation

- Neural network: $M = 1000$ parameters

$$\text{TIME}(F(\theta)) = 0.01 \text{ seconds}$$

An example: naive gradient computation

- Neural network: $M = 1000$ parameters

$$\text{TIME}(F(\theta)) = 0.01 \text{ seconds}$$

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An example: naive gradient computation

- Neural network: $M = 1$ billion parameters

$$\text{TIME}(F(\theta)) = 60 \text{ seconds}$$

An example: naive gradient computation

- Neural network: $M = 1$ billion parameters

$$\text{TIME}(F(\theta)) = 60 \text{ seconds}$$

$$\text{TIME}(\nabla F(\theta)) \sim 31 \text{ years}$$

An example: backpropagation scaling

- Neural network: $M = 1$ billion parameters

$$\text{TIME}(F(\theta)) = 60 \text{ seconds}$$

$$\text{TIME}(\nabla F(\theta)) \sim 5 \text{ minutes}$$

Relative complexity

$$\begin{aligned} \text{TIME}(\nabla F(\theta)) &\leq \omega_1 \text{ TIME}(F(\theta)) \\ \text{MEMORY}(\nabla F(\theta)) &\leq \omega_2 \text{ MEMORY}(F(\theta)) \end{aligned} \quad \omega_1, \omega_2 \in [2, 4]$$

Quantum backpropagation?

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$$F(\theta) = \langle \psi(\theta) | O | \psi(\theta) \rangle$$

Quantum backpropagation?

$$\text{TIME}(\nabla F(\theta)) \leq \omega_1 \text{TIME}(F(\theta)) \quad \omega_1 = O(\log(M))$$

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Simple variational model

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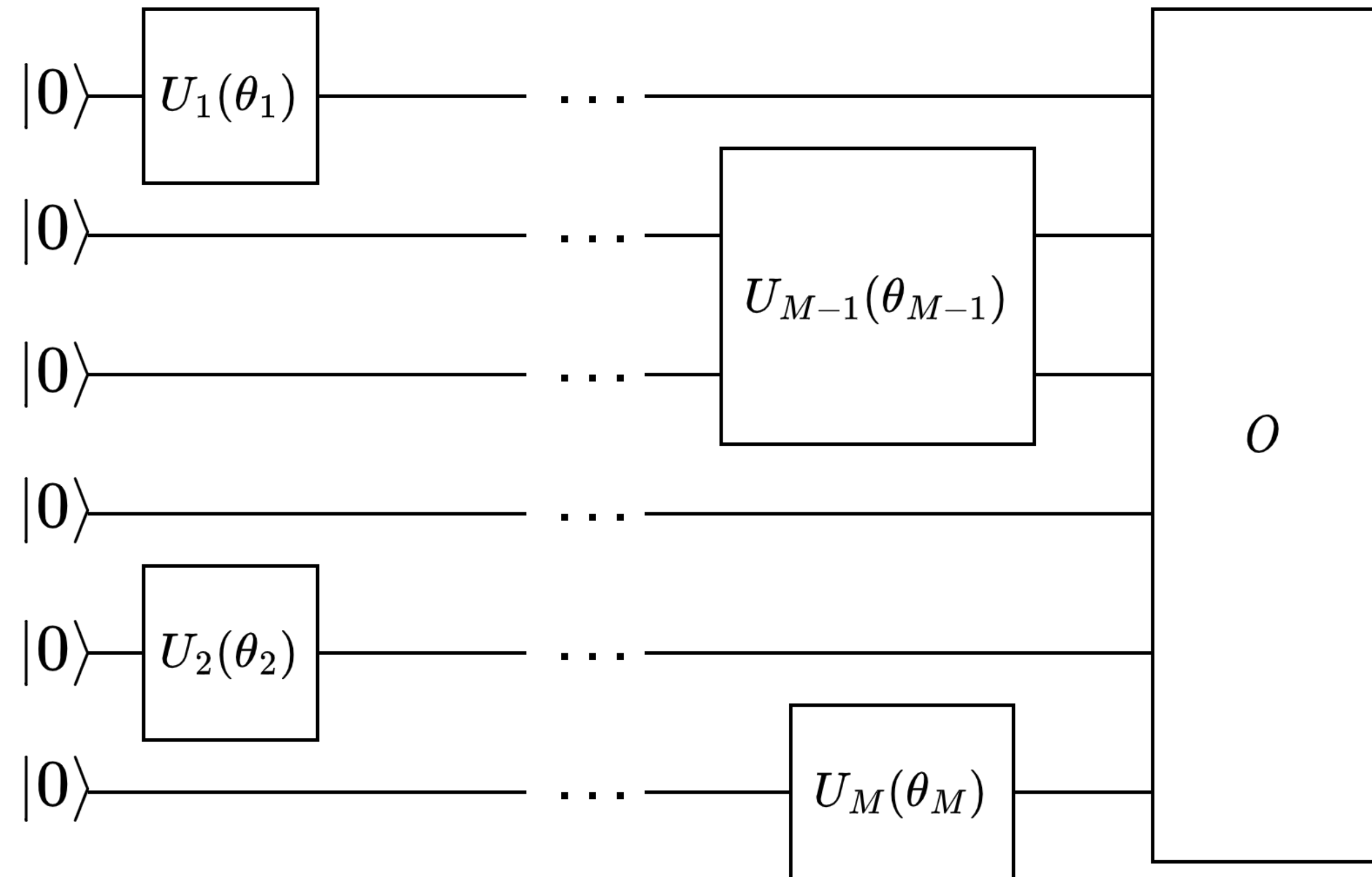
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Naive quantum gradient scaling

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- Unit cost for each parameterised unitary, of which there are M of them

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$$\text{TIME}(\nabla F(\theta)) = M \cdot M/\epsilon^2 = M \cdot \text{TIME}(F(\theta))$$

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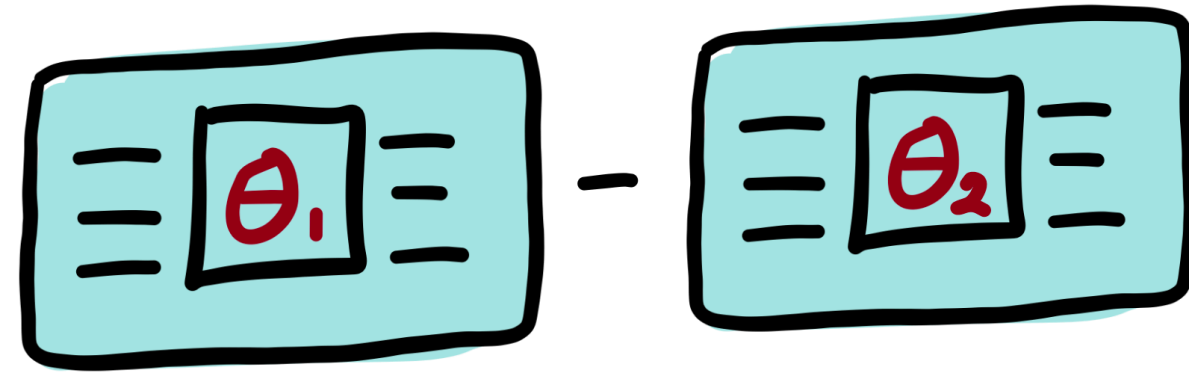
$$\text{TIME}(\nabla F(\theta)) = M \cdot M/\epsilon^2 = M \cdot \text{TIME}(F(\theta))$$

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Quantum backpropagation?

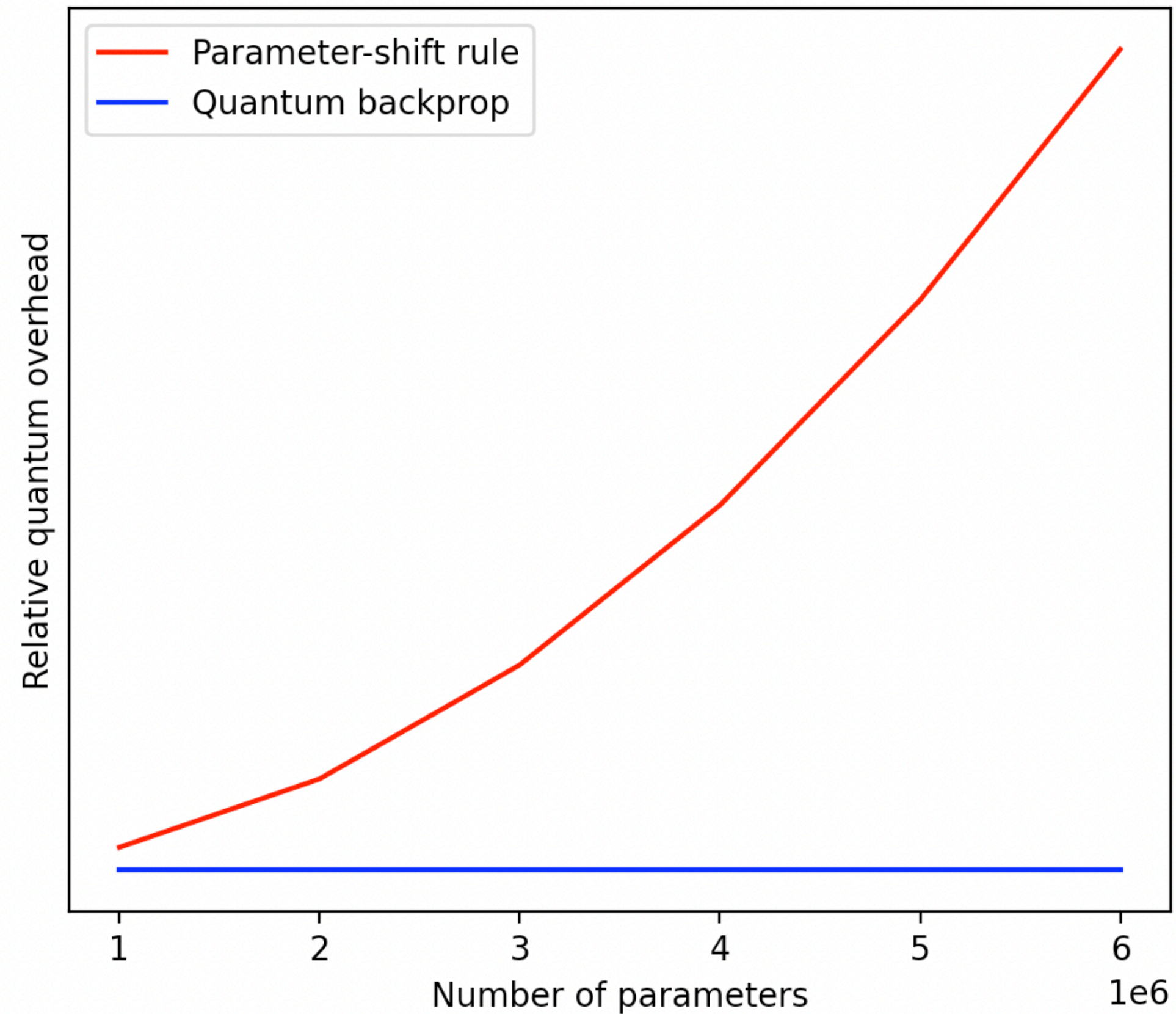
- Parameter shift rule does not yield backprop scaling

$$\nabla_{\theta} f = f(\theta_1) - f(\theta_2)$$



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- SPSA (meant to be dimension independent) will fail

Quantum backpropagation?

- Parameter shift rule does not yield backprop scaling
- SPSA (meant to be dimension independent) will fail
- *All known* methods fail (unless special case models are considered)

Is there something more fundamental
preventing us from achieving
backpropagation scaling?

$$[F'(\theta)]_{\theta_k} = -2 \operatorname{Im} \langle \psi(\theta) | O \frac{\partial}{\partial \theta_k} | \psi(\theta) \rangle$$

No cloning theorem

$$[F'(\theta)]_{\theta_k} = -2 \operatorname{Im} \langle \psi(\theta) | O \frac{\partial}{\partial \theta_k} | \psi(\theta) \rangle$$

Measurement collapse

Connecting the gradient problem to a more general open problem

Simplifying gradients

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Let $\vec{\theta} = 0$ and $O = I$

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$$[F'(\theta)]_{\theta_k} = 2 \operatorname{Re} \langle 0 | P_k | 0 \rangle$$

Simplifying gradients

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Task: Estimate the above expectation value for all $k = 1, \dots, M$ using as little resources as possible

Shadow tomography

Shadow Tomography of Quantum States*

Scott Aaronson[†]

Problem 1 (Shadow Tomography) *Given an unknown D -dimensional quantum mixed state ρ , as well as known 2-outcome measurements E_1, \dots, E_M , each of which accepts ρ with probability $\text{Tr}(E_i \rho)$ and rejects ρ with probability $1 - \text{Tr}(E_i \rho)$, output numbers $b_1, \dots, b_M \in [0, 1]$ such that $|b_i - \text{Tr}(E_i \rho)| \leq \varepsilon$ for all i , with success probability at least $1 - \delta$. Do this via a measurement of $\rho^{\otimes k}$, where $k = k(D, M, \varepsilon, \delta)$ is as small as possible.*

Reusing quantum states

Reusing quantum states

- Partially destructive measurements

$$\|\rho - \rho'\|_{\text{tr}}$$

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$$\|\rho - \rho'\|_{\text{tr}} \leq \alpha$$

Reusing quantum states

- Partially destructive measurements

$$\|\rho - \rho'\|_{\text{tr}} \leq \alpha$$

- Gentle measurements

Can we use shadow tomography
for gradients?

Yes... *and no.*

Shadow tomography

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- Leading techniques require storage of exponentially large offline models

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$$\text{MEMORY}(\nabla F(\theta)) \neq \omega \text{ MEMORY}(F(\theta))$$

Recap

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- Information reuse in quantum models is not easy and is an inhibitor of true backprop scaling

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- Information reuse in quantum models is not easy and is an inhibitor of true backprop scaling
- Current gradient methods for quantum backprop do not achieve the desired scaling of resources

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- There may be some restricted settings, perhaps where models are not as universal or powerful, where backprop scaling is attainable
- Shadow tomography: true computational complexity is still unknown
- Is there a more general computational argument to rule out backprop?
- New models or methods for optimisation? — If QML is to compete with classical ML